Art Of Problem Solving - AMC 10
# Week 7

Patrick & James Toche

July 24, 2021

Abstract

Notes on the AMC-10 Course by Art Of Problem Solving (AOPS). Copyright restrictions may apply. Written for personal use. Please report typos and errors over at https://github.com/ptoche/Math/tree/master/aops.

1.

Rolly wishes to secure his dog with an 8-foot rope to a square shed that is 16 feet on each side. His preliminary drawings are shown.
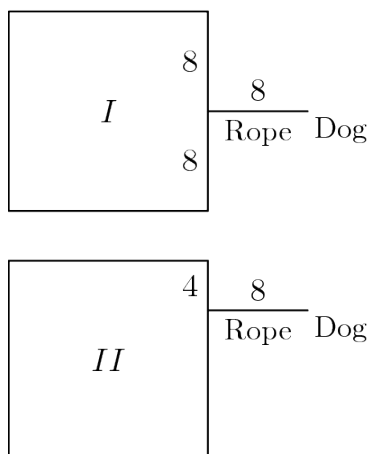
Which of these arrangements gives the dog the greater area to roam, and by how many square feet?

(A) I, by 8π (B) I, by 6π (C) II, by 4π (D) II, by 8π (E) II, by 10π

# 2.

A square of side length 1 and a circle of radius $\sqrt{3}/3$ share the same center. What is the area inside the circle, but outside the square?

(A) $\dfrac{\pi}{3} - 1$ (B) $\dfrac{2\pi}{9} - \dfrac{\sqrt{3}}{3}$ (C) $\dfrac{\pi}{18}$ (D) $\dfrac{1}{4}$ (E) $\dfrac{2\pi}{9}$

3.

Two distinct lines pass through the center of three concentric circles of radii 3, 2, and 1. The area of the shaded region in the diagram is 8/13 of the area of the unshaded region. What is the radian measure of the acute angle formed by the two lines?
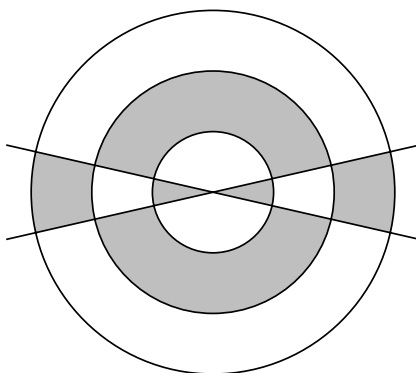
(A) $\dfrac{\pi}{8}$ (B) $\dfrac{\pi}{7}$ (C) $\dfrac{\pi}{6}$ (D) $\dfrac{\pi}{5}$ (E) $\dfrac{\pi}{4}$

4.

Many Gothic cathedrals have windows with portions containing a ring of congruent circles that are circumscribed by a larger circle. In the figure shown, the number of smaller circles is four. What is the ratio of the sum of the areas of the four smaller circles to the area of the larger circle?
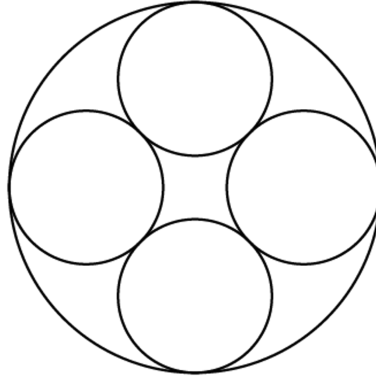
(A) $3 - 2\sqrt{2}$ (B) $2 - \sqrt{2}$ (C) $4(3 - 2\sqrt{2})$ (D) $\dfrac{1}{2}(3 - \sqrt{2})$ (E) $2\sqrt{2} - 2$

5.

Circles A, B, and C are externally tangent to each other and internally tangent to circle D. Circles B and C are congruent. Circle A has radius 1 and passes through the center of D. What is the radius of circle B?
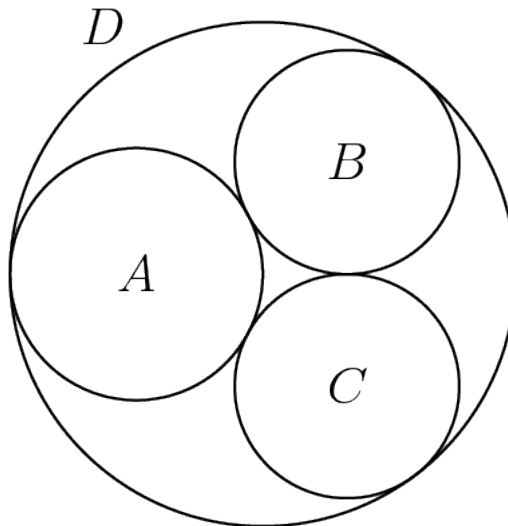
(A) $\dfrac{2}{3}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\dfrac{7}{8}$ (D) $\dfrac{8}{9}$ (E) $\dfrac{1 + \sqrt{3}}{3}$

6.

Let $\overline{AB}$ be a diameter of a circle and C be a point on $\overline{AB}$ with $2 \cdot AC = BC$. Let D and E be points on the circle such that $\overline{DC} \perp \overline{AB}$ and $\overline{DE}$ is a second diameter. What is the ratio of the area of triangle DCE to the area of triangle ABD?

3
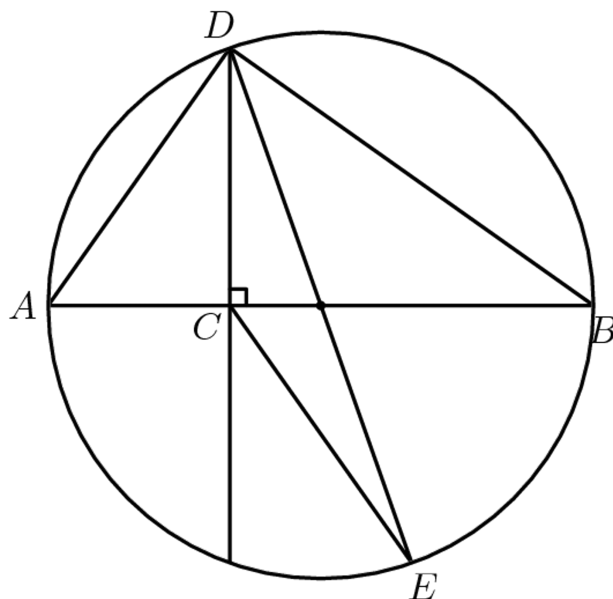
(A) $\dfrac{1}{6}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{3}$ (D) $\dfrac{1}{2}$ (E) $\dfrac{2}{3}$

7.

Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 feet and revolves at the constant rate of one revolution per minute. How many seconds does it take a rider to travel from the bottom of the wheel to a point 10 vertical feet above the bottom?

(A) 5   (B) 6   (C) 7.5   (D) 10   (E) 15

8.

In triangle ABC we have $AB = 7$, $AC = 8$, and $BC = 9$. Point D is on the circumscribed circle of the triangle so that $\overline{AD}$ bisects $\angle BAC$. What is the value of AD/CD?

(A) $\dfrac{9}{8}$ (B) $\dfrac{5}{3}$ (C) 2   (D) $\dfrac{17}{7}$ (E) $\dfrac{5}{2}$
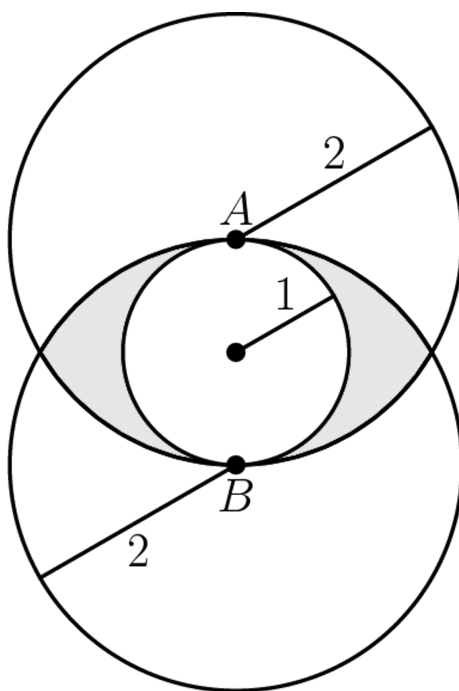
9.

Circles with centers O and P have radii 2 and 4, respectively, and are externally tangent. Points A and B are on the circle centered at O, and points C and D are on the circle centered at P, such that $\overline{AD}$ and $\overline{BC}$ are common external tangents to the circles. What is the area of hexagon $AOBCPD$?
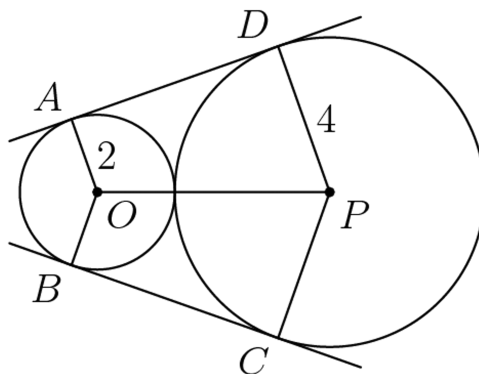
(A) $18\sqrt{3}$ (B) $24\sqrt{2}$ (C) 36 (D) $24\sqrt{3}$ (E) $32\sqrt{2}$

10.

A circle of radius 1 is internally tangent to two circles of radius 2 at points A and B, where AB is a diameter of the smaller circle. What is the area of the region, shaded in the figure, that is outside the smaller circle and inside each of the two larger circles?

(A) $\dfrac{5}{3}\pi - 3\sqrt{2}$ (B) $\dfrac{5}{3}\pi - 2\sqrt{3}$ (C) $\dfrac{8}{3}\pi - 3\sqrt{3}$ (D) $\dfrac{8}{3}\pi - 3\sqrt{2}$ (E) $\dfrac{8}{3}\pi - 2\sqrt{3}$