

# Magnetic Prandtl Number Dependence of the Kinetic-to-Magnetic Dissipation Ratio

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2014

# Overview

## Magnetohydrodynamics

- What exactly is MHD?

- Examples

## Reynolds Numbers and the Magnetic Prandtl Number

- Reynolds Numbers

- Magnetic Prandtl Number

## DNS of Turbulent Dynamos

- Governing Equations

- Results

## Shell and 1D Models

- Shell Model

- Driven 1D Model

## Conclusions

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- ▶ Electrically conducting fluids.
  - ▶ Plasmas.
  - ▶ Liquid metals.
  - ▶ Electrolytes.

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# Magnetohydrodynamics

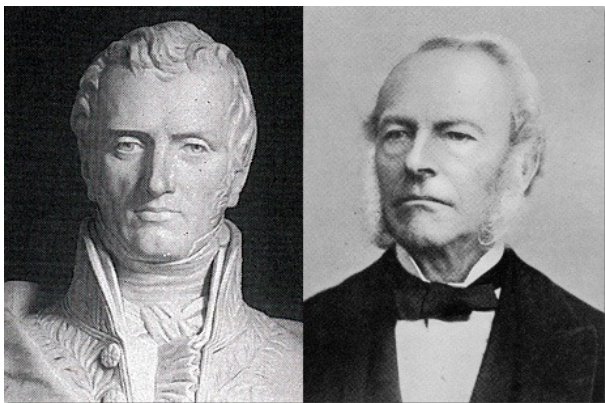
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- ▶ Moving  $\rho$  generate currents.
- ▶ Ampère's law:  $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ .
- ▶ Faraday's law:  $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$ .

# Magnetohydrodynamics

What exactly is MHD?

Navier-Stokes equations





# Magnetohydrodynamics

What exactly is MHD?

Maxwell equations



# Magnetohydrodynamics

What exactly is MHD?

Maxwell equations

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = - \frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

# Magnetohydrodynamics

What exactly is MHD?

## Numerical simulations

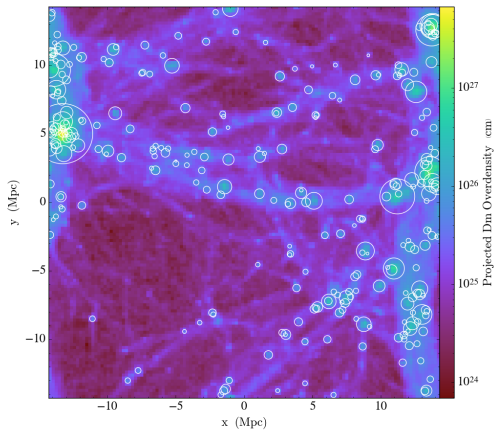
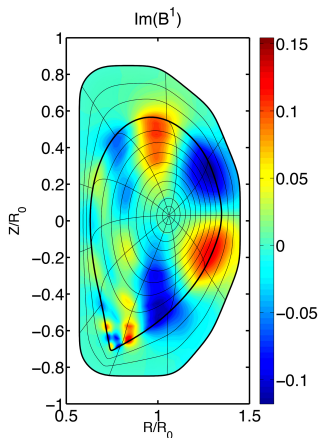


Figure: Cosmological simulation showing dark matter halos.

# Magnetohydrodynamics

What exactly is MHD?

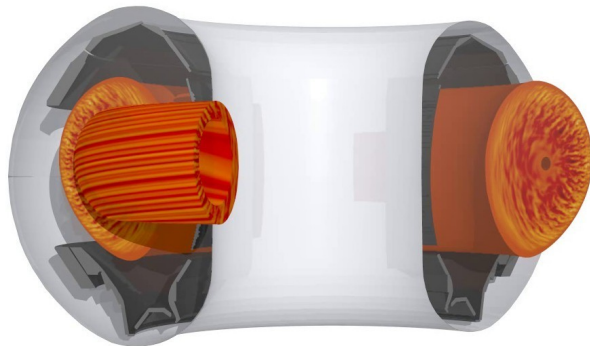
## Numerical simulations



**Figure:** Radial component of magnetic field amplitude in an unstable  $n=1$  kink mode in DIII-D. MHD Stability code MARS.

# Magnetohydrodynamics

Examples: Laboratory Plasma



**Figure:** Snapshot from a numerical simulation of plasma turbulence in the ASDEX Upgrade tokamak with the nonlinear gyrokinetic code GENE. Dr. Jenko

# Magnetohydrodynamics

Examples: Magnetic Dynamos - Astrophysical Scales

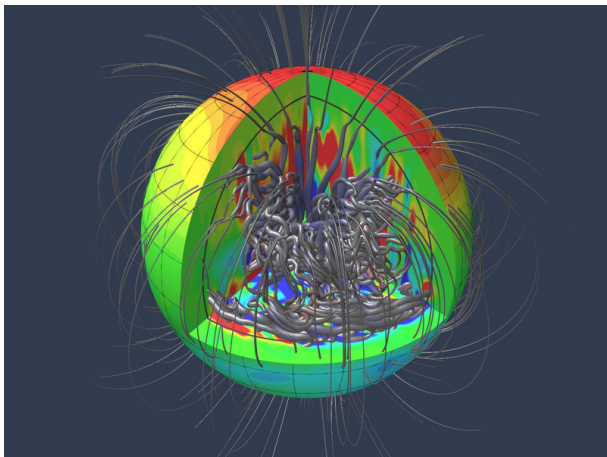
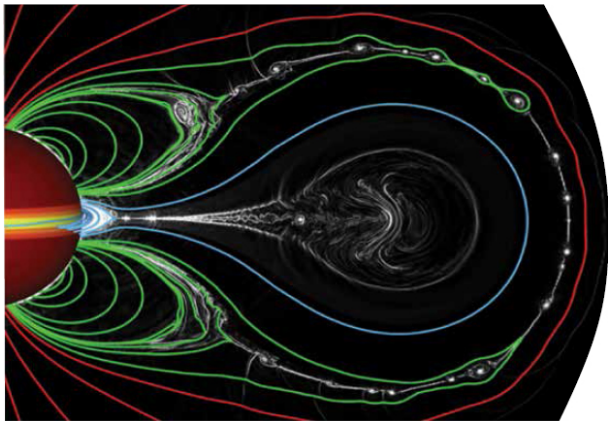


Figure: Jupiter cut open (2014). Dr. Krummheuer & Dr. Wicht

# Magnetohydrodynamics

Examples: MHD Turbulence - Astrophysical Scales



**Figure:** Ultra-high-resolution numerical simulation of a coronal mass ejection and associated flare. Solar and Space Physics (2010)

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Magnetic Prandtl Number

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## Shell and 1D Models

Shell Model

Driven 1D Model

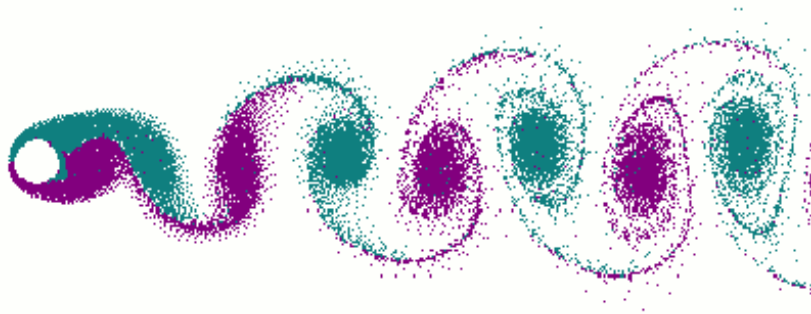
## Conclusions



# Reynolds Numbers and the Magnetic Prandtl Number

## Reynolds Number

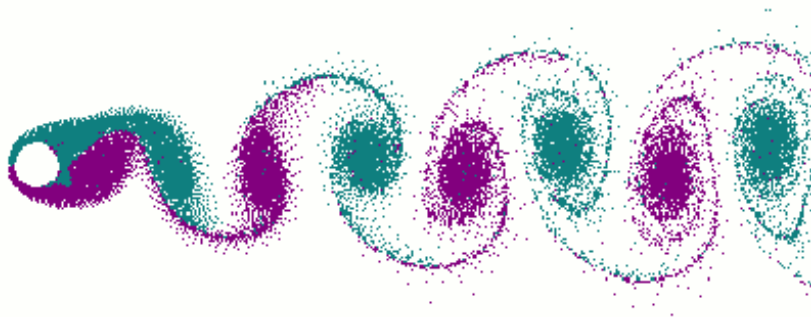
$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u L}{\nu}$$



# Reynolds Numbers and the Magnetic Prandtl Number

## Reynolds Number

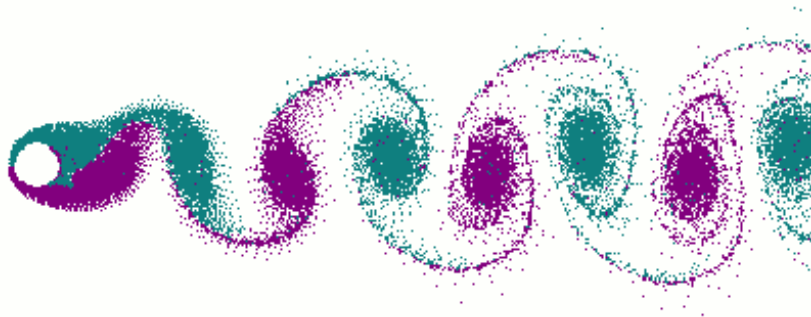
$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u L}{\nu}$$



# Reynolds Numbers and the Magnetic Prandtl Number

## Reynolds Number

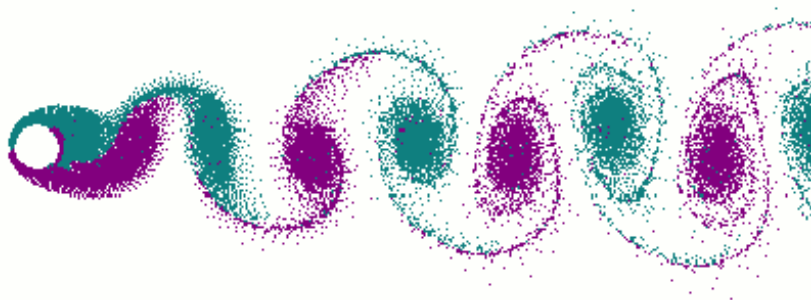
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# Reynolds Numbers and the Magnetic Prandtl Number

## Reynolds Number

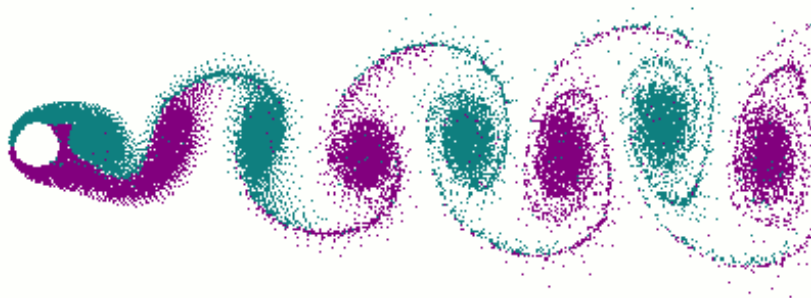
$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u L}{\nu}$$



# Reynolds Numbers and the Magnetic Prandtl Number

## Reynolds Number

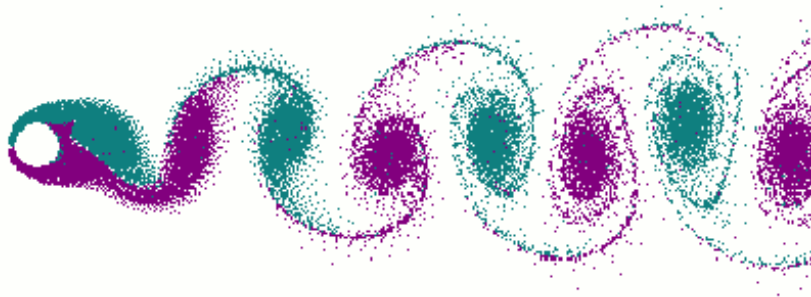
$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u L}{\nu}$$



# Reynolds Numbers and the Magnetic Prandtl Number

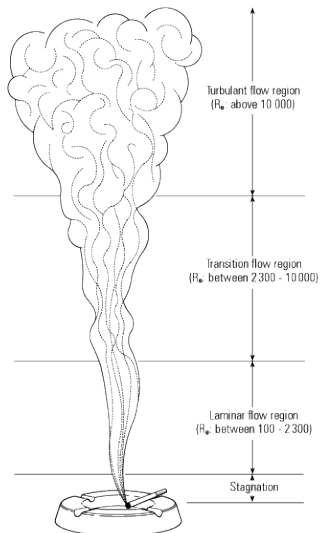
## Reynolds Number

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u L}{\nu}$$



# Reynolds Numbers and the Magnetic Prandtl Number

## Reynolds Number



# Reynolds Numbers and the Magnetic Prandtl Number

## Magnetic Reynolds Number

Ideal MHD equations: Perfectly conducting fluids.



# Reynolds Numbers and the Magnetic Prandtl Number

## Magnetic Reynolds Number

$$Re_M = \frac{\text{inertial forces}}{\text{diffusive forces}} = \frac{u L}{\eta}$$

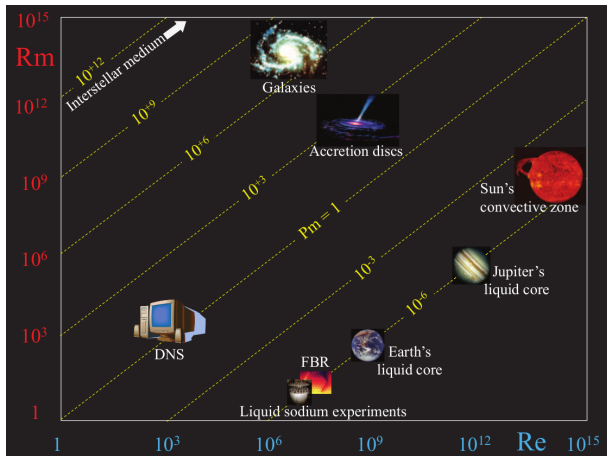
# Reynolds Numbers and the Magnetic Prandtl Number

## Magnetic Prandtl Number

$$Pr_M = \frac{Re_M}{Re} = \frac{\nu}{\eta}$$

# Reynolds Numbers and the Magnetic Prandtl Number

## Magnetic Prandtl Number



**Figure:** Map of “typical” objects in the plane  $(Re, Re_M)$ . Yellow dashed lines are  $Pr_M$  isolines. [1].

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# DNS of Turbulent Dynamos

## Governing Equations

Forced MHD turbulence of a gas with isothermal equation of state:  
 $p = \rho c_s^2$ .

$$\begin{aligned}\frac{D \ln \rho}{Dt} &= -\nabla \cdot \mathbf{u} \\ \frac{D \mathbf{u}}{Dt} &= -c_s^2 \nabla \ln \rho - 2\boldsymbol{\Omega} \times \mathbf{u} + \mathbf{f} \\ &\quad + \rho^{-1} [\mathbf{J} \times \mathbf{B} + \nabla \cdot (2\nu \rho \boldsymbol{\mathcal{S}})] \\ \frac{\partial \mathbf{A}}{\partial t} &= \mathbf{u} \times \mathbf{B} - \eta \mu_0 \mathbf{J}\end{aligned}$$

# DNS of Turbulent Dynamos

## Governing Equations

Kinetic and Magnetic energies.

$$\frac{d}{dt} \langle \rho u^2 / 2 \rangle = \langle p \nabla \cdot \mathbf{u} \rangle + \langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle + \langle \rho \mathbf{u} \cdot \mathbf{f} \rangle - \langle 2\rho\nu S^2 \rangle$$

$$\frac{d}{dt} \langle B^2 / 2\mu_0 \rangle = -\langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle - \langle \eta \mu_0 J^2 \rangle$$

# DNS of Turbulent Dynamos

## Governing Equations

Kinetic and Magnetic energies.

$$\begin{aligned}\frac{d}{dt}\langle \rho u^2/2 \rangle &= \langle p \nabla \cdot \mathbf{u} \rangle + \langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle + \langle \rho \mathbf{u} \cdot \mathbf{f} \rangle - \langle 2\rho\nu\mathcal{S}^2 \rangle \\ \frac{d}{dt}\langle B^2/2\mu_0 \rangle &= -\langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle - \langle \eta\mu_0 J^2 \rangle\end{aligned}$$

Dissipation rates.

$$\epsilon_K = \langle 2\rho\nu\mathcal{S}^2 \rangle, \quad \epsilon_M = \langle \eta\mu_0 J^2 \rangle$$

# DNS of Turbulent Dynamos

## Governing Equations

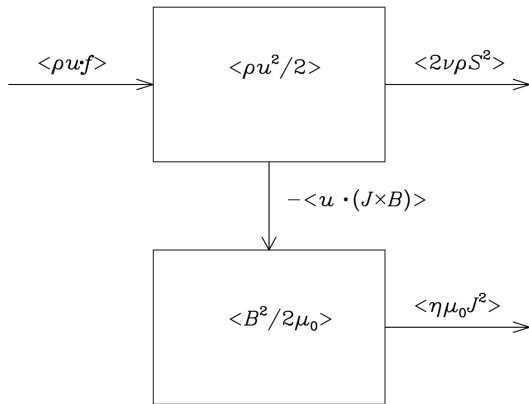


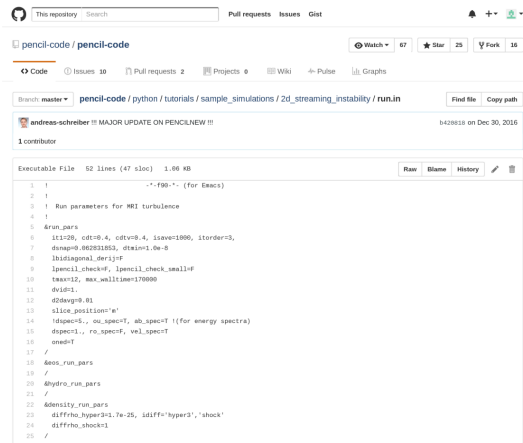
Figure: Flow of energy sketch [2].



# DNS of Turbulent Dynamos

## Simulations and Results

### Pencil code (NORDITA)



The screenshot shows the GitHub repository for Pencil-code. The repository name is "pencil-code / pencil-code". It has 67 watchers, 25 stars, and 16 forks. The main branch is "master". The file path shown is "pencil-code / python / tutorials / sample\_simulations / 2d\_streaming\_instability / run.in". The file is 52 lines (47 sloc) and 1.06 KB. The code is a shell script for running Pencil-code simulations. It sets various parameters for the simulation, including the number of processors, the domain size, the time step, and the output files. The code is as follows:

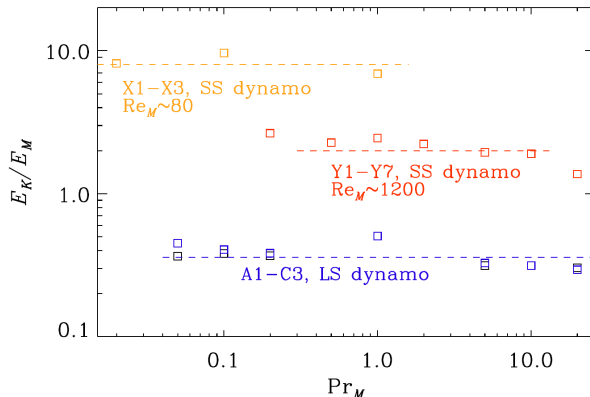
```
1 !  
2 !  
3 ! Run parameters for MRI turbulence  
4 !  
5 #run_pars  
6 lti=20, cdc=0.4, cdtv=8.4, lsave=1800, ltorder=3,  
7 dsnap=0.062831853, dtain=1.0e-8  
8 lbiadiagonal_derij=F  
9 lpencil_check=F, lpencil_check_small=F  
10 ltime=12, max_waltime=170000  
11 dvid=1  
12 d2avg=9.01  
13 slice_position='e'  
14 'dspec=5., ou_spec=T, ab_spec=T' ! (for energy spectra)  
15 dspec=1., ro_spec=F, vel_spec=T  
16 oned=T  
17 /  
18 #eos_run_pars  
19 /  
20 #hydro_run_pars  
21 /  
22 #density_run_pars  
23 diffrho_hyper3=1.7e-25, idiff='hyper3','shock'  
24 diffrho_shock=1  
25 /  
26 - - -
```

Figure: Snapshot of Pencil-code GitHub repository.

# DNS of Turbulent Dynamos

## Simulations and Results

Energy ratio approximately independent on  $Pr_M$ .

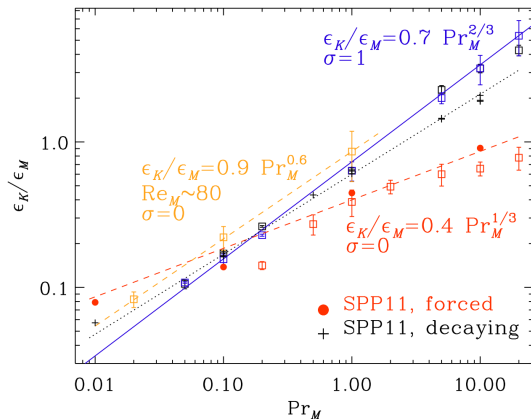


**Figure:** Energy ratio  $E_K/E_M$  dependence on  $Pr_M$  for large-scale dynamo (blue) and small-scale dynamos (orange and red) [2].

# DNS of Turbulent Dynamos

## Simulations and Results

Dissipation ratio dependency on  $Pr_M$ .



**Figure:** Dissipation ratio  $\epsilon_K/\epsilon_M$  dependence on  $Pr_M$  for non-helical forcing ( $\sigma = 0$ ) and for fully helical forcing ( $\sigma = 1$ ). [2].

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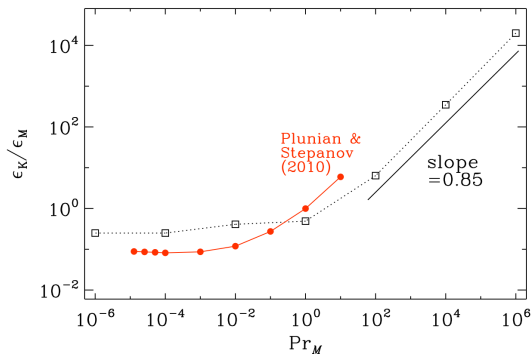
## Shell Model

Similar equations than before - same conserved quantities.  
Time integration scheme: Adams-Bashforth

# Shell and 1D Models

## Shell Model

Dissipation ratio dependency on  $Pr_M$ .



**Figure:** Dissipation ratio  $\epsilon_K/\epsilon_M$  dependence on  $Pr_M$  [2]. Red shows simulations made by Plunian and Stepanov [3].

# Shell and 1D Models

## Driven 1D Model

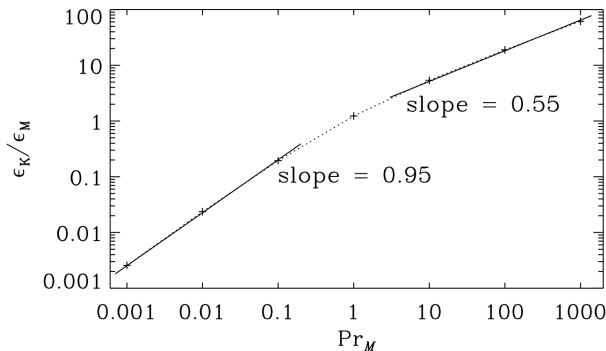
Neglecting gas pressure:

$$\begin{aligned}\frac{\partial u}{\partial t} &= -uu' - bb' + \tilde{\nu}u'' \\ \frac{\partial b}{\partial t} &= -ub' - bu' + \eta b''\end{aligned}$$

# Shell and 1D Models

## Driven 1D Model

Dissipation ratio dependency on  $Pr_M$ .



**Figure:** Dissipation ratio  $\epsilon_K/\epsilon_M$  dependence on  $Pr_M$  [2]. Red shows simulations made by Plunian and Stepanov [3].



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2. Discovered scaling laws ( $\epsilon_K/\epsilon_M \sim Pr_M^q$ ) for fully helical and non-helical forcing. Where  $q \approx 2/3$  for the former and  $q \approx [1/3 - 0.6]$  for the latter.

# Conclusions

1. Obtained relations for the kinetic-to-magnetic dissipation ratio for a broad range of  $Pr_M$  values.
2. Discovered scaling laws ( $\epsilon_K/\epsilon_M \sim Pr_M^q$ ) for fully helical and non-helical forcing. Where  $q \approx 2/3$  for the former and  $q \approx [1/3 - 0.6]$  for the latter.
3. Consistent results to previous simulations regarding the kinetic-to-magnetic dissipation ratio were acquired.

# References

- [1] F. Plunian, R. Stepanov, and P. Frick, "Shell models of magnetohydrodynamic turbulence," *Physics Reports*, vol. 523, no. 1, pp. 1–60, 2013.
- [2] A. Brandenburg, "Magnetic prandtl number dependence of the kinetic-to-magnetic dissipation ratio," *The Astrophysical Journal*, vol. 791, no. 1, p. 12, 2014.
- [3] F. Plunian and R. Stepanov, "Cascades and dissipation ratio in rotating magnetohydrodynamic turbulence at low magnetic prandtl number," *Physical Review E*, vol. 82, no. 4, p. 046311, 2010.

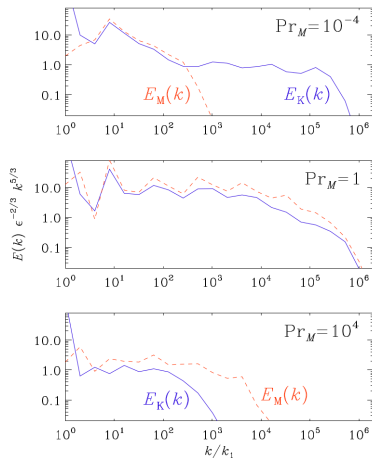
# Dependence on $Re$

Table

Run	$\nu k_1/c_s$	$\eta k_1/c_s$	$Re$	$Re_M$	$Pr_M$	$\sigma$	$u_{rms}/c_s$	$b_{rms}/c_s$	$\epsilon_K/\epsilon_T$	$\epsilon_M/\epsilon_T$	$C_\epsilon$	$k_\nu/k_1$	$k_\eta/k_1$	res.
A1	$5.0 \times 10^{-4}$	$2.5 \times 10^{-5}$	56	1123	20.00	1	0.087	0.158	0.81	0.19	1.83	38	247	1024 <sup>3</sup>
A2	$5.0 \times 10^{-4}$	$5.0 \times 10^{-5}$	57	568	10.00	1	0.088	0.157	0.76	0.24	1.80	37	156	512 <sup>3</sup>
A3	$5.0 \times 10^{-4}$	$1.0 \times 10^{-4}$	57	284	5.00	1	0.088	0.157	0.69	0.31	1.82	36	99	512 <sup>3</sup>
A4	$5.0 \times 10^{-5}$	$5.0 \times 10^{-5}$	587	587	1.00	1	0.091	0.128	0.39	0.61	1.75	179	201	512 <sup>3</sup>
A5	$5.0 \times 10^{-5}$	$2.5 \times 10^{-4}$	606	121	0.20	1	0.094	0.155	0.21	0.79	1.46	150	63	512 <sup>3</sup>
A6	$5.0 \times 10^{-5}$	$5.0 \times 10^{-4}$	594	59	0.10	1	0.092	0.149	0.15	0.85	1.60	139	38	512 <sup>3</sup>
A7	$5.0 \times 10^{-5}$	$1.0 \times 10^{-3}$	581	29	0.05	1	0.090	0.149	0.10	0.90	1.72	125	23	512 <sup>3</sup>
B1	$5.0 \times 10^{-5}$	$5.0 \times 10^{-5}$	587	587	1.00	1	0.091	0.128	0.39	0.61	1.75	179	201	512 <sup>3</sup>
B2	$2.5 \times 10^{-4}$	$5.0 \times 10^{-5}$	117	587	5.00	1	0.091	0.159	0.67	0.33	1.57	60	168	512 <sup>3</sup>
B3	$5.0 \times 10^{-4}$	$5.0 \times 10^{-5}$	57	568	10.00	1	0.088	0.157	0.76	0.24	1.80	37	156	512 <sup>3</sup>
B4	$1.0 \times 10^{-3}$	$5.0 \times 10^{-5}$	27	542	20.00	1	0.084	0.155	0.84	0.16	2.09	23	141	512 <sup>3</sup>
C1	$2.0 \times 10^{-5}$	$1.0 \times 10^{-4}$	1548	310	0.20	1	0.096	0.155	0.19	0.81	1.30	287	124	512 <sup>3</sup>
C2	$2.0 \times 10^{-5}$	$2.0 \times 10^{-4}$	1532	153	0.10	1	0.095	0.149	0.14	0.87	1.41	268	76	512 <sup>3</sup>
C3	$2.0 \times 10^{-5}$	$4.0 \times 10^{-4}$	1516	76	0.05	1	0.094	0.140	0.10	0.90	1.47	248	46	512 <sup>3</sup>
X1	$5.0 \times 10^{-4}$	$5.0 \times 10^{-4}$	56	56	1.00	0	0.113	0.043	0.46	0.54	0.35	28	29	256 <sup>3</sup>
X2	$3.5 \times 10^{-5}$	$3.5 \times 10^{-4}$	864	86	0.10	0	0.121	0.039	0.18	0.82	0.26	159	41	256 <sup>3</sup>
X3	$7.0 \times 10^{-6}$	$3.5 \times 10^{-4}$	4179	84	0.02	0	0.117	0.041	0.08	0.92	0.28	422	42	512 <sup>3</sup>
Y1	$1.0 \times 10^{-3}$	$5.0 \times 10^{-5}$	55	1093	20.00	0	0.082	0.070	0.44	0.56	2.35	16	164	512 <sup>3</sup>
Y2	$5.0 \times 10^{-4}$	$5.0 \times 10^{-5}$	121	1213	10.00	0	0.091	0.066	0.40	0.60	1.79	27	168	512 <sup>3</sup>
Y3	$2.5 \times 10^{-4}$	$5.0 \times 10^{-5}$	245	1227	5.00	0	0.092	0.066	0.38	0.62	1.64	44	167	512 <sup>3</sup>
Y4	$1.0 \times 10^{-4}$	$5.0 \times 10^{-5}$	647	1293	2.00	0	0.097	0.065	0.33	0.67	1.42	85	171	512 <sup>3</sup>
Y5	$5.0 \times 10^{-5}$	$5.0 \times 10^{-5}$	1293	1293	1.00	0	0.097	0.062	0.28	0.72	1.32	135	171	512 <sup>3</sup>
Y6	$2.5 \times 10^{-5}$	$5.0 \times 10^{-5}$	2533	1267	0.50	0	0.095	0.063	0.21	0.79	1.34	210	173	512 <sup>3</sup>
Y7	$1.0 \times 10^{-5}$	$5.0 \times 10^{-5}$	6400	1280	0.20	0	0.096	0.059	0.12	0.88	1.20	356	174	512 <sup>3</sup>

# Shell Models

## Energy profiles with shell model



**Figure:** Compensated time-averaged kinetic and magnetic energy spectra for shell models at three values of  $Pr_M$  [2].