Russian School of Math: Lesson 5

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Abstract

This note reviews a small number of problems from the Russian School of Math test. Written for personal use.

1

Find the last two digits of $2017^{20} + 2018^{20} + 2019^{20}$.

2

Find all n, $n \in \mathbb{N}$, such that $\varphi(n) = 2$.

3

Prove that if m and n are coprime, then $\varphi(m \cdot n) > \varphi(m) \cdot \varphi(n)$.

4

Find all ordered pairs (m, n), where $m, n \in \mathbb{N}$, $n > 1$ and $\varphi(\varphi(n^m)) = n$.

5

Let $d_1, d_2, \ldots, d_k$ be all natural divisors of n, $n \in \mathbb{N}$ such that $d_1 < d_2 < \ldots < d_k$. Prove that $\varphi(d_1) + \varphi(d_2) + \ldots + \varphi(d_k) = n$.