

# Art Of Problem Solving - AMC 10

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### **Abstract**

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1.

For how many integers  $n$  between 1 and 100 does  $x^2 + x - n$  factor into the product of two linear factors with integer coefficients?

- (A) 0   (B) 1   (C) 2   (D) 9   (E) 10

2.

Suppose that  $a$  and  $b$  are nonzero real numbers, and that the equation  $x^2 + ax + b = 0$  has solutions  $a$  and  $b$ . Then the pair  $(a, b)$  is

- (A)  $(-2, 1)$    (B)  $(-1, 2)$    (C)  $(1, -2)$    (D)  $(2, -1)$    (E)  $(4, 4)$

3.

Let  $f$  be the function defined by  $f(x) = ax^2 - \sqrt{2}$  for some positive  $a$ . If  $f(f(\sqrt{2})) = -\sqrt{2}$ , then  $a =$

- (A)  $\frac{2-\sqrt{2}}{2}$    (B)  $\frac{1}{2}$    (C)  $2 - \sqrt{2}$    (D)  $\frac{\sqrt{2}}{2}$    (E)  $\frac{2+\sqrt{2}}{2}$

4.

Both roots of the quadratic equation  $x^2 - 63x + k = 0$  are prime numbers. The number of possible values of  $k$  is

- (A) 0   (B) 1   (C) 2   (D) 4   (E) more than four

5.

Let  $@$  denote the “averaged with” operation:  $a@b = \frac{a+b}{2}$ . Which of the following distributive laws holds for all numbers  $x$ ,  $y$ , and  $z$ ?

- I.  $x@(y+z) = (x@y) + (x@z)$   
II.  $x + (y@z) = (x+y)@(x+z)$   
III.  $x@(y@z) = (x@y)@(x@z)$

- (A) I only   (B) II only   (C) III only   (D) I and III only   (E) II and III only

6.

If  $f(x) = ax^4 - bx^2 + x + 5$  and  $f(-3) = 2$ , then  $f(3) =$

- (A)  $-5$    (B)  $-2$    (C) 1   (D) 3   (E) 8

7.

What is the sum of the reciprocals of the roots of the equation

$$\frac{2003}{2004}x + 1 + \frac{1}{x} = 0?$$

(A)  $-\frac{2004}{2003}$    (B)  $-1$    (C)  $\frac{2003}{2004}$    (D)  $1$    (E)  $\frac{2004}{2003}$

**8.**

Let  $f$  be a polynomial function such that, for all real  $x$ ,

$$f(x^2 + 1) = x^4 + 5x^2 + 3$$

For all real  $x$ ,  $f(x^2 - 1)$  is

(A)  $x^4 + 5x^2 + 1$    (B)  $x^4 + x^2 - 3$    (C)  $x^4 - 5x^2 + 1$    (D)  $x^4 + x^2 + 3$    (E) none of these

**9.**

The polynomial  $x^3 - ax^2 + bx - 2010$  has three positive integer roots. What is the smallest possible value of  $a$ ?

(A) 78   (B) 88   (C) 98   (D) 108   (E) 118

**10.**

Let  $f$  be a function for which  $f(x/3) = x^2 + x + 1$ . Find the sum of all values of  $z$  for which  $f(3z) = 7$ .

(A)  $-1/3$    (B)  $-1/9$    (C)  $0$    (D)  $5/9$    (E)  $5/3$