Art Of Problem Solving - AMC 10
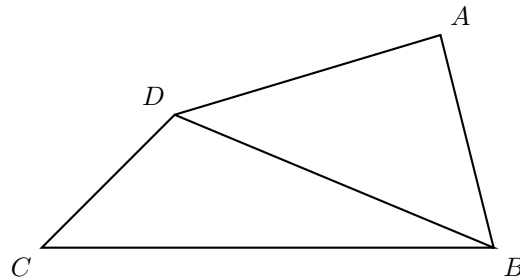# Week 6

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Abstract

Notes on the AMC-10 Course by Art Of Problem Solving (AOPS). Copyright restrictions may apply. Written for personal use. Please report typos and errors over at https://github.com/ptoche/Math/tree/master/aops.

1.

In quadrilateral $ABCD$, $AB = 5$, $BC = 17$, $CD = 5$, $DA = 9$, and BD is an integer. What is BD?

(A) 11   (B) 12   (C) 13   (D) 14   (E) 15

2.

Rectangle $ABCD$ has $AB = 4$ and $BC = 3$. Segment EF is constructed through B so that $EF \perp DB$, and A and C lie on DE and DF, respectively. What is EF?

(A) 9   (B) 10   (C) $\dfrac{125}{12}$ (D) $\dfrac{103}{9}$ (E) 12
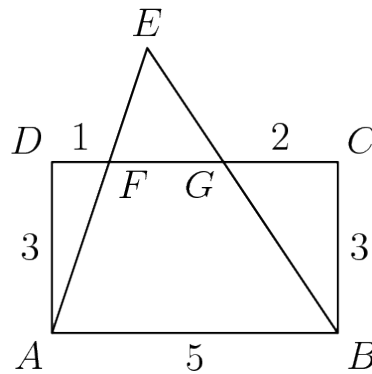
3.

Points A, B, C, D, E, and F lie, in that order, on AF, dividing it into five segments, each of length 1. Point G is not on line AF. Point H lies on GD, and point J lies on GF. The line segments HC, JE, and AG are parallel. Find HC/JE.
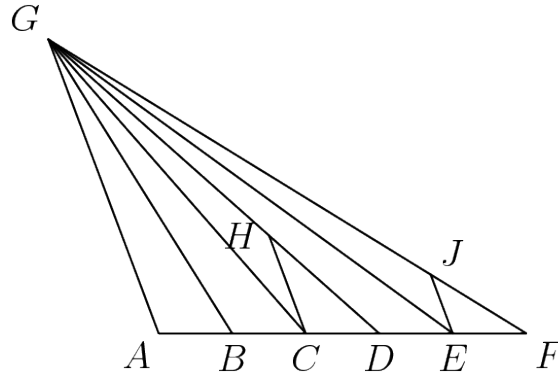
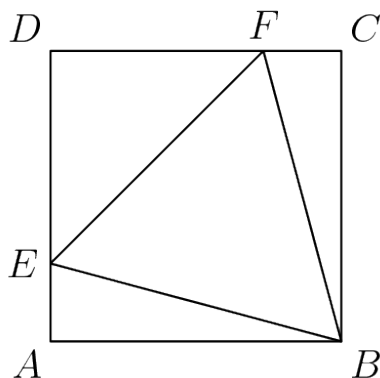(A) 5/4   (B) 4/3   (C) 3/2   (D) 5/3   (E) 2

4.

In rectangle $ABCD$, $AB = 5$ and $BC = 3$. Points F and G are on CD so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E. Find the area of triangle AEB.

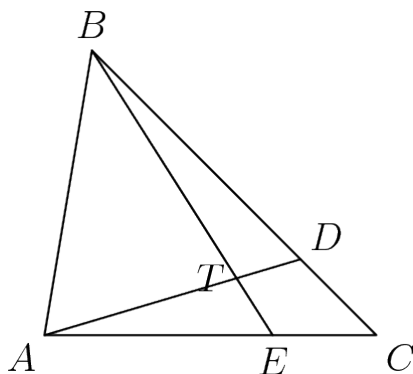(A) 10   (B) $\dfrac{21}{2}$ (C) 12   (D) $\dfrac{25}{2}$ (E) 15

5.

Points E and F are located on square $ABCD$ so that triangle BEF is equilateral. What is the ratio of the area of triangle DEF to that of triangle ABE?

$$\text{(A) } \frac{4}{3} \quad \text{(B) } \frac{3}{2} \quad \text{(C) } \sqrt{3} \quad \text{(D) } 2 \quad \text{(E) } 1 + \sqrt{3}$$
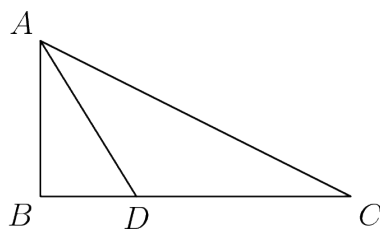
6.

In triangle ABC points D and E lie on BC and AC, respectively. If AD and BE intersect at T so that $AT/DT = 3$ and $BT/ET = 4$, what is CD/BD?

$$\text{(A) } \frac{1}{8} \quad \text{(B) } \frac{2}{9} \quad \text{(C) } \frac{3}{10} \quad \text{(D) } \frac{4}{11} \quad \text{(E) } \frac{5}{12}$$
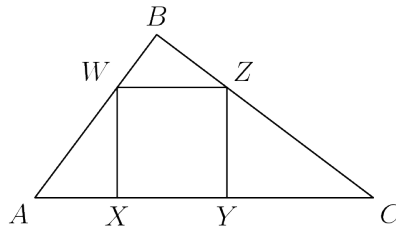
7.

Triangle ABC has a right angle at B, $AB = 1$, and $BC = 2$. The bisector of $\angle BAC$ meets BC at D. What is BD?

$$\text{(A) } \frac{\sqrt{3}-1}{2} \quad \text{(B) } \frac{\sqrt{5}-1}{2} \quad \text{(C) } \frac{\sqrt{5}+1}{2} \quad \text{(D) } \frac{\sqrt{6}+\sqrt{2}}{2} \quad \text{(E) } 2\sqrt{3}-1$$

8.

Right triangle ABC has $AB = 3$, $BC = 4$, and $AC = 5$. Square $XYZW$ is inscribed in triangle ABC with X and Y on AC, W on AB, and Z on BC. What is the side length of the square?

(A) $\dfrac{3}{2}$　(B) $\dfrac{60}{37}$　(C) $\dfrac{12}{7}$　(D) $\dfrac{23}{13}$　(E) 2

9.

A triangle with sides of 5, 12, and 13 has both an inscribed and a circumscribed circle. What is the distance between the centers of those circles?

(A) $\dfrac{3\sqrt{5}}{2}$　(B) $\dfrac{7}{2}$　(C) $\sqrt{15}$　(D) $\dfrac{\sqrt{65}}{2}$　(E) $\dfrac{9}{2}$

10.

In triangle ABC we have $AB = 25$, $BC = 39$, and $AC = 42$. Points D and E are on AB and AC respectively, with $AD = 19$ and $AE = 14$. What is the ratio of the area of triangle ADE to the area of the quadrilateral $BCED$?

(A) $\dfrac{266}{1521}$　(B) $\dfrac{19}{75}$　(C) $\dfrac{1}{3}$　(D) $\dfrac{19}{56}$　(E) 1