2021 Fall AMC 10B Problems/Problem 3

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Problem

The expression $\frac{2021}{2020} - \frac{2020}{2021}$ is equal to the fraction $\frac{p}{q}$ in which p and q are positive integers whose greatest common divisor is 1. What is p?

$(\mathbf{A})\ 1 \qquad (\mathbf{B})\ 9 \qquad (\mathbf{C})\ 2020 \qquad (\mathbf{D})\ 2021 \qquad (\mathbf{E})\ 4041$

Solution 1

We write the given expression as a single fraction:

$$\frac{2021}{2020} - \frac{2020}{2021} = \frac{2021 \cdot 2021 - 2020 \cdot 2020}{2020 \cdot 2021}$$

by cross multiplication. Then by factoring the numerator, we get

$$\frac{2021 \cdot 2021 - 2020 \cdot 2020}{2020 \cdot 2021} = \frac{(2021 - 2020)(2021 + 2020)}{2020 \cdot 2021}.$$

The question is asking for the numerator, so our answer is $2021 + 2020 = 4041,$ giving answer choice $\boxed{(\mathbf{E})}$.

~Aops-g5-gethsemanea2

Solution 2

Denote $a = 2020$. Hence,

$$\begin{aligned} \frac{2021}{2020} - \frac{2020}{2021} &= \frac{a+1}{a} - \frac{a}{a+1} \\ &= \frac{(a+1)^2 - a^2}{a(a+1)} \\ &= \frac{2a+1}{a(a+1)}. \end{aligned}$$

We observe that $\gcd(2a+1, a) = 1$ and $\gcd(2a+1, a+1) = 1$.

Hence, $\gcd(2a+1, a(a+1)) = 1$.

Therefore, $p = 2a + 1 = 4041$.

Therefore, the answer is $\boxed{\textbf{(E)}\ 4041}$.

~Steven Chen (www.professorchenedu.com)

Video Solution by Interstigation

https://youtu.be/p9_RH4s-kBA?t=160

See Also

2021 Fall AMC 10B (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
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