

# Art Of Problem Solving - AMC 10

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### **Abstract**

Notes on the AMC-10 Course by Art Of Problem Solving (AOPS). Copyright restrictions may apply. Written for personal use. Please report typos and errors over at <https://github.com/ptocher/Math/tree/master/aops>.

1.

In a collection of red, blue, and green marbles, there are 25% more red marbles than blue marbles, and there are 60% more green marbles than red marbles. Suppose that there are  $r$  red marbles. What is the total number of marbles in the collection?

- (A)  $2.85r$    (B)  $3r$    (C)  $3.4r$    (D)  $3.85r$    (E)  $4.25r$

2.

At the beginning of the school year, 50% of all students in Mr. Wells' math class answered "Yes" to the question "Do you love math", and 50% answered "No". At the end of the school year, 70% answered "Yes" and 30% answered "No". Altogether,  $x\%$  of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of  $x$ ?

- (A) 0   (B) 20   (C) 40   (D) 60   (E) 80

3.

The average of the numbers  $1, 2, 3, \dots, 98, 99$ , and  $x$  is  $100x$ . What is  $x$ ?

- (A)  $\frac{49}{101}$    (B)  $\frac{50}{101}$    (C)  $\frac{1}{2}$    (D)  $\frac{51}{101}$    (E)  $\frac{50}{99}$

4.

A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each of the juniors receive on the test?

- (A) 85   (B) 88   (C) 93   (D) 94   (E) 98

5.

Let  $a$  and  $b$  be relatively prime integers with  $a > b > 0$  and

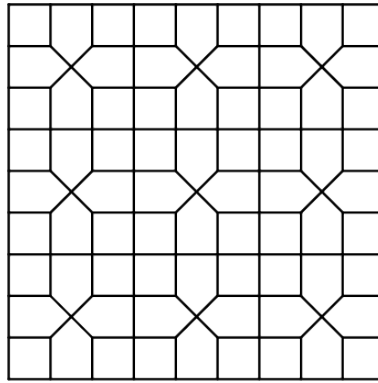
$$\frac{a^3 - b^3}{(a - b)^3} = \frac{73}{3}$$

What is  $a - b$ ?

- (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

6.

The plane is tiled by congruent squares of side length  $a$  and congruent pentagons of side lengths  $a$  and  $\frac{a\sqrt{2}}{2}$ , as arranged in the diagram below. The percent of the plane that is enclosed by the pentagons is closest to



- (A) 50   (B) 52   (C) 54   (D) 56   (E) 58

7.

Let  $A$ ,  $M$ , and  $C$  be non-negative integers such that  $A + M + C = 10$ . What is the maximum value of  $A \cdot M \cdot C + A \cdot M + M \cdot C + C \cdot A$ ?

- (A) 49   (B) 59   (C) 69   (D) 79   (E) 89

8.

The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is

- (A) 11   (B) 12   (C) 13   (D) 14   (E) 15

**Note:** The mode is the most frequent number, and the range is the difference between the lowest number and the highest number. For example, the mode of the numbers 2, 2, 7, 7, 7, 7, 13, 13, 13 is 7, and the range is  $13 - 2 = 11$ .

9.

A rectangular floor measures  $a$  feet by  $b$  feet, where  $a$  and  $b$  are positive integers with  $b > a$ . An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half the area of the entire floor. How many possibilities are there for the ordered pair  $(a, b)$ ?

- (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

10.

When 15 is appended to a list of integers, the mean is increased by 2. When 1 is appended to the enlarged list, the mean of the enlarged list is decreased by 1. How many integers were in the original list?

- (A) 4   (B) 5   (C) 6   (D) 7   (E) 8