Russian School of Math: Lesson 1

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Abstract

This note reviews a small number of problems from the Russian School of Math test. Written for personal use.

1

Find the number of solution-pairs in positive integers of the equation

$$3x + 4y = 93$$

Solution

Solution: 7

2

Solve the following DE

$$\frac{xy}{x+y} = 2017$$

Solution

$$(4034, 4034), \quad (2018, 4070306) \quad (2016, -4066272) \quad (-4066272, 2016) \quad (4070306, 2018)$$

3

Find the sum of all positive integers for which $n^2 - 19n + 99$ is a perfect square.

Solution

Solution: 38

4

Solve in integers:

$$3x + 3y + 5z = 1$$
$$4x + 5y - 2z = 4$$

Solution

Solution:

$$x = 8 - 31n, \quad y = -6 + 26n \quad z = -1 + 3n \quad z \in \mathbb{Z}$$

5

Prove that the equation $20x - 19y = 2019$ has no solution, where x and y are perfect squares of integers.

Solution

6

Solve in integers using factorization

$$x + y = xy$$

Solution

7

Solve in integers using factorization

$$y^3 - x^3 = 91$$

Solution

8

Solve in integers using factorization

$$xy = x + y + 3$$

Solution

9

Solve in integers using factorization

$$x^2 - y^2 = 2019$$

Solution

10

Solve in integers using factorization

$$2(x + 7)(y + 1) = 3xy$$

Solution

11

Solve in integers using factorization

$$x^3 - xy - 7x + 2y + 23 = 0$$

Solution