Art Of Problem Solving - AMC 10
Week 9

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Abstract

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1.

Henry's Hamburger Heaven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties, and any collection of condiments. How many different kinds of hamburgers can be ordered?

(A) 24 (B) 256 (C) 768 (D) $40,320$ (E) $120,960$

2.

At an inter-species dance party, each cat danced with exactly three dogs and each dog danced with exactly two cats. No one danced with anyone of their own species, and twelve cats attended the party. How many dogs attended the party?

(A) 8 (B) 12 (C) 16 (D) 18 (E) 24

3.

A restaurant offers three desserts, and exactly twice as many appetizers as main courses. A dinner consists of an appetizer, a main course, and a dessert. What is the least number of main courses that the restaurant should offer so that a customer could have a different dinner each night in the year 2003?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

4.

A license plate in a certain state consists of 4 digits, not necessarily distinct, and 2 letters, also not necessarily distinct. These six characters may appear in any order, except that the two letters must appear next to each other. How many distinct license plates are possible?

(A) $10^4 \cdot 26^2$ (B) $10^3 \cdot 26^3$ (C) $5 \cdot 10^4 \cdot 26^2$ (D) $10^2 \cdot 26^4$ (E) $5 \cdot 10^3 \cdot 26^3$

5.

A set of 25 square blocks is arranged into a 5×5 square. How many different combinations of 3 blocks can be selected from that set so that no two are in the same row or column?

(A) 100 (B) 125 (C) 600 (D) 2300 (E) 3600

6.

Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

(A) 22 (B) 25 (C) 27 (D) 28 (E) 729

7.

Seven distinct pieces of candy are to be distributed among three bags. The red bag and the blue bag must each receive at least one piece of candy; the white bag may remain empty. How many arrangements are possible?

(A) 1930 (B) 1931 (C) 1932 (D) 1933 (E) 1934

8.

Two subsets of the set $S = \{a, b, c, d, e\}$ are to be chosen so that their union is S and their intersection contains exactly two elements. In how many ways can this be done, assuming that the order in which the subsets are chosen does not matter?

(A) 20 (B) 40 (C) 60 (D) 160 (E) 320

9.

The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?

(A) 18 (B) 24 (C) 36 (D) 42 (E) 60

10.

A subset B of the set of integers from 1 to 100, inclusive, has the property that no two elements of B sum to 125. What is the maximum possible number of elements in B?

(A) 50 (B) 51 (C) 62 (D) 65 (E) 68