

Hypothesis Testing & Confidence Intervals: Weight & Height

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Textbook:

James H. Stock and Mark W. Watson, *Introduction to Econometrics*, 4th Edition, Pearson.

Other references:

Joshua D. Angrist and Jörn-Steffen Pischke, *Mostly Harmless Econometrics: An Empiricist's Companion*, 1st Edition, Princeton University Press.

Jeffrey M. Wooldridge, *Introductory Econometrics: A Modern Approach*, 7th Edition, Cengage Learning.

The textbook comes with online resources and study guides. Other references will be given from time to time.

Problems and Applications

Suppose a random sample of 200 20-year-old men is selected from a population and their heights and weights are recorded. A regression of weight on height yields:

$$\widehat{Weight} = -99.41 + 3.94 \text{ Height}, \quad R^2 = 0.81, \quad SER = 10.2$$

(2.15) (0.31)

where *Weight* is measured in pounds and *Height* is measured in inches. Two of your classmates differ in height by 1.5 inches. Construct a 99% confidence interval for the difference in their weights.

Confidence Interval for a Mean Difference

In the regression of Y on X , construct a confidence interval for the difference ΔY .

Let u denote the error in the regression of Y on X :

$$Y = \beta_0 + \beta_1 X + u$$

Consider two observations X_i and X_j and let the difference be $\Delta X = X_i - X_j$. Likewise for ΔY . By construction, the data and parameters satisfy:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

$$Y_j = \beta_0 + \beta_1 X_j + u_j$$

$$\implies \Delta Y = \beta_1 \Delta X + \Delta u$$

It follows that a confidence interval for ΔY is a confidence interval for $\beta_1 \Delta X$ and since ΔX is a constant, the confidence interval is just:

$$\left[\hat{\beta}_1 \pm t_{\alpha/2} \cdot \text{SE}(\hat{\beta}_1) \right] \cdot \Delta X$$

Test Scores

Construct a 99% confidence interval for the difference in their weights.

To construct a confidence interval for the difference in their weights, relate the expected difference in their weights to the observed difference in their height:

$$\widehat{\Delta Weight} = \Delta Height \hat{\beta}_1$$

The standard error for the expected difference is:

$$SE(\widehat{\Delta Weight}) = \Delta Height SE(\hat{\beta}_1)$$

A confidence interval for the difference in their weights may be constructed in a manner analogous to a confidence interval for the slope coefficient β_1 :

$$\begin{aligned} & \hat{\beta}_1 \cdot \Delta Height \pm t_{\alpha/2} \cdot SE(\hat{\beta}_1) \cdot \Delta Height \\ &= 3.94 \cdot 1.5 \pm 2.58 \cdot 0.31 \cdot 1.5 \end{aligned}$$

The critical t -value, $t_{\alpha/2} \approx 2.5758$, may be computed with `qnorm(1-0.01/2)`. A 99% confidence interval for the difference in their weights is therefore:

$$4.71 < \beta_1 < 7.11$$