Art Of Problem Solving - AMC 10
Week 4

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Abstract

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1.

For how many integers n between 1 and 100 does $x^2 + x - n$ factor into the product of two linear factors with integer coefficients?

(A) 0	(B) 1	(C) 2	(D) 9	(E) 10

2.

Suppose that a and b are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions a and b. Then the pair (a, b) is

(A) $(-2, 1)$	(B) $(-1, 2)$	(C) $(1, -2)$	(D) $(2, -1)$	(E) $(4, 4)$

3.

Let f be the function defined by $f(x) = ax^2 - \sqrt{2}$ for some positive a. If $f(f(\sqrt{2})) = -\sqrt{2}$, then $a =$

(A) $\dfrac{2-\sqrt{2}}{2}$	(B) $\dfrac{1}{2}$	(C) $2 - \sqrt{2}$	(D) $\dfrac{\sqrt{2}}{2}$	(E) $\dfrac{2+\sqrt{2}}{2}$

4.

Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. The number of possible values of k is

(A) 0	(B) 1	(C) 2	(D) 4	(E) more than four

5.

Let @ denote the "averaged with" operation: $a@b = \frac{a+b}{2}$. Which of the following distributive laws holds for all numbers x, y, and z?

I. $x@(y + z) = (x@y) + (x@z)$
III. $x + (y@z) = (x + y)@(x + z)$
III. $x@(y@z) = (x@y)@(x@z)$

(A) I only	(B) II only (C)	III only (D)	I and III only (E)	II and III only

6.

If $f(x) = ax^4 - bx^2 + x + 5$ and $f(-3) = 2$, then $f(3) =$

(A) -5	(B) -2	(C) 1	(D) 3	(E) 8

7.

What is the sum of the reciprocals of the roots of the equation

$$\frac{2003}{2004}x + 1 + \frac{1}{x} = 0?$$

(A) $-\dfrac{2004}{2003}$ (B) -1 (C) $\dfrac{2003}{2004}$ (D) 1 (E) $\dfrac{2004}{2003}$

8.

Let f be a polynomial function such that, for all real x,

$$f(x^2 + 1) = x^4 + 5x^2 + 3$$

For all real x, $f(x^2 - 1)$ is

(A) $x^4 + 5x^2 + 1$ (B) $x^4 + x^2 - 3$ (C) $x^4 - 5x^2 + 1$ (D) $x^4 + x^2 + 3$ (E) none of these

9.

The polynomial $x^3 - ax^2 + bx - 2010$ has three positive integer roots. What is the smallest possible value of a?

(A) 78 (B) 88 (C) 98 (D) 108 (E) 118

10.

Let f be a function for which $f(x/3) = x^2 + x + 1$. Find the sum of all values of z for which $f(3z) = 7$.

(A) $-1/3$ (B) $-1/9$ (C) 0 (D) 5/9 (E) 5/3