

# Art Of Problem Solving - AMC 10

## July 17, 2021

Patrick & James Toche

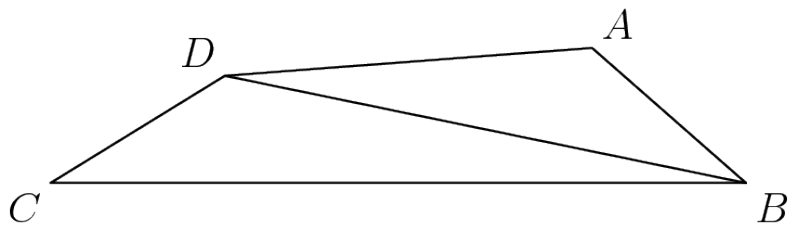
Revised: July 3, 2021

### **Abstract**

Notes on the AMC-10 Course by Art Of Problem Solving (AOPS). Copyright restrictions may apply. Written for personal use. Please report typos and errors over at <https://github.com/ptocher/Math/tree/master/aops>.

1.

In quadrilateral  $ABCD$ ,  $AB = 5$ ,  $BC = 17$ ,  $CD = 5$ ,  $DA = 9$ , and  $BD$  is an integer. What is  $BD$ ?



- |        |        |        |        |        |
|--------|--------|--------|--------|--------|
| (A) 11 | (B) 12 | (C) 13 | (D) 14 | (E) 15 |
|--------|--------|--------|--------|--------|

$x$

$x$
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**2.**

Rectangle  $ABCD$  has  $AB = 4$  and  $BC = 3$ . Segment  $\overline{EF}$  is constructed through  $B$  so that  $\overline{EF} \perp \overline{DB}$ , and  $A$  and  $C$  lie on  $\overline{DE}$  and  $\overline{DF}$ , respectively. What is  $EF$ ?

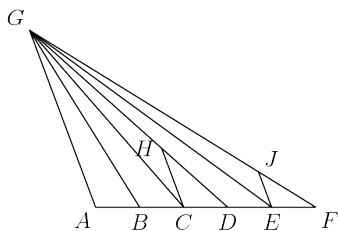
- |       |        |                      |                     |        |
|-------|--------|----------------------|---------------------|--------|
| (A) 9 | (B) 10 | (C) $\frac{125}{12}$ | (D) $\frac{103}{9}$ | (E) 12 |
|-------|--------|----------------------|---------------------|--------|

$x$

$x$
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3.

Points  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ , and  $F$  lie, in that order, on  $\overline{AF}$ , dividing it into five segments, each of length 1. Point  $G$  is not on line  $AF$ . Point  $H$  lies on  $\overline{GD}$ , and point  $J$  lies on  $\overline{GF}$ . The line segments  $\overline{HC}$ ,  $\overline{JE}$ , and  $\overline{AG}$  are parallel. Find  $HC/JE$ .



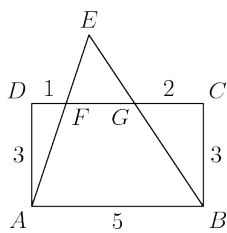
- |           |           |           |           |         |
|-----------|-----------|-----------|-----------|---------|
| (A) $5/4$ | (B) $4/3$ | (C) $3/2$ | (D) $5/3$ | (E) $2$ |
|-----------|-----------|-----------|-----------|---------|

$x$

$x$
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4.

In rectangle  $ABCD$ ,  $AB = 5$  and  $BC = 3$ . Points  $F$  and  $G$  are on  $\overline{CD}$  so that  $DF = 1$  and  $GC = 2$ . Lines  $AF$  and  $BG$  intersect at  $E$ . Find the area of triangle  $AEB$ .



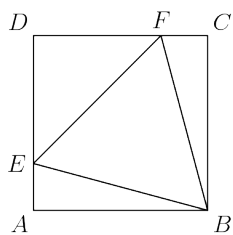
- (A) 10    (B)  $\frac{21}{2}$     (C) 12    (D)  $\frac{25}{2}$     (E) 15

$x$

$x$

5.

Points  $E$  and  $F$  are located on square  $ABCD$  so that triangle  $BEF$  is equilateral. What is the ratio of the area of triangle  $DEF$  to that of triangle  $ABE$ ?



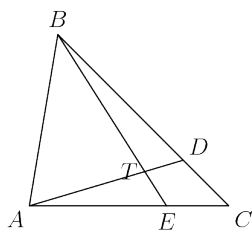
- (A)  $\frac{4}{3}$    (B)  $\frac{3}{2}$    (C)  $\sqrt{3}$    (D) 2   (E)  $1 + \sqrt{3}$

$x$

$x$

6.

In triangle  $ABC$  points  $D$  and  $E$  lie on  $\overline{BC}$  and  $\overline{AC}$ , respectively. If  $\overline{AD}$  and  $\overline{BE}$  intersect at  $T$  so that  $AT/DT = 3$  and  $BT/ET = 4$ , what is  $CD/BD$ ?



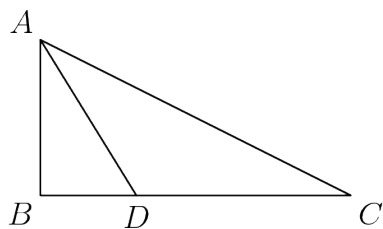
- |                   |                   |                    |                    |                    |
|-------------------|-------------------|--------------------|--------------------|--------------------|
| (A) $\frac{1}{8}$ | (B) $\frac{2}{9}$ | (C) $\frac{3}{10}$ | (D) $\frac{4}{11}$ | (E) $\frac{5}{12}$ |
|-------------------|-------------------|--------------------|--------------------|--------------------|

$x$

$x$
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7.

Triangle  $ABC$  has a right angle at  $B$ ,  $AB = 1$ , and  $BC = 2$ . The bisector of  $\angle BAC$  meets  $\overline{BC}$  at  $D$ . What is  $BD$ ?



- |                            |                            |                            |                                   |                   |
|----------------------------|----------------------------|----------------------------|-----------------------------------|-------------------|
| (A) $\frac{\sqrt{3}-1}{2}$ | (B) $\frac{\sqrt{5}-1}{2}$ | (C) $\frac{\sqrt{5}+1}{2}$ | (D) $\frac{\sqrt{6}+\sqrt{2}}{2}$ | (E) $2\sqrt{3}-1$ |
|----------------------------|----------------------------|----------------------------|-----------------------------------|-------------------|

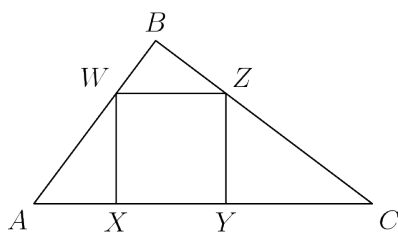
$x$

$x$
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8.

Right triangle  $ABC$  has  $AB = 3$ ,  $BC = 4$ , and  $AC = 5$ . Square  $XYZW$  is inscribed in triangle  $ABC$  with  $X$  and  $Y$  on  $\overline{AC}$ ,  $W$  on  $\overline{AB}$ , and  $Z$  on  $\overline{BC}$ . What is the side length of the square?



- |                   |                     |                    |                     |       |
|-------------------|---------------------|--------------------|---------------------|-------|
| (A) $\frac{3}{2}$ | (B) $\frac{60}{37}$ | (C) $\frac{12}{7}$ | (D) $\frac{23}{13}$ | (E) 2 |
|-------------------|---------------------|--------------------|---------------------|-------|

$x$

$x$
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**9.**

A triangle with sides of 5, 12, and 13 has both an inscribed and a circumscribed circle. What is the distance between the centers of those circles?

- |                           |                   |                 |                           |                   |
|---------------------------|-------------------|-----------------|---------------------------|-------------------|
| (A) $\frac{3\sqrt{5}}{2}$ | (B) $\frac{7}{2}$ | (C) $\sqrt{15}$ | (D) $\frac{\sqrt{65}}{2}$ | (E) $\frac{9}{2}$ |
|---------------------------|-------------------|-----------------|---------------------------|-------------------|

$x$

$x$
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**10.**

In triangle  $ABC$  we have  $AB = 25$ ,  $BC = 39$ , and  $AC = 42$ . Points  $D$  and  $E$  are on  $\overline{AB}$  and  $\overline{AC}$  respectively, with  $AD = 19$  and  $AE = 14$ . What is the ratio of the area of triangle  $ADE$  to the area of the quadrilateral  $BCED$ ?

- |                        |                     |                   |                     |       |
|------------------------|---------------------|-------------------|---------------------|-------|
| (A) $\frac{266}{1521}$ | (B) $\frac{19}{75}$ | (C) $\frac{1}{3}$ | (D) $\frac{19}{56}$ | (E) 1 |
|------------------------|---------------------|-------------------|---------------------|-------|

$x$

$x$
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