

Art Of Problem Solving - AMC 10

July 17, 2021

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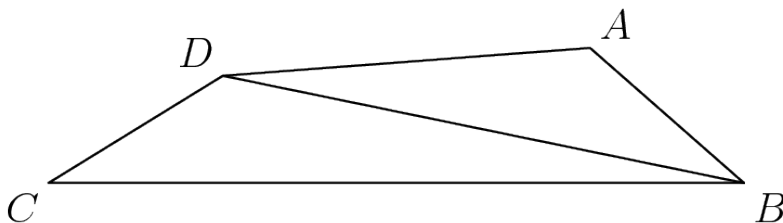
Revised: July 3, 2021

Abstract

Notes on the AMC-10 Course by Art Of Problem Solving (AOPS). Copyright restrictions may apply. Written for personal use. Please report typos and errors over at <https://github.com/ptocher/Math/tree/master/aops>.

1.

In quadrilateral $ABCD$, $AB = 5$, $BC = 17$, $CD = 5$, $DA = 9$, and BD is an integer. What is BD ?



- (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

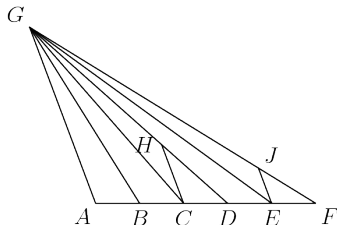
2.

Rectangle $ABCD$ has $AB = 4$ and $BC = 3$. Segment \overline{EF} is constructed through B so that $\overline{EF} \perp \overline{DB}$, and A and C lie on \overline{DE} and \overline{DF} , respectively. What is EF ?

- (A) 9 (B) 10 (C) $\frac{125}{12}$ (D) $\frac{103}{9}$ (E) 12

3.

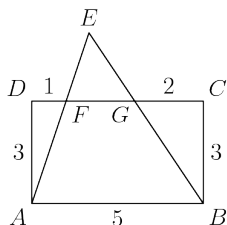
Points A, B, C, D, E , and F lie, in that order, on \overline{AF} , dividing it into five segments, each of length 1. Point G is not on line AF . Point H lies on \overline{GD} , and point J lies on \overline{GF} . The line segments \overline{HC} , \overline{JE} , and \overline{AG} are parallel. Find HC/JE .



- (A) $5/4$ (B) $4/3$ (C) $3/2$ (D) $5/3$ (E) 2

4.

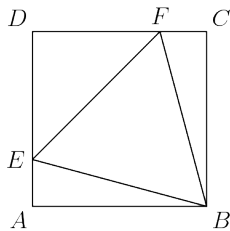
In rectangle $ABCD$, $AB = 5$ and $BC = 3$. Points F and G are on \overline{CD} so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E . Find the area of triangle AEB .



- (A) 10 (B) $\frac{21}{2}$ (C) 12 (D) $\frac{25}{2}$ (E) 15

5.

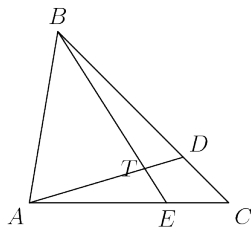
Points E and F are located on square $ABCD$ so that triangle BEF is equilateral. What is the ratio of the area of triangle DEF to that of triangle ABE ?



- (A) $\frac{4}{3}$ (B) $\frac{3}{2}$ (C) $\sqrt{3}$ (D) 2 (E) $1 + \sqrt{3}$

6.

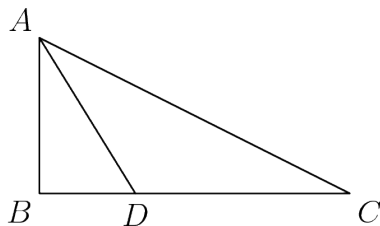
In triangle ABC points D and E lie on \overline{BC} and \overline{AC} , respectively. If \overline{AD} and \overline{BE} intersect at T so that $AT/DT = 3$ and $BT/ET = 4$, what is CD/BD ?



- (A) $\frac{1}{8}$ (B) $\frac{2}{9}$ (C) $\frac{3}{10}$ (D) $\frac{4}{11}$ (E) $\frac{5}{12}$

7.

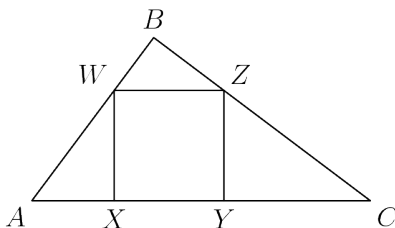
Triangle ABC has a right angle at B , $AB = 1$, and $BC = 2$. The bisector of $\angle BAC$ meets \overline{BC} at D . What is BD ?



- (A) $\frac{\sqrt{3}-1}{2}$ (B) $\frac{\sqrt{5}-1}{2}$ (C) $\frac{\sqrt{5}+1}{2}$ (D) $\frac{\sqrt{6}+\sqrt{2}}{2}$ (E) $2\sqrt{3} - 1$

8.

Right triangle ABC has $AB = 3$, $BC = 4$, and $AC = 5$. Square $XYZW$ is inscribed in triangle ABC with X and Y on \overline{AC} , W on \overline{AB} , and Z on \overline{BC} . What is the side length of the square?



- | | | | | |
|-------------------|---------------------|--------------------|---------------------|-------|
| (A) $\frac{3}{2}$ | (B) $\frac{60}{37}$ | (C) $\frac{12}{7}$ | (D) $\frac{23}{13}$ | (E) 2 |
|-------------------|---------------------|--------------------|---------------------|-------|

9.

A triangle with sides of 5, 12, and 13 has both an inscribed and a circumscribed circle. What is the distance between the centers of those circles?

- | | | | | |
|---------------------------|-------------------|-----------------|---------------------------|-------------------|
| (A) $\frac{3\sqrt{5}}{2}$ | (B) $\frac{7}{2}$ | (C) $\sqrt{15}$ | (D) $\frac{\sqrt{65}}{2}$ | (E) $\frac{9}{2}$ |
|---------------------------|-------------------|-----------------|---------------------------|-------------------|

10.

In triangle ABC we have $AB = 25$, $BC = 39$, and $AC = 42$. Points D and E are on \overline{AB} and \overline{AC} respectively, with $AD = 19$ and $AE = 14$. What is the ratio of the area of triangle ADE to the area of the quadrilateral $BCED$?

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|------------------------|---------------------|-------------------|---------------------|-------|
| (A) $\frac{266}{1521}$ | (B) $\frac{19}{75}$ | (C) $\frac{1}{3}$ | (D) $\frac{19}{56}$ | (E) 1 |
|------------------------|---------------------|-------------------|---------------------|-------|