Art Of Problem Solving - AMC 10
Week 8

Patrick & James Toche

July 31, 2021

Abstract

Notes on the AMC-10 Course by Art Of Problem Solving (AOPS). Copyright restrictions may apply. Written for personal use. Please report typos and errors over at https://github.com/ptoche/Math/tree/master/aops.

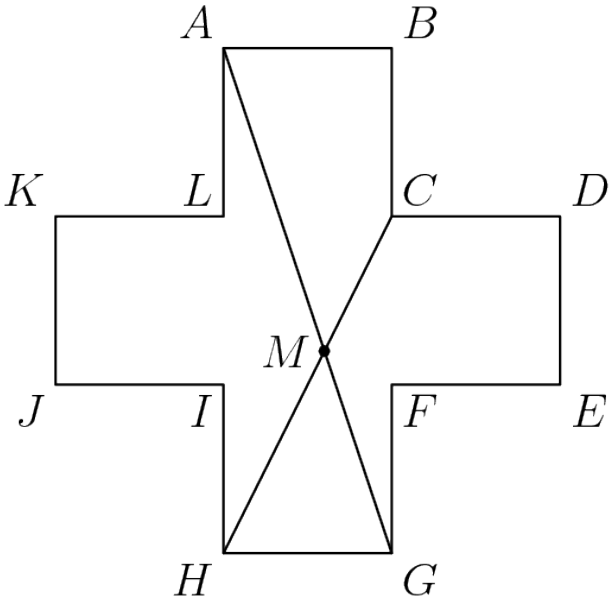
1.

A solid cube has side length 3 inches. A 2-inch by 2-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?

(A) 7 (B) 8 (C) 10 (D) 12 (E) 15

2.

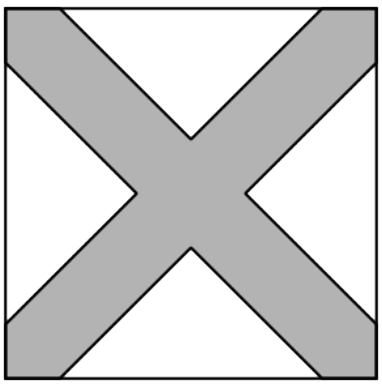
Consider the 12-sided polygon $ABCDEFGHIJKL$, as shown. Each of its sides has length 4, and each two consecutive sides form a right angle. Suppose that AG and CH meet at M. What is the area of quadrilateral $ABCM$?



(A) $\frac{44}{3}$ (B) 16 (C) $\frac{88}{5}$ (D) 20 (E) $\frac{62}{3}$

3.

A paint brush is swept along both diagonals of a square to produce the symmetric painted area, as shown. Half the area of the square is painted. What is the ratio of the side length of the square to the brush width?



(A) $2\sqrt{2}+1$ (B) $3\sqrt{2}$ (C) $2\sqrt{2}+2$ (D) $3\sqrt{2}+1$ (E) $3\sqrt{2}+2$

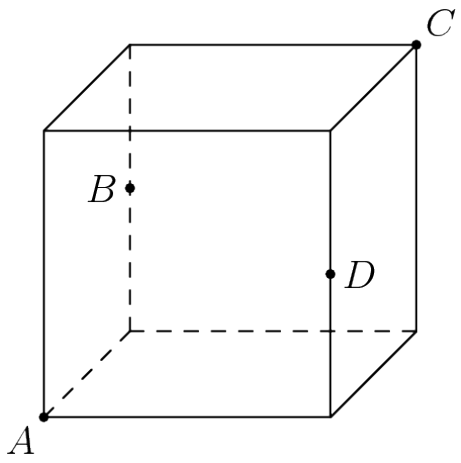
4.

A fly trapped inside a cubical box with side length 1 meter decides to relieve its boredom by visiting each corner of the box. It will begin and end in the same corner and visit each of the other corners exactly once. To get from a corner to any other corner, it will either fly or crawl in a straight line. What is the maximum possible length, in meters, of its path?

(A) $4+4\sqrt{2}$ (B) $2+4\sqrt{2}+2\sqrt{3}$ (C) $2+3\sqrt{2}+3\sqrt{3}$ (D) $4\sqrt{2}+4\sqrt{3}$ (E) $3\sqrt{2}+5\sqrt{3}$

5.

A cube with side length 1 is sliced by a plane that passes through two diagonally opposite vertices A and C and the midpoints B and D of two opposite edges not containing A or C, as shown. What is the area of quadrilateral $ABCD$?



(A) $\frac{\sqrt{6}}{2}$ (B) $\frac{5}{4}$ (C) $\sqrt{2}$ (D) $\frac{3}{2}$ (E) $\sqrt{3}$

6.

A pyramid with a square base is cut by a plane that is parallel to its base and is 2 units from the base. The surface area of the smaller pyramid that is cut from the top is half the surface area of the original pyramid. What is the altitude of the original pyramid?

(A) 2 (B) $2+\sqrt{2}$ (C) $1+2\sqrt{2}$ (D) 4 (E) $4+2\sqrt{2}$

7.

Convex quadrilateral $ABCD$ has $AB = 9$ and $CD = 12$. Diagonals AC and BD intersect at E, $AC = 14$, and triangles AED and BEC have equal areas. What is AE?

(A) $\frac{9}{2}$ (B) $\frac{50}{11}$ (C) $\frac{21}{4}$ (D) $\frac{17}{3}$ (E) 6

8.

Quadrilateral $ABCD$ has $AB = BC = CD$, $\angle ABC = 70^\circ$, and $\angle BCD = 170^\circ$. What is the degree measure of $\angle BAD$?

(A) 75 (B) 80 (C) 85 (D) 90 (E) 95

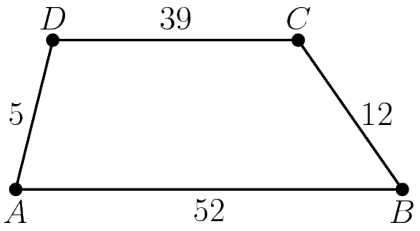
9.

Two distinct regular tetrahedra have all their vertices among the vertices of the same unit cube. What is the volume of the region formed by the intersection of the tetrahedra?

(A) $\frac{1}{12}$ (B) $\frac{\sqrt{2}}{12}$ (C) $\frac{\sqrt{3}}{12}$ (D) $\frac{1}{6}$ (E) $\frac{\sqrt{2}}{6}$

10.

In trapezoid $ABCD$ with bases AB and CD, we have $AB = 52$, $BC = 12$, $CD = 39$, and $DA = 5$. The area of $ABCD$ is



(A) 182 (B) 195 (C) 210 (D) 234 (E) 260