2021 Fall AMC 10B Problems/Problem 10

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Problem

Forty slips of paper numbered 1 to 40 are placed in a hat. Alice and Bob each draw one number from the hat without replacement, keeping their numbers hidden from each other. Alice says, "I can't tell who has the larger number." Then Bob says, "I know who has the larger number." Alice says, "You do? Is your number prime?" Bob replies, "Yes." Alice says, "In that case, if I multiply your number by 100 and add my number, the result is a perfect square. " What is the sum of the two numbers drawn from the hat?

$\textbf{(A)}\ 27 \qquad \textbf{(B)}\ 37 \qquad \textbf{(C)}\ 47 \qquad \textbf{(D)}\ 57 \qquad \textbf{(E)}\ 67$

Solution 1

Because Alice doesn't know who has the larger number, she doesn't have $1.$ Because Alice says that she doesn't know who has the larger number, Bob knows that she doesn't have $1.$ But Bob knows who has the larger number, this implies that Bob has the smallest possible number. Because Bob's number is prime, Bob's number is $2.$ Thus, the perfect square is in the $200's.$ The only perfect square is $225.$ Thus, Alice's number is $25.$ The sum of Alice's and Bob's number is $25+2=27.$ Thus the answer is $\boxed{\textbf{(A.)}}$.

~NH14

Solution 2

Denote by A and B the numbers drawn by Alice and Bob, respectively.

Alice's sentence ``I can't tell who has the larger number. *implies* $A \in \{2, \cdots, 39\}$.

Bob's sentence ``I know who has the larger number. *implies* $B \in \{1, 2, 39, 40\}$.

Their subsequent conversation that B is prime implies $B = 2$.

Then, Alice's next sentence ``In that case, if I multiply your number by 100 and add my number, the result is a perfect square. *implies* $200 + A$ *is a perfect square.* Hence, $A = 25$.

Therefore, the answer is $\boxed{\textbf{(A)}\ 27}$.

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Video Solution by Interstigation

https://youtu.be/p9_RH4s-kBA?t=1524

See Also

2021 Fall AMC 10B (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
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