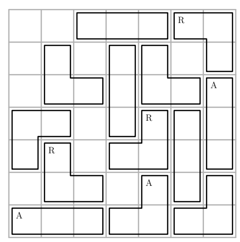
Student: James Toche
Username: jjlotoche
ID#: 44857

USA Mathematical Talent Search

Year	Round	Problem
36	2	1

Problem 1

Question

> Fill each cell with an integer from $1-7$ so each number appears exactly once in each row and column. In each "cage" of three cells, the three numbers must be valid lengths for the sides of a non-degenerate triangle. Additionally, if a cage has an "A", the triangle must be acute, and if the cage has an "R", the triangle must be right.
>
> 
>
> There is a unique solution, but you do not need to prove that your answer is the only one possible. You merely need to find an answer that satisfies the conditions of the problem. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)

3	1	7	6	2	4	5
4	7	2	5	6	1	3
7	3	5	2	1	6	4
1	2	6	4	5	3	7
2	5	1	3	4	7	6
6	4	3	1	7	5	2
5	6	4	7	3	2	1

Problem 2

Question

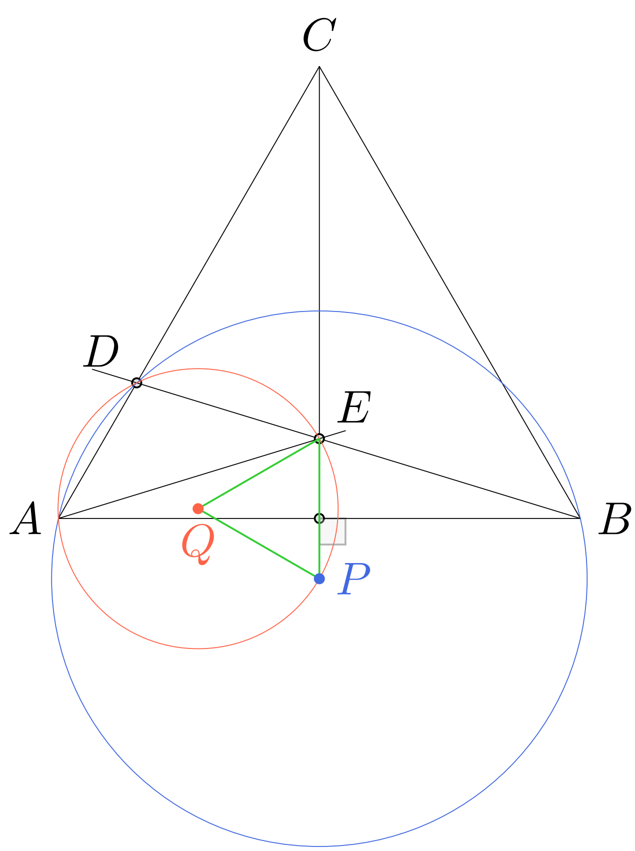
> In how many ways can a 3×3 grid be filled with integers from 1 to 12 such that all three of the following conditions are satisfied: (a) both 1 and 2 appear in the grid, (b) the grid contains at most 8 distinct values, and (c) the sums of the numbers in each row, each column, and both main diagonals are all the same? Rotations and reflections are considered the same.

TO DO

Problem 3

Question

> $\triangle ABC$ is an equilateral triangle. D is a point on AC, and E is a point on BD. Let P and Q be the circumcenters of $\triangle ABD$ and $\triangle AED$, respectively. Prove that $\triangle EPQ$ is an equilateral triangle if and only if $AB \perp CE$.



Student: James Toche
Username: jjlotoche
ID#: 44857

Problem 4

Question

> Let $x_1 < x_2 < \ldots < x_n$ (with $n \geq 2$) and let S be the set of all the x_i. Let T be a randomly chosen subset of S. What is the expected value of the indexed alternating sum of T? Express your answer in terms of the x_i.
> **Note:** We define the indexed alternating sum of T as
>
> $$\sum_{i=1}^{|T|}(-1)^{i+1}(i)T[i],$$
>
> where $T[i]$is the ith element of T when listed in increasing order. For example, if $T = \{1, 3, 5\}$, then the indexed alternating sum of T is
>
> $$1 \cdot 1 - 2 \cdot 3 + 3 \cdot 5 = 10$$
>
> Alternating sums of empty sets are defined to be 0.

TO DO

Problem 5

Question

> Prove that there is no polynomial $P(x)$ with integer coefficients such that
>
> $$P(\sqrt[3]{5}+\sqrt[3]{25}) = 2\sqrt[3]{5}+3\sqrt[3]{25}$$

TO DO

Acknowledgments

I acknowledge help with ...

Problem 1

- this

Problem 2

- this

Problem 3

- this

Problem 4

- this

Problem 5

- this