Russian School of Math: Lesson 8

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Abstract

This note reviews a small number of problems from the Russian School of Math test. Written for personal use.

1

How many odd positive factors does 30^{10} have?

(a) 121

(b) 1331

(c) 665

(d) 666

(e) 667

Solution

2

Let x and y be random numbers such that $x, y \in [0, 1]$. Find the probability that

$$|x - 0.5| + |y - 1.5| \leq 1$$

(a) $\frac{1}{4}$

(b) $\frac{3}{4}$

(c) $\frac{1}{6}$

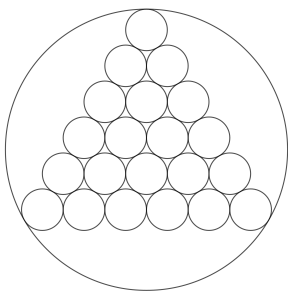
(d) $\frac{5}{6}$

(e) $\frac{1}{2}$

Solution

3

The figure shows 22 circles, 21 of which have radius 1. Any two neighboring circles are tangent, and so share a common point. What is the circumference of the largest circle?



(a) $\frac{2\pi}{3}\left(10\sqrt{3}+3\right)$

(b) $\frac{20\pi}{3}$

(c) $\frac{20\sqrt{3}\pi}{3}$

(d) 13π

(e) 20π

Solution

4

Complex numbers a and b are such that $(x^2-2x+3)(x^2-4x+12)=(x^2+ax+6)(x^2+bx+6)$ for all values of x. What is $|a-b|$?

(a) $2\sqrt{2}$

(b) $3\sqrt{2}$

(c) $6\sqrt{2}$

(d) 12

(e) 2

Solution

5

How many 8-digit numbers have the following property: the number is divisible by 11 and its digits are $1, 2, \ldots, 8$ without repetitions?

(a) 3456

(b) 1152

(c) 5040

(d) 40320

(e) 4608

Solution

6

A rectangular 2×10 grid is randomly covered by ten 1×2 rectangles (dominoes). What is the probability that the two squares in the fifth column from the left will be covered by different dominoes?

(a) $\frac{64}{89}$

(b) $\frac{49}{89}$

(c) $\frac{25}{89}$

(d) $\frac{40}{89}$

(e) $\frac{1}{2}$

Solution

7

Let (a_n) be a number sequence defined as follows:

$$a_1 = 1, \quad a_2 = 1, \quad a_3 = \frac{1}{2} \quad \text{and} \quad a_{n+1} = \frac{a_n^3 a_{n-2}}{a_{n-1}^3 + a_{n-2}a_{n-1}a_n}$$

where $n = 3, 4, \ldots$

Calculate

$$S_{10} = \frac{a_1}{a_2} + 2\frac{a_2}{a_3} + 3\frac{a_3}{a_4} + \ldots + 10\frac{a_{10}}{a_{11}}$$

(a) $11!$

(b) $11! - 1$

(c) $10! + 3$

(d) $10! + 11$

(e) $10 \cdot 11$

Solution