Welcome and thanks for being here,

My name is Andres Cathey, and I am going to be talking about the research conducted by Dr. Axel Brandenburg from Stockholm University. Specifically I’m going to be talking about a paper he wrote in 2014.

This paper studies the kinetic-to-magnetic dissipation rate’s dependance on the Magnetic Prandtl number in MHD scenarios.

Now I’m going to give you a brief overview of what I’m going to cover.

First I’ll try to explain what MHD is, what are some interesting systems that it studies. Then I’ll present a few values that are used to characterize MHD systems - namely the Reynold numbers and the Magnetic Prandtl number.

Afterwards we’ll delve directly into the paper at hand, starting with a brief introduction to it. The paper studies a few methods so we’ll try to cover each of them, along with their results and briefly the way that they were approached my Dr. Brandenburg.

And finally, a few concluding remarks will be made.

So, a couple of months ago I wasn’t too sure what MHD was (other than what the accronym meant), so first I will try to explain what MHD is. To do so, I’ll give a few examples and state why I think that it’s important.

OK!

MHD describes the macroscopic behavior of electrically conducting fluids, notably, but not limited to plasmas.

The fundamental concept behind MHD is that magnetic fields can [induce](https://en.wikipedia.org/wiki/Electromagnetic_induction) currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. The set of equations that describe MHD are a combination of the [Navier-Stokes equations](https://en.wikipedia.org/wiki/Navier-Stokes_equations) of [fluid dynamics](https://en.wikipedia.org/wiki/Fluid_dynamics) and [Maxwell's equations](https://en.wikipedia.org/wiki/Maxwell's_equations) of [electromagnetism](https://en.wikipedia.org/wiki/Electromagnetism). These [differential equations](https://en.wikipedia.org/wiki/Differential_equation) must be solved [simultaneously](https://en.wikipedia.org/wiki/Simultaneous_equation), either analytically or [numerically](https://en.wikipedia.org/wiki/Numerical_analysis).

For more general processes, however, numerical computations become the major tool, a trend observed in many branches of physics.

With present-day supercomputers the partial differential equations for many two-dimensional fluid dynamic problems can be solved "exactly" for interesting Reynolds numbers and arbitrary boundary conditions. Moreover by performing a series of computer runs with different values of the externally given parameters scaling laws can be obtained.

It might seem little in view of the beauty of exact analytical results; it should, however, be compared with the actually achievable, highly approximate analytical approaches often encountered.

A few examples of MHD are

The dynamo theory describes the process through which a rotating, [convecting](https://en.wikipedia.org/wiki/Convection), and [electrically](https://en.wikipedia.org/wiki/Electric) conducting fluid can maintain a magnetic field: magnetic fields in Sun, Earth, Jupiter, Mercury... Accretion disks, geomagnetic field reversal.

Reynolds number:

The Re is an important number in hydrodynamics - in broad terms, it is the relation between inertial forces (being the movement of the fluid itself) to viscous forces (interaction between the fluid’s constituents).

It is more helpful to understand Re as a measure of the turbulence of the system - namely, Re is used to predict when the onset of turbulence will occur, like it is shown in the picture. It is therefore a very important measure in meteorology.

Car, airplane and submarine manufacturers rely heavily on the Re to scale how these will interact with air or water.

Earth dynamo: Re ~ 10^9

Sun dynamo: Re ~ 10^13

Inside of the Sun: Re ~ 10^9

Solar corona: Re ~ 10^4

Magnetic Re:

Unsurprisingly, the Rem is useful to define MHD flow. It is also a dimensionless relation - between inertial (inductive) forces and diffusive forces. In order to understand MHD turbulence, both the Re and the Rem are very important.

Perfectly conductive fluids (infinite electric conductivity) have an infinite Rem. This is where ideal MHD equations can be used.

Earth dynamo: Rem ~ 10^2

Sun dynamo: Rem ~ 10^10

Inside of the Sun: Rem ~ 10^6

Solar corona: Rem ~ 10^8 - 10^12

Rm is small — which is never realised astrophysically.

In the context of magnetic dynamos,  For values larger than Rem ~ 10, a disk [dynamo](http://scienceworld.wolfram.com/physics/Dynamo.html) is self-sustaining.

In general, we can say that large values of Re are characteristic of turbulent flows, while large values of Rm mean that resistive effects are restricted to thin regions where the current density is large. However, if the plasma flows are turbulent, then Rm can be reduced significantly

Magnetic Prandtl number:

Earth dynamo: Pr\_M ~ 10^-7: very small.

Inside the Sun: Pr\_M ~ 10^-3: small

Solar corona: Pr\_M ~ 10^4 - 10^8: Large!

a magnetic Prandtl number of order unity is a widely accepted value for the thin accretion discs.

Pr\_M > 1: Viscosity is

Pr\_M >> 1: Viscosity dominates over diffusivity: Interference with dyanmics of B-field reconection.

Most energy dissipated viscously.

Pr\_M << 1: Diffusivity dominates over viscosity: No interference with dynamics

of B-field reconnection.

Paper:

magnetic fields provide an additional important path-

way for dissipating turbulent energy through Joule heating (\eta \mu\_0 J^2 term)

viscosity: term with \mu. There, S is the rate-of-strain tensor.

A – vector potential

The total (kinetic plus magnetic) energy is

sourced by <ρu · f>.

Hydromagnetic turbulence simulations exhibiting

dynamo action have shown that the values of energy

dissipation are then no longer constant, and that their

ratio scales with PrM

The important characteristics of this dependence are as follows:

**When PrM is > 1, then the dissipation is dominated by the kinetic term**

**When PrM is <= 1, then the dissipation is dominated by the magnetic one**

For small enough values of PrM (~10^-3 - 10^-5) the relation between the dissipations remains constant (the main component of the dissipation comes from joule heating).

All of the energy that is eventually dissipated comes

from the forcing in the momentum equation

dynamo process would be intimately linked to Joule dissipation

and one must therefore be concerned that it is also linked to the

physical or even numerical nature of energy dissipation.

combination

of different approaches to MHD turbulence ranging from direct

numerical simulations (DNS) of the MHD equations in three

dimensions and shell models of the turbulence capturing aspects

of the spectral cascade, to a simple one-dimensional model of

MHD

DNS:

Forced MHD turbulence of a gas that can be described by an isothermal equation of state. c\_s is the isothermal speed of sound. The turbulence’s forcing function is either fully helical or non-helical (with weak seed magnetic field which is then amplified by dynamo action).

Fully helical: large-scale dynamo (large-scale B)

Non-helical: small-scale dynamo (smaller B)

S\_ij = traceless rate-of-strain tensor.

D/Dt = advective/convective derivative. = partial\_t + u . Nabla

Consider triply periodic domain (for numerical purposes).

The total (kinetic plus magnetic) energy is

sourced by <rho u f>.

and dissipated by the sum of viscous and

Joule dissipation: epsilon\_T = epsilon\_K + epsilon\_M

<rho nabla u>: gas expansion work

<u J B >: Lorentz force work

Steady state Lorentz force work balances with epsilon\_M

Simulations:

Periodic boundary conditions. Low Mach number (u\_rms / c\_s) ~ 0.1

The Pencil Code is a high-order [finite-difference code](https://en.wikipedia.org/wiki/Finite-difference_method) for solving [partial differential equations](https://en.wikipedia.org/wiki/Partial_differential_equation), written in [Fortran 95](https://en.wikipedia.org/wiki/Fortran).

Results:

First of all, note that in

all cases, the **energy ratio EK/EM** is roughly **independent** of

**PrM** but it varies with ReM - proven for small-scale dynamos by Haugen et al. 2003.

The presence of **helicity in the forcing function** can lead to magnetic

field generation at the largest scale of the system. It is therefore

also referred to as a **large-scale dynamo.**

**Non-helical** forcing leads to magnetic fields on scales that are typically somewhat **smaller** than the energy-carrying scale of the turbulent motions

The simulations show that for both σ = 1 and 0, the ratio \_x0007\_epsilon\_K /epsilon\_M **scales** with PrM: epsilon\_K/epsilon\_M ~ PrM^**q**

For σ = 1, we find **q** ≈ **2/3** for both small and large values of PrM ,

while for σ = 0, we find **q** ≈ **0.6** for PrM **<** 1 with Re ≈ **80** and

**q** ≈ **0.3** for PrM **>** 1 with Re ≈ **460**.

The fraction of energy that is being diverted to magnetic energy through dynamo action depends on the term −<u · ( J × B)>

and that this must be equal to epsilon\_M in the statistically steady state.

==> epsilon\_M / epsilon\_T = efficiency of the dynamo.

Shell model:

Previous results: constant dissipation ratio for small $Pr\_M$ and show a sub-linear increase at large $Pr\_M$.

“random” number generator to generate the external forces.

Plunian and Stepanov their results are consistent with a constant dissipation ratio for small PrM and show a sub-linear increase at large PrM

1D model:

Conclusions:

1. showing an improvement from their previous findings. Confirmed earlier results that for large-scale dynamos, the ratio $\epsilon\_K / \epsilon\_M$ is proportionate to $Pr\_M^{0.6}$.

2. The slope is larger when the Reynolds numbers is smaller – more energy is dissipated through the viscosity.

3. similar results in shell model to Plunian & Stepanov, and previous results form brandenburg as well.

Extra:

table: showing changes with Re and Rem but same PrM.

Shell model: For $Pr\_M=1$, the magnetic and kinetic energy spectra are similar, while for large (small) values of $Pr\_M$, the kinetic (magnetic) energy spectrum is prematurely truncated, as is also the case in the DNS of Brandenburg (2009) for small $Pr\_M$.

very small values of PrM , most of the energy

is dissipated resistivity and there is not much kinetic energy

left at the end of the turbulent kinetic energy cascade. As a

consequence, it is then possible to decrease the value of ν further

and still dissipate the remaining kinetic energy, which implies

that the nominal value of the fluid Reynolds number can become

much larger than what is usually possible when there is no

additional resistive dissipation.