2021 Fall AMC 10B Problems/Problem 9

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Problem

The knights in a certain kingdom come in two colors. $\frac{2}{7}$ of them are red, and the rest are blue. Furthermore, $\frac{1}{6}$ of the knights are magical, and the fraction of red knights who are magical is 2 times the fraction of blue knights who are magical. What fraction of red knights are magical?

$\textbf{(A)}\ \frac{2}{9} \qquad \textbf{(B)}\ \frac{3}{13} \qquad \textbf{(C)}\ \frac{7}{27} \qquad \textbf{(D)}\ \frac{2}{7} \qquad \textbf{(E)}\ \frac{1}{3}$

Solution 1

Let k be the number of knights: then the number of red knights is $\frac{2}{7}k$ and the number of blue knights is $\frac{5}{7}k$.

Let b be the fraction of blue knights that are magical - then $2b$ is the fraction of red knights that are magical. Thus we can write the equation $b \cdot \frac{5}{7}k + 2b \cdot \frac{2}{7}k = \frac{k}{6} \implies \frac{5}{7}b + \frac{4}{7}b = \frac{1}{6} \implies \frac{9}{7}b = \frac{1}{6} \implies b = \frac{7}{54}$

We want to find the fraction of red knights that are magical, which is $2b = \frac{7}{27} = \boxed{C}$

~KingRavi

Solution 2

We denote by p the fraction of red knights who are magical.

Hence,

$$\frac{1}{6} = \frac{2}{7}p + \left(1 - \frac{2}{7}\right)\frac{p}{2}.$$

By solving this equation, we get $p = \frac{7}{27}$.

Therefore, the answer is $\boxed{\textbf{(C)}\ \frac{7}{27}}$.

~Steven Chen (www.professorchenedu.com)

Video Solution by Interstigation

https://youtu.be/p9_RH4s-kBA?t=1274

See Also

2021 Fall AMC 10B (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
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