Art Of Problem Solving - AMC 10
Week 1

Patrick & James Toche

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Abstract

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1.

The number of real values of x that satisfy the equation

$$\left(2^{6x+3}\right)\left(4^{3x+6}\right) = 8^{4x+5}$$

is:

zero,	one,	two,	three,	greater than 3

2.

For which of the following values of k does the equation

$$\frac{x-1}{x-2} = \frac{x-k}{x-6}$$

have no solution for x?

$k=1,$	$k=2,$	$k=3,$	$k=4,$	$k=5$

3.

How many ordered triples (a, b, c) of nonzero real numbers have the property that each number is the product of the other two?

1,	2,	3,	4,	5

4.

Two non-zero real numbers, a and b, satisfy $ab = a - b$. Which of the following is a possible value of $\frac{a}{b} + \frac{b}{a} - ab$?

$-2,$	$-1/2,$	$1/3,$	$1/2,$	2

5.

If $a + 1 = b + 2 = c + 3 = d + 4 = a + b + c + d + 5$, then $a + b + c + d$ is

$-5,$	$-10/3,$	$-7/3,$	$5/3,$	5

6.

Let a, b, c, and d be real numbers with $|a - b| = 2$, $|b - c| = 3$, and $|c - d| = 4$. What is the sum of all possible values of $|a - d|$?

9,	12,	15,	18,	24

7.

If x and y are nonzero numbers such that $x = 1 + \frac{1}{y}$ and $y = 1 + \frac{1}{x}$, then y equals

$x-1,$	$1-x,$	$1+x,$	$-x,$	x

8.

A right triangle has perimeter 32 and area 20. What is the length of its hypotenuse?

$\frac{57}{4}$,	$\frac{59}{4}$,	$\frac{61}{4}$,	$\frac{63}{4}$,	$\frac{65}{4}$

9.

Let a, b, c be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. Then $a^2 - b^2 + c^2$ is

0,	1,	4,	7,	8

10.

Suppose that the number a satisfies the equation $4 = a + a^{-1}$. What is the value of $a^4 + a^{-4}$?

164,	172,	192,	194,	212