2021 AMC 12A Problems/Problem 7

The following problem is from both the 2021 AMC 10A #9 and 2021 AMC 12A #7, so both problems redirect to this page.

Contents

- 1 Problem
- 2 Solution 1 (Expand)
- 3 Solution 2 (Expand and then Factor)
- 4 Solution 3 (Beyond Overkill)
- 5 Video Solution (Simple & Quick)
- 6 Video Solution by Aaron He (Trivial Inequality)
- 7 Video Solution by North America Math Contest Go Go (Trivial Inequality, Simon's Favourite Packing Theorem)
- 8 Video Solution by Hawk Math
- 9 Video Solution (Trivial Inequality, Simon's Favorite Factoring)
- 10 Video Solution 6
- 11 Video Solution by TheBeautyofMath
- 12 Video Solution by The Learning Royal
- 13 See also

Problem

What is the least possible value of $(xy-1)^2+(x+y)^2$ for real numbers x and y?

$$(\mathbf{A}) \ 0 \qquad (\mathbf{B})$$

(B)
$$\frac{1}{4}$$
 (C) $\frac{1}{2}$ (D) 1 (E) 2

Solution 1 (Expand)

Expanding, we get that the expression is $x^2+2xy+y^2+x^2y^2-\underline{2xy+1}$ or $x^2+y^2+x^2y^2+1$. By the Trivial Inequality (all squares are nonnegative) the minimum value for this is $|\mathbf{D}| 1$, which can be achieved at x=y=0.

~aop2014

Solution 2 (Expand and then Factor)

We expand the original expression, then factor the result by grouping:

$$(xy-1)^{2} + (x+y)^{2} = (x^{2}y^{2} - 2xy + 1) + (x^{2} + 2xy + y^{2})$$

$$= x^{2}y^{2} + x^{2} + y^{2} + 1$$

$$= x^{2}(y^{2} + 1) + (y^{2} + 1)$$

$$= (x^{2} + 1)(y^{2} + 1).$$

Clearly, both factors are positive. By the Trivial Inequality, we have

$$(x^2+1)(y^2+1) \ge (0+1)(0+1) = \boxed{\textbf{(D) 1}}$$

Note that the least possible value of $(xy-1)^2+(x+y)^2$ occurs at x=y=0.

~MRENTHUSIASM

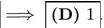
Solution 3 (Beyond Overkill)

Like solution 1, expand and simplify the original equation to $x^2+y^2+x^2y^2+1$ and let $f(x,y)=x^2+y^2+x^2y^2+1$. To find local extrema, find where $\nabla f(x,y)=\mathbf{0}$. First, find the first partial derivative with respect to x and y and find where they are 0:

$$\frac{\partial f}{\partial x} = 2x + 2xy^2 = 2x(1+y^2) = 0 \implies x = 0$$

$$\frac{\partial \hat{f}}{\partial y} = 2y + 2yx^2 = 2y(1+x^2) = 0 \implies y = 0$$

Thus, there is a local extremum at (0,0). Because this is the only extremum, we can assume that this is a minimum because the problem asks for the minimum (though this can also be proven using the partial second derivative test) and the global minimum since it's the only minimum, meaning f(0,0) is the minimum of f(x,y). Plugging (0,0) into f(x,y), we find 1



~ DBlack2021

Video Solution (Simple & Quick)

https://youtu.be/2CZ1u4J9yk4

~ Education, the Study of Everything

Video Solution by Aaron He (Trivial Inequality)

https://www.youtube.com/watch?v=xTGDKBthWsw&t=6m58s

Video Solution by North America Math Contest Go Go (Trivial Inequality, Simon's Favourite Packing Theorem)

https://www.youtube.com/watch?v=PbJK4KKfQjY&list=PLexHyfQ8DMuKqltG3cHT7Di4jhVl6L4YJ&index=8

Video Solution by Hawk Math

https://www.youtube.com/watch?v=P5al76DxyHY

Video Solution (Trivial Inequality, Simon's Favorite Factoring)

https://youtu.be/DP0ppuQzFPE

~ pi_is_3.14

Video Solution 6

https://youtu.be/hm0GYmVmY1c

~savannahsolver

Video Solution by TheBeautyofMath

https://youtu.be/s6E4E06XhPU?t=640 (for AMC 10A)

https://youtu.be/cckGBU2x1zg?t=95 (for AMC 12A)

~IceMatrix

Video Solution by The Learning Royal

https://youtu.be/AWjOeBFyeb4

See also

:	3))
Preceded by	Followed by
Problem 8	Problem 10
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 1	4 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25
All AMC 10 Prob	lems and Solutions
`	s (http://www.artofproblemsolving.com/community/c ² 3))
Preceded by	Followed by
Problem 6	Problem 8

All AMC 12 Problems and Solutions

The problems on this page are copyrighted by the Mathematical Association of America (http://www.maa.org)'s American

Mathematics Competitions (http://amc.maa.org).

(8)

 $Retrieved\ from\ "https://artofproblemsolving.com/wiki/index.php?title=2021_AMC_12A_Problems/Problem_7\&oldid=169357"$

Copyright © 2022 Art of Problem Solving