2018-2019 Exam

Question 1, part (ii)

Everything is right through

$$e^z = \frac{-1 \pm \sqrt{3}i}{2}$$

Taking ln, the real part is

$$\ln |\frac{-1 \pm \sqrt{3}i}{2}| = \ln 1 = 0$$

not ln(2) as in the notes. The imaginary part is fine.

Question 1, part (ii)

The question asks for the roots of $z^7 + 1 = 0$, i.e., the 7th roots of -1; the solutions work with $z^-1 = 0$, i.e., the seventh roots of 1. The 7th roots of -1 are $e^{\pi i/7}, e^{3\pi i/7}, e^{5\pi i/7}, e^{7\pi i/7} = -1, e^{9\pi i/7} = e^{-5\pi/7}, e^{11\pi i/7} = e^{-3\pi i/7}, e^{13\pi i/7} = e^{-\pi i/7}$.

Hence

$$\begin{aligned} z^7 + 1 &= (z+1)(z - e^{\pi i/7})(z - e^{-\pi i/7})(z - e^{3\pi i/7})(z - e^{-3\pi i/7})(z - e^{\pi 5i/7})(z - e^{-\pi 5i/7}) \\ &= (z+1)(z^2 - 2z\cos(\pi/7) + 1)(z^2 - 2z\cos(3\pi/7) + 1)(z^2 - 2z\cos(5\pi/7) + 1) \end{aligned}$$

Question 2, Part (iii)

The functions the question deal with are slightly wrong / in the wrong order.

Part (a) on the actual exam is answered as Part (b) in the solutions.

Part (b) on the actual exam is not harmonic – the first term $2\cos(x)\cosh(y)$ is, but $2x^2$ is not. Hence it cannot be the real part of an analytic function.

Part (c) on the actual exam is very nearly, but not quite, the function dealt with in the solutions – $\sinh(2y)$ on the exam has become $\cosh(2y)$ in the solutions. The general method of solution is the same, except we find

$$f'(z) = 3 + 10\sin(2x)\sinh(2y) - 10\cos(2x)\cosh(2y)$$

setting $y = 0$, $\sinh(2y) = 0$ and $\cosh(2y) = 1$, so we guess

$$f'(z) = 3 - 10\cos(2z)$$

, and thus $f(z) = 3z - 5\sin(2z) + i\alpha$, where $\alpha \in \mathbb{R}$.

Question 4, Part (ii)

These look right, but I wanted to point out a subtlety in Part (c) that could trip you up – the Laurent expansion of $2m(z)$ is found, which has residue $-\pi^2/48$, dividing by 2 gives the desired residue of $m(z)$.