Russian School of Math: Lesson 11

James & Patrick

Revised: December 13, 2024

Abstract

This note reviews a small number of problems from the Russian School of Math test. Written for personal use.

1

w is a circumcircle of right $\triangle ABC$. BE is an altitude and F is midpoint of leg AB. Point M, N, E, and F are collinear, where MN is a chord of w. Given that the length of hypotenuse AC is 16 and $\angle C = 30°$, find MN.

Solution

2

$ABCD$ is a trapezoid with $BC \perp AD$. Circle w is tangent to line CD and passes through points A, B, C. Given $AB = 5$, $BC = 4$ and $AC = \sqrt{65}$, find CD.

Solution

3

w is a circumcircle af $\triangle ABC$ with center 0. Line l is a tangent to w passing through point B. Lines AO and l intersect at point M. Given $AB = BC = 40$, $AC = 48$, find BM.

Solution

4

A circle with diameter PQ of length 10 is internally tangent at P to a circle of radius 20. Square $ABCD$ is constructed with A and B on the larger, CD tangent at Q to the smaller circle, and the smaller circle outside $ABCD$. The length of AB can be written in the form $m + \sqrt{n}$, where m and n are integers. Find $m + n$.

Solution

5

The sides of the triangle RSM are $RM = 16$, $SM = 12$. The circle with diameter RM crosses the side RS at the point K. It is known that $\angle SRM = \angle SMK$. Find SK.

Solution

6

In triangle ABC, the medians AD and CE have lengths 18 and 27 respectively and $AB = 24$. Extend CE to intersect the circumcircle of ABC at F. The area of triangle AFB is $m\sqrt{n}$, where m and n are positive integers and n is not divisible by the square of any prime. Find $m + n$.

Solution