Magnetic Prandtl Number Dependence of the Kinetic-to-Magnetic Dissipation Ratio

Presented by Andrés Cathey

Axel Brandenburg

KTH Royal institute of Technology and Stockholm University

2014

Overview

Overview

- Electrically conducting fluids.
  - Plasmas.
  - Liquid metals.
  - Electrolytes.

- Electrically conducting fluids.
  - Plasmas.
  - Liquid metals.
  - Electrolytes.
- Moving ρ generate currents.

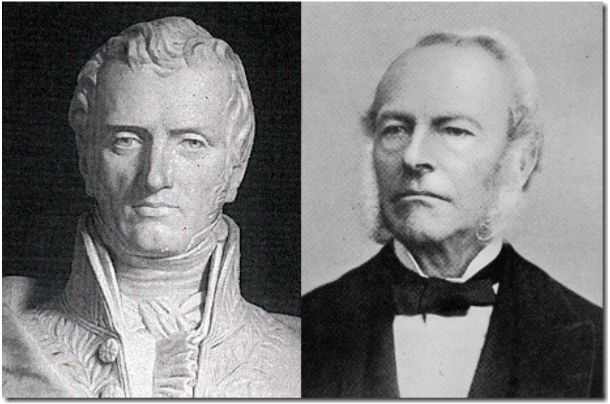
- Electrically conducting fluids.
  - Plasmas.
  - Liquid metals.
  - Electrolytes.
- Moving ρ generate currents.
- Ampére's law: $\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J}$.

Magnetohydrodynamics

What exactly is MHD?

- Electrically conducting fluids.
  - Plasmas.
  - Liquid metals.
  - Electrolytes.
- Moving ρ generate currents.
- Ampére's law: $\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J}$.
- Faraday's law: $\boldsymbol{\nabla} \times \mathbf{E} = -\partial_t \mathbf{B}$.

Navier-Stokes equations



Maxwell equations

Maxwell equations

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

Numerical simulations

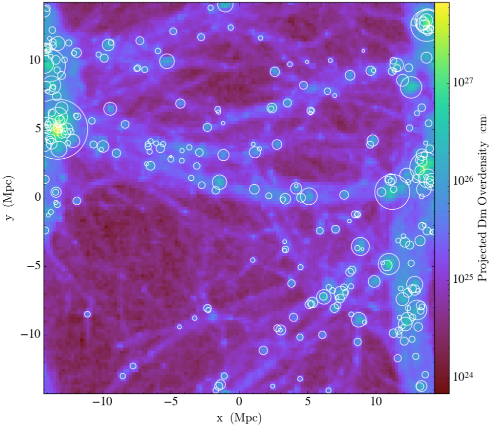


Figure: Cosmological simulation showing dark matter halos.

Numerical simulations

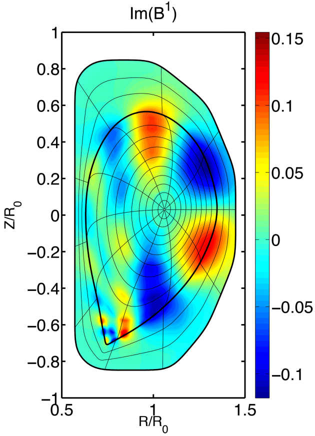


Figure: Radial component of magnetic field amplitude in an unstable n=1 kink mode in DIII-D. MHD Stability code MARS.

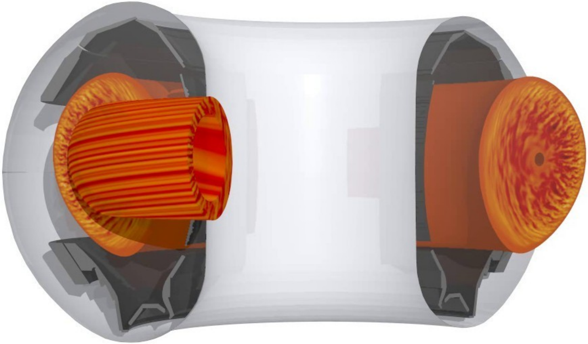


Figure: Snapshot from a numerical simulation of plasma turbulence in the ASDEX Upgrade tokamak with the nonlinear gyrokinetic code GENE. Dr. Jenko

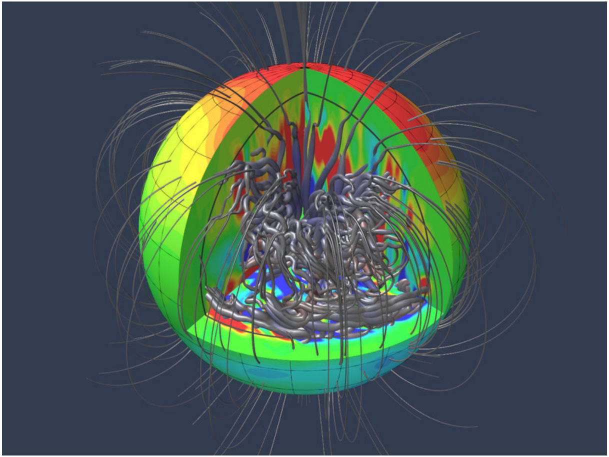


Figure: Jupiter cut open (2014). Dr. Krummheuer & Dr. Wicht

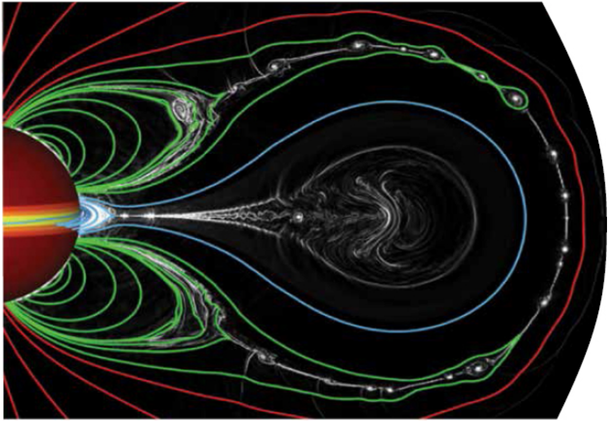
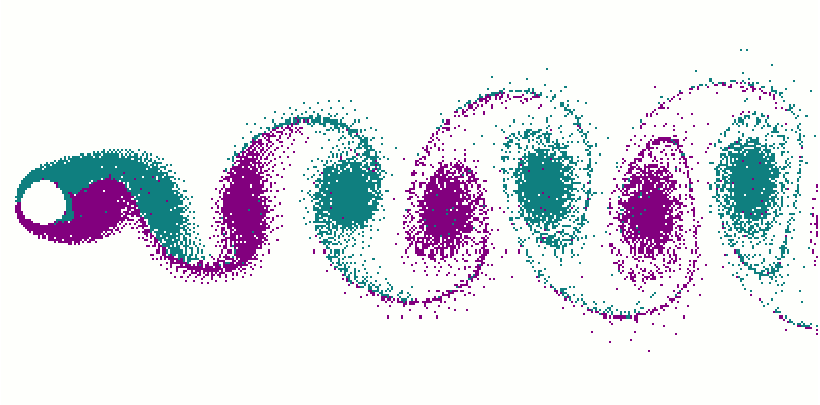


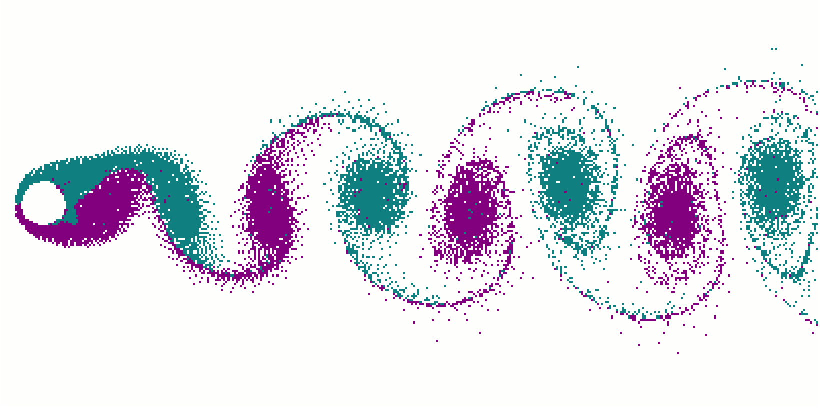
Figure: Ultra-high-resolution numerical simulation of a coronal mass ejection and associated flare. Solar and Space Physics (2010)

Overview

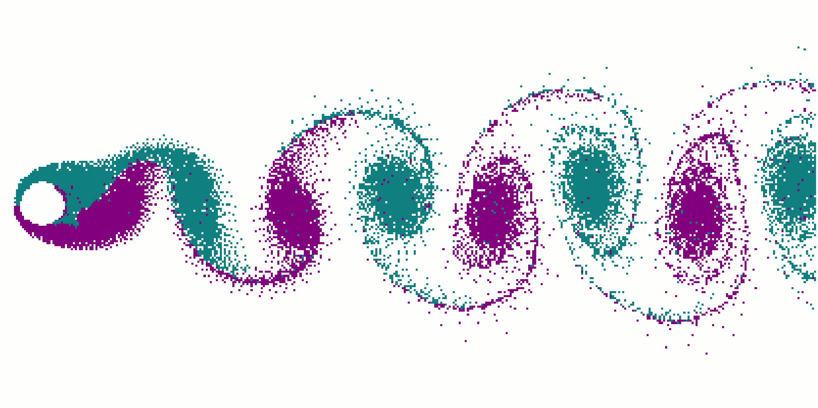
$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u\,L}{\nu}$$



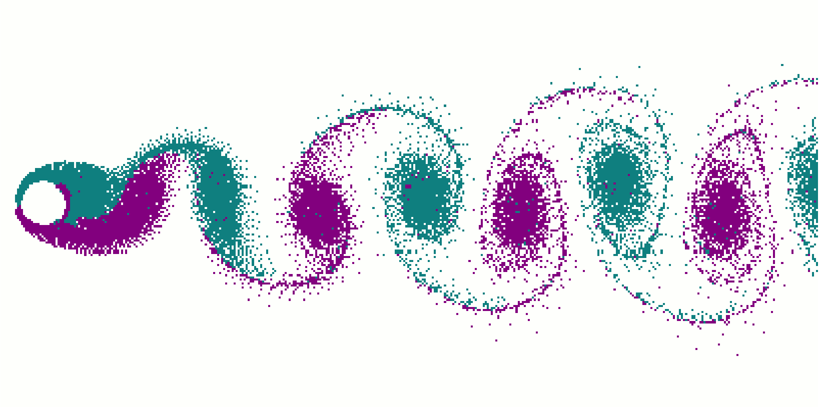
$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u\,L}{\nu}$$



$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u\,L}{\nu}$$



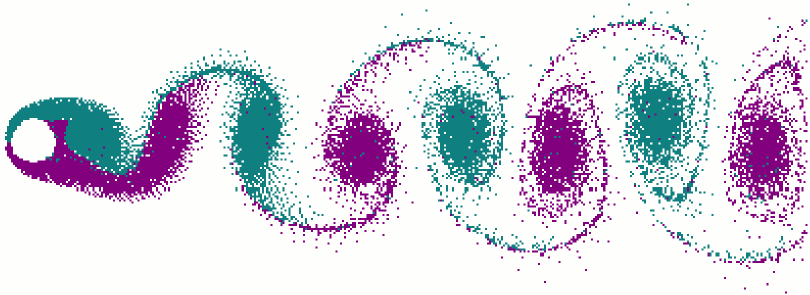
$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u\,L}{\nu}$$



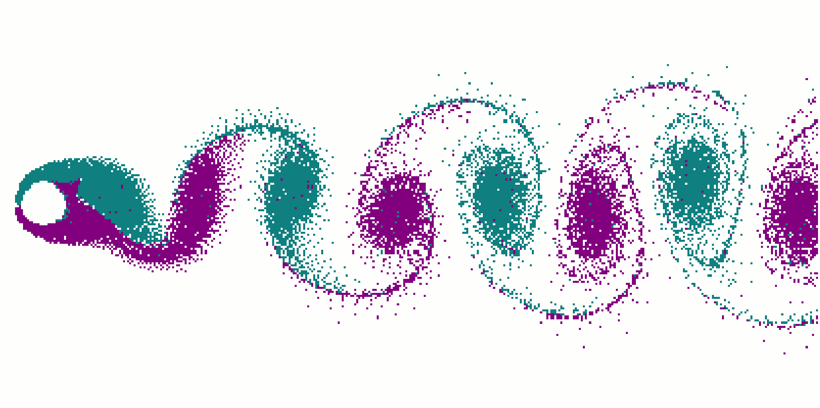
Reynolds Numbers and the Magnetic Prandtl Number

Reynolds Number

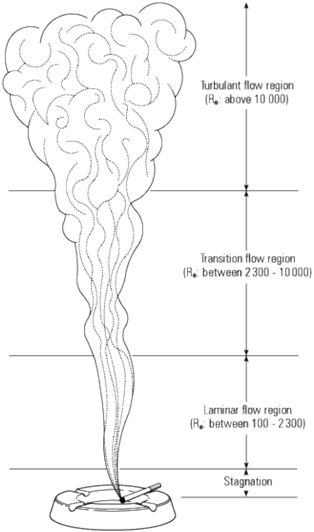
$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u\,L}{\nu}$$



$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{u\,L}{\nu}$$



Wikimedia Commons (2014)



Turbulant flow region
(Rₑ: above 10 000)
Transition flow region
(Rₑ: between 2 300 - 10 000)
Laminar flow region
(Rₑ: between 100 - 2 300)
Stagnation

Ideal MHD equations: Perfectly conducting fluids.

$$Re_M = \frac{\text{inertial forces}}{\text{diffusive forces}} = \frac{u\,L}{\eta}$$

$$Pr_M = \frac{Re_M}{Re} = \frac{\nu}{\eta}$$

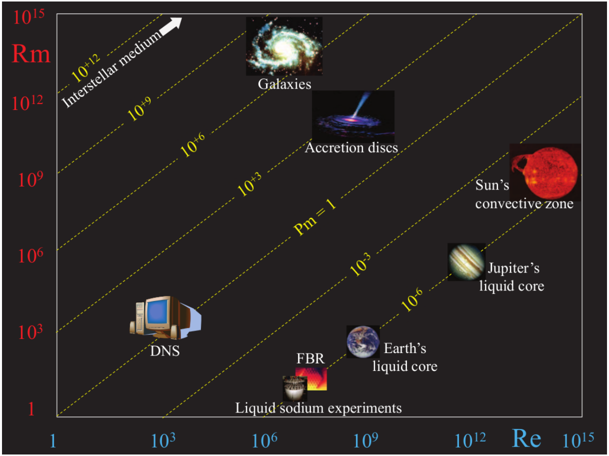


Figure: Map of "typical" objects in the plane (Re, Re_M). Yellow dashed lines are Pr_M isolines. [1].

Reynolds Numbers and the Magnetic Prandtl Number

Energy dependence on Pr_M

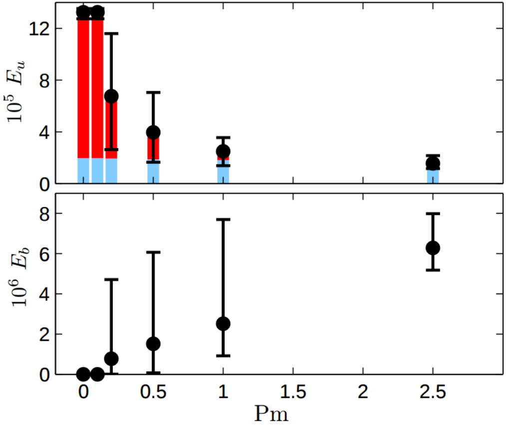


Figure: Kinetic E_u and magnetic E_b energies as a function of Pr_M in the dynamic phase [2].

Reynolds Numbers and the Magnetic Prandtl Number

Energy dependence on Pr_M

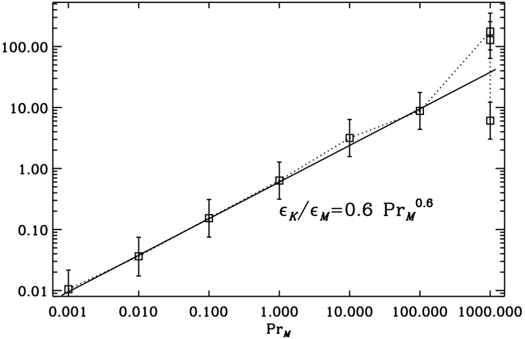


Figure: Kinetic ϵ_K to magnetic ϵ_M dissipation rate as a function of Pr_M "after" asymptotical regime [3].

Overview

Forced MHD turbulence of a gas with isothermal equation of state: $p = \rho c_s^2$.

$$
\begin{aligned}
\frac{D ln\rho}{Dt} &= -\,\boldsymbol{\nabla}\cdot\mathbf{u}\\
\frac{D\mathbf{u}}{Dt} &= -\,c_s^2\boldsymbol{\nabla} ln\rho - 2\boldsymbol{\Omega}\times\mathbf{u} + \mathbf{f}\\
&\quad + \rho^{-1}[\mathbf{J}\times\mathbf{B} + \boldsymbol{\nabla}\cdot(2\nu\rho\boldsymbol{\mathcal{S}})]\\
\frac{\partial\mathbf{A}}{\partial t} &= \mathbf{u}\times\mathbf{B} - \eta\mu_0\mathbf{J}
\end{aligned}
$$

Kinetic and Magnetic energies.

$$\frac{d}{dt}\langle \rho u^2/2 \rangle = \langle p \boldsymbol{\nabla} \cdot \mathbf{u} \rangle + \langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle + \langle \rho \mathbf{u} \cdot \mathbf{f} \rangle - \langle 2\rho\nu \mathcal{S}^2 \rangle$$

$$\frac{d}{dt}\langle B^2/2\mu_0 \rangle = -\langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle - \langle \eta\mu_0 J^2 \rangle$$

Kinetic and Magnetic energies.

$$\frac{d}{dt}\langle \rho u^2/2 \rangle = \langle p \boldsymbol{\nabla} \cdot \mathbf{u} \rangle + \langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle + \langle \rho \mathbf{u} \cdot \mathbf{f} \rangle - \langle 2 \rho \nu \mathcal{S}^2 \rangle$$

$$\frac{d}{dt}\langle B^2/2\mu_0 \rangle = -\langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle - \langle \eta \mu_0 J^2 \rangle$$

Dissipation rates.

$$\epsilon_K = \langle 2 \rho \nu \mathcal{S}^2 \rangle, \qquad \epsilon_M = \langle \eta \mu_0 J^2 \rangle$$

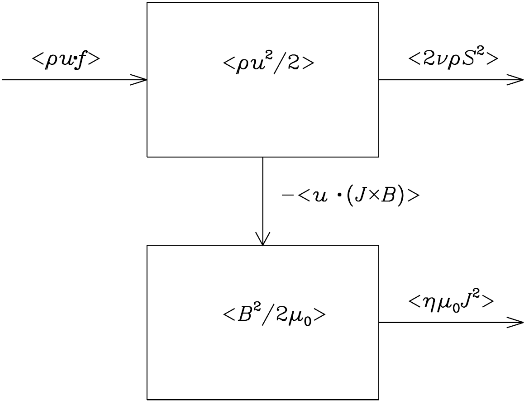
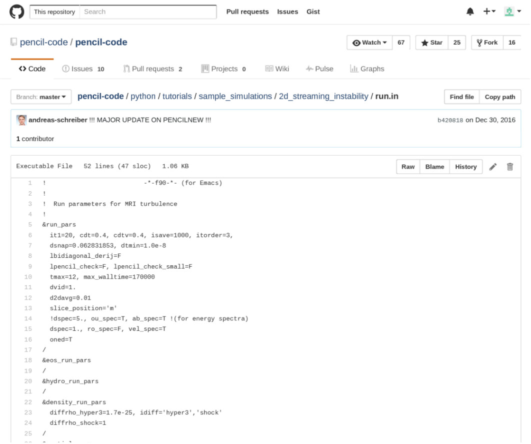


Figure: Flow of energy sketch [4].

Pencil code (NORDITA)



This repository | Search | Pull requests | Issues | Gist

pencil-code / **pencil-code**

Watch 67 | Star 25 | Fork 16

Code | Issues 10 | Pull requests 2 | Projects 0 | Wiki | Pulse | Graphs

Branch: master | **pencil-code** / python / tutorials / sample_simulations / 2d_streaming_instability / **run.in** | Find file | Copy path

andreas-schreiber !!! MAJOR UPDATE ON PENCILNEW !!! b420818 on Dec 30, 2016

1 contributor

Executable File 52 lines (47 sloc) 1.06 KB | Raw | Blame | History

```
!                          -*-f90-*- (for Emacs)
!
!  Run parameters for MRI turbulence
!
&run_pars
  it1=20, cdt=0.4, cdtv=0.4, isave=1000, itorder=3,
  dsnap=0.062831853, dtmin=1.0e-8
  lbidiagonal_derij=F
  lpencil_check=F, lpencil_check_small=F
  tmax=12, max_walltime=170000
  dvid=1.
  d2davg=0.01
  slice_position='m'
  !dspec=5., ou_spec=T, ab_spec=T !(for energy spectra)
  dspec=1., ro_spec=F, vel_spec=T
  oned=T
/
&eos_run_pars
/
&hydro_run_pars
/
&density_run_pars
  diffrho_hyper3=1.7e-25, idiff='hyper3','shock'
  diffrho_shock=1
/
```

Figure: Snapshot of Pencil-code GitHub repository.

Energy ratio approximately independent on Pr_M.

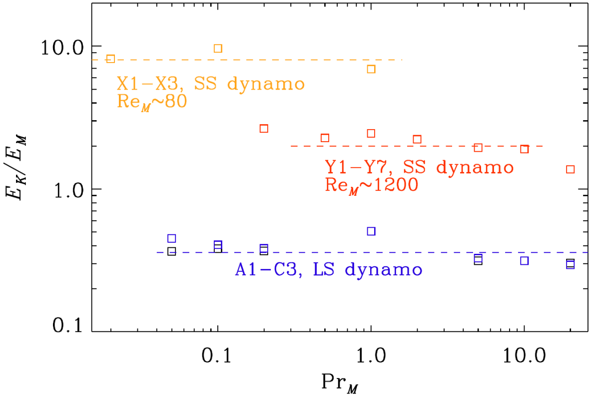


Figure: Energy ratio E_K/E_M dependence on Pr_M for large-scale dynamo (blue) and smal-scale dynamos (orange and red) [4].

Dissipation ratio dependency on Pr_M.

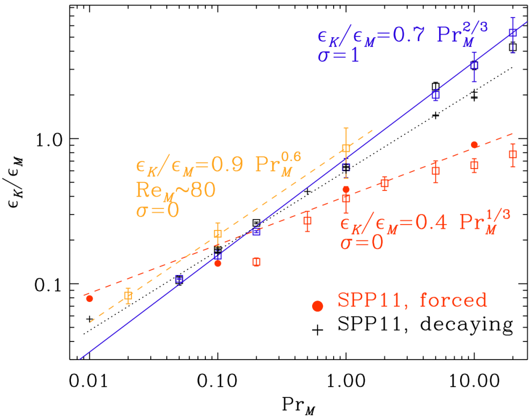


Figure: Dissipation ratio ϵ_K/ϵ_M dependence on Pr_M for non-helical forcing ($\sigma = 0$) and for fully helical forcing ($\sigma = 1$). [4].

Overview

Similar equations than before - same conserved quantities.
Time integration scheme: Adams-Bashforth

Dissipation ratio dependency on Pr_M.

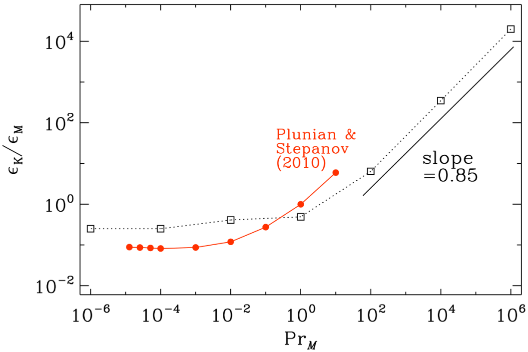


Figure: Dissipation ratio ϵ_K/ϵ_M dependence on Pr_M [4]. Red shows simulations made by Plunian and Stepanov [5].

Neglecting gas pressure:

$$\frac{\partial u}{\partial t} = -uu' - bb' + \tilde{\nu} u''$$
$$\frac{\partial b}{\partial t} = -ub' - bu' + \eta b''$$

Dissipation ratio dependency on Pr_M.

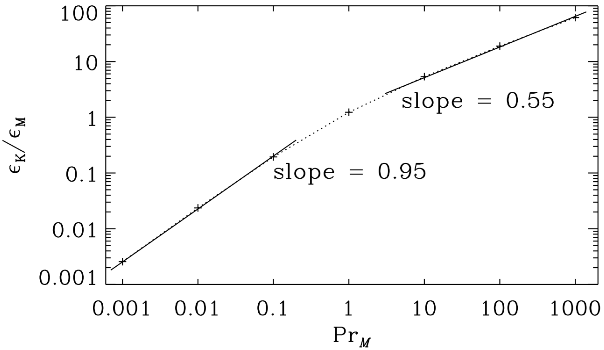


Figure: Dissipation ratio ϵ_K/ϵ_M dependence on Pr_M [4]. Red shows simulations made by Plunian and Stepanov [5].

Overview

Conclusions

1. Obtained relations for the kinetic-to-magnetic dissipation ratio for a broad range of Pr_M values.

Conclusions

1. Obtained relations for the kinetic-to-magnetic dissipation ratio for a broad range of Pr_M values.
2. Confirmed earlier results that for large-scale dynamos, the ratio ϵ_K/ϵ_M is proportionate to $Pr_M^{0.6}$.

Conclusions

1. Obtained relations for the kinetic-to-magnetic dissipation ratio for a broad range of Pr_M values.
2. Confirmed earlier results that for large-scale dynamos, the ratio ϵ_K/ϵ_M is proportionate to $Pr_M^{0.6}$.
3. Discovered scaling laws ($\epsilon_K/\epsilon_M \sim Pr_M^q$) for fully helical and non-helical forcing. Where $q \approx [0.6 - 2/3]$ for the former and $q \approx 1/3$ for the latter.

Conclusions

1. Obtained relations for the kinetic-to-magnetic dissipation ratio for a broad range of Pr_M values.
2. Confirmed earlier results that for large-scale dynamos, the ratio ϵ_K/ϵ_M is proportionate to $Pr_M^{0.6}$.
3. Discovered scaling laws ($\epsilon_K/\epsilon_M \sim Pr_M^q$) for fully helical and non-helical forcing. Where $q \approx [0.6 - 2/3]$ for the former and $q \approx 1/3$ for the latter.
4. Consistent results to previous simulations regarding the kinetic-to-magnetic dissipation ratio were acquired.

References

[1] F. Plunian, R. Stepanov, and P. Frick, “Shell models of magnetohydrodynamic turbulence,” *Physics Reports*, vol. 523, no. 1, pp. 1–60, 2013.

[2] C. Guervilly, D. W. Hughes, and C. A. Jones, “Generation of magnetic fields by large-scale vortices in rotating convection,” *Physical Review E*, vol. 91, no. 4, p. 041001, 2015.

[3] A. Brandenburg, “Dissipation in dynamos at low and high magnetic prandtl numbers,” *Astronomische Nachrichten*, vol. 332, no. 1, pp. 51–56, 2011.

[4] A. Brandenburg, “Magnetic prandtl number dependence of the kinetic-to-magnetic dissipation ratio,” *The Astrophysical Journal*, vol. 791, no. 1, p. 12, 2014.

[5] F. Plunian and R. Stepanov, “Cascades and dissipation ratio in rotating magnetohydrodynamic turbulence at low magnetic prandtl number,” *Physical Review E*, vol. 82, no. 4, p. 046311, 2010.

Shell Models

Energy profiles with shell model

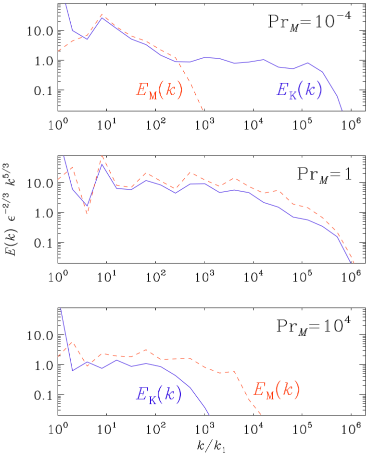


Figure: Compensated time-averaged kinetic and magnetic energy spectra for shell models at three values of Pr_M [4].