MATHCOUNTS®

2019
■ State Competition ■
Sprint Round
Problems 1−30

	0
	1
	2
	3
	4
	5
	6
	7
	8
	9

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

Raytheon

2019 MATHCOUNTS
National Competition Sponsor

03-S19SPR

1. _____ times
The height of Hyperion, the tallest redwood tree, is 380 feet. The three-year old redwood tree in Paul's backyard is 19 feet tall. The height of Hyperion is how many times the height of Paul's tree?

2. _____
If $y = 2x + 7$ and $x = 2$, what is the value of y?

3. _____
A bag has one white, two black, three red and four green marbles. If a single marble is randomly drawn from this bag, what is the probability it is black or white? Express your answer as a common fraction.

4. $ _____
If Jessica works at a fixed hourly rate of pay, as represented by the double number line shown, what is Jessica's total pay for 8 hours of work?
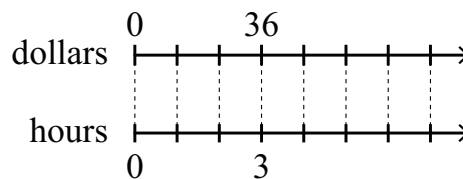
5. _____ points
So far, Ricardo has scores of 13, 17, 19 and 21 points for the first four rounds of a dice game. What does he need the total score to be for the next two rounds combined in order to achieve an average score of 20 points per round for all six rounds?

6. _____ cm   The length of a particular rectangle is $\frac{2}{5}$ its perimeter. If the length of the rectangle is 12 cm, what is its width?

7. _____   Rico showed his friends a card quickly before hiding it. Dee said the card was the 8 of spades. Todd said it was the 7 of hearts. Nia said it was the 6 of spades. Dee, Todd and Nia were each correct about either the number or the suit, but not both. What number was on the card Rico showed?

8. _____ units²   Two vertices of a triangle are at A(1, 5) and B(5, 5) in the coordinate plane. The third vertex, C, is the translation of point B three units down and one unit left. What is the area of triangle ABC?

9. _____   All of Bethany's math quiz scores are displayed in the stem and leaf plot shown. What is the absolute difference between the median and the mode of the 10 scores?

```
6 | 9
7 | 0 5 6
8 | 3 7 7 8
9 | 2 4
```

10. _____   What value of n satisfies $(n + 1)! - n! = 4320$?

11. _____ January 1, 2018 was a Monday. What is the next year in which January 1 will fall on a Monday?

12. _____ miles On weekdays Sally rides her bike m miles each day. On each of the two weekend days she rides it 5 miles farther. If Sally rides 94 miles each week, how many miles does she ride each weekday?

13. _____ What is the greatest integer p such that 33! has 3^p as a factor?

14. _____ What is the value of $\dfrac{5! + 6!}{4! + 3!}$?

15. _____ feet A rectangle is twice as long as it is wide. When the lengths of all sides are increased by 3 feet, the area of the new rectangle is triple that of the original rectangle. What is the length of the new rectangle?

16. _____ The sum of six consecutive integers, the least of which is 30, can also be written as a sum of five consecutive integers. What is the greatest of these five integers?

17. _____ If a, b, c and d are positive integers such that $1 - \dfrac{1}{324} = \dfrac{a}{b} \cdot \dfrac{c}{d}$, what is the least possible value of $a + c$?

18. _____ If C is a digit such that the product of the three-digit numbers 2C8 and 3C1 is the five-digit number 90C58, what is the value of C?

19. _____ The graph of the equation $\dfrac{(x-3)(y-7)}{(x+1)(2y-5)} = \dfrac{1}{2}$ is a line missing two points. What is the slope of the line? Express your answer as a common fraction.

20. _____ What is the sum of all the integers x for which $x^2 + 4x \le 1$?

21. _____ Two points are randomly placed on a number line from −1 to +1. What is the probability that the origin lies between the two points? Express your answer as a common fraction.

22. _____ m/s A canary flies directly east 4000 meters at a speed of 20 m/s. It then immediately turns and flies directly north for 3000 meters at a speed of 30 m/s. The canary then flies back to its starting point in a straight line in 100 seconds. What is the average speed of the canary over the entire trip?

23. _____ If a, b and c are positive integers such that $a - bc = 19$ and $a + bc = 99$, what is the least possible value of $a + b + c$?

24. _____ Eliza creates a custom 6-sided die by randomly choosing six distinct integers from 1 to 7, inclusive, to paint onto the sides of a blank cube. She tells Philip that the faces of her die have a sum of 24. Philip rolls the die. What is the probability that Philip's die shows a prime number on the top face? Express your answer as a common fraction.

25. _____ units What is the greatest possible radius of a circle that passes through the points (1, 2) and (4, 5) and whose interior is contained in the first quadrant of the coordinate plane? Express your answer in simplest radical form.

26. _____ cm   A regular tetrahedron is a solid with four equilateral triangular faces. What is the height of a regular tetrahedron with edges of length 5 cm? Express your answer as a common fraction in simplest radical form.

27. _____ cm³   A cone with base radius 12 cm is sliced parallel to its base, as shown, to remove a smaller cone of height 15 cm. If the height of the smaller cone is three-fourths that of the original cone, what is the volume of the remaining frustum? Express your answer in terms of π.
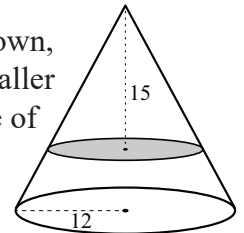
28. _____ units   The circles given by the equations $x^2 + y^2 = 169$ and $x^2 + (y - 14)^2 = 225$ have a common chord. How many units long is that chord?

29. _____   Chris flips a coin 16 times. Given that exactly 12 of the flips land heads, what is the probability that Chris never flips five heads in a row? Express your answer as a common fraction.

30. _____ units   Point P is the instersection of five distinct chords in circle O as shown. If the distance from P to the circle along each segment of the five chords is an integer, what is the least possible integer radius of circle O?