Problems and Applications

Suppose a random sample of 200 20-year-old men is selected from a population and their heights and weights are recorded. A regression of weight on height yields:

$$\underset{}{\widehat{Weight}} = \underset{(2.15)}{-99.41} + \underset{(0.31)}{3.94}\ Height, \quad R^2 = 0.81, \quad SER = 10.2$$

where $Weight$ is measured in pounds and $Height$ is measured in inches. Two of your classmates differ in height by 1.5 inches. Construct a 99% confidence interval for the difference in their weights.

Test Scores

Construct a 99% confidence interval for the difference in their weights.

To construct a confidence interval for the difference in their weights, relate the expected difference in their weights to the observed difference in their height:

$$\Delta \widehat{Weight} = \Delta\, Height\, \hat{\beta}_1$$

The standard error for the expected difference is:

$$\text{SE}(\Delta \widehat{Weight}) = \Delta\, Height\ \text{SE}(\hat{\beta}_1)$$

A confidence interval for the difference in their weights may be constructed in a manner analogous to a confidence interval for the slope coefficient β_1:

$$\begin{aligned} &\hat{\beta}_1 \cdot \Delta\, Height \pm t_{\alpha/2} \cdot \text{SE}(\hat{\beta}_1) \cdot \Delta\, Height \\ = \quad & 3.94 \cdot 1.5 \pm 2.58 \cdot 0.31 \cdot 1.5 \end{aligned}$$

The critical t-value, $t_{\alpha/2} \approx 2.5758$, may be computed with `qnorm(1-0.01/2)`. A 99% confidence interval for the difference in their weights is therefore:

$$4.71 < \beta_1 < 7.11$$

Confidence Interval for a Mean Difference

In the regression of Y on X, construct a confidence interval for the difference ΔY.

Let u denote the error in the regression of Y on X:

$$Y = \beta_0 + \beta_1 X + u$$

Consider two observations X_i and X_j and let the difference be $\Delta X = X_i - X_j$. Likewise for ΔY. By construction, the data and parameters satisfy:

$$\begin{aligned} Y_i &= \beta_0 + \beta_1 X_i + u_i \\ Y_j &= \beta_0 + \beta_1 X_j + u_j \\ \implies \Delta Y &= \beta_1\, \Delta X + \Delta u \end{aligned}$$

It follows that a confidence interval for ΔY is a confidence interval for $\beta_1\, \Delta X$ and since ΔX is a constant, the confidence interval is just:

$$\left[\hat{\beta}_1 \pm t_{\alpha/2} \cdot \text{SE}(\hat{\beta}_1)\right] \cdot \Delta X$$