2021 AMC 12A Problems/Problem 5

The following problem is from both the 2021 AMC 10A #8 and 2021 AMC 12A #5, so both problems redirect to this page.

Contents

Problem

When a student multiplied the number 66 by the repeating decimal, $1.\underline{a}\ \underline{b}\ \underline{a}\ \underline{b}\ldots = 1.\overline{\underline{a}\ \underline{b}}$, where a and b are digits, he did not notice the notation and just multiplied 66 times $1.\underline{a}\ \underline{b}$. Later he found that his answer is 0.5 less than the correct answer. What is the 2-digit number $\underline{a}\ \underline{b}$?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 60 \qquad \textbf{(E)}\ 75$

Solution 1

We are given that $66\left(1.\overline{\underline{a}\ \underline{b}}\right) - 0.5 = 66\left(1.\underline{a}\ \underline{b}\right)$, from which

$$\begin{aligned} 66\left(1.\overline{\underline{a}\ \underline{b}}\right) - 66\left(1.\underline{a}\ \underline{b}\right) &= 0.5 \\ 66\left(1.\overline{\underline{a}\ \underline{b}} - 1.\underline{a}\ \underline{b}\right) &= 0.5 \\ 66\left(0.\underline{0}\ \underline{0}\ \overline{\underline{a}\ \underline{b}}\right) &= 0.5 \\ 66\left(\frac{1}{100}\cdot 0.\overline{\underline{a}\ \underline{b}}\right) &= \frac{1}{2} \\ 0.\overline{\underline{a}\ \underline{b}} &= \frac{25}{33} \\ 0.\overline{\underline{a}\ \underline{b}} &= 0.\overline{75} \\ \underline{a}\ \underline{b} &= \boxed{\textbf{(E)}\ 75}. \end{aligned}$$

~MRENTHUSIASM

Solution 2

It is known that $\underline{0}.\overline{\underline{a}\ \underline{b}} = \frac{\underline{a}\ \underline{b}}{99}$ and $\underline{0}.\underline{a}\ \underline{b} = \frac{\underline{a}\ \underline{b}}{100}$.

Let $x = \underline{a}\ \underline{b}$. We have

$$66\left(1+\frac{x}{99}\right) - 66\left(1+\frac{x}{100}\right) = 0.5.$$

Expanding and simplifying give $\frac{x}{150} = 0.5$, so $x = \boxed{\textbf{(E)}\ 75}$.

~aop2014 ~BakedPotato66 ~MRENTHUSIASM

Solution 3 (Similar to Solution 2)

We have

$$66 \cdot \left(1+\frac{10a+b}{100}\right) + \frac{1}{2} = 66 \cdot \left(1+\frac{10a+b}{99}\right).$$

Expanding both sides, we have

$$66 + \frac{33(10\bar{a}+b)}{50} + \frac{\bar{1}}{2} = 66 + \frac{2(10\bar{a}+b)}{3}.$$

Subtracting 66 from both sides, we have

$$\frac{33(10\bar{a}+b)}{50} + \frac{\bar{1}}{2} = \frac{2(10\bar{a}+b)}{3}.$$

Multiplying both sides by $50 \cdot 3 = 150$, we have

$$99(10a+b) + 75 = 100(10a+b).$$

Thus, the answer is $10a+b = \boxed{\textbf{(E)}\ 75}$.

By letting $x = \underline{a}\ \underline{b} = 10a+b$, this solution is similar to Solution 2. In this solution, we solve for $10a+b$ as a whole.

-mathboy282 (Solution)

~MRENTHUSIASM (Minor Revision)

Video Solution (Simple & Quick)

https://youtu.be/9HI79V-vtCU

~ Education, the Study of Everything

Video Solution by Aaron He

https://www.youtube.com/watch?v=xTGDKBthWsw&t=4m12s

Video Solution (Use of Properties of Repeating Decimals)

https://www.youtube.com/watch?v=zS1u-ohUDzQ&list=PLexHyfQ8DMuKqltG3cHT7Di4jhVl6L4YJ&index=6\

~North America Math Contest Go Go Go

Video Solution by Punxsutawney Phil

https://youtube.com/watch?v=MUHja8TpKGw&t=359s

Video Solution by Hawk Math

https://www.youtube.com/watch?v=P5al76DxyHY

Video Solution (Using Repeating Decimal Properties)

https://youtu.be/vQZ13WiL4WU

~ pi_is_3.14

Video Solution

https://youtu.be/DOF3FYUsXsU

~savannahsolver

Video Solution by TheBeautyofMath

https://youtu.be/s6E4E06XhPU?t=360 (AMC 10A)

https://youtu.be/rEWS75W0Q54?t=511 (AMC 12A)

~IceMatrix

Video Solution by The Learning Royal

https://youtu.be/AWjOeBFyeb4

See also

2021 AMC 10A (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
Preceded by **Problem 7**	Followed by **Problem 9**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

2021 AMC 12A (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
Preceded by **Problem 4**	Followed by **Problem 6**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 12 Problems and Solutions	