МАТСТАТ-1

Зад. 1

$$\bar{X} = \frac{3,4 + 4,0 + \dots 4,6}{20} = \frac{113,3}{20} = 5,665$$

$$MED = X\left(\left[\frac{n+1}{2}\right]\right) = \frac{5,1+5,3}{2} = 5.2$$

Мода - нельзя вычислить, 3 значения встречаются по 2 раз (наиб. часто)

$$RANGE = X_{max} - X_{min} = 8,6 - 3,4 = 5.2$$

$$UP = X_{([0,75(n+1)])} = 7.5$$

$$LP = X([0,25(n+1)]) = 4.6$$

$$IQR = UP - LP = 7.5 - 4.6 = 2.9$$

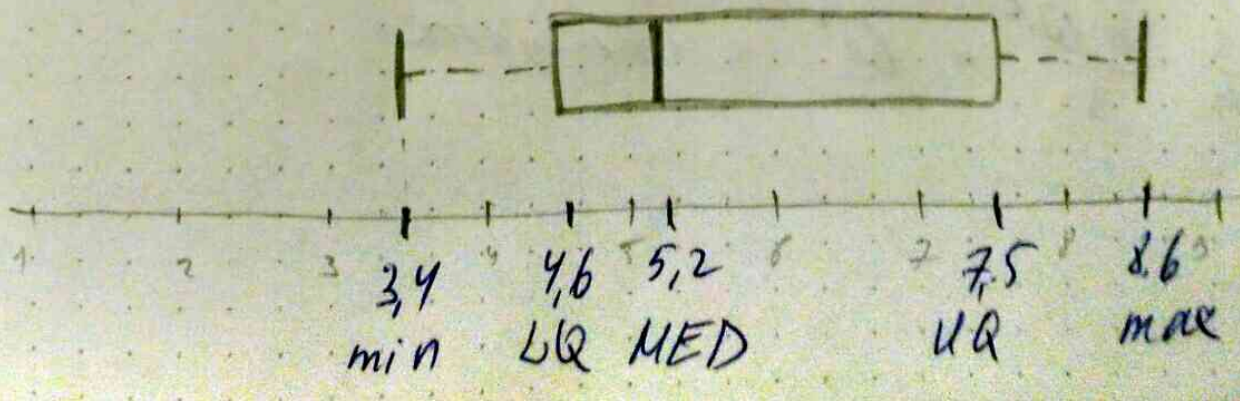
$$VarX = \frac{1}{n-1}\left(\sum_{i=1}^{n} X_i^2 - n\bar{X}^2\right) =$$

$$= \frac{1}{19}\left(\underbrace{\sum X_i^2}_{697.33} - 20 \cdot (5.665)^2\right) \approx 2.92$$

$$SD = \sqrt{VarX} \approx 1,709$$

Границы выбросов:

$[(X_{25} - 1.5 \cdot IQR),$
$(X_{75} - 1.5\,IQR)]$
$[0,25, 11.85]$
→ выбросов нет.



Зад. 2 $X_1, \dots X_n \sim U[0, \Theta]$

$\mathbb{E}X = \frac{a+b}{2} = \frac{\Theta}{2}$

а) $\hat{\Theta}(X_1, \dots, X_n) = 6$

$E_\Theta \cdot 6 = 6 \cdot E_\Theta = 6\frac{\Theta}{2} = 3\Theta$ - смещена

б) $\hat{\Theta}(X_1, \dots, X_n) = 2X_n$

$E_\Theta 2X_n = 2\mathbb{E}X_n = 2\frac{\Theta}{2} = \Theta$ - не смещена

с) $\hat{\Theta}(X_1, \dots X_n) = 2X_1 + 2X_n$

$E_\Theta(2X_1 + 2X_n) = 2E_\Theta(X_1 + X_n) = 2(E_\Theta(X_1) + E_\Theta(X_n)) =$

$= 2\left(\frac{\Theta}{2} + \frac{\Theta}{2}\right) = 2\Theta$ - смещена

д) $\hat{\Theta}(X_1, \dots, X_n) = 2(X_1 + \dots + X_n)/n$

$E_\Theta \frac{2(X_1 + \dots + X_n)}{n} = \frac{2}{n} E_\Theta(X_1 + \dots + X_n) =$

$= \frac{2}{n}\left(E_\Theta(X_1) + \dots + E_\Theta(X_n)\right) = \frac{2}{n}\left(\underbrace{\frac{\Theta}{2} + \dots + \frac{\Theta}{2}}_{n \text{ шт}}\right) =$

$= \frac{2}{n} \cdot \frac{\Theta n}{2} = \Theta$ - не смещена

е) $\hat{\Theta} = \frac{3(X_1^2 + \dots + X_n^2)}{n}$

$E_\Theta \frac{3(X_1^2 + \dots X_n^2)}{n} = \frac{3}{n}\left(\mathbb{E}(X_1^2) + \mathbb{E}(X_2^2) + \dots + \mathbb{E}(X_n^2)\right) =$

$E_\Theta(X^2) = \int_0^\Theta x^2 \frac{1}{\Theta} dx = \frac{1}{\Theta}\frac{x^3}{3}\Big|_0^\Theta = \frac{\Theta^3}{3\Theta} = \frac{\Theta^2}{3}$

$= \frac{3}{n} \cdot \frac{n\Theta^2}{3} = \Theta^2$ - смещена