Д/З Матан 2 : 12 сентября

Зад. 1

a) $e^{20}x^n \overset{?}{=} \bar{o}(x^{n+1}),\ x\to+\infty$

$\lim\limits_{x\to+\infty} \frac{e^{20}x^n}{x^{n+1}} = e^{20}\lim\limits_{x\to+\infty}\frac{x^n}{x^{n+1}} = e^{20}\lim\limits_{x\to+\infty}\frac{1}{n} = 0.$

$\Rightarrow e^{20}x^n = \bar{o}(x^{n+1}),\ x\to+\infty$

b) $e^{20}x^n \overset{?}{\neq} \bar{o}(x^{n-1}),\ x\to 0$

$\lim\limits_{x\to0}\frac{e^{20}x^n}{x^{n-1}} = e^{20}\lim\limits_{x\to0}\frac{x^n}{x^n\cdot x^{-1}} = e^{20}\lim\limits_{x\to0}\frac{1}{x^{-1}} = e^{20}\lim\limits_{x\to0} x = 0$

$\Rightarrow e^{20}x^n = \bar{o}(x^{n-1})$

c) $(x-1)^2\left(5+\sin\frac{1}{x-1}\right) \overset{?}{=} \underline{O}\left((x-1)^2+(x-1)^3\right),\ x\to1$

$t = x-1,\ t\to0$

$f(t) = \frac{t^2\left(5+\sin\frac1t\right)}{t^2+t^3},\ t\to0$, $f(t)$ – ограничена?

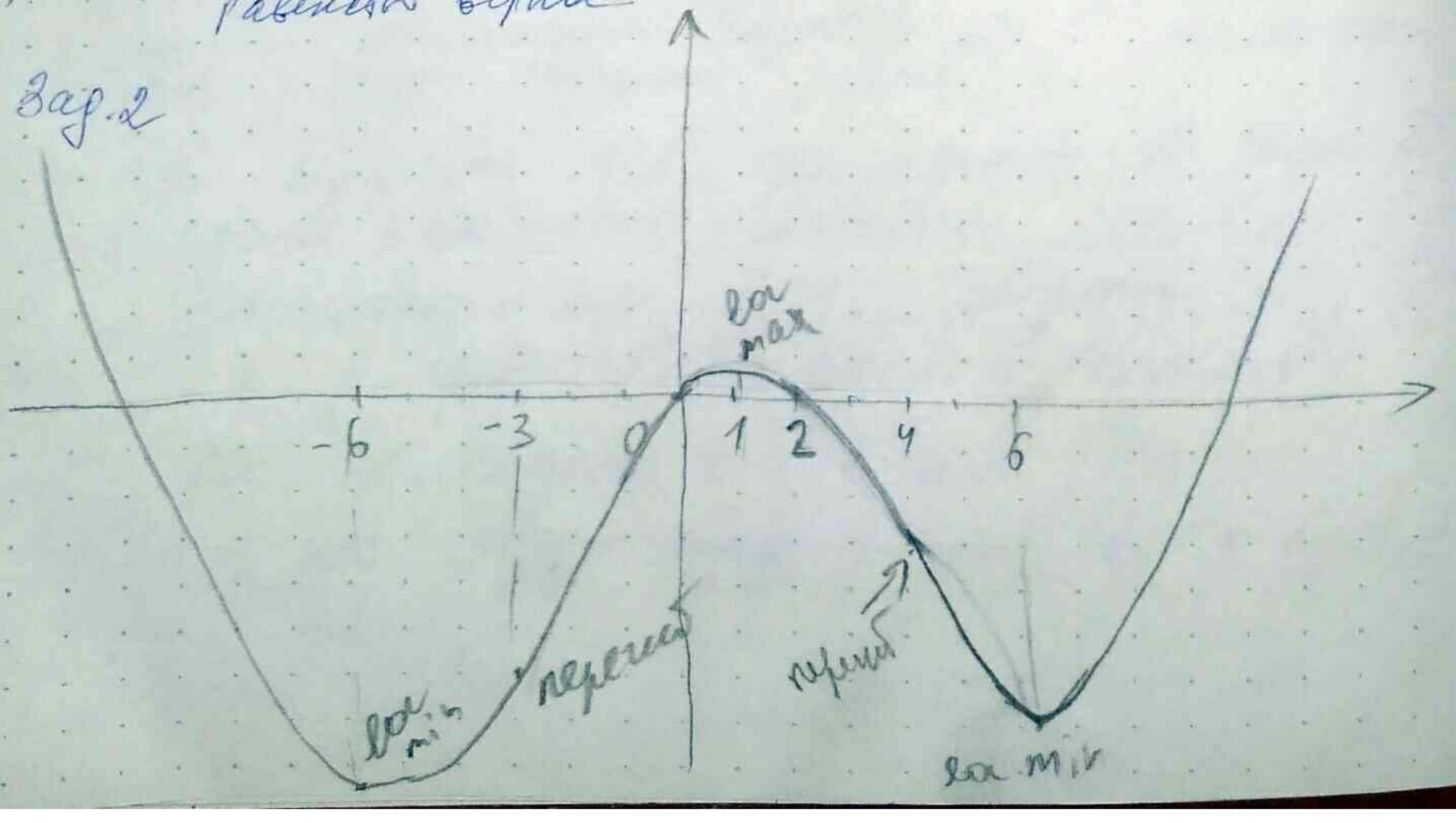
$f(t) = \frac{5+\sin\frac1t}{1+t}$; $4\le 5+\sin\frac1t\le 6$, $t\ne0 \Rightarrow t\to0$

$0.5<1+t<1.5$

$\frac{4}{1.5} < f(t) < \frac{6}{0.5}$, $\frac83 < f(t) < 12$ – ограничена $\Rightarrow$

равенство верное

Зад.2



Зад. 3.

a) $f(x) = \ln \sin(x^2)$

$f'(x) = \ln' \sin(x^2) \cdot (\sin(x^2))' = \frac{1}{\sin(x^2)} \cdot \sin'(x^2) \cdot 2x =$

$= 2x \cdot \frac{\cos(x^2)}{\sin(x^2)} = 2x \, ctg(x^2).$

b) $f(x) = \arctan \sin(x^2)$

$f'(x) = \arctan' \sin(x^2) \cdot (\sin(x^2))' = -\frac{1}{1+\sin^2 x^2} \cdot 2x \cdot \cos(x^2) =$

$= -\frac{2x \cdot \cos(x^2)}{1+\sin^2 x^2}$

c) $f(x) = x \cdot e^{-x^2}$

$f'(x) = x' \cdot e^{-x^2} + x(e^{-x^2})' = e^{-x^2} + x \cdot (e^{-x^2} \cdot (-x^2)') =$

$= e^{-x^2} - x^3 \cdot e^{-x^2} = e^{-x^2}(1 - x^3)$

d) $f(x) = (\sin x)^{\cos x}$;

$f'(x) = (e^{\ln(\sin x) \cdot \cos x})' = e^{\ln(\sin x) \cdot \cos x} \cdot (\ln(\sin x) \cdot \cos x)'$

$(\ln(\sin x) \cdot \cos x)' =$

$= \ln' \sin x \cdot \cos x + \ln(\sin x)(-\sin x) = \frac{\cos^2 x}{\sin x} -$

$- \sin x \cdot \ln(\sin x)$

$f'(x) = (\sin x)^{\cos x} \cdot \left(\frac{\cos^2 x}{\sin x} - \sin x \cdot \ln(\sin x) \right)$

Зад. 4 Ур-е касат.: $f(x) = \frac{2x}{x+1}$ в т. $(1, 1)$

$y = f'(a)(x-a) + f(a)$

$f(a) = f(1) = \frac{2}{1+1} = 1$

$y = f'(1)\cdot(x-1) + 1 = (x-1)' + 1 = 2$

Ур-е касат.: $y = 2$.

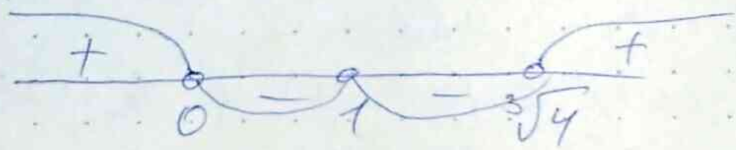
Зад. 5

а) $f(x) = \frac{x^4}{x^3-1}$

1. Област определение $x \in (-\infty; 1); (1, +\infty)$.
2. Нули: $f(x) = 0$, $x = 0$.
3. $f'(x) = \frac{4x^3(x^3-1) - x^4\cdot 3x^2}{(x^3-1)^2} = \frac{4x^6 - 4x^3 - 3x^6}{(x^3-1)^2} =$

$= \frac{x^6 - 4x^3}{(x^3-1)^2} = \frac{x^3(x^3-4)}{(x^3-1)^2} > 0, x \neq 1$

$x^3(x^3-4)$



$f'(x) = 0$: $x = 0$, $x = \sqrt[3]{4}$

$x = 0$ - лок. минимум

$x = \sqrt[3]{4}$ - лок. максимум

4. $f''(x) = \frac{(x^6-4x^3)'(x^3-1)^2 - (x^6-4x^3)((x^3-1)^2)'}{(x^3-1)^4} =$

$= \frac{(6x^5-12x^2)(x^3-1)^2 - (x^6-4x^3)2(x^3-1)3x^2}{(x^3-1)^4} = [x \neq 1]$

$= \frac{\cancel{(x^3-1)}((6x^5-12x^2)(x^3-1) - 6x^2(x^6-4x^3))}{(x^3-1)^{\cancel{4}\,3}} =$

$= \frac{\cancel{6x^8} - 6x^5 - 12x^5 + 12x^3 - \cancel{6x^8} + 24x^5}{(x^3-1)^3} = \frac{6x^5 + 12x^2}{(x^3-1)^3} =$

$= \frac{\overset{\geq 0}{6x^2}(x^3+2)}{(x^3-1)^3}$

$x > 1 \;:\; f''(x) > 0 \Rightarrow f(x)$ вогнута

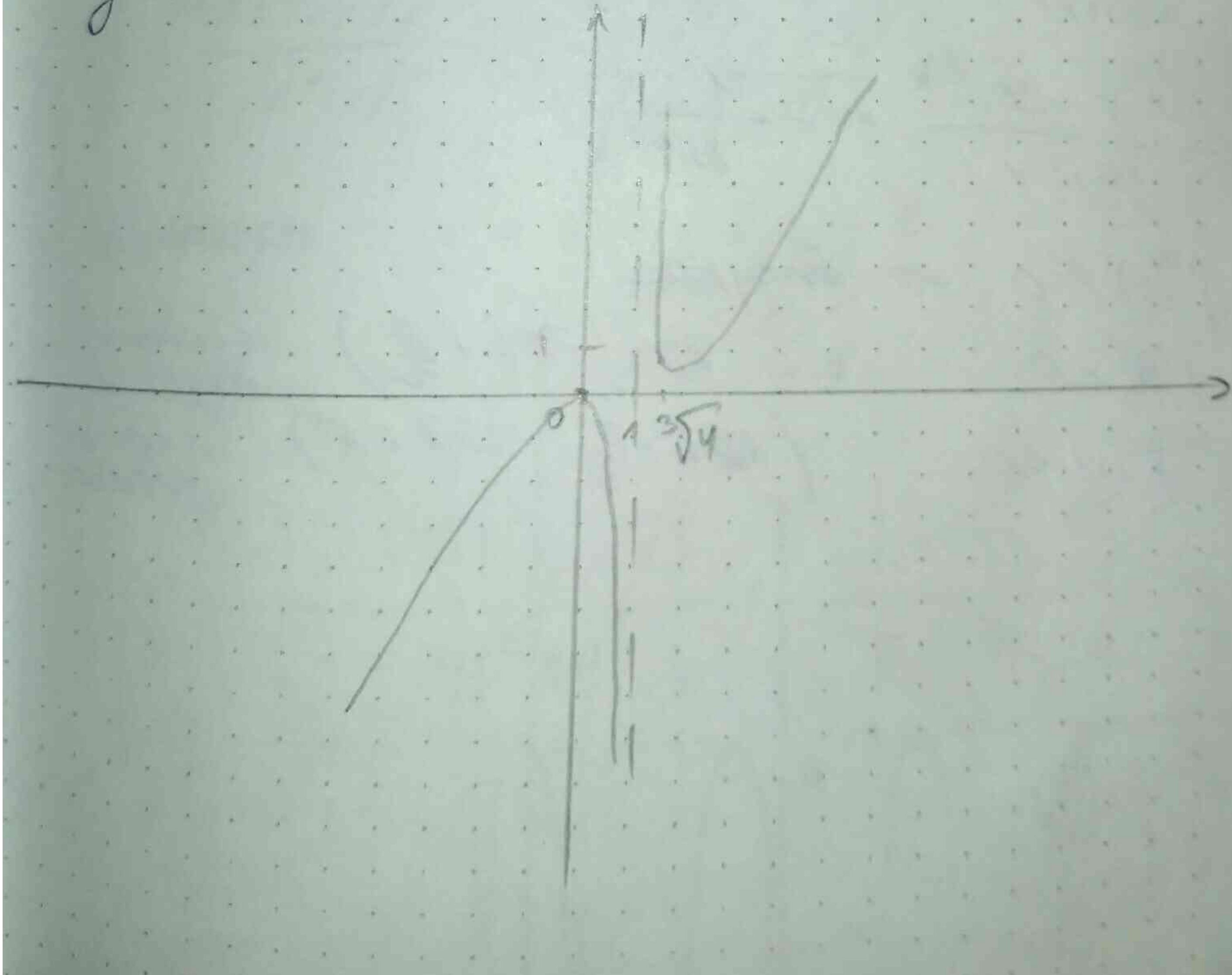
$x < 1 \;:\; f''(x) < 0 \Rightarrow f(x)$ выпукла

5. Асимптоты

$$\lim_{x \to 1-0} f(x) = \frac{\lim\limits_{x \to 1-0} x^4}{\lim\limits_{x \to 1-0} (x^3 - 1)} = \frac{1}{-0} = -\infty$$

$$\lim_{x \to 1+0} f(x) = \frac{\lim\limits_{x \to 1+0} x^4}{\lim\limits_{x \to 1+0} (x^3 - 1)} = +\infty$$

$y = 1$: асимптота

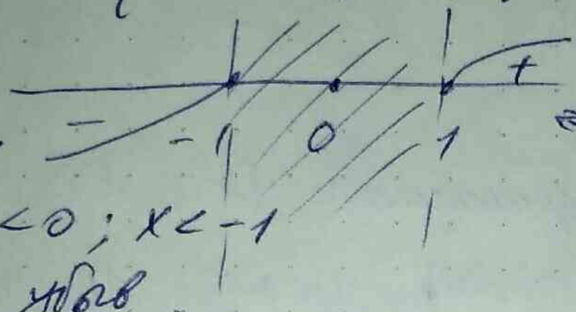


c) $\ln(x^4-1)$

Обл. опред.: $x^4-1>0$; $x^4>1$; $\begin{cases} x>1 \\ x<-1 \end{cases}$

$f'(x)=\frac{4x^3}{x^4-1}$

$f'(x)>0$ $\begin{cases} 4x^3>0 \\ x^4>1 \end{cases}$ $\begin{cases} 4x^3<0 \\ x^4<1 \end{cases}$



$f'(x)>0$; $x>1$ монот. возраст.

$f'(x)<0$; $x<-1$ монот. убыв.

$$f''(x)=\frac{12x^2(x^4-1)-4x^3\cdot 4x^3}{(x^4-1)^2}=\frac{4x^2(3x^2(x^4-1)-4x^5)}{(x^4-1)^2}=$$

$$=\frac{-4x^6-12x^2}{(x^4-1)^2}=\frac{-4x^2(x^4+3)}{(x^4-1)^2}<0 \Rightarrow \text{выпукла}$$

Асимптоты: $x=1$; $x=-1$.

Нули:
$\ln 1=0$
$x^4-1=1$
$x^4=2$
$x=\pm\sqrt[4]{2}$