Ask Math Anything

Daily Challenge with Po-Shen Loh

16 June 2020

Abstract

Professor Po-Shen Loh solves problems on his YouTube channel. A selection for practice.
Reference: Ask Math Anything - Daily Challenge with Po-Shen Loh

2020/06/19

1 The Golden Rectangle and Golden Ratio

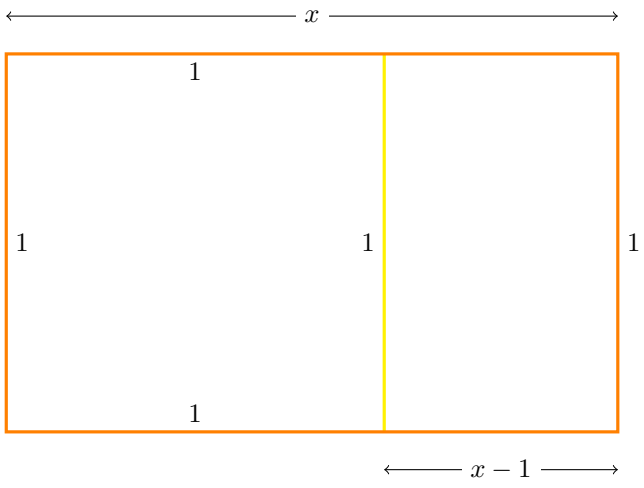
The golden rectangle has the property that if you cut a square inside of it, the remaining rectangle is "similar" to the original rectangle, that is

$$\text{The ratio } \frac{\text{length}}{\text{width}} \text{ are equal}$$

If x denotes the length of the rectangle (to be determined), the proportions of the orignial and inscribed rectangles must satifsy:

$$\frac{x}{1} = \frac{1}{x-1}$$

The Golden rectangle is:



Rearranging the equation yields a quadratic equation,

$$x^2 - x - 1 = 0$$

This quadratic equation has two real solutions, one positive, one negative (both irrational):

$$(x - \phi)(x - \psi) = 0$$

where ϕ (pronounced 'phi') is the famous golden ratio – sometimes called the golden section:

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618034$$

and ψ is the not-at-all famous negative solution of the quadratic equation:

$$\psi = \frac{1-\sqrt{5}}{2} \approx -0.618034$$

Note that the repeating decimals are the same. If you cannot see why, look up Professor Po-Shen Loh's video on the quadratic equation. The golden ratio is sometimes denoted φ (also pronounced 'phi').

2 The Silver Rectangle and Silver Ratio

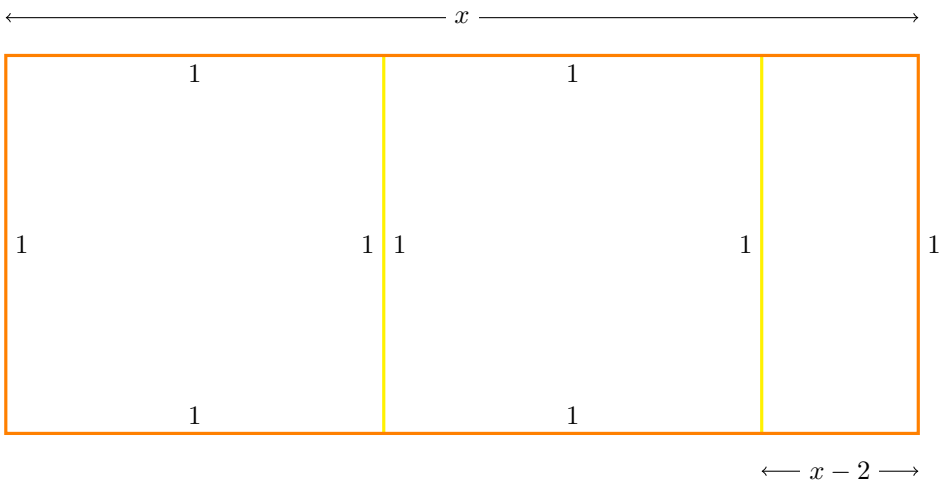
The golden rectangle has the property that if you cut two squares inside of it, the remaining rectangle is "similar" to the original rectangle, that is

$$\text{The ratio} \quad \frac{\text{length}}{\text{width}} \quad \text{are equal}$$

If x denotes the length of the rectangle (to be determined), the proportions of the orignial and inscribed rectangles must satifsy:

$$\frac{x}{1} = \frac{1}{x-2}$$

The Silver rectangle is:



Rearranging the equation yields a quadratic equation,

$$x^2 - 2x - 1 = 0$$

This quadratic equation has two real solutions, one positive, one negative (both irrational). The silver ratio – sometimes called the silver mean – is:

$$\delta = 1 + \sqrt{2} \approx 2.414214$$

Beware of the signs: The silver quadratic equation almost looks like $x^2 - 2x + 1 = (x-1)^2$.