

Magnetic Prandtl Number Dependence of the Kinetic-to-Magnetic Dissipation Ratio

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2014

Overview

Magnetohydrodynamics

- What exactly is MHD?

- Examples

Reynolds Numbers and the Magnetic Prandtl Number

- Reynolds Numbers

- Magnetic Prandtl Number

DNS of Turbulent Dynamos

- Governing Equations

- Results

Shell and 1D Models

- Shell Model

- Driven 1D Model

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- ▶ Electrically conducting fluids.
 - ▶ Plasmas.
 - ▶ Liquid metals.
 - ▶ Electrolytes.

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Magnetohydrodynamics

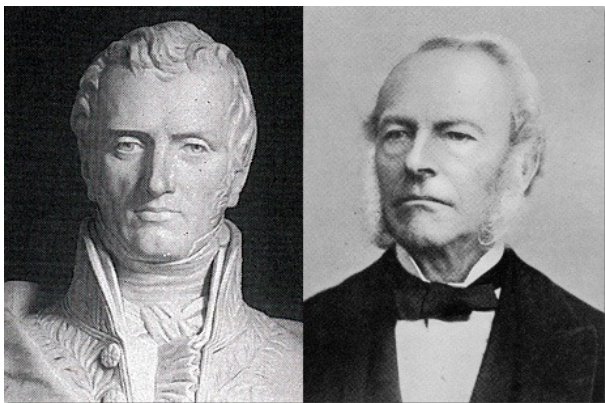
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- ▶ Ampère's law: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$.
- ▶ Faraday's law: $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$.

Magnetohydrodynamics

What exactly is MHD?

Navier-Stokes equations



Magnetohydrodynamics

What exactly is MHD?

Maxwell equations



Magnetohydrodynamics

What exactly is MHD?

Maxwell equations

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = - \frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

Magnetohydrodynamics

What exactly is MHD?

Numerical simulations

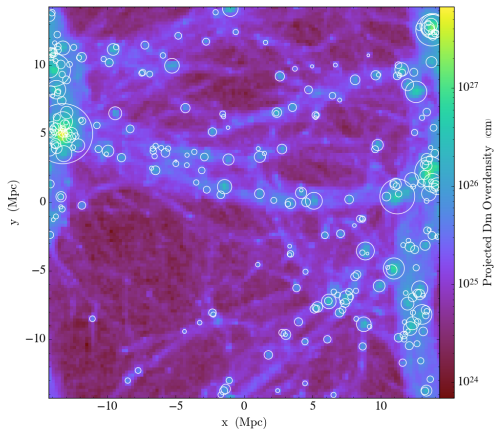


Figure: Cosmological simulation showing dark matter halos.

Magnetohydrodynamics

What exactly is MHD?

Numerical simulations

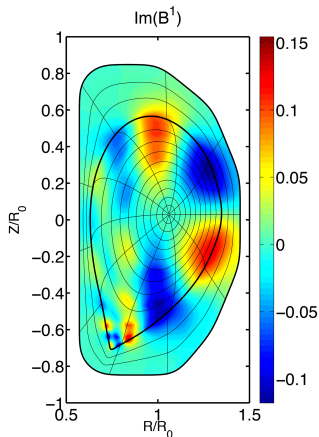


Figure: Radial component of magnetic field amplitude in an unstable $n=1$ kink mode in DIII-D. MHD Stability code MARS.

Magnetohydrodynamics

Examples: Laboratory Plasma

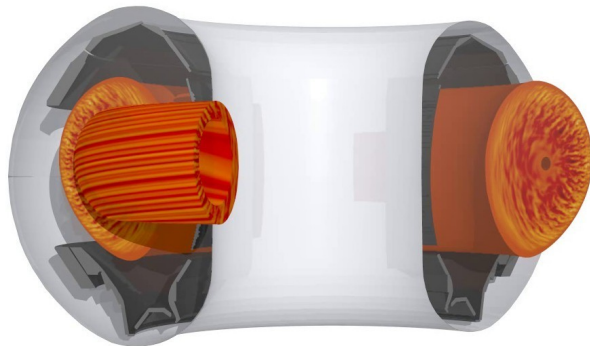


Figure: Snapshot from a numerical simulation of plasma turbulence in the ASDEX Upgrade tokamak with the nonlinear gyrokinetic code GENE. Dr. Jenko

Magnetohydrodynamics

Examples: Magnetic Dynamos - Astrophysical Scales

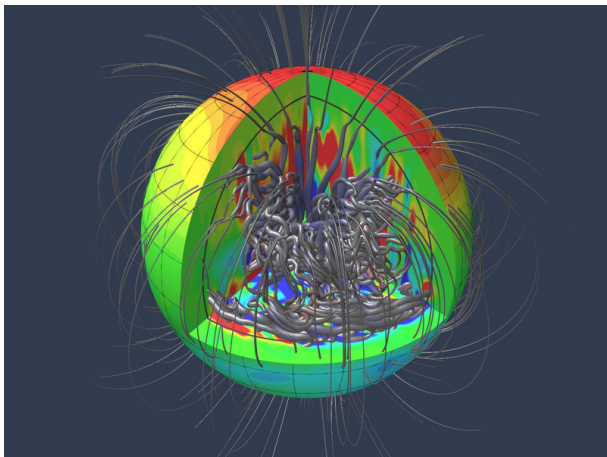


Figure: Jupiter cut open (2014). Dr. Krummheuer & Dr. Wicht

Magnetohydrodynamics

Examples: MHD Turbulence - Astrophysical Scales

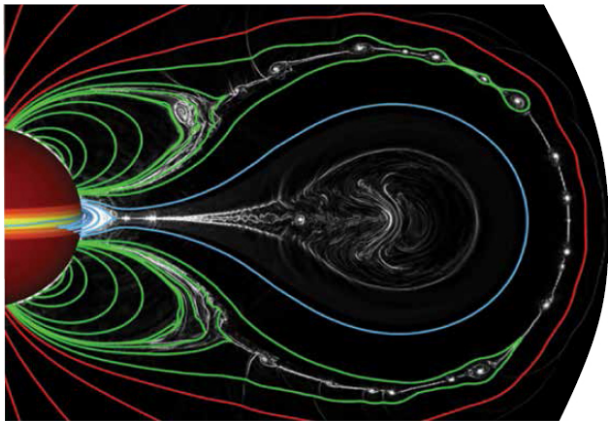


Figure: Ultra-high-resolution numerical simulation of a coronal mass ejection and associated flare. Solar and Space Physics (2010)

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Reynolds Numbers

Magnetic Prandtl Number

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Shell Model

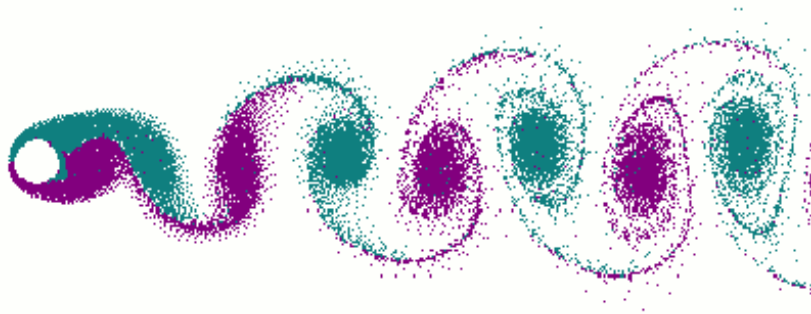
Driven 1D Model

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Reynolds Numbers and the Magnetic Prandtl Number

Reynolds Number

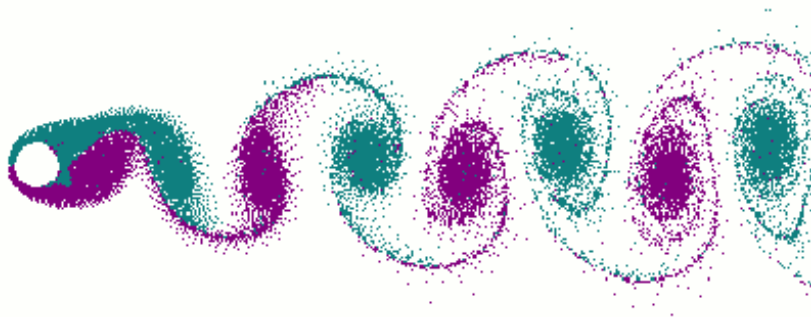
$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u L}{\nu}$$



Reynolds Numbers and the Magnetic Prandtl Number

Reynolds Number

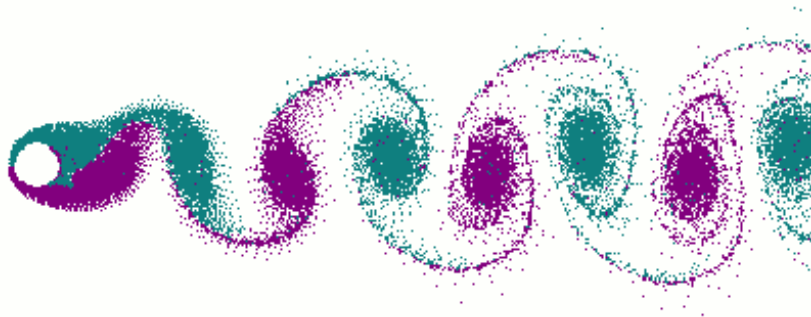
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Reynolds Numbers and the Magnetic Prandtl Number

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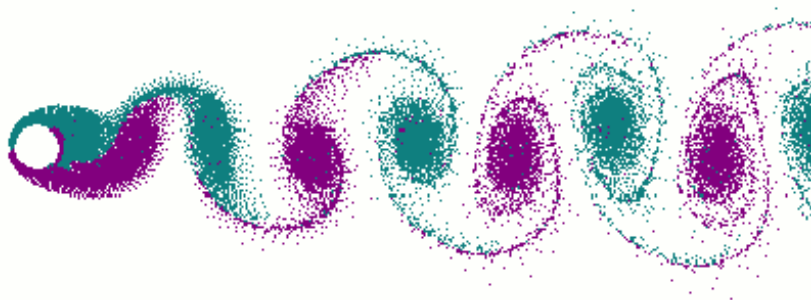
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Reynolds Numbers and the Magnetic Prandtl Number

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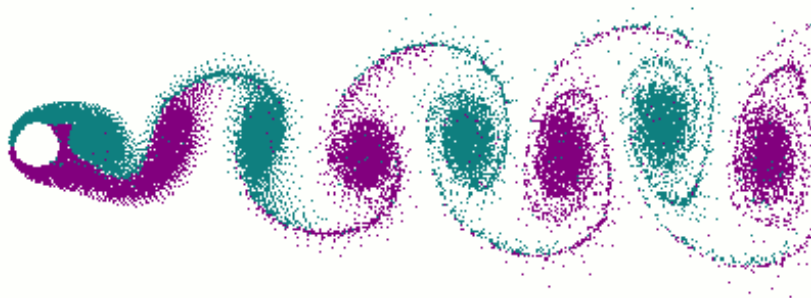
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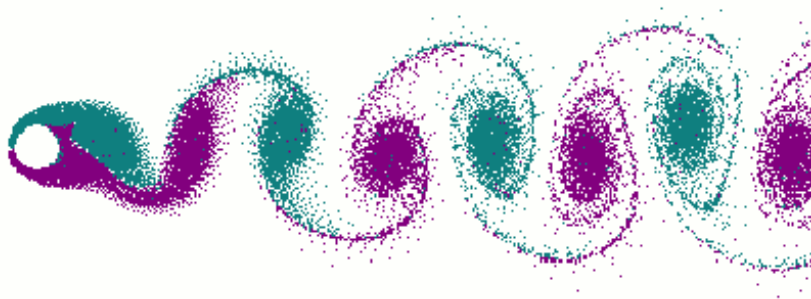
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Reynolds Numbers and the Magnetic Prandtl Number

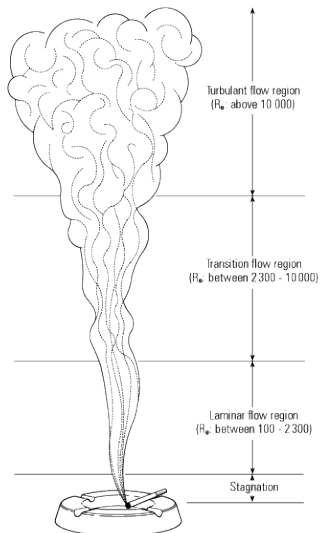
Reynolds Number

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u L}{\nu}$$



Reynolds Numbers and the Magnetic Prandtl Number

Reynolds Number



Reynolds Numbers and the Magnetic Prandtl Number

Magnetic Reynolds Number

Ideal MHD equations: Perfectly conducting fluids.

Reynolds Numbers and the Magnetic Prandtl Number

Magnetic Reynolds Number

$$Re_M = \frac{\text{inertial forces}}{\text{diffusive forces}} = \frac{u L}{\eta}$$

Reynolds Numbers and the Magnetic Prandtl Number

Magnetic Prandtl Number

$$Pr_M = \frac{Re_M}{Re} = \frac{\nu}{\eta}$$

Reynolds Numbers and the Magnetic Prandtl Number

Magnetic Prandtl Number

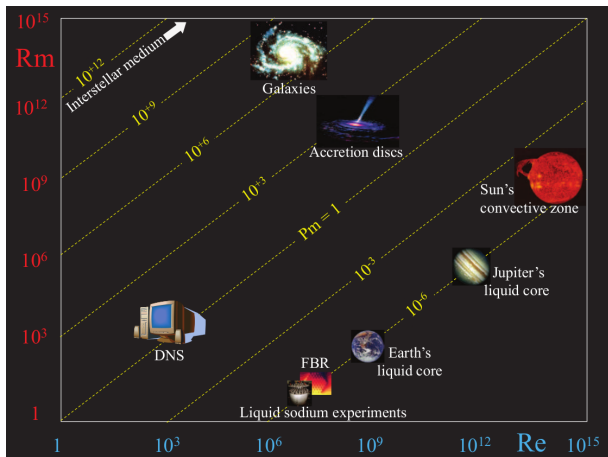


Figure: Map of “typical” objects in the plane (Re, Re_M) . Yellow dashed lines are Pr_M isolines. [1].

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Governing Equations

Forced MHD turbulence of a gas with isothermal equation of state:
 $p = \rho c_s^2$.

$$\begin{aligned}\frac{D \ln \rho}{Dt} &= -\nabla \cdot \mathbf{u} \\ \frac{D \mathbf{u}}{Dt} &= -c_s^2 \nabla \ln \rho - 2\boldsymbol{\Omega} \times \mathbf{u} + \mathbf{f} \\ &\quad + \rho^{-1} [\mathbf{J} \times \mathbf{B} + \nabla \cdot (2\nu \rho \boldsymbol{\mathcal{S}})] \\ \frac{\partial \mathbf{A}}{\partial t} &= \mathbf{u} \times \mathbf{B} - \eta \mu_0 \mathbf{J}\end{aligned}$$

DNS of Turbulent Dynamos

Governing Equations

Kinetic and Magnetic energies.

$$\frac{d}{dt} \langle \rho u^2 / 2 \rangle = \langle p \nabla \cdot \mathbf{u} \rangle + \langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle + \langle \rho \mathbf{u} \cdot \mathbf{f} \rangle - \langle 2\rho\nu S^2 \rangle$$

$$\frac{d}{dt} \langle B^2 / 2\mu_0 \rangle = -\langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle - \langle \eta \mu_0 J^2 \rangle$$

DNS of Turbulent Dynamos

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Dissipation rates.

$$\epsilon_K = \langle 2\rho\nu \mathcal{S}^2 \rangle, \quad \epsilon_M = \langle \eta\mu_0 J^2 \rangle$$

DNS of Turbulent Dynamos

Governing Equations

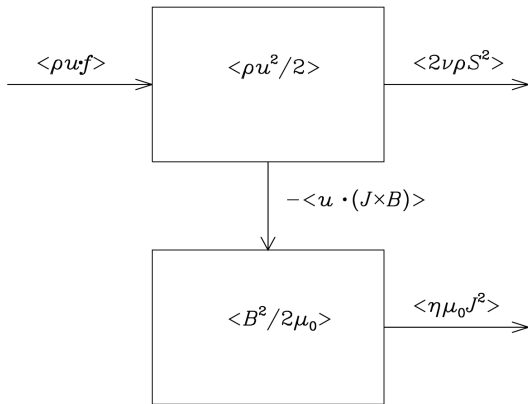
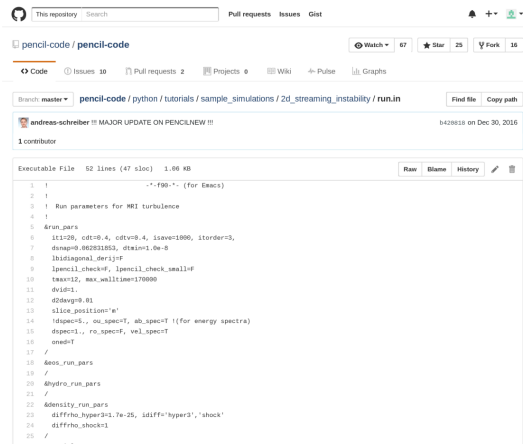


Figure: Flow of energy sketch [2].

DNS of Turbulent Dynamos

Simulations and Results

Pencil code (NORDITA)



The screenshot shows the GitHub repository for 'pencil-code'. The repository is owned by 'andreas-schreiber' and has 67 watchers, 25 stars, and 16 forks. The current branch is 'master'. The file path shown is 'pencil-code / python / tutorials / sample_simulations / 2d_streaming_instability / run.in'. The file is 1.06 KB and contains 52 lines of code. The code is a shell script for running Pencil code simulations. It sets various parameters for the simulation, including the number of processors, the domain size, the time step, and the output files. The script also sets the verbosity level and the location of the input files.

```
1 #! /bin/sh
2 #
3 # Run parameters for MRI turbulence
4 #
5 #run_pars
6 lti=20, cdc=0.4, cdtv=8.4, lsave=1000, ltorder=3,
7 dsnap=0.002831853, dtain=1.0e-8
8 lbdiaagonal_derj=F
9 lpencil_check=F, lpencil_check_small=F
10 lsave=12, max_waltime=170000
11 dvid=1
12 d2avg=9.01
13 slice_position='e'
14 ldspec=5., ou_spec=T, ab_spec=T !((for energy spectra)
15 dspec=1., ro_spec=F, vel_spec=T
16 oned=T
17 /
18 aeos_run_pars
19 /
20 ahydro_run_pars
21 /
22 adensity_run_pars
23 diffrho_hyper3=1.7e-25, idiff='hyper3','shock'
24 diffrho_shock=1
25 /
26 - - -
```

Figure: Snapshot of Pencil-code GitHub repository.

DNS of Turbulent Dynamos

Simulations and Results

Energy ratio approximately independent on Pr_M .

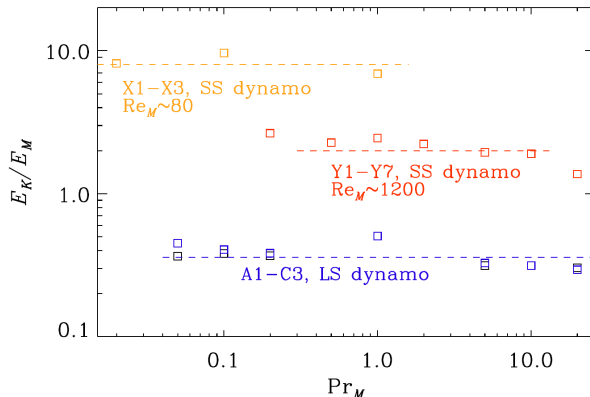


Figure: Energy ratio E_K/E_M dependence on Pr_M for large-scale dynamo (blue) and small-scale dynamos (orange and red) [2].

DNS of Turbulent Dynamos

Simulations and Results

Dissipation ratio dependency on Pr_M .

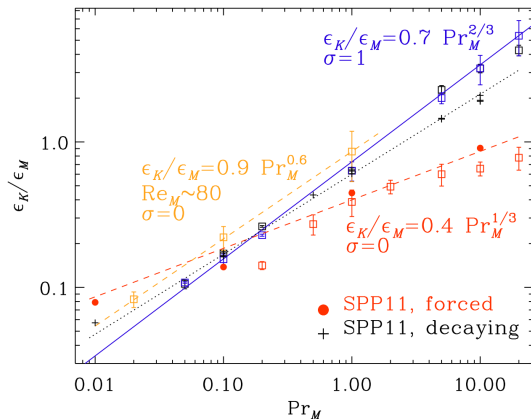


Figure: Dissipation ratio ϵ_K/ϵ_M dependence on Pr_M for non-helical forcing ($\sigma = 0$) and for fully helical forcing ($\sigma = 1$). [2].

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Similar equations than before - same conserved quantities.
Time integration scheme: Adams-Bashforth

Shell and 1D Models

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Dissipation ratio dependency on Pr_M .

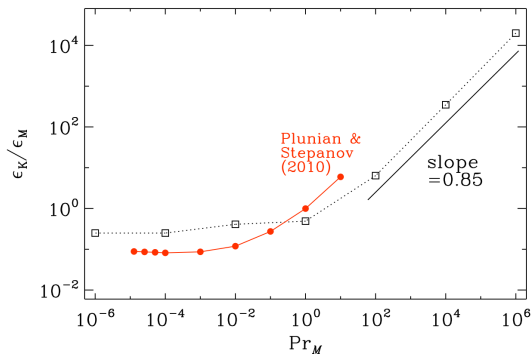


Figure: Dissipation ratio ϵ_K/ϵ_M dependence on Pr_M [2]. Red shows simulations made by Plunian and Stepanov [3].

Shell and 1D Models

Driven 1D Model

Neglecting gas pressure:

$$\begin{aligned}\frac{\partial u}{\partial t} &= -uu' - bb' + \tilde{\nu}u'' \\ \frac{\partial b}{\partial t} &= -ub' - bu' + \eta b''\end{aligned}$$

Shell and 1D Models

Driven 1D Model

Dissipation ratio dependency on Pr_M .

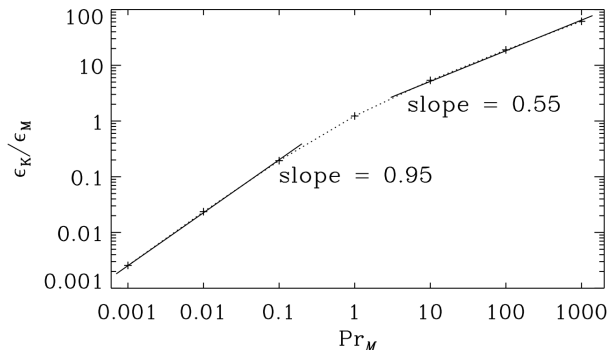


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4. Consistent results to previous simulations regarding the kinetic-to-magnetic dissipation ratio were acquired.

References

- [1] F. Plunian, R. Stepanov, and P. Frick, "Shell models of magnetohydrodynamic turbulence," *Physics Reports*, vol. 523, no. 1, pp. 1–60, 2013.
- [2] A. Brandenburg, "Magnetic prandtl number dependence of the kinetic-to-magnetic dissipation ratio," *The Astrophysical Journal*, vol. 791, no. 1, p. 12, 2014.
- [3] F. Plunian and R. Stepanov, "Cascades and dissipation ratio in rotating magnetohydrodynamic turbulence at low magnetic prandtl number," *Physical Review E*, vol. 82, no. 4, p. 046311, 2010.

Shell Models

Energy profiles with shell model

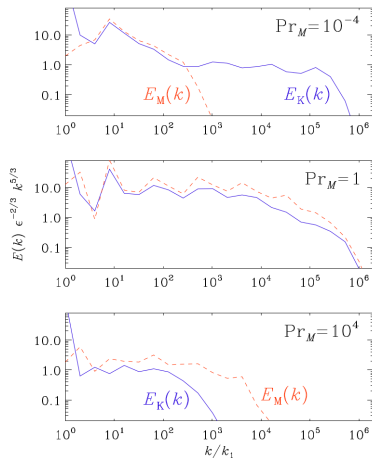


Figure: Compensated time-averaged kinetic and magnetic energy spectra for shell models at three values of Pr_M [2].