2021 AMC 10A Problems/Problem 4

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Problem

A cart rolls down a hill, travelling 5 inches the first second and accelerating so that during each successive 1-second time interval, it travels 7 inches more than during the previous 1-second interval. The cart takes 30 seconds to reach the bottom of the hill. How far, in inches, does it travel?

$\textbf{(A)}\ 215 \qquad \textbf{(B)}\ 360 \qquad \textbf{(C)}\ 2992 \qquad \textbf{(D)}\ 3195 \qquad \textbf{(E)}\ 3242$

Solution 1 (Arithmetic Series)

Since

$$\text{Distance} = \text{Speed} \cdot \text{Time},$$

we seek the sum

$$5 \cdot 1 + 12 \cdot 1 + 19 \cdot 1 + 26 \cdot 1 + \cdots = 5 + 12 + 19 + 26 + \cdots,$$

in which there are 30 terms.

The last term is $5 + 7 \cdot (30 - 1) = 208.$ Therefore, the requested sum is

$$5 + 12 + 19 + 26 + \cdots + 208 = \frac{5 + 208}{2} \cdot 30 = \boxed{\textbf{(D)}\ 3195}.$$

Recall that to find the sum of an arithmetic series, we take the average of the first and last terms, then multiply by the number of terms:

$$\text{Sum} = \frac{\text{First} + \text{Last}}{2} \cdot \text{Count}$$

~MRENTHUSIASM

Solution 2 (Arithmetic Series)

The distance (in inches) traveled within each 1-second interval is:

$$5, 5 + 1(7), 5 + 2(7), \ldots, 5 + 29(7).$$

This is an arithmetic sequence so the total distance travelled, found by summing them up is:

$$\text{number of terms}\cdot\text{average of terms} = \text{number of terms}\cdot\frac{\text{first term} + \text{last term}}{2}.$$

Or,

$$30 \cdot \frac{5 + 5 + 29(7)}{2} = 15 \cdot 213 = \boxed{\textbf{(D) } 3195}.$$

~BakedPotato66

Solution 3 (Answer Choices and Modular Arithmetic)

From the 30-term sum

$$5 + 12 + 19 + 26 + \cdots$$

in Solution 1, taking modulo 10 gives

$$5+12+19+26+\cdots \equiv 3\cdot(5+2+9+6+3+0+7+4+1+8) = 3\cdot 45 \equiv 5 \pmod{10}.$$

The only answer choices congruent to 5 modulo 10 are $\textbf{(A)}$ and $\textbf{(D)}$. By a quick estimation, $\textbf{(A)}$ is too small, leaving us with $\boxed{\textbf{(D) } 3195}$.

~MRENTHUSIASM

Solution 4 (Motion With Constant Acceleration)

This problem can be solved by physics method. The average speed increases 7 in/s per second. So, the acceleration $a = 7\text{ in/s}^2$. The average speed of the first second is 5 in/s. We can know the initial velocity $v_0 = 5 - 0.5 \cdot 7 = 1.5$. The displacement at $t = 30$ is

$$s = \frac{1}{2}at^2 + v_0t = \frac{1}{2} \cdot 7 \cdot 30^2 + 1.5 \cdot 30 = \boxed{\textbf{(D) } 3195}.$$

~Bran_Qin

Video Solution (Simple and Quick)

https://youtu.be/qLDkSnxLvxM

~ Education, the Study of Everything

Video Solution (Arithmetic Sequence but in a Different Way)

https://www.youtube.com/watch?v=sJa7uB-UoLc&list=PLexHyfQ8DMuKqltG3cHT7Di4jhVl6L4YJ&index=4

~ North America Math Contest Go Go Go

Video Solution (Using Arithmetic Sequence)

https://youtu.be/7NSfDCJFRUg

~ pi_is_3.14

Video Solution

https://youtu.be/aO-GklwkBfI

~savannahsolver

Video Solution by TheBeautyofMath

https://youtu.be/50CThrk3RcM?t=262

~IceMatrix

Video Solution by The Learning Royal

https://youtu.be/slVBYmcDMOI

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Preceded by **Problem 3**	Followed by **Problem 5**
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