2021 Fall AMC 12B Problems/Problem 19

The following problem is from both the 2021 Fall AMC 10B #21 and 2021 Fall AMC 12B #19, so both problems redirect to this page.

Problem

Regular polygons with $5, 6, 7,$ and 8 sides are inscribed in the same circle. No two of the polygons share a vertex, and no three of their sides intersect at a common point. At how many points inside the circle do two of their sides intersect?

$\textbf{(A)}\ 52 \qquad \textbf{(B)}\ 56 \qquad \textbf{(C)}\ 60 \qquad \textbf{(D)}\ 64 \qquad \textbf{(E)}\ 68$

Solution 1

Imagine we have 2 regular polygons with m and n sides and $m > n$ inscribed in a circle without sharing a vertex. We see that each side of the polygon with n sides (the polygon with fewer sides) will be intersected twice. (We can see this because to have a vertex of the m-gon on an arc subtended by a side of the n-gon, there will be one intersection to "enter" the arc and one to "exit" the arc. ~KingRavi)

This means that we will end up with 2 times the number of sides in the polygon with fewer sides.

If we have polygons with $5, 6, 7,$ and 8 sides, we need to consider each possible pair of polygons and count their intersections.

Throughout 6 of these pairs, the 5-sided polygon has the least number of sides 3 times, the 6-sided polygon has the least number of sides 2 times, and the 7-sided polygon has the least number of sides 1 time.

Therefore the number of intersections is $2 \cdot (3 \cdot 5 + 2 \cdot 6 + 1 \cdot 7) = \boxed{\textbf{(E)}\ 68}$.

~kingofpineapplz

See Also

2021 Fall AMC 10B (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
Preceded by **Problem 20**	Followed by **Problem 22**
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All AMC 10 Problems and Solutions	

2021 Fall AMC 12B (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
Preceded by **Problem 18**	Followed by **Problem 20**
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All AMC 12 Problems and Solutions	

Mathematics Competitions (http://amc.maa.org).