2021 Fall AMC 10B Problems/Problem 6

Contents

Problem

The least positive integer with exactly 2021 distinct positive divisors can be written in the form $m \cdot 6^k$, where m and k are integers and 6 is not a divisor of m. What is $m + k$?

$(\textbf{A})\ 47 \qquad (\textbf{B})\ 58 \qquad (\textbf{C})\ 59 \qquad (\textbf{D})\ 88 \qquad (\textbf{E})\ 90$

Solution 1

Let this positive integer be written as $p_1^{e_1} \cdot p_2^{e_2}$. The number of factors of this number is therefore $(e_1 + 1) \cdot (e_2 + 1)$, and this must equal 2021. The prime factorization of 2021 is $43 \cdot 47$, so $e_1 + 1 = 43 \implies e_1 = 42$ and $e_2 + 1 = 47 \implies e_2 = 46$. To minimize this integer, we set $p_1 = 3$ and $p_2 = 2$. Then this integer is $3^{42} \cdot 2^{46} = 2^4 \cdot 2^{42} \cdot 3^{42} = 16 \cdot 6^{42}$. Now $m = 16$ and $k = 42$ so $m + k = 16 + 42 = 58 = \boxed{B}$

~KingRavi

Solution 2

Recall that 6^k can be written as $2^k \cdot 3^k$. Since we want the integer to have 2021 divisors, we must have it in the form $p_1^{42} \cdot p_2^{46}$, where p_1 and p_2 are prime numbers. Therefore, we want p_1 to be 3 and p_2 to be 2. To make up the remaining 2^4, we multiply $2^{42} \cdot 3^{42}$ by m, which is 2^4 which is 16. Therefore, we have $42 + 16 = \boxed{(B) 58}$

~Arcticturn

Solution 3

If a number has prime factorization $p_1^{k_1} p_2^{k_2} \cdots p_m^{k_m}$, then the number of distinct positive divisors of this number is $(k_1 + 1)(k_2 + 1) \cdots (k_m + 1)$.

We have $2021 = 43 \cdot 47$. Hence, if a number N has 2021 distinct positive divisors, then N takes one of the following forms: p_1^{2020}, $p_1^{42} p_2^{46}$.

Therefore, the smallest N is $3^{42} 2^{46} = 2^4 \cdot 6^{42} = 16 \cdot 6^{42}$.

Therefore, the answer is $\boxed{\textbf{(B) } 58}$.

~Steven Chen (www.professorchenedu.com)

Video Solution by Interstigation

https://youtu.be/p9_RH4s-kBA?t=530

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Preceded by **Problem 5**	Followed by **Problem 7**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
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