# MAS439 COMMUTATIVE ALGEBRA
# PROBLEM SET 1

DUE FRIDAY, OCTOBER 18TH

## Question 1

**Introduction – nothing to prove here.** If $R$ and $S$ are commutative rings, then $R \times S$ is a commutative ring, where addition and multiplication are defined by:

$$(r_1, s_1) + (r_2, s_2) = (r_1 + r_2, s_1 + s_2)$$
$$(r_1, s_1) \cdot (r_2, s_2) = (r_1 \cdot r_2, s_1 \cdot s_2)$$

**The actual questions – one point each.**

(1) Find all the units in $\mathbb{Z} \times \mathbb{Z}$
(2) Find all the nilpotent elements of $\mathbb{Z} \times \mathbb{Z}$
(3) Find all the zero divisors in $\mathbb{Z} \times \mathbb{Z}$

Be sure that your answer justifies why you have found *all* of the elements asked for.

## Question 2

**The question, worth three points.** Prove that if $R$ is an integral domain, then $R[x]$ is an integral domain.

**A hint.** If you're having trouble, a good place to get started and/or get partial credit is to pick some specific elements of $R[x]$ and understand why they aren't zero divisors.

## Question 3

**Worth one point each.** How many different homomorphisms $\varphi : R \to S$ are there when

(1) $R = \mathbb{Z}$ and $S = \mathbb{Z}[x]$
(2) $R = \mathbb{Z}/7$ and $S = \mathbb{Z}/49$
(3) $R = \mathbb{Z}/14$ and $S = \mathbb{Z}/7$
(4) $R = \mathbb{Z} \times \mathbb{Z}$ and $S = \mathbb{Z}/12$

As always, be sure to justify your answers.