Investigating Amplitude Attenuation and Frequency Dispersion For a Lumped Transmission Line

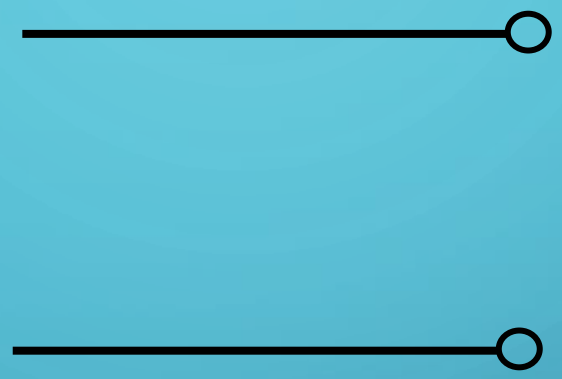
Leonardo Rotondi

Outline:

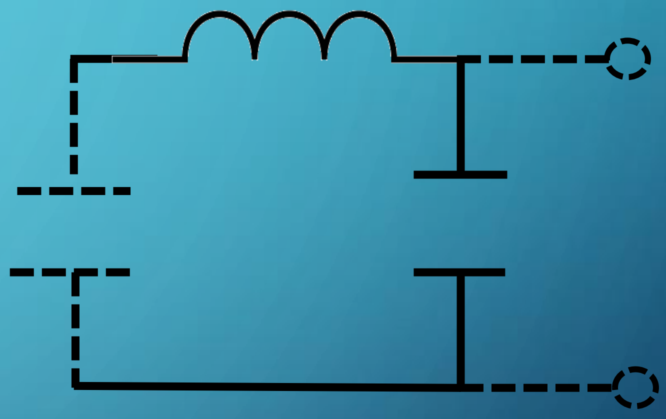
WHY DO WE CARE?

- Wires are everywhere!
- Not as simple as they seem
- The design of wires gives rise to inherent capacitance and inductance in the wires.
- These can hinder our ability to send electrical signals over long distances.

<u>Appearance of parallel wires</u>



Behaviour of parallel wires



1 section

- Behaves as an inductor due to induced magnetic field
- Parallel wires will act as capacitors
- Lumped transmission line has 40 sections
- 10 nodes
- L $= 330 \pm 20\%\ \mu H$
- C $= 0.015 \pm 10\%\ \mu F$
- Variable resistor placed in parallel with a switch at node 10

REFLECTION OF PULSES

- Impedance of transmission line due to components of the line
- Gives rise to reflected pulses
- The cause various complications in the line

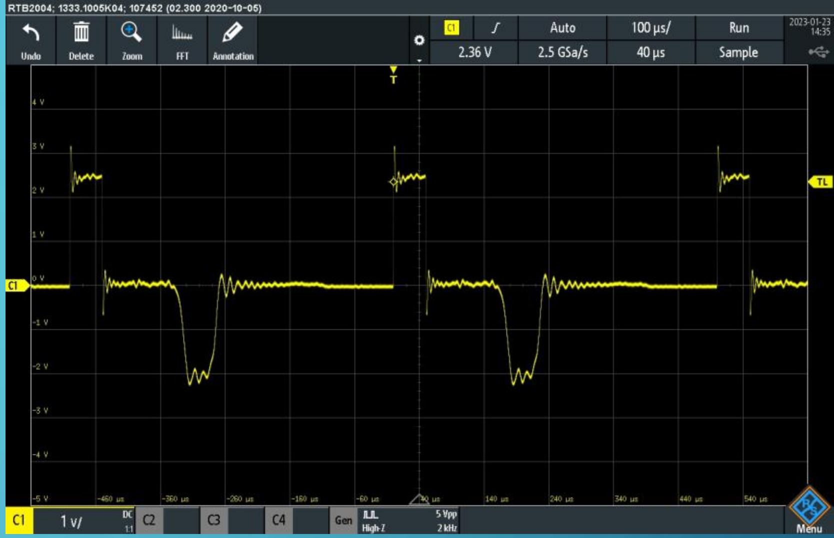
$$R = \frac{V_r}{V_i} = \frac{(Z_b - Z_a)}{(Z_b + Z_a)}$$

- Impedances must be equal to cancel reflected signal
- Variable resistor = 156.48 ± 1.73 Ω

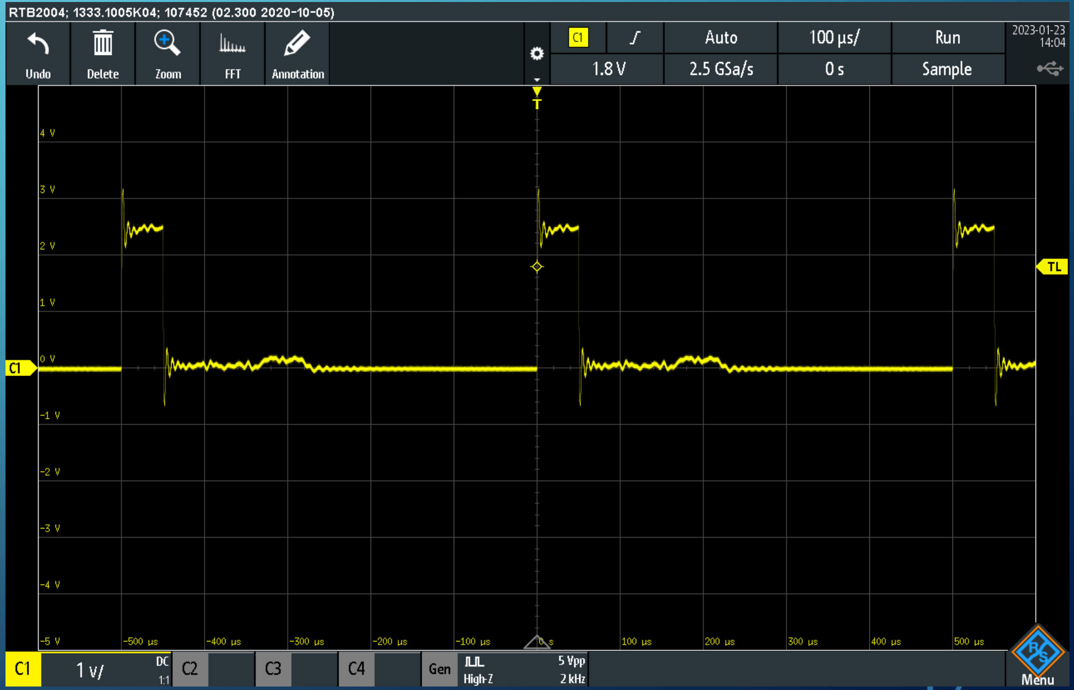
$$Z_0 \approx \sqrt{L/C}$$

- Theoretical impedance = 148.32 ± 16.58 Ω
- Expressions for the capacitor and inductor impedances contain angular frequencies (ω)
- We expect the ratio of the voltages to change

Solely reflected



Reflected and inverted



No reflection

FIRST HINTS OF FREQUENCY ATTENUATION

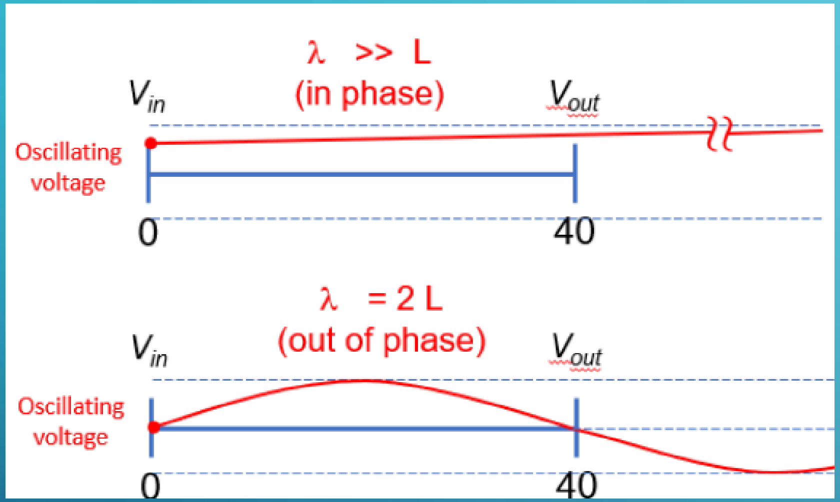
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$$

$$a_n = \frac{1}{L}\int_{2L} f(x)\cos\frac{n\pi x}{L}\,dx \qquad b_n = \frac{1}{L}\int_{2L} f(x)\sin\frac{n\pi x}{L}\,dx$$

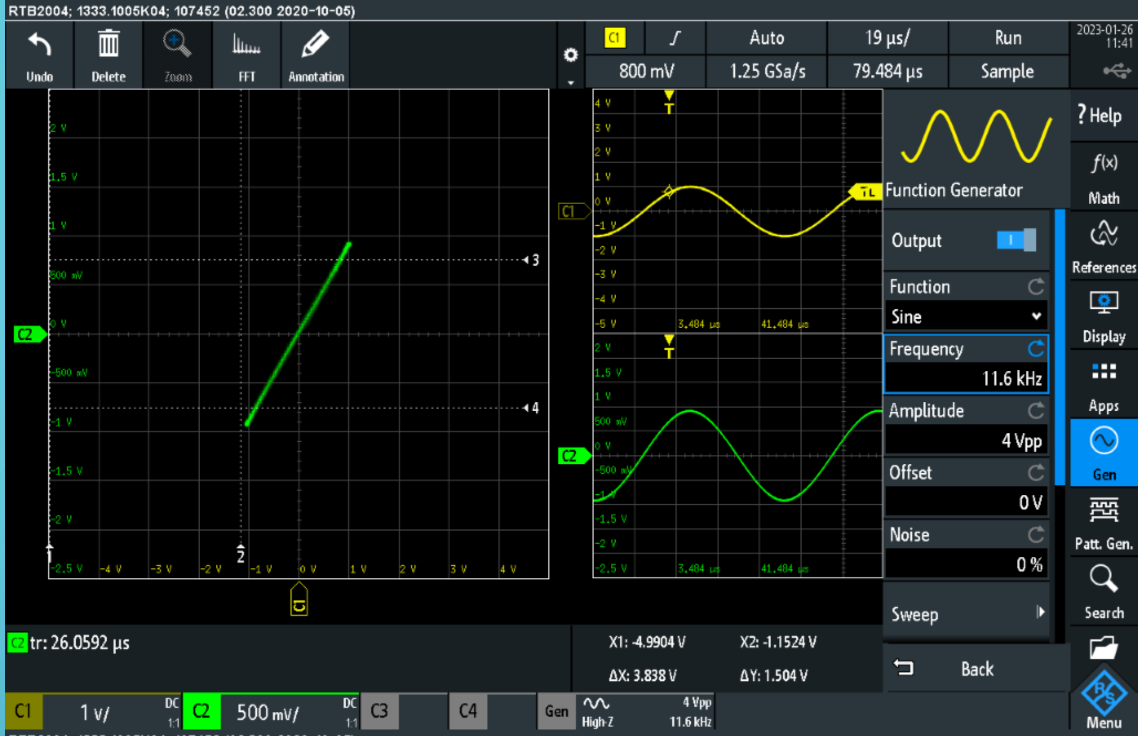
- Square waves appear to stretch as they propagate through the line
- Square waves can be considered a Fourier series to the nth harmonic
- Longer rise times imply higher frequency harmonics are supressed
- Lower frequency harmonics are less affected
- Effect becomes more evident further along the lumped transmission line

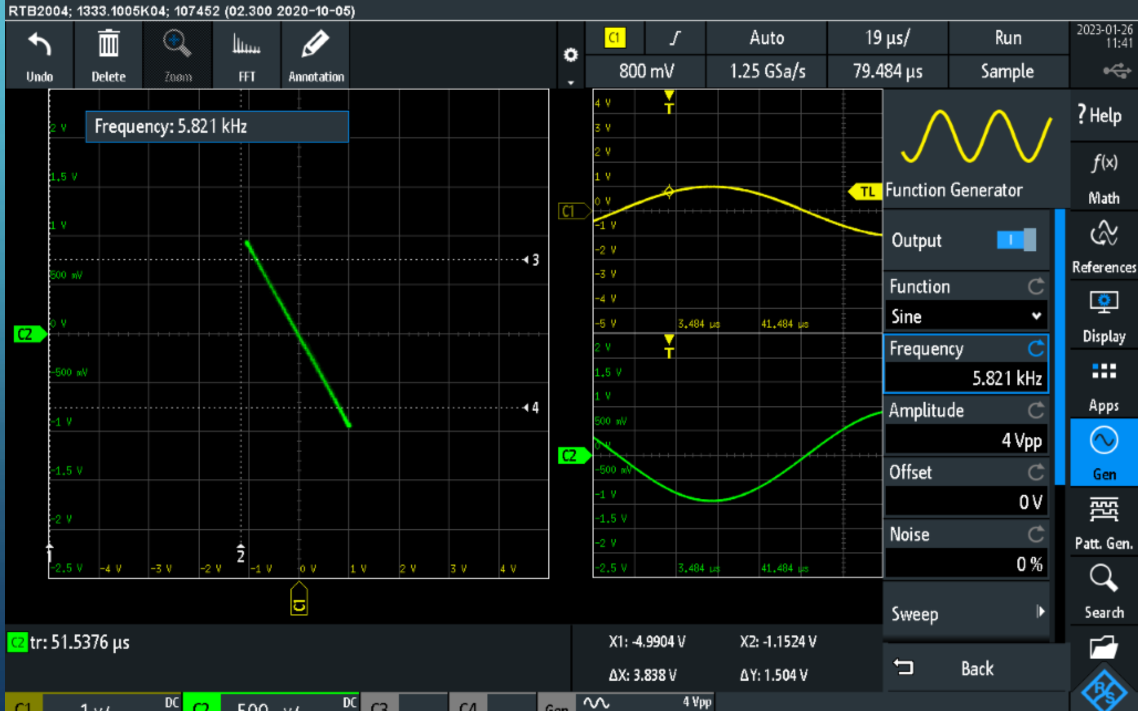
INVESTIGATING FREQUENCY RELATION

- Square wave gave an insight on frequency attenuation
- Setting up for taking data with frequency as the independent variable using sinusoidal waves
- Lissajous figures allow to identify when the two waveforms are either in or out of phase.



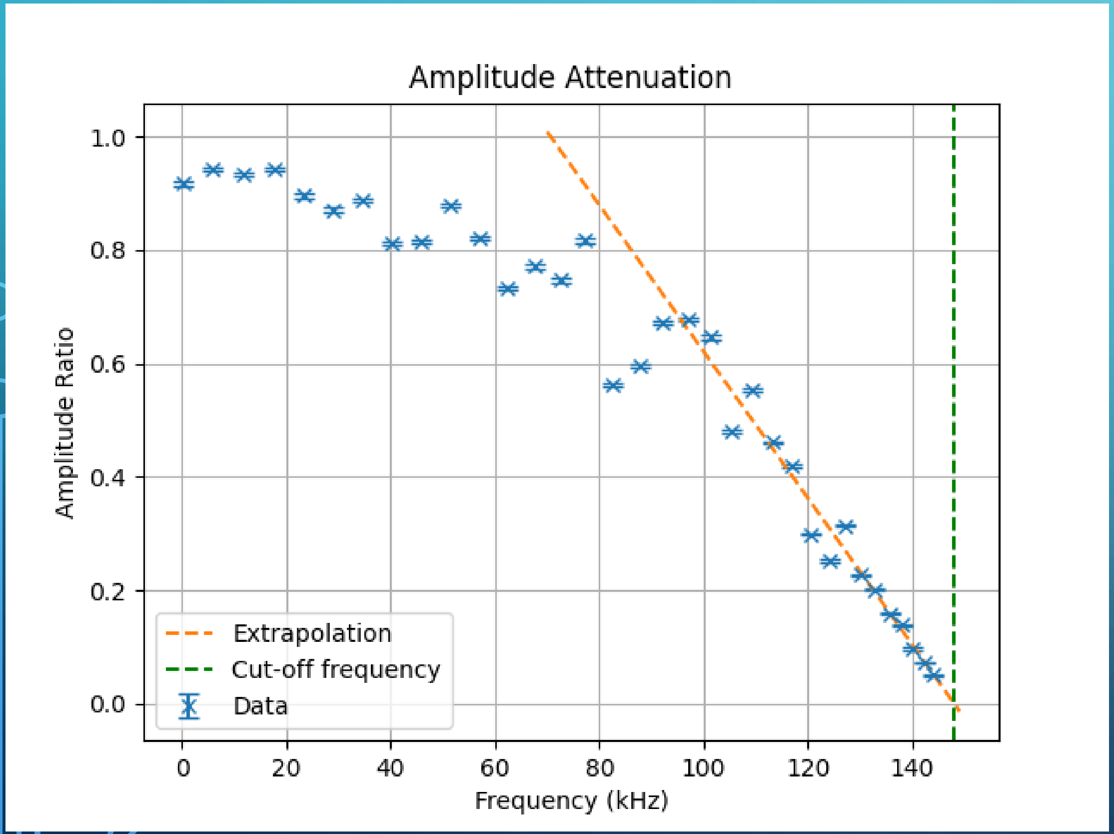
- $\lambda = \frac{2L}{n}$
- At n = 0 the wavelength is approximately infinity (and in phase)
- Starting frequency of 20kHz

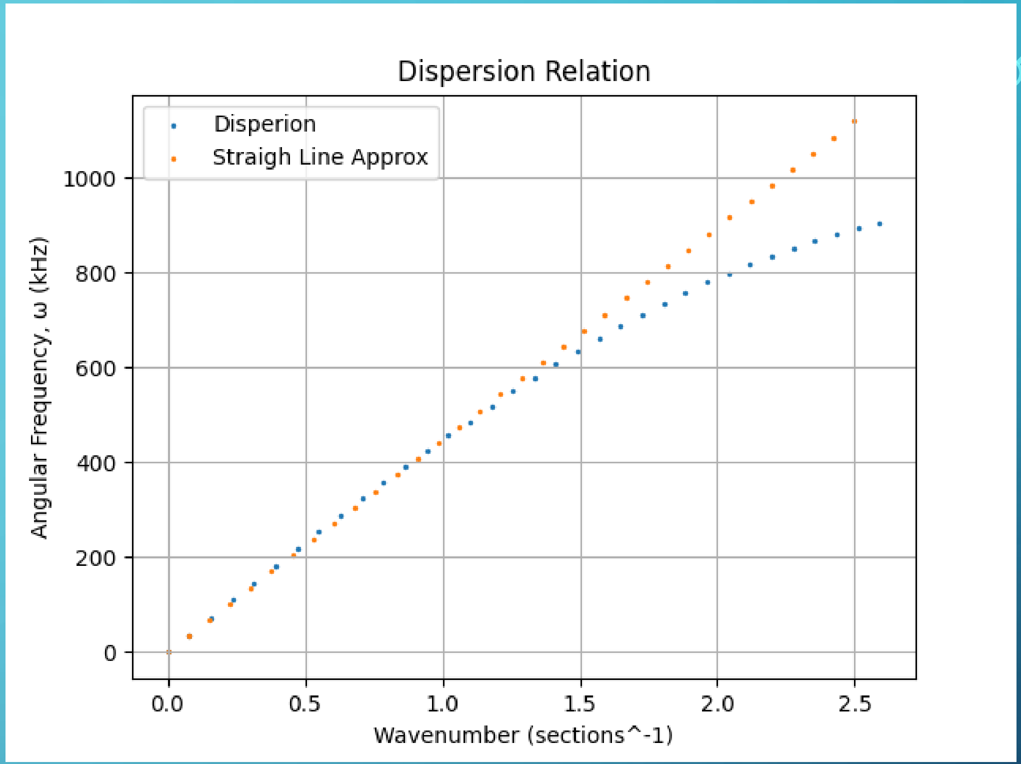




DATA

- Clear decrease in amplitude ratio as frequency increases
- Smothering of higher frequency in the transmission line
- Cut-off frequency = 148.03 ± 6.90 kHz
- $\omega_c = \sqrt{\frac{4}{LC}}$ (theoretical value)
- $f_c = 143.1 \pm 31.5$ kHz

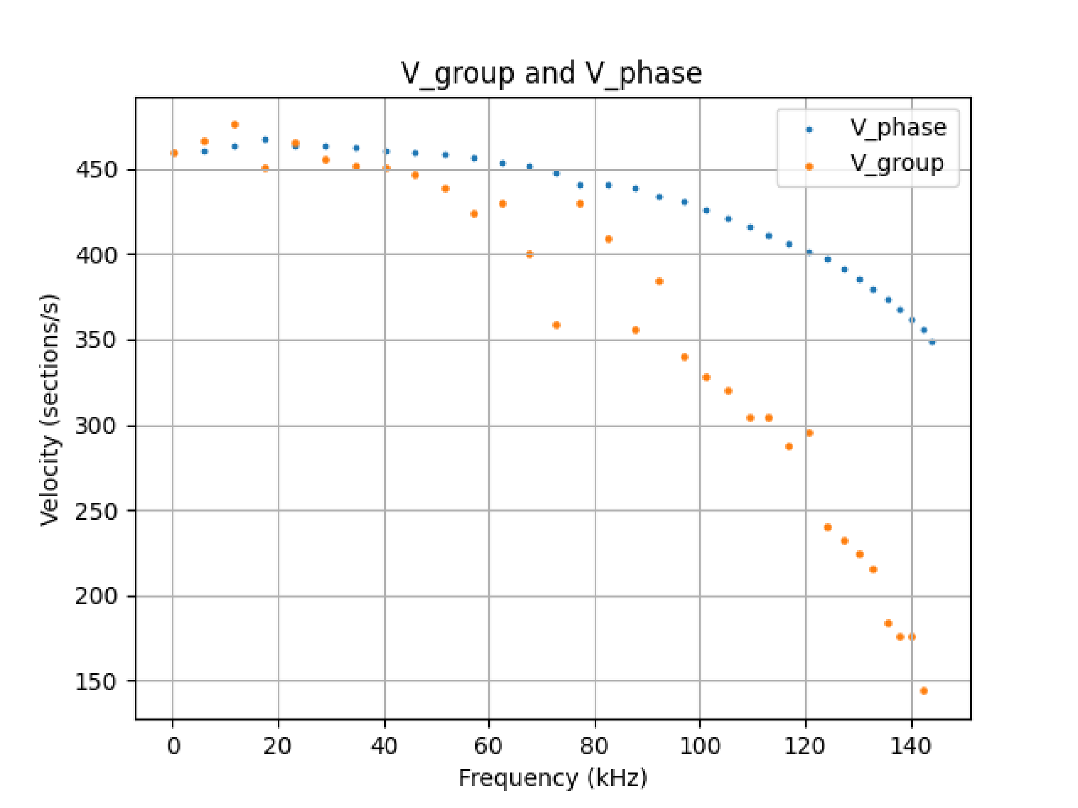


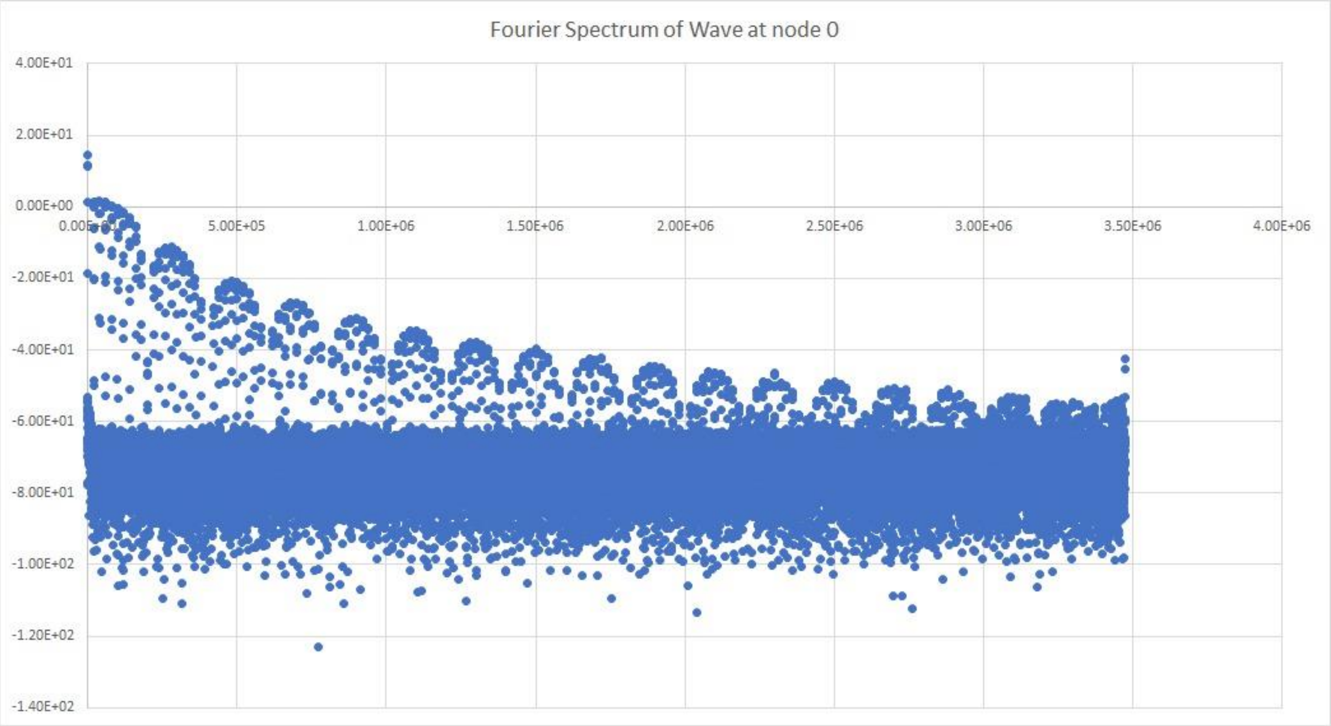


- Group velocity decreases
- Gradient of straight line gives phase velocity of lower frequency terms
- $V_{phase} = 448.0 \pm 2.1$ sections/s
- Theoretical $V_{phase} = 1/\sqrt{LC}$
- $V_{phase} = 449.5 \pm 100.5$ sections/s

GROUP VELOCITY AND FURTHER ANALYSIS

- V_{phase} decreases
- V_{group} obtained from dispersion relation
- $V_{group} = \frac{\Delta\omega}{\Delta k}$
- Decays at a faster rate than V_{phase}





- Taking the Fourier transform of the signal at node 0
- Shows clear amplitude attenuation
- Successive spikes decrease in amplitude

CONCLUSIONS AND DISCUSSIONS

- Amplitude attenuation as frequency increases
- Higher frequency signals travel slower
- Group velocity decreases as more frequencies are added

- Short cables barely are affected
- Longer cables such as transmission lines will experience these effects
- Complicated signals will be smothered
- Single, low frequencies are most efficient
- Dimensions can change inherent characteristics of wires