NT0 — Cross-Base Substrate Entry: Gate → Field → Mechanism → Survivor

(An Unavoidable Opening to the Number-Theory Track)

Justin Grieshop

Abstract

This paper is the opening gate to the Marithmetics number-theory track. Its purpose is not to persuade by rhetoric, but to force immediate credibility by making the substrate (i) visible, (ii) cross-base, and (iii) mechanically interpretable in under five minutes. The strategic constraint is adversarial: a skeptical reader must not retain the usual escape hatches ("base-10 artifact," "cherry-picked pattern," "visual coincidence"). We therefore proceed by a strict narrative arc—Gate → Field → Mechanism → Survivor—using deterministic, law-first witnesses. The four anchor figures are: NT0-00 (Unit/Non-Unit Gate), NT0-01 (Dimensionless Order Field $\hat{\chi}$), NT0-02 (Identity-Hit City Motif), and NT0-03 (Identity Pillars for fixed $n = 137$). Each figure is generated from a single residue-law predicate over $d = b - 1$ and admits immediate verification without tuning, smoothing, or learned embeddings. Where appropriate, we provide Authority-of-Record (AoR) audit surfaces for cross-base invariance discipline and designed-fail controls.

Keywords: finite residue substrate, cross-base invariance, units, multiplicative order, Carmichael function, identity hits, DRPT, auditability

1 Executive Reading Protocol (≤ 5 minutes)

1.1 The only inputs used in this entry

Choose a numeral base $b \geq 2$. Define the induced modulus

$$d := b - 1.$$

All figures in NT0 are computed in $\mathbb{Z}_d$ and depend only on: (i) the base b, (ii) a contiguous integer window of n, and (iii) a contiguous exponent window of k. No fitted parameters appear.

The cross-base panels below use the illustrative base set $\{8, 16, 12, 10\}$ (i.e., $d \in \{7, 15, 11, 9\}$). Nothing in the definitions privileges this choice; it is used because it mixes prime and composite d while remaining visually compact.

1.2 What each figure accomplishes (and which escape hatch it closes)

Gate (NT0-00): Unit vs non-unit partition.
Closes: "empty regions mean missing data," "identity patterns were highlighted by omission," and the strongest version of "cherry-picking," because non-units are formally ineligible to generate identity cycles.

Field (NT0-01): Dimensionless order field $\hat{\chi}(n) = \mathrm{ord}_d(n)/\lambda(d)$.
Closes: "pretty rows" and "base-dependent scaling," by upgrading pattern to a base-portable structural metric.

Mechanism (NT0-02): Identity-hit predicate rendered across bases.
Closes: "base-10 artifact" and "visual coincidence," because the same deterministic predicate generates structured identity geometry in multiple bases on the same n-window.

Survivor (NT0-03): Fixed-row witness for $n = 137$.
Closes: "you can always make a heatmap look structured," by stripping the city down to one row and showing the same mechanism persists across bases.

Scope note (what NT0 does not contain). NT0 is substrate-only. It does not introduce or assume any of the downstream constructions used in later demos and physics overlays, including: lane predicate families (moduli/residue sets/thresholds), Φ-channel lawbooks, Palette-B exponent sets, cosmology monomials, or ALQ dressing maps. Those belong to the admissible continuation layer and are governed by the ACS in Appendix B.

2 Preliminaries: The Residue Environment Induced by Base

2.1 Base induces modulus $d = b - 1$

For any base $b \geq 2$, the standard digit-sum congruence implies

$$b \equiv 1 \pmod{b-1},$$

and therefore all base-b "digital-root" phenomena reduce to arithmetic modulo $d = b - 1$. NT0 does not require digital-root conventions; it requires only the induced finite ring $\mathbb{Z}_d$.

2.2 Units and non-units

Fix $d \geq 2$. An integer n is a *unit* mod d iff

$$\gcd(n, d) = 1,$$

equivalently the residue class $[n] \in \mathbb{Z}_d$ is invertible. If $\gcd(n, d) > 1$, then n is a *non-unit* (a zero-divisor class). NT0 uses this partition as the first gate because identity structure lives in the multiplicative group of units.

2.3 Multiplicative order and the Carmichael exponent

For a unit n mod d, define the *multiplicative order*

$$\mathrm{ord}_d(n) := \min\{k \geq 1 : n^k \equiv 1 \pmod{d}\}.$$

Let $\lambda(d)$ denote the Carmichael function, i.e., the exponent of the unit group $(\mathbb{Z}/d\mathbb{Z})^\times$. Then for every unit n,

$$\mathrm{ord}_d(n) \mid \lambda(d), \qquad \text{and} \qquad n^{\lambda(d)} \equiv 1 \pmod{d}.$$

This is the canonical tile scale: $\lambda(d)$ is the smallest universal period guarantee for the unit sector.

3 Gate: Unit/Non-Unit Eligibility Across Bases

3.1 The gate is not aesthetic; it is necessary

A skeptical reader may ask why NT0 draws stark partitions. The reason is purely algebraic:

Proposition 3.1 (Non-units cannot hit identity)**.** *If* $\gcd(n, d) > 1$, *then* $n^k \not\equiv 1 \pmod{d}$ *for every* $k \geq 1$.

Proof. Let $g = \gcd(n, d) > 1$. Then $n \equiv 0 \pmod{g}$, so $n^k \equiv 0 \pmod{g}$ for all k. But $1 \not\equiv 0 \pmod{g}$. Hence $n^k \not\equiv 1 \pmod{d}$. □

Thus any visualization of identity hits that does not first display the unit/non-unit gate invites misreading: blank regions are not "missing signal"; they are mathematically ineligible.

3.2 Figure NT0-00 (the gate)

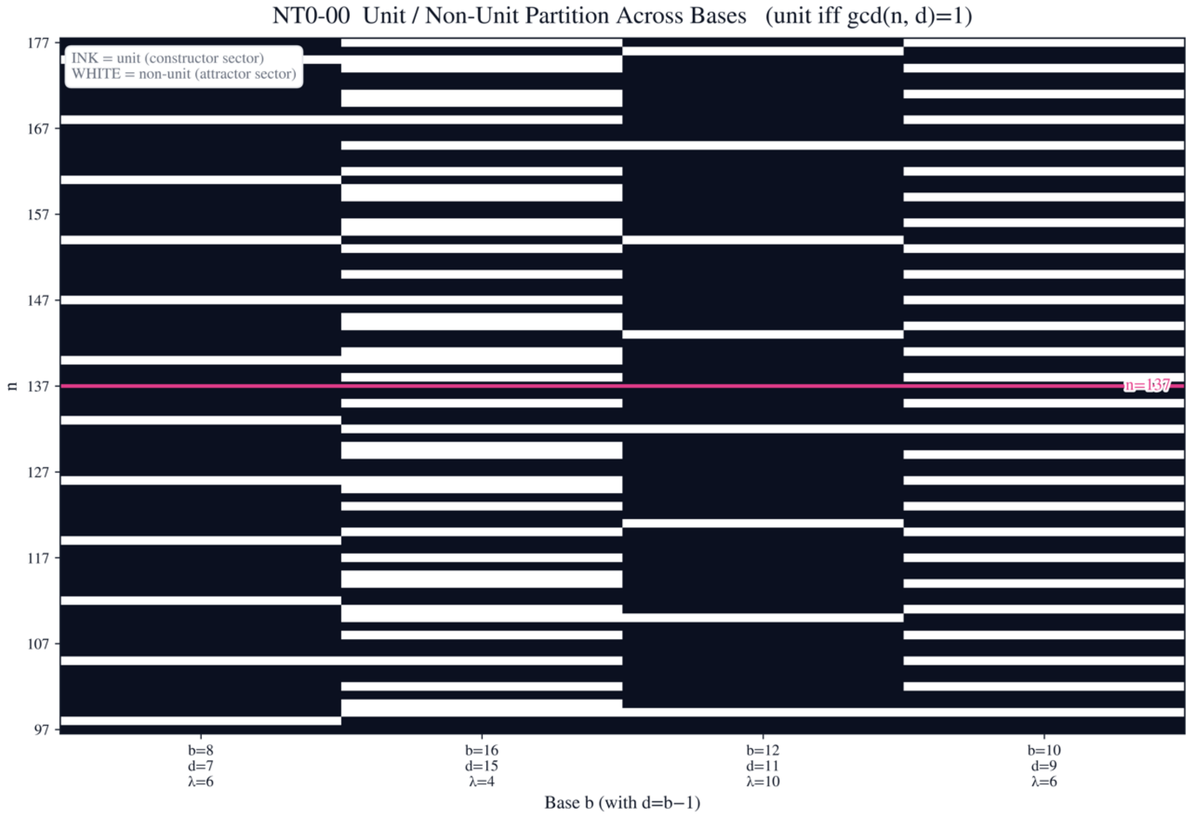

Figure 1: **Unit/Non-Unit Partition Across Bases (the gate).** For each base b, we work in the residue ring $\mathbb{Z}_d$ with $d = b - 1$. A row index n is a unit iff $\gcd(n, d) = 1$. Only units admit inverses and therefore can participate in identity cycles; non-units belong to the attractor sector and cannot return to the unit group. This panel is the prerequisite for every downstream DRPT visual: it explains which n are eligible to generate identity structure and prevents misreading empty regions as missing signal.

4 Field: A Cross-Base Structural Metric $\hat{\chi}$ That Cannot Be Dismissed as "Pretty"

4.1 Why a dimensionless field is the correct next step

Even after the unit gate, a skeptic can argue: "you have drawn a partition, but you have not shown a quantitative invariant."

The first invariant must be:

- intrinsic to the residue environment $\mathbb{Z}_d$,
- defined without reference to digit strings, and
- comparable across bases.

The multiplicative order satisfies the first two constraints. To satisfy the third, we normalize order by the universal tile scale $\lambda(d)$.

4.2 Definition of $\hat{\chi}$

For each base b (with $d = b - 1$), define the *dimensionless order field*

$$\hat{\chi}_b(n) := \begin{cases} \mathrm{ord}_d(n)/\lambda(d), & \gcd(n,d) = 1, \\ \text{undefined}, & \gcd(n,d) > 1. \end{cases}$$

By construction, $\hat{\chi}_b(n) \in (0,1]$ for units, and is undefined for non-units.

Interpretation.

- $\hat{\chi}_b(n) = 1$ means n achieves the full group exponent scale: it is maximally cyclic relative to $\lambda(d)$.
- Smaller $\hat{\chi}_b(n)$ means identity returns occur on a finer lattice.
- Non-units are excluded, not because they are uninteresting, but because "order" is not defined there.

4.3 Figure NT0-01 (field witness)

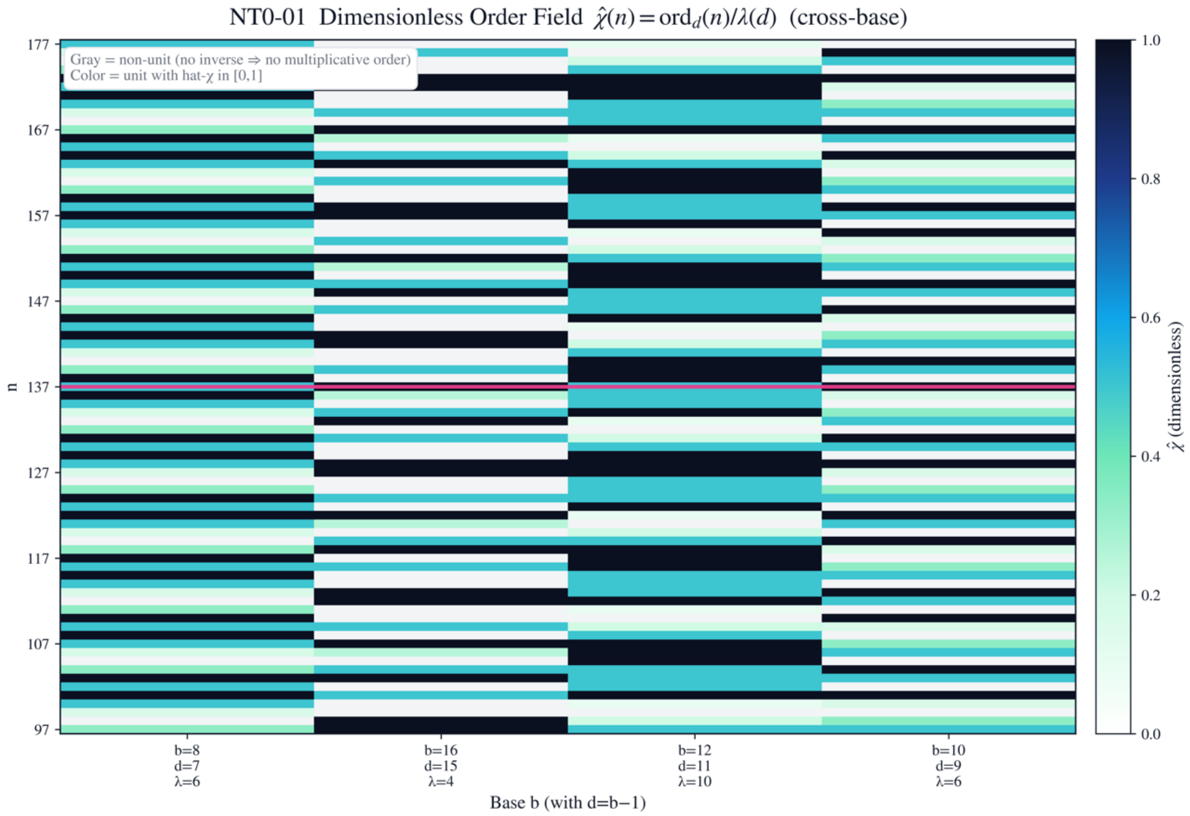

Figure 2: **Dimensionless Order Field** $\hat{\chi}(n) = \mathrm{ord}_d(n)/\lambda(d)$ **(cross-base).** For each base b (with $d = b - 1$), we compute the multiplicative order $\mathrm{ord}_d(n)$ for unit rows and normalize by the Carmichael exponent $\lambda(d)$, producing the dimensionless invariant $\hat{\chi} \in [0,1]$. Gray denotes non-units (no inverse, no order). This figure upgrades "pattern" into an audit-grade structural metric: it quantifies cycle strength in a base-portable way and demonstrates the base-as-gauge discipline (comparisons are made in normalized invariants, not digit labels).

5 Mechanism: Identity Hits Are Not Coincidences; They Are the Order Lattice

5.1 Identity-hit predicate (law-first)

Define the *identity-hit indicator* for base b (with $d = b - 1$):

$$\mathbf{1}_{\mathrm{id}}^{(b)}(n,k) := \begin{cases} 1, & \gcd(n,d) = 1 \text{ and } n^k \equiv 1 \pmod{d}, \\ 0, & \text{otherwise.} \end{cases}$$

This predicate is deterministic, discrete, and entirely internal to $\mathbb{Z}_d$.

5.2 The key mechanism (why "city lanes" must appear)

For a unit n, let $t = \mathrm{ord}_d(n)$. Then:

$$n^k \equiv 1 \pmod{d} \iff t \mid k.$$

Proposition 5.1 (Identity hits occur on a divisor lattice). *For any unit n mod d, the set of identity-hit exponents is exactly*

$$\{k \geq 1 : n^k \equiv 1 \pmod{d}\} = \{t, 2t, 3t, \ldots\}, \quad t = \mathrm{ord}_d(n).$$

Therefore, the identity-hit matrix is not a "texture." It is the superposition of many divisor lattices $t\mathbb{N}$ stacked over n. Columns with many divisors in the realized order set naturally accumulate hits (vertical lanes); columns with fewer such divisors naturally sparse out (structured gaps). This is a mechanism statement, not a visual metaphor.

5.3 Figure NT0-02 (mechanism made visible across bases)

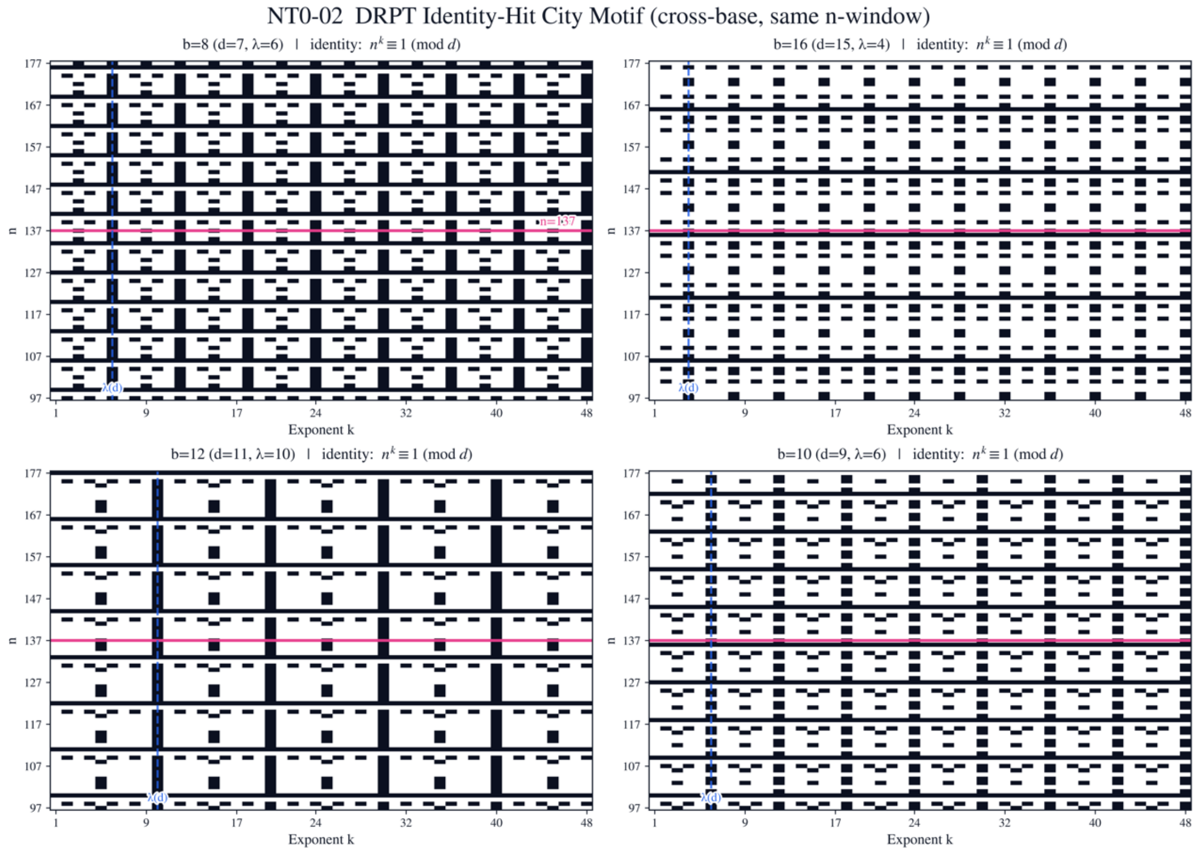

Figure 3: **DRPT Identity-Hit City Motif (cross-base, same n-window).** Identity hits are defined by the deterministic predicate $n^k \equiv 1 \pmod d$ in $\mathbb{Z}_d$ with $d = b - 1$. For the same n-window and the same exponent window, we render the identity-hit matrix across multiple bases. The resulting "city" geometry (vertical lanes, repeated tilings, and structured gaps) is not a base-10 artifact: the same law produces structured identity architecture in each gauge base, visible without statistical smoothing or learned embedding.

6 Survivor: A Single-Row Witness Eliminates "Heatmap Ambiguity"

6.1 Why a fixed-row witness is mandatory

A skeptical reader may concede that a dense binary matrix can look structured "by chance," especially when plotted with strong contrast. NT0 therefore strips the mechanism down to a single row n, leaving no degrees of freedom:

- no row aggregation,
- no density effects,
- no visual compositing.

The only remaining object is the identity-hit lattice $t\mathbb{N}$ for that n.

6.2 The fixed-row predicate

For fixed n, the identity-hit plot is simply:

$$k \mapsto \mathbf{1}\{n^k \equiv 1 \pmod d\},$$

with the same $d = b - 1$ rule.

6.3 The specific witness $n = 137$ (illustrative, not a rule change)

For the base set shown in the figures:

- $b = 8$, $d = 7$, $137 \equiv 4 \pmod 7$, $\mathrm{ord}_7(4) = 3$, $\lambda(7) = 6$, so $\hat{\chi} = 3/6 = 1/2$.
- $b = 16$, $d = 15$, $137 \equiv 2 \pmod{15}$, $\mathrm{ord}_{15}(2) = 4$, $\lambda(15) = 4$, so $\hat{\chi} = 4/4 = 1$.
- $b = 12$, $d = 11$, $137 \equiv 5 \pmod{11}$, $\mathrm{ord}_{11}(5) = 5$, $\lambda(11) = 10$, so $\hat{\chi} = 5/10 = 1/2$.
- $b = 10$, $d = 9$, $137 \equiv 2 \pmod 9$, $\mathrm{ord}_9(2) = 6$, $\lambda(9) = 6$, so $\hat{\chi} = 6/6 = 1$.

Nothing about this computation changes the governing rule. It is included because a reader can verify it mentally or on paper in minutes, which is the intended function of NT0.

6.4 Figure NT0-03 (survivor lane)

Figure 4: **The Survivor Lane: Identity Pillars Across Bases for fixed $n = 137$.** We isolate a single row $n = 137$ and plot its identity hits ($n^k \equiv 1 \pmod d$) across the same gauge bases. This removes all visual ambiguity: the identity mechanism is present in every base and produces clean "pillars" of recurrence. For this particular n and base set, the induced orders realize a simple progression across bases ($\mathrm{ord} = 3, 4, 5, 6$ across $d = 7, 15, 11, 9$), strengthening interpretability while keeping the rule fixed. This is a deterministic witness that the substrate mechanism is cross-base and reproducible.

Reader Note (Scope Boundary; avoiding "engineering" misreads). In NT0, the terms *gate*, *field*, *city*, *lanes*, and *pillars* are expository labels for deterministic sets defined by elementary arithmetic in $\mathbb{Z}_d$ with $d = b - 1$. No additional selection is performed in this paper beyond the unit eligibility predicate $\gcd(n, d) = 1$ and the identity-hit predicate $n^k \equiv 1 \pmod d$.

Specifically:

- The "gate" (NT0-00) is the unit/non-unit partition $\gcd(n, d) = 1$ vs $\gcd(n, d) > 1$. It is an eligibility boundary: non-units are provably ineligible for identity hits.
- The "lanes" in NT0-02 refer to the deterministic superposition of divisor-lattices $k \in \mathrm{ord}_d(n)\mathbb{N}$ across n. They are not lane filters.

- The "pillars" in NT0-03 are the multiples of $\mathrm{ord}_d(137)$ for a fixed row $n = 137$, rendered across multiple bases.

NT0 therefore contains no lane rule lists, no modulus/residue threshold filters, no monomials, and no dressing maps. Readers who wish to audit how later rule families are constructed (and why that construction is constrained rather than ad hoc) should consult Appendix B, which formalizes the Admissible Continuation Spec (ACS): the admissibility constraints, falsification requirements, and minimal necessity tests required of any downstream rule system. Appendix B is not required to verify NT0's substrate claims.

Fast Verification Path (AoR index; for skeptical readers). If you want to verify that the substrate visuals in NT0 are not a base-10 artifact and that downstream claims are not "engineered," use the AoR index surfaces rather than prose. Start with: (i) `demo_index.csv` (what ran, where the logs/artifacts are), (ii) `constants_master.csv` (what values were produced and where they came from), and (iii) `falsification_matrix.csv` (which deliberate violations must fail, and the expected outcomes). The GUM report is the human-readable map that points into these same artifacts.

- `demo_index.csv`: https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/tables/demo_index.csv
- `constants_master.csv`: https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/tables/constants_master.csv
- `falsification_matrix.csv`: https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/tables/falsification_matrix.csv
- GUM report (index PDF): https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/report/GUM_Report_v32_2026-01-25_04-42-51Z.pdf

7 Closing the Standard Escape Hatches (Explicitly)

7.1 "Base-10 artifact"

NT0 does not compare digit strings across bases. It compares residue-law invariants under $d = b - 1$. The unit gate is defined by $\gcd(n, d) = 1$, the order field by $\mathrm{ord}_d(n)$, the tile scale by $\lambda(d)$, and identity hits by $n^k \equiv 1 \pmod d$. These statements are meaningful in every base because they are not statements about representation; they are statements about $\mathbb{Z}_d$.

7.2 "Cherry-picked pattern"

The "pattern" is a predicate, not a selection. Given b, n-window, and k-window, the figures are forced. Moreover, NT0 begins with the unit/non-unit gate precisely to prevent selection by omission: non-units are provably ineligible for identity hits (Proposition 3.1). Blank regions are not curated; they are required.

7.3 "Visual coincidence"

The identity-hit city is not a freeform texture: it is the superposition of divisor lattices $t\mathbb{N}$, where $t = \mathrm{ord}_d(n)$. The "city lanes" are a visualization of divisibility structure and the realized order distribution in $(\mathbb{Z}/d\mathbb{Z})^{\times}$. The fixed-row witness (NT0-03) removes aggregation entirely; what remains is the exact lattice of multiples of $\mathrm{ord}_d(n)$.

8 AoR Audit Hooks (For Readers Who Want Immediate Custody Verification)

This paper's core claims are classical and do not require AoR to be true. AoR is included for readers who want proof that the broader program enforces cross-base discipline and designed-fail controls at scale.

AoR demo index (navigation):
https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/tables/demo_index.csv

Base-gauge invariance audit surface (example):
DEMO-64 stdout:
https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/logs/substrate__demo-64-base-gauge-invariance-integer-selector.out.txt
DEMO-64 stderr:
https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/logs/substrate__demo-64-base-gauge-invariance-integer-selector.err.txt

Designed-fail catalog (matrix of required failures):
https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/tables/falsification_matrix.csv

Determinism/replay posture (run reproducibility table):
https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/tables/run_reproducibility.csv

9 Reproducibility Protocol for NT0 (How to Regenerate Each Figure in Minutes)

NT0 is designed so that a reader can reproduce the core experience without installing a full repository, without running a complex pipeline, and without trusting any interpretive narrative. This section declares the minimal computation needed for each figure and the exact parameters used in the canonical panels.

9.1 Canonical window declaration (the "same n-window" requirement)

To prevent subtle cherry-picking accusations, NT0 treats the n-window as a declared part of the object.

Canonical window used in Figures NT0-00 through NT0-02:

$$n \in \{97, 98, \ldots, 177\} \quad \text{(inclusive)}$$

Canonical exponent window used in Figures NT0-02 and NT0-03:

$$k \in \{1, 2, \ldots, 48\}$$

These choices are not special. They are declared because: (i) the window is wide enough to contain multiple unit/non-unit alternations for composite d, and (ii) the exponent window is wide enough to display several identity pillars for the $\lambda(d)$ scales in the example base set.

9.2 Canonical base set (gauge bases; illustrative)

The cross-base panels shown in NT0 use:

$$b \in \{8, 16, 12, 10\}, \quad \text{hence} \quad d = b - 1 \in \{7, 15, 11, 9\}.$$

For this base set:

- $\lambda(7) = 6$ (prime modulus; cyclic unit group)
- $\lambda(15) = 4$
- $\lambda(11) = 10$ (prime modulus; cyclic unit group)
- $\lambda(9) = 6$

Nothing in NT0 depends on these specific values beyond display scaling. A reader may substitute any base set; the only requirement for the NT0 claim boundary is that the same definitions be applied in each base.

9.3 Figure NT0-00 (unit mask atlas): exact construction

For each base b with $d = b - 1$, compute the binary eligibility predicate:

$$U_b(n) := \begin{cases} 1, & \gcd(n, d) = 1, \\ 0, & \gcd(n, d) > 1. \end{cases}$$

Plot the matrix $U_b(n)$ over the declared n-window as a column panel for each base, using a consistent legend (unit vs non-unit).

This figure is purely Euclidean-algorithm arithmetic. It contains no modular exponentiation.

9.4 Figure NT0-01 ($\hat{\chi}$ atlas): exact construction

For each base b with $d = b - 1$:

- If $\gcd(n, d) > 1$, mark n as non-unit (gray; undefined).
- If $\gcd(n, d) = 1$, compute $\mathrm{ord}_d(n)$ by finding the smallest $t \geq 1$ such that $n^t \equiv 1 \pmod{d}$.
- Compute $\hat{\chi}(n) = \mathrm{ord}_d(n)/\lambda(d)$ and render $\hat{\chi}$ as a continuous field in $[0, 1]$.

This figure depends on: gcd, modular exponentiation, and $\lambda(d)$. For the small moduli in the canonical base set, $\lambda(d)$ can be computed directly from the structure of $(\mathbb{Z}/d\mathbb{Z})^\times$ or by brute-force exponent search:

$$\lambda(d) = \mathrm{lcm}\big(\mathrm{ord}_d(u) : \gcd(u, d) = 1\big).$$

9.5 Figure NT0-02 (identity-hit city): exact construction

For each base b with $d = b - 1$, compute the identity-hit indicator on the declared windows:

$$\mathbf{1}_{\mathrm{id}}^{(b)}(n, k) = \mathbf{1}\{\gcd(n, d) = 1 \text{ and } n^k \equiv 1 \pmod{d}\}.$$

Plot this as a binary field over $n \in [97, 177]$ and $k \in [1, 48]$, with black for "hit" and white for "no hit."

This figure is the flagship mechanism witness because it is a direct rendering of one predicate. There is no smoothing, no frequency analysis, and no learned embedding.

9.6 Figure NT0-03 (fixed $n = 137$ identity pillars): exact construction

Fix $n = 137$. For each base b with $d = b - 1$, compute:

$$\mathbf{1}\{137^k \equiv 1 \pmod{d}\} \quad \text{for} \quad k = 1, \ldots, 48.$$

Plot the resulting hit positions as vertical pillars.

This is the "strip down to one lane" witness. If a reader distrusts dense heatmaps, they can ignore NT0-02 and verify only NT0-03; the mechanism is then unambiguous: pillars occur precisely at multiples of $\mathrm{ord}_d(137)$.

9.7 Minimal pseudocode (language-agnostic)

```
Input: base set B, n-window [n_min, n_max], k-window [1, K]

For each base b in B:
    d <- b - 1
    Compute lambda(d) (either known formula or brute lcm of unit orders)
    For n in [n_min..n_max]:
        If gcd(n, d) != 1:
            unit_mask[n] = 0; hat_chi[n] = None
        Else:
            unit_mask[n] = 1
            t <- smallest k >= 1 with powmod(n, k, d) = 1
            hat_chi[n] <- t / lambda(d)
        For k in [1..K]:
            id_hit[n, k] <- (powmod(n, k, d) == 1)
```

This is sufficient to regenerate all four figures.

10 How NT0 Constrains and Clarifies the Rest of the NT Track

The number-theory track is not a gallery of patterns. It is a progressive tightening of what may count as structure, culminating in selection rules and invariants that are (i) base-portable, (ii) operator-admissible, and (iii) audit-verifiable.

NT0's job is to make three constraints unavoidable from the first page.

10.1 Constraint A — eligibility must be explicit (units vs non-units)

Later constructions that rely on invertibility (identity hits, cycle transport, unit-group statistics, character decompositions) must explicitly state the unit gate. If a later argument uses "order" or "inverse," it inherits NT0-00 and Proposition 3.1 automatically.

10.2 Constraint B — cross-base comparability must use normalized invariants

Later papers will introduce richer invariants than $\hat{\chi}$. NT0 sets the discipline: base-to-base comparisons are made in invariants that are meaningful in each $\mathbb{Z}_{b-1}$, not in digit strings. $\hat{\chi}$ is the first "hat" invariant because it is:

- dimensionless,
- bounded,
- and mechanically interpretable (cycle strength relative to a canonical tile).

10.3 Constraint C — mechanisms must be stated as predicates, not narratives

NT0-02 and NT0-03 introduce a standard of evidence for the substrate: the reader must be able to point to a predicate ("hit iff $n^k \equiv 1 \pmod d$") and watch structure appear. Later NT papers are expected to maintain this style: state a law; render its witness; give the reader a simple verification pathway.

10.4 What the reader should expect next (bridge forward)

With the gate, field, and mechanism established, the next NT papers expand in the following direction:

- from identity hits to full DRPT structure (value cycles, attractors, and residue transport),
- from order fields to family structure (how integers cluster by shared residue signatures), and
- from visual motifs to admissible selection (why some structures survive under lawful transforms and why others are excluded).

NT0 is therefore not "a preface." It is the rule that later NT papers must satisfy: if a later claim cannot be made visible in the residue substrate under a stated predicate, it does not belong in the NT track.

11 Claim Boundary for NT0 (What Is Proved Here, What Is Merely Shown)

11.1 What is proved (deductive layer)

The following statements are purely classical and require no AoR:

- Non-units cannot produce identity hits (Proposition 3.1).
- Unit identity hits occur exactly on the multiples of the multiplicative order (Proposition 5.1).
- Normalization by $\lambda(d)$ yields a dimensionless order ratio $\hat{\chi} \in (0, 1]$ for units.
- Cross-base comparison may be made in these invariants because they are internal to $\mathbb{Z}_{b-1}$ and do not depend on digit labels.

11.2 What is shown (witness layer)

The four figures are witnesses that the above deductive facts are visible and interpretable at a glance under the declared windows and bases.

Critically: NT0 does not ask the reader to infer structure from images. It asks the reader to verify predicates and then notice that the predicates produce structured geometry.

11.3 What is not claimed (to avoid premature overreach)

NT0 does not claim:

- that $n = 137$ is "special" in any base-independent sense beyond the demonstrated residue facts in the displayed bases;
- that identity-hit geometry alone determines any physical constant;
- that visual similarity across bases implies equality of digit sequences or of value distributions.

NT0's contribution is stricter: it establishes an unavoidable substrate and a standard of cross-base evidence.

Conclusion: NT0 as an Unavoidable Opening

NT0 is designed so that a hostile reader cannot dismiss the substrate without contradicting elementary finite arithmetic.

- The unit/non-unit gate is not interpretive; it is eligibility.
- The order field $\hat{\chi}$ is not aesthetic; it is a normalized structural invariant.
- The identity-hit city is not coincidence; it is a divisor lattice rendered across bases.
- The fixed-row witness removes heatmap ambiguity and leaves only the mechanism.

With these four witnesses, the reader is compelled into the only productive next question:

> *Given that this substrate is real, cross-base, and mechanically interpretable, what lawful operators and selections are permitted—and what invariants can be extracted without introducing hidden degrees of freedom?*

That question is the bridge into NT-1 and the remainder of the suite. The admissible continuation contract governing downstream rule construction is specified in Appendix B.

A Short Proofs (For Readers Who Want the Algebra Without Detours)

A.1 Proof of Proposition 3.1 (non-units cannot hit identity)

Let $g = \gcd(n, d) > 1$. Then $n \equiv 0 \pmod g$, hence $n^k \equiv 0 \pmod g$ for all k. If $n^k \equiv 1 \pmod d$, then in particular $n^k \equiv 1 \pmod g$, contradiction. Therefore $n^k \not\equiv 1 \pmod d$ for all $k \geq 1$. □

A.2 Proof of Proposition 5.1 (identity hits form a divisor lattice)

Let $t = \mathrm{ord}_d(n)$. By definition, $n^t \equiv 1 \pmod d$ and t is minimal with this property. If $t \mid k$, write $k = mt$. Then $n^k = (n^t)^m \equiv 1^m \equiv 1 \pmod d$. Conversely, if $n^k \equiv 1 \pmod d$, then by the standard order lemma in finite groups, the order t divides k. □

A.3 Why $\hat{\chi} \in (0, 1]$ for units

If n is a unit, $\mathrm{ord}_d(n) \mid \lambda(d)$, so $\mathrm{ord}_d(n) \leq \lambda(d)$. Therefore $\hat{\chi} = \mathrm{ord}_d(n)/\lambda(d) \in (0, 1]$. □

B Admissible Continuation Spec (ACS): From Substrate Predicates to Canonical Rule Families

B.1 Purpose

NT0 makes the finite residue substrate visible and cross-base, using only predicates internal to $\mathbb{Z}_d$ with $d = b - 1$. A separate question—often the first question asked by hostile readers—is whether later "rule constructions" (lane predicates, selection engines, lawbooks, dressing maps) are forced continuations of the substrate or well-engineered filters.

This appendix closes that conviction gap by doing one thing only: it defines the *admissibility contract* that any downstream rule family must satisfy in order to be treated as a lawful continuation of NT0 rather than an arbitrary engineering choice.

The appendix is written as a specification: it defines inputs, outputs, admissibility constraints, required falsification witnesses, and minimal necessity tests. It also provides Authority-of-Record (AoR) audit hooks where these constraints are enforced and recorded.

B.2 Inputs (substrate primitives available to continuation)

Fix a base $b \geq 2$ and $d = b - 1$. The admissible continuation layer may depend only on the following substrate objects (and objects derived from them by DOC-admissible operators):

B.2.1 Core substrate objects

- **Unit gate:** $U_d(n) = \mathbf{1}\{\gcd(n, d) = 1\}$.
- **Multiplicative order (units only):** $\mathrm{ord}_d(n)$.
- **Carmichael exponent:** $\lambda(d)$.
- **Dimensionless order field (units only):** $\hat{\chi}_d(n) = \mathrm{ord}_d(n)/\lambda(d) \in (0, 1]$.
- **Identity-hit predicate:** $\mathbf{1}\{U_d(n) = 1 \wedge n^k \equiv 1 \pmod{d}\}$.

B.2.2 Permitted derived summaries (examples; non-exhaustive)

- Order histograms over an n-window, or over all units.
- Tile-aligned aggregates over k within a fixed multiple of $\lambda(d)$.
- Residue-class transport signatures, provided the transport map is injective on the intended domain (CRT hygiene; non-injective maps must be treated as falsifiers).

B.3 Outputs (what continuation may produce)

Continuation produces deterministic rules and selectors that will later be used to identify survivors (e.g., candidate rows, triples, families) and to derive invariant outputs.

The ACS recognizes four output categories:

(A) Lane predicates: boolean predicates on integers n built from bounded-complexity modular conditions and substrate metrics.

(B) Canonical selectors: deterministic selection engines that take substrate summaries as input and return a discrete survivor set (e.g., an integer, a finite family, or a finite tuple).

(C) Lawbooks: deterministic maps from certified discrete survivors to rational or algebraic outputs (e.g., dimensionless constants).

(D) Dressing classes: deterministic correction maps constrained to a strictly admissible finite family (no continuous fit) applied only where explicitly permitted by DOC/Foundations tiering.

B.4 Admissibility constraints (ACS-1 through ACS-6)

A downstream rule family $\mathcal{R}$ (lane rules, selectors, lawbooks, dressing) is admissible only if it satisfies all of the following.

B.4.1 ACS-1: Base-as-gauge invariance (representation independence)

Let $\mathcal{B}$ be a declared base set. A rule family $\mathcal{R}$ is *base-gauge invariant* on $\mathcal{B}$ if, for all $b, b' \in \mathcal{B}$, the outputs of $\mathcal{R}$ agree exactly for discrete quantities (integers, tuples, rationals) when computed from their respective $\mathbb{Z}_{b-1}$ substrate objects under the same declared algorithm and admissible representation transforms.

Operational requirement: The AoR must include an explicit base-gauge invariance audit recording PASS/FAIL outcomes over the declared $\mathcal{B}$.

AoR audit surface (example):

DEMO-64 stdout (base-gauge invariance audit):
https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/logs/substrate__demo-64-base-gauge-invariance-integer-selector.out.txt

DEMO-64 stderr:
https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/logs/substrate__demo-64-base-gauge-invariance-integer-selector.err.txt

B.4.2 ACS-2: DOC legality for any averaging, smoothing, windowing, or transfer

If $\mathcal{R}$ uses any operator beyond raw modular predicates (e.g., windows, averages, transforms, filters), those operators must be DOC-admissible. At minimum, admissible averaging/transfer operators must satisfy:

- nonnegativity (no signed cancellation as a hidden degree of freedom),
- normalization/unit mass (no hidden scaling),
- boundedness/non-amplification (no injected high-frequency energy), and
- declared symmetry/commutation constraints when relevant.

B.4.3 ACS-3: Designed-fail separation (negative controls are mandatory)

A rule family is not admissible unless it fails under explicitly constructed non-admissible transforms. At minimum, the AoR must include designed-fail tests in three classes:

- **digit-level corruption:** digit drift/injection in representation layer must FAIL,
- **non-injective residue transport:** CRT collisions under non-coprime moduli must be exhibited, and
- **illegal operator class:** sharp/signed windows or kernels must FAIL by producing forbidden signatures such as negative lobes, overshoot, or mass violation.

Operational requirement: The AoR must enumerate these falsifiers and record required outcomes.

AoR falsifier catalog (canonical matrix):
https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/tables/falsification_matrix.csv

Representative designed-fail audit bundle (example surface):
DEMO-69 stdout:
https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/logs/controllers__demo-69-oatb-operator-admissibility-transfer-bridge.out.txt

B.4.4 ACS-4: Rigidity / non-genericity under near-neighbor perturbations

If $\mathcal{R}$ is a lawful continuation rather than a tuned artifact, it must be *rigid*: small perturbations of the rule family should typically destroy the outputs or cause explosion (non-uniqueness).

Rigidity is certified operationally by a *near-neighbor sweep*: enumerate a bounded neighborhood $\mathcal{N}(\mathcal{R})$ of rule variants and record whether they reproduce the same outputs. Admissibility requires that "same output" events are rare unless the variants are provably equivalent under a declared equivalence relation.

B.4.5 ACS-5: Necessity under ablation (remove a constraint $\rightarrow$ system breaks)

A subset of the constraints used by $\mathcal{R}$ must be shown necessary by explicit ablation: removing a constraint produces either:

- drift (violates ACS-1), or
- illegality (violates ACS-2/ACS-3), or
- explosion (loss of uniqueness; violates ACS-4).

Example of an ablation-style certificate recorded in AoR (illustrative of the requirement pattern):
DEMO-34 stdout includes explicit explosion counts under dropped coupling locks (uniqueness breaks when a gate is removed):
https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/logs/bridge__demo-34-omega-sm-master-flagship-v1.out.txt

B.4.6 ACS-6: Canonical tie-break: minimal description length (MDL) / complexity ordering

If more than one rule family survives ACS-1 through ACS-5, the program must select a canonical representative by a declared complexity ordering (e.g., minimal parameter count, minimal modulus size subject to constraints, minimal residue set complexity, minimal threshold encoding cost, minimal composition depth). The ordering must be declared and applied mechanically.

B.5 Bounded candidate class (how "lane rules" are made non-arbitrary)

To avoid open-ended engineering degrees of freedom, lane predicates and related filters must be drawn from a *bounded grammar*.

One admissible generic lane predicate schema is:

- choose a modulus m from a declared finite set $\mathcal{M}$,
- choose a residue subset $R \subseteq (\mathbb{Z}/m\mathbb{Z})^\times$ from a declared finite set $\mathcal{R}(m)$,
- optionally include a substrate metric threshold (e.g., $\hat{\chi}_d(n) \geq \tau$ for τ from a bounded rational grid),
- include any side constraints only if they are themselves substrate-expressible and bounded-complexity (e.g., valuation constraints like $v_2(n-1)$ bounded by a declared small integer).

Example grammar (schematic):

$$L(n) = U_d(n) \cdot \mathbf{1}\{n \bmod m \in R\} \cdot \mathbf{1}\{\hat{\chi}_d(n) \geq \tau\} \cdot \mathbf{1}\{\text{side constraints}(n)\}.$$

This appendix does not assert which (m, R, τ) are correct. It asserts the rule-construction posture: candidates are enumerated from a bounded grammar and must survive ACS constraints. That is what converts "rules we chose" into "rules that survived."

B.6 Minimal necessity tests required for publication-grade credibility

To preempt "engineering" accusations, the following minimal tests must be recorded for any published rule family $\mathcal{R}$:

Test N1 (Base invariance). Run $\mathcal{R}$ over the declared base set $\mathcal{B}$ and show exact agreement of discrete outputs. Provide PASS/FAIL and record any drift.
(Example audit surface pattern: DEMO-64.)

Test N2 (Designed-fail: digit and transport corruption). Apply digit injection/drift and non-injective residue transport falsifiers and require FAIL with explicit witnesses (collision examples; representation failure signatures).
(Required catalog: falsification_matrix.csv; example audit pattern: DEMO-69.)

Test N3 (Ablation and near-neighbor rigidity). Remove each major component of $\mathcal{R}$ one at a time (ablation) and perturb it within a declared neighborhood (near-neighbor sweep). Require explosion/drift/illegality as appropriate. Provide the explosion counts and/or drift metrics.
(Example ablation-certificate pattern recorded in AoR: DEMO-34.)

B.7 AoR audit navigation (how a reader verifies we are not "cheating")

A reader auditing continuation should be able to do the following without interpretation:

1. **Locate the relevant demo(s)** via demo_index.csv.
   https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/tables/demo_index.csv
2. **Confirm base-gauge invariance audits exist** and record deterministic PASS/FAIL (example: DEMO-64).
   DEMO-64 stdout:
   https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/logs/substrate__demo-64-base-gauge-invariance-integer-selector.out.txt
3. **Confirm designed-fail requirements are enumerated** (falsification_matrix.csv) and that at least one representative designed-fail bundle is recorded (example: DEMO-69).
   falsification_matrix.csv:
   https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/tables/falsification_matrix.csv
   DEMO-69 stdout:
   https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/logs/controllers__demo-69-oatb-operator-admissibility-transfer-bridge.out.txt
4. **Confirm determinism posture** via run_reproducibility.csv.
   https://github.com/public-arch/Marithmetics/blob/release-aor-20260125T043902Z/gum/authority_archive/AOR_20260125T043902Z_52befea/GUM_BUNDLE_v30_20260125T043902Z/tables/run_reproducibility.csv

B.8 Interpretation rule (how to read later rule systems in good faith)

When a later paper introduces a specific rule set (lane predicates, selection engine configuration, monomials, dressing), the default interpretation is not "assumed law." The correct interpretation under this suite's posture is:

- the rule set is a candidate from a bounded grammar,
- it is constrained by ACS-1 through ACS-6, and
- its admissibility is evidenced by AoR audits (base invariance, designed-fail, rigidity/ablation) and by deterministic custody.

This is the standard that prevents later continuation from being read as engineering.