2021 Fall AMC 10B Problems/Problem 11

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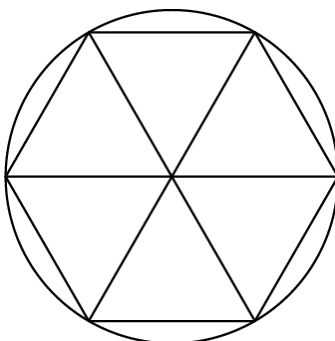
Problem

A regular hexagon of side length 1 is inscribed in a circle. Each minor arc of the circle determined by a side of the hexagon is reflected over that side. What is the area of the region bounded by these 6 reflected arcs?

$\textbf{(A)}\ \dfrac{5\sqrt{3}}{2} - \pi \qquad \textbf{(B)}\ 3\sqrt{3} - \pi \qquad \textbf{(C)}\ 4\sqrt{3} - \dfrac{3\pi}{2} \qquad \textbf{(D)}\ \pi - \dfrac{\sqrt{3}}{2} \qquad \textbf{(E)}\ \dfrac{\pi + \sqrt{3}}{2}$

Solution 1

This is the graph of the original Hexagon. After reflecting each minor arc over the sides of the hexagon it will look like this;
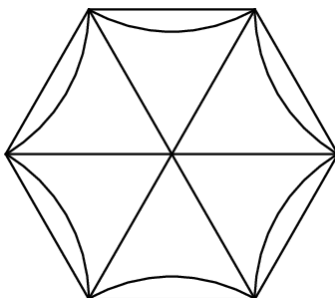
This bounded region is the same as the area of the hexagon minus the area of each of the reflect arcs. From the first diagram, the area of each arc is the area of the $60°$ sector minus the area of the equilateral triangle, so each arc has an area of

$$\frac{\pi r^2}{6} - \frac{s^2 \sqrt{3}}{4} \implies \frac{\pi}{6} - \frac{\sqrt{3}}{4}.$$

There are 6 total arcs, so the total area of the arcs is $6 \cdot \left(\dfrac{\pi}{6} - \dfrac{\sqrt{3}}{4} \right) = \pi - \dfrac{3\sqrt{3}}{2}$.

The area of the hexagon is $6 \cdot \dfrac{\sqrt{3}}{4} = \dfrac{3\sqrt{3}}{2}$, so the area of the bounded region is:

$$\frac{3\sqrt{3}}{2} - \left(\pi - \frac{3\sqrt{3}}{2}\right) = 3\sqrt{3} - \pi = \boxed{B}$$

~KingRavi

Solution 2

Let the hexagon described be of area H and let the circle's area be C. Let the area we want to aim for be A. Thus, we have that $C - H = H - A$, or $A = 2H - C$. By some formulas, $C = \pi r^2 = \pi$ and

$$H = 6 \cdot \frac{1}{2} \cdot 1 \cdot \left(\frac{1}{2} \cdot \sqrt{3}\right) = \frac{3\sqrt{3}}{2}.$$ Thus, $A = 3\sqrt{3} - \pi$ or $\boxed{(\mathbf{B})}$.

~Hefei417, or 陆畅 Sunny from China

Solution 3

Denote by O the center of this circle. Hence, the radius of this circle is 1. Denote this hexagon as $ABCDEF$.

We have $\angle AOB = 60°$. Hence, the area of the region formed between segment $\overline{AB}$ and the minor arc formed by A and B, denoted as M, is

$$\begin{aligned} M &= \pi 1^2 \frac{60}{360} - \frac{\sqrt{3}}{4} 1^2 \\ &= \frac{\pi}{6} - \frac{\sqrt{3}}{4}. \end{aligned}$$

Therefore, the area of the region that this problem asks us to compute is

$$\pi 1^2 - 12M = 3\sqrt{3} - \pi.$$

Therefore, the answer is $\boxed{(\mathbf{B})\ 3\sqrt{3} - \pi}$.

~Steven Chen (www.professorchenedu.com)

Video Solution by Interstigation

https://youtu.be/xKlsLPzXsOM

See Also

2021 Fall AMC 10B (**Problems** · **Answer Key** · Resources (http://www.artofproblemsolving.com/community/c13))	
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