ME 561/EECS 561 Winter 2021
Homework 5

Prof. Vasudevan

Assigned: 19 Feb 2021
Due: 25 Feb 2021 at 10pm

Submission Details

Use this PDF only as a reference for the questions. All answers should be submitted via Gradescope before the due deadline.

Problem 1: [12 point(s)]

Find the modified Z transform of the following functions by hand.

1.1 [6 point(s)]

$$E(s) = \frac{20}{(s+2)(s+5)}$$

1.2 [6 point(s)]

$$E(s) = \frac{5}{s(s+1)}$$

Problem 2: [12 point(s)]

Find the Z transform of the following functions by hand. The results from 1 may be helpful.

2.1 [6 point(s)]

$$E(s) = \frac{20\mathrm{e}^{-0.3Ts}}{(s+2)(s+5)}$$

2.2 [6 point(s)]

$$E(s) = \frac{5\mathrm{e}^{-0.6Ts}}{s(s+1)}$$

Problem 3: [21 point(s)]

Consider the following system in Fig. 1. (Note that epsilon in the diagram is equivalent to e. $\mathcal{E}^{-Ts} = \mathrm{e}^{-Ts}$)

3.1 [9 point(s)] Find the output $c(kT)$ by hand for the system for $e(t)$ equal to a unit-step function.

3.2 [6 point(s)] Explain the effect of the sampler and data hold on $c(kT)$ in the upper path.

3.3 [6 point(s)] Sketch the unit-response $c(t)$ of the system. This sketch can be made without mathematically solving for $C(s)$.

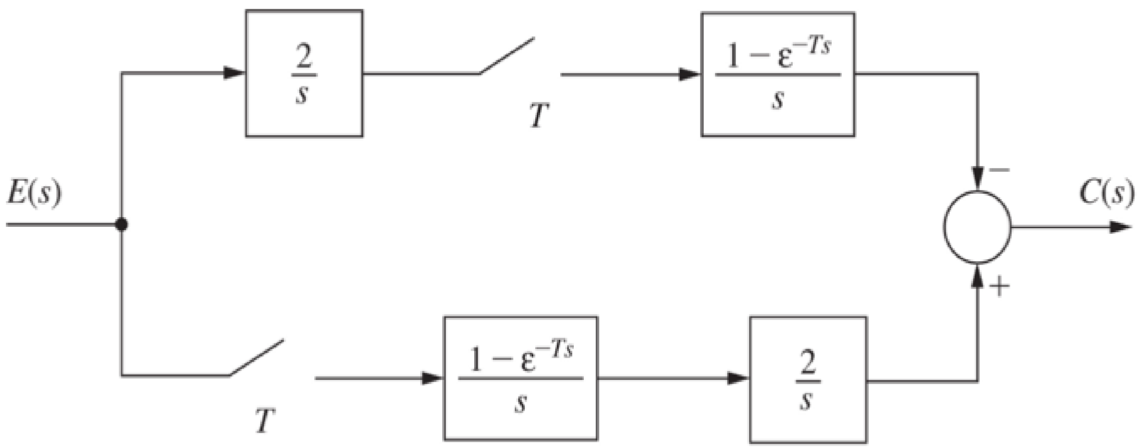
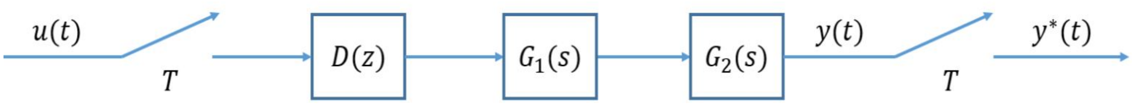


Figure 1: Split sample-and-hold system.

Problem 4: [30 point(s)]

Consider the following system with sampling period $T = 1$s.



4.1 [2 point(s)] List those transfer functions in the block diagram that contain the transfer function of a data hold.

4.2 [8 point(s)] Suppose

$$D(z) = \frac{z-1}{z}, \quad G_1(s)G_2(s) = \frac{s(s+1)}{s^2+2s+2} \text{ (not including data hold) }, \quad u(t) = t$$

and we use a zero-order hold. Solve for $y(t)$ by hand.

4.3 [6 point(s)] Find the pulse transfer function of the whole system by hand.

4.4 [6 point(s)] Using the pulse transfer function, find $y^*(t)$ by hand.

4.5 [6 point(s)] Sketch $y(t)$ and $y^*(t)$.

4.6 [2 point(s)] The dc gain is defined as the steady-state output when the input is the Heaviside function. What is the dc gain of the system?

Problem 5: [25 point(s)]

In this problem, we will compute the pulse transfer function in two different ways. Consider the simplified quarter car model depicted in the Fig. 2. Here v is the longitudinal speed of the car.

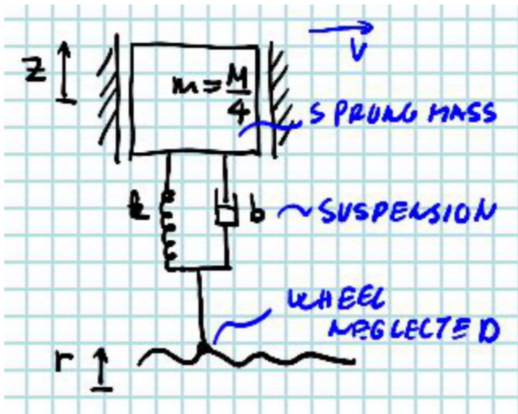


Figure 2: Simplified quarter car model

The mass m represents a fourth of the mass of the car. The stiffness k and damping b represent the spring and shock. The term $r(t)$ is the road excitation, and serves as an input to the system. Using the states $p = z - r$ and $w = \dot{z}$, the dynamics can be written as

$$\begin{bmatrix} \dot{p} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix} \begin{bmatrix} p \\ w \end{bmatrix} + \begin{bmatrix} -1 \\ b/m \end{bmatrix} \dot{r}(t)$$

The velocity of the car body is considered as the output

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ w \end{bmatrix}$$

For this problem consider the values

$$m = 300[\mathrm{kg}] \quad k = 18000[\mathrm{N/m}] \quad b = 1200[\mathrm{N \cdot s/m}]$$

Suppose we want to know the response of the system given the input $u = \dot{r}(t)$. In the Fig. 3, $D(z)$ is the digital filter, and $G(s)$ is the transfer function of the system. For the sake of simplicity, let's assume $D(z) = 1$, and sampling period $T = 1$s. Answer the following questions:

5.1 [6 point(s)] Find the (analog) transfer function $G_0(s) = G_{\mathrm{ZOH}} \cdot G(s)$.

5.2 [6 point(s)] Find the Z transform of $G_0(s)$.

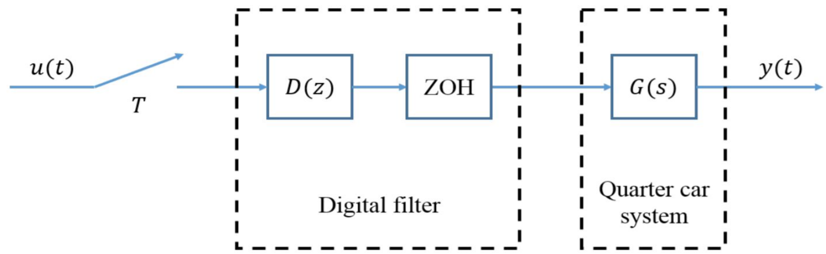


Figure 3: Signal flow of system

5.3 [7 point(s)] Find the discrete-time, state space representation of the system. Remember in homework 1, we proved:

$$A_d = e^{AT}, \quad B_d = \left(\int_0^T e^{Av} dv\right) B, \quad C_d = C$$

5.4 [6 point(s)] Find the transfer function of the discretized system.