2021 AMC 12A Problems/Problem 12

The following problem is from both the 2021 AMC 12A #12 and 2021 AMC 10A #14, so both problems redirect to this page.

Contents

Problem

All the roots of the polynomial $z^6 - 10z^5 + Az^4 + Bz^3 + Cz^2 + Dz + 16$ are positive integers, possibly repeated. What is the value of B?

$(\textbf{A})\ -88 \qquad (\textbf{B})\ -80 \qquad (\textbf{C})\ -64 \qquad (\textbf{D})\ -41 \qquad (\textbf{E})\ -40$

Solution 1

By Vieta's formulas, the sum of the six roots is 10 and the product of the six roots is 16. By inspection, we see the roots are $1, 1, 2, 2, 2,$ and $2,$ so the function is $(z-1)^2(z-2)^4 = (z^2 - 2z + 1)(z^4 - 8z^3 + 24z^2 - 32z + 16).$ Therefore, calculating just the z^3 terms, we get $B = -32 - 48 - 8 = \boxed{(\textbf{A})\ -88}$.

~JHawk0224

Solution 2

Using the same method as Solution 1, we find that the roots are $2, 2, 2, 2, 1,$ and $1.$ Note that B is the negation of the 3rd

symmetric sum of the roots. Using casework on the number of 1's in each of the $\binom{6}{3} = 20$ products $r_a \cdot r_b \cdot r_c,$ we obtain

$$B = -\left(\binom{4}{3}\binom{2}{0} \cdot 2^3 + \binom{4}{2}\binom{2}{1} \cdot 2^2 \cdot 1 + \binom{4}{1}\binom{2}{2} \cdot 2\right) = -(32 + 48 + 8) = \boxed{(\textbf{A})\ -88}.$$

~ike.chen

Video Solution by Hawk Math

https://www.youtube.com/watch?v=AjQARBvdZ20

Video Solution by OmegaLearn (Using Vieta's Formulas & Combinatorics)

https://youtu.be/5U4MJTo3F5M

~ pi_is_3.14

Video Solution by Power Of Logic (Using Vieta's Formulas)

https://youtu.be/rl6QtVnIbdU

Video Solution by TheBeautyofMath

https://youtu.be/t-EEP2V4nAE?t=1080 (for AMC 10A)

https://youtu.be/ySWSHyY9TwI?t=271 (for AMC 12A)

~IceMatrix

See also

2021 AMC 10A (**Problems** · **Answer Key** · Resources (http://www.artofproblemsolving.com/community/c13))	
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2021 AMC 12A (**Problems** · **Answer Key** · Resources (http://www.artofproblemsolving.com/community/c13))	
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