

Art Of Problem Solving - AMC 10

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Abstract

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1.

For how many integers n between 1 and 100 does $x^2 + x - n$ factor into the product of two linear factors with integer coefficients?

- (A) 0 (B) 1 (C) 2 (D) 9 (E) 10

2.

Suppose that a and b are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions a and b . Then the pair (a, b) is

- (A) $(-2, 1)$ (B) $(-1, 2)$ (C) $(1, -2)$ (D) $(2, -1)$ (E) $(4, 4)$

3.

Let f be the function defined by $f(x) = ax^2 - \sqrt{2}$ for some positive a . If $f(f(\sqrt{2})) = -\sqrt{2}$, then

- (A) $\frac{2-\sqrt{2}}{2}$ (B) $\frac{1}{2}$ (C) $2 - \sqrt{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{2+\sqrt{2}}{2}$

4.

Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. The number of possible values of k is

- (A) 0 (B) 1 (C) 2 (D) 4 (E) more than four

5.

Let $@$ denote the “averaged with” operation: $a@b = \frac{a+b}{2}$. Which of the following distributive laws holds for all numbers x , y , and z ?

- I. $x@(y+z) = (x@y) + (x@z)$
II. $x + (y@z) = (x+y)@(x+z)$
III. $x@(y@z) = (x@y)@(x@z)$

- (A) I only (B) II only (C) III only (D) I and III only (E) II and III only

6.

If $f(x) = ax^4 - bx^2 + x + 5$ and $f(-3) = 2$, then $f(3) =$

- (A) -5 (B) -2 (C) 1 (D) 3 (E) 8

7.

What is the sum of the reciprocals of the roots of the equation

$$\frac{2003}{2004}x + 1 + \frac{1}{x} = 0?$$

- (A) $-\frac{2004}{2003}$ (B) -1 (C) $\frac{2003}{2004}$ (D) 1 (E) $\frac{2004}{2003}$

8.

Let f be a polynomial function such that, for all real x ,

$$f(x^2 + 1) = x^4 + 5x^2 + 3.$$

- | | | | | |
|----------------------|---------------------|----------------------|---------------------|-------------------|
| (A) $x^4 + 5x^2 + 1$ | (B) $x^4 + x^2 - 3$ | (C) $x^4 - 5x^2 + 1$ | (D) $x^4 + x^2 + 3$ | (E) none of these |
|----------------------|---------------------|----------------------|---------------------|-------------------|

9.

The polynomial $x^3 - ax^2 + bx - 2010$ has three positive integer roots. What is the smallest possible value of a ?

- | | | | | |
|--------|--------|--------|---------|---------|
| (A) 78 | (B) 88 | (C) 98 | (D) 108 | (E) 118 |
|--------|--------|--------|---------|---------|

10.

Let f be a function for which $f(x/3) = x^2 + x + 1$. Find the sum of all values of z for which $f(3z) = 7$.

- | | | | | |
|------------|------------|-------|-----------|-----------|
| (A) $-1/3$ | (B) $-1/9$ | (C) 0 | (D) $5/9$ | (E) $5/3$ |
|------------|------------|-------|-----------|-----------|