AMC 10 Problem Series (2804)

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Friday
Jun 4, 2021 - Aug 20, 2021
7:30 - 9:00 PM ET (4:30 - 6:00 PM PT)

Homework

Lesson: 1 **2** 3 4 5 6 7 8 9 10 11 12

Homework: Lesson 2

You have completed **10** of **10** challenge problems.
Past Due **Jun 19**.

Readings

Lesson 2 Transcript: Fri, Jun 11

Challenge Problems

Total Score: 60 / 60

Problem 1 – Correct! – Score: 6 / 6 (2845)

Problem: Report Error

Yan is somewhere between his home and the stadium. To get to the stadium he can walk directly to the stadium, or else he can walk home and then ride his bicycle to the stadium. He rides 7 times as fast as he walks, and both choices require the same amount of time. What is the ratio of Yan's distance from his home to his distance from the stadium?

(A) $\frac{2}{3}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{6}{7}$

Solution:
Let x and y be the distance between Yan and his home and Yan and the stadium, respectively. Let w be the rate at which Yan can walk. Then the rate at which Yan can ride his bicycle is $7w$.

The time it takes for Yan to walk to the stadium is y/w, and the time it takes for Yan to go home and then ride his bicycle to the stadium is

$$\frac{x}{w} + \frac{x+y}{7w},$$

so

$$\frac{x}{w} + \frac{x+y}{7w} = \frac{y}{w}.$$

Multiplying both sides by $7w$, we get

$$7x + x + y = 7y,$$

which simplifies to $8x = 6y$. Then $x/y = 6/8 = \boxed{3/4}$. The answer is (B).

Your Response(s):

B

Problem 2 – Correct! – Score: 6 / 6 (2846)

Problem:

Report Error

Angelina drove at an average rate of 80 kph and then stopped 20 minutes for gas. After the stop, she drove at an average rate of 100 kph. Altogether she drove 250 km in a total trip time of 3 hours including the stop. Which equation could be used to solve for the time t in hours that she drove before her stop?

(A) $80t + 100(8/3 - t) = 250$ **(B)** $80t = 250$ **(C)** $100t = 250$ **(D)** $90t = 250$ **(E)** $80(8/3 - t) + 100t = 250$

Solution:

Before the stop, while Angelina was driving at 80 kph, she covered a distance of $80t$. She stopped for 20 minutes, which is 1/3 of an hour, so after the stop, she drove for $3 - t - 1/3 = 8/3 - t$ more hours. She drove at 100 kph, so she covered a distance of $100(8/3 - t)$. Hence, $\boxed{80t + 100(8/3 - t) = 250}$. The answer is (A).

Your Response(s):

- A

Problem 3 – Correct! – Score: 6 / 6 (2847)

Problem:

Report Error

When a bucket is two-thirds full of water, the bucket and water weigh a kilograms. When the bucket is one-half full of water the total weight is b kilograms. In terms of a and b, what is the total weight in kilograms when the bucket is full of water?

(A) $\frac{2}{3}a + \frac{1}{3}b$ **(B)** $\frac{3}{2}a - \frac{1}{2}b$ **(C)** $\frac{3}{2}a + b$ **(D)** $\frac{3}{2}a + 2b$ **(E)** $3a - 2b$

Solution:

Let x be the weight of the (empty) bucket, and let y be the weight of the water in a full bucket. Then $x + 2y/3 = a$ and $x + y/2 = b$. Subtracting these equations, we get $y/6 = a - b$, so $y = 6a - 6b$. Substituting into the second equation, we get $x + 3a - 3b = b$, so $x = 4b - 3a$. Hence, the weight of a full bucket is $x + y = (6a - 6b) + (4b - 3a) = \boxed{3a - 2b}$. The answer is (E).

Your Response(s):

- E

Problem 4 – Correct! – Score: 6 / 6 (2848)

Problem:

Report Error

On a 50-question multiple choice math contest, students receive 4 points for a correct answer, 0 points for an answer left blank, and -1 point for an incorrect answer. Jesse's total score on the contest was 99. What is the maximum number of questions that Jesse could have answered correctly?

(A) 25 **(B)** 27 **(C)** 29 **(D)** 31 **(E)** 33

Solution:

Let x, y, and z be the number of questions that Jesse answered correctly, left blank, and answered incorrectly, respectively. Then $x + y + z = 50$ and $4x - z = 99$.

From the second equation, $z = 4x - 99$. Substituting into the first equation, we get $x + y + 4x - 99 = 50$, which simplifies to $5x + y = 149$. Then $x \le 149/5 = 29 + 4/5$, so $x \le 29$.

For $x = 29$, $y = 4$, and $z = 17$, the conditions $x + y + z = 50$ and $4x - z = 99$ are satisfied, so the maximum number of questions that Jesse could have answered correctly is $\boxed{29}$. The answer is (C).

Your Response(s):

C

Problem 5 – Correct! – Score: 6 / 6 (2849)

Problem: Report Error

Roy bought a new battery-gasoline hybrid car. On a trip the car ran exclusively on its battery for the first 40 miles, then ran exclusively on gasoline for the rest of the trip, using gasoline at a rate of 0.02 gallons per mile. On the whole trip he averaged 55 miles per gallon. How long was the trip in miles?

(A) 140 **(B)** 240 **(C)** 440 **(D)** 640 **(E)** 840

Solution:

Let d be the length of the trip in miles. Roy used no gasoline for the 40 first miles, then used 0.02 gallons of gasoline per mile on the remaining $d - 40$ miles, for a total of $0.02(d - 40)$ gallons. Hence, his average mileage was

$$\frac{d}{0.02(d-40)} = 55.$$

Multiplying both sides by $0.02(d - 40)$, we get

$$d = 55 \cdot 0.02 \cdot (d - 40) = 1.1d - 44.$$

Then $0.1d = 44$, so $d = \boxed{440}$. The answer is (C).

Your Response(s):

C

Problem 6 – Correct! – Score: 6 / 6 (2850)

Problem: Report Error

Sarah pours four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then transfers half the coffee from the first cup to the second and, after stirring thoroughly, transfers half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

(A) 1/4 **(B)** 1/3 **(C)** 3/8 **(D)** 2/5 **(E)** 1/2

Solution:

Initially, the first cup has four ounces of coffee, and the second cup has four ounces of cream. After she transfers half the coffee from the first cup to the second cup, the first cup has two ounces of coffee, and the second cup contains a mixture of two ounces of coffee and four ounces of cream. In other words, the second cup contains six ounces, which is 1/3 coffee and 2/3 cream.

Sarah then transfers half the mixture in the second cup back to the first cup. Sarah transfers three ounces, which is $3 \cdot 1/3 = 1$ ounce coffee, and $3 \cdot 2/3 = 2$ ounces cream. Hence, the first cup now contains $2 + 1 = 3$ ounces coffee, and 2 ounces cream, so the fraction that is cream is $\boxed{2/5}$. The answer is (D).

Your Response(s):

D

Problem 7 – Correct! – Score: 6 / 6 (2851)

Problem: Report Error

Andrea and Lauren are 20 kilometers apart. They bike toward one another with Andrea traveling three times as fast as Lauren, and the distance between them decreasing at a rate of 1 kilometer per minute. After 5 minutes, Andrea stops biking because of a flat tire and waits for Lauren. After how many minutes from the time they started to bike does Lauren reach Andrea?

(A) 20 **(B)** 30 **(C)** 55 **(D)** 65 **(E)** 80

Solution:

Since Andrea and Lauren are initially 20 kilometers apart, and the distance between them decreases at 1 kilometer per minute, they are due to meet in 20 minutes. Since Andrea travels three times as fast as Lauren, she would cover three times more distance as Lauren. Hence, Andrea would cover 15 kilometers, and Lauren would cover 5 kilometers. It follows that Andrea travels at $15/20 = 3/4$ kilometers per minute, and Lauren travels at $5/20 = 1/4$ kilometers per minute.

After 5 minutes, when Andrea gets her flat tire, the distance between Andrea and Lauren is still 15 kilometers. It takes Lauren another $15/(1/4) = 60$ minutes to reach Andrea. Therefore, from the time they started to bike, it takes Lauren $5 + 60 = \boxed{65}$ minutes to reach Andrea. The answer is (D).

Your Response(s):

D

Problem 8 – Correct! – Score: 6 / 6 (2852)

Problem: Report Error

Andy's lawn has twice as much area as Beth's lawn and three times as much area as Carlos' lawn. Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as Andy's mower. If they all start to mow their lawns at the same time, who will finish first?

(A) Andy **(B)** Beth **(C)** Carlos **(D)** Andy and Carlos tie for first. **(E)** All three tie.

Solution:

Let k be the area of Andy's lawn. Then Beth's lawn has area $k/2$, and Carlos's lawn has area $k/3$.

Let r be the rate of Carlos's mower. Then Beth's mower has a rate of $2r$, and Andy's mower has a rate of $3r$.

Hence, Andy can cut his lawn in a time of

$$\frac{k}{3r},$$

Beth can cut her lawn in a time of

$$\frac{k/2}{2r} = \frac{k}{4r},$$

and Carlos can cut his lawn in a time of

$$\frac{k/3}{} \quad \frac{k}{}$$

$$\frac{k/3}{r} = \frac{k}{3r}.$$

Since $\frac{k}{4r} < \frac{k}{3r}$, $\boxed{\text{Beth}}$ finishes her lawn first. The answer is (B).

Your Response(s):

B

Problem 9 – Correct! – Score: 6 / 6 (2853)

Problem: Report Error

It takes A algebra books (all the same thickness) and H geometry books (all the same thickness, which is greater than that of an algebra book) to completely fill a certain shelf. Also, S of the algebra books and M of the geometry books would fill the same shelf. Finally, E of the algebra books alone would fill this shelf. Given that A, H, S, M, and E are distinct positive integers, it follows that E is

(A) $\frac{AM+SH}{M+H}$ (B) $\frac{AM^2+SH^2}{M^2+H^2}$ (C) $\frac{AH-SM}{M-H}$ (D) $\frac{AM-SH}{M-H}$ (E) $\frac{AM^2-SH^2}{M^2-H^2}$

Solution:

Let x and y be the thicknesses of an algebra book and geometry book, respectively, and let z be the length of the shelf. Then from the given information,

$$\begin{aligned} Ax + Hy &= z, \\ Sx + My &= z, \\ Ex &= z. \end{aligned}$$

From the third equation, $x = z/E$. Substituting into the first two equations, we get

$$\begin{aligned} \frac{A}{E}z + Hy &= z, \\ \frac{S}{E}z + My &= z. \end{aligned}$$

From the first equation,

$$Hy = z - \frac{A}{E}z = \frac{E-A}{E}z,$$

so

$$\frac{y}{z} = \frac{E-A}{EH}.$$

From the second equation,

$$My = z - \frac{S}{E}z = \frac{E-S}{E}z,$$

so

$$\frac{y}{z} = \frac{E-S}{ME}.$$

Hence,

$$\frac{E-A}{EH} = \frac{E-S}{ME}.$$

Multiplying both sides by HME, we get $ME - AM = HE - HS$. Then $(M-H)E = AM - HS$, so

$$E = \boxed{\frac{AM-HS}{M-H}}.$$

The answer is (D).

Your Response(s):

D

Problem 10 – Correct! – Score: 6 / 6 (2854)

Problem: Report Error

One morning each member of Angela's family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

Solution:

Let x and y be the total amounts of milk and coffee (in cups) consumed by Angela's family each morning. Each family member drinks a cup of coffee, so the number of people in Angela's family is $x+y$.

Angela's cup contains a quarter of the milk and a sixth of the coffee, so $x/4 + y/6 = 1$. Multiplying this equation by 4, we get

$$x + \frac{2y}{3} = 4.$$

Hence, $x + y > x + 2y/3 = 4$. Multiplying the equation by $3/2$, we get

$$\frac{3x}{2} + y = 6.$$

Hence, $x + y < 3x/2 + y = 6$. The only possible value of $x+y$ is $\boxed{5}$. The answer is (C).

Your Response(s):

C