2021 AMC 12A Problems/Problem 4

The following problem is from both the 2021 AMC 10A #7 and 2021 AMC 12A #4, so both problems redirect to this page.

Contents

Problem

Tom has a collection of 13 snakes, 4 of which are purple and 5 of which are happy. He observes that

- all of his happy snakes can add,
- none of his purple snakes can subtract, and
- all of his snakes that can't subtract also can't add.

Which of these conclusions can be drawn about Tom's snakes?

$\textbf{(A)}$ Purple snakes can add.

$\textbf{(B)}$ Purple snakes are happy.

$\textbf{(C)}$ Snakes that can add are purple.

$\textbf{(D)}$ Happy snakes are not purple.

$\textbf{(E)}$ Happy snakes can't subtract.

Solution 1 (Comprehensive Explanation of Logic)

We are given that

$$\text{happy} \implies \text{can add}, \tag{1}$$
$$\text{purple} \implies \text{cannot subtract}, \tag{2}$$
$$\text{cannot subtract} \implies \text{cannot add}. \tag{3}$$

Two solutions follow from here:

Solution 1.1 (Intuitive)

Combining (2) and (3) gives

$$\text{happy} \implies \text{can add}, \tag{1}$$

$$\underbrace{\text{purple} \implies \overbrace{\text{cannot subtract}}^{(3)}}_{(2)} \implies \text{cannot add}. \tag{$*$}$$

Clearly, the answer is $\boxed{\textbf{(D)}}$.

~MRENTHUSIASM (credit given to abhinavg0627)

Solution 1.2 (Rigorous)

Recall that every conditional statement $\bar{p} \implies \bar{q}$ is always logically equivalent to its contrapositive $\neg\bar{q} \implies \neg\bar{p}$.

Combining (1), (2) and (3) gives

$$\underbrace{\text{purple} \implies \overbrace{\text{cannot subtract}}^{(3)}}_{(2)} \implies \underbrace{\text{cannot add} \implies \text{not happy}}_{\text{Contrapositive of } (1)}. \tag{$**$}$$

Applying the hypothetical syllogism to $(**)$, we conclude that

$$\text{purple} \implies \text{not happy},$$

whose contrapositive is

$$\text{happy} \implies \text{not purple}.$$

Therefore, the answer is $\boxed{\textbf{(D)}}$.

Remark

The conclusions in the other choices do not follow from $(**)$:

$\textbf{(A)}$ $\text{purple} \implies \text{can add}$

$\textbf{(B)}$ $\text{purple} \implies \text{happy}$

$\textbf{(C)}$ $\text{can add} \implies \text{purple}$

$\textbf{(E)}$ $\text{happy} \implies \text{cannot subtract}$

~MRENTHUSIASM

Solution 2 (Process of Elimination)

From Solution 1.1, we can also see this through the process of elimination. Statement A is false because purple snakes cannot add. B is false as well because since happy snakes can add and purple snakes can not add, purple snakes are not happy snakes. E is false using the same reasoning, purple snakes are not happy snakes so happy snakes can subtract since purple snakes cannot subtract. C is false since snakes that can add are happy, not purple. That leaves statement D. $\boxed{\textbf{(D)}}$ is the only correct statement.

~Bakedpotato66

Solution 3 (Rigorous)

We first convert each statement to "If X, then Y" form:

- If a snake is happy, then it can add.
- If a snake is purple, then it can't subtract.
- If a snake can't subtract, then it can't add.

Now, we simply check the truth value for each statement:

A. Combining the last two propositions, we have
- If a snake is purple, then it can't add.

Thus, $\mathbf{(A)}$ is never true.

B. From the last part, we found that
- If a snake is purple, then it can't add.

Also, since the contrapositive of a proposition has the same truth value as the proposition itself, we know, from the first statement, that
- If a snake can't add, then it isn't happy.

Combining these two propositions, we find that
- If a snake is purple, then it isn't happy. Purple snakes are not happy.

Thus, $\mathbf{(B)}$ is never true.

C. From part $\mathbf{(A)}$, we found that "If a snake is purple, then it can't add." This implies its contrapositive, "If a snake can add, then it is not purple." is true, meaning $\mathbf{(C)}$ is NEVER true. [Thanks again to MRENTHUSIASM for pointing this out!]

D. From the first statement, we have
- If a snake is happy, then it can add.

From the contrapositive of the third statement, we have
- If a snake can add, then it can subtract.

Then, from the contrapositive of the second statement, we have
- If a snake can subtract, then it is not purple.

Combining all of these yields
- If a snake is happy, then it is not purple.

Thus, $\mathbf{(D)}$ is always true.

E. From the first proposition, we have
- If a snake is happy, then it can add.

From the contrapositive of the third proposition, we have
- If a snake can add, then it can subtract.

Combining these two propositions gives
- If a snake is happy, then it can subtract.

Thus, $\mathbf{(E)}$ is never true.

Therefore, $\boxed{\mathbf{(D)}}$ is our answer.

~ Peace09 (My First Wiki Solution!)

~ MRENTHUSIASM (Revision Suggestions and Code Adjustments)

Video Solution (Simple & Quick)

https://youtu.be/hJKHaIcyIxA

~ Education the Study of Everything

Video Solution by Aaron He (Sets)

https://www.youtube.com/watch?v=xTGDKBthWsw&t=164

Video Solution by Punxsutawney Phil

https://youtube.com/watch?v=MUHja8TpKGw&t=259s (Note that there's a slight error in the video I corrected in the description)

Video Solution by Hawk Math

https://www.youtube.com/watch?v=P5al76DxyHY

Video Solution (Using Logic to Eliminate Choices)

https://youtu.be/Mofw3VXHPyg

~ pi_is_3.14

Video Solution

https://youtu.be/uDJv06-cNrI

~savannahsolver

Video Solution by TheBeautyofMath

https://youtu.be/s6E4E06XhPU?t=202 (AMC10A)

https://youtu.be/rEWS75W0Q54?t=353 (AMC12A)

~IceMatrix

Video Solution by The Learning Royal

https://youtu.be/AWjOeBFyeb4

See also

2021 AMC 10A (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
Preceded by **Problem 6**	Followed by **Problem 8**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2021 AMC 12A (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
Preceded by **Problem 3**	Followed by **Problem 5**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	

Mathematics Competitions (http://amc.maa.org).