Central Limit Theorem

In a population, $\mu = 100$ and $\sigma^2 = 43$. Use the central limit theorem to answer the question:

a. In a random sample of size $n = 100$, find Pr$(\overline{Y} < 101)$.

$$\begin{aligned} \mathrm{P}\left[\overline{Y} < 101\right] &= \mathrm{P}\left[\frac{\overline{Y} - \mu}{\sigma/\sqrt{n}} < \frac{101 - \mu}{\sigma/\sqrt{n}}\right] \\ &= \mathrm{P}\left[Z < \frac{101 - 100}{\sqrt{43}/\sqrt{100}}\right] \\ &\approx \mathrm{P}[Z < 1.525] \\ &\approx 0.94 \end{aligned}$$

In R, you would compute the probability with `pnorm(1.525)`.

Central Limit Theorem

In a population, $\mu = 100$ and $\sigma^2 = 43$. Use the central limit theorem to answer the question:

b. In a random sample of size $n = 165$, find Pr$(\overline{Y} > 98)$.

$$\begin{aligned} \mathrm{P}\left[\overline{Y} > 98\right] &= 1 - \mathrm{P}\left[\overline{Y} < 98\right] \\ &= 1 - \mathrm{P}\left[\frac{\overline{Y} - \mu}{\sigma/\sqrt{n}} < \frac{98 - \mu}{\sigma/\sqrt{n}}\right] \\ &= 1 - \mathrm{P}\left[Z < \frac{98 - 100}{\sqrt{43}/\sqrt{165}}\right] \\ &\approx 1 - \mathrm{P}[Z < -3.92] \\ &\approx 0.99995 \\ &\approx 1 \end{aligned}$$

R code: `1-pnorm(-3.9178)`.

Central Limit Theorem

In a population, $\mu = 100$ and $\sigma^2 = 43$. Use the central limit theorem to answer the question:

c. In a random sample of size $n = 64$, find Pr$(101 < \overline{Y} < 103)$.

$$\begin{aligned} \mathrm{P}\left[101 < \overline{Y} < 103\right] &= \mathrm{P}\left[\frac{101 - \mu}{\sigma/\sqrt{n}} < \frac{\overline{Y} - \mu}{\sigma/\sqrt{n}} < \frac{103 - \mu}{\sigma/\sqrt{n}}\right] \\ &= \mathrm{P}\left[\frac{101 - 100}{\sqrt{43}/\sqrt{64}} < Z < \frac{103 - 100}{\sqrt{43}/\sqrt{64}}\right] \\ &\approx \mathrm{P}[1.220 < Z < 3.660] \\ &\approx 1 - 0.89 \\ &\approx 0.11 \end{aligned}$$

R code: `pnorm(3.660)-pnorm(1.220)`.