Review of Probability: Tabulation

Dr. Patrick Toche

Textbook:

James H. Stock and Mark W. Watson, *Introduction to Econometrics*, 4th Edition, Pearson.

Other references:

Joshua D. Angrist and Jörn-Steffen Pischke, *Mostly Harmless Econometrics: An Empiricist's Companion*, 1st Edition, Princeton University Press.

Jeffrey M. Wooldridge, *Introductory Econometrics: A Modern Approach*, 7th Edition, Cengage Learning.

The textbook comes with online resources and study guides. Other references will be given from time to time.

Probability & Unemployment

The following table gives the joint probability distribution between employment status and college graduation among those either employed or looking for work (unemployed) in the working-age U.S. population for September 2017.

1. Compute $\mathrm{E}(Y)$.
2. The unemployment rate is the fraction of the labor force that is unemployed. Show that the unemployment rate is given by $1 - \mathrm{E}(Y)$.
3. Calculate $\mathrm{E}(Y|X = 1)$ and $\mathrm{E}(Y|X = 0)$.
4. Calculate the unemployment rate for (i) college graduates and (ii) non-college graduates.
5. A randomly selected member of this population reports being unemployed. What is the probability that this worker is a college graduate? A non–college graduate?
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Probability & Unemployment

Employment & College Graduation (Population aged 25 and above, September 2017)

	Unemployed $Y = 0$	Employed $Y = 1$	Total
Non-College Graduates ($X = 0$)	0.026	0.576	0.602
College Graduates ($X = 1$)	0.009	0.389	0.398
Total	0.035	0.965	1.000

Probability & Unemployment

1. Compute $\mathrm{E}(Y)$.

$$\begin{aligned}\mathrm{E}[Y] &= 0 \times \Pr(Y = 0) + 1 \times \Pr(Y = 1) \\ &= 0 \times 0.035 \quad + 1 \times 0.965 \\ &= 0.965\end{aligned}$$

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2. Show that the unemployment rate is given by $1 - \mathrm{E}(Y)$.

The probability of unemployment is also the unemployment rate $u = \Pr(Y = 0)$.

$$\Pr(Y = 0) = 0.035$$

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Probability & Unemployment

3. Calculate $\mathrm{E}(Y|X=1)$ and $\mathrm{E}(Y|X=0)$.

$$\begin{aligned}\mathrm{E}(Y|X=1) &= 0 \times \Pr(Y=0|X=1) + 1 \times \Pr(Y=1|X=1) \\ &= \Pr(Y=1|X=1) \\ &= \frac{\Pr(Y=1, X=1)}{P(X=1)} = \frac{0.389}{0.398} = 0.977\end{aligned}$$

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Probability & Unemployment

4. Calculate the unemployment rate for (i) college graduates and (ii) non-college graduates.

(i) Unemployment rate for college graduates:

$$1 - \mathrm{E}[Y|X = 1] = 1 - 0.977 = 0.023$$

(ii) Unemployment rate for non-college graduates:

$$1 - \mathrm{E}[Y|X = 0] = 1 - 0.957 = 0.043$$

Probability & Unemployment

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Probability & Unemployment

5. What is the probability that this unemployed worker is a college graduate? A non–college graduate?

Probability this unemployed person is a college graduate:

$$\Pr[X = 1|Y = 0] = \frac{\Pr[X = 1, Y = 0]}{\Pr[Y = 0]} = \frac{0.009}{0.035} = 0.257$$

Probability this unemployed person is not a college graduate:

$$\Pr[X = 0|Y = 0] = \frac{\Pr[X = 0, Y = 0]}{\Pr[Y = 0]} = \frac{0.026}{0.035} = 0.743$$

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Probability & Unemployment

6. Are educational achievement and employment status independent? Explain.

Let's check the plausibility of independence for non-college graduates:

$$\Pr[X = 0, Y = 0] = 0.026$$
$$\Pr[X = 0] \times \Pr[Y = 0] = 0.602 \times 0.035 = 0.022$$

Since $0.022 \approx 0.026$, the hypothesis is still plausible.
Let's check it for college graduates:

$$\Pr[X = 1, Y = 0] = 0.009$$
$$\Pr[X = 1] \times \Pr[Y = 0] = 0.398 \times 0.035 = 0.014$$

Since $0.009 \ll 0.014$, the independence hypothesis is shaky.
Even more convincing evidence against independence:

$$\Pr[X = 0|Y = 0] = 0.743 \neq 0.602 = \Pr[X = 0]$$
$$\Pr[X = 1|Y = 0] = 0.257 \neq 0.398 = \Pr[X = 1]$$

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