Ask Math Anything

Daily Challenge with Po-Shen Loh

17 June 2020

Abstract

Professor Po-Shen Loh solves problems on his YouTube channel. A selection for practice.
Reference: Ask Math Anything - Daily Challenge with Po-Shen Loh
(I didn't realize the streams are deleted after a week, so unfortunately I no longer have access to the video to complete these notes...)

Squaring Large Numbers

Calculate:

$$(111,111,111)^2$$

Try with smaller numbers, say $(111)^2$:

$$\begin{array}{r} \,1\,1\,1 \\ \times\,1\,1\,1 \\ \hline \,1\,1\,1 \\ \,1\,1\,1 \\ 1\,1\,1 \\ \hline 1\,2\,3\,2\,1 \end{array}$$

and $(1,111)^2$:

$$\begin{array}{r} \,1\,1\,1\,1 \\ \times\,1\,1\,1\,1 \\ \hline \,1\,1\,1\,1 \\ \,1\,1\,1\,1 \\ \,1\,1\,1\,1 \\ 1\,1\,1\,1 \\ \hline 1\,2\,3\,4\,3\,2\,1 \end{array}$$

See the pattern? Yes, the answer is:

$$(111,111,111)^2 = 12,345,678,987,654,321$$

Calculate the even larger 11-digit square of ones:

$$(11,111,111,111)^2$$

We could write it out:

$$
\begin{array}{r}
\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1 \\
\times\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1 \\
\hline
\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1 \\
\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\, \\
1\,1\,1\,1\,1\,1\,1\,1\,1\,1\, \\
1\,1\,1\,1\,1\,1\,1\,1\,1\,1\, \\
1\,1\,1\,1\,1\,1\,1\,1\,1\,1\, \\
1\,1\,1\,1\,1\,1\,1\,1\,1\,1\, \\
1\,1\,1\,1\,1\,1\,1\,1\,1\,1\, \\
1\,1\,1\,1\,1\,1\,1\,1\,1\,1\, \\
1\,1\,1\,1\,1\,1\,1\,1\,1\,1\, \\
1\,1\,1\,1\,1\,1\,1\,1\,1\,1\, \\
1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1 \\
\hline
1\,2\,3\,4\,5\,6\,7\,9\,0\,1\,2\,0\,9\,8\,7\,6\,5\,4\,3\,2\,1
\end{array}
$$

but that's a little insane.

Instead, we can exploit the pattern we noticed:

$$
\begin{aligned}
(11, 111, 111, 111)^2 &= 123456789 \\
&\qquad\qquad\qquad\qquad\quad 10 \\
&\qquad\qquad\qquad\qquad\quad\ 11 \\
&\qquad\qquad\qquad\qquad\ \ 0987654321 \\
&= 12345679012 0987654321
\end{aligned}
$$

The digits that are written within the same column are carried from the