

# **CSCI-4113**

## **Assignment 2**

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### **Contents**

<b>1</b>	<b>Question 1</b>	<b>3</b>
<b>2</b>	<b>Question 2</b>	<b>6</b>
<b>3</b>	<b>Question 3</b>	<b>7</b>

**Note:** To avoid confusing myself, i redefined  $y$  as  $w$  so the resulting LP  $P$  is:

$$Mazimize(3x + 4w + 2z)$$

Such That:

$$4x + w - 2z \leq 20 \tag{1}$$

$$3x - w + 2 \geq 1 \tag{2}$$

$$x - 2w + 2z = -8 \tag{3}$$

$$4x - w + 3z \leq 21 \tag{4}$$

and

$$x \geq 0$$

$$w \leq 0$$

## 1 Question 1

Given the LP  $P$  that is clearly not in Canonical Form as it has a Minimization Objective Function, Lower Bound and equality Constraints, and negative/unbounded variables.

We define a sequence of equivalent LPs  $P^{(i)}, 0 \leq i \leq 3$  such that  $P^{(3)} = P'$  is an LP equivalent to  $P$  and in Canonical form:

### First

We need to define an LP  $P^{(0)}$  that is equivalent to  $P$  but with the a negated Objective Function for  $P^{(0)}$  to be a maximization LP.

1. Since  $P$  and  $P^{(0)}$  have the same variables and constraints, this means that they have the same set of feasible solutions.
2. Minimizing  $f(x)$  is the same as maximizing  $f'(x)$ , thus  $P$  and  $P'$  have the same Objective Function value.

$P^{(0)}$  will have an objective Function:

$$\text{Maximize}(-3x - 4w - 2z)$$

### Second

We now construct a new LP  $P^{(1)}$  that is equivalent to  $P^{(0)}$  but with no equality constraints. That is, we replace all the equalities with inequalities.

1. Since  $a = b$  is clearly equivalent to  $a \leq b$  &&  $a \geq b$
2. Thus  $P^{(0)}$  and  $P^{(1)}$  are equivalent since they have the same set of variables, the same objective function and equivalent constraints.

Thus we replace constraint (3) in  $P^{(0)}$  with  $\downarrow$  in  $P^{(1)}$

$$x - 2w + 2z \leq -8 \tag{3.1}$$

$$x - 2w + 2z \geq -8 \tag{3.2}$$

### Third

The next step is to replace all the lower bound inequality constraints in  $P^{(1)}$  to upper bound constraints, thus constructing a new LP  $P^{(2)}$ .

1. An inequality  $a \leq b$  is equivalent to its negation  $-a \geq -b$
2. Just like in the prev step, we are not changing the Objective function and replacing some constraints with equivalent one's thus both  $P^{(2)}$  and  $P^{(1)}$  have the same Objective Function values and set of feasible solutions and are thus equivalent.

So we replace constraint (1) and (3.2) respectively with:

$$-3x + w - z \leq -1 \quad (1.1)$$

$$-x + 2w - 2z \leq 8 \quad (3.2.1)$$

This gives us an LP  $P^{(2)}$

### Fourth

We now need to add non-negativity constraints to  $P^{(2)}$ .

1.  $x$  already has the constraint so we don't need to modify it.
2.  $w$  has a non-positivity constraint, which is equivalent to:

$$-w' \geq 0$$

Thus we replace every instance of  $w$  with  $-w'$  (note that  $w = w'$ ) and the constraint  $w \leq 0$  with  $w' \geq 0$ .

3. The variable  $z$  is unbounded. We can add a non-negativity constraint on it by representing it as the difference of two non-negative numbers. It is clear to see that this holds for any number. So:

$$z = z' - z''$$

And add the below constraints to the LP

$$z' \geq 0, z'' \geq 0$$

We then replace all instances of  $z$  with  $z' - z''$  in  $P^{(2)}$  Thus giving us a new LP  $P^{(3)}$ .

Notice that all the LP's in the sequence 0,1,2,3 are equivalent and since  $P$  is equivalent to  $P^{(0)}$ . This means that it is also equivalent to  $P^{(3)}$ . And since  $P^{(3)}$  is a maximization LP whose constraints all define upper bounds, and has non-negativity constraints on all its variables this means that it is in Canonical form, so  $P = P^{(3)} = P'$  (The equality here denotes equivalence).

Then  $P'$  is our Canonical LP such that (im reusing equation numbers, a hard reset):

$$\text{Maximize}(-3x + 4w - 2z' + 2z'')$$

Such that:

$$4x - w - 2z' + 2z'' \leq 20 \tag{1}$$

$$-3x - w - z' + z'' \leq -1 \tag{2}$$

$$x + 2w + 2z' - 2z'' \leq 8 \tag{3}$$

$$-x - 2w - 2z' + 2z'' \leq -8 \tag{4}$$

$$4x + w + 3z' - 3z'' \leq 21 \tag{5}$$

$$x \geq 0, w \geq 0, z' \geq 0, z'' \geq 0$$

## 2 Question 2

An LP is in standard form if it follows all the restrictions of the Canonical form with the changed restriction of having constraints be equality instead of inequalities. We do this by adding a slack variable  $y_i$  for the  $i^{th}$  constraint.

So LP equivalent to  $P$  and  $P'$  but in standard form is  $P''$  such that:

$$\text{Maximize}(-3x + 4w - 2z' + 2z'' + d)$$

Such that:

$$4x - w - 2z' + 2z'' + y_1 = 20 \quad (1)$$

$$-3x - w - z' + z'' + y_2 = -1 \quad (2)$$

$$x + 2w + 2z' - 2z'' + y_3 = 8 \quad (3)$$

$$-x - 2w - 2z' + 2z'' + y_4 = -8 \quad (4)$$

$$4x + w + 3z' - 3z'' + y_5 = 21 \quad (5)$$

$$x \geq 0, w \geq 0, z' \geq 0, z'' \geq 0$$

$$y_i \geq 0 \text{ where } 1 \leq i \leq 5 \quad (2.0.1)$$

### 3 Question 3

$b$	$y_1$	$y_2$	$y_3$	$y_4$	$y_5$	$x$	$w$	$z'$	$z''$
20	1					4	-1	-2	2
-1		1				-3	-1	-1	1
8			1			1	2	2	-2
-8				1		-1	-2	-2	2
21					1	4	1	3	-3
						-3	4	-2	2