Time-Regularized Blind Deconvolution Approach for Radio Interferometry

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Fourier imaging and calibration problem:

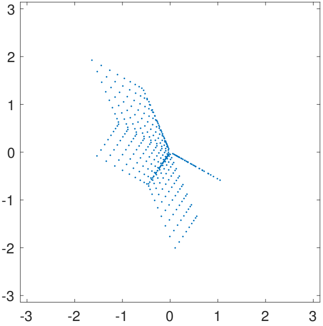
$$\mathbf{y} = \overline{\mathbf{G}}\mathcal{F}(\overline{\mathbf{x}}) + \mathbf{w}$$

OBJECTIVE: Find an estimate of the original image $\overline{\mathbf{x}}$ from the observations $\mathbf{y} \in \mathbb{C}^M$.

- $\overline{\mathbf{x}}$ is the original unknown image
- $\mathcal{F}$ is the 2D Fourier transform operator
- $\overline{\mathbf{G}}$ contains the unknown convolution kernels centred at the measured frequencies
- $\mathbf{w}$ is a realization of an additive i.i.d. Gaussian noise

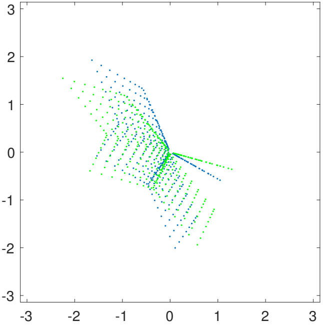
Radio interferometric imaging

VLA radio telescope (27 antennas)
Credit: NRAO



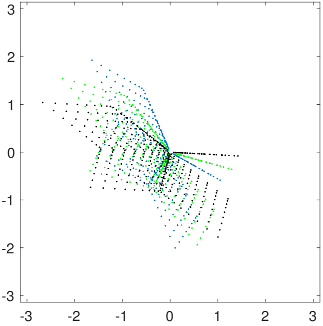
Fourier sampling at instant $t = 1$

VLA radio telescope (27 antennas)
Credit: NRAO



Fourier sampling at instant $t = 20$

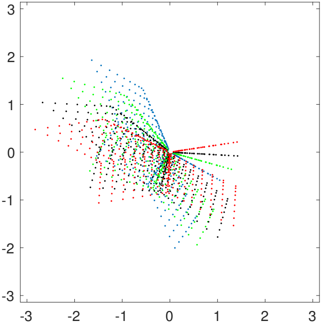
VLA radio telescope (27 antennas)
Credit: NRAO



Fourier sampling at instant $t = 40$

Radio interferometric imaging

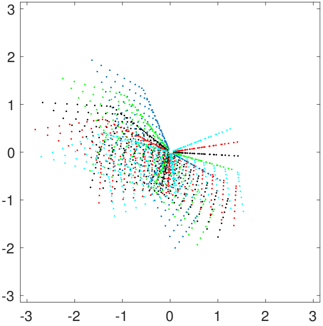
VLA radio telescope (27 antennas)
Credit: NRAO



Fourier sampling at instant $t = 60$

Radio interferometric imaging

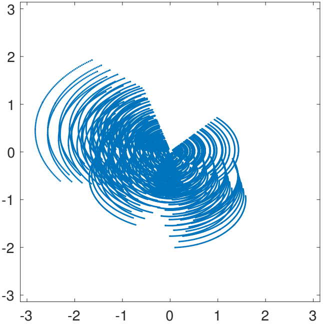
VLA radio telescope (27 antennas)
Credit: NRAO



Fourier sampling at instant $t = 80$

Radio interferometric imaging

VLA radio telescope (27 antennas)
Credit: NRAO



Fourier sampling for $T = 100$ time instant measurements

Observation model

- Interferometer with $\boxed{n_a \text{ antennas}}$ $\rightsquigarrow$ $M = n_a(n_a - 1)/2$ antenna pairs
- $T \geqslant 1$ measurements per antenna pair

$\rightsquigarrow$ TM measurements
indexed by t, α, β, with $t \in \{1, \ldots, T\}$ and $1 \leqslant \alpha < \beta \leqslant n_a$

Measurement acquired by the antenna pair (α, β) at the time instant t, at the spatial frequency $\mathbf{k}_{t,\alpha,\beta} = \mathbf{k}_{t,\alpha} - \mathbf{k}_{t,\beta}$:

$$y_{t,\alpha,\beta} = \sum_{n=-N/2}^{N/2-1} \overline{d}_{t,\alpha}(n)\overline{d}^*_{t,\beta}(n)\overline{x}(n)e^{-2i\pi(k_{t,\alpha}-k_{t,\beta})\frac{n}{N}} + w_{t,\alpha,\beta}$$

Observation model

$$\mathrm{y}_{t,\alpha,\beta} = \sum_{n=-N/2}^{N/2-1} \overline{\mathrm{d}}_{t,\alpha}(n)\overline{\mathrm{d}}^*_{t,\beta}(n)\overline{\mathrm{x}}(n)\mathrm{e}^{-2\mathrm{i}\pi(k_{t,\alpha}-k_{t,\beta})\frac{n}{N}} + \mathrm{w}_{t,\alpha,\beta}$$

- $\overline{\mathbf{x}} = (\overline{\mathrm{x}}(n))_n \in \mathbb{R}^N$
  $\rightsquigarrow$ unknown original image.

- $\overline{\mathbf{d}}_{t,\alpha} = (\overline{\mathrm{d}}_{t,\alpha}(n))_n \in \mathbb{C}^N$
  $\rightsquigarrow$ unknown direction-dependent effect (DDE) related to antenna $\alpha \in \{1, \dots, n_a\}$ and depending on the time instant $t \in \{1, \dots, T\}$

- $\mathbf{w} = \left(\mathrm{w}_{t,\alpha,\beta}\right)_{t,\alpha,\beta} \in \mathbb{C}^{TM}$
  $\rightsquigarrow$ realization of an additive i.i.d. Gaussian noise

Observation model

$$\boxed{\mathrm{y}_{t,\alpha,\beta} = \sum_{n=-N/2}^{N/2-1} \overline{\mathrm{d}}_{t,\alpha}(n)\overline{\mathrm{d}}^*_{t,\beta}(n)\overline{\mathrm{x}}(n)\mathrm{e}^{-2\mathrm{i}\pi(k_{t,\alpha}-k_{t,\beta})\frac{n}{N}} + \mathrm{w}_{t,\alpha,\beta}}$$

Particular case:

If, for every $n \in \{-N/2, \ldots, N/2-1\}$, $\overline{\mathrm{d}}_{t,\alpha}(n) = \delta_{t,\alpha} \in \mathbb{C}$, then $\overline{\mathbf{d}}_{t,\alpha}$ reduces to a direction-independent effect (DIE).

$\rightsquigarrow$ Approximation usually considered when the DIEs and DDEs need to be calibrated.

Observation model

$$y_{t,\alpha,\beta} = \sum_{n=-N/2}^{N/2-1} \overline{\mathrm{d}}_{t,\alpha}(n)\overline{\mathrm{d}}^*_{t,\beta}(n)\overline{\mathrm{x}}(n)\mathrm{e}^{-2\mathrm{i}\pi(k_{t,\alpha}-k_{t,\beta})\frac{n}{N}} + \mathrm{w}_{t,\alpha,\beta}$$

OBJECTIVE: Find an estimate of

- $(\forall t \in \{1,\ldots,T\})(\forall \alpha \in \{1,\ldots,n_a\})$ the DDEs $\overline{\mathbf{d}}_{t,\alpha}$,
- the original image $\overline{\mathbf{x}}$.

State of the art calibration approaches

- DIEs can be approximately known by performing calibration transfer
- StEFCal: Bilinear approach to solve the least squares minimization problem associated with the DIE calibration problem when the source is approximately known [Salvini & Wijnholds (2014)]
- Faceting: Divide the field of view into facets and see each facet as an image with unknown DIEs [Tasse (2014), Smirnov & Tasse (2015), van Weeren et al. (2016)]
  - ⇝ piecewise constant DDEs
  - ⇝ method designed for images with mainly point sources

State of the art imaging approaches

Two main *families* of approaches:

- CLEAN based methods: greedy-based approaches [Högbom (1974), Schwarz (1978), Thompson et al. (2001)]

  $\rightsquigarrow$ ℓ_1 regularization on the image space

- Use of compressive sensing theory and convex optimization methods [Wiaux et al. (2009), Wenger et al. (2010), Li et al. (2011), McEwen & Wiaux (2011), Carrillo et al. (2012), Onose et al. (2017)]

  $\rightsquigarrow$ ℓ_1 regularization on a sparsity basis

Joint calibration and imaging method for RI?

- **Current methods?**

  Alternate between the estimation of the DIEs/DDEs and the estimation of the image, using the preferred methods of the user.

  $\rightsquigarrow$ No theoretical convergence guarantee

  $\rightsquigarrow$ For DDEs, only works for images with point sources

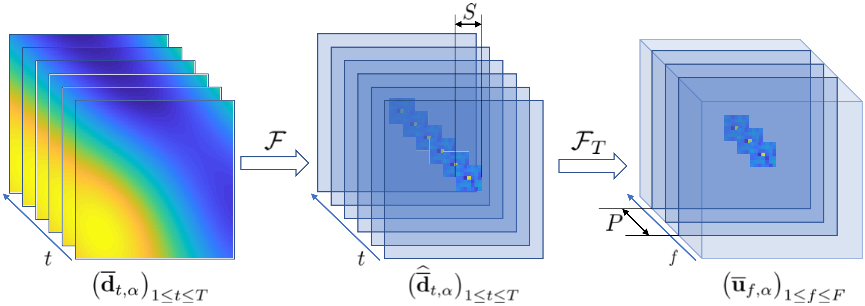
- **Proposed approach?**

  Design a global algorithm to solve the joint calibration and imaging problem, with convergence guarantees, and able to estimate sophisticated images with complex structures

  $\rightsquigarrow$ Use non-convex optimization techniques

Prior information on the DDEs

The DDEs are smooth both in space and in time



Prior information on the DDEs

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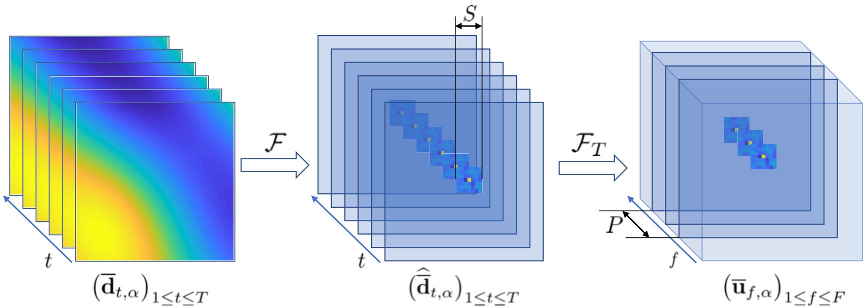


OBJECTIVE:

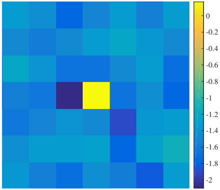
For every $\alpha \in \{1, \ldots, n_a\}$, find an estimate $\mathbf{u}_\alpha^\star \in \mathbb{C}^{S^2 \times P}$ of the non-zero Fourier coefficients $\overline{\mathbf{u}}_\alpha \in \mathbb{C}^{S^2 \times P}$ of the DDEs $(\overline{\mathbf{d}}_{t,\alpha})_{1 \leqslant t \leqslant T}$.

Assume that calibration transfer has been performed:

DIEs are normalized and are approximately known

i.e., for every $\alpha \in \{1, \ldots, n_a\}$ and $t \in \{1, \ldots, T\}$,

- the central coefficient of $\overline{\mathbf{u}}_{t,\alpha}$ belongs to a complex neighbourhood of 1.
- the other coefficients of $\overline{\mathbf{u}}_{t,\alpha}$ belong to a complex neighbourhood of 0.



Prior information of the image

Compute a preprocessing step to obtain a first approximation of the image

Solve the imaging problem considering normalized DIEs (without DDEs)

$$\mathbf{y} = \overline{\mathbf{G}}\mathcal{F}(\mathbf{x}) + \mathbf{w} \simeq \mathcal{S}\big(\mathcal{F}(\mathbf{x})\big) + \mathbf{w}$$

where

- $\mathcal{F}\colon \mathbb{C}^N \to \mathbb{C}^K$ represents the 2D Fourier transform operator
- $\overline{\mathbf{G}} \in \mathbb{C}^{TM \times K}$ contains the unknown antenna-based gains centred at the frequencies measured by the antenna pairs
- $\mathcal{S}\colon \mathbb{C}^K \to \mathbb{C}^{TM}$ linear operator selecting the frequencies measured by the antenna pairs

Prior information of the image

Compute a preprocessing step to obtain a first approximation of the image

Solve the imaging problem considering normalized DIEs (without DDEs)

$$\mathbf{y} = \overline{\mathbf{G}}\mathcal{F}(\mathbf{x}) + \mathbf{w} \simeq \mathcal{S}\big(\mathcal{F}(\mathbf{x})\big) + \mathbf{w}$$

where

- $\mathcal{F}\colon \mathbb{C}^N \to \mathbb{C}^K$ represents the 2D Fourier transform operator
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- $\mathcal{S}\colon \mathbb{C}^K \to \mathbb{C}^{TM}$ linear operator selecting the frequencies measured by the antenna pairs

Remark: $\mathbf{y} \simeq \mathcal{S}\big(\mathcal{F}(\mathbf{x})\big) + \mathbf{w}$ is the approximated model used by state of the art imaging methods when DDEs are unknown

Prior information of the image

Compute a preprocessing step to obtain a first approximation of the image

Solve the imaging problem considering normalized DIEs (without DDEs)

Let $\mathbf{x}_0^\star$ be a solution to $\underset{\mathbf{x}\in\mathbb{R}^N}{\text{minimize}} \ \frac{1}{2}\|\mathcal{S}\big(\mathcal{F}(\mathbf{x})\big) - \mathbf{y}\|_2^2 + \widetilde{r}(\mathbf{x})$

where $\widetilde{r}\colon \mathbb{R}^N \to]-\infty, +\infty]$ is a regularization function, e.g.,

- $\widetilde{r}(\mathbf{x}) = \nu\|\boldsymbol{\Psi}^\dagger\mathbf{x}\|_1 + \iota_{[0,+\infty[^N}(\mathbf{x})$
  where $\nu > 0$ and $\boldsymbol{\Psi}^\dagger \in \mathbb{R}^{Q\times N}$ is a given sparsity basis
- ...

$\rightsquigarrow$ Can be solved with any convex optimization algorithm, e.g. forward-backward [Combettes & Wajs (2005)], primal-dual algorithms [Condat (2013), Vu (2013), Combettes & Pesquet (2012)], ...

Compute a preprocessing step to obtain a first approximation of the image

Solve the imaging problem considering normalized DIEs (without DDEs)

Let $\mathbf{x}_0^\star$ be a solution to $\underset{\mathbf{x}\in\mathbb{R}^N}{\text{minimize}} \;\; \frac{1}{2}\|\mathcal{S}\big(\mathcal{F}(\mathbf{x})\big) - \mathbf{y}\|_2^2 + \widetilde{r}(\mathbf{x})$

$\rightsquigarrow$ $\mathbf{x}_0^\star$ can be used as prior information

Prior information of the image

Compute a preprocessing step to obtain a first approximation of the image

Solve the imaging problem considering normalized DIEs (without DDEs)

Let $\mathbf{x}_0^\star$ be a solution to $\underset{\mathbf{x}\in\mathbb{R}^N}{\text{minimize}} \ \frac{1}{2}\|\mathcal{S}\big(\mathcal{F}(\mathbf{x})\big) - \mathbf{y}\|_2^2 + \widetilde{r}(\mathbf{x})$

- ~~$\mathbf{x}_0^\star$ can be used as prior information~~
- $\mathbf{x}_0^\star$ will contain artefacts (e.g. noisy background, wrong amplitude, wrong source detection, etc.)
- Use a thresholded version of $\mathbf{x}_0^\star$, denoted by $\mathbf{x}_0$, where low amplitude coefficients have been removed:

The original image can be decomposed as $\overline{\mathbf{x}} = \mathbf{x}_0 + \overline{\boldsymbol{\epsilon}}$
where $\overline{\boldsymbol{\epsilon}} \in \mathbb{R}^N$ is the error image to be estimated

Non-linear inverse problem

OBJECTIVE: Find an estimate of $\overline{\epsilon}$ and $\overline{\mathbf{U}}$, where $\overline{\mathbf{U}} = \left(\overline{\mathbf{u}_\alpha}\right)_{1 \leqslant \alpha \leqslant n_a}$, from

$$\boxed{\mathbf{y} = \Phi(\overline{\mathbf{U}}, \overline{\epsilon}) + \mathbf{w}}$$

with Φ the measurement operator associated with the RI inverse problem.

Non-linear inverse problem

OBJECTIVE: Find an estimate of $\overline{\boldsymbol{\epsilon}}$ and $\overline{\mathbf{U}}$, where $\overline{\mathbf{U}} = \left(\overline{\mathbf{u}_\alpha}\right)_{1 \leqslant \alpha \leqslant n_a}$, from

$$\mathbf{y} = \boldsymbol{\Phi}(\overline{\mathbf{U}}, \overline{\boldsymbol{\epsilon}}) + \mathbf{w}$$

with $\boldsymbol{\Phi}$ the measurement operator associated with the RI inverse problem.

- ⋆ Linear problem w.r.t. $\overline{\boldsymbol{\epsilon}}$

Non-linear inverse problem

OBJECTIVE: Find an estimate of $\overline{\epsilon}$ and $\overline{\mathbf{U}}$, where $\overline{\mathbf{U}} = \left(\overline{\mathbf{u}_\alpha}\right)_{1 \leqslant \alpha \leqslant n_a}$, from

$$\mathbf{y} = \mathbf{\Phi}(\overline{\mathbf{U}}, \overline{\epsilon}) + \mathbf{w}$$

with $\mathbf{\Phi}$ the measurement operator associated with the RI inverse problem.

- ⋆ Linear problem w.r.t. $\overline{\epsilon}$
- ⋆ Non-linear problem w.r.t. $\overline{\mathbf{U}}$

Non-linear inverse problem

OBJECTIVE: Find an estimate of $\overline{\boldsymbol{\epsilon}}$ and $\overline{\mathbf{U}}$, where $\overline{\mathbf{U}} = \left(\overline{\mathbf{u}_\alpha}\right)_{1 \leqslant \alpha \leqslant n_a}$, from

$$\mathbf{y} = \boldsymbol{\Phi}(\overline{\mathbf{U}}, \overline{\boldsymbol{\epsilon}}) + \mathbf{w}$$

with $\boldsymbol{\Phi}$ the measurement operator associated with the RI inverse problem.

- Linear problem w.r.t. $\overline{\boldsymbol{\epsilon}}$
- Non-linear problem w.r.t. $\overline{\mathbf{U}}$
  - ⇝ Use a bi-linear approach by introducing $\overline{\mathbf{U}}_1 = \overline{\mathbf{U}}_2 = \overline{\mathbf{U}}$

Problem reformulation: $\mathbf{y} = \boldsymbol{\Phi}(\overline{\mathbf{U}}_1, \overline{\mathbf{U}}_2, \overline{\boldsymbol{\epsilon}}) + \mathbf{w}$

Minimization problem

$$\boxed{\underset{\boldsymbol{\epsilon}\in\mathbb{R}^N,\,(\mathbf{U}_1,\mathbf{U}_2)\in(\mathbb{C}^{n_a\times S})^{2T}}{\operatorname{minimize}} \quad h\big(\boldsymbol{\epsilon},\mathbf{U}_1,\mathbf{U}_2\big) + r(\boldsymbol{\epsilon}) + p\big(\mathbf{U}_1,\mathbf{U}_2\big)}$$

- h is the least squares data fidelity term associated with the data model

$$h\big(\boldsymbol{\epsilon},\mathbf{U}_1,\mathbf{U}_2\big) = \frac{1}{2}\|\boldsymbol{\Phi}(\mathbf{U}_1,\mathbf{U}_2,\boldsymbol{\epsilon}) - \mathbf{y}\|^2$$

- r is the regularization term for the image

- p is the regularization term for the DDEs

Minimization problem

$$\underset{\epsilon \in \mathbb{R}^N,\, (\mathbf{U}_1, \mathbf{U}_2) \in (\mathbb{C}^{n_a \times S})^{2T}}{\text{minimize}} \quad h\big(\epsilon, \mathbf{U}_1, \mathbf{U}_2\big) + r(\epsilon) + p\big(\mathbf{U}_1, \mathbf{U}_2\big)$$

- $r(\epsilon) = \lambda \|\boldsymbol{\Psi}^\dagger(\mathbf{x}_0 + \epsilon)\|_1 + \iota_{\mathbb{E}}(\epsilon)$

  $\boldsymbol{\Psi} \in \mathbb{R}^{Q \times N}$ is a given sparsity basis

  $\lambda > 0$ is a regularization parameter

  $\mathbb{E}$ is a closed, convex and non-empty subset of $\mathbb{R}^N$ defined as
  $\mathbb{E} = \Big\{\epsilon \in \mathbb{R}^N \,\Big|\, (\forall n \in \mathbb{S}_0) \; -\vartheta x_0(n) \leqslant \epsilon(n) \leqslant \vartheta x_0(n),$
  and $(\forall n \in \mathbb{S}_0^c)\; 0 \leqslant \epsilon(n)\Big\}$
  with $\mathbb{S}_0$ the support of $\mathbf{x}_0$, $\mathbb{S}_0^c$ its complementary set, and $\vartheta \in [0, 1]$ representing the percentage error we assume on $\mathbf{x}_0$

Minimization problem

$$\underset{\epsilon \in \mathbb{R}^N,\, (\mathbf{U}_1, \mathbf{U}_2) \in (\mathbb{C}^{n_a \times S})^{2T}}{\text{minimize}} \quad h(\epsilon, \mathbf{U}_1, \mathbf{U}_2) + r(\epsilon) + p(\mathbf{U}_1, \mathbf{U}_2)$$

- $p(\mathbf{U}_1, \mathbf{U}_2) = \eta \|\mathbf{U}_1 - \mathbf{U}_2\|_2^2 + \iota_{\mathbb{D}}(\mathbf{U}_1) + \iota_{\mathbb{D}}(\mathbf{U}_2)$

  $\eta > 0$ is a regularization parameter

  $$\mathbb{D} = \big\{ \mathbf{U} \in \mathbb{C}^{T n_a \times S} \,|\, (\forall t \in \{1, \ldots, T\})(\forall \alpha \in \{1, \ldots, n_a\}) \\ \mathrm{u}_{t,\alpha}(0) \in B_\infty(1; \upsilon) \quad \text{and} \quad (\forall s \neq 0) \quad \mathrm{u}_{t,\alpha}(s) \in B_\infty(0; \upsilon) \big\}$$

Minimization problem

$$\boxed{\underset{\epsilon \in \mathbb{R}^N,\, (\mathbf{U}_1, \mathbf{U}_2) \in (\mathbb{C}^{n_a \times S})^{2T}}{\text{minimize}} \quad h\big(\epsilon, \mathbf{U}_1, \mathbf{U}_2\big) + r(\epsilon) + p\big(\mathbf{U}_1, \mathbf{U}_2\big)}$$

- Use a block coordinate forward-backward algorithm, alternating between the estimation of $\overline{\epsilon}$, $\overline{\mathbf{U}}_1$, and $\overline{\mathbf{U}}_2$
  [Bolte *et al.* (2014), Frankel *et al.* (2015), Chouzenoux *et al.* (2016)]

Block coordinate forward-backward algorithm

For $i = 0, 1, \ldots$

Choose to update either the DDEs $\big(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\big)$, or the image $\boldsymbol{\epsilon}^{(i)}$.

If the DDEs are updated:

$\mathbf{U}_1^{(i,0)} = \mathbf{U}_1^{(i)}$, $\mathbf{U}_2^{(i,0)} = \mathbf{U}_2^{(i)}$.

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_1^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\Big(\mathbf{U}_1^{(i,\ell)} - \Gamma_1^{(i)} \cdot \Big(\nabla_{\mathbf{U}_1} h\big(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i,\ell)}, \mathbf{U}_2^{(i)}\big) - \eta\big(\mathbf{U}_1^{(i,\ell)} - \mathbf{U}_2^{(i)}\big)\Big)\Big)$$

$\mathbf{U}_1^{(i+1)} = \mathbf{U}_1^{(i,L^{(i)})}$.

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_2^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\Big(\mathbf{U}_2^{(i,\ell)} - \Gamma_2^{(i)} \cdot \Big(\nabla_{\mathbf{U}_2} h\big(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i,\ell)}\big) - \eta\big(\mathbf{U}_2^{(i,\ell)} - \mathbf{U}_1^{(i+1)}\big)\Big)\Big)$$

$\mathbf{U}_2^{(i+1)} = \mathbf{U}_2^{(i,L^{(i)})}$.

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i)}$.

If the image is updated:

$\boldsymbol{\epsilon}^{(i,0)} = \boldsymbol{\epsilon}^{(i)}$.

For $j = 0, \ldots, J^{(i)} - 1$

$$\boldsymbol{\epsilon}^{(i,j+1)} = \operatorname{prox}_{\tau^{(i)} r}\Big(\boldsymbol{\epsilon}^{(i,j)} - \tau^{(i)} \nabla_{\boldsymbol{\epsilon}} h\big(\boldsymbol{\epsilon}^{(i,j)}, \mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\big)\Big)$$

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i,J^{(i)})}$.

$\big(\mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i+1)}\big) = \big(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\big)$.

Block coordinate forward-backward algorithm

For $i = 0, 1, \ldots$

Choose to update either the DDEs $(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)})$, or the image $\boldsymbol{\epsilon}^{(i)}$. (essentially cyclic updating rule)

If the DDEs are updated:

$\mathbf{U}_1^{(i,0)} = \mathbf{U}_1^{(i)}, \ \mathbf{U}_2^{(i,0)} = \mathbf{U}_2^{(i)}$.

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_1^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\left(\mathbf{U}_1^{(i,\ell)} - \Gamma_1^{(i)} \cdot \left(\nabla_{\mathbf{U}_1} h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i,\ell)}, \mathbf{U}_2^{(i)}) - \eta(\mathbf{U}_1^{(i,\ell)} - \mathbf{U}_2^{(i)})\right)\right)$$

$\mathbf{U}_1^{(i+1)} = \mathbf{U}_1^{(i,L^{(i)})}$.

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_2^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\left(\mathbf{U}_2^{(i,\ell)} - \Gamma_2^{(i)} \cdot \left(\nabla_{\mathbf{U}_2} h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i,\ell)}) - \eta(\mathbf{U}_2^{(i,\ell)} - \mathbf{U}_1^{(i+1)})\right)\right)$$

$\mathbf{U}_2^{(i+1)} = \mathbf{U}_2^{(i,L^{(i)})}$.

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i)}$.

If the image is updated:

$\boldsymbol{\epsilon}^{(i,0)} = \boldsymbol{\epsilon}^{(i)}$.

For $j = 0, \ldots, J^{(i)} - 1$

$$\boldsymbol{\epsilon}^{(i,j+1)} = \operatorname{prox}_{\tau^{(i)} r}\left(\boldsymbol{\epsilon}^{(i,j)} - \tau^{(i)} \nabla_{\boldsymbol{\epsilon}} h(\boldsymbol{\epsilon}^{(i,j)}, \mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)})\right)$$

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i,J^{(i)})}$.

$(\mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i+1)}) = (\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)})$.

Block coordinate forward-backward algorithm

For $i = 0, 1, \ldots$

Choose to update either the DDEs $\left(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\right)$, or the image $\boldsymbol{\epsilon}^{(i)}$.

If the DDEs are updated:

$\mathbf{U}_1^{(i,0)} = \mathbf{U}_1^{(i)}, \ \mathbf{U}_2^{(i,0)} = \mathbf{U}_2^{(i)}.$

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_1^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}} \left(\mathbf{U}_1^{(i,\ell)} - \Gamma_1^{(i)} \cdot \left(\nabla_{\mathbf{U}_1} h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i,\ell)}, \mathbf{U}_2^{(i)}) - \eta (\mathbf{U}_1^{(i,\ell)} - \mathbf{U}_2^{(i)}) \right) \right)$$

$\mathbf{U}_1^{(i+1)} = \mathbf{U}_1^{(i,L^{(i)})}.$

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_2^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}} \left(\mathbf{U}_2^{(i,\ell)} - \Gamma_2^{(i)} \cdot \left(\nabla_{\mathbf{U}_2} h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i,\ell)}) - \eta (\mathbf{U}_2^{(i,\ell)} - \mathbf{U}_1^{(i+1)}) \right) \right)$$

$\mathbf{U}_2^{(i+1)} = \mathbf{U}_2^{(i,L^{(i)})}.$

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i)}.$

If the image is updated:

$\boldsymbol{\epsilon}^{(i,0)} = \boldsymbol{\epsilon}^{(i)}.$

For $j = 0, \ldots, J^{(i)} - 1$

$$\boldsymbol{\epsilon}^{(i,j+1)} = \operatorname{prox}_{\tau^{(i)} r} \left(\boldsymbol{\epsilon}^{(i,j)} - \tau^{(i)} \nabla_{\boldsymbol{\epsilon}} h(\boldsymbol{\epsilon}^{(i,j)}, \mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}) \right)$$

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i,J^{(i)})}.$

$(\mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i+1)}) = (\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}).$

Block coordinate forward-backward algorithm

For $i = 0, 1, \ldots$

Choose to update either the DDEs $\left(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\right)$, or the image $\boldsymbol{\epsilon}^{(i)}$.

If the DDEs are updated:

$\mathbf{U}_1^{(i,0)} = \mathbf{U}_1^{(i)}, \; \mathbf{U}_2^{(i,0)} = \mathbf{U}_2^{(i)}.$

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_1^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\left(\mathbf{U}_1^{(i,\ell)} - \Gamma_1^{(i)} \cdot \left(\nabla_{\mathbf{U}_1} h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i,\ell)}, \mathbf{U}_2^{(i)}) - \eta(\mathbf{U}_1^{(i,\ell)} - \mathbf{U}_2^{(i)})\right)\right)$$

$\mathbf{U}_1^{(i+1)} = \mathbf{U}_1^{(i,L^{(i)})}.$

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_2^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\left(\mathbf{U}_2^{(i,\ell)} - \Gamma_2^{(i)} \cdot \left(\nabla_{\mathbf{U}_2} h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i,\ell)}) - \eta(\mathbf{U}_2^{(i,\ell)} - \mathbf{U}_1^{(i+1)})\right)\right)$$

$\mathbf{U}_2^{(i+1)} = \mathbf{U}_2^{(i,L^{(i)})}.$

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i)}.$

If the image is updated:

$\boldsymbol{\epsilon}^{(i,0)} = \boldsymbol{\epsilon}^{(i)}.$

For $j = 0, \ldots, J^{(i)} - 1$

$$\boldsymbol{\epsilon}^{(i,j+1)} = \operatorname{prox}_{\tau^{(i)} r}\left(\boldsymbol{\epsilon}^{(i,j)} - \tau^{(i)} \nabla_{\boldsymbol{\epsilon}} h(\boldsymbol{\epsilon}^{(i,j)}, \mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)})\right)$$

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i,J^{(i)})}.$

$\left(\mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i+1)}\right) = \left(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\right).$

Block coordinate forward-backward algorithm

For $i = 0, 1, \ldots$

Choose to update either the DDEs $\big(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\big)$, or the image $\boldsymbol{\epsilon}^{(i)}$.

If the DDEs are updated:

$\mathbf{U}_1^{(i,0)} = \mathbf{U}_1^{(i)}$, $\mathbf{U}_2^{(i,0)} = \mathbf{U}_2^{(i)}$.

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_1^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\Big(\mathbf{U}_1^{(i,\ell)} - \Gamma_1^{(i)} \cdot \Big(\nabla_{\mathbf{U}_1} h\big(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i,\ell)}, \mathbf{U}_2^{(i)}\big) - \eta\big(\mathbf{U}_1^{(i,\ell)} - \mathbf{U}_2^{(i)}\big)\Big)\Big)$$

$\mathbf{U}_1^{(i+1)} = \mathbf{U}_1^{(i,L^{(i)})}$.

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_2^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\Big(\mathbf{U}_2^{(i,\ell)} - \Gamma_2^{(i)} \cdot \Big(\nabla_{\mathbf{U}_2} h\big(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i,\ell)}\big) - \eta\big(\mathbf{U}_2^{(i,\ell)} - \mathbf{U}_1^{(i+1)}\big)\Big)\Big)$$

$\mathbf{U}_2^{(i+1)} = \mathbf{U}_2^{(i,L^{(i)})}$.

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i)}$.

If the image is updated:

$\boldsymbol{\epsilon}^{(i,0)} = \boldsymbol{\epsilon}^{(i)}$.

For $j = 0, \ldots, J^{(i)} - 1$

$$\boldsymbol{\epsilon}^{(i,j+1)} = \mathrm{prox}_{\tau^{(i)} r}\Big(\boldsymbol{\epsilon}^{(i,j)} - \tau^{(i)} \nabla_{\boldsymbol{\epsilon}} h\big(\boldsymbol{\epsilon}^{(i,j)}, \mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\big)\Big)$$

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i,J^{(i)})}$.

$\big(\mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i+1)}\big) = \big(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\big)$.

$$(\forall \widetilde{\mathbf{U}} \in \mathbb{C}^{n_a \times S}) \quad \mathcal{P}_{\mathbb{D}}(\widetilde{\mathbf{U}}) = \underset{\mathbf{U} \in \mathbb{D}}{\operatorname{argmin}} \; \|\mathbf{U} - \widetilde{\mathbf{U}}\|_2^2$$

Block coordinate forward-backward algorithm

For $i = 0, 1, \dots$

Choose to update either the DDEs $\big(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\big)$, or the image $\boldsymbol{\epsilon}^{(i)}$.

If the DDEs are updated:

$\mathbf{U}_1^{(i,0)} = \mathbf{U}_1^{(i)}, \ \mathbf{U}_2^{(i,0)} = \mathbf{U}_2^{(i)}.$

For $\ell = 0, \dots, L^{(i)} - 1$

$$\mathbf{U}_1^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\left(\mathbf{U}_1^{(i,\ell)} - \Gamma_1^{(i)} \cdot \left(\nabla_{\mathbf{U}_1} h\big(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i,\ell)}, \mathbf{U}_2^{(i)}\big) - \eta\big(\mathbf{U}_1^{(i,\ell)} - \mathbf{U}_2^{(i)}\big)\right)\right)$$

$\mathbf{U}_1^{(i+1)} = \mathbf{U}_1^{(i,L^{(i)})}.$

For $\ell = 0, \dots, L^{(i)} - 1$

$$\mathbf{U}_2^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\left(\mathbf{U}_2^{(i,\ell)} - \Gamma_2^{(i)} \cdot \left(\nabla_{\mathbf{U}_2} h\big(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i,\ell)}\big) - \eta\big(\mathbf{U}_2^{(i,\ell)} - \mathbf{U}_1^{(i+1)}\big)\right)\right)$$

$\mathbf{U}_2^{(i+1)} = \mathbf{U}_2^{(i,L^{(i)})}.$

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i)}.$

If the image is updated:

$\boldsymbol{\epsilon}^{(i,0)} = \boldsymbol{\epsilon}^{(i)}.$

For $j = 0, \dots, J^{(i)} - 1$

$$\boldsymbol{\epsilon}^{(i,j+1)} = \mathrm{prox}_{\tau^{(i)} r}\left(\boldsymbol{\epsilon}^{(i,j)} - \tau^{(i)} \nabla_{\boldsymbol{\epsilon}} h\big(\boldsymbol{\epsilon}^{(i,j)}, \mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\big)\right)$$

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i,J^{(i)})}.$

$\big(\mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i+1)}\big) = \big(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\big).$

Block coordinate forward-backward algorithm

For $i = 0, 1, \ldots$

Choose to update either the DDEs $\big(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\big)$, or the image $\boldsymbol{\epsilon}^{(i)}$.

If the DDEs are updated:

$\mathbf{U}_1^{(i,0)} = \mathbf{U}_1^{(i)}$, $\mathbf{U}_2^{(i,0)} = \mathbf{U}_2^{(i)}$.

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_1^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\left(\mathbf{U}_1^{(i,\ell)} - \Gamma_1^{(i)} \cdot \left(\nabla_{\mathbf{U}_1} h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i,\ell)}, \mathbf{U}_2^{(i)}) - \eta(\mathbf{U}_1^{(i,\ell)} - \mathbf{U}_2^{(i)})\right)\right)$$

$\mathbf{U}_1^{(i+1)} = \mathbf{U}_1^{(i,L^{(i)})}$.

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_2^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\left(\mathbf{U}_2^{(i,\ell)} - \Gamma_2^{(i)} \cdot \left(\nabla_{\mathbf{U}_2} h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i,\ell)}) - \eta(\mathbf{U}_2^{(i,\ell)} - \mathbf{U}_1^{(i+1)})\right)\right)$$

$\mathbf{U}_2^{(i+1)} = \mathbf{U}_2^{(i,L^{(i)})}$.

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i)}$.

If the image is updated:

$\boldsymbol{\epsilon}^{(i,0)} = \boldsymbol{\epsilon}^{(i)}$.

For $j = 0, \ldots, J^{(i)} - 1$

$$\boldsymbol{\epsilon}^{(i,j+1)} = \mathrm{prox}_{\tau^{(i)} r}\left(\boldsymbol{\epsilon}^{(i,j)} - \tau^{(i)} \nabla_{\boldsymbol{\epsilon}} h(\boldsymbol{\epsilon}^{(i,j)}, \mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)})\right)$$

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i,J^{(i)})}$.

$\big(\mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i+1)}\big) = \big(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\big)$.

$$(\forall \widetilde{\boldsymbol{\epsilon}} \in \mathbb{R}^N) \quad \mathrm{prox}_r(\widetilde{\boldsymbol{\epsilon}}) = \underset{\boldsymbol{\epsilon} \in \mathbb{R}^N}{\operatorname{argmin}} \quad r(\boldsymbol{\epsilon}) + \frac{1}{2}\|\boldsymbol{\epsilon} - \widetilde{\boldsymbol{\epsilon}}\|_2^2$$

Block coordinate forward-backward algorithm

For $i = 0, 1, \ldots$

Choose to update either the DDEs $(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)})$, or the image $\boldsymbol{\epsilon}^{(i)}$.

If the DDEs are updated:

$\mathbf{U}_1^{(i,0)} = \mathbf{U}_1^{(i)}, \ \mathbf{U}_2^{(i,0)} = \mathbf{U}_2^{(i)}$.

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_1^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\Big(\mathbf{U}_1^{(i,\ell)} - \boxed{\Gamma_1^{(i)}} \cdot \big(\nabla_{\mathbf{U}_1} h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i,\ell)}, \mathbf{U}_2^{(i)}) - \eta(\mathbf{U}_1^{(i,\ell)} - \mathbf{U}_2^{(i)})\big)\Big)$$

$\mathbf{U}_1^{(i+1)} = \mathbf{U}_1^{(i,L^{(i)})}$.

For $\ell = 0, \ldots, L^{(i)} - 1$

$$\mathbf{U}_2^{(i,\ell+1)} = \mathcal{P}_{\mathbb{D}}\Big(\mathbf{U}_2^{(i,\ell)} - \boxed{\Gamma_2^{(i)}} \cdot \big(\nabla_{\mathbf{U}_2} h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i,\ell)}) - \eta(\mathbf{U}_2^{(i,\ell)} - \mathbf{U}_1^{(i+1)})\big)\Big)$$

$\mathbf{U}_2^{(i+1)} = \mathbf{U}_2^{(i,L^{(i)})}$.

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i)}$.

If the image is updated:

$\boldsymbol{\epsilon}^{(i,0)} = \boldsymbol{\epsilon}^{(i)}$.

For $j = 0, \ldots, J^{(i)} - 1$

$$\boldsymbol{\epsilon}^{(i,j+1)} = \mathrm{prox}_{\tau^{(i)} r}\Big(\boldsymbol{\epsilon}^{(i,j)} - \boxed{\tau^{(i)}} \nabla_{\boldsymbol{\epsilon}} h(\boldsymbol{\epsilon}^{(i,j)}, \mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)})\Big)$$

$\boldsymbol{\epsilon}^{(i+1)} = \boldsymbol{\epsilon}^{(i,J^{(i)})}$.

$(\mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i+1)}) = (\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)})$.

- $(\forall i \in \mathbb{N})$ $\Gamma_1^{(i)}$ and $\Gamma_2^{(i)}$ $\rightsquigarrow$ preconditioning matrices
- $(\forall i \in \mathbb{N})$ $\tau^{(i)}$ $\rightsquigarrow$ step-size

Synthetic data

Simulation settings:

$n_a = 27$ antennas of the VLA telescope

each antenna pair acquires $T = 200$ snapshots

time interval of 12 hours

images of dimension $N = 256 \times 256$

DDEs: spatial Fourier support of size $S = 5 \times 5$

temporal Fourier support of size $P = 3$

Comparison between the following methods:

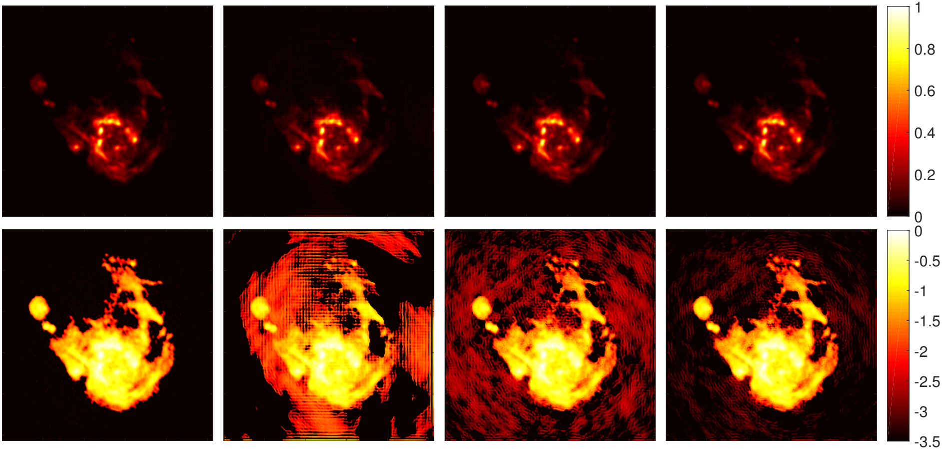
Imaging with the ground truth DDEs

Imaging with normalized DIEs

Joint imaging and DDE calibration without time regularization

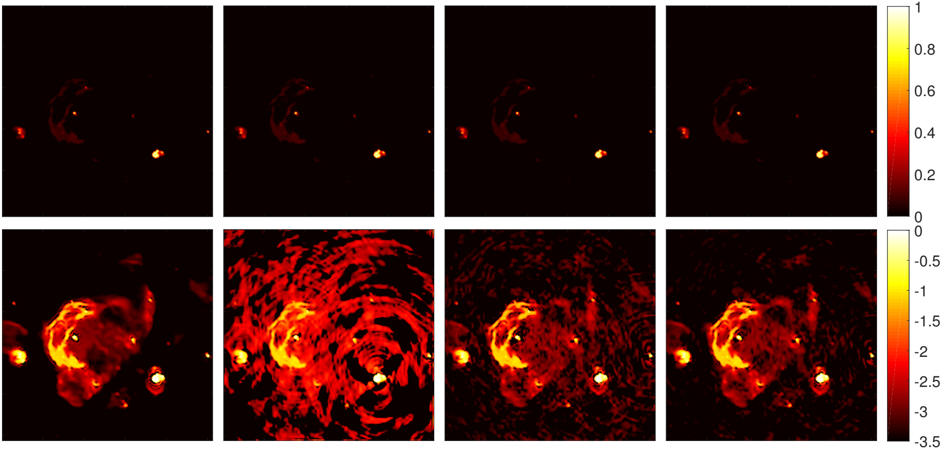
Proposed approach

Image of M31



SNR = 38.26 dB | SNR = 17.32 dB | SNR = 26.77 dB | SNR = 28.51 dB

Image of W28



SNR = 20.12 dB   SNR = 17.20 dB   SNR = 20.73 dB   SNR = 20.91 dB

Conclusions

- Joint DDE calibration and imaging model
  - DDEs are modelled as smooth images in space and time
    $\rightsquigarrow$ Estimation of the non-zero Fourier coefficients
  - Images with sophisticated structures can be considered
    $\rightsquigarrow$ Adapted choice of the regularization
- Flexible block coordinate forward backward approach
  - Essentially cyclic updating rule
  - Variable metric strategy
  - Global convergence to a critical point
  - Adapted initialization using calibration transfer
- Good performances on realistic synthetic data
- Results on real datasets from VLA telescope (ongoing work)

Thank you!

References:

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- S. Salvini and S. J. Wijnholds. **Fast gain calibration in radio astronomy using alternating direction implicit methods: Analysis and applications**. A&A, 2014.
- O.M. Smirnov. **Revisiting the radio interferometer measurement equation. II. Calibration and direction-dependent effects**. A & A, 2015.
- E. Chouzenoux, J.-C. Pesquet and A. Repetti. **A Block Coordinate Variable Metric Forward-Backward Algorithm**. J. Global Optim., 2016.
- A. Repetti, J. Birdi, A. Dabbech and Y. Wiaux. **Non-convex optimization for self-calibration of direction-dependent effects in radio interferometric imaging**. MNRAS, 2017.

Backup slide: convergence guarantees

$$\boxed{0 < \tau^{(i)} < 1/\|\boldsymbol{\Phi}(\mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}, .)\|^2}$$

$\Gamma_1^{(i)} = (\gamma_{1,\alpha}^{(i)} \mathbf{1}_{S^2 \times P})_{1 \leqslant \alpha \leqslant n_a} \in \mathbb{R}^{S^2 \times P \times n_a}$ with $\boxed{0 < \gamma_{1,\alpha}^{(i)} < 1/(\eta + \zeta_{1,\alpha}^{(i)})}$

$\zeta_{1,\alpha}^{(i)} \rightsquigarrow$ Lipschitz constant of the partial derivative of $h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)})$ w.r.t. $\mathbf{u}_{1,\alpha}^{(i)}$

$\Gamma_2^{(i)} = (\gamma_{2,\alpha}^{(i)} \mathbf{1}_{S^2 \times P})_{1 \leqslant \alpha \leqslant n_a} \in \mathbb{R}^{S^2 \times P \times n_a}$ with $\boxed{0 < \gamma_{2,\alpha}^{(i)} < 1/(\eta + \zeta_{2,\alpha}^{(i)})}$

$\zeta_{2,\alpha}^{(i)} \rightsquigarrow$ Lipschitz constant of the partial derivative of $h(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i+1)}, \mathbf{U}_2^{(i)})$ w.r.t. $\mathbf{u}_{2,\alpha}^{(i)}$

- ⋆ The sequence of iterates $\left(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\right)_{i \in \mathbb{N}}$ generated by the BCFB algorithm converges to a critical point $\left(\boldsymbol{\epsilon}^\star, \mathbf{U}_1^\star, \mathbf{U}_2^\star\right)$ of f
- ⋆ $\left(f\left(\boldsymbol{\epsilon}^{(i)}, \mathbf{U}_1^{(i)}, \mathbf{U}_2^{(i)}\right)\right)_{i \in \mathbb{N}}$ is a non-increasing sequence converging to $f\left(\boldsymbol{\epsilon}^\star, \mathbf{U}_1^\star, \mathbf{U}_2^\star\right)$

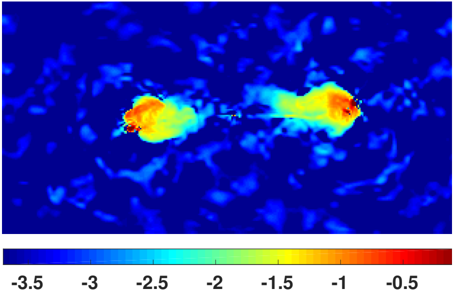
Backup slide: Observation of Cygnus A (real data)

- VLA data at 8.4 GHz with $n_a = 26$ active antennas
- $T = 625$ snapshots
- $M = 185648$ measurements

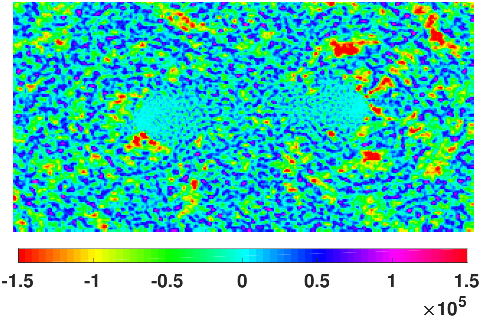
Reconstruction with normalized DIEs

$\text{SNR}_{dirty} = 44.45$ dB

$\boldsymbol{x}^\star$



Residual image: $\boldsymbol{\Phi}^\dagger(\boldsymbol{\Phi}\boldsymbol{x}^\star - \boldsymbol{y})$



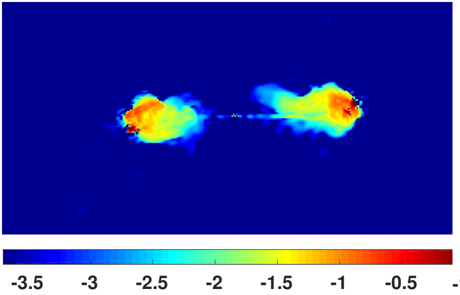
Backup slide: Observation of Cygnus A (real data)

- VLA data at 8.4 GHz with $n_a = 26$ active antennas
- $T = 625$ snapshots
- $M = 185648$ measurements

Joint imaging and DDE calibration

$\mathrm{SNR}_{dirty} = 50.21$ dB

$\boldsymbol{x}^\star$



Residual image: $\boldsymbol{\Phi}^\dagger(\boldsymbol{\Phi}\boldsymbol{x}^\star - \boldsymbol{y})$