

Math Miscellani

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Abstract

This note reviews tips and tricks and selected problems to prepare for middle school math competitions. Written for personal use. Please report typos and errors over at <https://github.com/ptoché/Math/tree/master/mathcounts>.

Triangle

Calculate the height and the area of an equilateral triangle with side length a .

From Words to Equations

There were 9 adults and 11 children at the movie at 11:45a.m. By 11:50a.m., 7 more adults and 8 more children were at the movie. At 12:00p.m., there were 60 adults and children, and the ratio of adults to children was the same as at 11:45a.m. How many more children came to the movie between 11:50a.m. and 12:00p.m.?

Mean Cats

The mean number of cats living in each of the 50 apartments in a particular apartment building is 0.44 cats. A total of 32 apartments in the building are cat-free. What is the mean number of cats in the apartments that have at least one cat? Express your answer to the nearest tenth.

Mean Grades

The mean of Danielle's test scores is 85. If Danielle's lowest test score, which is 61 were to be discarded, the mean of her remaining test scores would be 88. How many tests did Danielle take?

Median in a Range

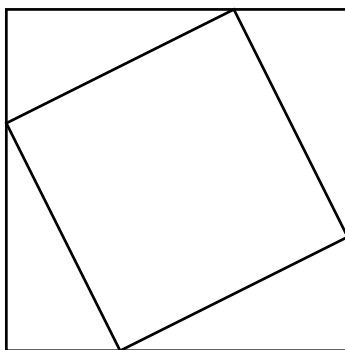
What is the median of the integers between 1 and 1000 that are divisible by 28?

Median in a Range

What is the median of the integers between 1 and 1000 that are *not* divisible by 28?

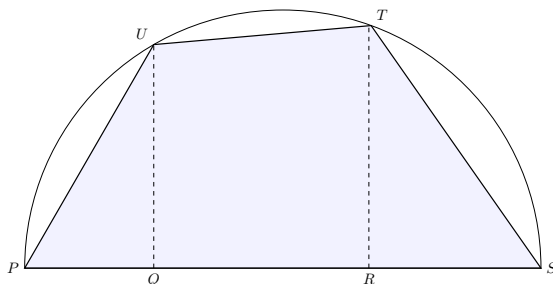
Rotated Square

The vertices of the smaller square in the figure are at trisection points of the sides of the larger square. What is the ratio of the area of the smaller square to the area of the larger square? Express your answer as a common fraction.



Area of a Quadrilateral

Quadrilateral $PSTU$ is inscribed in semicircle O , as shown, with $PQ = 3$ units, $QR = 5$ units and $RS = 4$ units. What is the area of quadrilateral $PSTU$? Express your answer as a decimal to the nearest tenth.



Mystery Integer

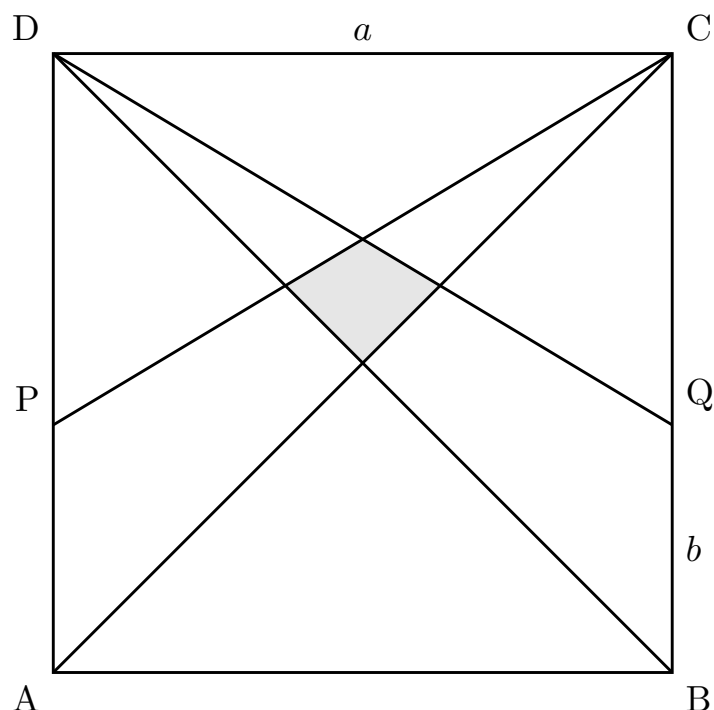
What positive four-digit integer has its thousands and hundreds digits add up to the tens digit, its hundreds and tens digits add up to its ones digit and its tens and ones digits add up to the two-digit number formed by the thousands and hundreds digits?

Mystery Sum

If $2015 = 101a + 19b$, for positive integers a and b , what is the value of $a + b$?

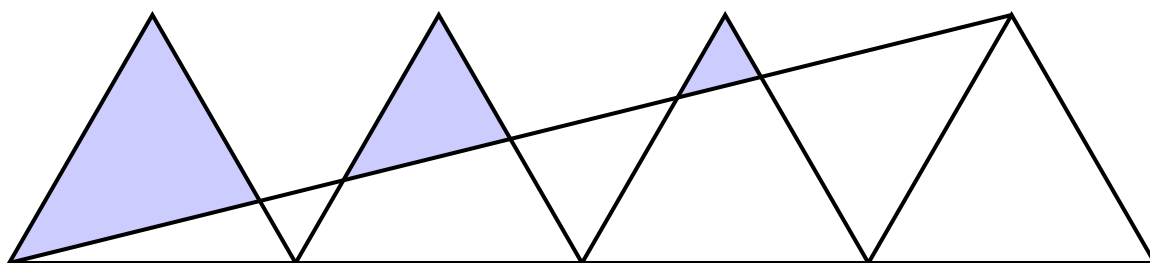
Based On 2021 Chapter Invitational Sprint Q29.

Square $ABCD$, shown here, has side length a units. Points P and Q are located on sides AD and BC , respectively, with $AP = BQ = b$ units. Triangles ACP and BDQ overlap in the square to form the shaded quadrilateral. What is the area of the shaded quadrilateral? Express your answer as a common fraction.



Four Equilateral Triangles

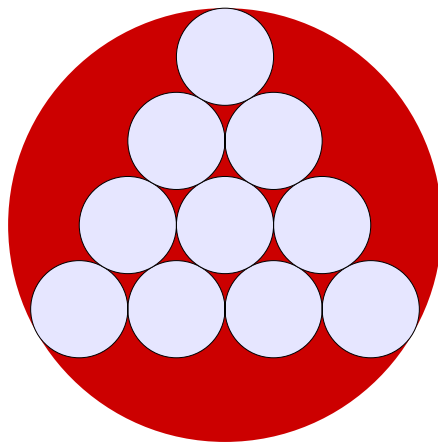
If a is the side length of one of the four equilateral triangles, calculate the shaded area.



A Circle Packing Problem

The problem presented here was originally proposed by Batsui Mitsunao, a fifteen-year-old boy, and written on a tablet hung in 1812 at the Nishihirokami Hachiman shrine in Izumi city, Toyama prefecture, Japan (Fukagawa Hidetoshi and Tony Rothman, *Sacred Mathematics: Japanese Temple Geometry*, Princeton University Press, 2008).

Several circles of radius r are packed to form a pyramid, using n circles along each side, as shown in the figure below in the special case $n = 4$. A larger circle circumscribes the pyramid. Express the radius of the larger circle in terms of r and n . Express the ratio of the areas of the n smaller circles to the area of the larger circle in terms of n . Calculate the limit of the ratio as $n \rightarrow \infty$.



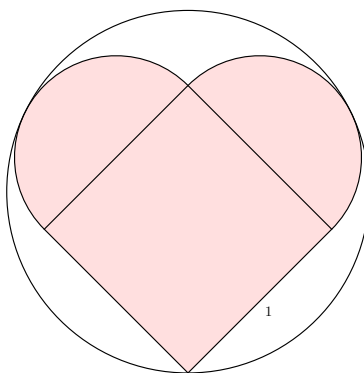
A Hidden Gold Nugget

Solve for the real values of x in the equation:

$$\left(x - \frac{1}{x}\right)^{\frac{1}{2}} + \left(1 - \frac{1}{x}\right)^{\frac{1}{2}} = x$$

Radius of a Heart

Calculate the radius of the circle the heart is inscribed in.



Quadratic Equations

The sum of a number x and its reciprocal equals $-\frac{17}{4}$. What is the sum of all possible values of x ? Express your answer as a common fraction.

Quadratic Forms: The Vertex

Consider the quadratic equation with real coefficients a , b , c :

$$ax^2 + bx + c = 0$$

Find the x - and y -coordinates of the vertex. Under what condition is the vertex a minimum?

Quadratic Forms: The Vertex Form

Consider the quadratic function

$$x^2 + 2x + 3$$

Find the graph's turning point.

Quadratic Forms: The Vertex Form

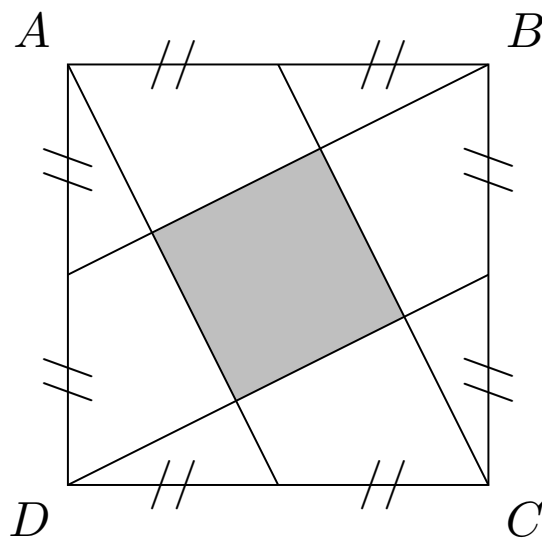
Consider the quadratic function

$$-4x^2 + 2x + 3$$

Find the graph's turning point.

Square inscribed inside a square

The shaded square region is created by connecting each vertex to a midpoint. What fraction of square $ABCD$ is shaded?

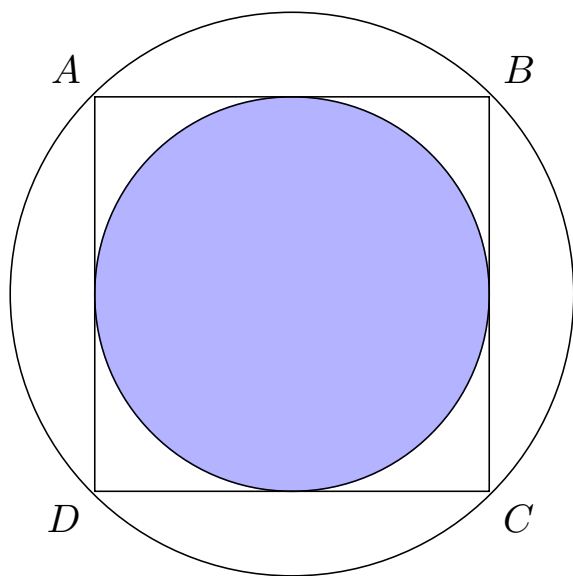
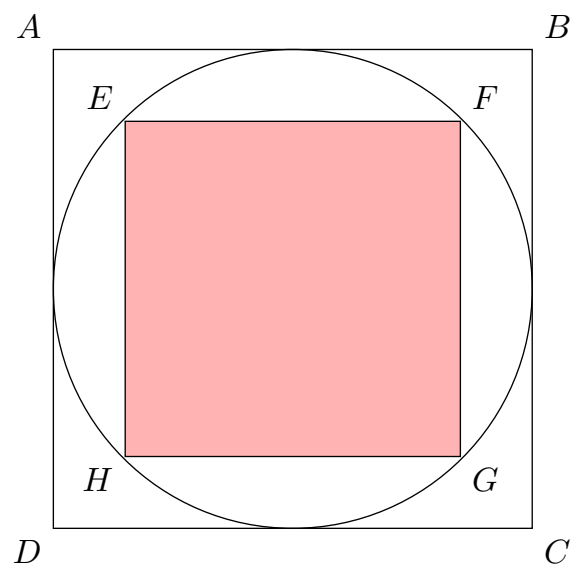


Inscribed Squares and Circles

Calculate the shaded area as a percentage of the area of the largest geometric figure.

Annual Coincidences

Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren?



Mystery Pattern

Can you find the pattern?

