2021 AMC 12A Problems/Problem 3

The following problem is from both the 2021 AMC 10A #3 and 2021 AMC 12A #3, so both problems redirect to this page.

Contents

Problem

The sum of two natural numbers is $17{,}402$. One of the two numbers is divisible by 10. If the units digit of that number is erased, the other number is obtained. What is the difference of these two numbers?

$\textbf{(A)}\ 10{,}272 \qquad \textbf{(B)}\ 11{,}700 \qquad \textbf{(C)}\ 13{,}362 \qquad \textbf{(D)}\ 14{,}238 \qquad \textbf{(E)}\ 15{,}426$

Solution 1 (Algebra)

The units digit of a multiple of 10 will always be 0. We add a 0 whenever we multiply by 10. So, removing the units digit is equal to dividing by 10.

Let the smaller number (the one we get after removing the units digit) be a. This means the bigger number would be $10a$.

We know the sum is $10a + a = 11a$ so $11a = 17402$. So $a = 1582$. The difference is $10a - a = 9a$. So, the answer is $9(1582) = \boxed{\textbf{(D)}\ 14{,}238}$.

~abhinavg0627

Solution 2 (Arithmetic)

Since the ones place of a multiple of 10 is 0, this implies the other integer has to end with a 2 since both integers sum up to a number that ends with a 2. Thus, the ones place of the difference has to be $10 - 2 = 8$, and the only answer choice that ends with an 8 is $\boxed{\textbf{(D)}\ 14{,}238}$.

Another quick solution is to realize that the sum is represents a number n added to $10n$. The difference is $9n$, which is $\frac{9}{11}$ of the given sum.

~CoolJupiter 2021

Solution 3 (Vertical Addition and Logic)

Let the larger number be $\underline{AB,CD0}$. It follows that the smaller number is $\underline{A,BCD}$. Adding vertically, we have

$$\begin{array}{cccccc} & \bar{A} & \bar{B} & \bar{C} & \bar{D} & \bar{0} \\ + & & A & B & C & D \\ \hline & 1 & 7 & 4 & 0 & 2 \end{array}$$

Working from right to left, we get

$$D = 2 \implies C = 8 \implies B = 5 \implies A = 1.$$

The larger number is $15{,}820$ and the smaller number is $1{,}582$. Their difference is $15{,}820 - 1{,}582 = \boxed{\textbf{(D)}\ 14{,}238}$.

~MRENTHUSIASM

Video Solutions

Video Solution (Simple)

https://youtu.be/SEp9flDYm2c

~ Education, the study Of Everything

Video Solution by North America Math Contest Go Go Go

https://www.youtube.com/watch?v=hMqA8i8a2SQ&list=PLexHyfQ8DMuKqltG3cHT7Di4jhVl6L4YJ&index=3

Video Solution by Aaron He

https://www.youtube.com/watch?v=xTGDKBthWsw&t=1m28s

Video Solution by Punxsutawney Phil

https://youtube.com/watch?v=MUHja8TpKGw&t=143s

Video Solution by Hawk Math

https://www.youtube.com/watch?v=P5al76DxyHY

Video Solution (Using Algebra and Meta-solving)

https://youtu.be/d2musztzDjw

-pi_is_3.14

Video Solution by WhyMath

https://youtu.be/VpYmQEKcBpA

~savannahsolver

Video Solution by TheBeautyofMath

https://youtu.be/50CThrk3RcM?t=107 (for AMC 10A)

Video Solution by IceMatrix

https://youtu.be/rEWS75W0Q54?t=198 (for AMC 12A)

~IceMatrix

Video Solution (Problems 1-3)

https://youtu.be/CupJpUzKPB0

~MathWithPi

Video Solution by The Learning Royal

https://youtu.be/slVBYmcDMOI

See also

2021 AMC 10A (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
Preceded by **Problem 2**	Followed by **Problem 4**
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All AMC 10 Problems and Solutions	

2021 AMC 12A (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
Preceded by **Problem 2**	Followed by **Problem 4**
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All AMC 12 Problems and Solutions	