

A Faceted Prior for Scalable Wideband Computational Imaging

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1. Context and proposed approach

- **Objective:** form extreme size wideband image \mathbf{X} from incomplete data

$$\begin{bmatrix} \mathbf{Y} \\ \vdots \\ \mathbf{Y} \end{bmatrix} = \Phi \begin{pmatrix} \mathbf{X} \\ \vdots \\ \mathbf{X} \end{pmatrix} + \mathbf{N}, \quad \mathbf{X} \in \mathbb{R}^{N \times L}, \quad \mathbf{Y}, \mathbf{N} \in \mathbb{C}^{M \times L}$$

$$\rightsquigarrow \text{optimization problem: } \underset{\mathbf{X} \in \mathbb{R}_{+}^{N \times L}}{\text{minimize}} \underbrace{f(\mathbf{Y}, \Phi(\mathbf{X}))}_{\text{data fitting}} + \underbrace{r(\mathbf{X})}_{\text{regularization}}$$

- Spectral correlations: low-rankness and joint-sparsity regularization

$$r(\mathbf{X}) = \lambda \|\mathbf{X}\|_* + \mu \|\Psi^\dagger \mathbf{X}\|_{2,1}, \quad \Psi^\dagger \in \mathbb{R}^{P \times N} \text{ sparsifying dictionary}$$

rightsquigarrow **prohibitive cost:** radio-astronomy, $L \approx 10^3$ channels, $N \approx 10^{14}$ pixels

- **Proposed approach:** facet-based low-rankness and joint sparsity prior

$$r(\mathbf{X}) = \sum_{i=1}^I \lambda_i \|\mathbf{W}_i \tilde{\mathbf{S}}_i \mathbf{X}\|_* + \mu_i \|\Psi_i^\dagger \mathbf{S}_i \mathbf{X}\|_{2,1}$$

rightsquigarrow define dictionary Ψ_i^\dagger based on the structure of Ψ^\dagger ;
(exact decomposition when Ψ^\dagger is a wavelet dictionary [1])
rightsquigarrow **more scalable, promotes local spectral correlations**;
rightsquigarrow $\mathbf{S}_i, \tilde{\mathbf{S}}_i$ selection operators, weights \mathbf{W}_i to mitigate faceting artifacts.

2. Application to radio-astronomy

- **Measurement operator:**

$$\Phi(\mathbf{X}) = \{\Phi_\ell \mathbf{x}_\ell\}_{\ell=1}^L, \quad \Phi_\ell = \Theta_\ell \mathbf{G}_\ell \mathbf{F} \mathbf{Z}$$

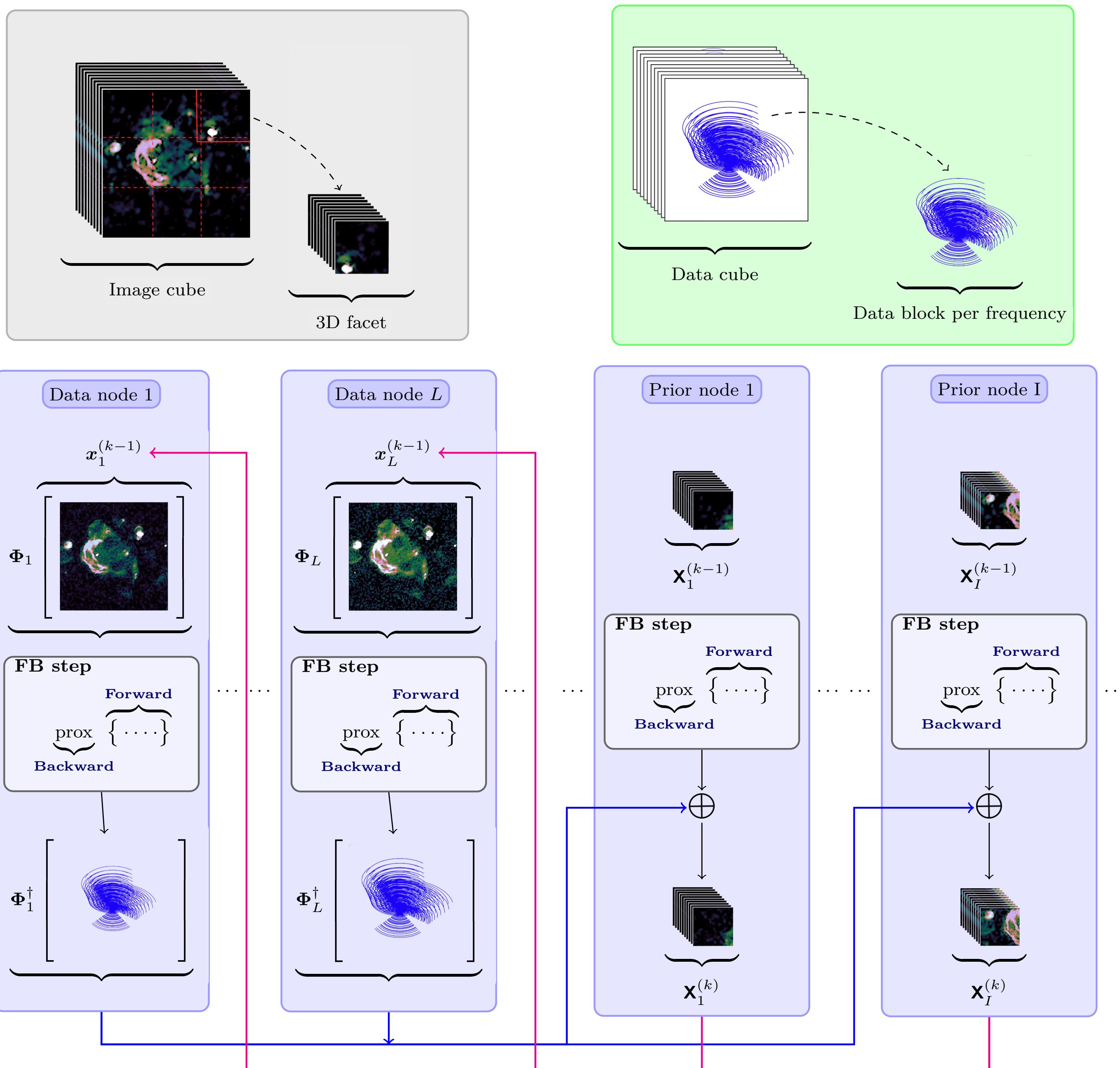
$\mathbf{x}_\ell \in \mathbb{R}^N$ image in channel ℓ (column of \mathbf{X})
 $\mathbf{F} \in \mathbb{C}^{K \times K}$ Fourier transform
 $\mathbf{Z} \in \mathbb{R}^{K \times N}$ zero-padding and scaling operator
 $\Theta_\ell \in \mathbb{R}^{M \times M}$ natural weighting matrix
 $\mathbf{G}_\ell \in \mathbb{C}^{M \times K}$ interpolation and calibration kernels

- **Problem formulation:** extension of HyperSARA [2] – a wideband radio-interferometric (RI) imaging approach

rightsquigarrow data fitting term: per channel ℓ_2 constraint, controlled by ε_ℓ

$$\underset{\mathbf{X} \in \mathbb{R}_{+}^{N \times L}}{\text{minimize}} \underbrace{\sum_{\ell=1}^L \iota_B(\mathbf{y}_\ell, \varepsilon_\ell)(\Phi_\ell \mathbf{x}_\ell)}_{f(\mathbf{Y}, \Phi(\mathbf{X}))} + \underbrace{\sum_{i=1}^I \|\mathbf{W}_i \tilde{\mathbf{S}}_i \mathbf{X}\|_* + \mu_i \|\Psi_i^\dagger \mathbf{S}_i \mathbf{X}\|_{2,1}}_{r(\mathbf{X})}$$

- **Imaging algorithm:** preconditioned primal-dual algorithm [3].



3. Illustration on synthetic data

- **Simulation settings:**

- synthetic wideband image of the W28 supernova remnant;
- $L = 20$ spectral channels, $N = 1024 \times 1024$ pixels;
- $M \approx 0.5N$ measurements per channel, SNR = 60 dB;
- faceted HyperSARA compared to HyperSARA [2] and single channel reconstruction (SARA [4]):
- ▷ quality comparable to HyperSARA, much lower computing time;
- ▷ overall reconstruction improvement of very low intensity emissions;
- ▷ limited performance of SARA: single channel rightsquigarrow limited range of spatial frequencies exploited (nature of RI Fourier sampling).

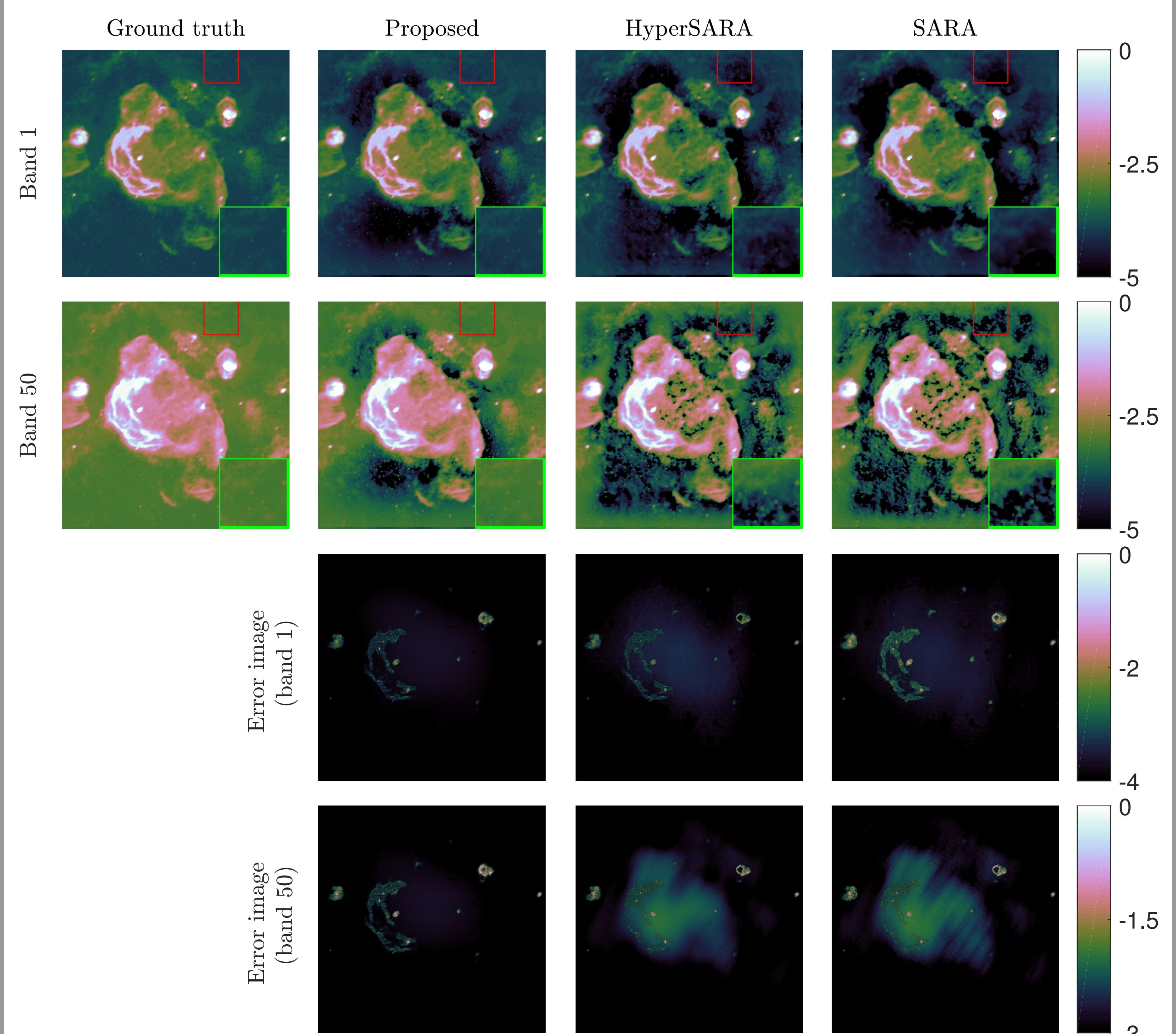


Figure 1: Reconstructed and error images (in rows) displayed in log scale for the different approaches. The reconstruction SNR for the displayed channels and per iteration computing time are: (proposed) [36.97/36.97 dB, 7.8 s], (HyperSARA) [SNR = 38.83/37.83 dB, 45 s], (SARA) [27.78/37.06 dB, 2.2 s].

4. Conclusions and perspectives

- **Conclusions:**

- ✓ **faceted prior for scalable wideband imaging**;
- ✓ promote **local spectral correlations** via a facet-based nuclear norm
rightsquigarrow improved reconstruction of faint emissions in RI, compared to HyperSARA.

- **Perspectives:**

- control number of facets handled at each iteration: randomization;
- investigate faceting of Fourier transform involved in RI measurement operator: preserve both data and image locality.

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