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Adv Opt for Machine Learning
Homework 5
Mirror Descent for Constrained Logistic Regression

The objective of this assignment was to apply mirror descent to solve the simplex-constrained least-mean-square problem defined as:

$$min_{x \in \Delta} f(x) = \frac{1}{2}\|Ax - b\|_2^2, \text{ where } x \in R^n, A \in R^{m \times n}, b \in R^m.$$

The simplex constraint is defined as $\Delta := \{x \in R^n_+, \sum_{i=1}^{n} x_i = 1\}$.

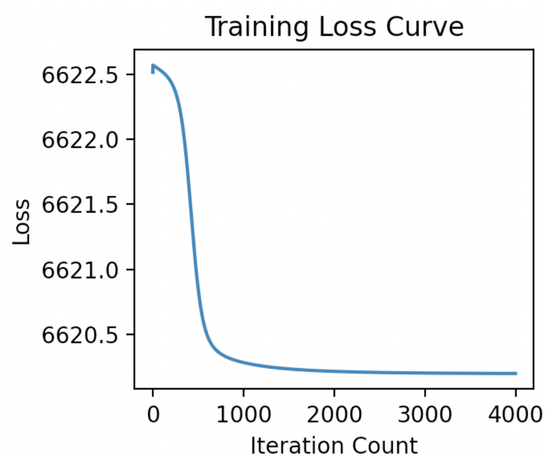
The gradient for the definition above is given as $\nabla f(x) = A^T(Ax - b)$. The underlying signal (z), and the initialization of x_0 were generated randomly using the numpy.random.randn function. The initial noise variance was chosen to be 0.1.

Using a divergence function of $\varphi(x) = -\sum_{i=1}^{n} x_i \log x_i$, the following closed-form update method can be applied:

$$[x_{k+1}] = \frac{[x_k]_i \exp(-\eta[\nabla f(x_t)]_i)}{\sum_{j=1}^{n} [x_t]_j \exp(-\eta[\nabla f(x_t)]_j)}, \quad 1 \le i \le n$$

where η is the learning rate and i is the entry in a vector.

Using n = 600, m = 200, η= 1e-2, and an iteration count of 4000, the training loss curve of the implementation is shown below.



The training loss curve follows an expected trajectory. Other than a slight initial spike, which I presume is from the randomization of the variable initializations, the loss decreases as iterations go on.