数分期中复习答案

八、

18-19、 1. 切向量 $v_1=(1,-1,0)\times(1,1,1)=(-1,-1,2)$ $\quad v_2=(2,1,0)\times(1,0,1)=(1,-2,-1)$ $\Rightarrow$ 平面法向量 $n=v_1\times v_2=(5,1,3)$

设 $5x+y+3z=\lambda$. 取 $(0,0,1)\in L_1$, $(0,1,0)\in L_2$ $\quad d_1=d_2\Rightarrow |3-\lambda|=|1-\lambda| \Rightarrow \lambda=2$. $\Rightarrow 5x+y+3z-2=0$.

19-20. 1. L_1: $r=\overset{r_1}{(0,0,-1)}+\lambda\overset{v}{(1,-1,2)}$. $\quad L_2$: $r=\overset{r_2}{(0,0,1)}+\lambda\overset{v}{(1,-1,2)}$ $\quad$ 旋转面 $|(r-r_2)\times v|=|(r_1-r_2)\times v| \Leftrightarrow (x+y)^2+(2x-z+1)^2+(2y+z-1)^2=8$.

九、

12-13 一、 1. (1). $y=kx^3$. 不连续. $\frac{\partial}{\partial x}f(0,0)=0$. $\frac{\partial}{\partial y}f(0,0)=0$ $\quad$ 不可微.

(2). 连续. $\frac{\partial}{\partial x}f(0,0)=\frac{\partial}{\partial y}f(0,0)=0$. $\quad \left|\frac{f(x,y)}{\rho}\right|\le\sqrt{\frac{|x||y|}{2}}\to 0$. $\quad f(x,y)=o(\rho)$. 可微.

4. $\frac{\partial z}{\partial x}=2\frac{\partial z}{\partial u}+\frac{\partial z}{\partial v}$. $\quad \frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}+c\frac{\partial z}{\partial v}$. $\quad \frac{\partial^2 z}{\partial x^2}=4\frac{\partial^2 z}{\partial u^2}+4\frac{\partial^2 z}{\partial u\partial v}+\frac{\partial^2 z}{\partial v^2}$ $\quad \frac{\partial^2 z}{\partial x\partial y}=2\frac{\partial^2 z}{\partial u^2}+(2c+1)\frac{\partial^2 z}{\partial u\partial v}+c\frac{\partial^2 z}{\partial v^2}$ $\quad \frac{\partial^2 z}{\partial y^2}=\frac{\partial^2 z}{\partial u^2}+2c\frac{\partial^2 z}{\partial u\partial v}+c^2\frac{\partial^2 z}{\partial v^2}$

$\Rightarrow 2\cdot(4,4,1)-5(2,2c+1,c)+2(1,2c,c^2)=(0,t,0)$ $\quad t\ne 0$. $\Rightarrow c=2$.

13-14. 2. 连续 $\frac{\partial}{\partial x}f(0,0)=\frac{\partial}{\partial y}f(0,0)=0$. $\quad f(x,y)=\rho^2\sin\frac{1}{\rho}=o(\rho)$ $\quad$ 可微.

19-20. 2.3 练习.

7. $g(\theta)\triangleq f(\cos\theta,\sin\theta)$ $\quad g'(\theta)=-f_x\sin\theta+f_y\cos\theta$. $\quad \exists\,\theta_1\in(0,\frac{\pi}{2})$, $\theta_2\in(\frac{\pi}{2},2\pi)$. $\quad g'(\theta_1)=g'(\theta_2)=0$.

18-19. 3. $u\triangleq\frac{y}{x}\Rightarrow u=2\arctan u\Rightarrow\frac{y}{x}=\text{const.}\Rightarrow\frac{d^2y}{dx^2}=0$

12-13 一. 6. $a\,dx+b\,dy+c\,dz=2x\varphi'dx+2y\varphi'dy+2z\varphi'dz$. $\Rightarrow \frac{\partial z}{\partial x}=\frac{2x\varphi'-a}{c-2z\varphi'}$. $\quad\frac{\partial z}{\partial y}=\frac{2y\varphi'-b}{c-2z\varphi'}$ $\quad$ 代入

18-19. 4. $\begin{cases}2x+y^2-1=0\\2xy=0\end{cases}\Rightarrow$ 驻点 $(\frac{1}{2},0)$. $-\frac{1}{4}$. $\quad (0,\pm1)$. 0. $\quad$ 边界 $f=x^2+x(2-x^2)-x=-x^3+x^2+x$ $\quad \varphi(1)=1$. $\varphi(-\frac{1}{3})=-\frac{5}{27}$. $\varphi(2)=-2$. $\varphi(-2)=10$.

$\Rightarrow \max=10$. $\min=-2$.

12-13 一. 2. $\begin{cases}y-\frac{50}{x^2}=0\\x-\frac{20}{y^2}=0\end{cases}\Rightarrow\begin{cases}x=5\\y=2\end{cases}$ $\quad xy+\frac{50}{x}+\frac{20}{y}\ge 3\sqrt[3]{50\times20}=30$ $\quad$ 极小值. 最小值

13-14. 3. $\begin{cases}-(1+e^y)\sin x=0\\e^y\cos x-e^y-ye^y=0\end{cases}\Rightarrow \overset{(i)}{\begin{cases}x=2k\pi\\y=0\end{cases}}$. $f=2$. or $\overset{(ii)}{\begin{cases}x=(2k+1)\pi\\y=-2\end{cases}}$. $f=1+2e^{-2}$ $\quad Q=(-(1+e^y)\cos x)h^2+2(-e^y\sin x)hk+e^y(\cos x-2-y)k^2$

(i) $Q=-2h^2-k^2<0$. 极大值 $\quad$ (ii) $Q=(1+e^{-2})h^2-e^yk^2$ 不定 非极值点.

4. $\begin{cases}\frac{1}{a}+2\lambda x=0\\\frac{1}{b}+2\lambda y=0\\x^2+y^2=1\end{cases}\Rightarrow\begin{cases}x=-\frac{1}{2\lambda a}\\y=-\frac{1}{2\lambda b}\\\lambda=\pm\frac{1}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}\end{cases}$ $\quad (x^2+y^2)(\frac{1}{a^2}+\frac{1}{b^2})\ge(\frac{x}{a}+\frac{y}{b})^2$

19-20. 6. $|f(x,y,z)|=\left|1+\int_0^1\frac{d}{d\theta}f(\theta x,\theta y,\theta z)\,d\theta\right|\le 1+\int_0^1\left|\frac{d}{d\theta}f(\theta x,\theta y,\theta z)\right|d\theta=1+\int_0^1|\nabla f\cdot(x,y,z)|\,d\theta\le 1+|(x,y,z)|\le 4$.

十、

11-12. 6. $LHS=\int_0^1 x_1dx_1\int_{x_1}^1 x_2dx_2\cdots\int_{x_{n-1}}^1 x_ndx_n=\frac{1}{2^n}\int_0^1 dx_1^2\int_{x_1^2}^1 dx_2^2\cdots\int_{x_{n-1}^2}^1 dx_n^2=\frac{1}{2^n}\int_0^1 dt_1\int_{t_1}^1 dt_2\cdots\int_{t_{n-1}}^1 dt_n=\frac{1}{2^n}\int_0^1 dt_n\int_0^{t_n}dt_{n-1}\cdots\int_0^{t_2}dt_1=\cdots=\frac{1}{2^n n!}$

12-13 二、 2. (1) $=\iint_{r\le\sqrt{2}\cos(\theta-\frac{\pi}{4})} r^2\,dr\,d\theta=\int_{-\frac{\pi}{4}}^{\frac{3}{4}\pi}d\theta\int_0^{\sqrt{2}\cos(\theta-\frac{\pi}{4})}r^2dr=\frac{2\sqrt{2}}{3}\int_{-\frac{\pi}{4}}^{\frac{3}{4}\pi}\cos^3(\theta-\frac{\pi}{4})\,d\theta=\frac{2\sqrt{2}}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^3\theta\,d\theta=\frac{8\sqrt{2}}{9}$

(2) $=\int_0^{\frac{\pi}{2}}\int_0^1 r\,dr\,d\theta\int_r^{\sqrt{2-r^2}}z^2dz=\frac{\pi}{2}\int_0^1 r\,dr\int_r^{\sqrt{2-r^2}}z^2dz=\frac{\pi}{2}\left(\int_0^1\int_0^z rz^2\,dr\,dz+\int_1^{\sqrt{2}}\int_0^{\sqrt{2-z^2}}rz^2\,dr\,dz\right)=\frac{\pi}{2}\left(\frac{1}{10}+\frac{8\sqrt{2}}{15}-\frac{7}{15}\right)=\left(\frac{8\sqrt{2}}{15}-\frac{11}{30}\right)\frac{\pi}{2}$.

(3) $=\int_0^1 dx\int_0^x\frac{y}{\sqrt{1+x^3}}dy=\frac{1}{2}\int_0^1\frac{x^2}{\sqrt{1+x^3}}dx=\frac{1}{3}(\sqrt{2}-1)$.

(4) $=0$. (对称性).

4. $x^2+y^2=z^{\frac{3}{2}}-z^2$. $\quad z^{\frac{3}{2}}-z^2=0\Rightarrow z=0,1$. $\quad V=\int_0^1\pi\cdot(z^{\frac{3}{2}}-z^2)\cdot dz=(\frac{2}{5}-\frac{1}{3})\pi=\frac{\pi}{15}$

5. 归纳 $n=1$. $\int_a^x dx_1\int_a^{x_1}f(x_2)dx_2=\int_a^x dx_2\int_{x_2}^x f(x_2)dx_1=\int_a^x(x-y)f(y)dy$. $\quad$ 设 $n-1$ 时成立. $\quad LHS=\int_a^x dx_1\cdot\frac{1}{(n-1)!}\int_a^{x_1}(x_1-y)^{n-1}f(y)dy=\frac{1}{(n-1)!}\int_a^x dy\int_y^x(x_1-y)^{n-1}f(y)dx_1$

$=\frac{1}{n!}\int_a^x(x-y)^n f(y)dy$.

6. $I_1=\int_{-1}^1\cos x\cdot\pi(1-x^2)\,dx=2\pi\int_0^1(1-x^2)\cos x\,dx=2\pi\int_0^1 2x\sin x\,dx=4\pi(\sin 1-\cos 1)$. $\quad$ 旋转坐标轴 s.t. $x'=ax+by+cz$. $\Rightarrow I_2=I_1$.

18-19. 6. $x=\sqrt{2}u$. $y=\sqrt{3}v$. $z=2w$. $\Rightarrow$ 单位球被 $\sqrt{2}u+\sqrt{3}v+2w=1$ 截. $\quad d=\frac{1}{\sqrt{2+3+4}}=\frac{1}{3}$. $\quad V'=\int_{\frac{1}{3}}^1\pi(1-h^2)dh=\frac{28}{81}\pi$ $\quad V=2\sqrt{6}\,V'=\frac{56}{81}\sqrt{6}\pi$.

7. $u=x+y$. $v=x-y$ $\quad I=\frac{1}{2}\int_{[-1,1]^2}(uv)^m\,du\,dv=\frac{1}{2}\left(\int_{-1}^1 u^m du\right)^2=\begin{cases}0. & m.\ \text{odd}\\2/(m+1)^2 & m.\ \text{even}\end{cases}$

8. $D\cap E=\frac{1}{8}m(E\cap[-1,1]^3)$ $\quad m(E\setminus[-1,1]^3)=6V(\ )=6\int_1^{\sqrt{a}}\pi\cdot(a-h^2)\,dh=6\pi\left(a(\sqrt{a}-1)-\frac{1}{3}(\sqrt{a}^3-1)\right)=2\pi(2a\sqrt{a}-3a+1)$

$m(E\cap[-1,1]^3)=\frac{4}{3}\pi a\sqrt{a}-2\pi(2a\sqrt{a}-3a+1)=\pi(-\frac{8}{3}a\sqrt{a}+6a-2)$. $\quad V=\pi(-\frac{1}{3}a\sqrt{a}+\frac{3}{4}a-\frac{1}{4})$