Investigating Amplitude Attenuation and Frequency Dispersion For a Lumped Transmission Line

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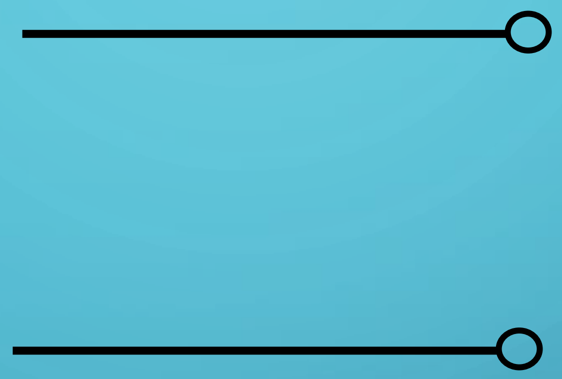
Outline:

1. Why do we care?
2. Reflection of pulses
3. Hints of frequency attenuation
4. Investigating frequency relation
5. Data
6. Group velocity and further analysis
7. Conclusions and discussions

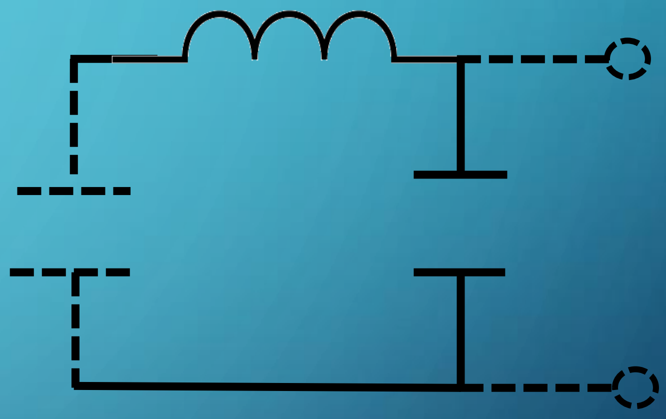
WHY DO WE CARE?

- Wires are everywhere!
- Not as simple as they seem
- The design of wires gives rise to inherent capacitance and inductance in the wires.
- These can hinder our ability to send electrical signals over long distances.

Appearance of parallel wires



Behaviour of parallel wires



1 section

- Behaves as an inductor due to induced magnetic field
- Parallel wires will act as capacitors
- Lumped transmission line has 40 sections
- 10 nodes
- $L = 330 \pm 20\%\ \mu H$
- $C = 0.015 \pm 10\%\ \mu F$
- Variable resistor placed in parallel with a switch at node 10

REFLECTION OF PULSES

- Impedance of transmission line due to components of the line
- Gives rise to reflected pulses
- The cause various complications in the line

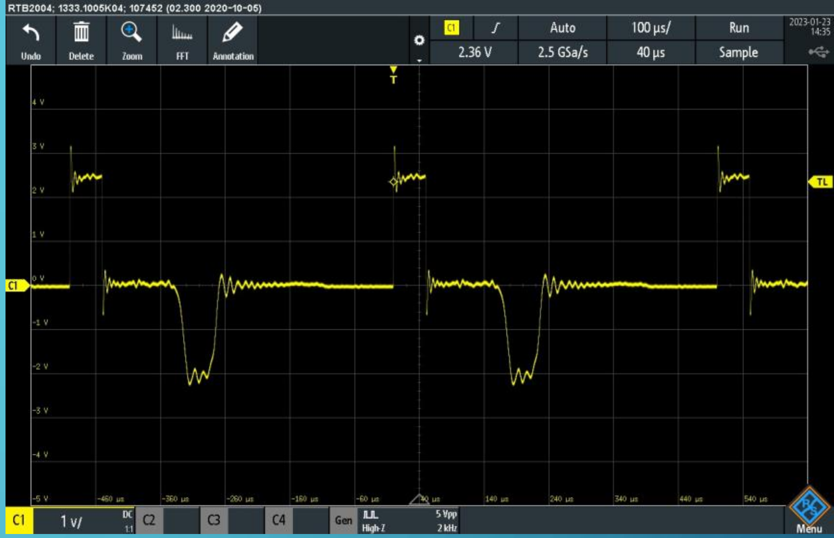
$$R = \frac{V_r}{V_i} = \frac{(Z_b - Z_a)}{(Z_b + Z_a)}$$

- Impedances must be equal to cancel reflected signal
- Variable resistor = 156.48 ± 1.73 Ω

$$Z_0 \approx \sqrt{L/C}$$

- Theoretical impedance = 148.32 ± 16.58 Ω
- Expressions for the capacitor and inductor impedances contain angular frequencies (ω)
- We expect the ratio of the voltages to change

Solely reflected



Reflected and inverted

No reflection

HINTS OF FREQUENCY ATTENUATION

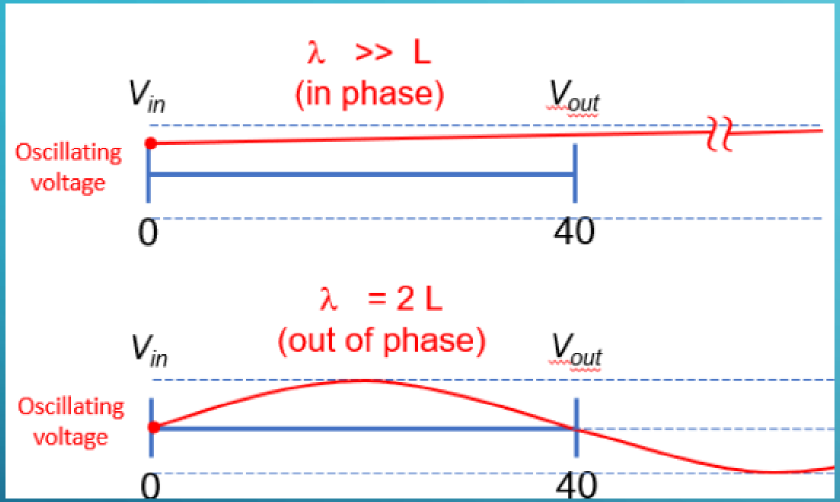
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$$

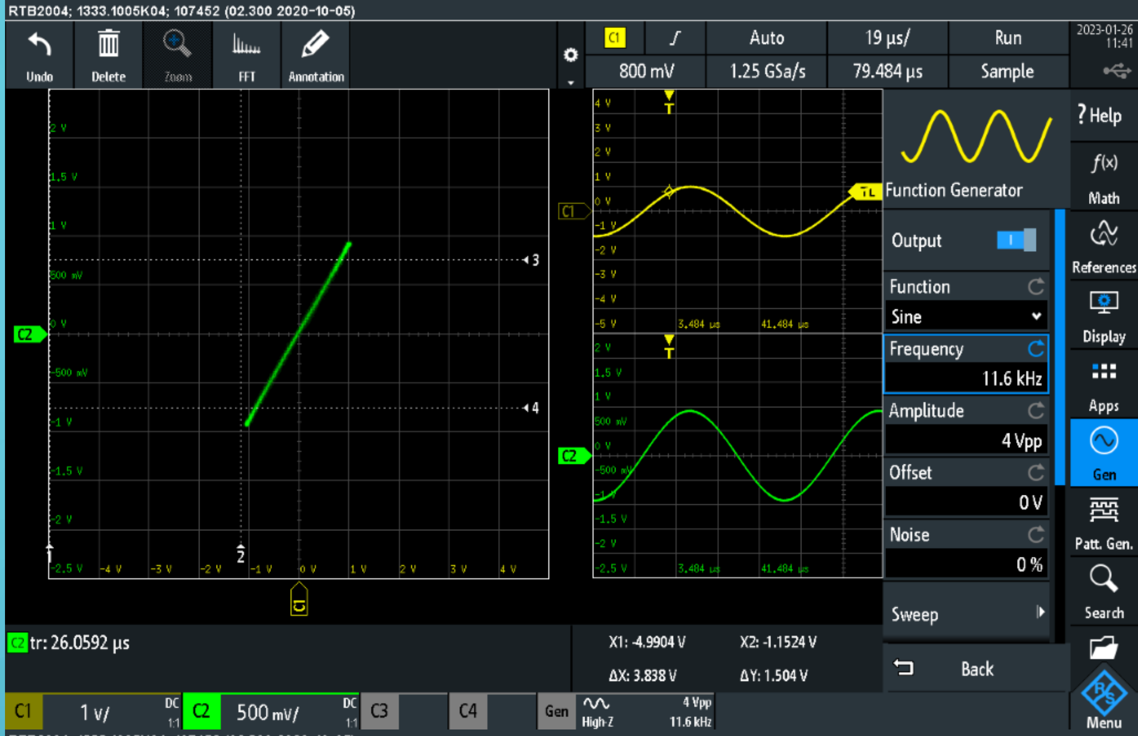
$$a_n = \frac{1}{L}\int_{2L} f(x)\cos\frac{n\pi x}{L}\,dx \qquad b_n = \frac{1}{L}\int_{2L} f(x)\sin\frac{n\pi x}{L}\,dx$$

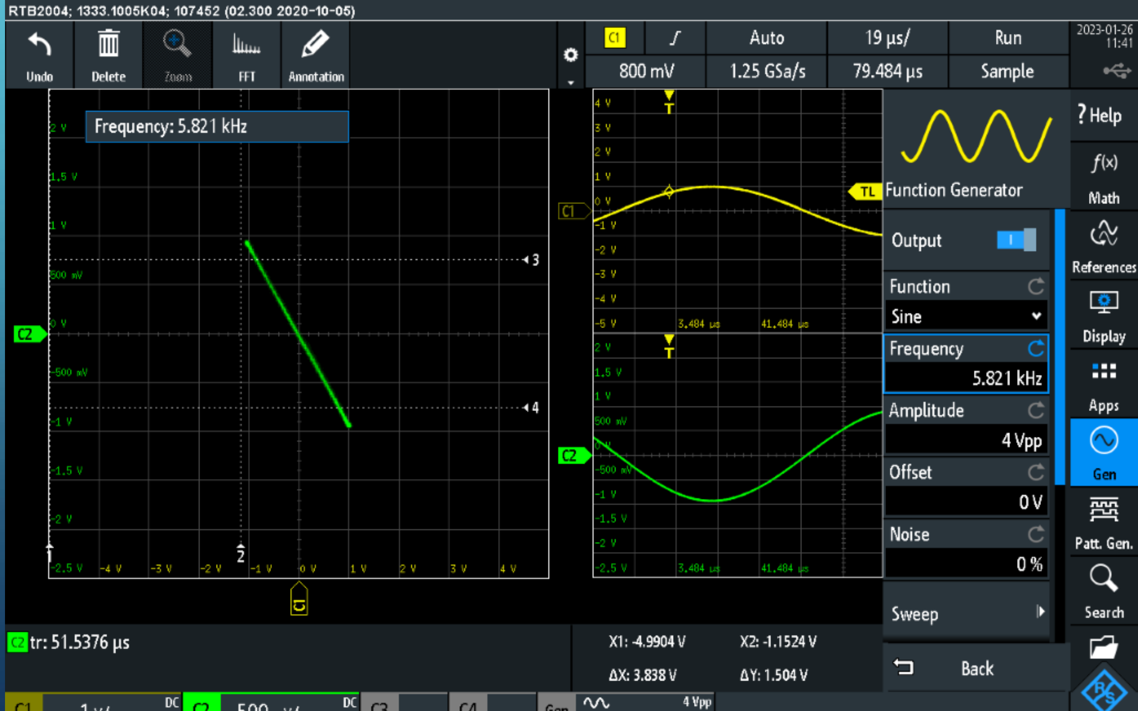
- Square waves appear to stretch as they propagate through the line
- Square waves can be considered a Fourier series to the nth harmonic
- Longer rise times imply higher frequency harmonics are supressed
- Lower frequency harmonics are less affected
- Effect becomes more evident further along the lumped transmission line

INVESTIGATING FREQUENCY RELATION

- Square wave gave an insight on frequency attenuation
- Setting up for taking data with frequency as the independent variable using sinusoidal waves
- Lissajous figures allow to identify when the two waveforms are either in or out of phase.



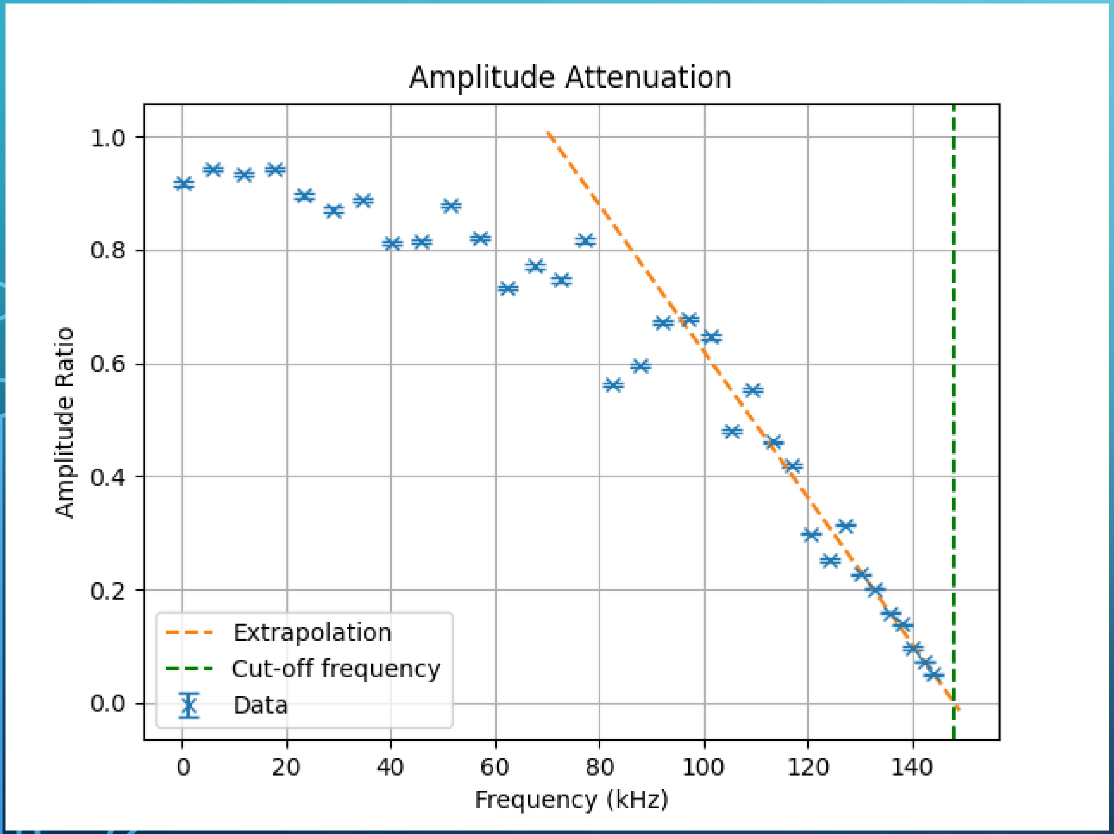


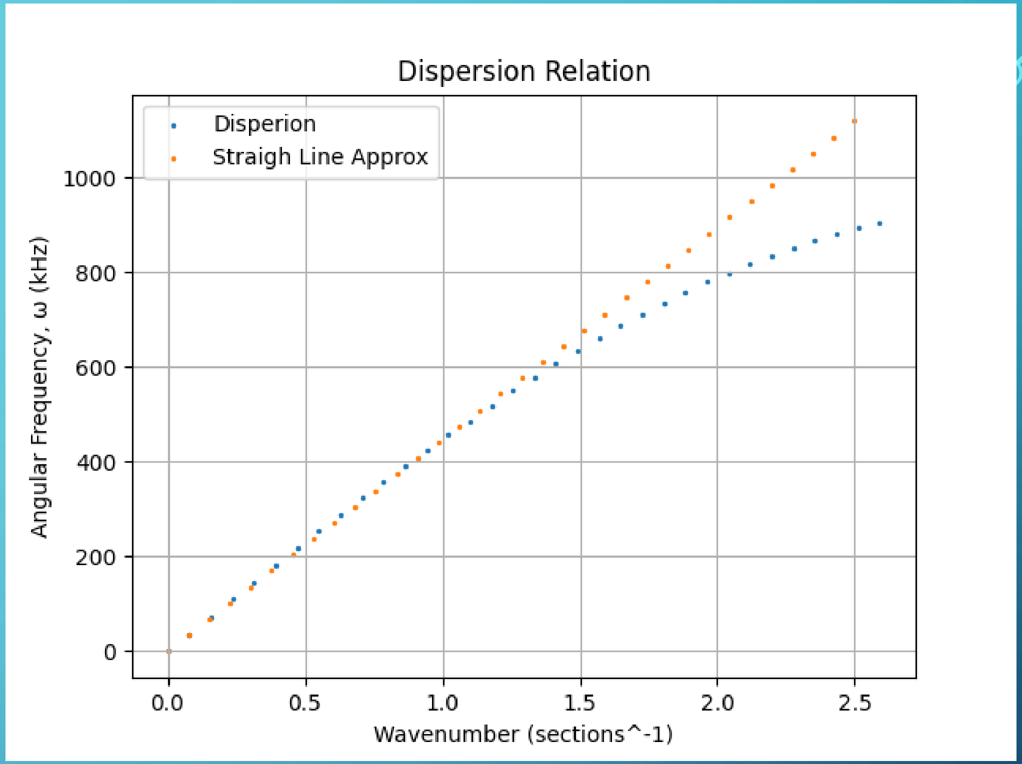


- $\lambda = \frac{2L}{n}$
- At n = 0 the wavelength is approximately infinity (and in phase)
- Starting frequency of 20kHz

DATA

- Clear decrease in amplitude ratio as frequency increases
- Smothering of higher frequency in the transmission line
- Cut-off frequency = 148.03 ± 6.90 kHz
- $\omega_c = \sqrt{\frac{4}{LC}}$ (theoretical value)
- $f_c = 143.1 \pm 31.5$ kHz

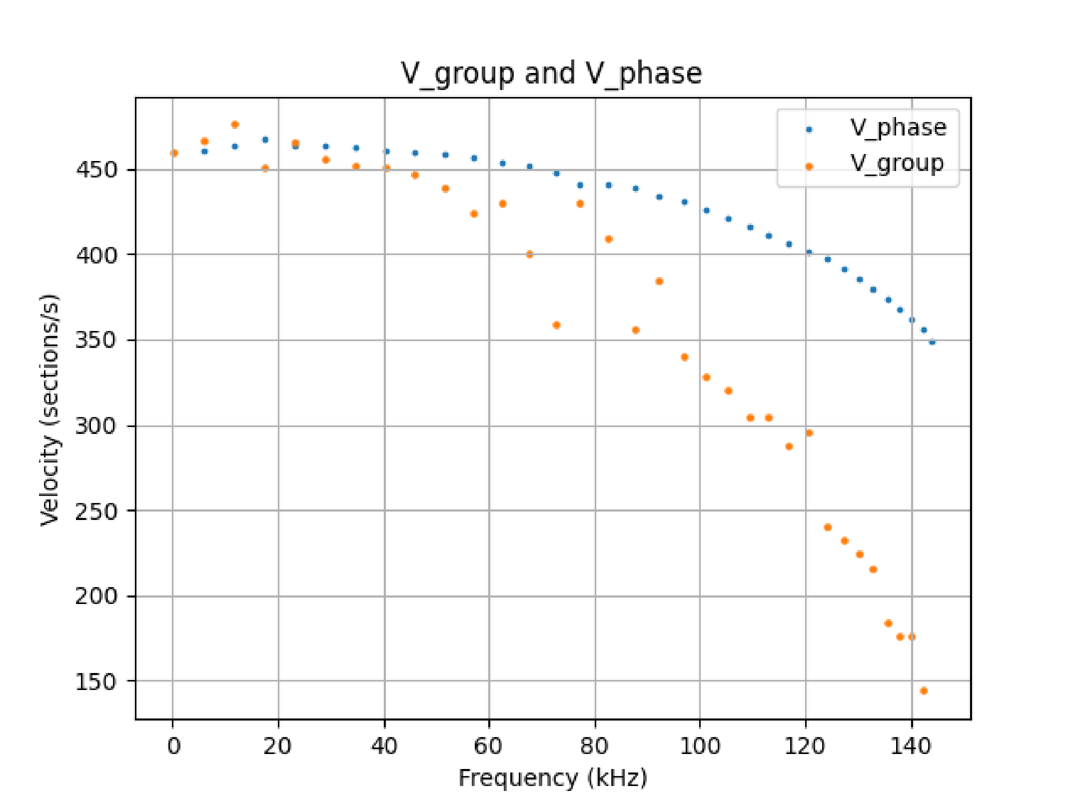


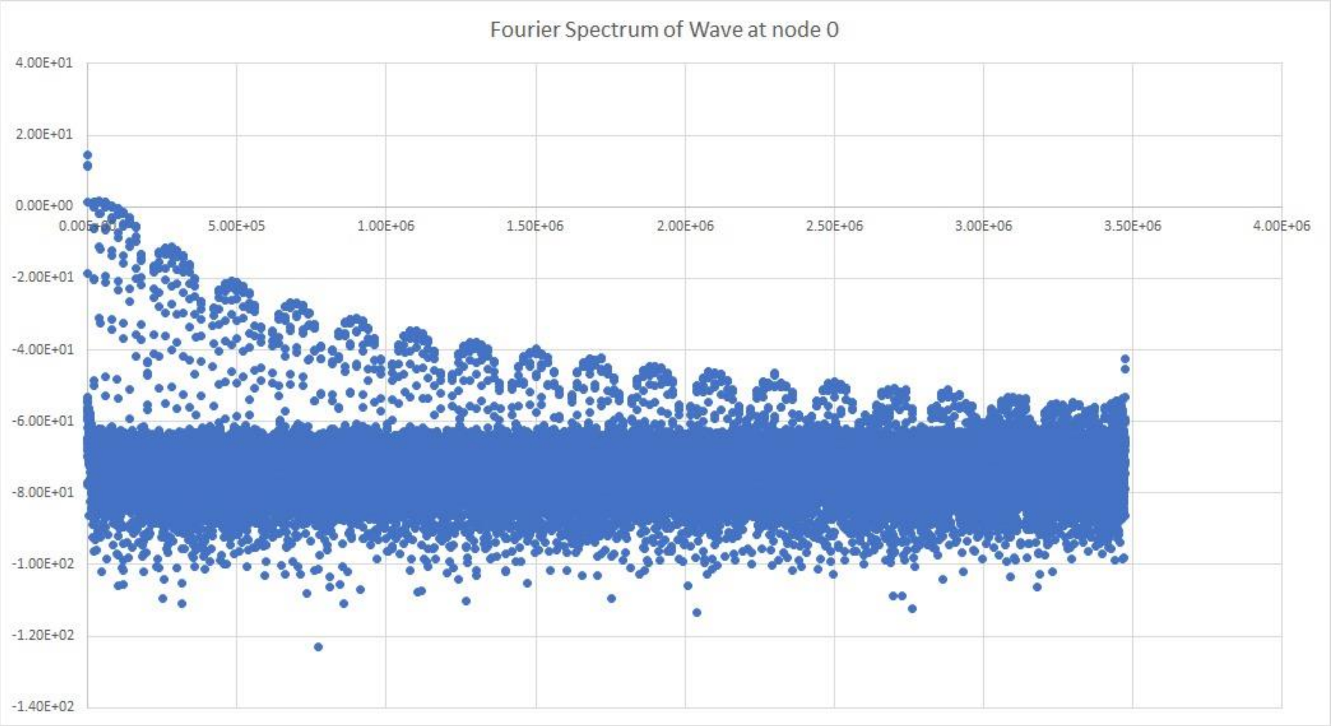


- Group velocity decreases
- Gradient of straight line gives phase velocity of lower frequency terms
- $V_{phase} = 448.0 \pm 2.1$ sections/s
- Theoretical $V_{phase} = 1/\sqrt{LC}$
- $V_{phase} = 449.5 \pm 100.5$ sections/s

GROUP VELOCITY AND FURTHER ANALYSIS

- V_{phase} decreases
- V_{group} obtained from dispersion relation
- $V_{group} = \frac{\Delta\omega}{\Delta k}$
- Decays at a faster rate than V_{phase}





- Taking the Fourier transform of the signal at node 0
- Shows clear amplitude attenuation
- Successive spikes decrease in amplitude

CONCLUSIONS AND DISCUSSIONS

- Amplitude attenuation as frequency increases
- Higher frequency signals travel slower
- Group velocity decreases as more frequencies are added

- Short cables barely are affected
- Longer cables such as transmission lines will experience these effects
- Complicated signals will be smothered
- Single, low frequencies are most efficient
- Dimensions can change inherent characteristics of wires