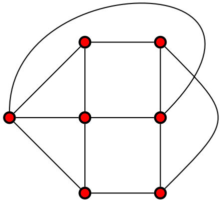
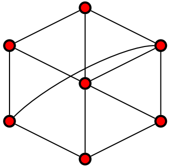
# Which graphs are/aren't isomorphic? Prove it.

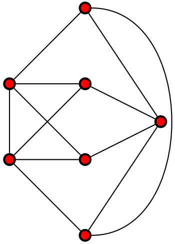
$A$



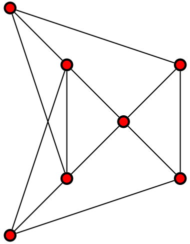
$B$



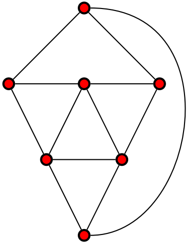
$C$



$D$

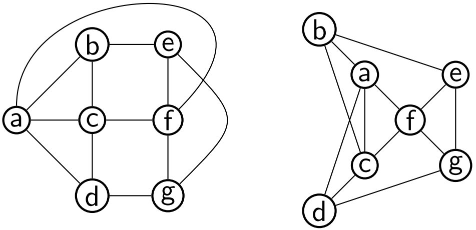


$E$



# One solution to warm-up

- Graph $B$ has a vertex of degree 5; others have degree sequence $[4, 4, 4, 3, 3, 3, 3]$, so none are isomorphic to $B$.
- In $A, D, E$, the three vertices of degree 4 all touch, but not in $C$, so none are isomorphic to $C$.
- In $A, D$, every vertex is adjacent to a vertex of degree 4, but not in $E$, so none are isomorphic to $E$.
- But we see below $A$ is isomorphic to $D$:



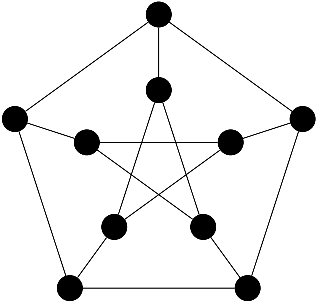
# Basic graphs and concepts

- The *empty graph* $E_n$ has $n$ vertices and no edges
- The *complete graph* $K_n$ has $n$ vertices, and each vertex is connected to every other.
- The *path graph* $P_n$ has $n$ vertices $\{1, \ldots, n\}$ with an edge between $i$ and $i + 1$
- The *cycle graph* $C_n$ has $n$ vertices $\{1, \ldots, n\}$ with an edge between $i$ and $i + 1$ and between $n$ and 1.

## Definition

Let $G$ be a simple graph with vertex set $V$. Its *complement* $G^c$ is another graph with vertex set $V$, where two vertices $v, w \in V$ are adjacent in $G^c$ if and only if they are not adjacent in $G$.

# Obligatory Petersen graph



What does it mean for a graph to be connected?

# Connected means we can "get from any vertex to another"

## Definition (Walk)

Let $G$ be a simple graph. A *walk* in $G$ is a sequence of vertices $v_1, v_2, \ldots, v_n$ so that $v_i$ is adjacent to $v_{i+1}$. We we say the walk goes from $v_1$ to $v_n$.

## Definition (Connected)

A graph $G$ is *connected* if there is a walk between any two vertices $v$ and $w$ in $G$.

## Definitions I won't use without explaining

- A *trail* is a walk that doesn't repeat any edges
- A *path* is a walk that doesn't repeat any vertices

# Bipartite graphs

## Definition (Bipartite graphs)

A graph $G$ is *bipartite* if we can colour every vertex either blue or red so that every edge goes between a blue vertex and a red vertex.

## Definition (Complete bipartite graphs)

The *complete bipartite graph* $K_{m,n}$ consists of $m + n$ vertices, $m$ coloured red, $n$ coloured blue, and an edge between any red vertex and and any blue vertex.

## Examples

# Another way to characterise bipartite graphs

## Lemma

*A graph $G$ is bipartite if and only if it doesn't have any cycles of odd length (i.e., subgraphs of the form $C_{2k+1}$).*

## Bipartite $\implies$ no odd cycles:

Subgraphs of bipartite graphs are bipartite

## No odd cycles $\implies$ Bipartite:

Try to colour $G$ by distance from $v$

## Definition (Distance)

Let $G$ be connected, and let $v, w$ be two vertices. The *distance from $v$ to $w$* is the least number of edges in any walk from $v$ to $w$.