A quaternion-indicative solution for null points illustrating negative low-energy states for the electron

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Abstract

This letter proposes a mathematical hypothesis for null points in an electrical circuital arrangement under the influence of magnetic field induction. The formalism is used a means to describe uniquely coupled fields that manifest a planar waves perpendicular to the angle of propagation. The quaternion method for computing the longitudinal wave is supported by the differential method. The solution indicates the circuital arrangement satisfies Dirac's contention of negative-energy electron states where charges are posited at their lowest energy at minimal velocity.

The circuital model

Consider the circuital configuration shown in Fig.1 of the case of two solenoids: The first consists of a sinusoidal winding at a given angle between successive turns along its length; the second consists of a single strand of wire affixed in such a manner as to have regular divisions cross along the length. The first coil, the primary, has leads on each end while the second coil, the secondary, has both leads toward one end while at its extremity, loops back.

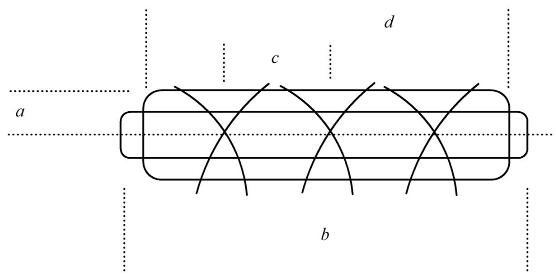

Fig.1. Circuital approximation of coupled solenoids.

The approximation consists of a primary coil tightly wound with wire of a radius much smaller than its length such that the distance between turns is nearly zero and that the angle, $\theta = 30^0$. Connections to each end of the primary coil extend outward so they can be connected to external equipment and components. The secondary coil is wound with a wide spacing between turns: The first traversal along the length coincides with the center line of the structure; the second traversal loops back along the length crossing the same center line such that there are a series of crossed wires, as shown in Fig.1. Connections to the same end of the secondary coil extend outward from the coil where they can be connected to external equipment and components.

A series of experiments are conducted to illustrate the contention of electromagnetic phenomena at the interface of the secondary coil as sinusoidal currents are applied to the primary, following the mathematical analysis. The experiment strives to answer the following questions: Due to Faraday, as the emission of energy from a primary-secondary sinusoidal coil is well-known, what is the case when the secondary has such a winding? When a continuous electromagnetic signal is applied to the primary, what is the character of the electromagnetic wave induced in the secondary? What kind of special character, if any, would the secondary exhibit patterns that have not been accounted for in Faraday's research but that which might be still covered by Maxwell? Does the wave exhibit this character because of the influence upon electrons moving in contradictory motion at the crossing of the wire in the secondary? Can the mathematical framework of this special character be described by ordinary vector analysis or it is necessary to call upon other analytical tools from the period? Can the analysis yield the contention that there are observable null points in this circuital configuration?

The mathematical model

In consideration of the circuital arrangement described in the previous section, the applied electromagnetic wave at the primary, via a signal generator in the range of 2 - 9MHz, is imagined to consist as a plane wave of the general form of the three-dimensional wave equation

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}=0. \tag{1}$$

The wave equation can be derived from Maxwell's equations in free space and in the absence of charges and currents, beginning with Faraday's equation. The spatial frequency of the plane wave is given by

$$\begin{aligned} E(x,z) &= \exp\left\{i\frac{2\pi}{\lambda}\left(x\sin\theta+z\cos\theta\right)\right\} \\ &= \exp\left\{i2\pi\left(\frac{x}{\Lambda}\right)\right\}, \end{aligned} \tag{2}$$

where,

$$\Lambda \approx \frac{\lambda}{\sin\theta}. \tag{3}$$

As the wave induced in the secondary is similar in character to the one in the primary, there is a minimum of superpositioning where,

$$a\cos\left(k_1 z-\omega_1 t\right)\approx a\cos\left(k_2 z-\omega_2 t\right). \tag{4}$$

Given the rather ordinary treatment, what can be said about the result of these waves interacting? Let's examine the arrangement without the "scaffolding" Ampère introduced into the process. Given Tait [1] (§453-456) the equation of coaxial circular currents [2, 3] is

$$\frac{ii'dsds'}{r^2}\left(\sin\theta\sin\theta'\cos\omega+k\cos\theta\cos\theta'\right), \tag{5}$$

and given the statement by Ampère that: "No closed circuit can set in motion an element of a circular conductor about an axis through the center of the circle and perpendicular to its plane," in consideration of the three independent variables expressed in (1) and given form in (2), the total effect of the closed circuit is zero, a complete differential form. As such and given by Tait [1] (§455) any element of the primary circuit, α, or the secondary circuit, α', where a vector connects the two at their midpoints, as described by Ampère, is the quaternion

$$-\frac{2C\alpha\alpha_1 U\alpha}{(T\alpha)^{\frac{1}{2}}} dd_1 (T\alpha)^{\frac{1}{2}}, \tag{6}$$

where U is the unit-vector of α, T the tensor of α, d, d_1 the differentials of the primary and secondary circuital expressions accordingly, and C a constant, namely the magnetic permeability, μ_0. The expression given in (6) is the same expression as (5) without the "scaffolding" in place. Now it is possible to intuitively construct relationship between the plane wave traveling in the primary and the induced plane wave in the secondary by stating that a singular plane wave is represented in completion by a quaternion of the form, the primary,

$$q_1 = w + ix + jy + kz, \tag{7}$$

and the secondary,

$$q_2 = w' + ix' + jy' + kz', \tag{8}$$

where the norm of either is $w^2 + x^2 + y^2 + z^2$ and if any or all of the components be imaginary, as in the algebraic sense of $\sqrt{-1}$, the norm becomes $w^2 + x^2 + y^2 + z^2 = 0$. In such cases as in the one illustrated by the circuit, the simplification of the primary to the secondary is expressed as $Aq + q_1 B = C$. For consistency, $aqb = c$ where a is the primary, b the secondary, and c the solution consequential to their product. If any of these are not null, the solution is $q = a^{-1}cb^{-1}$, however, if some are null, given a and c to be null, then it yields $aq = cb^{-1}$. A peculiarity arises in the equation $aq = c$ when both a and c are null, the solution is indeterminate, following Tait [1] (p.153): "…if Q be a solution, then $Q + \bar{a}R$ where R is an arbitrary quaternion, is also a solution. Similarly, for the equation $qb = c$, if Q be a solution, then $Q + S\bar{b}$ (S an arbitrary quaternion) is also a solution: and in like manner for the equations if Q be a solution then also $q + aR + Sb$ (R, S arbitrary quaternions) is a solution."

The above analysis implies the following. In the case of $aqb = c$ where all quaternions a, b and c are null (their norms equal to zero) the result is quite remarkable. From the condition that a, b are nullitats, it follows that ab, aib, ajb, akb are scalar multiples of one and the same nullitat, say of ab. When such a condition is satisfied at points along the travel of the plane wave in the primary, the secondary wave at the points where they cross each other, expresses the solution c a scalar multiple of it, as $c = \lambda ab$. Then the equation $aqb = \lambda ab$ has the solution $q = \lambda$ and the general solution is $q = \lambda + aR + Sb$, at the points. Hence, the mathematical model indicates there are no vectors operating in the regions under the points and the emission of radiation is essentially scalar in nature, traversing in a longitudinal fashion along the direction of propagation. The secondary plane wave has been transformed into a different kind of wave at the points, which can be detailed quantitatively.

The secondary plane wave moves perpendicular to the primary magnetic field, following the application of electrical currents in such a way as that the wave is moving in the direction of its oscillation, contrary to the ordinary understanding of the strict adherence to transverse electromagnetic waves. While not violating any of Maxwell's equations, the orientation and design of the secondary gives rise to the phenomenon. As the wave is considered harmonic since the placement of each cross of the secondary winding is evenly-spaced, meaning that $c_1 = c_2 = \ldots c_n$. Therefore, the frequency and wavelength can be given by

$$y(x,t) = y_0 \cos\left(\omega\left(t - \frac{x}{c}\right)\right), \tag{9}$$

where y is the displacement, x the distance from the source (the last crossing point), y_0 the amplitude (applied at the primary coil), c the speed and ω the angular frequency. The quantity x/c is the time the wave takes to travel the distance between points. The general wave solution for this case is [4],

$$\left[\frac{W}{c} + \frac{e}{c}A_0 + \rho_1\left(\boldsymbol{\sigma}, \mathbf{p} + \frac{e}{c}\mathbf{A}\right) + \rho_3 mc\right]\psi = 0, \tag{10}$$

for the (desired and non-desired) solution of both positive and negative kinetic energy of the electron, that have the periodic solution of the form,

$$\psi = \mu e^{-iEt/h}, \tag{11}$$

where μ is independent of t and E has both positive and negative values. Compared to the Hamiltonian of (10),

$$\left(\frac{W}{c} + \frac{e}{c}A_0\right)^2 - \left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 - m^2c^2 = 0, \tag{12}$$

there is ambiguity in the sign of W or, as Dirac [4] states, of $W + e\mathbf{A}_0$ including the potentials [6]. As the dilemma can be deduced that all negative electron energy states are occupied save a few of small velocity, consequential to the Pauli exclusion principle, a given arrangement can force electrons into certain negative-energy "holes" where positrons can co-exist in a single beam or propagating longitudinal wave at the interface. This is because the circuital arrangement satisfies the (third) Maxwell equation $\operatorname{div}\mathbf{E} = -4\pi\rho$ producing an electric field of infinite divergence, as set as a condition by Dirac [4], setting the insight that all particles that go into their states of lowest energy will result in the charge being of the opposite sign.

Its wavelength is determined by the distance, c, between the crossover points on the secondary coil. Its power is relative to the input on the primary coil and saturates the secondary if the primary is longer than the secondary, or, $b > d$. However, given the ratio of the curvature to the radius of the coil there is a limit based on the relationship between the radius and the curvature at the surface of the solenoid. This ratio could be altered if the coil were a curved solenoid.

The longitudinal wave traverses the interface formed by the secondary coil wherein the low-energy electron states exist at the points where the wires cross.

Discussion

The transformation of the secondary plane wave into a longitudinal wave could have interesting physical applications [7, 8], implying a particle of displacement, and particle velocity propagated in an elastic medium (the field) parallel to the secondary coil extending away from its surface. Further research is indicated where circuital construction and analysis be performed to understand the nature and character of electromagnetic energy emission from such an arrangement.

propagating in an anisotropic dielectric, given the points where the wires cross as disturbing what would ordinarily be a secondary magnetic field manifesting on the surface of a solenoid. However, given the influence on isotropy by the configuration, the solution lends itself to the possibility that I agrees experimentally with the concept of singularities noted by Dirac [4] in describing “holes” where the contention of the negative-energy states being completely occupied and unobservable, while he predicted negative kinetic energy values for an electron. It thus appears that we must abandon the identification of the holes with protons and must find some other interpretation for them. Following Oppenheimer [5], we can assume that in the world as we know it, all, and not merely nearly all, of the negative-energy states for electrons are occupied. A hole, if there were one, would be a new kind of particle, unknown to experimental physics, having the same mass and opposite charge to an electron. We may call such a particle an anti-electron. We should not expect to find any of them in nature, on account of their rapid rate of recombination with electrons, but if they could be produced experimentally…they would be quite stable and amenable to observation.

Words.

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