



Code	Subject Title	Cr. Hrs	Semester
BBA-403	Mathematics (Advanced)	3	VII
Year	Discipline		
4	Business Administration		

Objectives:

The primary goal of this course is to teach the techniques of integral calculus that is likely to be encountered in business and economics courses in college and in subsequent professional activities. The course is designed to provide a sound, intuitive understanding of the basic concepts without sacrificing mathematical accuracy. Student performance will be evaluated on group projects handed out in class, pop quizzes and term exams.

Course Contents:**Introduction to Ant- differentiation:**

The indefinite integral integration by substitution area and the definite integral. Applications to business and economics, Integration by parts; integral tables. Review on integration.

Further Topics in Integration:

The definite integral as the limit of a sum improper integrals. Probability density functions numerical integration review on further topics on integration.

Functions of Two Variables:

Functions of two variables surfaces and level curves. Partial derivatives the chain rule; Approximation by the total differential relative maxima and minima. Lagrange multipliers. The method of least squares. Double integrals review on functions of two variables.

The Trigonometric Functions:

Introduction to trigonometric functions; Differentiation and integration of trigonometric functions; Additional applications involving trigonometric functions.

Differential Equations:

Differential equations an introduction; Second order differential equations; Difference equations; Reviewing differential & difference equations.

Recommended Books:



UNIVERSITY OF THE PUNJAB

Seventh Semester – 2019

Examination: B.S. 4 Years Program

Roll No. in Fig.

Roll No. in Words.

PAPER: Mathematics (Advance)

Course Code: BBA-403 Part-I (Compulsory)

MAX. TIME: 30 Min.

MAX. MARKS: 10

Signature of Supdt.:

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Division of marks is given in front of each question.

This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Encircle the right answer, cutting and overwriting is not allowed. (1x10=10)

- (i) If $f(x) = \sin x + \cos x$, then $f'(0) = \dots$
(a) -1 (b) 1 (c) 0 (d) None of these
- (ii) If $y = x^4 + 2x^2 + 2$, then $dy/dx = \dots$
(a) $x\sqrt{y-1}$ (b) $2x\sqrt{y-1}$ (c) $4x\sqrt{y-1}$ (d) None of these
- (iii) For a function $f(x)$, if $f'(c) = 0$ & $f''(c) > 0$ then $f(x)$ has ... at $x = c$
(a) relative maxima (b) relative minima (c) point of inflection (d) None of these
- (iv) If $y = e^{10x}$ then $y''' = \dots$
(a) $10e^{10x}$ (b) $100e^{10x}$ (c) $1000e^{10x}$ (d) None of these
- (v) If $f(x) = 3 + x$, then
(a) $f'(0) \neq f'(1)$ (b) $f'(0) = f'(1)$ (c) $f'(0) > f'(1)$ (d) None of these
- (vi) If $\int \frac{\sec^2 x}{\tan x} dx = \dots$
(a) $\ln(\tan x) + c$ (b) $\ln(\sec x) + c$ (c) $\sec \frac{1}{x} + c$ (d) None of these
- (vii) $\int \frac{e^x}{e^x - 1} dx = \dots$
(a) $\ln|1 - e^x| + c$ (b) $\ln|1 + e^x| + c$ (c) $\ln|e^x - 1| + c$ (d) None of these
- (viii) $\int_a^b f(x)f'(x)dx = \dots$
(a) $f(b) - f(a)$ (b) 0 (c) $1/2(f(b)^2 - f(a)^2)$ (d) None of these
- (ix) If $\int \frac{f'(x)}{f(x)} dx$ is equal to
(a) $\ln(f(x)) + c$ (b) $\ln(f'(x)) + c$ (c) $\frac{(f(x))^2}{2} + c$ (d) None of these
- (x) $\int_0^1 \frac{dx}{1+x^2} = \dots$
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) None of these



UNIVERSITY OF THE PUNJAB

Seventh Semester – 2019

Examination: B.S. 4 Years Program

Roll No.

PAPER: Mathematics (Advance)

Course Code: BBA-403 Part – II

MAX. TIME: 2 Hrs. 30 Min.

MAX. MARKS: 50

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Q.2. Give short answers to the following questions.

(10x2=20)

- (i) Find $\frac{dy}{dx}$ if $x^2 + y^2 = 1$.
- (ii) Differentiate $\sin\sqrt{x} + \sqrt{\sin x}$ w.r.t. x .
- (iii) Find $\frac{dy}{dx}$ if $y = \ln(x + \sqrt{x^2 + 1})$.
- (iv) Find the first order partial derivative of $f(x, y) = xysinx$.
- (v) Differentiate $\sin x$ w.r.t. $\cot x$.
- (vi) Solve the differential equation $\frac{dy}{dx} = 3x^2$; $y(1) = -1$.
- (vii) Find the change in $y = x^2 + 2x$ when x changes from 2 to 2.18.
- (viii) If $y = e^{-x}\sin 3x$ find y'' .
- (ix) Evaluate the integral $\int_0^1 (\sqrt{x} + 1)^2 dx$.
- (x) Evaluate the integral $\int (x-1)(x-3)dx$.

Q.3 State 2nd derivative test and find the extreme values of the function $f(x) = 3x^2 - 4x + 5$.

(6)

Q.4 Find the extreme values of $f(x, y, z) = x + y + z$ subject to $g(x, y, z) = x^2 + y^2 + z^2 - 4 = 0$ by using Lagrange Multipliers.

(6)

Q.5 If $x = a\cos^3\theta$, $y = b\sin^3\theta$ then show that $a\frac{dy}{dx} + b\tan\theta = 0$.

(6)

Q.6 Use numerical integration to find the approximate value of $\int_0^1 \frac{1}{\sqrt{x^2+1}} dx$ with $n = 4$.

(6)

Q.7 A random variable X has probability density function

(6)

$$f(x) = \begin{cases} c\sqrt{x}, & \text{if } 0 \leq x \leq 1, \\ 0, & \text{if otherwise.} \end{cases}$$

Find (i) Find c (ii) $P(\frac{1}{2} \leq x \leq 1)$.



UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program :Seventh Semester – 2020

Paper: Mathematics (Advance)

Course Code: BBA-403

Part – I (Compulsory)

Time: 30Min. Marks: 10

Roll No. in Fig.

Roll No. in Words.

Attempt this Paper on this Question Sheet only.

Division of marks is given in front of each question.

This Paper will be collected back after expiry of time limit mentioned above.

Signature of Supdt.:

Q.1. Encircle the right answer cutting and overwriting is not allowed. (10x1=10)

- (i) If $y = e^{\sin x}$ then $\frac{dy}{dx} = \dots$
 (a) $e^{\cos x}$ (b) $\cos x e^{\cos x}$ (c) $\cos x e^{\sin x}$ (d) None of these
- (ii) $\frac{d}{dx}(\cos^{-1} \frac{x}{a}) = \dots$
 (a) $\frac{1}{\sqrt{a^2 - x^2}}$ (b) $\frac{-1}{\sqrt{a^2 - x^2}}$ (c) $\frac{-a}{x\sqrt{a^2 - x^2}}$ (d) None of these
- (iii) If $y = a^{\sqrt{x}}$ then $\frac{dy}{dx} = \dots$
 (a) $a^{\sqrt{x}} \frac{\ln a}{2\sqrt{x}}$ (b) $a^{\frac{\ln a}{2\sqrt{x}}}$ (c) $a^{\sqrt{x}} \frac{1}{2\sqrt{x}}$ (d) None of these
- (iv) If $x^2 + y^2 = a^2$, then $y_2 = \dots$
 (a) $\frac{-a^2}{y^3}$ (b) $\frac{a^2}{y^3}$ (c) $\frac{-a^2}{y^2}$ (d) None of these
- (v) The value of $\int_1^2 (x^2 + 1)dx = \dots$
 (a) 10 (b) $\frac{8}{3}$ (c) $\frac{10}{3}$ (d) None of these
- (vi) The value of the integral $\int \cos 2x dx = \dots$
 (a) $\frac{1}{2} \cos 2x + c$ (b) $\frac{1}{2} \sin 2x + c$ (c) $\frac{-1}{2} \sin 2x + c$ (d) None of these
- (vii) The value of the integral $\int \sec^2 x dx = \dots$
 (a) $2 \sec x$ (b) $2 \tan x + c$ (c) $\tan x + c$ (d) None of these
- (viii) The value of $\int_a^a f(x) dx = \dots$
 (a) 0 (b) 1 (c) $f(b) - f(a)$ (d) None of these
- (ix) If $f(x) = 3x^2 + 4x + 7$ then the value of $f'(0)$ is equal to
 (a) 4 (b) 10 (c) 20 (d) None of these
- (x) The area between x -axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$ is equal to
 (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{10}{3}$ (d) None of these



ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Q.2. Solve the following:

(5x4=20)

- (i) If $y = e^{-x} \cos x$ find y' and y'' .
(ii) The revenue from selling x units of desks is $R(x) = 2000(1 - \frac{1}{x})$. Find the marginal revenue $R'(x)$ on selling x units of desks. Also find the marginal revenue $R'(x)$ on selling 9th desk.
(iii) Evaluate $\int \frac{1}{(1+x^2) \tan^{-1} x} dx$.
(iv) Find the area between the x -axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$.
(v) Use numerical integration with $n = 4$ to approximate $\int_0^4 \sqrt{x^2 + 1}$

Solve the following:

(5x6=30)

- Q.3 If $x = a \cos^3 \theta$, $y = b \sin^3 \theta$ then show that $a \frac{dy}{dx} + b \tan \theta = 0$.
Q.4 Solve the first order differential equation $(x^2 - yx^2) \frac{dy}{dx} + y^2(1 + x) = 0$.
Q.5 Evaluate the integral $\int \tan^4 x dx$.
Q.6 Use the method of least squares to find least squares linear regression line $y = ax + b$ for the data

x	1	3	4	6	7	8
y	4.5	5.8	6.1	6.9	7.4	7.8

Q.7 A random variable X has probability density function (p.d.f.)

$$f(x) = \begin{cases} kx, & \text{if } 0 \leq x \leq 2, \\ 0, & \text{if otherwise.} \end{cases}$$

Find (i) k (ii) $P(x \geq 1)$ (iii) $P(\frac{1}{2} \leq x \leq \frac{3}{2})$.



Q.1. Solve the following:

(5x6=30)

- (i) Find the first and second order derivatives of $y = (1 + \sqrt{\cos x})^2$ at $x = 0, \frac{\pi}{2}$.
- (ii) Then find the area between the curve $y = x(x-1)(x+1)$ and x-axis.
- (iii) Find the first order partial derivative of $f(x, y) = 4 + \sqrt{xy}$.
- (iv) Evaluate the integrals (i) $\int x \sin x dx$ (ii) $\int \sin^2 x \cos x dx$.
- (v) Use numerical integration with $n = 4$ to approximate $\int_0^3 \sqrt{3-x} dx$

Q.2. Solve the following:

(5x6=30)

- (a) If $y = e^{ax} \sin bx$ then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Also show that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$.
- (b) Evaluate the definite integrals $\int_0^1 (x^2 + x) dx$ and $\int_0^\pi \cos^2 x \sin x dx$.
- (c) Maximize $f(x, y) = xy$ subject to the constraint $x + y = 10$.
- (d) A random variable X has probability density function (p.d.f.)

$$f(x) = \begin{cases} k(1+x^2), & \text{if } 0 \leq x \leq 1, \\ 0, & \text{if otherwise.} \end{cases}$$

Find (i) k (ii) $P(0 \leq x \leq \frac{1}{2})$ (iii) $P(\frac{1}{2} \leq x \leq \frac{1}{3})$.

- (e) Use the method of least squares to find least squares linear regression line $y = ax + b$ for the data given below. Estimate the value of y when $x = 30$.

x	3	5	6	9	10	12	15	20	22	28
y	10	12	15	18	20	22	27	30	32	34

**THE ANSWERS MUST BE ATTEMPTED ON THE ANSWER SHEET PROVIDED****Q.1. Solve the following:****(5x6=30)**

- (i) If $y = (1 + \sqrt{\sin x})^2$ then find its first and second order derivatives at $x = 0, \frac{\pi}{4}$.
- (ii) Given the cost function $C(x) = ax^2 + bx + c$. Then find the actual expression for $C(x)$ by using conditions $C(0) = 0$, $C'(1) = 1$ and $C'(-1) = 1$.
- (iii) Evaluate the integral $\int x^2 \sin x dx$.
- (iv) Then find the area between the curve $y = \sin 2x$ and x -axis from 0 to $\frac{\pi}{3}$.
- (v) Maximize $f(x, y) = xy$ subject to the constraint $x + y = 12$.

Q.2. Solve the following:**(5x6=30)**

- (a) If $y = e^x \sin x$ then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Also show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.
- (b) If $x = a \cos \theta$, $y = a \sin \theta$ then first and second order derivatives w. r. t. x .
- (c) Evaluate the definite integrals $\int_1^2 (x^2 + 1)dx$ and $\int_0^{\frac{\pi}{6}} \cos^3 x dx$.
- (d) A random variable X has probability density function (p.d.f.)

$$f(x) = \begin{cases} kx(2-x), & \text{if } 0 \leq x \leq 2, \\ 0, & \text{if otherwise.} \end{cases}$$

Find (i) k (ii) $P(0 \leq x \leq 1)$ (iii) $P(1 \leq x \leq 2)$.

- (e) Use the method of least squares to find least squares linear regression line $y = ax + b$ for the data given below. Estimate the value of y when $x = 30$.

x	5	6	8	10	12	13	15	16	17
y	16	19	23	28	36	41	44	45	50



THE ANSWERS MUST BE ATTEMPTED ON THE ANSWER SHEET PROVIDED

Q.1. Solve the following:

(5x6=30)

- (i) Differentiate $y = x \cos y$ w.r.t x to find $\frac{dy}{dx}$.
- (ii) Evaluate the integrals $\int x \cos^2 x dx$.
- (iii) Let $f(x, y) = xy + \sqrt{xy}$. Find the first order partial derivative $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
- (iv) Find the area bounded the curve $y = 4 - x^2$ and x -axis.
- (v) Maximize $f(x, y) = xy$ subject to the constraint $x + y = 12$.

Q.2. Solve the following:

(5x6=30)

- (a) If $y = \tan(p \tan^{-1} x)$, then show that $(1 + x^2)y' - p(1 + y^2) = 0$.
- (b) If $x = a \cos \theta$, $y = a \sin \theta$ then find first and second order derivatives $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
- (c) Evaluate the definite integrals $\int_1^2 \frac{x}{x^2+1} dx$.
- (d) A random variable X has probability density function $f(x) = kx(1 - x)$, if $x \in [0, 1]$, $f(x) = 0$ if $x \notin [0, 1]$. Find k and $P(\frac{1}{3} \leq x \leq \frac{2}{3})$.
- (e) Estimate the linear regression line $y = ax + b$ by using the method of least squares for the data given below. Estimate the value of y when $x = 30$.

x	3	5	6	9	10	12	15	20	22	28
y	10	12	15	18	20	22	27	30	32	34