Art Of Problem Solving - AMC 10
Week 12

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Abstract

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1.

A parabola with equation $y = x^2 + bx + c$ passes through the points $(2, 3)$ and $(4, 3)$. What is c?

(A) 2 (B) 5 (C) 7 (D) 10 (E) 11

2.

If a, $b > 0$ and the triangle in the first quadrant bounded by the coordinate axes and the graph of $ax + by = 6$ has area 6, then $ab =$

(A) 3 (B) 6 (C) 12 (D) 108 (E) 432

3.

The lines $x = \frac{1}{4}y + a$ and $y = \frac{1}{4}x + b$ intersect at the point $(1, 2)$. What is $a + b$?

(A) 0 (B) $\frac{3}{4}$ (C) 1 (D) 2 (E) $\frac{9}{4}$

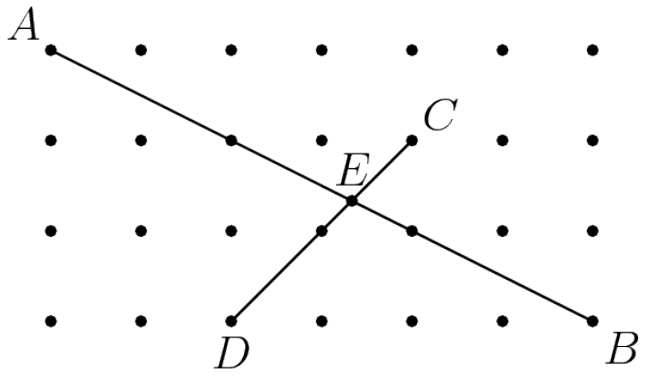
4.

Triangle OAB has $O = (0, 0)$, $B = (5, 0)$, and A in the first quadrant. In addition, $\angle ABO = 90^\circ$ and $\angle AOB = 30^\circ$. Suppose that $\overline{OA}$ is rotated 90° counterclockwise about O. What are the coordinates of the image of A?

(A) $\left(-\frac{10}{3}\sqrt{3}, 5\right)$ (B) $\left(-\frac{5}{3}\sqrt{3}, 5\right)$ (C) $\left(\sqrt{3}, 5\right)$ (D) $\left(\frac{5}{3}\sqrt{3}, 5\right)$ (E) $\left(\frac{10}{3}\sqrt{3}, 5\right)$

5.

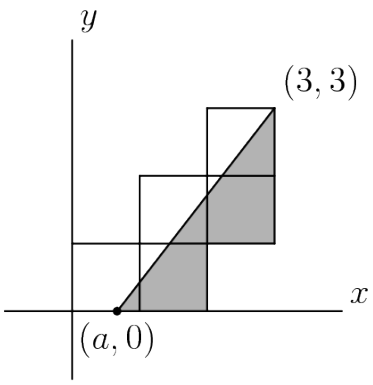
The diagram shows 28 lattice points, each one unit from its nearest neighbors. Segment AB meets segment CD at E. Find the length of segment AE.



(A) $4\sqrt{5}/3$ (B) $5\sqrt{5}/3$ (C) $12\sqrt{5}/7$ (D) $2\sqrt{5}$ (E) $5\sqrt{65}/9$

6.

Five unit squares are arranged in the coordinate plane as shown, with the lower left corner at the origin. The slanted line, extending from $(a, 0)$ to $(3, 3)$, divides the entire region into two regions of equal area. What is a?



(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{4}{5}$

7.

In rectangle $ABCD$, we have $A = (6, -22)$, $B = (2006, 178)$, and $D = (8, y)$, for some integer y. What is the area of rectangle $ABCD$?

(A) 4000 (B) 4040 (C) 4400 (D) 40,000 (E) 40,400

8.

If (a, b) and (c, d) are two points on the line whose equation is $y = mx + k$, then the distance between (a, b) and (c, d), in terms of a, c, and m, is

(A) $|a - c|\sqrt{1 + m^2}$ (B) $|a + c|\sqrt{1 + m^2}$ (C) $\frac{|a - c|}{\sqrt{1 + m^2}}$ (D) $|a - c|(1 + m^2)$ (E) $|a - c||m|$

9.

A lattice point is a point in the plane with integer coordinates. How many lattice points are on the line segment whose endpoints are $(3, 17)$ and $(48, 281)$? (Include both endpoints of the segment in your count.)

(A) 2 (B) 4 (C) 6 (D) 16 (E) 46

10.

The number of distinct points in the xy-plane common to the graphs of $(x + y - 5)(2x - 3y + 5) = 0$ and $(x - y + 1)(3x + 2y - 12) = 0$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4