Art Of Problem Solving - AMC 10
Week 11

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Abstract

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1.

Mrs. Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs. Walter noticed that after each score was entered, the average was always an integer. The scores (listed in ascending order) were 71, 76, 80, 82, and 91. What was the last score Mrs. Walter entered?

(A) 71	(B) 76	(C) 80	(D) 82	(E) 91

2.

The digits 1, 2, 3, 4, 5, 6, 7, and 9 are used to form four two-digit prime numbers, with each digit used exactly once. What is the sum of these four primes?

(A) 150	(B) 160	(C) 170	(D) 180	(E) 190

3.

Suppose that m and n are positive integers such that $75m = n^3$. What is the minimum possible value of $m + n$?

(A) 15	(B) 30	(C) 50	(D) 60	(E) 5700

4.

What is the largest integer that is a divisor of $(n+1)(n+3)(n+5)(n+7)(n+9)$ for all positive even integers n?

(A) 3	(B) 5	(C) 11	(D) 15	(E) 165

5.

Let S be the set of the 2005 smallest positive multiples of 4, and let T be the set of the 2005 smallest positive multiples of 6. How many elements are common to S and T?

(A) 166	(B) 333	(C) 500	(D) 668	(E) 1001

6.

For how many positive integers n less than or equal to 24 is $n!$ evenly divisible by $1+2+\cdots+n$?

(A) 8	(B) 12	(C) 16	(D) 17	(E) 21

7.

A finite sequence of three-digit integers has the property that the tens and units digits of each term are, respectively, the hundreds and tens digits of the next term, and the tens and units digits of the last term are, respectively, the hundreds and tens digits of the first term. For example, such a sequence might begin with terms 247, 475, and 756 and end with the term 824. Let S be the sum of all the terms in the sequence. What is the largest prime number that always divides S?

(A) 3	(B) 7	(C) 13	(D) 37	(E) 43

8.

Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

(A) 8	(B) 9	(C) 10	(D) 11	(E) 12

9.

Let x and y be two-digit integers such that y is obtained by reversing the digits of x. The integers x and y satisfy $x^2 - y^2 = m^2$ for some positive integer m. What is $x + y + m$?

(A) 88	(B) 112	(C) 116	(D) 144	(E) 154

10.

A high school basketball game between the Raiders and the Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?

(A) 30	(B) 31	(C) 32	(D) 33	(E) 34