Use of Direct Numerical Simulations for Studies on Magnetohydrodynamics

Andrés Cathey

The University of Edinburgh

June 20, 2017

- Theory review of MHD and turbulent processes.
- Crash course on ARCHER and Scientific Computing.
- Become familiar with available MHD code (eDNS).
- Determine onset of dynamo action (threshold value of magnetic Reynolds number (Re_M)) in forced MHD simulations.

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- Determine onset of dynamo action (threshold value of magnetic Reynolds number (Re_M)) in forced MHD simulations.
- Study behaviour for different magnetic Prandtl number ($Pr_M = Re_M/Re = \nu/\eta$).
- (If time allows) Study effect of helicity's effect on the inverse cascade.

- ARCHER is the largest UK National Supercomputing Service.
- Run parallelised simulations of cubic boxes describing MHD turbulence.
- Larger boxes require more cores to store in-code memory.
- Boxes of lattice size $N^3 > 32^3$ cannot realistically be worked on regular laptops/desktops (eDNS allows to get to up to 2048^3).
- Lattice size is directly proportional to the maximum wavenumber of the simulations, and it has to be set up such that the kinetic and magnetic Kolmogorov length scales are resolved.

$$\partial_t \mathbf{u} = -\frac{1}{\rho}\boldsymbol{\nabla} p - (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} + \frac{1}{\rho}(\boldsymbol{\nabla}\times\mathbf{b})\times\mathbf{b} + \nu\nabla^2\mathbf{u} + \mathbf{f}_u \tag{1}$$

$$\partial_t \mathbf{b} = (\mathbf{b}\cdot\boldsymbol{\nabla})\mathbf{u} - (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{b} + \eta\nabla^2\mathbf{b} + \mathbf{f}_b \tag{2}$$

$$\boldsymbol{\nabla}\cdot\mathbf{u} = 0 \tag{3}$$

$$\boldsymbol{\nabla}\cdot\mathbf{b} = 0 \tag{4}$$

$$k_{max} \approx \frac{N}{3} \qquad k_{max} > k_\eta,\ k_\nu \tag{5}$$

$$k_\eta = \left(\frac{\varepsilon}{\eta^3}\right)^{1/4} \qquad k_\nu = \left(\frac{\varepsilon}{\nu^3}\right)^{1/4} \tag{6}$$

Current Work

Onset of Dynamo Action

Keep Pr_M constant, vary ν. $Re_M = 28.27$

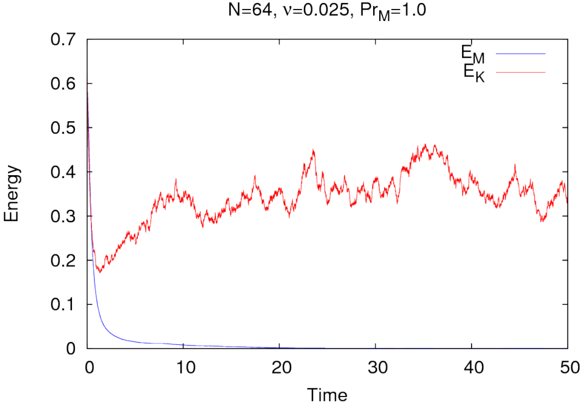

Figure: eDNS simulation with 64 lattice size and $\nu = 0.025$ and $\eta = 0.025$ ($Pr_M = 1.0$).

Keep Pr_M constant, vary ν. $Re_M = 36.14$

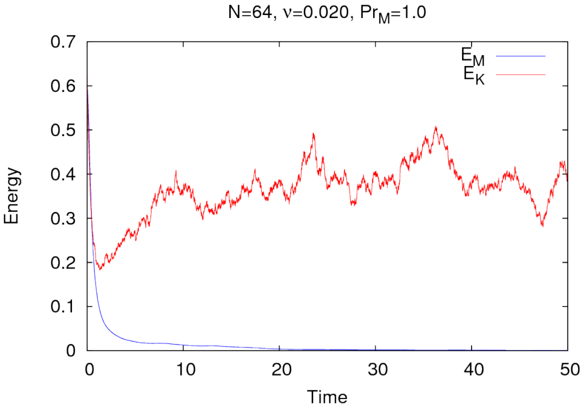

Figure: eDNS simulation with 64 lattice size and $\nu = 0.020$ and $\eta = 0.020$ ($Pr_M = 1.0$).

Keep Pr_M constant, vary ν. $Re_M = 47.16$

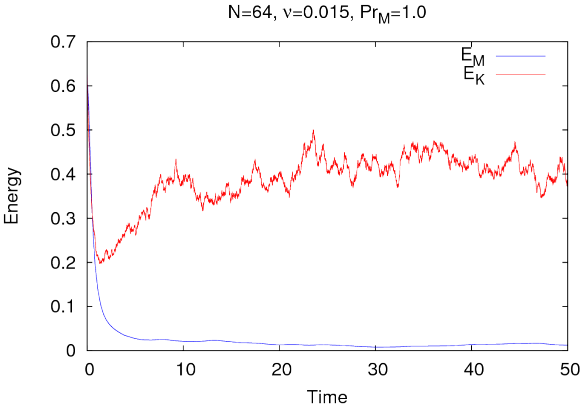

Figure: eDNS simulation with 64 lattice size and $\nu = 0.015$ and $\eta = 0.015$ ($Pr_M = 1.0$).

Keep Pr_M constant, vary ν. $Re_M = 69.58$

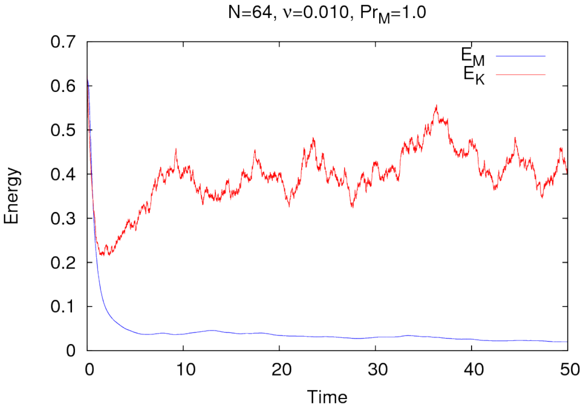

Figure: eDNS simulation with 64 lattice size and $\nu = 0.010$ and $\eta = 0.010$ ($Pr_M = 1.0$).

Keep ν constant, vary Pr_M. $Re_M = 29.47$

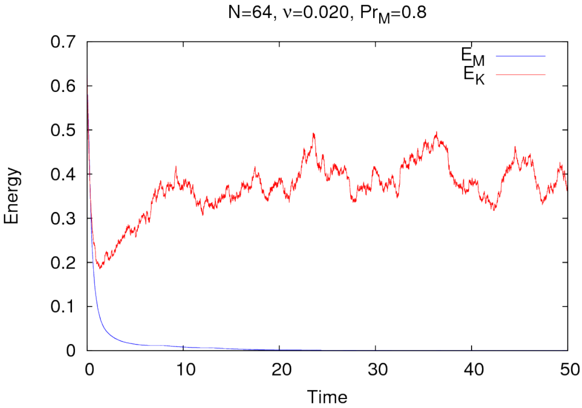

Figure: eDNS simulation with 64 lattice size and $\nu = 0.020$ and $\eta = 0.025$ ($Pr_M = 0.8$).

Keep ν constant, vary Pr_M. $Re_M = 36.14$

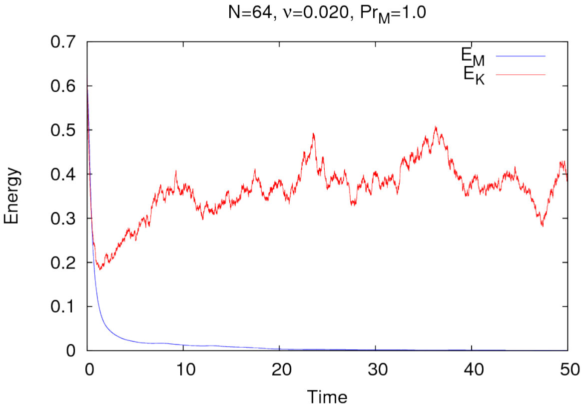

Figure: eDNS simulation with 64 lattice size and $\nu = 0.020$ and $\eta = 0.020$ ($Pr_M = 1.0$).

Keep ν constant, vary Pr_M. $Re_M = 48.17$

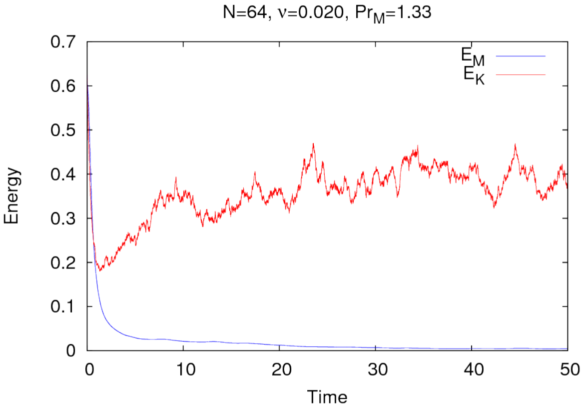

Figure: eDNS simulation with 64 lattice size and $\nu = 0.020$ and $\eta = 0.015$ ($Pr_M = 1.33$).

Keep ν constant, vary Pr_M. $Re_M = 70.23$

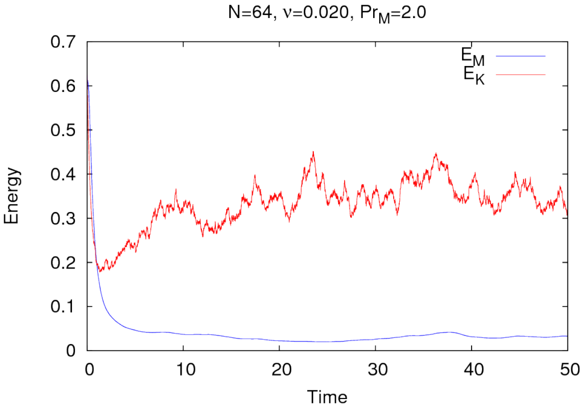

Figure: eDNS simulation with 64 lattice size and $\nu = 0.020$ and $\eta = 0.010$ ($Pr_M = 2.0$).

- Both methods have similar behaviour - magnetic Reynolds number is the key component for the onset of Dynamo action.
- Pick only one method.
- Look at the energies in the end of the simulation as a function of wavenumber.

Wavenumber summed energy and energy spectra. $\nu = 0.020$. $Pr_M = 0.8$. $Re_M = 29.47$

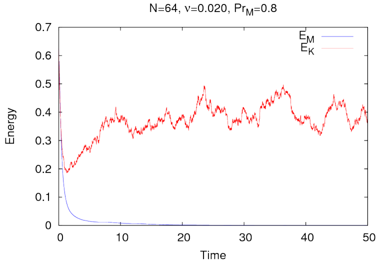

Figure: k-summed kinetic and magnetic Energy for $N = 64$, $\nu = 0.020$, $\eta = 0.025$.

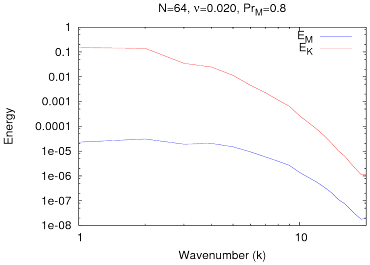

Figure: Kinetic and magnetic Energy spectra for $N = 64$, $\nu = 0.020$, $\eta = 0.025$.

Current Work

Onset of Dynamo Action

Wavenumber summed energy and energy spectra. $\nu = 0.020$. $Pr_M = 1.0$. $Re_M = 36.14$

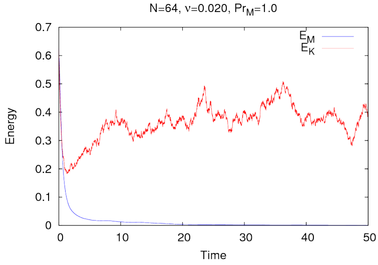

Figure: k-summed kinetic and magnetic Energy for $N = 64$, $\nu = 0.020$, $\eta = 0.010$.

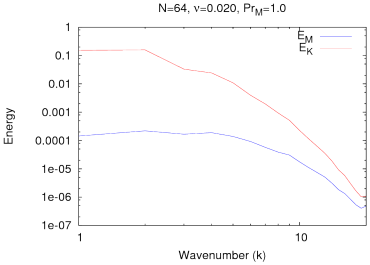

Figure: Kinetic and magnetic Energy spectra for $N = 64$, $\nu = 0.020$, $\eta = 0.010$.

Wavenumber summed energy and energy spectra. $\nu = 0.020$. $Pr_M = 1.0$. $Re_M = 36.14$

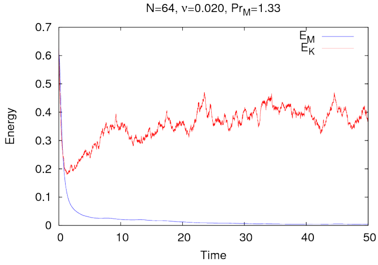

Figure: k-summed kinetic and magnetic Energy for $N = 64$, $\nu = 0.020$, $\eta = 0.015$.

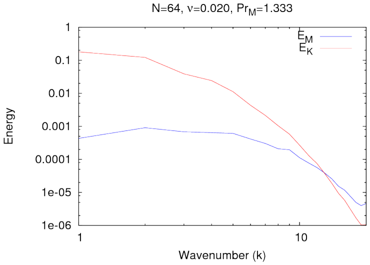

Figure: Kinetic and magnetic Energy spectra for $N = 64$, $\nu = 0.020$, $\eta = 0.015$.