Art Of Problem Solving - AMC 10
Week 10

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August 14, 2021

Abstract

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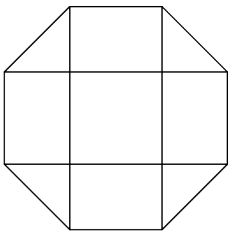
1.

Two fair coins are to be tossed once. For each head that results, one fair die is to be rolled. What is the probability that the sum of the die rolls is odd? (Note that if no die is rolled, the sum is 0.)

(A) $\frac{3}{8}$ (B) $\frac{1}{2}$ (C) $\frac{43}{72}$ (D) $\frac{5}{8}$ (E) $\frac{2}{3}$

2.

A dart board is a regular octagon divided into regions as shown. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is the probability that the dart lands within the center square?



(A) $\frac{\sqrt{2}-1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{2-\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{4}$ (E) $2-\sqrt{2}$

3.

Bernardo randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

(A) $\frac{47}{72}$ (B) $\frac{37}{56}$ (C) $\frac{2}{3}$ (D) $\frac{49}{72}$ (E) $\frac{39}{56}$

4.

Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1,2,3,\ldots,2010\}$. What is the probability that $abc+ab+a$ is divisible by 3?

(A) $\frac{1}{3}$ (B) $\frac{29}{81}$ (C) $\frac{31}{81}$ (D) $\frac{11}{27}$ (E) $\frac{13}{27}$

5.

Two points on the circumference of a circle of radius r are selected independently and at random. From each point a chord of length r is drawn in a clockwise direction. What is the probability that the two chords intersect?

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

6.

A bag contains two red beads and two green beads. You reach into the bag and pull out a bead, replacing it with a new red bead regardless of the color you pulled out. What is the probability that all beads in the bag are red after three such replacements?

(A) $\frac{1}{8}$	(B) $\frac{5}{32}$	(C) $\frac{9}{32}$	(D) $\frac{3}{8}$	(E) $\frac{7}{16}$

7.

For a particular peculiar pair of dice, the probabilities of rolling 1, 2, 3, 4, 5, and 6 on each die are in the ratio $1 : 2 : 3 : 4 : 5 : 6$. What is the probability of rolling a total of 7 on the two dice?

(A) $\frac{4}{63}$	(B) $\frac{1}{8}$	(C) $\frac{8}{63}$	(D) $\frac{1}{6}$	(E) $\frac{2}{7}$

8.

A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

(A) $\frac{3}{10}$	(B) $\frac{2}{5}$	(C) $\frac{1}{2}$	(D) $\frac{3}{5}$	(E) $\frac{7}{10}$

9.

Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, \ldots, 10\}$. The probability that Sergio's number is larger than the sum of the two numbers chosen by Tina is

(A) $\frac{2}{5}$	(B) $\frac{9}{20}$	(C) $\frac{1}{2}$	(D) $\frac{11}{20}$	(E) $\frac{24}{25}$

10.

A bug starts at one vertex of a cube and moves along the edges of the cube according to the following rule. At each vertex the bug will choose to travel along one of the three edges emanating from that vertex. Each edge has equal probability of being chosen, and all choices are independent. What is the probability that after seven moves the bug will have visited every vertex exactly once?

(A) $\frac{1}{2187}$	(B) $\frac{1}{729}$	(C) $\frac{2}{243}$	(D) $\frac{1}{81}$	(E) $\frac{5}{243}$