

# Math Bits

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## First Few Squares

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$6^2 = 36$$

$$7^2 = 49$$

$$8^2 = 64$$

$$9^2 = 81$$

$$10^2 = 100$$

$$11^2 = 121$$

$$12^2 = 144$$

$$13^2 = 169$$

$$14^2 = 196$$

$$15^2 = 225$$

$$16^2 = 256$$

$$17^2 = 289$$

$$18^2 = 324$$

$$19^2 = 361$$

$$20^2 = 400$$

$$21^2 = 441$$

$$22^2 = 484$$

$$23^2 = 529$$

$$24^2 = 576$$

$$25^2 = 625$$

$$30^2 = 900$$

$$35^2 = 1225$$

$$40^2 = 1600$$

$$45^2 = 2025$$

$$50^2 = 2500$$

$$55^2 = 3025$$

$$60^2 = 3600$$

$$65^2 = 4225$$

$$70^2 = 4900$$

$$75^2 = 5625$$

$$80^2 = 6400$$

$$85^2 = 7225$$

$$90^2 = 8100$$

$$95^2 = 9025$$

$$100^2 = 10000$$

## Useful Cubes

$$2^3 = 8$$

$$3^3 = 27$$

$$4^3 = 64$$

$$5^3 = 125$$

$$6^3 = 216$$

$$7^3 = 343$$

$$8^3 = 512$$

$$9^3 = 729$$

$$10^3 = 1000$$

## Useful Fourth Powers

$$2^4 = 16$$

$$3^4 = 81$$

$$4^4 = 256$$

$$5^4 = 625$$

$$6^4 = 1296$$

$$7^4 = 2401$$

$$8^4 = 4096$$

$$9^4 = 6561$$

$$10^4 = 10000$$

## More Useful Powers

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

$$2^8 = 256$$

$$2^9 = 512$$

$$2^{10} = 1024$$

$$2^{11} = 2048$$

$$2^{12} = 4096$$

# Highly Composite Numbers

A positive integer with more divisors than any smaller positive integer has. Plato set 5,040 as the ideal number of citizens in a city. In bold, superior highly composite numbers.

<b>2</b>	2
4	$2^2$
<b>6</b>	$2 \cdot 3$
<b>12</b>	$2^2 \cdot 3$
24	$2^3 \cdot 3$
36	$2^2 \cdot 3^2$
48	$2^4 \cdot 3$
<b>60</b>	$2^2 \cdot 3 \cdot 5$
<b>120</b>	$2^3 \cdot 3 \cdot 5$
180	$2^2 \cdot 3^2 \cdot 5$
240	$2^4 \cdot 3 \cdot 5$
<b>360</b>	$2^3 \cdot 3^2 \cdot 5$
720	$2^4 \cdot 3^2 \cdot 5$
840	$2^3 \cdot 3 \cdot 5 \cdot 7$
1,260	$2^2 \cdot 3^2 \cdot 5 \cdot 7$
1,680	$2^4 \cdot 3^1 \cdot 5 \cdot 7$
<b>2,520</b>	$2^3 \cdot 3^2 \cdot 5 \cdot 7$
<b>5,040</b>	$2^4 \cdot 3^2 \cdot 5 \cdot 7$
7,560	$2^3 \cdot 3^3 \cdot 5 \cdot 7$
10,080	$2^5 \cdot 3^2 \cdot 5 \cdot 7$
15,120	$2^4 \cdot 3^3 \cdot 5 \cdot 7$
20,160	$2^6 \cdot 3^2 \cdot 5 \cdot 7$
25,200	$2^4 \cdot 3^2 \cdot 5^2 \cdot 7$
27,720	$2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$
45,360	$2^4 \cdot 3^4 \cdot 5 \cdot 7$
50,400	$2^5 \cdot 3^2 \cdot 5^2 \cdot 7$
<b>55,440</b>	$2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$
83,160	$2^3 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11$
110,880	$2^5 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$
166,320	$2^4 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11$
221,760	$2^6 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$
277,200	$2^4 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11$
332,640	$2^5 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11$
498,960	$2^4 \cdot 3^4 \cdot 5 \cdot 7 \cdot 11$
554,400	$2^5 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11$
665,280	$2^6 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11$
<b>720,720</b>	$2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13$

## Superior Highly Composite Numbers

2	2
6	$2 \cdot 3$
12	$2^2 \cdot 3$
60	$2^2 \cdot 3 \cdot 5$
120	$2^3 \cdot 3 \cdot 5$
360	$2^3 \cdot 3^2 \cdot 5$
2,520	$2^3 \cdot 3^2 \cdot 5 \cdot 7$
5,040	$2^4 \cdot 3^2 \cdot 5 \cdot 7$
55,440	$2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$
720,720	$2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13$

## Perfect numbers

A number equal to the sum of its proper divisors.

6	$2^1(2^2 - 1)$
28	$2^2(2^3 - 1)$
496	$2^4(2^5 - 1)$
8128	$2^6(2^7 - 1)$
33550336	$2^{12}(2^{13} - 1)$
8589869056	$2^{16}(2^{17} - 1)$
137438691328	$2^{18}(2^{19} - 1)$

Euclid's Perfect Number Theorem: If  $2^p - 1$  is prime, then  $2^{p-1}(2^p - 1)$  is perfect. Euler's Perfect Number Theorem: If  $n$  is an even perfect number, it is of the form  $n = 2^{p-1}(2^p - 1)$  for some prime  $p$  and Mersenne prime  $2^p - 1$ .

## Prime Quadruplets

The first eight prime quadruplets are:

5, 7, 11, 13
11, 13, 17, 19
101, 103, 107, 109
191, 193, 197, 199
821, 823, 827, 829
1481, 1483, 1487, 1489
1871, 1873, 1877, 1879
2081, 2083, 2087, 2089

## Prime Quintuplets

The first few quintuplets that are not also sextuplets are:

5, 7, 11, 13, 17  
1481, 1483, 1487, 1489, 1493  
3457, 3461, 3463, 3467, 3469  
5647, 5651, 5653, 5657, 5659  
15727, 15731, 15733, 15737, 15739  
21011, 21013, 21017, 21019, 21023  
22271, 22273, 22277, 22279, 22283  
55331, 55333, 55337, 55339, 55343  
79687, 79691, 79693, 79697, 79699  
88807, 88811, 88813, 88817, 88819

## Prime Sextuplets

The first five sextuplets are:

7, 11, 13, 17, 19, 23  
97, 101, 103, 107, 109, 113  
16057, 16061, 16063, 16067, 16069, 16073  
19417, 19421, 19423, 19427, 19429, 19433  
43777, 43781, 43783, 43787, 43789, 43793

## Mersenne Primes

Prime numbers of the form  $2^p - 1$ , where  $p$  is some prime number. Examples:

$p$	$2^p - 1$
2	3
3	7
5	31
7	127
11	$23 \times 89$
13	8, 191
17	131, 071
19	524, 287
23	$47 \times 178481$

51 Mersenne primes are known. The largest known prime number is a Mersenne prime with  $p = 82,589,933$ .

## Rectangle

Any rectangle of sides  $a$  and  $b$ :

$$P = 2 \times (a + b)$$

$$A = a \times b$$

## Triangle

Any triangle of height  $h$  and base  $b$ :

$$A = \frac{1}{2}(h \times b)$$

## Pythagorean Theorem

For any right-triangle with hypotenuse length  $c$  and legs  $a$ ,  $b$ :

$$c^2 = a^2 + b^2$$

Example:

$$5^2 = 3^2 + 4^2$$

## Pythagorean Triples

Famous Pythagorean triple:

$$(3, 4, 5)$$

More triples:

$$\begin{array}{lll} (5, 12, 13) & (8, 15, 17) & (7, 24, 25) \\ (20, 21, 29) & (12, 35, 37) & (9, 40, 41) \\ (28, 45, 53) & (11, 60, 61) & (16, 63, 65) \\ (33, 56, 65) & (48, 55, 73) & (13, 84, 85) \\ (36, 77, 85) & (39, 80, 89) & (65, 72, 97) \end{array}$$

Non-Pythagorean triple:  $(1, 1, \sqrt{2})$  ( $\sqrt{2}$  is irrational!)

## Circle

Any circle of radius  $r$  (or diameter  $d = 2r$ ):

$$P = \tau r = 2\pi r = \pi d \approx 6.28r$$

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 \approx 3.14r^2$$

## Interesting Natural Numbers

$$3435 = 3^3 + 4^4 + 3^3 + 5^5$$

## Interesting Irrational Numbers

$$\pi \approx 3.14159265358979311600 \dots$$

$$\varphi \approx 1.61803398874989490253 \dots$$

$$\psi \approx 3.35988566624317755317 \dots$$

$$\sqrt{2} \approx 1.41421356237309514548 \dots$$

$$\sqrt{3} \approx 1.73205080756887719318 \dots$$

$$\sqrt{5} \approx 2.23606797749978980505 \dots$$

$$\sqrt{6} \approx 2.44948974278317788134 \dots$$

$$\sqrt{7} \approx 2.64575131106459071617 \dots$$

$$\sqrt{8} \approx 2.82842712474619029095 \dots$$

$$e \approx 2.71828182845904509080 \dots$$

$$\gamma \approx 0.57721566490153286061 \dots$$

## More Interesting Numbers

$$\tau = 2\pi$$

$$\varphi = \frac{\sqrt{5}}{2}$$

$$\psi = \frac{1}{\varphi}$$

$$F_n = \frac{\varphi^n - (1 - \varphi)^n}{\sqrt{5}}$$

$$e = \frac{1}{1} + \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \frac{1}{1 \times 2 \times 3 \times 4} + \dots = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$$i = \sqrt{-1}$$

$$\text{googol} = 10^{100}$$

$$\text{googolplex} = 10^{\text{googol}} = 10^{10^{100}}$$

$$C_{10} = 0.123456789101112131415116171819202122232425 \dots$$



## Riemann zeta Numbers

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \dots$$

$$\zeta(1) = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \rightarrow \infty$$

$$\zeta(2) = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \dots \approx 1.64493406684822643647$$

$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \dots \approx 1.20205690315959428540$$

$$\zeta(4) = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \dots \approx 1.08232323371113819152$$

$$\zeta(1/2) = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \dots - 1.46035450880958681289$$

$$\zeta(0) = 1 + 1 + 1 + 1 + 1 + \dots = -\frac{1}{2}$$

$$\zeta(-1) = 1 + 2 + 3 + 4 + 5 + \dots = -\frac{1}{12}$$