2021 AMC 10A Problems/Problem 19

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Problem

The area of the region bounded by the graph of

$$x^2+y^2 = 3|x-y| + 3|x+y|$$

is $m+n\pi$, where m and n are integers. What is $m+n$?

$\textbf{(A)}\ 18\qquad\textbf{(B)}\ 27\qquad\textbf{(C)}\ 36\qquad\textbf{(D)}\ 45\qquad\textbf{(E)}\ 54$

Solution 1

In order to attack this problem, we need to consider casework:

Case 1: $|x-y|=x-y, |x+y|=x+y$

Substituting and simplifying, we have $x^2-6x+y^2=0$, i.e. $(x-3)^2+y^2=3^2$, which gives us a circle of radius 3 centered at $(3,0)$.

Case 2: $|x-y|=y-x, |x+y|=x+y$

Substituting and simplifying again, we have $x^2+y^2-6y=0$, i.e. $x^2+(y-3)^2=3^2$. This gives us a circle of radius 3 centered at $(0,3)$.

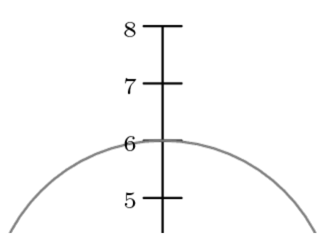
Case 3: $|x-y|=x-y, |x+y|=-x-y$

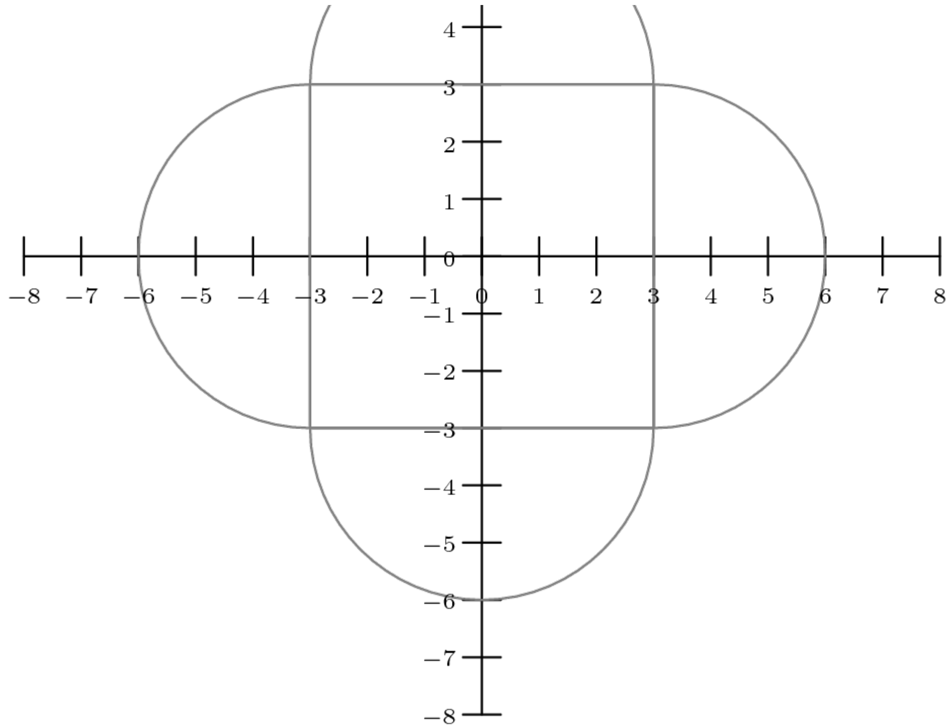
Doing the same process as before, we have $x^2+y^2+6y=0$, i.e. $x^2+(y+3)^2=3^2$. This gives us a circle of radius 3 centered at $(0,-3)$.

Case 4: $|x-y|=y-x, |x+y|=-x-y$

One last time: we have $x^2+y^2+6x=0$, i.e. $(x+3)^2+y^2=3^2$. This gives us a circle of radius 3 centered at $(-3,0)$.

After combining all the cases and drawing them on the Cartesian Plane, this is what the diagram looks like:



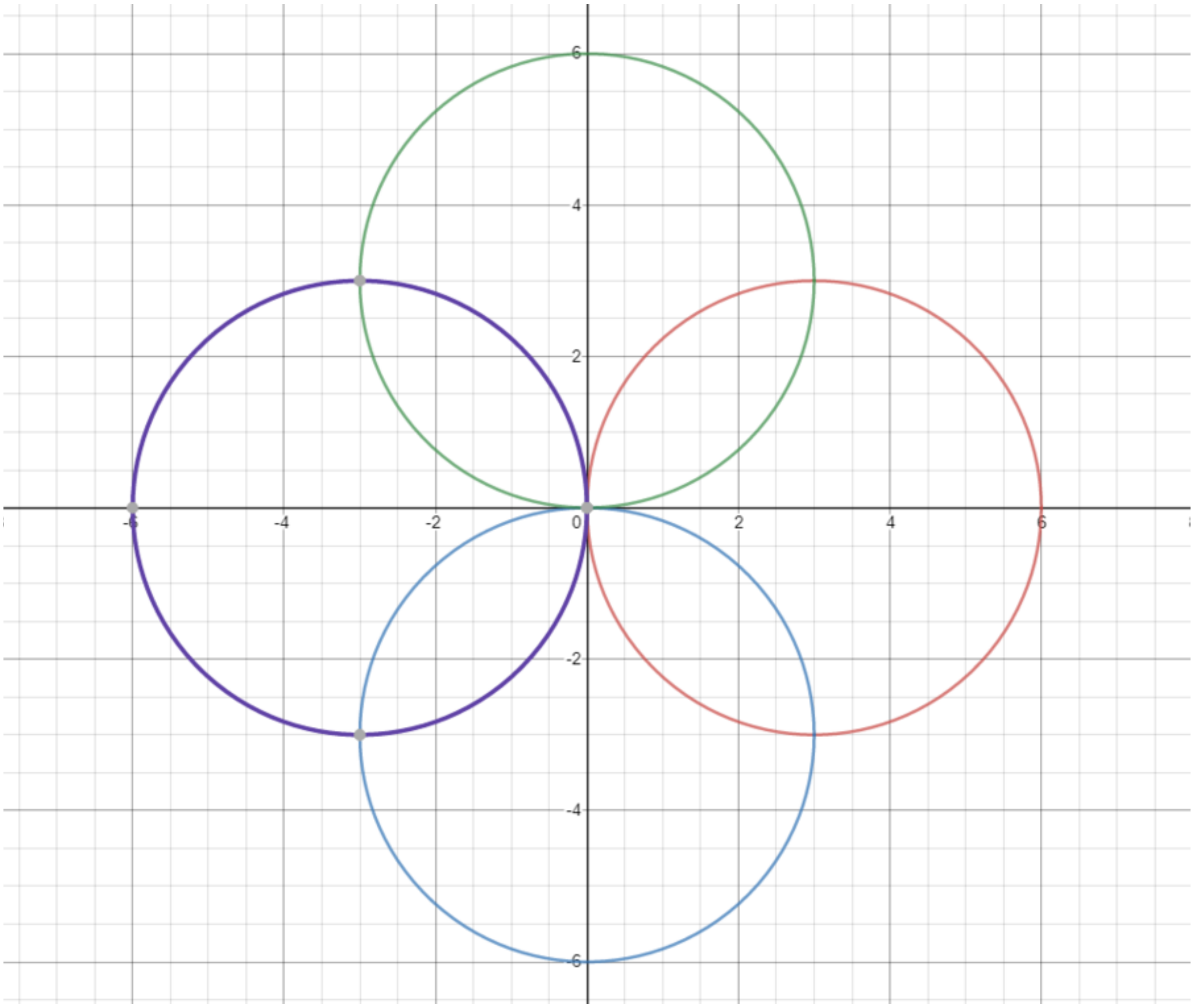


Now, the area of the shaded region is just a square with side length 6 with four semicircles of radius 3. The area is $6 \cdot 6 + 4 \cdot \frac{9\pi}{2} = 36 + 18\pi$. The answer is $36 + 18$ which is $\boxed{\textbf{(E) } 54}$

Solution by Bryguy

Remark

This problem asks for the area of **the union of these four circles**:



Solution 2 (Guessing)

Assume y = 0. We get that x = 6. That means that this figure must contain the points $(0,6)$, $(6,0)$, $(0,-6)$, $(-6,0)$. Now, assume that x = y. We get that x = $3\sqrt{3}$. We get the points $(3,3)$, $(3,-3)$, $(-3,3)$, $(-3,-3)$.

Since this contains $x^2 + y^2$, assume that there are circles. Therefore, we can guess that there is a center square with area $6 \cdot 6$ = 36 and 4 semicircles with radius 3. We get 4 semicircles with area 4.5π, and therefore the answer is $36 + 18$ = $\boxed{(E) 54}$

~Arcticturn

Video Solution (Using Absolute Value Properties to Graph)

https://youtu.be/EHHpB6GIGPc

~ pi_is_3.14

Video Solution by The Power Of Logic (Graphing)

https://youtu.be/-pa72wBA85Y

Video Solution by TheBeautyofMath

https://youtu.be/U6obY_kio0g

~IceMatrix

See Also

2021 AMC 10A (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
Preceded by **Problem 18**	Followed by **Problem 20**
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