Chapter # 6

Q# ~~25~~ 25

With replacement:-

1. Both ball will be white

White balls = 4

Rea balls = 2

P (White) = ?

$$P(A) = \frac{n(A)}{n(S)} = \frac{\text{Total no. of favourable outcomes}}{\text{T. No.. OF outcomes in sample space}}$$

$n(S) = 4+2 = 6$

$$P(\text{White}) = \frac{\cancel{4}^{2}}{\cancel{6}_{3}} = \frac{2}{3}$$

$P(\text{both white}) = P(\text{White}) \cdot P(\text{white})$

$$= \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9} \quad \text{Ans}$$

2. Both ball will Same Colours:-

$P(\text{both red}) = P(\text{Red}) \cdot P(\text{Red})$

$$= \frac{2}{6} \cdot \frac{2}{6} = \frac{1}{9}$$

$P(\text{same colour}) = P(\text{both white}) + P(\text{B.Red})$

$$= \frac{4}{9} + \frac{1}{9} = \frac{5}{9}$$

3. That Both are different Colours:-

- First draw the white, second draw red.
- First draw red, second draw white

$P(\text{diff Colours}) = P(W) \cdot P(R) + P(R) + P(W)$

$= \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} \Rightarrow \frac{2}{9} + \frac{2}{9}$

$= \frac{4}{9}$

WITHOUT REPLACEMENT:

1. Both balls white:.

$P(\text{White}) = \frac{4}{6} = \frac{2}{3}$

$\Rightarrow$ (ek ball niklanay kay bad 5)

$P(\text{2nd w. ball}) = \frac{3}{5}$

$P(\text{Both white}) = \frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$

2. Both are same colours:.

$P(\text{White}) = \frac{4}{6} = \frac{2}{3}$

$P(\text{both white}) = \frac{2}{5}$

$P(\text{Red balls}) = \frac{2}{6} \cdot \frac{1}{5} = \frac{1}{15}$

$P(\text{same colours}) = \frac{2}{5} + \frac{1}{15} = \frac{7}{15}$

3. Both are different (one red, one white):-

$P(\text{d. colours}) = 1 - P(\text{same colours})$

$= 1 - \frac{7}{15} = \frac{8}{15}$

Q No# 26

(b) Total eggs = 12

Good eggs = 7

Rotten eggs = 5

(i) First 2 are goods and remaning 3 are rotten?

Good eggs $= P_1 = \frac{7}{12}$

$P_2 = \frac{6}{11}$ (one good egg is removed)

$P_3 = \frac{5}{10}$ (rotten eggs)

$P_4 = \frac{4}{9}$ (seond rotten)

$P_T = P_1 + P_2 + P_3 + P_4$

$= \frac{7}{12} + \frac{6}{11} + \frac{5}{10} + \frac{4}{9}$

Fifth egg rotten:-

$$P_5 = \frac{3}{8}$$

$$P_T = \frac{7.6.5.4.3}{12.11.10.9.8} = \frac{2520}{95040} = 0.2647$$

$P_T = 0.2647$ Answer

(ii) Any two are good and remaining three are rotten:-

Select two goods eggs out of 7.

$$^7C_2 = \frac{7!}{2!(7-2)!} = \frac{7!}{2!(5!)}$$

$$= \frac{7.6.\cancel{5!}}{2!.\cancel{5!}} = \frac{7.6}{2!} = 21$$

Total select eggs out of 12 is

$$^{12}C_5 = \frac{12!}{5!(12-5)!} = \frac{12!}{5!(7!)}$$

$$= \frac{12.11.10.9.8.\cancel{7!}}{5!\cancel{7!}} = \frac{12.11.10.9.8}{5.4.3.2.1}$$

$$^{12}C_5 = \frac{95040}{120} = 792 \text{ Ans}$$

Q# 27
(a)

Total no.of Students = 100
S' with defectied eyes = 60 = E
S with defected ears = 50 = F

1) Propabality of defected ears and defectiv eyes:

$|E \cup F| = |E| + |F| - |E \cap F|$

$|E \cup F| = 100$

$100 = 60 + 50 - |E \cap F|$

$|E \cap F| = 60 + 50 - 100 = 10$

2)

$P(\text{D.eyes but not D ears}) = |E| - |E \cap F|$

$= 60 - 10 = 50$

$P(A) = \frac{50}{100} = 0.5$

$P(\text{D.eyes and De. Ears}) = \frac{|E \cap F|}{100}$

$P(B) = \frac{10}{100}$

$P(B) = 0.1$

(b)

1. Probability that all five cards are red.

Red cards = 26 (13 hearts, 13 diamonds)

⇒ The probability of selecting a red card on each draw, without replacement, is calculated as:-

$\Rightarrow P(1) = \frac{26}{52}$ ∵ 52 = Total cards

$\Rightarrow P(2) = \frac{25}{51}$ when one card - (minus)

$\Rightarrow P(3) = \frac{24}{50}$ (already 2 cards been drawn)

$\Rightarrow P(\text{all red}) = \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} \times \frac{23}{49} \times \frac{22}{48}$

$P(\text{all red}) = \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} = 0.0517$

2. Probability that all five cards are diamonds:-

There are 13 diamonds:

$P(1) = \frac{13}{52}$, $P(2) = \frac{12}{51}$

$P(3) = \frac{11}{50}$, $P(4) = \frac{10}{49}$

$P(5) = \frac{9}{48}$

$P(\text{all diamonds}) = \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48}$

$= 0.0032$ Ans

Q NO: 28

(a) Tossing a fair coin twice

Event A = A head occurs on the first toss

Event B = The same face does not occurs on both tosses. {~~H H. HT~~}

Two events are indepent is:-

$P(A \cap B) = P(A) \times P(B)$ $\Rightarrow$ {H.H, HT}

$P(A) = \frac{1}{2}$ (A happness first toss is a head)

$P(B) = ?$ (B is happnes two toss)

(HH (both heads), (HT), (TH), (TT))

→ If Two toss so faces are (HT) on (TH), there are 2 outcomes out of 4 possible outcomes.

P

Q#28 (a) Fair coin tossed twice

Event A: A head occurs on the first toss. $A = \{H \cdot H \cdot HT\}$

Event B: The same face does not occurs on both tosses.

$B = \{HT, TH\}$

$S = \{HH, HT, TH, TT\}$

$P(\text{each outcomes}) = \frac{1}{4}$

$P(A) = \frac{\cancel{2}}{\cancel{4}_2}$, $(HH, HT) = A$

$P(A) = \frac{1}{2}$

$B = (HT, TH)$

$P(B) = \frac{2}{4}, \frac{1}{2}$

$A \cap B = HT$

$P(A \cap B) = \frac{1}{4}$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

(b) Two balls drawn without replacement:

Urn balls = 1, 2, 3. 4

Event A: The first ball drawn has a 1 on it.

$A = \{(1,2), (1,3), (1,4)\}$

Event B: The second ball drawn has a 1 on it.

$B = \{(2,1), (3,1), (4,1)\}$

$S = \{(1,2), (1,3), (1,4), (2,1). (3,1), (4,1)\}$

S = All possible outcomes

$S = \{(1,2)(1,3)(1,4)(2,1)(2,3)(2,4),(3,1)(3,2)$
$\cancel{(3,3)}(3,4),(4,1)(4,2)(4,3)\}$

$P(A) = \dfrac{3}{12}$ $\qquad$ $A = \{(1,2)(1,3)(1,4)\}$

$n(E) = 3, \quad n(S) = 12.$

$P(A) = \dfrac{\cancel{3}\,1}{\cancel{12}\,4} = \dfrac{1}{4}$

$P(B) = \dfrac{3}{12} = \dfrac{1}{4}$

$P(A \cap B) = 0$

$P(A) \cdot P(B) = \dfrac{1}{4} \cdot \dfrac{1}{4} = \dfrac{1}{16}$

(a) A and B are independent

(b) A and B are not independent

(Question # 29)

A = The event that the first die shows a, 1.
B = The event that the second die show a 6.
C = The event that the sum of two dice is 7.

$|S| = 6 \times 6 = 36$

$A = \{(1,1)(1,2)(1,3),(1,4),(1,5)(1,6)\}$

$A = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

$C \Rightarrow n(C) = 6, \quad n(S) = 36)$

$$P(C) = \frac{6}{36} = \frac{1}{16}$$

1. $P(A \cap B)$

The first die is 1 and the second is 6. $= \{1, 6\}$

$$P(A \cap B) = \frac{1}{36}$$

2. $P(A \cap C)$:

$(A \cap C)$ The first die is 1 and the sum is 7. $= (1, 6)$

$$P(A \cap C) = \frac{1}{36}$$

3. $$P(B \cap C) = \frac{1}{36}$$

4. $P(A \cap B \cap C)$ The first die is 1 the second die is 6 and their sum is 7. $(1, 6)$

$$P(A \cap B \cap C) = \frac{1}{36}$$

$$P(A) \cdot P(B) \cdot P(C) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$$

$P(A \cap B \cap C) \neq P(A) \cdot P(B) \cdot P(C)$ the events are not independent.

Q#31 (a) The probability of getting a head from a coin toss is $\frac{1}{2}$

The probability of nüber less then 5 $(1, 2, 3, 4)$ from a die roll is $\frac{4}{6} = \frac{2}{3}$

$$P(A) = \frac{1}{2}, \quad P(B) = \frac{2}{3}$$

$$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$

(b)

$$P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{3}$$

$$P(C) = \frac{1}{4}$$

$$P(A' \cap B' \cap C') = P(A') \cdot P(B') \cdot P(C')$$

$$= \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{4}\right)$$

$$= \left(\frac{1}{2}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{3}{4}\right) = \frac{6}{24} = \frac{1}{4}$$

The probability that at least one student solves the problem is

$$1 - P(A' \cap B' \cap C') = 1 - \frac{1}{4} = \frac{3}{4}$$

Question # 32

(a)

4 Kings in a deck of 52 cards.

(i) First Card Replaced.

Kings = 4

first King picking = $\frac{4}{52}$

$P(K) = \frac{4}{52}$

After replacement $P(Q) = \frac{4}{52}$

(4 Queen in a deck)

$P(K) \cdot P(Q) = \frac{4}{52} \cdot \frac{4}{52} \Rightarrow = \frac{16}{2704}$

We could pick a queen first then a King

$= 2 \times \frac{16}{2704} = \frac{32}{2704}$

$= \frac{1}{85}$

(ii) Not Replaced:-

$P(K_1) = \frac{4}{52}$ (after picking a King there are 51 cards left and 4 Queen)

$P(Q_2) = \frac{4}{51}$

$P(K) \cdot P(Q) = \frac{4}{52} \times \frac{4}{51} = \frac{16}{2652}$

Total probability $= \frac{16}{2652} + \frac{16}{2652}$

$P(\text{King then queen}) = \cancel{P(Q)} + \cancel{P(Q)}$

$P(K) \cdot P(Q) = \frac{16}{2652}$

$P(\text{Queen then King}) = P(Q) \cdot P(K)$

$= \frac{16}{2652}$

$P(\text{total}) = \frac{16}{2652} + \frac{16}{(2652)} = \frac{32}{2652}$

$= \frac{8}{663}$

(b)

Total num of people = 10+8+9+6

$T = 33$

$^{33}C_6 = \frac{33!}{6!(33-6)!} = \frac{33!}{6!(27)!}$

$= \frac{33!}{6!\,27!} = \frac{33.32.31.30.29.28.\cancel{27!}}{6!\,\cancel{27!}}$

$= \frac{33.32.31.30.29.28}{6.5.4.3.2.1} = 1,107568$

(i) 2 = professer from 10

$^{10}C_2 = \frac{10!}{2!(10-2)!} = \frac{10.9.\cancel{8!}}{2!\,\cancel{8!}}$

$= \frac{10.9}{2.1} = \frac{\cancel{90}^{45}}{\cancel{2}} = 45$

2 = businessman from 8

$^8C_2 = \Rightarrow 28$

1 = Doctor from 6

$^6C_1 = 6$

1 = lawyer from 9

$^9C_1 = 9$

$= 45 \times 25 \times 6 \times 9$

$= 68040$

So, $= \dfrac{\text{Num of ways to specific commuity}}{\text{Total num of ways}}$

$= \dfrac{68040}{1.107568} = 0.0614$

(ii) No professer:-

Committe of 6 from the non-professers $= 8+9+6 = 23$

$^{23}C_6 = 100947$

$= \dfrac{\text{No. of ways to select of commity}}{\text{Total num select any committe}}$

$= \dfrac{100947}{1.107568} = 0.0911$

(iii) At least one Professer.

P = 1 - no professers

$= 1 - 0.0911$

$= 0.9089$ Ans

Q #35

Total cards = 80

1 to 80 between

1, 4, 9, 16, 25, 36, 49, 64

There are 8 perfect squares.

Probabality = $\frac{\text{num of perfect square}}{\text{Total no of cards}}$

$= \frac{8}{80} = \frac{1}{10}$

$= 0.1$ Answer

Question # 36

(a)

Three item from tweenty

$= {}^{20}C_3 = \frac{20!}{3!\,7!}$

${}^{20}C_3 = 1140 = n(S)$

If there is no faulty items

= faulty items - ~~no faulty items~~

$= 3 - 20 = 20 - 3$

$= 17$ no faulty items.

${}^{17}C_3 = \frac{17!}{3!\,14!} = 686$

If At least one faulty

= No. of ways − no-faulty items

= 1140 − 680

$n(A) = 460$

probability $= \frac{460}{1140} = \frac{23}{57}$

(b)

Aprial = 30 days

1st F is not born same day as the host 29/30

2nd friend is not born on the same day at host is 29/30.

probability of 5 friends are

$= \left(\frac{29}{30}\right)^5$

$= 0.9836$

Question # 37

INDEPENDEN EVENTS:-

These events are where the occurance of one event does't affect the probability of the occurance of another events.

Mutually exclusiv events are events where the occurance of one event prevents the occurance of another event. They can't happen at the same time.

Question # 40

(C)

Total light bulbs = 15
Defective bulbs = 5
Non-defective bulbs = 15 - 5 = 10
No. of bulb chosen = 3

$$^{15}C_3 = \frac{15!}{3!(15-3)!} = \frac{15!}{3!\,12!}$$

$$^{15}C_3 = \frac{15.14.13}{3.2.1} = 455 = n(S)$$

(a) None is Defective:

To have no defective bulb, all 3 three bulbs must come from 10.

$$^{10}C_3 = \frac{10!}{3!(10-3)!} = \frac{10.9.8}{3.2.1}$$

$^{10}C_3 = 120 \Rightarrow n(A)$

$P(\text{none is defective}) = \frac{120}{455} = \frac{n(A)}{n(S)}$

$P(n.D) = \frac{24}{91}$

(b) At least one is defective

$P(\text{one.D}) = 1 - P(\text{non-D})$

$= 1 - \frac{24}{91} = 1 - \frac{24}{91}$

$= \frac{91-24}{91} = \frac{67}{91}$

$P(\text{one.De}) = \frac{67}{91}$ Answer.

Question # 41

(i) One is an Ace and the other a Jack:-

Total no. of Cards = 52

No. of Aces = 4

No. of Jacks = 4

choose 1 Ace = $^4C_1 = 4$

$^4C_1 = \frac{4!}{1!(4-1)!} = \frac{4.\cancel{3!}}{1!(\cancel{3!})} = 4$

Choose 1 Jack $= {}^4C_1 = 4$

Total outcomes $= 4 \times 4$

$n(A) = 16$

$\Rightarrow$ total num. of ways to draw 2 cards from the deck is

$${}^{52}C_2 = \frac{52!}{2!(52-2)!} = \frac{52!}{2!\,50!} =$$

$$= \frac{52 \cdot 51 \cdot \cancel{50!}}{2!\,\cancel{50!}} = 1326 = n(S)$$

$$P(\text{one Ace and one Jack}) = \frac{n(A)}{n(S)} = \frac{16}{1326}$$

$$= \frac{8}{663}$$

(ii) First is an Ace an the other is a Jack:

$$P(\text{First is an Ace}) = \frac{4}{52}$$

after drawing 1 ace 51 cards left including 4 Jacks.

$$P(\text{second card is Jack}) = \frac{4}{51}$$

$$P(\text{1st Ace. 2nd Jack}) = \frac{4}{52} \times \frac{4}{51}$$

$$\frac{2652}{4} = 663$$

Question # 42

(b)

Choosing three distint integers from first 15 positive integers:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
13, 14, 15

odd num = 7
even num = 8

(i) Sum is odd:

We can select 1 or 3.

⇒ choosing 1 odd num

$^7C_1 = 7$ ways

⇒ Choos 2 even num

$^8C_2 = 28$ ways

$7 \times 28 = 196$

Choosing 3 odd numbers:

$^7C_3 = 35$ ways

$196 + 35 = 231$

$$^{15}C_3 = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} = 455$$

→ Probability that sum is odd

$= \frac{431}{455} = \frac{21}{41}$

(ii) PRODUCT IS ODD:

3 odd num from the 7 available odd num

${}^7C_3 = \frac{7!}{3!(7-3)!} = 35$

$\frac{35}{455} = \frac{1}{13}$

(iii) PRODUCT IS EVEN:

The product is even if

$455 - 56 = 399$

$P(\text{Product even}) = \frac{399}{455}$

(iv) 'THEIR SUM IS EVEN'

odd num 8 from choosing 2.
There are 7 and choose 1.

${}^8C_2 \times {}^7C_1 = \frac{8 \times 7}{2 \times 1} = 7$

$= 28 \times 7 = 196$

$P(\text{sum is even}) = \frac{196}{455}$

Question # 42

(C) WINNING THE GAME WHEN ROLLIN A DIE:-

You win if outcome is either even or divisible by 3.

outcomes are = {1, 2, 3, 4, 5, 6}

⇒ Even Numbers = (3, 4, 6) 3 outcomes

⇒ Divisible by 3 = (3, 6) 2 " "

The num 6 is both even and divisible by 3, we not double count.

2, 3, 4 and 6

$$P(\text{win}) = \frac{4}{6} = \frac{2}{3}$$

Question # 41