

# Attitude Propagator in Python

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## 1 Attitude Kinematics and Dynamics Analysis

A short explanation of the system of equations is presented to better understand the kinematics and dynamics it represents. In the given case, it can be written as

$$\begin{cases} \dot{\mathbf{q}}(t) = \frac{1}{2} \boldsymbol{\Omega}(\boldsymbol{\omega}(t)) \mathbf{q}(t), \\ \dot{\boldsymbol{\omega}}(t) = \mathbf{J}^{-1}(\boldsymbol{\tau} - \boldsymbol{\omega} \times (\mathbf{J}\boldsymbol{\omega})), \end{cases} \quad (1)$$

where

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} -[\boldsymbol{\omega}]_{\times} & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^{\top} & 0 \end{bmatrix} \quad (2)$$

and the skew-symmetric matrix generated by  $\boldsymbol{\omega}$  equals

$$[\boldsymbol{\omega}]_{\times} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}. \quad (3)$$

The system consists of two first-order ODEs. Both of them are nonlinear - the first due to its dependency of  $\boldsymbol{\omega}$  and the second due to the present cross-product term  $\boldsymbol{\omega} \times (\mathbf{J}\boldsymbol{\omega})$ . The dynamics equation is self-contained, whereas the kinematics equation is coupled to it via  $\boldsymbol{\omega}$ . Considering these two characteristics and the present unit norm constraint imposed on quaternions  $\|\mathbf{q}\| = 1$ , the system can be classified as a **constrained nonlinear ODE system**.

Furthermore, in this specific case, the disturbance torque is considered constant and equal to  $\boldsymbol{\tau}_{d, \text{const}} = [0.001, 0.0015, 0.0]$  N m. Regarding the inertia matrix of the rigid-body satellite, it equals

$$\mathbf{J} = \begin{bmatrix} 1.0 & 0.1 & 0.1 \\ 0.1 & 2.0 & 0.1 \\ 0.1 & 0.1 & 0.3 \end{bmatrix} \text{ kg m}^2, \quad (4)$$

which is relevant for the following sections.

## 2 Angular Rates Analysis

As stated in the first section, the dynamics equation is independent of the kinematics, which is why it is analyzed first. Due to the presence of non-zero off-diagonal elements in the inertia matrix in equation (4), the rotation about any of the three axes induces motion along the others as well. This explains the results in Figure 1. In the case of a purely diagonal matrix, the plot would have shown three constant lines instead of the three coupled oscillations observed in the figure. Considering that the y-axis is the axis of largest inertia, another distinctive feature of the graph is the wave along the y-axis appearing to be more stable compared to the ones along the other axes.

Regarding the perturbed dynamics case, the intuitive approach is based on recognizing that the initially dominating term in the dynamics equation is the torque-induced one ( $\mathbf{J}^{-1}\boldsymbol{\tau}$ ) instead of the one containing the inertia matrix ( $\mathbf{J}^{-1}(\boldsymbol{\omega} \times (\mathbf{J}\boldsymbol{\omega}))$ ), which becomes quadratically small due to  $\boldsymbol{\omega}$ . This, combined with the y-element of the disturbance vector  $\boldsymbol{\tau}_{d, \text{const}}$  being the most pronounced one of the three, sets the tendency for the angular rate evolution during the rest of the simulation time. Lastly, due to the aforementioned coupling between the axes' dynamics, the z-component also indirectly experiences the applied disturbance torque.

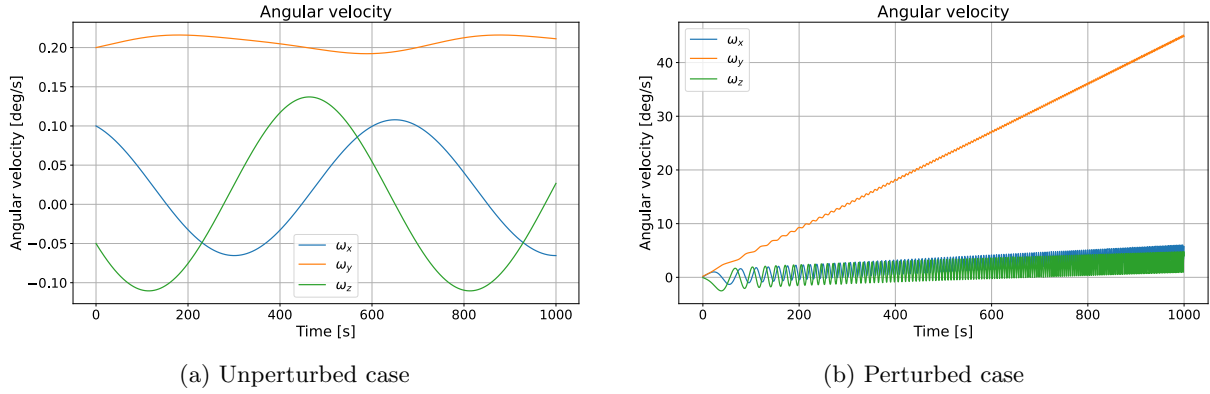


Figure 1: Comparison between the angular rates without and in the presence of a disturbance torque

### 3 Attitude Representation Analysis

This work focuses on three attitude representations for analysis and comparison: quaternion, Euler angles and Modified Rodrigues Parameters. By looking at the kinematics equation, defined in the first section, as well as the matrix  $\Omega(\omega)$ , the equation behavior can be expressed in a simple way - the more the angular rates increase, the more pronounced the attitude variations become as well. Regardless of the currently chosen attitude representation on the output level, the attitude changes should become more rapid as the simulation time advances in the case of an added constant disturbance torque  $\tau_{d, \text{const}}$ . This tendency is confirmed by all the plots in Figure 2.

Regarding the quaternion attitude representation, the scalar part represents the evolution of the rotation angle, whereas the other three parts contain the rotation angle evolution scaled by corresponding factors. Based on the results in Figure 2, one can deduce that the dominating axis, about which the rotation mostly takes place, is the y-axis. This is also consistent with the angular rate results in the previous section. Given that the implemented propagator is based solely on quaternions, the quaternion-related graph results from Figure 2 can be considered the most important.

Moreover, the mapping from quaternion to Modified Rodrigues Parameters is straightforward:

$$\sigma_{1/2/3} = \frac{q_{1/2/3}}{1 + q_4}, \quad (5)$$

where  $q_4$  is the scalar part of the quaternion. Therefore, considering the graph results up to now, it makes sense for the  $\sigma_2$  component to be the most pronounced of the three. This is confirmed by the corresponding graph in Figure 2.

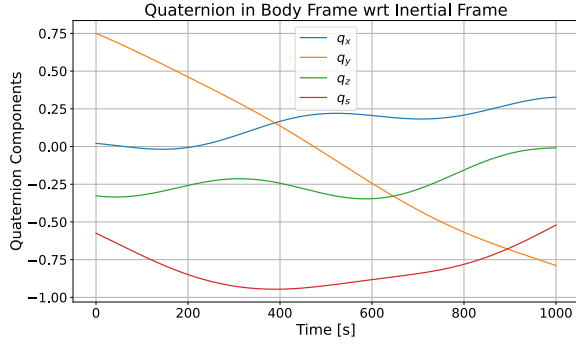
Unfortunately, the mapping from quaternion to Euler angles is not as intuitive as the one from quaternion to Modified Rodrigues Parameters. Nevertheless, no particular reason was identified for the figure of the Euler angles' time evolution in the presence of a disturbance torque to be considered less trustworthy than those of quaternion and Modified Rodrigues Parameters.

### 4 Outro

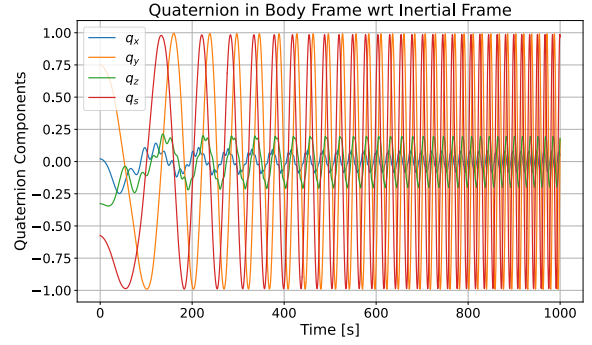
Working on the provided task was a pleasure. I thank you for the opportunity to learn more about the kinematics and dynamics of the different attitude representations, and for the chance to practice Python after a few years of not having used it for projects.

I hope my work convinces you to invite me to an online meeting, so that we can go more into detail about my exact implementation of the attitude propagator as well as discuss any further topics related to aerospace or other relevant fields. :)

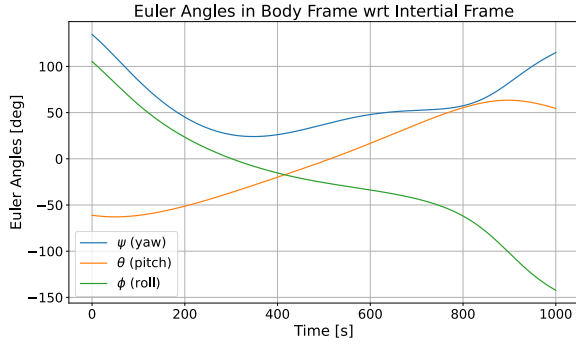
Petar Ivanov



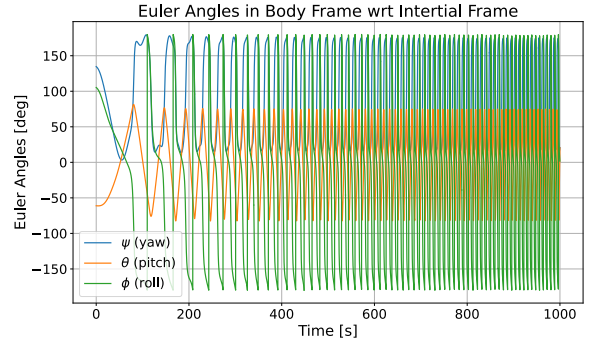
(a) Unperturbed quaternion



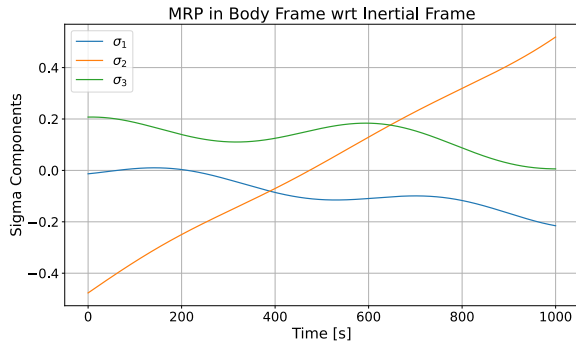
(b) Perturbed quaternion



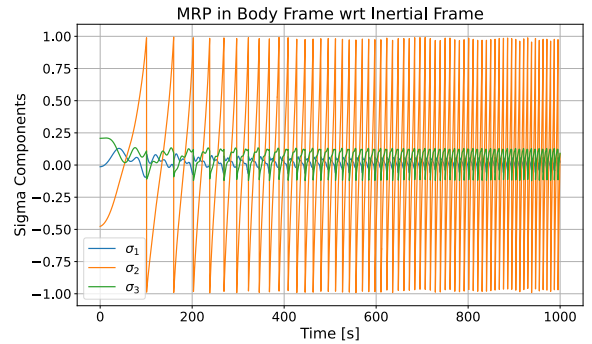
(c) Unperturbed Euler angles



(d) Perturbed Euler angles



(e) Unperturbed MRP



(f) Perturbed MRP

Figure 2: Comparison between different attitude representations in the perturbed and unperturbed case