

2021 AMC 10A Problems/Problem 11

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Problem

For which of the following integers b is the base- b number $2021_b - 221_b$ not divisible by 3?

(A) 3 (B) 4 (C) 6 (D) 7 (E) 8

Solution 1 (Factor)

We have

$$\begin{aligned} 2021_b - 221_b &= (2021_b - 21_b) - (221_b - 21_b) \\ &= 2000_b - 200_b \\ &= 2b^3 - 2b^2 \\ &= 2b^2(b - 1), \end{aligned}$$

which is divisible by 3 *unless* $b \equiv 2 \pmod{3}$. The only choice congruent to 2 modulo 3 is **(E) 8**.

~MRENTHUSIASM

Solution 2 (Vertical Subtraction)

Vertically subtracting $2021_b - 221_b$, we see that the ones place becomes 0, and so does the b^1 place. Then, we perform a carry (make sure the carry is in base b). Let $b - 2 = A$. Then, we have our final number as

$$1A00_b.$$

Now, when expanding, we see that this number is simply $b^3 - (b - 2)^2$.

Now, notice that the final number will only be congruent to

$$b^3 - (b - 2)^2 \equiv 0 \pmod{3}.$$

If either $b \equiv 0 \pmod{3}$, or if $b \equiv 1 \pmod{3}$ (because note that $(b - 2)^2$ would become $\equiv 1 \pmod{3}$, and b^3 would become $\equiv 1 \pmod{3}$ as well, and therefore the final expression would become $1 - 1 \equiv 0 \pmod{3}$). Therefore, b must be $\equiv 2 \pmod{3}$. Among the answers, only 8 is $\equiv 2 \pmod{3}$, and therefore our answer is **(E) 8**.

~icecreamrolls8

Solution 3 (Answer Choices)

By the definition of bases, we have

$$2021_b - 221_b = (2b^3 + 2b + 1) - (2b^2 + 2b + 1).$$

For values b_1 and b_2 such that $b_1 \equiv b_2 \pmod{3}$, we get

$$(2b_1^3 + 2b_1 + 1) - (2b_1^2 + 2b_1 + 1) \equiv (2b_2^3 + 2b_2 + 1) - (2b_2^2 + 2b_2 + 1) \pmod{3}.$$

Note that answer choices (A), (B), (C), (D), (E) are congruent to 0, 1, 0, 1, 2 modulo 3, respectively. So, (A) and (C) are either both correct or both incorrect. Since there is only one correct answer, (A) and (C) are both incorrect. Similarly, (B) and (D) are both incorrect. This leaves us with (E) 8, the answer choice with a unique residue modulo 3.

~emerald_block ~MRENTHUSIASM

Video Solution (Simple and Quick)

<https://youtu.be/1TZ1ul9z8fU>

~ Education, the Study of Everything

Video Solution

<https://www.youtube.com/watch?v=XBfRVYx64dA&list=PLexHyfQ8DMuKqltG3cHT7Di4jhVI6L4YJ&index=10>

~North America Math Contest Go Go Go

Video Solution

<https://youtu.be/zYluBXDhJJA>

~savannahsolver

Video Solution by TheBeautyofMath

<https://youtu.be/t-EEP2V4nAE>

~IceMatrix

See Also

2021 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/community/c1333))	
Preceded by Problem 10	Followed by Problem 12
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