数学分析 B2 第三次作业

9.1.17 (1) 在 $x \neq y$ 处连续, $x = y$ 处不连续. 注意 $(0,0)$ 处不连续. (3) 连续.

9.1.18 略.

9.1.19 证明: $\forall (x_0, y_0) \in D, \forall \varepsilon > 0, \exists \delta_1 > 0$, 使得当 $|y - y_0| < \delta_1$ 时, 有 $|f(x_0, y) - f(x_0, y_0)| < \frac{\varepsilon}{2}$. 又 $\exists \delta_2 > 0$, 使得当 $|x - x_0| < \delta_2$ 时, 有 $|f(x, y_0 + \delta_1) - f(x_0, y_0 + \delta_1)| < \frac{\varepsilon}{2}$ 且 $|f(x, y_0 - \delta_1) - f(x_0, y_0 - \delta_1)| < \frac{\varepsilon}{2}$. 取 $\delta < \min\{\delta_1, \delta_2\}$ 使得当 $|x - x_0|, |y - y_0| < \delta$ 时 $(x, y) \in D$. 由 $f(x, y)$ 关于 y 的单调性, 有

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\begin{aligned}
&|f(x,y) - f(x_0,y_0)| \\
\leq &\max\{|f(x, y_0 + \delta_1) - f(x_0, y_0)|, |f(x, y_0 - \delta_1) - f(x_0, y_0)|\} \\
\leq &\max\{|f(x, y_0 + \delta_1) - f(x_0, y_0 + \delta_1)| + |f(x_0, y_0 + \delta_1) - f(x_0, y_0)|, \\
&\quad |f(x, y_0 - \delta_1) - f(x_0, y_0 - \delta_1)| + |f(x_0, y_0 - \delta_1) - f(x_0, y_0)|\} \\
\leq &\frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.
\end{aligned}
$$

9.1.23 证明: 取 $\varepsilon_0 = 1, \forall \delta > 0$, 令 $\delta' = \min\{\delta, \frac{1}{2}\}$, 取 $(x_1, y_1) = (1 - \delta', 1), (x_2, y_2) = (1 - \frac{\delta'}{2}, 1)$, 则 $\rho((x_1, y_1), (x_2, y_2)) = \frac{\delta'}{2} < \delta$, 但 $|f(x_1, y_1) - f(x_2, y_2)| = \frac{1}{\delta'} > 1$. 所以不一致收敛.

9.2.1 $(1)\frac{2}{5}$.

9.2.2 $(3) z_x = \frac{1}{y} \cos\frac{x}{y} \cos\frac{y}{x} + \frac{y}{x^2} \sin\frac{x}{y} \sin\frac{y}{x}, z_y = -\frac{x}{y^2} \cos\frac{x}{y} \cos\frac{y}{x} - \frac{1}{x} \sin\frac{x}{y} \sin\frac{y}{x}$.

$(5) u_x = -\frac{y}{x^2+y^2}, u_y = \frac{x}{x^2+y^2}$.

$(7) u_x = y^z x^{y^z - 1}, u_y = (\ln u)_y u = x^{y^z} y^{z-1} z \ln x, u_z = (\ln u)_z u = x^{y^z} y^z \ln x \ln y$.

9.2.3 $f_x = \frac{2 \sin x^2 y}{x}, f_y = \frac{\sin x^2 y}{y}$.

9.2.4 $f_x(0,0) = \lim\limits_{x \to 0} \frac{0 \cdot \sin \frac{1}{x^2+0} - 1}{x - 0} = 0, f_y = \lim\limits_{y \to 0} \frac{y \sin \frac{1}{y^2} - 0}{y - 0} = \lim\limits_{y \to 0} \sin \frac{1}{y^2}$ 不存在.

9.2.5 因为 $\lim\limits_{x \to 0} \frac{\sqrt{x^2+0^2}}{x - 0}$ 不存在, 所以 $\frac{\partial z}{\partial x}(0,0)$ 不存在, 同理 $\frac{\partial z}{\partial y}(0,0)$ 不存在. 注意, 要证明在 $(0,0)$ 处的偏导数不存在, 不是证明偏导数在这一点处的极限不存在.