Russian School of Math: Lesson 6

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Abstract

This note reviews a small number of problems from the Russian School of Math test. Written for personal use.

1

Convert 100_{b+1} to base b, where $b \geq 3$.

2

The repeating decimals of $0.\overline{ab}$ and $0.\overline{abc}$ satisfy $0.\overline{ab} + 0.\overline{abc} = \frac{33}{37}$, where a, b, c are (not necessarily distinct) digits. Find the three-digit number $\overline{abc}$.

3

Find the number of ending zeros of 2018! in base 9. Give your answer in base 9.

4

How many natural decimal numbers are 3-digit numbers when written in base 12 and 4-digit numbers when written in base 8.

5

A number N has three digits when expressed in base 7. When N is expressed in base 9 the digits are reversed. Find the middle digit in either representation of N.

6

The number n can be written in base 14 as $\overline{abc}_{14}$; it can be written in base 15 as $\overline{acb}_{15}$; and in base 6 as $\overline{acac}_6$, where $a > 0$. Find the base 10 representation of n.

7

What is the largest positive integer n less than $10,000$ such that in base 4, n and $3n$ have the same number of digits; in base 8, n and $7n$ have the same number of digits; and in base 16, n and $15n$ have the same number of digits? Express your answer in base 10.

8

Let $b(n)$ be the number of digits in the base-4 representation of n. Evaluate

$$\sum_{i=1}^{2013} b(i)$$