2021 Fall AMC 12B Problems/Problem 6

The following problem is from both the 2021 Fall AMC 10B #8 and 2021 Fall AMC 12B #6, so both problems redirect to this page.

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Problem

The largest prime factor of 16384 is 2 because $16384 = 2^{14}$. What is the sum of the digits of the greatest prime number that is a divisor of 16383?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 22$

Solution

We have

$$\begin{aligned} 16383 &= 2^{14} - 1 \\ &= \left(2^7 + 1\right)\left(2^7 - 1\right) \\ &= 129 \cdot 127 \\ &= 3 \cdot 43 \cdot 127. \end{aligned}$$

Therefore, the greatest prime divisor of 16383 is 127. The sum of its digits is $1 + 2 + 7 = \boxed{\textbf{(C)}\ 10}$.

~Steven Chen (www.professorchenedu.com) ~NH14 ~kingofpineapplz ~Arcticturn

Video Solution by Interstigation

https://youtu.be/p9_RH4s-kBA?t=1121

See Also

2021 Fall AMC 10B (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
Preceded by **Problem 7**	Followed by **Problem 9**
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2021 Fall AMC 12B (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
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All AMC 12 Problems and Solutions