2021 Fall AMC 12B Problems/Problem 2
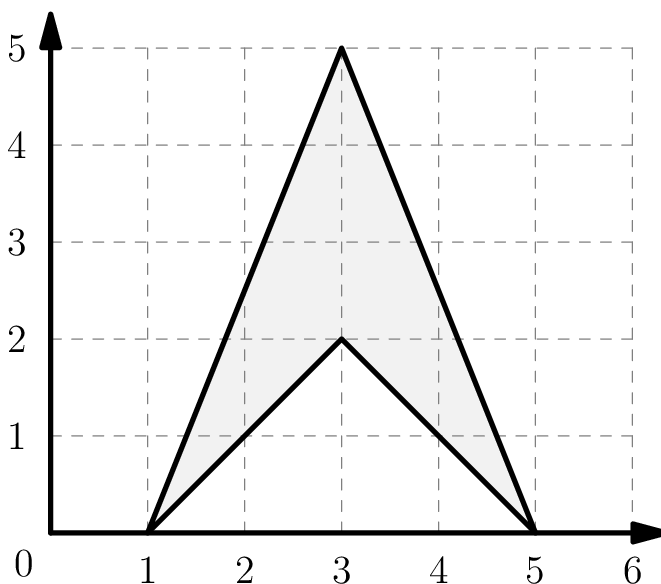
The following problem is from both the 2021 Fall AMC 10B #2 and 2021 Fall AMC 12B #2, so both problems redirect to this page.

Contents

Problem

What is the area of the shaded figure shown below?

$\textbf{(A) } 4 \qquad \textbf{(B) } 6 \qquad \textbf{(C) } 8 \qquad \textbf{(D) } 10 \qquad \textbf{(E) } 12$

Solution 1 (Area Addition)

The line of symmetry divides the shaded figure into two congruent triangles, each with base 3 and height $2.$

Therefore, the area of the shaded figure is

$$2 \cdot \left(\frac{1}{2} \cdot 3 \cdot 2 \right) = 2 \cdot 3 = \boxed{\textbf{(B) } 6}.$$

~MRENTHUSIASM ~Wilhelm Z

Solution 2 (Area Subtraction)

To find the area of the shaded figure, we subtract the area of the smaller triangle (base 4 and height 2) from the area of the larger triangle (base 4 and height 5):

$$\frac{1}{2} \cdot 4 \cdot 5 - \frac{1}{2} \cdot 4 \cdot 2 = 10 - 4 = \boxed{\textbf{(B) } 6}.$$

~MRENTHUSIASM ~Steven Chen (www.professorchenedu.com)

Solution 3 (Shoelace Theorem)

The consecutive vertices of the shaded figure are $(1, 0), (3, 2), (5, 0),$ and $(3, 5).$ By the Shoelace Theorem, the area is

$$\frac{1}{2} \cdot |(1 \cdot 2 + 3 \cdot 0 + 5 \cdot 5 + 3 \cdot 0) - (0 \cdot 3 + 2 \cdot 5 + 0 \cdot 3 + 5 \cdot 1)| = \frac{1}{2} \cdot 12 = \boxed{\textbf{(B) } 6}.$$

~Taco12 ~I-AM-DA-KING

Solution 4 (Pick's Theorem)

We have 4 lattice points in the interior and 6 lattice points on the boundary. By Pick's Theorem, the area of the shaded figure is

$$4 + \frac{6}{2} - 1 = 4 + 3 - 1 = \boxed{\textbf{(B) } 6}.$$

~danprathab

See Also

2021 Fall AMC 10B (**Problems · Answer Key ·** Resources (http://www.artofproblemsolving.com/community /c13))	
Preceded by **Problem 1**	Followed by **Problem 3**
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2021 Fall AMC 12B (**Problems · Answer Key ·** Resources (http://www.artofproblemsolving.com/community /c13))	
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