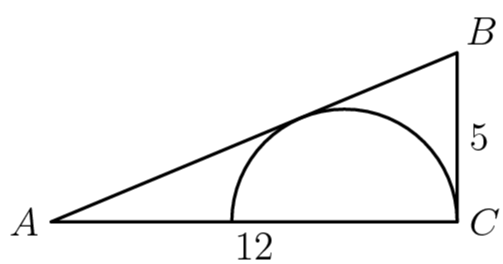
2017 AMC 8 Problems/Problem 22

Contents

Problem

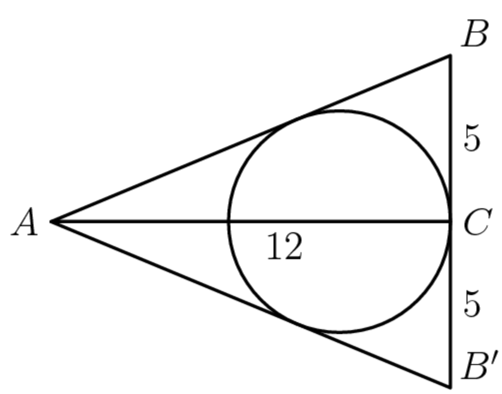
In the right triangle ABC, $AC = 12$, $BC = 5$, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?



$\textbf{(A) }\frac{7}{6}\qquad\textbf{(B) }\frac{13}{5}\qquad\textbf{(C) }\frac{59}{18}\qquad\textbf{(D) }\frac{10}{3}\qquad\textbf{(E) }\frac{60}{13}$

Solution 1

We can reflect triangle ABC over line $AC.$ This forms the triangle $AB'C$ and a circle out of the semicircle. Let us call the center of the circle $O.$



We can see that Circle O is the incircle of $ABB'.$ We can use a formula for finding the radius of the incircle. The area of a triangle $= \text{Semiperimeter} \cdot \text{inradius}$. The area of ABB' is $12 \times 5 = 60.$ The semiperimeter is $\dfrac{10 + 13 + 13}{2} = 18.$ Simplifying $\dfrac{60}{18} = \dfrac{10}{3}.$ Our answer is therefore $\boxed{\textbf{(D)}\ \frac{10}{3}}$.

Asymptote diagram by Mathandski

Solution 2

We immediately see that $AB = 13$, and we label the center of the semicircle O and the point where the circle is tangent to the triangle D. Drawing radius OD with length x such that OD is perpendicular to AB, we immediately see that $ODB \cong OCB$ because of HL congruence, so $BD = 5$ and $DA = 8$. By similar triangles ODA and BCA, we see that $\frac{8}{12} = \frac{x}{5} \implies 12x = 40 \implies x = \frac{10}{3} \implies \boxed{\textbf{(D)}\ \frac{10}{3}}$.

Solution 3

Let the center of the semicircle be O. Let the point of tangency between line AB and the semicircle be F. Angle BAC is common to triangles ABC and AFO. By tangent properties, angle AFO must be 90 degrees. Since both triangles ABC and AFO are right and share an angle, AFO is similar to ABC. The hypotenuse of AFO is $12 - r$, where r is the radius of the circle. (See for yourself) The short leg of AFO is r. Because $\triangle AFO$ ~ $\triangle ABC$, we have $r/(12-r) = 5/13$ and solving gives $r = \boxed{\textbf{(D)}\ \frac{10}{3}}$

Solution 4

Let the tangency point on AB be D. Note

$$AD = AB - BD = AB - BC = 8$$

By Power of a Point,

$$12(12 - 2r) = 8^2$$

Solving for r gives

$$r = \boxed{\textbf{(D)}\ \frac{10}{3}}$$

Solution 5

Let us label the center of the semicircle O and the point where the circle is tangent to the triangle D. The area of $\triangle ABC$ = the areas of $\triangle ABO$ + $\triangle ACO$, which means $(12*5)/2 = (13*r)/2 + (5*r)/2$. So it gives us $r = \boxed{\textbf{(D)}\ \frac{10}{3}}$.----LarryFlora

Video Solution

https://youtu.be/Y0JBJgHsdGk

https://youtu.be/3VjySNobXLI - Happytwin

https://youtu.be/KtmLUlCpj-I - savannahsolver

https://youtu.be/FDgcLW4frg8?t=3837 - pi_is_3.14

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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