Russian School of Math: Lesson 10

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Abstract

This note reviews a small number of problems from the Russian School of Math test. Written for personal use.

1

For certain real numbers a, b, and c, the polynomial $g(x) = x^3 + ax^2 + x + 10$ has three distinct roots and each root of $g(x)$ is also a root of the polynomial $f(x) = x^4 + x^3 + bx^2 + 100x + c$. Calculate $f(1)$.

Solution

2

Consider the polynomials $P(x) = x^6 - x^5 - x^3 - x^2 - x$ and $Q(x) = x^4 - x^3 - x^2 - 1$. Given that z_1, z_2, z_3, and z_4 are the roots of $Q(x) = 0$, find $P(z_1) + P(z_2) + P(z_3) + P(z_4)$.

Solution

3

Find the smallest positive integer n with the property that the polynomial $x^4 - nx + 63$ can be written as the product of two non-constant polynomials with integer coefficients.

Solution

4

For some integer m, the polynomial $x^3 - 2011x + m$ has three integer roots a, b, and c. Find $|a| + |b| + |c|$.

Solution