# 2021 AMC 10A Problems/Problem 25

## Contents



## Problem

How many ways are there to place $3$ indistinguishable red chips, $3$ indistinguishable blue chips, and $3$ indistinguishable green chips in the squares of a $3 \times 3$ grid so that no two chips of the same color are directly adjacent to each other, either vertically or horizontally?

$\textbf{(A)}\ 12 \qquad\textbf{(B)}\ 18 \qquad\textbf{(C)}\ 24 \qquad\textbf{(D)}\ 30 \qquad\textbf{(E)}\ 36$

## Solution 1 (Casework on the Center's Color Chip's Configurations)

Call the different colors A,B,C. There are $3! = 6$ ways to rearrange these colors to these three letters, so $6$ must be multiplied after the letters are permuted in the grid. WLOG assume that A is in the center.

$$\begin{matrix} ? & ? & ? \\ ? & A & ? \\ ? & ? & ? \end{matrix}$$

In this configuration, there are two cases, either all the A's lie on the same diagonal:

$$\begin{matrix} ? & ? & A \\ ? & A & ? \\ A & ? & ? \end{matrix}$$

or all the other two A's are on adjacent corners:

$$\begin{matrix} A & ? & A \\ ? & A & ? \\ ? & ? & ? \end{matrix}$$

In the first case there are two ways to order them since there are two diagonals, and in the second case there are four ways to order them since there are four pairs of adjacent corners.

In each case there is only one way to put the three B's and the three C's as shown in the diagrams.

$$\begin{matrix} C & B & A \\ B & A & C \\ A & C & B \end{matrix}$$

$$\begin{matrix} A & B & A \\ C & A & C \end{matrix}$$

$$\mathrm{B} \quad \mathrm{C} \quad \mathrm{B}$$

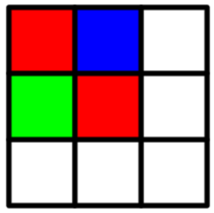
This means that there are $4+2=6$ ways to arrange A,B, and C in the grid, and there are $6$ ways to rearrange the colors. Therefore, there are $6\cdot 6=36$ ways in total, which is $\boxed{\textbf{(E) } 36}$.

-happykeeper

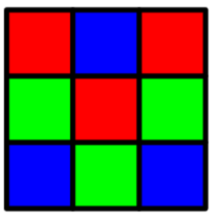
## Solution 2 (Casework on the Top-Center and Center-Left Chips)

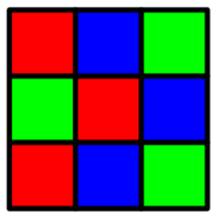
Without the loss of generality, we fix the top-left square with a red chip. We apply casework to its two adjacent chips:

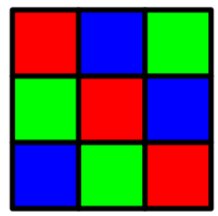
**<u>Case (1): The top-center and center-left chips have different colors.</u>**



There are three subcases for Case (1):

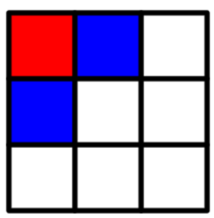




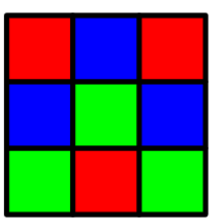


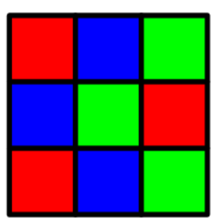
As there are $3!=6$ permutations of the three colors, each subcase has $6$ ways. So, Case (1) has $3\cdot 6=18$ ways in total.

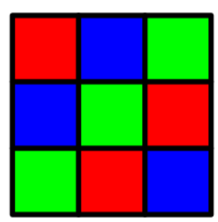
**<u>Case (2): The top-center and center-left chips have the same color.</u>**



There are three subcases for Case (2):







As there are $3!=6$ permutations of the three colors, each subcase has $6$ ways. So, Case (2) has $3\cdot 6=18$ ways in total.

**<u>Answer</u>**

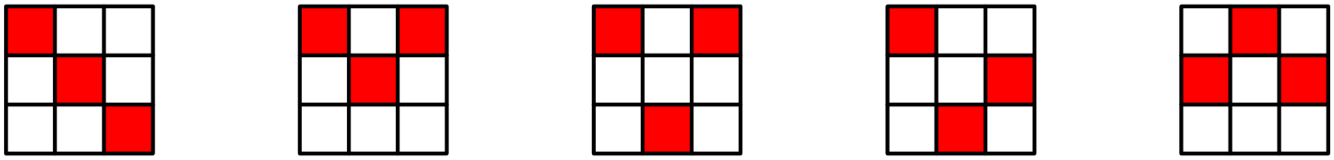
Together, the answer is $18+18=\boxed{\textbf{(E) } 36}$.

~MRENTHUSIASM

## Solution 3 (Casework on the Red Chips' Configurations)

We consider all possible configurations of the red chips for which rotations matter:

Rotational Symmetry



2 Configurations 4 Configurations 4 Configurations 4 Configurations 4 Configurations

As there are $2! = 2$ permutations of blue and green for each configuration, the answer is $2 \cdot (2+4+4+4+4) = \boxed{\textbf{(E) } 36}$.

~MRENTHUSIASM (credit given to FlameKhoEmberish)

## Solution 4 (Casework and Symmetry)

$$(1) \quad \begin{array}{ccc} \text{R} & \text{G} & ? \\ \text{B} & \text{R} & ? \\ ? & ? & ? \end{array} \implies \begin{array}{ccc} \text{R} & \text{G} & \text{B} \\ \text{B} & \text{R} & \text{G} \\ \text{R} & \text{G} & \text{B} \end{array} \implies \begin{array}{ccc} \text{R} & \text{G} & \text{R} \\ \text{B} & \text{R} & \text{B} \\ \text{G} & \text{B} & \text{G} \end{array} \qquad 3 \cdot 2 \cdot 2 = 12$$

There are $3$ choices for $R$, $2$ choices for $G$. $R$ on the down left corner can be switched with $B$ on the upper right corner.

$$(2) \quad \begin{array}{ccc} \text{R} & \text{G} & ? \\ \text{B} & \text{R} & ? \\ ? & ? & ? \end{array} \implies \begin{array}{ccc} \text{R} & \text{G} & \text{B} \\ \text{B} & \text{R} & \text{G} \\ \text{G} & \text{B} & \text{R} \end{array} \qquad 3 \cdot 2 = 6$$

There are $3$ choices for $R$, $2$ choices for $G$.

$$(3) \quad \begin{array}{ccc} \text{G} & \text{R} & ? \\ \text{R} & \text{B} & ? \\ ? & ? & ? \end{array} \implies \begin{array}{ccc} \text{G} & \text{R} & \text{B} \\ \text{R} & \text{B} & \text{G} \\ \text{G} & \text{R} & \text{B} \end{array} \implies \begin{array}{ccc} \text{G} & \text{R} & \text{G} \\ \text{R} & \text{B} & \text{R} \\ \text{B} & \text{G} & \text{B} \end{array} \qquad 3 \cdot 2 \cdot 2 = 12$$

Note that $(3)$ is a $180^\circ$ rotation of $(1)$.

$$(4) \quad \begin{array}{ccc} \text{G} & \text{R} & ? \\ \text{R} & \text{B} & ? \\ ? & ? & ? \end{array} \implies \begin{array}{ccc} \text{G} & \text{R} & \text{B} \\ \text{R} & \text{B} & \text{G} \\ \text{B} & \text{G} & \text{R} \end{array} \qquad 3 \cdot 2 = 6$$

Note that $(4)$ is a $90^\circ$ rotation of $(2)$.

Therefore, the answer is $2 \cdot (12+6) = \boxed{\textbf{(E) } 36}$.

~isabelchen (https://artofproblemsolving.com/wiki/index.php/User:Isabelchen)

## Solution 5 (Casework and Derangements)

**Case (1)**: We have a permutation of R, B, and G as all of the rows. There are $3!$ ways to rearrange these three colors. After finishing the first row, we move onto the second. Notice how the second row must be a derangement of the first one. By the derangement formula, $\frac{3!}{e} \approx 2$, so there are two possible permutations of the second row. (Note: You could have also found the number of derangements of PIE). Finally, there are $2$ possible permutations for the last row. Thus, there are $3! \cdot 2 \cdot 2 = 24$ possibilities.

**Case (2)**: All of the rows have two chips that are the same color and one that is different. There are obviously $3$ possible configurations for the first row, $2$ for the second, and $2$ for the third. Thus, there are $3 \cdot 2 \cdot 2 = 12$ possibilities.

Therefore, our answer is $24 + 12 = \boxed{\textbf{(E) } 36}$.

~michaelchang1

## Video Solution (Easiest)

https://www.youtube.com/watch?v=UPUrYN1YuVA ~ MathEx

## Video Solution by OmegaLearn (Symmetry, Casework, and Reflections/Rotations)

https://youtu.be/wKJ9ppl-8Ew ~ pi_is_3.14

## Video Solution by The Power of Logic

https://www.youtube.com/watch?v=TEsHuvXA9Ic

## Video Solution by MRENTHUSIASM (English & Chinese)

https://www.youtube.com/watch?v=_2hCBZHb3SA

~MRENTHUSIASM

## See Also


| **2021 AMC 10A** (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13)) | |
|---|---|
| Preceded by<br>**Problem 24** | Followed by<br>**Last Problem** |
| 1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25 | |
| **All AMC 10 Problems and Solutions** | |