20171646 박태은.

(고급 소프트웨어 실습 I 2주차 과제)

$$r_{xy} = \frac{\dfrac{\sum_i^n (X_i - \bar{X})(Y_i - \bar{Y})}{n}}{\sqrt{\dfrac{\sum_i^n (X_i - \bar{X})^2}{n}}\sqrt{\dfrac{\sum_i^n (Y_i - \bar{Y})^2}{n}}}$$

$$= \frac{\dfrac{\sum_i^n (X_i - \bar{X})(Y_i - \bar{Y})}{n}}{\dfrac{\sqrt{\sum_i^n (X_i - \bar{X})^2} \cdot \sqrt{\sum_i^n (Y_i - \bar{Y})^2}}{n}}$$

$$= \frac{\sum_i^n (X_i - \bar{X})(Y_i - \bar{Y}) \text{ --- (분자)}}{\sqrt{\sum_i^n (X_i - \bar{X})^2} \cdot \sqrt{\sum_i^n (Y_i - \bar{Y})^2} \text{ -- (분모)}}$$

*분자와 분모로 나누어 계산해보겠다.

$$\sum_i^n (X_i - \bar{X})(Y_i - \bar{Y})$$ (분자)

$$= \sum_i^n (X_i Y_i - X_i \bar{Y} - \bar{X} Y_i + \bar{X}\,\bar{Y})$$

$$= \sum_i^n (X_i Y_i) - \sum_i^n (X_i \bar{Y}) - \sum_i^n (\bar{X} Y_i) + \sum_i^n (\bar{X}\bar{Y})$$

$$= \sum_i^n (X_i Y_i) - \bar{Y} \cdot \sum_i^n X_i - \bar{X} \sum_i^n Y_i + n \cdot (\bar{X}\bar{Y})$$

$$= \sum_i^n (X_i Y_i) - n \cdot \bar{Y} \cdot \sum_i^n \frac{X_i}{n} - n \cdot \bar{X} \sum_i^n \frac{Y_i}{n} + n(\bar{X}\bar{Y})$$

$$= \sum_i^n (X_i Y_i) - n \cdot \bar{Y} \cdot \bar{X} \cancel{- n\bar{X} \cdot \bar{Y} + n \cdot (\bar{X} \cdot \bar{Y})}$$

$$= \sum_i^n (X_i Y_i) - n \cdot \bar{Y}\bar{X}$$

$$\sqrt{\sum_i^n (X_i-\overline{X})^2}\sqrt{\sum_i^n (Y_i-\overline{Y})^2}$$ (분모)

$$=\sqrt{\sum_i^n (X_i^2-2X_i\overline{X}+\overline{X}^2)}\sqrt{\sum_i^n (Y_i^2-2Y_i\overline{Y}+\overline{Y}^2)}$$

$$=\sqrt{\sum_i^n X_i^2-2\sum_i^n X_i\cdot\overline{X}+\sum_i^n\overline{X}^2}\sqrt{\sum_i^n Y_i^2-2\sum_i^n Y_i\cdot\overline{Y}+\sum_i^n\overline{Y}^2}$$

여기서, $\sum_i^n X_i\cdot\overline{X}=n\cdot\sum_i^n\frac{X_i}{n}\cdot\overline{X}=n\cdot\overline{X}^2$

그리고 $\sum_i^n\overline{X}^2=n\cdot\overline{X}^2$ 이므로 . 위 식은

$$=\sqrt{\sum_i^n X_i^2-2\cdot n\cdot\overline{X}^2+n\cdot\overline{X}^2}\sqrt{\sum_i^n Y_i^2-2\cdot n\cdot\overline{Y}^2+n\cdot\overline{Y}^2}$$

$$=\sqrt{\sum_i^n X_i^2-n\cdot\overline{X}^2}\sqrt{\sum_i^n Y_i^2-n\cdot\overline{Y}^2}$$

$$=\sqrt{\sum_i^n X_i^2-n\cdot\left(\frac{\sum_i^n X_i}{n}\right)^2}\cdot\sqrt{\sum_i^n Y_i^2-n\cdot\left(\frac{\sum_i^n Y_i}{n}\right)^2}$$

$$=\sqrt{\sum_i^n X_i^2-\frac{1}{n}\cdot\left(\sum_i^n X_i\right)^2}\cdot\sqrt{\sum_i^n Y_i^2-\frac{1}{n}\left(\sum_i^n Y_i\right)^2}$$

정리를 하면,

$$r_{XY} = \frac{\sum_i^n (X_i Y_i) - n \cdot \overline{X} \cdot \overline{Y}}{\sqrt{\sum_i^n X_i^2 - \frac{1}{n}\left(\sum_i^n X_i\right)^2} \sqrt{\sum_i^n Y_i^2 - \frac{1}{n}\left(\sum_i^n Y_i\right)^2}}$$

$$= \frac{n \sum_i^n (X_i Y_i) - n^2 \cdot \left(\frac{\sum_i^n X_i}{n}\right)\left(\frac{\sum_i^n Y_i}{n}\right)}{\sqrt{n \sum_i^n X_i^2 - \left(\sum_i^n X_i\right)^2} \sqrt{n \sum_i^n Y_i^2 - \left(\sum_i^n Y_i\right)^2}}$$

$$= \frac{n\left(\sum XY\right) - \left(\sum X\right)\left(\sum Y\right)}{\sqrt{n \sum X^2 - \left(\sum X\right)^2} \sqrt{n \sum Y^2 - \left(\sum Y\right)^2}}$$