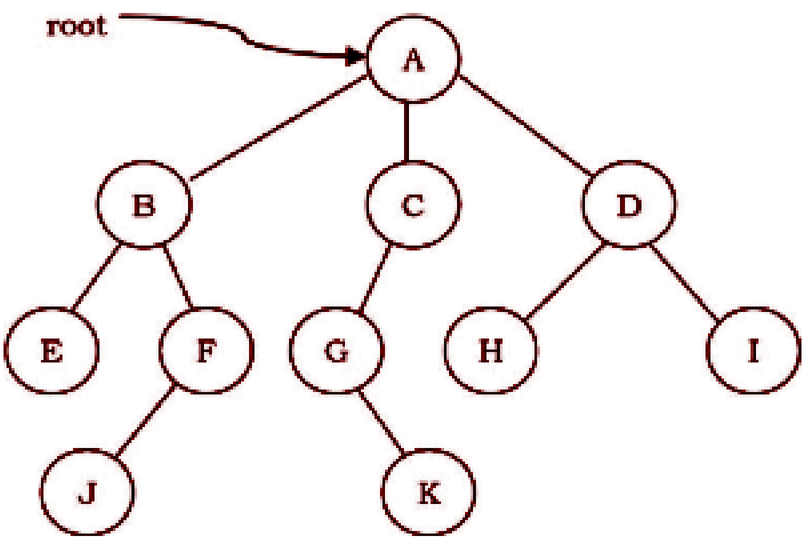
Binary Trees- 1

What is A Tree?

- A tree is a data structure similar to a linked list but instead of each node pointing simply to the next node in a linear fashion, each node points to several nodes.
- A tree is an example of a non- linear data structure.
- A tree structure is a way of representing the hierarchical nature of a structure in a graphical form.

Terminology Of Trees

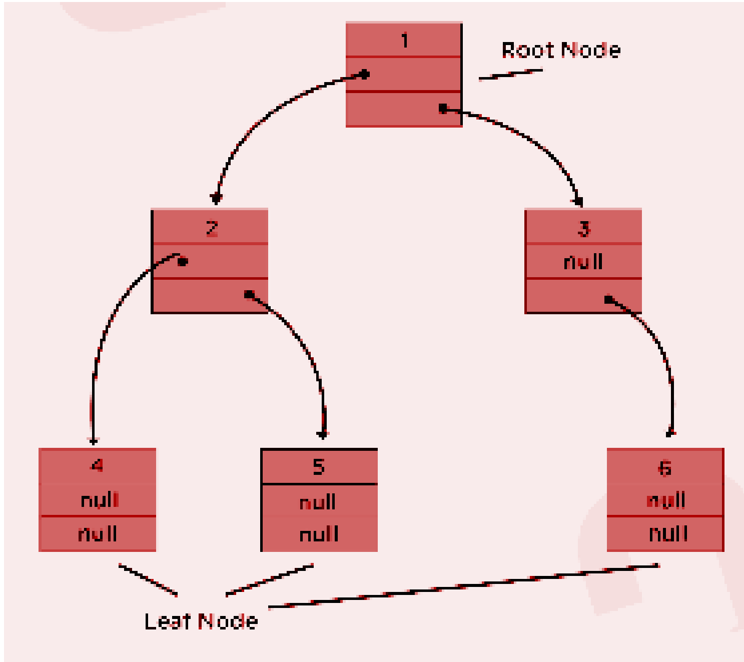


- The root of a tree is the node with no parents. There can be at most one root node in a tree **(node A in the above example)**.
- An **edge** refers to the link from a parent to a child **(all links in the figure)**.
- A node with no children is called a **leaf node** **(E, J, K, H, and I)**.
- The children nodes of the same parent are called **siblings** **(B, C, D are siblings of parent A and E, F are siblings of parent B)**.

- The set of all nodes at a given depth is called the **level** of the tree **(B, C, and D are the same level)**. The root node is at level zero.
- The **depth** of a node is the length of the path from the root to the node **(depth of G is 2, *A -> C –> G*)**.
- The **height** of a node is the length of the path from that node to the deepest node.
- The **height** of a tree is the length of the path from the root to the deepest node in the tree.
- A (rooted) tree with only one node (the root) has a height of zero.

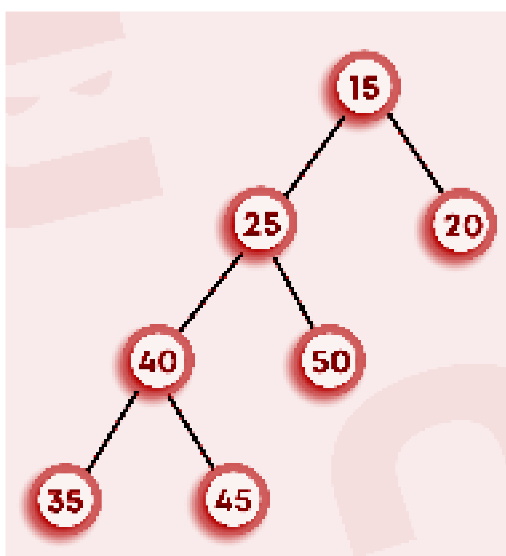
Binary Trees

- A generic tree with at most two child nodes for each parent node is known as a binary tree.
- A binary tree is made of nodes that constitute a **left** pointer, a **right** pointer, and a data element. The **root** pointer is the topmost node in the tree.
- The left and right pointers recursively point to smaller **subtrees** on either side.
- An empty tree is also a valid binary tree.
- *A formal definition is:* A **binary tree** is either empty (represented by a None pointer), or is made of a single node, where the left and right pointers (recursive definition ahead) each point to a **binary tree**.



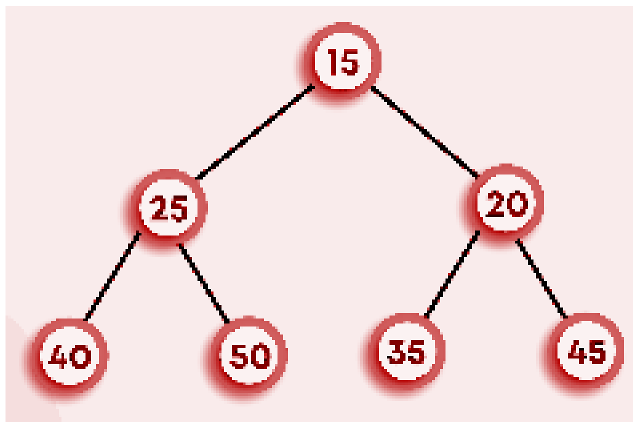
Types of binary trees:

Full binary trees: A binary tree in which every node has 0 or 2 children is termed as a full binary tree.

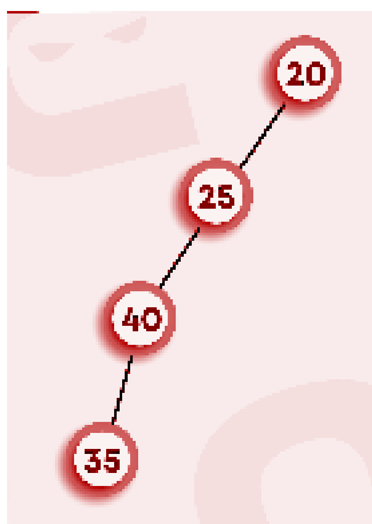


Complete binary tree: A complete binary tree has all the levels filled except for the last level, which has all its nodes as much as to the left.

Perfect binary tree: A binary tree is termed perfect when all its internal nodes have two children along with the leaf nodes that are at the same level.

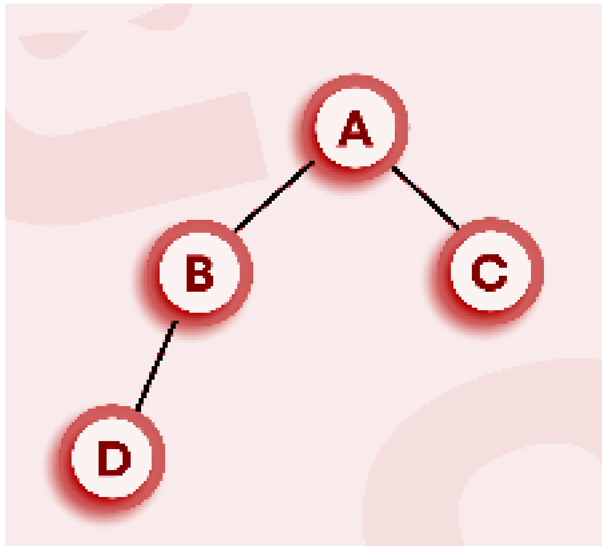


A degenerate tree: In a degenerate tree, each internal node has only one child.



The tree shown above is degenerate. These trees are very similar to linked-lists.

Balanced binary tree: A binary tree in which the difference between the depth of the two subtrees of every node is at most one is called a balanced binary tree.



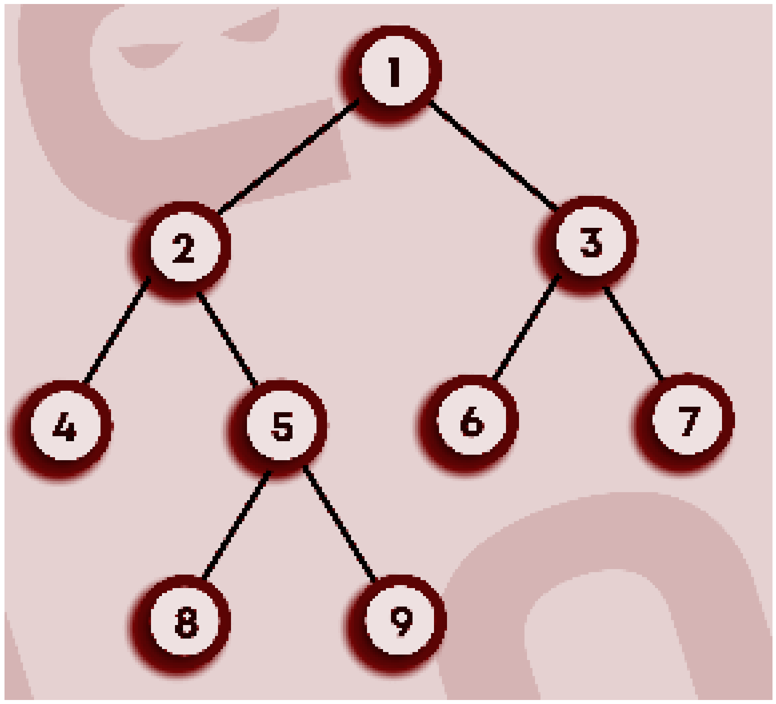
Binary tree representation:

Binary trees can be represented in two ways:

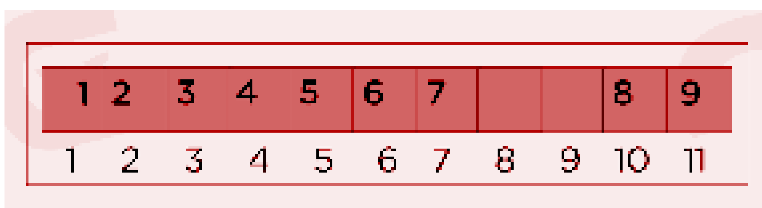
Sequential representation

- This is the most straightforward technique to store a tree data structure. An array is used to store the tree nodes.
- The number of nodes in a tree defines the size of the array.
- The root node of the tree is held at the first index in the array.
- In general, if a node is stored at the $\mathbf{i^{th}}$ location, then its **left** and **right** child are kept at $\mathbf{(2i)^{th}}$ and $\mathbf{(2i+1)^{th}}$ locations in the array, respectively.

Consider the following binary tree:



The array representation of the above binary tree is as follows:



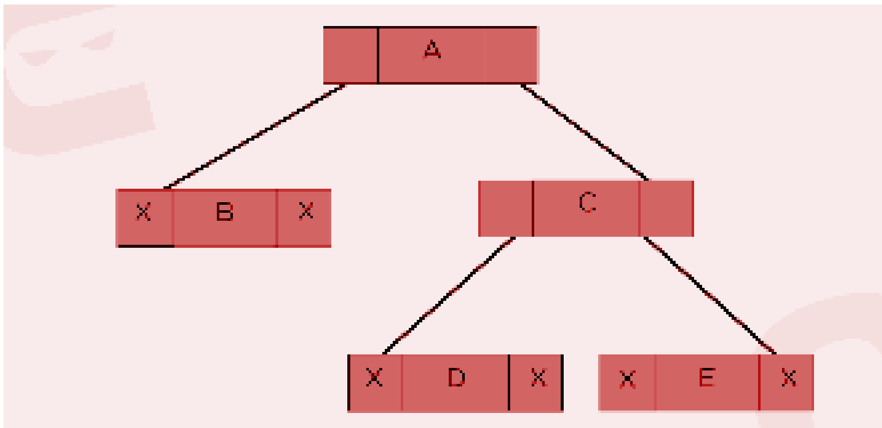
As discussed above, we see that the left and right child of each node is stored at locations **2*(nodePosition)** and **2*(nodePosition)+1**, respectively.

For Example, The location of node 3 in the array is 3. So its left child will be placed at **2*3 = 6**. Its right child will be at the location **2*3 +1 = 7**. As we can see in the array, children of 3, which are 6 and 7, are placed at locations 6 and 7 in the array.

Note: The sequential representation of the tree is not preferred due to the massive amount of memory consumption by the array.

Linked list representation:

In this type of model, a linked list is used to store the tree nodes. The nodes are connected using the parent-child relationship like a tree. The following diagram shows a linked list representation for a tree.

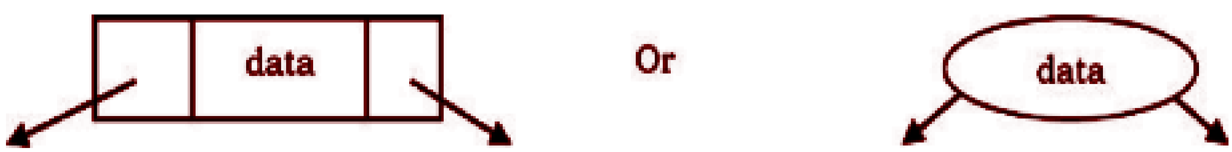


As shown in the above representation, each linked list node has three components:

- Pointer to the left child
- Data
- Pointer to the right child

Note: If there are no children for a given node (leaf node), then the left and right pointers for that node are set to **None**.

Let's now check the implementation of the **Binary tree class**.




```
class BinaryTreeNode:
    def __init__(self, data):
        self.left = None #To store data
        self.right = None #For storing the reference to left pointer
        self.data = data #For storing the reference to right pointer
```

Operations on Binary Trees

Basic Operations

- Inserting an element into a tree

- Deleting an element from a tree
- Searching for an element
- Traversing the tree

Auxiliary Operations

- Finding the size of the tree
- Finding the height of the tree
- Finding the level which has the maximum sum and many more...

Print Tree Recursively

Let's first write a program to print a binary tree recursively. Follow the comments in the code below:

```
def printTree(root):
      root == None: #Empty tree
            return
      print(root.data, end ":") #Print root data
      if root.left != None:
            print("L", root.left.data, end=",") #Print left child
      if root.right != None:
            print("R", root.right.data, end="") #Print right child
      Print #New Line
      printTree(root.left) #Recursive call to print left subtree
      printTree(root.right) #Recursive call to print right subtree
```

Input Binary Tree

We will be following the level-wise order for taking input and -1 denotes the **None** pointer.

```
def treeInput():
```

```
    rootData = int(input())
    if rootData == -1: #Leaf Node is denoted by -1
        return None

    root = BinaryTreeNode(rootData) #Create a tree node
    leftTree =treeInput() #Take input for left subtree
    rightTree = treeInput() #Take input for right subtree
    root.left = leftTree #Assign the left subtree to the left child
    root.right = rightTree #Right subtree to the right child
    return root
```

Count nodes

- Unlike the Generic trees, where we need to traverse the children vector of each node, in binary trees, we just have at most left and right children for each node.
- Here, we just need to recursively call on the right and left subtrees independently with the condition that the node pointer is not None.
- Follow the comments in the upcoming code for better understanding:

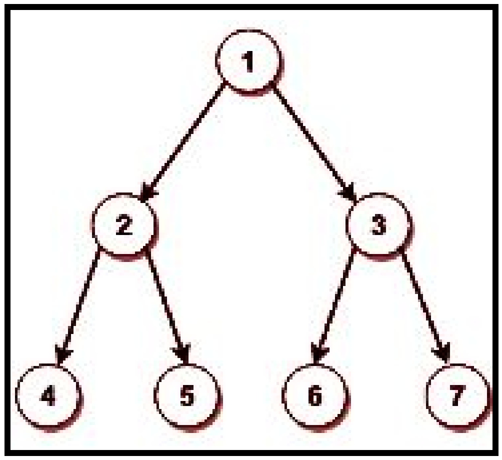
```
def count_nodes(node):
    if node is None: #Check if root node is None
        return 0
    return 1 + count_nodes(node.left) + count_nodes(node.right)
#Recursively count number of nodes in left and right subtree and add
```

Binary tree traversal

Following are the ways to traverse a binary tree and their orders of traversal:

- **Preorder traversal** : ROOT -> LEFT -> RIGHT
- **Postorder traversal** : LEFT -> RIGHT-> ROOT
- **Inorder traversal** : LEFT -> ROOT -> RIGHT

Some examples of the above-stated traversal methods:



- **Preorder traversal:** 1, 2, 4, 5, 3, 6, 7
- **Postorder traversal:** 4, 5, 2, 6, 7, 3, 1
- **Inorder traversal:** 4, 2, 5, 1, 6, 3, 7

Let's look at the code for inorder traversal, below:

```
# A function to do inorder tree traversal
def printInorder(root):
    if root:#If tree is not empty
        printInorder(root.left) # First recur on left child
        print(root.val)# Then print the data of the node
        printInorder(root.right) # Now recur on right child
```

Now, from this inorder traversal code, try to code preorder and postorder traversal yourselves. If you get stuck, refer to the solution tab for the same.

Node with the Largest Data

In a Binary Tree, we must visit every node to figure out the maximum. So the idea is to traverse the given tree and for every node return the maximum of 3 values:

- Node's data.
- Maximum in node's left subtree.
- Maximum in node's right subtree.

Below is the implementation of the above approach.

```
def findMaximum(root):
    # Base case
    if (root == None):
        return float('-inf') #**

    # Return maximum of 3 values:
    # 1) Root's data 2) Max in Left Subtree
    # 3) Max in right subtree
    max = root.data
    lmax = findMaximum(root.left) #Maximum of left subtree
    rmax = findMaximum(root.right) #Maximum of right subtree
    if (lmax > max):
        max = lmax
    if (rmax > max):
        max = rmax
    return max
```

Note**: In python, float values can be used to represent an infinite integer. One can use **float('-inf')** as an integer to represent it as “Negative ”infinity or the smallest possible integer.