

---

# MATHCOUNTS®

---

2021

■ **Mock National Competition** ■  
**Target Round**  
**Problems 1 & 2**

---

Name \_\_\_\_\_

State \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

**Author:** Puddles\_Penguin, CT17, Mathletesv, Grizzly, kred9, and usernameyourself

***L<sup>A</sup>T<sub>E</sub>X*** by: scrabblers94

**Test-solved by:** v4913, djmathman, dchenmathcounts, i3435

---

Problem 1	Problem 2	Scorer's Initials

1. \_\_\_\_\_ A special rock lies in the ocean with weight 20 pounds. Every **minute**, the rock loses 25 percent of its weight, and then gains 20 pounds. After **5 years**, the weight of the rock is closest to what integer?

2. \_\_\_\_\_ Jerry thinks of a finite arithmetic sequence of integers with first term 1 and last term 19. Then, Laura computes the sum of all of the numbers in Jerry's sequence. What is the sum of all possible numbers Laura could obtain?

---

# MATHCOUNTS®

---

2021

■ **Mock National Competition** ■  
**Target Round**  
**Problems 3 & 4**

---

Name \_\_\_\_\_

State \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

---

Problem 3	Problem 4	Scorer's Initials

3. \_\_\_\_\_ How many ordered triples  $(a, b, c)$  of (not necessarily distinct) positive integers not exceeding 10 satisfy  $\frac{a}{b} \times \frac{a}{c} = \frac{a}{b} - \frac{a}{c}$ ?

4. \_\_\_\_\_ What is the sum of the three smallest positive, odd integers  $n > 1$  for which there exists a non-negative integer  $k$  such that  $2^k n$  has exactly  $n$  divisors?

---

# MATHCOUNTS®

---

2021

■ **Mock National Competition** ■  
**Target Round**  
**Problems 5 & 6**

---

Name \_\_\_\_\_

State \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

---

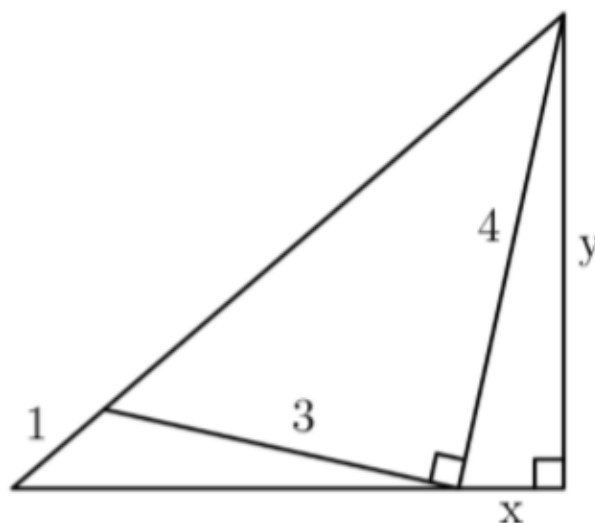
Problem 5	Problem 6	Scorer's Initials

5. \_\_\_\_\_

An integer  $a$  is selected between 3 and 5, inclusive, and an integer  $b$  is selected between 30 and 50, inclusive. How many distinct possible values are there for the product  $ab$ ?

6. \_\_\_\_\_

In the figure shown, a right triangle with leg lengths 3 and 4 is inscribed in a larger right triangle. What is the value of  $\frac{x}{y}$ ? Express your answer as a common fraction.



---

# MATHCOUNTS®

---

2021

■ Mock National Competition ■  
Target Round  
Problems 7 & 8

---

Name \_\_\_\_\_

State \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

---

Problem 7	Problem 8	Scorer's Initials

7. \_\_\_\_\_ What is the sum of all positive integers  $n$  less than 50 such that the sum of the base 6 and base 9 representations of  $n$  are the same?

8. \_\_\_\_\_ Four basketball teams with distinct fixed skill levels participate in a special tournament. The manager of the tournament knows that whenever two teams play, the team with the higher skill level always wins, but he has no prior knowledge of the skill levels of the four teams. Every day, the manager randomly chooses two teams to play each other and records the winner. What is the expected number of days until the manager can determine with certainty the order of the four teams' skill levels? Note: Two teams might play each other more than once.