Gamma Mixture Model (mixing at gamma level, 2 comp.).
(j^{th} column of y)

$$f(y_j \mid \underline{\alpha}_j, \underline{\beta}_j) = \lambda \, Ga(y_j \mid \alpha_{j1}, \beta_{j1}) + (1-\lambda) Ga(y_j \mid \alpha_{j2}, \beta_{j2})$$

$$= \int_{\gamma_j} \left(\lambda_j Ga(y_j \mid \alpha_{j1}, \beta_{j1})\right)^{\gamma_j} \left((1-\lambda_j) Ga(y_j \mid \alpha_{j2}, \beta_{j2})\right)^{1-\gamma_j} d\gamma_j$$

$$L(\underline{y}_j \mid \underline{\alpha}_j, \underline{\beta}_j) = \prod_{i=1}^{n} \left(\lambda_j \frac{\beta_{j1}^{\alpha_{j1}}}{\Gamma(\alpha_{j1})} (y_{ij})^{\alpha_{j1}-1} \exp\{-\beta_{j1} y_{ij}\}\right)^{\gamma_{ij}} \left((1-\lambda_j) \frac{\beta_{j2}^{\alpha_{j2}}}{\Gamma(\alpha_{j2})} (y_{ij})^{\alpha_{j2}-1} \exp\{-\beta_{j2} y_{ij}\}\right)^{1-\gamma_{ij}}$$

$$= (\lambda_j)^{n_j} \frac{\beta_{j1}^{\alpha_{j1} n_j}}{\Gamma(\alpha_{j1})^{n_j}} \left(\prod_{\{i:\gamma_{ij}=1\}} y_{ij}\right)^{\alpha_{j1}-1} \exp\Big\{-\beta_{j1} \sum_{\{i:\gamma_{ij}=1\}} y_{ij}\Big\}$$

$\Rightarrow$

$$\pi(\gamma_j \mid -) = Ber\left(\gamma_j \,\Big|\, \frac{\lambda_j Ga(y_j \mid \alpha_{j1}, \beta_{j1})}{\lambda_j Ga(y_j \mid \alpha_{j1}, \beta_{j1}) + (1-\lambda_j) Ga(y_j \mid \alpha_{j2}, \beta_{j2})}\right)$$

$$\pi(\lambda_j \mid -) = Beta\left(a_0 + \sum \gamma_j, \, b_0 + \sum (1-\gamma_j)\right)$$

$$\pi(\beta_{jl} \mid -) \propto \beta_{jl}^{n_j \alpha_{jl}} \exp\Big\{-\beta_{jl} \sum_{\{i:\gamma_{ijl}=1\}} y_{ij}\Big\} \times \pi(\beta)$$

$$= Ga\Big(n_{jl}\alpha_{jl} + a_0, \sum_{i:\gamma_{ijl}=1} y_{ij} + b_0\Big)$$

$\Rightarrow \quad n \rightarrow n_{jl} \quad \Rightarrow \quad$ njl (scalar)

$\sum_{i=1}^{n} y_{ij} \rightarrow \sum_{\{i:\gamma_{ijl}=1\}} y_{ij} \quad \Rightarrow \quad$ yjl.sum (scalar)

$\prod_{i=1}^{n} y_{ij} \rightarrow \prod_{\{i:\gamma_{ijl}=1\}} y_{ij} \Rightarrow$ yjl.prod (scalar)