Russian School of Math: Lesson 2

James & Patrick

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Abstract

This note reviews a small number of problems from the Russian School of Math test. Written for personal use.

1

Find all right triangles with integer sides ($a \leq b \leq c$) such that the area and perimeter are equal.

Solve the problem without using the Pythagorean triplets theorem.

Solution

Solution: $(a, b, c) \in \{(5, 12, 13), (6, 8, 10)\}$

2

Solve in integers

$$(x - y)^3 + (y - z)^3 + (z - x)^3 = 30$$

Solution

3

Solve in integers

$$x + y = x^2 - xy + y^2$$

Solution

4

Find $3x^2y^2$ if x and y are integers such that:

$$y^2 + 3x^2y^2 = 30x^2 + 517$$

Solution

5

Prove that all integer solutions to the equation

$$xy = zw$$

are $x = mn$, $y = pq$, $z = mp$, $w = nq$, where m, n, p, q are integers and $\text{GCD}(n, p) = 1$.

Solution

6

Solve the following equation in positive integers:

$$x^{-2} + y^{-2} = z^{-2}$$

Solution