Russian School of Math: Lesson 7

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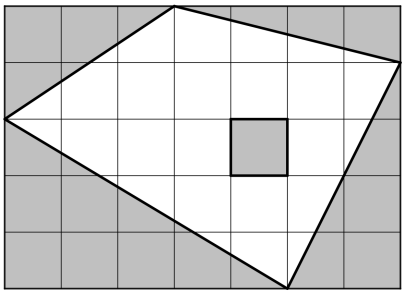
Revised: October 27, 2024

Abstract

This note reviews a small number of problems from the Russian School of Math test. Written for personal use.

1

Find the difference of the area of the shaded parts and the area of the white part, if the side length of each square is 1.



(a) -2
(b) -1
(c) 0
(d) 1
(e) 2

2

Triangle ABC has side lengths $AB = 5$, $BC = 6$, and $AC = 7$. Two bugs start simultaneously from A and crawl along the sides of the triangle in opposite directions at the same speed. They meet at point D. What is BD?

(a) 1

(b) 2

(c) 4

(d) 5

(e) 8

3

Define $a@b = ab - b^2$ and $a\#b = a + b - ab^2$. Calculate $\dfrac{6@2}{6\#2}$.

(a) -1

(b) $-\dfrac{1}{2}$

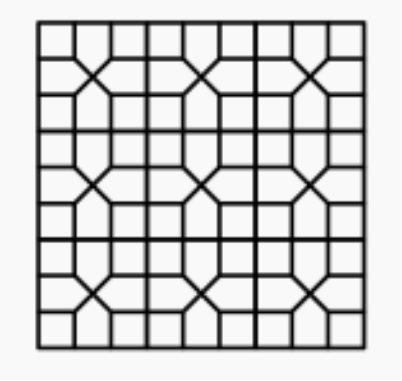
(c) $\dfrac{1}{8}$

(d) $\dfrac{1}{4}$

(e) $\dfrac{1}{2}$

4

The plane is tiled by congruent squares and congruent pentagones as indicated. The percent of the plane that is enclosed by the pentagons is closest to:



(a) 54
(b) 56
(c) 58
(d) 60
(e) 62

5

If m and b are real numbers and $mb > 0$, then the line whose equation is $y = mx + b$ cannot contain the point:

(a) $(0, 1997)$

(b) $(1997, 0)$

(c) $(19, 97)$

(d) $(19, -97)$

(e) $(0, -1997)$

6

Points A, B, C, and D lie on a line, in that order, with $AB = CD$ and $BC = 12$. Point E is not on the line and $BE = CE = 10$. The perimeter of $\triangle AED$ is twice the perimeter of $\triangle BEC$. Find AB.

(a) $\frac{17}{2}$

(b) 9

(c) $\frac{19}{2}$

(d) 10

(e) $\frac{12}{2}$

7

Square $ABCD$ has side length s, a circle centered at E has radius r, where r and s are both rational. The circle passes through D, where D lies on $\overline{BE}$. Point F lies on the circle, on the same side of $\overline{BE}$ as A. Segment AF is tangent to the circle and $AF = \sqrt{9 + 5\sqrt{2}}$. Calculate r/s.

(a) $\frac{1}{2}$

(b) $\frac{4}{7}$

(c) $\frac{2}{3}$

(d) $\frac{5}{9}$

(e) $\frac{9}{5}$

8

There is a smallest positive real number a such that there exists a positive real number b such that all the roots of the polynomial $x^3 - ax^2 + bx - a$ are real. In fact, for this value of a, the value of b is unique. Find the value of b.

(a) 6

(b) 7

(c) 8

(d) 9

(e) 10