Assignment:

Govt. Post Graduate College
for women

BS Islamiyat B3-S2

Subject:
Quantitative Reasoning I
(GQR-101)

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Factorial notation:

"Factorial notation is a mathemati
notation used to represent the prod
of all positive integers from 1 up
to a given number"

It denoted by ! mark

$$n! = n \times (n-1) \times (n-2) \times \dots 321.$$

:- Exercise :-

1. $4!$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

2. $6!$

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 =$

3. $\dfrac{8!}{7!}$

$= \dfrac{8 \cdot \cancel{7!}}{\cancel{7!}} = 8$

4. $\dfrac{10!}{7!}$

$= \dfrac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}} = 720$

5. $\dfrac{11!}{4! \times 7!}$

$= \dfrac{11 \cdot 10 \cdot \overset{3}{\cancel{9}} \cdot \overset{2}{\cancel{8}} \cdot \cancel{7!}}{4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{7!}} = 11 \cdot 10 \cdot 3 = 330$

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6 $\frac{8!}{4! \times 2!}$

$= \frac{8 \cdot 7 \cdot \cancel{6}^{3} \cdot 5 \cdot \cancel{4!}}{\cancel{4!} \cdot \cancel{2} \cdot 1}$

$= 8 \cdot 7 \cdot 3 \cdot 5 = 840$

7 $\frac{3!}{0!}$

$= \frac{3 \cdot 2 \cdot 1}{1} = 6$

8 $\frac{11!}{2! \, 4! \, 5!}$

$= \frac{11 \cdot 10 \cdot \cancel{9}^{3} \cdot \cancel{8}^{4} \cdot 7 \cdot \cancel{6}^{2} \cdot \cancel{5!}}{\cancel{2} \cdot 1 \cdot \cancel{4} \cdot 3 \cdot \cancel{2} \cdot 1 \cdot \cancel{5!}}$

$11 \cdot 10 \cdot 9 \cdot 7 = 6930$

9 $\frac{9!}{2!(9-2)!}$

$= \frac{9 \cdot 8 \cdot 7!}{2!(7)!}$

$= \frac{9 \cdot \cancel{8}^{4} \cdot \cancel{7!}}{\cancel{2} \cdot 1 \cdot \cancel{7!}}$

$9 \cdot 4 = 36$

10 $\frac{15!}{15!(15-15)!}$

$= \frac{15!}{\cancel{15!}(0)!}$

$= \frac{1}{1 \cdot 1} = 1$

11 $\frac{6!}{3! \, 3!}$

$= \frac{\cancel{6}^{2} \cdot 5 \cdot \cancel{4}^{2} \cdot \cancel{3!}}{\cancel{3} \cdot 2 \cdot 1 \cdot \cancel{3!}}$

$= 2 \cdot 5 \cdot 2 = 20$

12 $4! \cdot 0! \cdot 1!$

$4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 24$

each following factorial form

1 $6 \cdot 5 \cdot 4$

$6!$

2 $\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$

$\frac{8!}{3!}$

3 $12 \cdot 11 \cdot 10$

$12!$

5 $20 \cdot 19 \cdot 18 \cdot 17$

$20!$

4 $\frac{52 \cdot 51 \cdot 50 \cdot 49}{4 \cdot 3 \cdot 2 \cdot 1}$

$\frac{52!}{4!}$

6 $\frac{10 \cdot 9}{2 \cdot 1}$

$\frac{10!}{2!}$

8 $n(n-1)(n-2)$

$n!$

7 $(n+2)(n+1)(n)$

$\frac{(n+1)!}{(n-1)!}$

Permutation:

"The arrangement of finite number of object one or more at a time is called Permutation."

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

Exercise :-

$^{n}P_r = \frac{n!}{(n-r)!}$

1. $^{20}P_3$

$^{20}P_3 = \frac{20!}{(20-3)!} = \frac{20!}{17!} = \frac{20\cdot19\cdot18\cdot\cancel{17!}}{\cancel{17!}}$

$= 20\cdot19\cdot18 = 6840$

2. $^{16}P_4$

$^{16}P_4 = \frac{16!}{(16-4)!} = \frac{16\cdot15\cdot14\cdot13\cdot\cancel{12!}}{\cancel{12!}}$

$= 16\cdot15\cdot14\cdot13 = 43680$

3. $^{12}P_5$

$^{12}P_5 = \frac{12!}{(12-5!)} = \frac{12\cdot11\cdot10\cdot9\cdot8\cdot\cancel{7!}}{\cancel{7!}}$

$= 12\cdot11\cdot10\cdot9\cdot8 = 95040$

4. $^{10}P_7$

$^{10}P_7 = \frac{10!}{(10-7!)} = \frac{10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot\cancel{3!}}{\cancel{3!}}$

$= 10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4 = 604800$

5. $^{9}P_8$

$^{9}P_8 = \frac{9!}{(9-8)!} = \frac{9\cdot8\cdot7\cdot6\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot\cancel{1!}}{\cancel{1!}}$

$9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2 = 362880$

CamScanner

Find the value n.

1. ${}^{11}P_n = 11\cdot10\cdot9$

$$\frac{11!}{(11-n)!} = 11\cdot10\cdot9$$

$$= \frac{11!}{(11-n)!} = 990 \Rightarrow (11-n)! = \frac{11!}{11\cdot10}$$

$$= (11-n)! = 8!$$

$$11-n = 8 \Rightarrow 11-8 \Rightarrow n=3$$

2. ${}^{n}P_4 : {}^{n-1}P_3 = 9:1$

$$\frac{{}^{n}P_4}{{}^{n-1}P_3} = \frac{9}{1} \Rightarrow \frac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-1-3)}} = 9$$

$$\frac{n!}{(n-4)!} \cdot \frac{(n-4)!}{(n-1)!} = 9$$

$$= \frac{n!}{(n-1)!} = 9 \Rightarrow \frac{n(n-1)!}{(n-1)!} = 9$$

$$n = 9$$

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* How many signals can be given by 5 flags of different colours, using 3 flags at a time?

Total number of flags = 5
Number of signals using = 3 flags
$= {}^5P_3$

* How many signals can be given by 6 flags of different colours when any number of them are used at a time

Total number of flags = 6

$\frac{6!}{n!} = \frac{6 \cdot 5 \cdot 4!}{4!} = 6 \cdot 5 = 30$

Number of signal using one flage $= {}^6P_1$

$= \frac{6!}{(6-1)!} = \frac{6 \cdot 5!}{5!} = 6$

Number of signal 2 flags $= {}^6P_2$

$= \frac{6!}{(6-2)!} = \frac{6 \cdot 5 \cdot 4!}{4!} = 6 \cdot 5 = 30$

Number of signal 3 flags $= {}^6P_3$

$\frac{6!}{(6-3)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 6 \cdot 5 \cdot 4 = 120$

Number of signal 4 flage = $^6P_4$

$\frac{6!}{(6-4)!} \quad \frac{6\cdot5\cdot4\cdot3\cdot2!}{2!} = 6\cdot5\cdot4\cdot3 = 360$

Number of signal 5 flage = $^6P_5$

$\frac{6!}{(6-5)!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1.}{1!.} = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$

Number of signal 6 flags = $^6P_6$

$\frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{6!}{1} \quad [\because 0! = 1]$

$6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$

Total number of signals

$6+30+120+360+720+720 = 1956.$

★ How many words can be formed from the letter of the following words using all letter when no letter is to be repeated

1. PLANE 2. OBJECT 3. FASTING

⇒ PLANE $^5P_5$

$\frac{5!}{(5-5)!} = \frac{5!}{0!} = 5\cdot4\cdot3\cdot2\cdot1 = 120$

⇒ OBJECT

$\frac{6!}{(6-6)!} = \frac{6!}{0!} = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$

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⇒ FASTING

$$^7P_7 = \frac{7!}{(7-7)!} = \frac{7!}{0!} = 7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = 5040$$

* How many 3-digit numbers can be formed by using each one of the digits 2, 3, 5, 7, 9 only once?

Total number of digits = 5

$$^5P_3 = \frac{5!}{(5-3)!} = \frac{5\cdot4\cdot3\cdot\cancel{2!}}{\cancel{2!}} = 5\cdot4\cdot3 = 60$$

* The Governor of the Punjab call a meeting of 12 officers In how many ways can they seated at a round table?

12 officers can be seated around the round table in 11! ways

11.10.9.8.7.6.5.4.3.2.1 = 39916800 ways

* The D.C.Os of 10 districts meets to discuss the law and order situation in their districts In how many ways can they seated at a round table when Two particular D.C.Os insist on sitting Together?

9 ways

Date:

$= 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1 =$ [illegible]

* 11 members of a cube form 4 committes of 3, 4, 3, 2, 2 members so than one committed. Find the number of committeed.

Total members = 11

3, 4, 2, 2 members of 4 committeed and not repeated.

$$= \frac{11!}{3!\,4!\,2!\,2!}$$

$$= \frac{11 \cdot \cancel{10}^{5} \cdot \cancel{9}^{3} \cdot \cancel{8}^{4} \cdot 7 \cdot \cancel{6}^{3} \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{2} \cdot 1 \cdot \cancel{2} \cdot \cancel{1}}$$

$11 \cdot 5 \cdot 3 \cdot 4 \cdot 3 \cdot 7 \cdot 5 = 69300$

Find the numbers of ways in which 5 men and 5 women can be seated at a round table in such a way that no Two persons of the same sex sit together.

Let us first seat the mean at a round table. We fix the seat of one man the remaining 4 of them can be steated in 4! ways. Now we can seat 5 women one between Two of the men in 5! ways.

Thus 5 men and 5 women can be alternately seated at a round table in $4! \times 5!$

$= 4\cdot3\cdot2\cdot1 \cdot 5\cdot4\cdot3\cdot2\cdot1$

$= 2880$ ways

Fatima invites 14 people to a dinner There are 9 males and 5 females who are seated at two different tables so that guests of one sex sit at one round table and the guest of the other sex at the second table Find the number of ways in which all guests are seated.

9 males can be seated at a round table in $8!$ ways

5 females can be seated at a round round table in $4!$ ways

Both males and females can be seated in $8! \times 4!$ ways.

$= 8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1\cdot4\cdot3\cdot2\cdot1$

$= 967680$ ways.

Combination

An arrangement of n-differ object taken "r" at a time with any order is called combination

$$^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

:- Exercise :-

$^{12}C_{3}$

$$\frac{12!}{3!\,(12-3)!} = \frac{\overset{4}{\cancel{12}} \cdot 11 \cdot \overset{5}{\cancel{10}} \cdot \cancel{9!}}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{9!}}$$

$$4 \cdot 11 \cdot 5 = 220$$

$^{20}C_{17}$

$$\frac{20!}{17!\,(20-17)!} = \frac{\overset{10}{\cancel{20}} \cdot 19 \cdot \overset{6}{\cancel{18}} \cdot \cancel{17!}}{\cancel{17!} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$$

$$10 \cdot 19 \cdot 6 = 1140$$

$^{n}C_{4}$

$$\frac{n!}{4!\,(n-4)!} = \frac{n(n-1)(n-2)(n-3)\cancel{(n-4)!}}{4 \cdot 3 \cdot 2 \cdot 1\,\cancel{(n-4)!}}$$

$$\frac{n(n-1)(n-2)(n-3)}{24}$$

$$nC_{10} = \frac{12 \times 11}{2!}$$

$$^nC_{10} = \frac{12\cdot 11}{2!} = \frac{12\cdot 11\cdot 10!}{2! \times 10!} = \frac{12!}{2!(12-2)}$$

$$^nC_{10} = {}^{12}C_2$$

$$^nC_{n-10} = {}^{12}C_2$$

$$n - 10 = 2 \Rightarrow n = 12$$

$$^5C_2$$

$$\frac{5!}{2!(5-2)!} = \frac{5\cdot \cancel{4}^{2}\cdot \cancel{3!}}{\cancel{2}\cdot 1\cdot \cancel{3!}} = 5\cdot 2 = 10$$

$$^6C_3$$

$$\frac{6!}{3!(6-3)} = \frac{\cancel{6}^{2}\cdot 5\cdot \cancel{4}^{2}\cdot \cancel{3!}}{\cancel{3!} \times \cancel{3}\cdot \cancel{2}_{1}\cdot 1} = 2\cdot 5\cdot 2 = 20$$

$$^{16}C_{11}$$

$$\frac{16!}{11!(16-11)!} = \frac{\cancel{16}^{4}\cdot \cancel{15}^{3}\cdot \cancel{14}^{7}\cdot 13\cdot \cancel{11!}}{\cancel{11!}\,\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} \quad 4\cdot 7\cdot 13 = 364$$

$$^{17}C_{11}$$

$$\frac{17!}{11!(17-11)!} = \frac{17\cdot \cancel{16}^{4}\cdot \cancel{15}^{5}\cdot 14\cdot 13\cdot \cancel{12}^{8}\cdot \cancel{11!}}{\cancel{11!}\,\cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = 17\cdot 4\cdot 14\cdot 13 = 12376$$

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How many (a) diagonals and (b) triangles can be formed by joining the vertices of the polygon having.

① 5 sides ② 8 sides ③ 12 sides

(a) Diagonals

Number of diagonals of 5 sided polygon:

$$^5C_2 - 5 = \frac{5!}{(5-2)!\,2!} - 5 = \frac{5\cdot4\cdot3!}{3!\cdot2!} - 5$$

$$= \frac{5\cdot4}{2\cdot1} - 5 = \frac{\cancel{20}^{10}}{\cancel{2}} - 5 = 10 - 5 = 5$$

Number of 8 sided polygon

$$= {}^8C_2 - 8 = \frac{8!}{2!\,(8-2)!} = \frac{8\cdot7\cdot6!}{2!\,6!} - 8 = \frac{8\cdot7}{2\cdot1} - 8$$

$$= \frac{\cancel{56}^{28}}{\cancel{2}} - 8 = 28 - 8 = 20$$

Number 12 sided polygon $^{12}C_2 - 12$

$$= \frac{12!}{(12-2)!\,2!} - 12 = \frac{12\cdot11\cdot10!}{10!\,2!} - 12 = \frac{12\cdot11}{2\cdot1} = \frac{\cancel{132}^{66}}{\cancel{2}} -$$

$$66 - 12 = 54$$

(b) triangles

$^5C_3$

$= \frac{5!}{(5-3)!\,3!} = \frac{5\cdot4\cdot3!}{2!\,3!} = \frac{\overset{10}{\cancel{20}}}{\cancel{2}} = 10$

$^8C_3$

$= \frac{8!}{(8-2)!\,2!} = \frac{8\cdot7\cdot6!}{6!\,2\cdot1} = \frac{8\cdot7}{2} = \frac{\overset{28}{\cancel{56}}}{\cancel{2}} = 28$

$^{12}C_3$

$= \frac{12!}{(12-3)!\,3!} = \frac{\overset{4}{\cancel{12}}\cdot11\cdot10\cdot\cancel{9!}}{\cancel{9!}\,\cancel{3}\cdot2\cdot1} = \frac{4\cdot11\cdot10}{2} = \frac{\overset{220}{\cancel{440}}}{\cancel{2}} = 220$

The members of a cub are 12 boys and 8 girls In how many ways can a committeed of 3 boys and 2 girls be formed

$^{12}C_3$ ways $\times$ $^8C_2$ ways

$= \frac{12!}{3!\,(12-3)!} \times \frac{8!}{2!\,(8-2)!}$

$= \frac{12\cdot11\cdot10\cdot9!}{3\cdot2\cdot1\cdot9!} \times \frac{8\cdot7\cdot6!}{2\cdot1\cdot6!}$

$$\frac{\cancel{12}^{4} \cdot 11 \cdot 10}{\cancel{3} \cdot 2 \cdot 1} \times \frac{8 \cdot 7}{2}$$

$$= \frac{\cancel{440}}{\cancel{2}} \times \frac{\cancel{56}}{\cancel{2}}$$

$$= 220 \times 28 = 6160$$

How many committeed of 5 members can be chosen from a group 8 person when each committeed must include 2 particular persons?

We have select 3 members out of 6 person and this can be done in ${}^{6}C_3$ ways.

$$\frac{6!}{(6-3)!\,3!} \quad \frac{\cancel{6}^{2} \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 5 \cdot 4 = 20$$

Circular permutation

Circular permutation refer to arrangement of object in a circle where the position of object are considered equivalent if one can be rotated into another

$P = (n-1)!$

$P \cdot (n-1)!$

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Exercise:

Find the number of different ways that a family of 6 can be seated around a circular Table with 6 chairs.

$P = (n-1)!$
$= (6-1)!$
$= 5!$
$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 120$ ways.

In how many ways can you arrange 5 different colored beads in a bracelets?

$P = \frac{(n-1)!}{2}$

$= \frac{(5-1)!}{2}$

$= \frac{4!}{2}$ $= 4 \cdot 3 \cdot 1 = 12$ ways

$= \frac{4 \cdot 3 \cdot \cancel{2} \cdot 1}{\cancel{2}}$

Date: __/__/20__ Day: ____

There are 8 people in a dinner gathering In how many ways can the host (one of the 8) arrange his guests around a dining table if

(a) they can sit on any of the chairs?

(b) 3 people insist on sitting beside each other

(c) 2 people refuse to sit beside each other

$n = 8$

(a) $P = (n-1)!$

$= (8-1)!$

$= 7!$

$= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 5040$

$5 + 1 = 6$

(b) $P = (6-1)! \times 3!$

$= 5! \times 3!$

$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1$

$= 720$

(c) $= (7-1)!\ 2!$

$= 6! \cdot 2!$

$= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1$

$= 1440$

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How many ways can 7 people be seated at a round table if 3 people refuse to sit next to each other.

$P = (n-1)$

$P = (7-1)! = 6!$

Now, let us assume these there particular people always sit together

A B C D $\underbrace{EFG}_{3}$

$(5-1)!\ 3!$

$(4)!\ 3!$

So therefore,

$= 6! - (4)!(3)!$

$= 720 - 144$

$= 576$ ways

How many ways can 4 people sit around a circular table?

$n = 4$

$(n-1)! = (4-1)!$

$3! = 3.2.1 = 6$

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How many way can 4 boys 2 girls be seated at a round table?

$n=6$

$P=(n-1)$

$P=(6-1)$

$=5!$

$=5\cdot4\cdot3\cdot2\cdot1$

$=120$

How many way can 3 girls and 5 boys be seated around a circular table if all girls must sit together?

3 girls = 1 group

1 group seat and 5 seats

$n=6$

$=(n-1)!=(6-1)!$

3 girls $=3!$

$=(6-1)!\cdot3!$

$=5!\cdot3!$

$=5\cdot4\cdot3\cdot2\cdot1\cdot3\cdot2\cdot1$

$=720$

How many ways can 5 girls and 5 boys be seated at a round table if No restriction

$P = (n-1)!$

$P = (10-1)!$

$= 9!$

$= 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 362,880$ ways

A circular table has 5 chairs How many different seating arrangements are possible for 5 people?

$P = (n-1)!$

$P = (5-1)!$

$= 4!$

$= 4 \cdot 3 \cdot 2 \cdot 1$

$= 24$

A music band has 6 members They want to stand in a circular formation on stage. How many different arrangement are possible?

$P = (n-1)!$

$(6-1)!$

$= 5!$

$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 120$

A company has 8 employees They want to arrange themselves in a circular formation for a team photo. How many different arrangement are possible?

$P = (n-1)!$

$= (8-1)!$

$= 7!$

$= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 5040$

A circular track has 10 lanes How many different arrangement are possible for 10 athletes running in a circular formation.

$P = (n-1)$

$= (10-1)!$

$= 9!$

$= 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 362880$

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A circular table has 9 chair. How many different seating arrangement are possible for 9 people if 3 of the people ~~for 9 p~~ are family and want to sit together?

$$\frac{(9-1)}{(3-1)} = \frac{8!}{2!} = \frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot\cancel{2}\cdot1)}{\cancel{2}\cdot1}$$

$= 20160$

A group of 7 friend want to sit around a circular table. However 2 of the friends are rivals and cannot sit next to each other. How many different seating arrangement are possible?

$= (7-1)! - (2\times5!)$

$= 6! - 240$

$= 360$

$=$

A circular track has 12 lanes How many different arrangements are possible for 12 athletes running in a circular formation if 4 of the athlethes are from the same team and want to run together?

$$= \frac{(12-1)!}{(4-1)!}$$

$$= \frac{11!}{3!}$$

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{\cancel{3 \cdot 2 \cdot 1}}$$

$$= 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$$

$$= 1320$$