Problems and Applications

Data were collected from a random sample of 220 home sales from a community in 2013. Let $Price$ denote the selling price (in $1000s), BDR denote the number of bedrooms, $Bath$ denote the number of bathrooms, $Hsize$ denote the size of the house (in square feet), $Lsize$ denote the lot size (in square feet), Age denote the age of the house (in years), and $Poor$ denote a binary variable that is equal to 1 if the condition of the house is reported as "poor." An estimated regression yields:

$$\widehat{Price} = 119.2 + 0.485\, BDR + 23.4\, Bath + 0.156\, Hsize$$
$$+\, 0.002\, Lsize + 0.090\, Age - 48.8\, Poor$$
$$\bar{R}^2 = 0.72, \quad SER = 41.5$$

a. Suppose a homeowner converts part of an existing family room in her house into a new bathroom. What is the expected increase in the value of the house?
b. Suppose a homeowner adds a new bathroom to her house, which increases the size of the house by 100 square feet. What is the expected increase in the value of the house?
c. What is the loss in value if a homeowner lets his house run down, so that its condition becomes "poor"?
d. Compute the R^2 for the regression.

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a. Suppose a homeowner converts part of an existing family room in her house into a new bathroom. What is the expected increase in the value of the house?

The number of bathrooms $Bath$ increases by 1. Since the other regressors are unchanged, only the slope coefficient on $Bath$ affects the expected increase in the house value:

$$\Delta\widehat{Price} = 23.4\,\Delta BATH$$
$$= 23.4 \cdot 1$$
$$= 23.4$$

The expected increase in house value is $\$23,400$.

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b. Suppose a homeowner adds a new bathroom to her house, which increases the size of the house by 100 square feet. What is the expected increase in the value of the house?

The number of bathrooms $Bath$ increases by 1 and the size of the house $Hsize$ increases by 100 square feet. The slope coefficients on $Bath$ and $Hsize$ both matter:

$$\Delta\widehat{Price} = 23.4\,\Delta BATH + 0.156\,\Delta Hsize$$
$$= 23.4 \cdot 1 + 0.156 \cdot 100$$
$$= 39.0$$

The expected increase in house value is $\$39,000$.

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c. What is the loss in value if a homeowner lets his house run down, so that its condition becomes "poor"?

The categorical variable $Poor$ changes from 0 to 1. Other regressors are unchanged.

$$\Delta\widehat{Price} = -48.8\,\Delta Poor$$
$$= -48.8 \cdot 1$$
$$= -48.8$$

The expected loss in house value is $\$48,800$.

d. Compute the R^2 for the regression.

The adjusted $\bar{R}^2$ is given, $\bar{R}^2 = 0.72$. The formula for the adjusted $\bar{R}^2$ in terms of the raw R^2 may be inverted to solve for R^2. The sample size is $n = 220$ and the number of regressors is $p = 6$.

$$\bar{R}^2 = 1 - \frac{n-1}{n-1-p}\left(1 - R^2\right)$$

$$\implies R^2 = 1 - \frac{n-1-p}{n-1}\left(1 - \bar{R}^2\right)$$

$$= 1 - \frac{220 - 1 - 6}{220 - 1}\left(1 - 0.72\right)$$

$$\approx 0.72767$$

The raw R^2 is about 0.73, not very different from the adjusted $\bar{R}^2$.