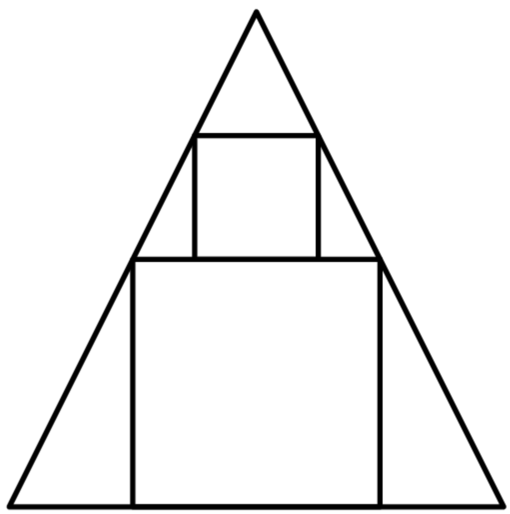
# 2021 Fall AMC 10B Problems/Problem 13

## Contents



## Problem

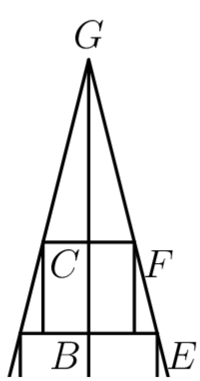
A square with side length $3$ is inscribed in an isosceles triangle with one side of the square along the base of the triangle. A square with side length $2$ has two vertices on the other square and the other two on sides of the triangle, as shown. What is the area of the triangle?

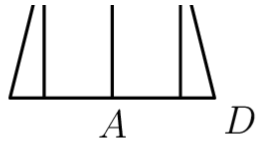


$(\textbf{A})\ 19\frac{1}{4} \qquad (\textbf{B})\ 20\frac{1}{4} \qquad (\textbf{C})\ 21\frac{3}{4} \qquad (\textbf{D})\ 22\frac{1}{2} \qquad (\textbf{E})\ 23\frac{3}{4}$

## Solution 1

Let's split the triangle down the middle and label it:






We see that $\triangle ADG \sim \triangle BEG \sim \triangle CFG$ by AA similarity. $BE = \frac{3}{2}$ because $AK$ cuts the side length of the square in half; similarly, $CF = 1$. Let $CG = h$: then by side ratios,

$$\frac{h+2}{h} = \frac{\frac{3}{2}}{1} \implies 2(h+2) = 3h \implies h = 4$$

.

Now the height of the triangle is $AG = 4 + 2 + 3 = 9$. By side ratios,

$$\frac{9}{4} = \frac{AD}{1} \implies AD = \frac{9}{4}$$

.

The area of the triangle is $AG \cdot AD = 9 \cdot \frac{9}{4} = \frac{81}{4} = \boxed{B}$

~KingRavi

## Solution 2

By similarity, the height is $3 + \frac{3}{1} \cdot 2 = 9$ and the base is $\frac{9}{2} \cdot 1 = 4.5$. Thus the area is $\frac{9 \cdot 4.5}{2} = 20.25 = 20\frac{1}{4}$, or $\boxed{\textbf{(B)}}$.

~Hefei417, or 陆畅 Sunny from China

## Solution 3 (With two different endings)

This solution is based on this figure: Image:2021_AMC_10B_(Nov)_Problem_13,_sol.png

Denote by $O$ the midpoint of $AB$.

Because $FG = 3$, $JK = 2$, $FJ = KG$, we have $FJ = \frac{1}{2}$.

We observe $\triangle ADF \sim \triangle FJH$. Hence, $\frac{AD}{FJ} = \frac{FD}{HJ}$. Hence, $AD = \frac{3}{4}$. By symmetry, $BE = AD = \frac{3}{4}$.

Therefore, $AB = AD + DE + BE = \frac{9}{2}$.

Because $O$ is the midpoint of $AB$, $AO = \frac{9}{4}$.

We observe $\triangle AOC \sim \triangle ADF$. Hence, $\frac{OC}{DF} = \frac{AO}{AD}$. Hence, $OC = 9$.

Therefore, $\text{Area } \triangle ABC = \frac{1}{2} AB \cdot OC = \frac{81}{4} = 20\frac{1}{4}$.

Therefore, the answer is $\boxed{\textbf{(B)}\ 20\frac{1}{4}}$.

~Steven Chen (www.professorchenedu.com)

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Alternatively, we can find the height in a slightly different way.

Following from our finding that the base of the large triangle $AB = \frac{9}{2}$, we can label the length of the altitude of $\triangle CHI$ as $x$. Notice that $\triangle CHI \sim \triangle CAB$. Hence, $\frac{HI}{AB} = \frac{x}{CO}$. Substituting and simplifying, $\frac{HI}{AB} = \frac{x}{CO} \Rightarrow \frac{2}{\frac{9}{2}} = \frac{x}{x+5} \Rightarrow \frac{x}{x+5} = \frac{4}{9} \Rightarrow x = 4 \Rightarrow CO = 4 + 5 = 9$. Therefore, the area of the triangle is $\frac{\frac{9}{2} \cdot 9}{2} = \frac{81}{4} = \boxed{\textbf{(B)}\ 20\frac{1}{4}}$.

~mahaler

## Video Solution by Interstigation

https://www.youtube.com/watch?v=mq4e-s9ENas

## See Also


| **2021 Fall AMC 10B** (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13)) | |
|---|---|
| Preceded by<br>**Problem 12** | Followed by<br>**Problem 14** |
| 1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25 | |
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