MathCounts Chapter Invitational February 2021 Target Round

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Abstract

Notes on Target Round of MathCounts Invitational Competition, February 2021. Questions are from MathCounts Foundation (https://www.mathcounts.org/). Copyright restrictions may apply. Written for personal use. Please report typos and errors over at https://github.com/ptoche/Math/tree/master/mathcounts.

Target Round

1.

Max is buying hot dogs and buns for dinner. If hot dogs are sold in packages of six and buns are sold in packages of eight, what is the least positive number of hot dogs that Max must buy so that he can purchase exactly the same number of buns?

| | hot dogs

The least common multiple of 6 and 8 can be found with one of these approaches:

1. list multiples of 6 until one of them is a multiple of 8.
2. list multiples of 8 until one of them is a multiple of 6.

The second approach would be a little quicker. We have 8, 16, 24, stop.

> 24 hot dogs

2.

The square card shown in Figure A, with area 16 units^2, is folded twice along the dashed lines to get the square shown in Figure B. Then, scissors are used to cut through the folded square, and one-fourth of the folded card, shown shaded in Figure C, is removed. When unfolded, what is the area of what remains of the card after the removal?
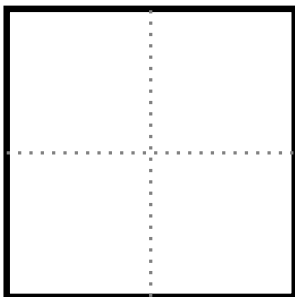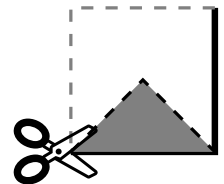
Figure A

Figure B

Figure C

| | units^2

3.

Cindy has been given four chores to complete today: vacuum, mop, dust and laundry. Because Cindy finds doing laundry especially unpleasant, she wants to do that chore last. Given this, in how many different orders can Cindy complete her chores?

[   ] orders

One of the 4 objects has its position predetermined (laundry), so that we are looking for the number of ways to permute 3 objects.

$3! = 3 \times 2 = 6$

6 orders

4.

In the dice game Zero, a player rolls six standard six-sided dice and then scores points for certain combinations, as shown in the table. A die cannot be used in more than one combination. For example, a player who rolls 1 5 6 4 4 6 scores 50 points, while a player who rolls 1 4 4 4 1 4 scores 250 points. What is the probability that a player scores 0 points on a roll? Express your answer as a common fraction.

Player's Roll	Points Scored
Three of a kind (ex. 2 2 2 4 6 3)	100
Four of a kind	200
Five of a kind	400
Six of a kind	900
Three pairs (ex. 2 2 3 3 4 4)	200
Six distinct (ex. 1 2 3 4 5 6)	300
The number 1	25
The number 5	25

The configurations that yield no points are 2 3 4 6 with two numbers repeated but no triple. The number of ways to choose 2 objects from 6 is

$$^{2}C_6 = \frac{6!}{2!(6-2)!} = \frac{6 \times 5}{2} = 15$$

To properly calculate the probability, we also need to take into account permutations of these six rolls (6!). The number of ways to roll six six-sided dice is 6^6. Thus, the probability is the ratio of these:

$$\frac{15 \times 6!}{6^6} = \frac{5}{216}$$

$$\boxed{\frac{5}{216}}$$

5.

A U.S. quarter is 0.069 inches thick. Clayton Kershaw earned 33 million dollars for the 2017 Major League Baseball season. If Kershaw received his entire 2017 salary in the form of a single stack of quarters, what would be the height of the stack, in miles? There are 5280 feet in a mile. Express your answer as a decimal to the nearest hundredth.

$\boxed{}$ units^2

A dollar in quarters is

$$4 \times 0.069 = 0.276 \text{ inches}$$

33 million of those is

$$33 \times 0.276 = 9,108,000 \text{ inches}$$

Converted to feet:

$$9,108,000 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} = 759,000 \text{ feet}$$

Converted to miles:

$$759,000 \text{ feet} \times \frac{1 \text{ mile}}{5280 \text{ feet}} = 143.75 \text{ miles}$$

$$\boxed{143.75 \text{ miles}}$$

6.

In rectangle $ABCD$ the length of side AB is less than the length of side BC. When diagonal AC is drawn, and a line segment from vertex B to diagonal AC is drawn so that the segment meets AC at a right angle, rectangle $ABCD$ is divided into three regions whose areas form an arithmetic progression. The ratio AB:BC, expressed as a common fraction in simplest radical form is $\dfrac{\sqrt{a}}{b}$. What is the value of ab?

TO DO: COPY MY NOTES Applying the Pythagorean theorem,

$$= \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{2}$$

It follows that

$$\frac{\sqrt{a}}{b} = \frac{\sqrt{2}}{2} \Rightarrow a = b = 2 \Rightarrow ab = 4$$

$$\boxed{4}$$

7.

If C is the temperature in degrees Celsius, the temperature in degrees Farenheit is given by the formula $F = \frac{9}{5}C + 32$. On a cold winter day, the temperature in degrees Farenheit is the additive inverse of the temperature in degrees Celsius. What is the temperature in degrees Celsius? Express your answer as a decimal to the nearest tenth.

$\boxed{}$ °C

The main difficulty here is knowing that the "additive inverse" of x is the number y such that $x + y = 0$, in other words the additive inverse of x is $-x$, its "opposite." The verbal statement yields a system of two equations in two unknowns:

$$F = \frac{9}{5}C + 32$$
$$F = -C$$

which may be solved by substitution:

$$-C = \frac{9}{5}C + 32 \Rightarrow \left(\frac{9}{5} + 1\right)C = -32 \Rightarrow C = -32 \times \frac{5}{14} \approx -11.429$$

$$\boxed{-11.4 \text{ °C}}$$

8.

How many ways are there to color the 4×4 grid shown, so that each unit square is red, blue, green, or yellow, and so that each square is the same color as exactly two of the squares that share a side with it?

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

$\boxed{}$ ways

TO DO: COPY MY NOTES

96 ways