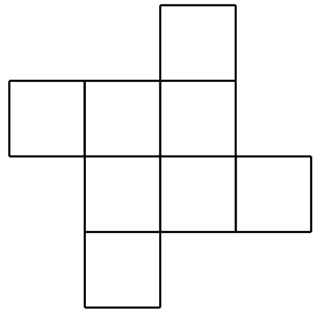
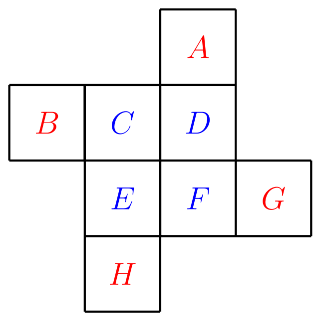
Place the first eight positive integers in the grid below such that sums along blocks of three squares are equal.



For convenience, label the boxes with letters:



First note that the grid can be rotated four ways: $A \longrightarrow G$, $A \longrightarrow H$, $A \longrightarrow B$. Thus by symmetry, there are 4 equivalent arrangement of numbers.

There are 48 permutations which keep the row-sums and column-sums constant. We divide by 4 to remove solutions that are obtained by rotation. We are therefore looking for 12 distinct solutions!

Let S denote the sum along blocks of three.

$$\begin{aligned} S &= B + C + D \\ &= E + F + G \\ &= C + E + H \\ &= A + D + F \end{aligned}$$

Let M denote the sum of the middle four squares (the blue squares).

$$M = C + D + E + F$$

Adding the sums gives

$$4S = A + B + C + D + E + F + G + H + M$$

We also have

$$A + B + C + D + E + F + G + H = 1 + 2 + \ldots + 7 + 8 = \frac{8 \times 9}{2} = 36$$

Combining these, we get

$$\begin{aligned} 4S &= 36 + M \\ M &= 4(S - 9) \end{aligned}$$

M must be a multiple of 4. Moreover, the lowest possible value for M is $1+2+3+4 = 10$, and the highest possible value is $8+7+6+5 = 26$. This gives possible values for M of 12, 16, 20, 24. To these correspond possible values of S of $9 + M/4 = 12$, 13, 14, 15.

Consider S = 15 (M = 24). Since $8 + 7 = 15$, the middle squares cannot have 8 and 7 aligned, because if they were the row-sums and column-sums would exceed $S = 15$. So the only possible arrangement is to have 8 and 7 aligned diagonally, that is ($C = 8$, $F = 7$) (ignoring equivalent rotations). Given this, there are two candidates for D and E, namely ($D = 6$, $E = 3$) and ($D = 5$, $E = 4$). But to get the sum $E + F + G = 15$ with $E = 4$ and $F = 7$ would require using 4 a second time, which is not allowed. Thus, for $S = 15$, we must have $C = 8$, $D = 6$, $E = 3$, $F = 7$, from which the other numbers are readily obtained. So a solution is:

Solution 1: S = 15

		2	
1	8	6	
	3	7	5
	4		

Another distinct solution may be obtained with the same middle numbers by exchanging 6 and 3:

Solution 2: S = 15

		5	
4	8	3	
	6	7	2
	1		

Consider S = 14 (M = 20). It is clear that if 8 and 7 are to be in the middle, they must be along a diagonal and they must be complemented by $(3, 2)$ or $(4, 1)$. However, $8 + 2 = 7 + 3$ rules out $(8, 7)$ and $(3, 2)$ in the middle. The other combination does work. So a solution with $S = 14$ is:

Solution 3: S = 14

		6	
5	8	1	
	4	7	3
	2		

Another solution may be found with the same middle numbers by exchanging 1 and 4:

Solution 4: S = 14

		3	
2	8	4	
	1	7	6
	5		

Are there other solutions for $M = 20$, $S = 14$? If 8 and 6 are in the middle, they must be along a diagonal and joined by $(5, 1)$ or $(4, 2)$. However, $(5, 1)$ can be ruled out since 8 and 1 would need to be aligned with 5 to sum to 14. And likewise $(4, 2)$ can be ruled out since 8 and 2 would need to be aligned with 4 to sum to 14. Now, if 8 and 5 are in the middle, it would have to be with $(4, 3)$. But 8 and 3 must be on a diagonal since they sum to 11 and if they were aligned would require another 3 to sum to 14, which is not allowed.

Solution 5: S = 14

		6	
1	8	5	
	4	3	7
	2		

Another distinct solution may be obtained with the same middle numbers by exchanging 5 and 4:

Solution 6: S = 14

		7	
2	8	4	
	5	3	6
	1		

Now if 7 and 6 are in the middle, it would have to be with 4 and 3. But since $7 + 3 = 6 + 4$, 7 and 6 would have to be arranged on the same row or column. But that won't work since $4 + 3 = 7$ and another 7 cannot be used. We do not need to look further down since $6 + 5 = 11 > 10 = M/2$, implying that if we place 6 and 5 in the middle, we cannot reach the sum $M = 20$ with smaller numbers like 4, 3.

Consider $\mathbf{S = 13}$ ($\mathbf{M = 16}$). There are two ways to arrange $(8, 5, 2, 1)$ in the middle.

Solution 7: $\mathbf{S = 13}$

		6	
3	8	2	
	1	5	7
	4		

Another distinct solution may be obtained with the same middle numbers by exchanging 1 and 2:

Solution 8: $\mathbf{S = 13}$

		7	
4	8	1	
	2	5	6
	3		

Likewise, there are two ways to arrange $6, 5, 4, 1$ in the middle.

Solution 9: $\mathbf{S = 13}$

		7	
2	6	5	
	4	1	8
	3		

Another solution may be found with the same middle numbers by exchanging 5 and 4:

Solution 10: $\mathbf{S = 13}$

		8	
3	6	4	
	5	1	7
	2		

Consider $\mathbf{S = 12}$ ($\mathbf{M = 12}$). These sums are small. Because $1+2+3=6$, candidates for the middle are $(6,3,2,1)$ and $(5,4,2,1)$ only. We can have $(6,1)$ and $(6,2)$ on the same row, but clearly not $(6,3)$. We must therefore have 6 and 3 on a diagonal:

Solution 11: S = 12

		8	
5	6	1	
	2	3	7
	4		

Another solution may be found with the same middle numbers by exchanging 1 and 2:

Solution 12: S = 12

		7	
4	6	2	
	1	3	8
	5		

To sum up, there are 12 solutions. We have 2 solutions that sum to 12 and 2 solutions that sum to 15. And we have 4 solutions that sum to 13 and 4 solutions that sum to 14. The solutions come in pairs, reflecting the existence of a fundamental symmetry not captured by a simple rotation. If we consider only the solutions characterized by a unique combination of middle numbers, there are 6 such solutions only.

The six "fundamental solutions" are: For $S = 12$: $(1, 2, 3, 6)$. For $S = 13$: $(1, 4, 5, 6)$ and $(1, 2, 5, 8)$. For $S = 14$: $(3, 4, 5, 8)$ and $(1, 4, 7, 8)$. For $S = 15$: $(3, 6, 7, 8)$.