数学分析 B2 第七次作业

10.1.1 (1)$\int_0^1 \mathrm{d}y \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\mathrm{d}x$;

(2)$\int_0^4 \mathrm{d}y \int_0^{\frac{y}{2}} f(x,y)\mathrm{d}x + \int_4^6 \mathrm{d}y \int_0^{6-y} f(x,y)\mathrm{d}x$;

(6)$\int_{\frac{1}{2}}^1 \mathrm{d}x \int_0^{\frac{1}{x}} f(x,y)\mathrm{d}y$.

10.1.2 (1)$\ln\frac{2+\sqrt{2}}{1+\sqrt{3}}$; (3)$-2$; (4)$\frac{2}{3}a^3$; (8)$\frac{\pi}{2}-1$.

10.1.3 (1)$\frac{8}{3}$.

10.1.5

$$\int_0^a \mathrm{d}x \int_0^x f(x)f(y)\mathrm{d}y = \int_{0\le y\le x\le a} f(x)f(y)\mathrm{d}x\mathrm{d}y$$

$$=\frac{1}{2}(\int_{0\le y\le x\le a} + \int_{0\le x\le y\le a})f(x)f(y)\mathrm{d}x\mathrm{d}y = \frac{1}{2}\int_{[0,a]^2} f(x)f(y)\mathrm{d}x\mathrm{d}y = \frac{1}{2}(\int_0^a f(x)\mathrm{d}x)^2.$$

$$\int_0^a \mathrm{d}x \int_0^x f(y)\mathrm{d}y = \int_0^a f(y)\mathrm{d}y \int_y^a \mathrm{d}x = \int_0^a (a-y)f(y)\mathrm{d}y = \int_0^a (a-x)f(x)\mathrm{d}x.$$

10.1.7 $f(0,0)$.

10.2.1 (1)$\frac{\pi}{4}[(1+R^2)\ln(1+R^2)-R^2]$; (5)$\frac{1}{2}R^3$.

10.2.2 (2)$\frac{8}{3}ab\arctan\frac{a}{b}$; (4)$\frac{1}{3}(a-b)(m-n)$; (6)$\frac{\pi}{4}$; (8)$\frac{1}{2}(1-\cos 1)$.

10.2.4 $RHS = \int_{[-\frac{\sqrt{\pi}}{2},\frac{\sqrt{\pi}}{2}]^2} e^{x^2+y^2}\mathrm{d}x\mathrm{d}y$. 记 $D_1 = B_1(0), D_2 = [-\frac{\sqrt{\pi}}{2},\frac{\sqrt{\pi}}{2}]^2, D = D_1\cap D_2$. 注意到 $S(D_1) = S(D_2)$, 而 $e^{x^2+y^2}|_{D_1\setminus D} \le e^{x^2+y^2}|_{D_2\setminus D}$ 即可.

10.2.5 $\int_0^1 e^{f(x)}\mathrm{d}x \int_0^1 e^{-f(y)}\mathrm{d}y = \int_{[0,1]^2} e^{f(x)-f(y)}\mathrm{d}x\mathrm{d}y \ge \int_{[0,1]^2}(f(x)-f(y)+1)\mathrm{d}x\mathrm{d}y = 1$. 也可以用 Cauchy 不等式.

10.2.6 $\int_{|x|+|y|\le 1} e^{f(x+y)}\mathrm{d}x\mathrm{d}y \ge \int_{|x|+|y|\le 1}(f(x+y)+1)\mathrm{d}x\mathrm{d}y = 2$.

10.3.1 (3)$\frac{\pi^2}{16}-\frac{1}{2}$; (4)$\frac{a^4}{8}$.

10.3.2 (1)$\frac{8}{9}a^2$.

10.3.3 (2)$\frac{\pi}{6}$; (3)$\frac{13}{4}\pi$; (5)$\frac{7}{216}$.

10.3.4 (4)$\frac{4}{15}\pi(R^5-r^5)$; (6)0.

10.3.5 (7)$\frac{4-2\sqrt{2}}{3}\pi abc$; (8)$\frac{\pi}{3}|a|^3$.

10.3.7 $F'(t) = 4\pi t^2 f(t^2)\operatorname{sgn}(t), t\ne 0. F'(0) = 0$.