2021 AMC 12A Problems/Problem 16

The following problem is from both the 2021 AMC 10A #16 and 2021 AMC 12A #16, so both problems redirect to this page.

Contents

Problem

In the following list of numbers, the integer n appears n times in the list for $1 \leq n \leq 200$.

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots, 200, 200, \ldots, 200$$

What is the median of the numbers in this list?

$\textbf{(A)}\ 100.5 \qquad\textbf{(B)}\ 134 \qquad\textbf{(C)}\ 142 \qquad\textbf{(D)}\ 150.5 \qquad\textbf{(E)}\ 167$

Solution 1

There are $1+2+..+199+200=\frac{(200)(201)}{2}=20100$ numbers in total. Let the median be k. We want to find the median k such that

$$\frac{k(k+1)}{2}=20100/2,$$

or

$$k(k+1)=20100.$$

Note that $\sqrt{20100} \approx 142$. Plugging this value in as k gives

$$\frac{1}{2}(142)(143)=10153.$$

$10153-142<10050$, so 142 is the 152nd and 153rd numbers, and hence, our desired answer. $\boxed{\textbf{(C)}\ 142}$.

Note that we can derive $\sqrt{20100} \approx 142$ through the formula

$$\sqrt{n}=\sqrt{a+b}\approx\sqrt{a}+\frac{b}{2\sqrt{a}+1},$$

where a is a perfect square less than or equal to n. We set a to 19600, so $\sqrt{a}=140$, and $b=500$. We then have

$n \approx 140 + \dfrac{500}{2(140)+1} \approx 142$. ~approximation by ciceronii

Note by Fasolinka (use answer choices): Once you know that the answer is in the 140s range by the approximation, it is highly improbable for the answer to be anything but C.

Solution 2

The xth number of this sequence is $\left|\left\lceil \dfrac{-1 \pm \sqrt{1+8x}}{2} \right\rceil\right|$ via the quadratic formula. We can see that if we halve x we end up getting $\left\lceil \dfrac{-1 \pm \sqrt{1+4x}}{2} \right\rceil$. This is approximately the number divided by $\sqrt{2}$. $\dfrac{200}{\sqrt{2}} = 141.4$ and since 142 looks like the only number close to it, it is answer $\boxed{(C) 142}$ ~Lopkiloinm

Solution 3 (answer choices)

We can look at answer choice C, which is 142 first. That means that the number of numbers from 1 to 142 is roughly the number of numbers from 143 to 200.

The number of numbers from 1 to 142 is $\dfrac{142(142+1)}{2}$ which is approximately 10000. The number of numbers from 143 to 200 is $\dfrac{200(200+1)}{2} - \dfrac{142(142+1)}{2}$ which is approximately 10000 as well. Therefore, we can be relatively sure the answer choice is $\boxed{(C)\ 142}$.

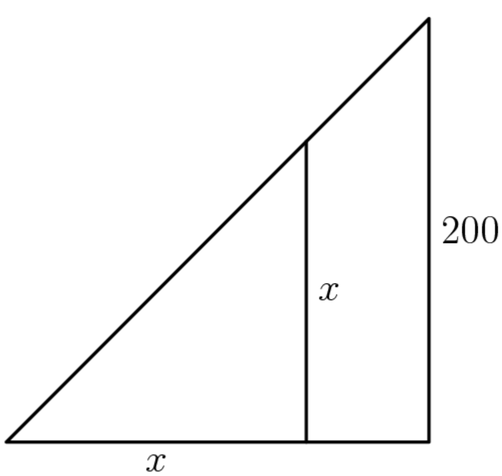
-PureSwag

Solution 4

We can arrange the numbers in the following pattern:

$$\begin{array}{ccccc} & & & & 200 \\ & & & 199 & 200 \\ & & \iddots & \vdots & \vdots \\ & 2 & \cdots & 199 & 200 \\ 1 & 2 & \cdots & 199 & 200 \end{array}$$

We can see this as a isosceles right triangle, with legs of length 200.



Let x be the side length such that both sides of the triangle have the same area. The desired answer is then around x because about half of the numbers in the list fall on each side.

Solving for x yields:

$$\begin{aligned}\frac{x^2}{2} &= \frac{1}{2} \cdot \frac{200^2}{2} \\ x^2 &= \frac{1}{2} \cdot 200^2 \\ x &= \frac{200}{\sqrt{2}} = 100\sqrt{2} \approx 141.\end{aligned}$$

We see that $\boxed{(C)\ 142}$ is the closest to x by far, and thus, can be relatively certain this is the answer.

~thinker123

Video Solution by Punxsutawney Phil

https://youtube.com/watch?v=vsE_ezaV4Xs

Video Solution by Hawk Math

https://www.youtube.com/watch?v=AjQARBvdZ20

Video Solution by Answer Choice

https://www.youtube.com/watch?v=YxWjDcUcaeQ&list=PLexHyfQ8DMuKqltG3cHT7Di4jhVl6L4YJ&index=13 ~North America Math Contest Go Go Go

Video Solution by pi_is_3.14 (Using Algebra)

https://youtu.be/HkwgH9Lc1hE

Video Solution by TheBeautyofMath

https://youtu.be/CTXQunZpBA4

~IceMatrix

See also

2021 AMC 10A (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
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2021 AMC 12A (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/community/c13))	
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