Cryptography and the World of the Mystery

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What we are going to do?

Let's explore the world of encryption!

What is cryptography?

a. Cryptography or cryptology is the practice and study of techniques for secure communication in the presence of third parties called adversaries.

b. More generally, cryptography is about constructing and analyzing protocols that prevent third parties or the public from reading private messages.

The link between Linear Algebra and Encryption

Because many types of encryption Matrix use the Math behind matrices to encrypt, Linear Algebra is required for Encryption and Decryption!

Is it complicated?

a. The idea behind encryption is not hard to understand at all!
b. Cipher matrix can be as simple as a 3x3 matrix composed of random integers that represent the characters in the plain-text.

A simple encryption method

Let's take a look at a simple encryption type :D

Say, I want to encrypt the word "MATH"

The general Idea

1. Convert a plain-text to a matrix
2. Encrypt the matrix
3. Decrypt the encrypted matrix

Plain-text to Matrix

Each **character** in plain-text must be denoted with a **numerical value** and placed into a matrix.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Plain-text to Matrix

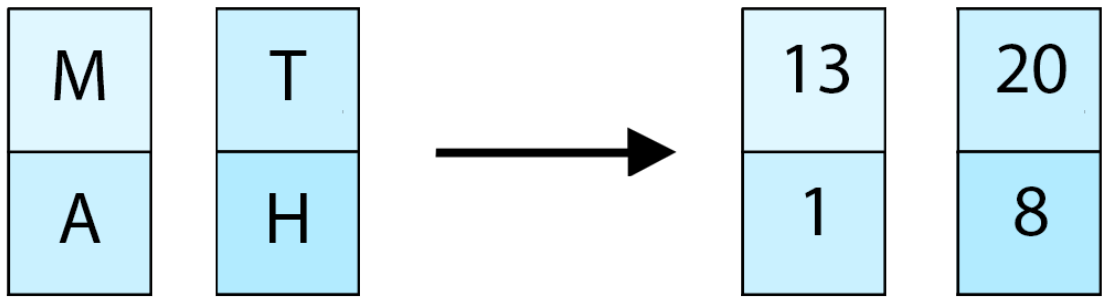
The **numerical values** are then separated into **vectors**, such that:

a. The number of **rows** of each **vector** is equivalent to the numbers of rows of the **cipher matrix**.

b. **Values** are placed **one at a time**, **going down** a row for each value.

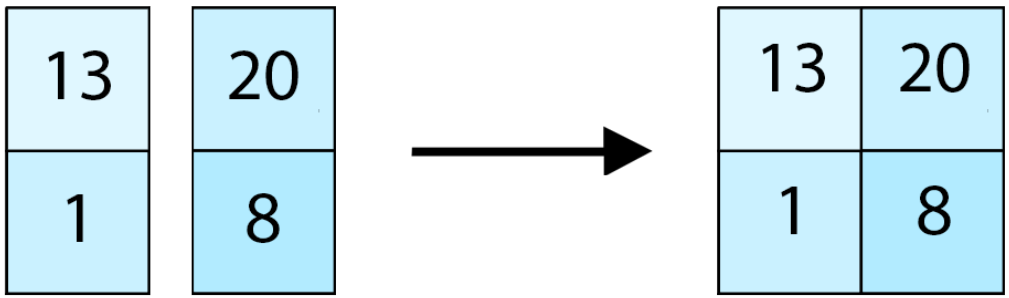
c. Vectors are filled **one to another**.

d. The remaining **empty entries** in the **last vector** is filled with **space**.



Plain-text to Matrix

The vectors are then **augmented** to form a **plain-text matrix**.



Encrypting the matrix

The plain-text matrix is then **multiplied** by another **cipher-matrix** to create the **encrypted matrix**.

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 13 & 20 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} 26 & 40 \\ 2 & 16 \end{bmatrix}$$

Decrypting the matrix

Let's take a look at this

$$
\begin{aligned}
& C \times P && = E \\
\iff \quad & C^{-1} \times C \times P && = C^{-1} \times E \\
\iff \quad & P && = C^{-1} \times E
\end{aligned}
$$

Decrypting the matrix

First, we need to find the **inverse** of the **cipher-matrix**.

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

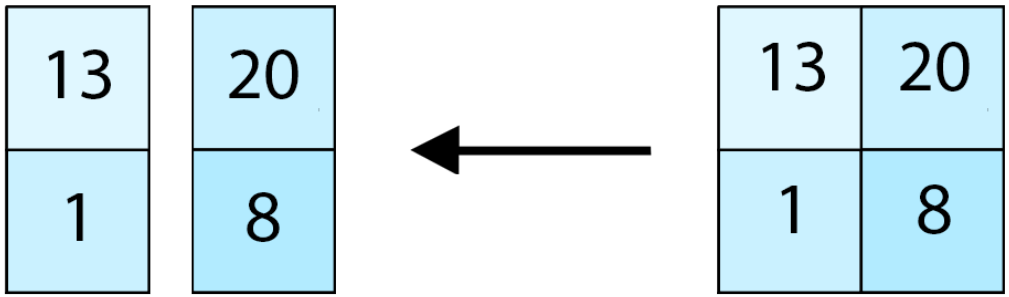
Decrypting the matrix

The **inverted matrix** is then **multiplied** with the **cipher-text matrix**. The **product** is the original **plain-text matrix**.

$$\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \times \begin{pmatrix} 26 & 40 \\ 2 & 16 \end{pmatrix} = \begin{pmatrix} 13 & 20 \\ 1 & 8 \end{pmatrix}$$

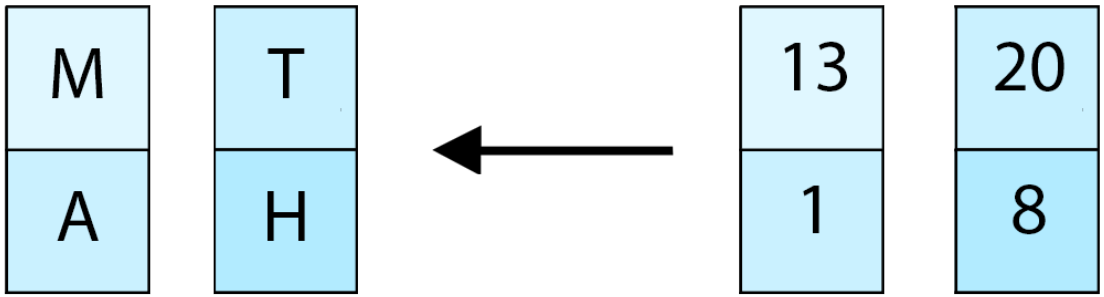
Decrypting the matrix

The **plain-text** can be found by **splitting** the **products** into **vectors**



Decrypting the matrix

And then use the **numerical rules** to convert the **numbers** back into their **letter forms**.



More advanced encryption

Let's take a look at AES, a more secure encryption type!

AES - ...

AES - Advanced Encryption Standard - is:

a. a symmetric encryption algorithm.
b. very powerful.
c. widely used in software and hardware throughout the world!

The general Process

a. AES operates on 4x4 matrix.
b. Each character in the plain-text is denoted with a corresponding numerical value

An example

Let's take a look at an example to understand the process

We'll encrypt and decrypt the plain-text:

"Come here I got cash"

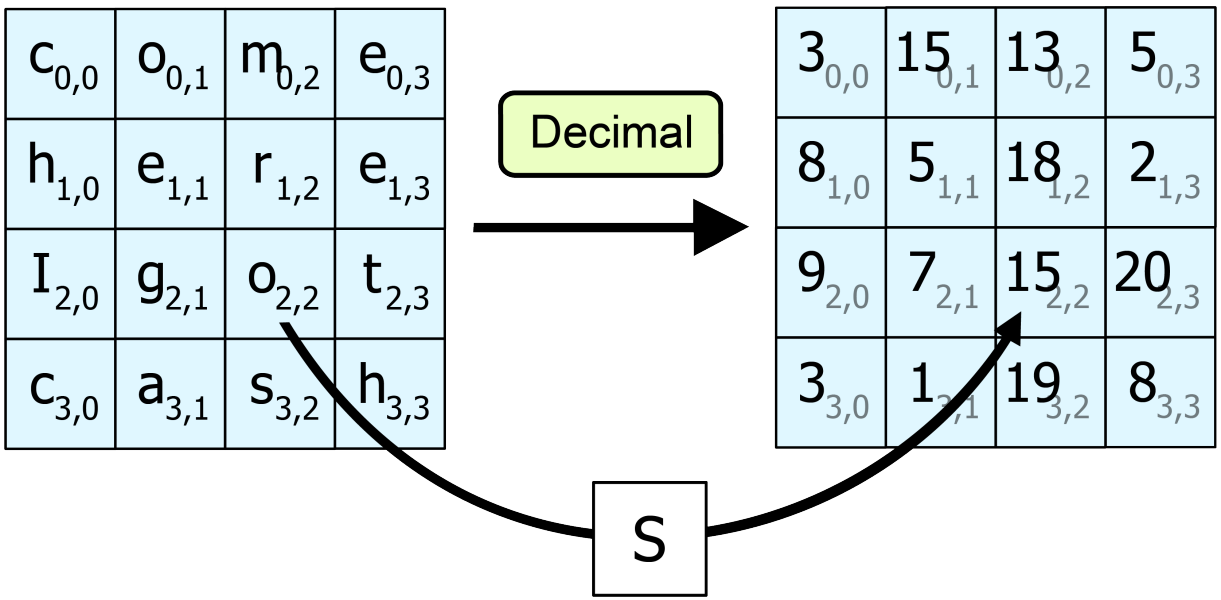
From text to plain-text matrix

Just like in the last example, we use the same rules to convert the text into a matrix.

$c_{0,0}$	$o_{0,1}$	$m_{0,2}$	$e_{0,3}$
$h_{1,0}$	$e_{1,1}$	$r_{1,2}$	$e_{1,3}$
$I_{2,0}$	$g_{2,1}$	$o_{2,2}$	$t_{2,3}$
$c_{3,0}$	$a_{3,1}$	$s_{3,2}$	$h_{3,3}$

Conversion Plain-text to Numerical Matrix

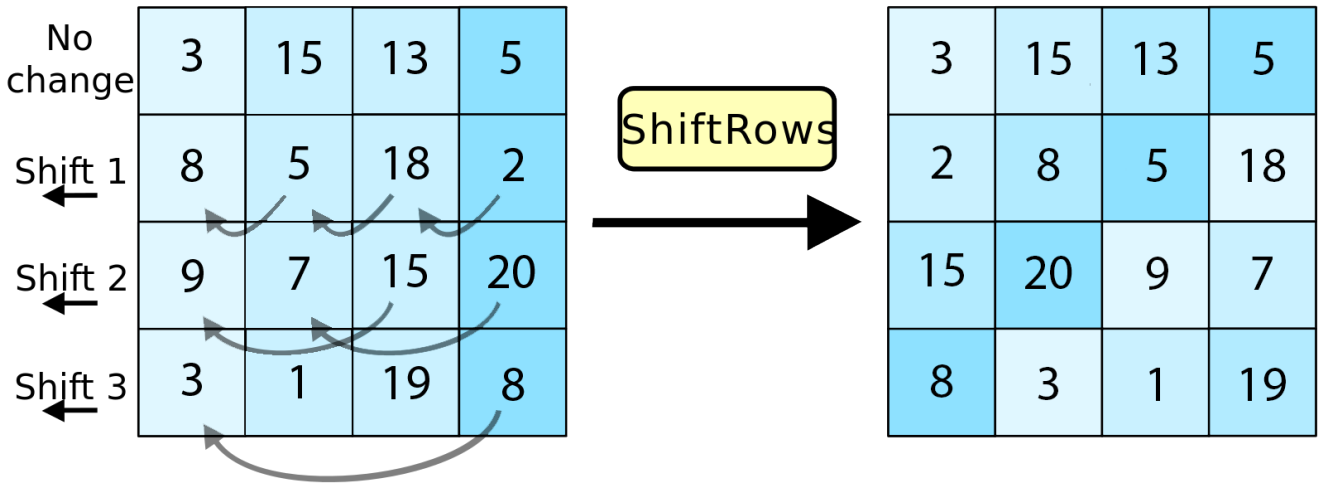
Then, we convert the plain-text into its corresponding numerical matrix



A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Shifting rows

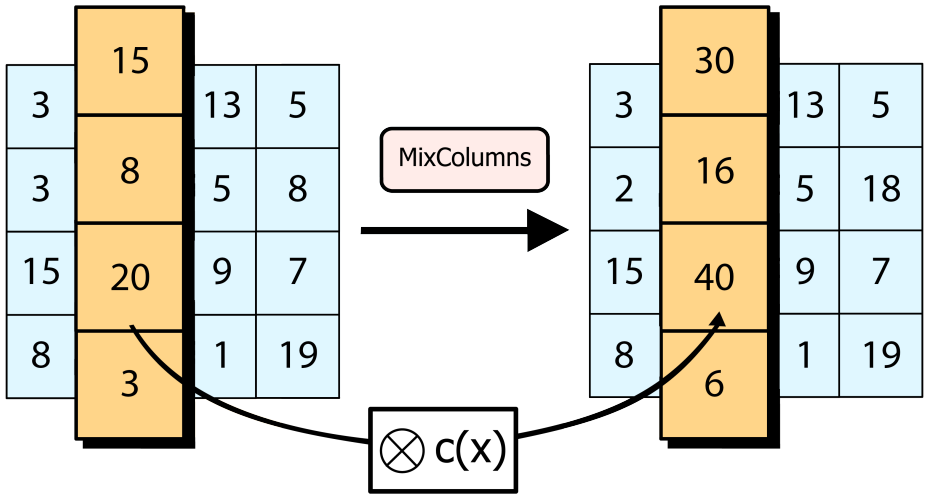
1. First row is unchanged
2. Second row is shifted to the left 1 time
3. Third row is shifted to the left 2 times
4. Fourth row is shifted to the left 3 times



Mixing the columns

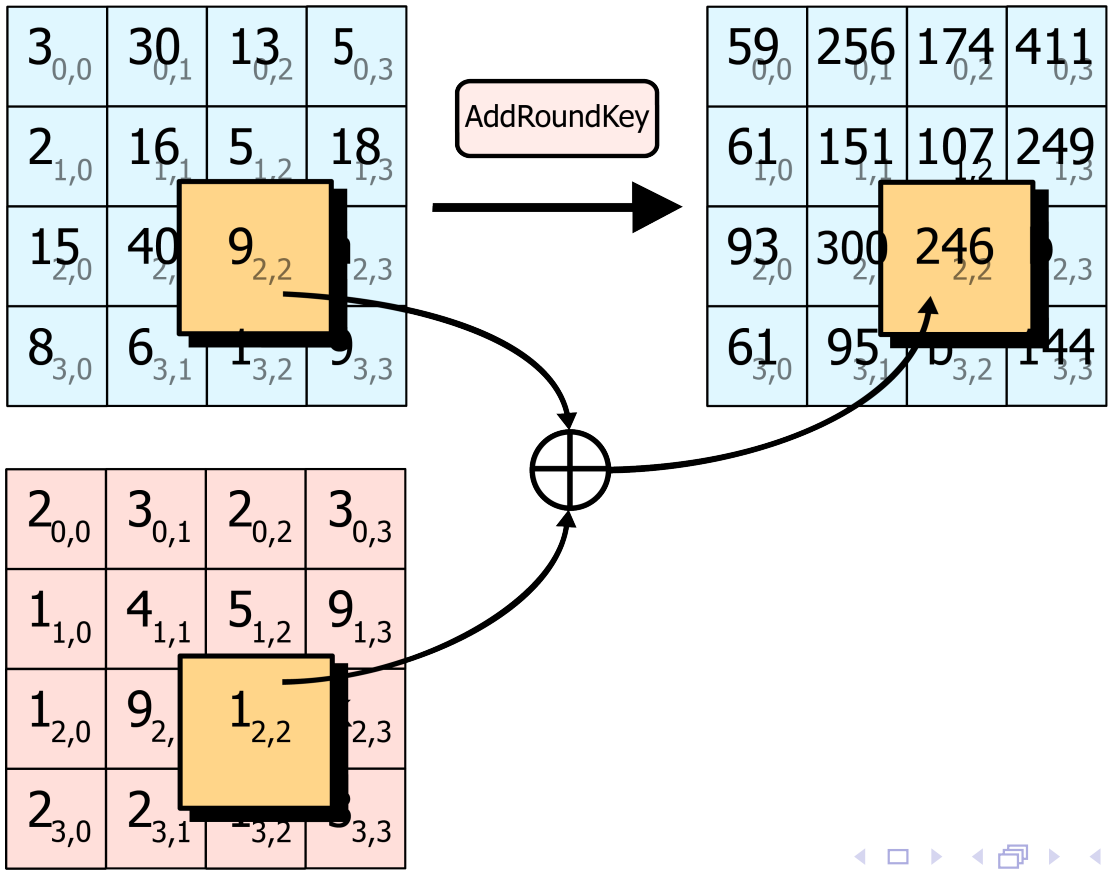
The four decimals in each column are transformed using a linear transformation

In this example, we'll scale the second column by 2



Adding round keys

We then multiply the matrix by another randomly generated invertible matrix, which is the private key



Sending and Deciphering

After encrypting the matrix, it is now secure to be sent to the recipient(s).
To decrypting, we can decipher the message using the private key that contains all the operation backward!

Conclusion

- Encryption plays an essential role in securing our private data.
- The examples use Linear Algebra to handle the Math, but there can be other methods!

Thank you very much for watching!

Reference

References :

https://searchsecurity.techtarget.com/definition/Advanced-Encryption-Standard Resources:

http://people.wku.edu/david.neal/307/Unit4/Crypt.pdf

http://web.csulb.edu/~jchang9/m247/m247_fa11_David_Diego_Alissa_Daniel.pdf