Russian School of Math: Lesson 4

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Abstract

This note reviews a small number of problems from the Russian School of Math test. Written for personal use.

1

Cagney can frost a cupcake every 20 seconds and Lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes?

(a) 20

(b) 25

(c) 30

(d) 35

(e) 40

Solution

> Every 60 seconds, Cagney and Lacey together frost $3+2=5$ cupcakes. In 5 minutes, therefore, they frost 25 cupcakes.

2

In $\triangle ABC$, we have $AC = BC = 7$ and $AB = 2$. Suppose that D is a point on line AB such that B lies between A and D and $CD = 8$. What is BD?

(a) 1

(b) $\sqrt{3}$

(c) 3

(d) 4

(e) $4\sqrt{2}$

Solution

> Solution: 3.

3

Let x be a real number such that $\sec x - \tan x = 2$. Then $\sec x + \tan x =?$

(a) 0.3

(b) 0.4

(c) 0.5

(d) 0.6

(e) 0.7

Solution

> Solution: 0.5.

4

Find positive consecutive integers starting with a whose average is b. What is the average of 5 consecutive integers that start with b?

(a) $a+2$

(b) $a+3$

(c) $a+4$

(d) $a+5$

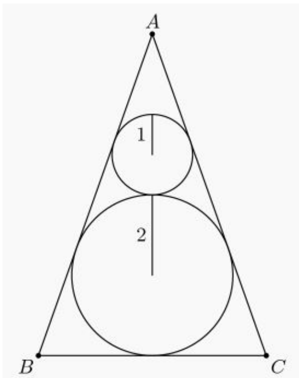
(e) $a+6$

Solution

> Solution: $a+4$.

5

A circle of radius 1 is tangent to a circle of radius 2. The sides of $\triangle ABC$ are tangent to the circles as shown, and the sides AB and AC are congruent. What is the area of $\triangle ABC$?



(a) $\frac{35}{2}$

(b) $16\sqrt{2}$

(c) $\frac{64}{3}$

(d) $18\sqrt{2}$

(e) 24

Solution

> Solution: $16\sqrt{2}$.

6

What is the value of the expression

$$\frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \ldots + \frac{1}{\log_{100} 100!}$$

(a) 0.1

(b) 1

(c) 10

(d) 100

(e) 1000

Solution

Solution: 1.

7

A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers chose spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park?

(a) $\frac{11}{20}$

(b) $\frac{4}{7}$

(c) $\frac{81}{140}$

(d) $\frac{17}{28}$

(e) $\frac{19}{28}$

Solution

The 12 cars leave 4 empty spaces. In the worst-case scenario, none of the 4 empty spaces are next to each other and Auntie Em will not be able to park; otherwise she will be able to park.
Let a star denote an occupied space and a bar denote an empty space. Auntie Em can park if she finds an arrangement like the following:

$$||*|*|**********$$

These are "favorable" arrangements. Auntie Em will not be able to park if she finds:

$$|*|*|*|*********$$

These are "unfavorable" arrangements.
Consider the "unfavorable" arrangements. The number of arrangements of 1 empty space after the 12 cars are parked is 13, because there are 11 potential "spaces" strictly between 12 cars and 2 more at either end. Once the first empty space is selected, there remain one less potential "space", since we cannot place this second space next to the existing empty space, that is the number of arrangements is 12 for the second space; and 11 for the third space; and 10 for the fourth and last space. Since the empty spaces can be permuted without affecting the parking possibilities, we correct for the overcounting by dividing by the number of ways to select 4 empty spaces, or $4 \cdot 3 \cdot 2$. The number of non-ordered unfavorable arrangements is therefore:

$$\frac{13 \cdot 12 \cdot 11 \cdot 10}{4 \cdot 3 \cdot 2} = \frac{13!}{9!4!} = \binom{13}{4} = 13 \cdot 11 \cdot 5$$

The total number of arrangements, including favorable and unfavorable arrangements, is:

$$\binom{16}{4} = \frac{16!}{4!12!} = \frac{16 \cdot 15 \cdot 14 \cdot 13}{4 \cdot 3 \cdot 2} = 2 \cdot 5 \cdot 14 \cdot 13$$

The fraction of unfavorable arrangements to the total is:

$$\frac{\binom{13}{4}}{\binom{16}{4}} = \frac{13 \cdot 11 \cdot 5}{2 \cdot 5 \cdot 14 \cdot 13} = \frac{11}{28}$$

Solution: $1 - \frac{11}{28} = \frac{17}{28} \approx 0.61$.

8

Let $ABCD$ be a trapezoid with $AB||CD$, $AB = 11$, $BC = 5$, $CD = 19$, and $DA = 7$. Bisectors of $\angle A$ and $\angle D$ meet at P, and bisectors of $\angle B$ and $\angle C$ meet at Q. What is the area of hexagon $ABQCDP$?

(a) $24\sqrt{3}$

(b) $28\sqrt{3}$

(c) $30\sqrt{3}$

(d) $35\sqrt{3}$

(e) $36\sqrt{3}$

Solution

Solution: $30\sqrt{3}$.