

# 2021 Fall AMC 10B Problems/Problem 16

## Problem

Five balls are arranged around a circle. Chris chooses two adjacent balls at random and interchanges them. Then Silva does the same, with her choice of adjacent balls to interchange being independent of Chris's. What is the expected number of balls that occupy their original positions after these two successive transpositions?

(A) 1.6      (B) 1.8      (C) 2.0      (D) 2.2      (E) 2.4

## Solution 1

After the first swap, we do casework on the next swap.

Case 1: Silva swaps the two balls that were just swapped

There is only one way for Silva to do this, and it leaves 5 balls occupying their original position.

Case 2: Silva swaps one ball that has just been swapped with one that hasn't swapped

There are two ways for Silva to do this, and it leaves 2 balls occupying their original positions.

Case 3: Silva swaps two balls that have not been swapped

There are two ways for Silva to do this, and it leaves 1 balls occupying their original positions.

Our answer is the average of all 5 possible swaps, so we get

$$\frac{5 + 2 \cdot 2 + 2 \cdot 1}{5} = \frac{11}{5} = \boxed{\text{(D) } 2.2}.$$

~kingofpineapplz

## See Also

2021 Fall AMC 10B (Problems • Answer Key • Resources ( <a href="http://www.artofproblemsolving.com/community/c13">http://www.artofproblemsolving.com/community/c13</a> ))	
Preceded by <b>Problem 15</b>	Followed by <b>Problem 17</b>
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All AMC 10 Problems and Solutions	

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