2021 Fall AMC 12B Problems/Problem 11

The following problem is from both the 2021 Fall AMC 10B #14 and 2021 Fall AMC 12B #11, so both problems redirect to this page.

Contents

Problem

Una rolls 6 standard 6-sided dice simultaneously and calculates the product of the 6 numbers obtained. What is the probability that the product is divisible by 4?

$\textbf{(A)}\ \frac{3}{4} \qquad \textbf{(B)}\ \frac{57}{64} \qquad \textbf{(C)}\ \frac{59}{64} \qquad \textbf{(D)}\ \frac{187}{192} \qquad \textbf{(E)}\ \frac{63}{64}$

Solution

We will use complementary counting to find the probability that the product is not divisible by 4. Then, we can find the probability that we want by subtracting this from 1. We split this into 2 cases.

Case 1: The product is not divisible by 2.

We need every number to be odd, and since the chance we roll an odd number is $\frac{1}{2}$, our probability is $\left(\frac{1}{2}\right)^6 = \frac{1}{64}$.

Case 2: The product is divisible by 2, but not by 4.

We need 5 numbers to be odd, and one to be divisible by 2, but not by 4. There is a $\frac{1}{2}$ chance that an odd number is rolled, a $\frac{1}{3}$ chance that we roll a number satisfying the second condition (only 2 and 6 work), and 6 ways to choose the order in which the even number appears.

Our probability is $\left(\frac{1}{2}\right)^5 \left(\frac{1}{3}\right) \cdot 6 = \frac{1}{16}$.

Therefore, the probability the product is not divisible by 4 is $\frac{1}{64} + \frac{1}{16} = \frac{5}{64}$.

Our answer is $1 - \frac{5}{64} = \boxed{\textbf{(C)}\ \frac{59}{64}}$.

~kingofpineapplz (Solution)

~stjwyl (Minor Edits)

Video Solution by Interstigation

https://www.youtube.com/watch?v=G_31gUNwkzQ

See Also

2021 Fall AMC 10B (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
Preceded by **Problem 13**	Followed by **Problem 15**
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2021 Fall AMC 12B (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
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