NT-3 — Structural Invariants, Rosetta Normalization, and Symmetry-Constrained Fixed-Point Selection

A base-portable invariant algebra on DRPT/CRT substrates,
and a finite selection calculus that forces unique survivors

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Abstract

This paper formalizes the invariant algebra that underlies the Marithmetics number theory track. Starting from explicitly finite substrates—residue rings $\mathbb{Z}_d$ with $d = b - 1$, digital-root power tables (DRPT), Chinese-remainder (CRT) wheels, and DOC-admissible operators—we define a structural alphabet of normalized invariants $(\hat{\chi}, \hat{\theta}, \hat{\Psi}, \hat{\kappa})$. These normalized quantities are constructed to be base-portable: they remove representation artifacts while preserving intrinsic cycle, density, circulation, and envelope structure. We then define the universal invariant template Φ as a rational assembly of $(\hat{\chi}, \hat{\theta}, \hat{\Psi}, \hat{\kappa})$, and present Symmetry-Constrained Fixed-Point selection (SCFP / SCFP++) as a finite elimination dynamics on the space of candidate assemblies. The SCFP constraints (principal survival, base invariance, inverse symmetry, maximal period with extremal circulation, minimal wheel exactness, and universal finite envelope) are stated as explicit predicates. Selection is deterministic on any bounded enumeration domain; when a survivor exists, it is uniquely determined as a finite object. This paper supplies the definitions and theorems required to treat "fixed-point emergence" as a mathematical procedure rather than an interpretive narrative. Computed demonstrations and ledgers are cited through a single Authority-of-Record archive.

Contents

1 Reader Contract

1.1 Scope and proof standard

All mathematical objects in this paper are explicitly finite: finite rings, finite groups, finite tables, finite operators, and finite enumerations. All theorems are therefore formalizable in ZFC as bounded constructions.

Deterministic Operator Calculus (DOC) is used as a discipline governing which operators are admissible when smoothing, measurement, or suppression is invoked. DOC is not used as an axiom system. Whenever DOC language is used, the underlying claim is a finite statement about a finite matrix (typically circulant, symmetric, positive semidefinite, and mass-preserving) and its spectrum.

1.2 Separation of claims

This paper distinguishes:

1. **Mathematical claims** (theorems and definitions on finite substrates).
2. **Evidence claims** (numerical logs, tables, plots, and certificates produced by code runs).

Mathematical claims stand on proof. Evidence claims are cited to a single, public Authority-of-Record (AoR) bundle through stable URLs.

1.3 Evidence capsule (single citation surface)

All citations to computed artifacts in this paper refer to the AoR with tag `aor-20260209T040755Z`, folder `gum/authority_archive/AOR_20260209T040755Z_0fc79a0`, and bundle SHA-256:

`c299b1a7a8ef77f25c3ebb326cb73f060b3c7176b6ea9eb402c97273dc3cf66c`

2 Introduction

2.1 The invariance problem in finite mathematics

Finite computations are vulnerable to non-intrinsic structure: patterns can depend on numeral base, indexing choices, or operator conventions. The problem addressed in this paper is to define a family of invariants that is:

1. **Finitary** (computed from bounded finite objects).
2. **Representation-stable** (invariant under admissible base/encoding changes).
3. **Operator-stable** (invariant under admissible smoothing/measurement operations).

The number-theory track uses digital-root residue rings $\mathbb{Z}_d$ (with $d = b - 1$) and the DRPT substrate (NT-2) as its base object. However, raw DRPT quantities are not automatically comparable across bases. A principled portability layer is required.

2.2 The four-letter substrate alphabet

In Marithmetics, the invariant algebra is organized around four families of quantities:

- χ: cycle structure (orders / periods) in the unit group of $\mathbb{Z}_d$.
- θ: survivor densities arising from CRT wheels (finite sieves).

- Ψ: circulation functionals on periodic sequences (cycle-weighted invariants).
- κ: universal finite envelopes controlling admissible compositions (operator bounds).

The central design move is *normalization*: we pass from raw $(\chi, \theta, \Psi, \kappa)$ to normalized $(\hat{\chi}, \hat{\theta}, \hat{\Psi}, \hat{\kappa})$ so that comparisons across bases and across admissible operator stacks have a common language.

2.3 From invariant templates to forced fixed points

Once a normalized alphabet is fixed, one may assemble dimensionless invariants by rational expressions. This is formalized as a template:

$$\Phi \;=\; F(\hat{\chi}, \hat{\theta}, \hat{\Psi}, \hat{\kappa}),$$

where F is an explicit rational map (or a finite family of such maps, indexed by channel constraints).

The critical question is not "can one assemble Φ," but "what forces a unique assembly." SCFP / SCFP++ answers this by imposing a small list of explicit constraints (Section **??**). Mathematically, the selection mechanism is a finite elimination system acting on a bounded candidate set; when a survivor exists, it is uniquely determined.

2.4 Position in the number theory track

- **NT-1** establishes admissible operator classes (DOC) on finite cyclic substrates and the transfer discipline used throughout the project.
- **NT-2** constructs DRPT geometry, splinter partitions, survivor fields, admissible window measurements, and finite tiling laws.
- **NT-3** (this paper) defines the normalized invariant alphabet, the universal template Φ, and SCFP selection as a deterministic finite elimination mechanism.
- Downstream papers use these invariants and the selection calculus to (i) state suppression and correlation bounds on CRT survivor sets, (ii) formalize finite-to-continuum residual laws, and (iii) transport results across bases without ambiguity.

3 Finite Substrates and Admissible Transforms

3.1 Base, modulus, and the residue substrate

Fix an integer base $b \geq 2$ and set

$$d \;:=\; b - 1.$$

All residue dynamics in this track occur in the finite ring $\mathbb{Z}_d$.

The unit group is

$$U_d \;:=\; (\mathbb{Z}_d)^\times \;=\; \{x \in \mathbb{Z}_d : \gcd(x, d) = 1\}.$$

The Carmichael exponent $\lambda(d)$ is the exponent of U_d: for every unit $u \in U_d$,

$$u^{\lambda(d)} \equiv 1 \pmod{d},$$

and $\lambda(d)$ is minimal with this property.

3.2 DRPT cycles and cycle orders

For $x \in \mathbb{Z}_d$, define the power sequence

$$P_d(x) := (x^k \bmod d)_{k\geq 1}.$$

If $x \in U_d$, the sequence is purely periodic, and its minimal period equals the multiplicative order $\mathrm{ord}_d(x)$, satisfying $\mathrm{ord}_d(x) \mid \lambda(d)$.

If $x \notin U_d$, the behavior is controlled by CRT splitting (NT-2, Section 2.6): powers collapse to 0 in each prime-power component where x is non-unit; globally, the sequence is eventually periodic and lands in the CRT ideal determined by the prime divisors of x.

3.3 CRT wheels and survivor densities

Let M be a squarefree modulus (typically a primorial wheel $M_y := \prod_{p\leq y} p$). Define the survivor indicator on $\mathbb{Z}$:

$$S_M(n) \;:=\; \mathbf{1}_{\gcd(n,M)=1}.$$

The survivor density is

$$\theta(M) \;:=\; \frac{\varphi(M)}{M} \;=\; \prod_{p\mid M}\left(1-\frac{1}{p}\right).$$

A wheel is called *window-exact up to the next prime square* if, on the interval $[2, p_{\text{next}}^2)$ where p_{next} is the next prime after the largest prime dividing M, the survivor set $\{n : S_M(n) = 1\}$ contains no composites; equivalently, it coincides with primes on that interval after excluding 1. This is the standard finite exactness property of the Eratosthenes sieve, expressed in CRT language.

3.4 Circulation functionals on periodic sequences

Let $C = (c_0, c_1, \ldots, c_{L-1})$ be a real periodic sequence of period L (typically a DRPT cycle of a unit element). Let $w = (w_0, \ldots, w_{L-1})$ be a nonnegative weight vector with $\sum_i w_i = 1$. Define the *weighted circulation functional*

$$\Psi_w(C) \;:=\; \sum_{i=0}^{L-1} w_i\, c_i,$$

or, when a signed circulation is required, replace c_i by a signed increment functional (for example, discrete derivative or oriented edge sum). The only requirement at the level of this paper is finiteness and explicitness: C and w are finite objects and $\Psi_w(C)$ is a finite rational (or algebraic) number when C is integer-valued and w is rational.

3.5 Envelopes and admissible operator stacks

Whenever the construction of Φ requires smoothing, suppression, or transfer, the operator stack must be DOC-admissible (NT-1). Concretely, this means the relevant finite matrices are required to be:

1. Circulant (convolution form on $\mathbb{Z}_M$).
2. Symmetric and positive semidefinite (Fourier multipliers in $[0, 1]$).
3. Mass-preserving ($T\mathbf{1} = \mathbf{1}$).

The envelope κ is a universal finite bound associated to an operator class or composition stack: it is defined so that admissible stacks satisfy a uniform inequality (e.g., an energy or residual bound) controlled by κ.

3.6 Rosetta normalization: base-portable invariants

The Rosetta layer (formalized in Section 5) supplies a deterministic pipeline that translates raw invariants across bases by normalization. The key point for the remainder of this paper is that every invariant used in Φ is replaced by a normalized version:

- $\hat{\chi}$: cycle order normalized by $\lambda(d)$.
- $\hat{\theta}$: survivor density normalized by a fixed reference density $\theta_\star$ appropriate to the channel.
- $\hat{\Psi}$: circulation normalized by a fixed reference circulation $\Psi_\star$ appropriate to the channel.
- $\hat{\kappa}$: envelope normalized by a fixed universal envelope $\kappa_\star$ appropriate to the operator class.

This yields a common alphabet for comparing bases and for enforcing SCFP constraints without representation leakage.

4 The Structural Invariant Alphabet $\mathcal{S}_{\text{struct}}$

4.1 Admissible equivalences and what "invariant" means here

This track uses invariance in a strict, finite sense.

Definition 4.1 (Admissible equivalence)**.** Fix a base $b \geq 2$ and $d = b - 1$. An *admissible equivalence* on any finite object X built from $\mathbb{Z}_d$, CRT wheels, DRPT plates, or DOC-admissible operators is generated by compositions of:

1. **Finite reindexings**: bijections of a finite index set (row/column relabelings, chart changes, CRT coordinate changes).
2. **CRT isomorphisms**: the canonical ring isomorphism $\mathbb{Z}_d \cong \prod_j \mathbb{Z}_{p_j^{a_j}}$ and its induced permutations of components.
3. **DOC-admissible operator transforms**: acting on a field by a DOC-admissible convolution operator on $\mathbb{Z}_M$ (NT-1), and then restricting/renormalizing to the finite index set under study (NT-2, Section 4.2).
4. **Rosetta base-gauge transforms**: changing base $b \mapsto b'$ and translating the construction through normalized invariants so that channel-meaning is preserved.

Definition 4.2 (Structural invariant)**.** A *structural invariant* is a function I from a class of finite objects to a codomain (typically $\mathbb{Q}$ or a finite set) such that

$$X \sim_{\text{adm}} Y \quad \Longrightarrow \quad I(X) = I(Y).$$

In this paper, "invariant" always means invariant under admissible equivalence.

4.2 The alphabet as four normalized families

The invariant alphabet used downstream is organized into four families, each with a raw form and a normalized ("hatted") form. The normalization is what makes cross-base comparison meaningful.

4.2.1 Cycle indices χ and normalized cycle indices $\hat{\chi}$

Let $U_d = (\mathbb{Z}_d)^\times$. For $u \in U_d$, define the multiplicative order

$$\chi_d(u) \;:=\; \mathrm{ord}_d(u) \qquad (\text{so } u^{\chi_d(u)} \equiv 1 \pmod{d} \text{ and } \chi_d(u) \text{ is minimal}).$$

Let $\lambda(d)$ be the Carmichael exponent of U_d.

Definition 4.3 (Normalized cycle index)**.** For $u \in U_d$,

$$\hat{\chi}_d(u) \;:=\; \frac{\chi_d(u)}{\lambda(d)} \;\in\; \mathbb{Q} \cap (0,1].$$

Lemma 4.4 (Existence of maximal-order units)**.** *Because U_d is a finite abelian group, there exists $u_\star \in U_d$ with*

$$\chi_d(u_\star) = \lambda(d),$$

and therefore $\hat{\chi}_d(u_\star) = 1$.

Proof. Write $U_d \cong C_{n_1} \times \cdots \times C_{n_k}$ with $n_1 \mid n_2 \mid \cdots \mid n_k$. The exponent equals n_k. The element whose coordinates are generators in each cyclic factor has order n_k. □

Definition 4.5 (Inverse-symmetric package)**.** A finite subset $P \subseteq U_d$ is *inverse-symmetric* if $u \in P \Rightarrow u^{-1} \in P$.

4.2.2 CRT survivor densities θ and normalized densities $\hat{\theta}$

Let $M \geq 1$ be a CRT wheel modulus (typically squarefree), with factorization $M = \prod_{p \mid M} p$. Define the wheel survivor indicator

$$S_M(n) \;:=\; \mathbf{1}_{\gcd(n,M)=1}.$$

Its density exists and equals

$$\theta(M) \;:=\; \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} S_M(n) \;=\; \frac{\varphi(M)}{M} \;=\; \prod_{p \mid M} \left(1 - \frac{1}{p}\right).$$

Definition 4.6 (Wheel exactness window)**.** Let $p_{\max}$ be the largest prime dividing M, and let p_{next} be the next prime after $p_{\max}$. We say the wheel M is *exact on* $[2, p_{\text{next}}^2)$ if

$$\{n \in [2, p_{\text{next}}^2) : \; S_M(n) = 1\}$$

contains no composite integers; equivalently, it coincides with the primes in that interval.

Definition 4.7 (Normalized survivor density)**.** Fix a reference wheel $M_\star$ appropriate to the channel (declared explicitly when the channel is declared). Define

$$\hat{\theta}(M) \;:=\; \frac{\theta(M)}{\theta(M_\star)}.$$

When the SCFP selector chooses $M = M_\star$ by minimality (Constraint C5), $\hat{\theta} = 1$ by construction.

4.2.3 Circulation functionals Ψ and normalized circulations $\hat{\Psi}$

Let $C = (c_0, \ldots, c_{L-1})$ be a finite periodic cycle, with entries in $\mathbb{Z}_d$ or in $\mathbb{R}$ after embedding $\mathbb{Z}_d \hookrightarrow \{0, 1, \ldots, d-1\} \subset \mathbb{R}$. Let $w = (w_0, \ldots, w_{L-1})$ be a weight vector with:

$$w_i \geq 0, \qquad \sum_{i=0}^{L-1} w_i = 1.$$

Definition 4.8 (Circulation functional)**.** The (weighted) circulation is

$$\Psi_w(C) \;:=\; \sum_{i=0}^{L-1} w_i\, c_i.$$

When an oriented circulation is required, c_i is replaced by an oriented increment $c_{i+1} - c_i$ (modulo d, then embedded into a symmetric integer interval); the choice is declared in the channel specification.

Definition 4.9 (Normalized circulation)**.** Fix a reference circulation value $\Psi_\star$ for the channel (declared explicitly). Define

$$\hat{\Psi} \;:=\; \frac{\Psi_w(C)}{\Psi_\star}.$$

As with $\hat{\theta}$, SCFP is designed to select the canonical pair (C, w) so that $\hat{\Psi} = 1$ when a reference gauge is chosen.

4.2.4 Universal envelopes κ and normalized envelopes $\hat{\kappa}$

The role of κ is to bound non-principal structure uniformly across admissible transforms.

Definition 4.10 (Envelope for an admissible family)**.** Let $\mathcal{G}$ be a finite family of admissible objects (kernels, windows, or bilinear phases), and let $E(g) \geq 0$ be a nonnegative "error size" extracted from $g \in \mathcal{G}$ by an explicit finite formula. A *universal envelope constant* $\kappa(\mathcal{G})$ is any bound satisfying

$$E(g) \;\leq\; \kappa(\mathcal{G}) \qquad \text{for all } g \in \mathcal{G}.$$

If the program declares a specific $\kappa_\star$ as the tightest validated universal envelope for a channel, we normalize by

$$\hat{\kappa} \;:=\; \frac{\kappa(\mathcal{G})}{\kappa_\star}.$$

4.3 Principal survival as a finite DOC theorem

SCFP Constraint C1 ("principal-character survival") is enforced in software as a filter, but it is rooted in a finite theorem in DOC: admissible smoothing and admissible measurement do not allow persistent non-principal Fourier mass unless that mass is structurally forced.

Let $K : \mathbb{Z}_M \to \mathbb{R}_{\geq 0}$ be a DOC-admissible kernel: $\sum_{x \in \mathbb{Z}_M} K(x) = 1$, $K(x) = K(-x)$, and its discrete Fourier multipliers satisfy $0 \leq \widehat{K}(\xi) \leq 1$ for all $\xi \in \mathbb{Z}_M$. Let T_K be convolution by K acting on $f : \mathbb{Z}_M \to \mathbb{C}$.

Lemma 4.11 (Fourier action of DOC kernels)**.** *For each frequency $\xi \in \mathbb{Z}_M$,*

$$\widehat{T_K f}(\xi) \;=\; \widehat{K}(\xi)\, \widehat{f}(\xi).$$

Proof. This is the standard diagonalization of circulant operators by the discrete Fourier basis. $\square$

Definition 4.12 (Principal component)**.** The *principal component* of f is its $\xi = 0$ Fourier coefficient $\widehat{f}(0)$, i.e. its mean:

$$\widehat{f}(0) = \sum_{x \in \mathbb{Z}_M} f(x).$$

Non-principal components are the coefficients $\widehat{f}(\xi)$ for $\xi \neq 0$.

Theorem 4.13 (Only-principal survival under strict contraction)**.** *Assume K is DOC-admissible and satisfies*

$$\rho(K) \;:=\; \max_{\xi \neq 0} \widehat{K}(\xi) \;<\; 1.$$

Then for every $f : \mathbb{Z}_M \to \mathbb{C}$,

$$\widehat{T_K^t f}(\xi) \;=\; \widehat{K}(\xi)^t\, \widehat{f}(\xi) \quad \textit{and hence} \quad \lim_{t \to \infty} \widehat{T_K^t f}(\xi) = 0 \textit{ for every } \xi \neq 0.$$

In particular, repeated admissible smoothing eliminates all non-principal Fourier mass.

Proof. Iterate Lemma 4.11:

$$\widehat{T_K^t f}(\xi) = \widehat{K}(\xi)^t\, \widehat{f}(\xi).$$

If $\xi \neq 0$, then $|\widehat{K}(\xi)| \leq \rho(K) < 1$, so $\widehat{K}(\xi)^t \to 0$. □

Remark 4.14 (Meaning for SCFP C1)**.** This theorem is the finite, operator-level content of "principal survival": if a construction requires persistent non-principal spectral mass to remain stable under admissible smoothing/measurement, then it is not invariant under the admissible transform class and is therefore excluded as non-structural.

4.4 Definition of $\mathcal{S}_{\text{struct}}$

Definition 4.15 (Structural invariant alphabet)**.** Let $\mathcal{S}_{\text{struct}}$ be the smallest collection of quantities containing:

1. **Cycle family**: $\lambda(d)$, $\chi_d(u)$, $\hat{\chi}_d(u)$, and inverse-symmetric package aggregates such as

$$\hat{\chi}(P) \;:=\; \sum_{u \in P} \hat{\chi}_d(u) \quad \text{for finite inverse-symmetric } P \subseteq U_d.$$

2. **Density family**: wheel densities $\theta(M) = \varphi(M)/M$, the exactness predicate on $[2, p_{\text{next}}^2)$, and normalized densities $\hat{\theta}(M) = \theta(M)/\theta(M_\star)$.

3. **Circulation family**: $\Psi_w(C)$ for declared admissible weight classes $\mathcal{W}_\Psi$ and declared cycle extraction rules C, and normalized circulations $\hat{\Psi} = \Psi/\Psi_\star$.

4. **Envelope family**: declared universal envelopes $\kappa(\mathcal{G})$ for declared admissible families $\mathcal{G}$, together with their normalized versions $\hat{\kappa} = \kappa/\kappa_\star$.

and closed under finite rational arithmetic (addition, subtraction, multiplication, division by nonzero rationals), provided the resulting expression is dimensionless.

This closure rule is not merely aesthetic: SCFP uniqueness is stated on a finite enumeration of such expressions.

5 The Universal Template Φ and Rosetta Base-Portability

5.1 The Rosetta alphabet $(\hat{\chi}, \hat{\theta}, \hat{\Psi}, \hat{\kappa})$

Fix a base $b \geq 2$ and write $d = b - 1$. The raw invariant alphabet of NT-3 contains cycle data (χ), sieve densities (θ), circulations (Ψ), and admissibility envelopes (κ). To compare constructions across bases without importing representational artifacts, we work with normalized (dimensionless) versions.

Definition 5.1 (Rosetta normalization, base b)**.** Let $d = b - 1$. Fix:

- a declared unit package $P \subseteq U_d$ (typically inverse-symmetric),
- a declared CRT wheel modulus M (squarefree unless otherwise stated),
- a declared circulation channel (C, w) (cycle extraction rule C with admissible weight w),
- a declared envelope class $\mathcal{G}$ with envelope functional E and global bound $\kappa(\mathcal{G})$.

Define the *normalized Rosetta invariants* by:

1. **Cycle normalization**:
$$\hat{\chi}_d(u) \;:=\; \frac{\mathrm{ord}_d(u)}{\lambda(d)} \in \mathbb{Q} \cap (0, 1] \qquad (u \in U_d),$$
and (when a package aggregate is required)
$$\hat{\chi}(P) \;:=\; \sum_{u \in P} \hat{\chi}_d(u).$$

2. **Density normalization**:
$$\theta(M) \;:=\; \frac{\varphi(M)}{M} \qquad \text{and} \qquad \hat{\theta}(M) \;:=\; \frac{\theta(M)}{\theta(M_\star)},$$
where $M_\star$ is a declared reference wheel for the channel (fixed globally, not per base).

3. **Circulation normalization**:
$$\Psi \;:=\; \Psi_w(C), \qquad \hat{\Psi} \;:=\; \frac{\Psi}{\Psi_\star},$$
where $\Psi_\star$ is a declared reference circulation (fixed globally for the channel).

4. **Envelope normalization**:
$$\kappa \;:=\; \kappa(\mathcal{G}), \qquad \hat{\kappa} \;:=\; \frac{\kappa}{\kappa_\star},$$
where $\kappa_\star$ is the declared universal envelope bound for the operator/analytic class used by the channel.

The quadruple
$$\mathcal{R}_b \;:=\; (\hat{\chi}, \hat{\theta}, \hat{\Psi}, \hat{\kappa})$$
is the *Rosetta signature* of the channel in base b.

5.2 Cross-base injectivity from CRT

Definition 5.2 (Cross-base digital-root signature)**.** Let $B = \{b_1, \ldots, b_s\}$ be a finite set of bases with $d_i := b_i - 1$. Define the *residue signature map*

$$\sigma_B : \mathbb{Z} \to \prod_{i=1}^{s} \mathbb{Z}_{d_i}, \qquad \sigma_B(n) \ := \ \big(n \bmod d_1, \ldots, n \bmod d_s\big).$$

Theorem 5.3 (CRT injectivity of cross-base signatures on a fundamental window)**.** *Assume the moduli $d_1, \ldots, d_s$ are pairwise coprime:*

$$\gcd(d_i, d_j) = 1 \quad (i \neq j).$$

Let

$$D \ := \ \prod_{i=1}^{s} d_i.$$

Then σ_B is injective on the integer interval $[0, D)$. Equivalently, if $0 \le n, m < D$ and $\sigma_B(n) = \sigma_B(m)$, then $n = m$.

Proof. If $\sigma_B(n) = \sigma_B(m)$, then $n \equiv m \pmod{d_i}$ for every i, hence $n \equiv m \pmod{D}$ by the Chinese remainder theorem. Since $0 \le n, m < D$, congruence modulo D forces equality, so $n = m$. ☐

5.3 Template grammar and dimensionless legality

Definition 5.4 (Admissible template grammar)**.** Let $\mathcal{A} = \{\hat{\chi}, \hat{\theta}, \hat{\Psi}, \hat{\kappa}\}$ denote the Rosetta alphabet for a declared channel. An *admissible template* is any expression Φ obtained from finitely many elements of $\mathcal{A}$ and finitely many rational constants by a finite number of operations from the set:

$$\{+,\ -,\ \times,\ /,\ (\cdot)^n \text{ for } n \in \mathbb{Z}\}$$

subject to:

1. division is only by nonzero rational expressions on the declared domain;
2. the resulting Φ is dimensionless (no anchors or observational constants appear as inputs).

We write

$$\Phi \ = \ F(\hat{\chi}, \hat{\theta}, \hat{\Psi}, \hat{\kappa})$$

to emphasize that Φ is determined by a finite rational map F applied to the normalized invariant signature.

5.4 Rosetta portability and Φ-invariance

Definition 5.5 (Rosetta portability across a base set)**.** Let B be a finite set of bases. A declared channel is *Rosetta-portable across* B if the normalized signature is constant across B:

$$\mathcal{R}_b \ = \ \mathcal{R}_{b'} \qquad \text{for all } b, b' \in B.$$

Theorem 5.6 (Template invariance under portable signatures)**.** *Let $\Phi = F(\hat{\chi}, \hat{\theta}, \hat{\Psi}, \hat{\kappa})$ be an admissible template (Definition 5.4). If the channel is Rosetta-portable across a base set B (Definition 5.5), then Φ is base-invariant across B:*

$$\Phi(b) = \Phi(b') \qquad \textit{for all } b, b' \in B.$$

Proof. If $\mathcal{R}_b = \mathcal{R}_{b'}$, then $F(\mathcal{R}_b) = F(\mathcal{R}_{b'})$ by determinism of evaluation of a rational expression on equal inputs. ☐

5.5 Anchor isolation

Principle 5.7 (Anchor isolation)**.** Anchors (unit conversions, SI calibrations, or observational scalings) are not permitted as inputs to the SCFP selection of a dimensionless Φ. Anchors may be applied only after a unique dimensionless $\Phi^\star$ is fixed by the admissible constraints. This enforces non-circularity: the constant is not smuggled in by a unit choice.

5.6 Derivation Path / Verification Protocol (DP/VP)

Definition 5.8 (Rosetta DP/VP protocol)**.** A *Rosetta protocol* is a finite sequence of stages $(R1, \ldots, R7)$ such that each stage produces:

1. a finite artifact (table, JSON, CSV, or text ledger),
2. a SHA-256 hash for that artifact,
3. an explicit declaration of the inputs and normalization gauges used.

A canonical instantiation is:

- **R1**: build DRPT objects and record cycle tables;
- **R2**: extract $\hat{\chi}$ from unit packages;
- **R3**: build CRT wheel(s) and compute $\hat{\theta}$;
- **R4**: compute circulation(s) and $\hat{\Psi}$;
- **R5**: certify the envelope class and compute $\hat{\kappa}$;
- **R6**: assemble Φ and persist formula + evaluation trace;
- **R7**: apply anchors (optional, and only downstream of R6).

C1. Principal-character survival (suppression). A candidate $\mathfrak{c}$ satisfies C1 if, for every DOC-admissible smoothing/measurement operator required by $\mathfrak{c}$, the construction is stable under principal-projection in the precise sense that no non-principal spectral component is required to persist.

C2. Base invariance (Rosetta portability). A candidate $\mathfrak{c}$ satisfies C2 if the normalized signature is constant across the declared base set B:

$$\mathcal{R}_b(\mathfrak{c}) \;=\; \mathcal{R}_{b'}(\mathfrak{c}) \qquad \text{for all } b, b' \in B.$$

Equivalently (by Theorem 5.6), every admissible template $\Phi_b(\mathfrak{c})$ is base-invariant across B.

C3. Inverse symmetry (unit reciprocity). A candidate $\mathfrak{c}$ satisfies C3 if its unit package $P \subseteq U_d$ is inverse-symmetric:

$$u \in P \;\Rightarrow\; u^{-1} \in P,$$

and if its circulation weighting respects reciprocity (weights equal on inverse-paired cycle positions under the cycle extraction rule).

C4. Maximal period with extremal circulation. Let $\lambda(d)$ be the exponent of U_d. A candidate $\mathfrak{c}$ satisfies C4 if, among all inverse-symmetric admissible packages in the library, its package P attains maximal cycle order (in the sense that at least one $u \in P$ has $\mathrm{ord}_d(u) = \lambda(d)$ when such a unit exists), and if, under the admissible weight class, the induced circulation $\Psi_w(C)$ is extremal among candidates that satisfy C1–C3.

C5. Minimal wheel exactness. A candidate $\mathfrak{c}$ satisfies C5 if its wheel M is (i) compatible with the symmetry/period structure required by P, (ii) window-exact up to the next prime-square cutoff (Definition 4.6), and (iii) minimal in the declared wheel library among wheels satisfying (i)–(ii).

C6. Universal finite envelope. A candidate $\mathfrak{c}$ satisfies C6 if it uses a declared universal envelope bound $\kappa_\star$ for its composition class. Any candidate requiring a weaker (larger) envelope bound that is not validated as universal is rejected.

5.7 SCFP selection as deterministic elimination

Define the *admissible survivor set* by

$$\mathcal{A} \;:=\; \{\mathfrak{c} \in \mathcal{C} :\ \mathfrak{c} \text{ satisfies C1–C6}\}.$$

Definition 5.9 (SCFP fixed point)**.** If $|\mathcal{A}| = 1$, write $\mathcal{A} = \{\mathfrak{c}^\star\}$. The resulting scalar

$$\Phi^\star \;:=\; \Phi_b(\mathfrak{c}^\star)$$

(which is independent of $b \in B$ by C2) is called the *SCFP fixed point* of the channel.

If $|\mathcal{A}| = 0$, the channel fails (a falsifier is triggered). If $|\mathcal{A}| > 1$, the channel is underconstrained (a different falsifier is triggered).

Proposition 5.10 (Finiteness of the candidate space)**.** *Fix a channel specification with bounded libraries $\mathcal{P}, \mathcal{M}, \mathcal{W}_\Psi, \mathcal{K}$ and a bounded template grammar depth producing a finite $\mathcal{F}$. Then the candidate space*

$$\mathcal{C} = \mathcal{P} \times \mathcal{M} \times \mathcal{W}_\Psi \times \mathcal{K} \times \mathcal{F}$$

is finite. Consequently, the survivor set $\mathcal{A} \subseteq \mathcal{C}$ is finite.

Proof. A finite Cartesian product of finite sets is finite. □

Proposition 5.11 (Determinism of SCFP filtering)**.** *Assume all libraries are enumerated in a fixed, declared order (lexicographic by a canonical encoding), and all predicates C1–C6 are evaluated by deterministic finite computations on the encoded objects. Then the SCFP survivor set $\mathcal{A}$ is a deterministic function of the declared inputs.*

Proof. Under the assumptions, each predicate evaluation is deterministic on each candidate $\mathfrak{c}$. The survivor set is defined by a finite conjunction of such predicates, hence is deterministic. □

Definition 5.12 (SCFP certificate)**.** An *SCFP certificate* for a channel is the finite data record consisting of:

1. **Spec**: the declared channel specification (base set B, libraries, normalization gauges, template grammar depth).
2. **Counts**: the cardinalities after each filter stage,

$$|\mathcal{C}| \;\to\; |\mathcal{C}_{C3}| \;\to\; |\mathcal{C}_{C4}| \;\to\; \cdots \;\to\; |\mathcal{A}|.$$

3. **Survivor list**: the full canonical encoding of every survivor $\mathfrak{c} \in \mathcal{A}$ (or the empty list).
4. **Evaluation traces**: for each survivor $\mathfrak{c}$, the explicit computed values of $\mathcal{R}_b(\mathfrak{c})$ for every $b \in B$, and the resulting $\Phi^\star$.
5. **Hashes**: SHA-256 hashes of all intermediate tables used to compute (2)–(4).

The certificate is PASS if $|\mathcal{A}| = 1$, and FAIL if $|\mathcal{A}| \neq 1$.

5.8 SCFP++: Stress Bases, Negative Controls, and Tie-Break Closure

SCFP++ is the version of the selection engine intended for external referees.

Definition 5.13 (Stress-base set)**.** A *stress-base set* B_{stress} is a finite set of bases used solely to pressure-test C2. It is declared separately from the minimal base set B used in the primary proof, and it is required to include moduli $d = b - 1$ of qualitatively different arithmetic types (prime, prime-power, highly composite).

Definition 5.14 (Negative controls)**.** A *negative control* is a deliberately inadmissible construction whose failure mode is predicted in advance. The SCFP++ certificate must include a catalog of negative controls and their failure logs.

Definition 5.15 (Auditable tie-break rule; SCFP++)**.** If the survivor set after C1–C6 is nonempty but contains more than one element due to declared equivalence, SCFP++ applies a deterministic tie-break:

$$\mathfrak{c}^\star = \arg\min_{\mathfrak{c}\in\mathcal{A}} \mathsf{DL}(\mathfrak{c}),$$

where DL is the bit-length of the canonical encoding of $\mathfrak{c}$ under the project's declared encoding scheme.

5.9 Designed FAIL and the falsifier schema

Definition 5.16 (SCFP falsifiers)**.** A channel is *falsified* if any of the following occur:

F1. Base drift: the normalized signature $\mathcal{R}_b$ is not constant across a declared base set (violation of C2).

F2. Multiplicity: $|\mathcal{A}| > 1$ after C1–C6 (underconstraint).

F3. Empty survivor: $|\mathcal{A}| = 0$ (overconstraint or incompatible channel).

F4. Non-principal dependence: a candidate requires persistent non-principal spectral mass (violation of C1).

F5. Wheel non-minimality: a strictly more minimal wheel exists satisfying the same declared exactness predicate (violation of C5).

F6. Non-universal envelope: a candidate requires an envelope bound not certified as universal for the declared operator/analytic class (violation of C6).

6 Worked Canonical Channels (Dimensionless Stage)

This section illustrates how the abstract apparatus reduces to concrete, finite, verifiable workflows. The emphasis here is dimensionless Φ selection.

6.1 Channel specification template (what must be declared)

A channel declaration must contain:

1. **Base set(s)**: a minimal base set B used for the proof, optionally a stress-base set B_{stress} used for SCFP++.
2. **Structural mapping rule (Rosetta rule)**: a rule for mapping "the same structural object" across bases, explicitly forbidding literal-digit transport.
3. **Libraries**: bounded libraries $\mathcal{P}, \mathcal{M}, \mathcal{W}_\Psi, \mathcal{K}$ with canonical enumerations.

4. **Normalization gauges**: declared reference objects $(M_\star, \Psi_\star, \kappa_\star)$ used to define $(\hat{\theta}, \hat{\Psi}, \hat{\kappa})$.
5. **Template grammar depth**: an explicit finite depth for enumerating admissible rational expressions F.

6.2 Example channel: maximal generator + inverse (a minimal SCFP demonstration)

Channel 6.2.A (Generator–inverse channel). Fix any base $b \geq 2$ and $d = b - 1$. Let $U_d = (\mathbb{Z}_d)^\times$ and $\lambda(d)$ its exponent. Declare:

- $\mathcal{P}$: all inverse-symmetric packages $P = \{u, u^{-1}\} \subseteq U_d$.
- $\mathcal{M}$: the singleton wheel $M_\star$ (declared reference; set $\hat{\theta} = 1$ by definition).
- $\mathcal{W}_\Psi$: the singleton reference circulation rule (declared reference; set $\hat{\Psi} = 1$ by definition).
- $\mathcal{K}$: the singleton universal envelope class (declared reference; set $\hat{\kappa} = 1$ by definition).
- $\mathcal{F}$: the singleton template
$$\Phi \ = \ \hat{\chi}(u) + \hat{\chi}(u^{-1}).$$

Theorem 6.1 (Forced value of Φ in Channel 6.2.A)**.** *If there exists $u \in U_d$ with $\mathrm{ord}_d(u) = \lambda(d)$, then SCFP yields the unique survivor package $P^\star = \{u, u^{-1}\}$ (up to the choice of a maximal-order unit), and the channel value is*
$$\Phi^\star \ = \ 2.$$

Proof. Let $u \in U_d$ satisfy $\mathrm{ord}_d(u) = \lambda(d)$. Then
$$\hat{\chi}(u) = \frac{\mathrm{ord}_d(u)}{\lambda(d)} = 1.$$
Since $\mathrm{ord}_d(u^{-1}) = \mathrm{ord}_d(u)$, we also have $\hat{\chi}(u^{-1}) = 1$. Therefore $\Phi = 1 + 1 = 2$.

C4 rejects any package $\{v, v^{-1}\}$ with $\mathrm{ord}_d(v) < \lambda(d)$, because then $\Phi < 2$. Hence every SCFP survivor must be a maximal-order inverse pair and yields $\Phi^\star = 2$. □

6.3 Concrete base-10 instantiation (explicit cycle table)

Let $b = 10$, so $d = b - 1 = 9$. The unit group is
$$U_9 = \{1, 2, 4, 5, 7, 8\}.$$

A short computation gives the orders:

- $2^1 \equiv 2,\ 2^2 \equiv 4,\ 2^3 \equiv 8,\ 2^4 \equiv 7,\ 2^5 \equiv 5,\ 2^6 \equiv 1 \pmod 9$, hence $\mathrm{ord}_9(2) = 6$.
- $5^1 \equiv 5,\ 5^2 \equiv 7,\ 5^3 \equiv 8,\ 5^4 \equiv 4,\ 5^5 \equiv 2,\ 5^6 \equiv 1 \pmod 9$, hence $\mathrm{ord}_9(5) = 6$.
- $4^1 \equiv 4,\ 4^2 \equiv 7,\ 4^3 \equiv 1 \pmod 9$, hence $\mathrm{ord}_9(4) = 3$.
- $7^1 \equiv 7,\ 7^2 \equiv 4,\ 7^3 \equiv 1 \pmod 9$, hence $\mathrm{ord}_9(7) = 3$.
- $8^1 \equiv 8,\ 8^2 \equiv 1 \pmod 9$, hence $\mathrm{ord}_9(8) = 2$.
- $\mathrm{ord}_9(1) = 1$.

Therefore $\lambda(9) = 6$. Inverses satisfy:

$$2^{-1} \equiv 5, \qquad 4^{-1} \equiv 7, \qquad 8^{-1} \equiv 8 \pmod 9.$$

Under C3–C4, the only inverse-symmetric package achieving maximal order is $\{2, 5\}$. Thus

$$\hat{\chi}(2) = \hat{\chi}(5) = \frac{6}{6} = 1, \qquad \Phi^\star = 2.$$

Negative control (label drift). If one attempts to reuse the labels "2" and "5" across a different base b' without mapping by maximal order, then in general $\mathrm{ord}_{b'-1}(2)$ and $\mathrm{ord}_{b'-1}(5)$ are not both maximal and $\hat{\chi}(2) + \hat{\chi}(5) \neq 2$. This is the canonical C2 falsifier: channel definitions transport structure, not numerals.

6.4 A-39 (Area Law) as the analytic envelope prototype

In SCFP, the envelope κ is the mechanism that prevents "context-dependent constants." In analytic number theory, the required envelope often takes the form of a uniform bound on bilinear exponential sums under window localization.

6.4.1 Setup (windows, arcs, and dyadic boxes)

Let M_1, M_2 be coprime positive integers and set

$$M := M_1 M_2, \qquad \gcd(M_1, M_2) = 1.$$

Let $I_M = [M, 2M] \cap \mathbb{Z}$ and $I_N = [N, 2N] \cap \mathbb{Z}$.

Fix a "band-limit" parameter $r \geq 1$. Fix reduced rationals a_i/q_i with $1 \leq q_i \leq Q$ and $(a_i, q_i) = 1$ for $i = 1, 2$. On the torus $\mathbb{T} = \mathbb{R}/\mathbb{Z}$, define the minor-arc neighborhoods

$$\mathcal{U}_i\Big(\frac{a_i}{q_i}\Big) := \left\{ t \in \mathbb{T} : \ \left| t - \frac{a_i}{q_i} \right| \leq \frac{C}{q_i r} \right\},$$

where $|\cdot|$ denotes distance in $\mathbb{T}$.

Assume we have nonnegative window majorants $W_i : \mathbb{T} \to [0, 1]$ such that:

- $W_i \geq \mathbf{1}_{\mathcal{U}_i(a_i/q_i)}$,
- $\operatorname{supp} \widehat{W_i} \subseteq \{\ell \in \mathbb{Z} : \ |\ell| \leq c\, r\}$ (band-limited),
- $\displaystyle\int_{\mathbb{T}} W_i(t)\, dt \asymp \frac{1}{q_i r}$.

Define the tensor window on $\mathbb{T}^2$ by

$$W(t_1, t_2) := W_1(t_1)\, W_2(t_2).$$

Then $\widehat{W}(\ell_1, \ell_2) = \widehat{W_1}(\ell_1)\widehat{W_2}(\ell_2)$, and

$$\iint_{\mathbb{T}^2} W \ \asymp \ \frac{1}{q_1 q_2 r^2}. \tag{1}$$

6.4.2 Statement (A-39": windowed area law for bilinear constraints)

Theorem 6.2 (A-39"). *Under the setup above, the number of pairs* $(m,n) \in I_M \times I_N$ *satisfying*

$$\frac{mn}{M_1} \in \mathcal{U}_1\Big(\frac{a_1}{q_1}\Big), \qquad \frac{mn}{M_2} \in \mathcal{U}_2\Big(\frac{a_2}{q_2}\Big)$$

obeys the bound

$$\#\{(m,n)\} \ \ll \ \frac{MN}{q_1 q_2\, r^2} \ + \ O\Big(\frac{M}{q_1 r} + \frac{N}{q_2 r} + 1\Big), \tag{2}$$

uniformly for $q_1, q_2 \le Q$. *The implied constants depend only on the fixed window family choices (through* C, c*).*

Interpretation. The main term $\asymp MN/(q_1 q_2 r^2)$ is the box area MN times the window area $\asymp (q_1 q_2 r^2)^{-1}$. The secondary terms are one-dimensional edge effects arising when one frequency coordinate is 0 in the Fourier expansion.

6.4.3 Proof sketch

Step 1 (Majorization by the smooth window).

$$\#\{(m,n)\} \le \sum_{m\in I_M} \sum_{n \in I_N} W\left(\frac{mn}{M_1}, \frac{mn}{M_2}\right).$$

Step 2 (Fourier expansion on $\mathbb{T}^2$). By Fourier inversion and band-limit:

$$W\left(\frac{mn}{M_1}, \frac{mn}{M_2}\right) = \sum_{|\ell_1|,|\ell_2| \le cr} \widehat{W_1}(\ell_1)\widehat{W_2}(\ell_2)\, e\left(\ell_1 \frac{mn}{M_1} + \ell_2 \frac{mn}{M_2}\right).$$

Step 3 (Main term from $(0,0)$ frequency). The $(\ell_1,\ell_2) = (0,0)$ term gives

$$\widehat{W}(0,0)\, S(0,0) = \left(\iint_{\mathbb{T}^2} W\right)\ (MN) \asymp \frac{MN}{q_1 q_2 r^2}.$$

Step 4 (Off-diagonal control via bilinear van der Corput). For $(\ell_1,\ell_2) \ne (0,0)$, the bilinear sum

$$S(\ell_1,\ell_2) = \sum_{m\in I_M} \sum_{n\in I_N} e\left(\ell_1 \frac{mn}{M_1} + \ell_2 \frac{mn}{M_2}\right)$$

satisfies $|S(\ell_1,\ell_2)| \ll \frac{MN}{r} + (M+N)r$ by standard bilinear cancellation.
Step 5 (Sum the off-diagonals). There are $O(r^2)$ frequency pairs. The window normalization implies $|\widehat{W}(\ell_1,\ell_2)| \ll \frac{1}{q_1 q_2 r^2}$. Summing gives the secondary terms in (2). □

6.4.4 How A-39 supplies κ

In SCFP language, A-39 provides a universal envelope for the bilinear windowed count:

$$\#\{(m,n)\} \ \le \ \mathrm{AreaMain}(M,N,q_1,q_2,r) \cdot \Big(1 + \kappa_{\mathrm{A39}}(\mathrm{edge})\Big),$$

with AreaMain $\asymp MN/(q_1 q_2 r^2)$ and edge envelope $\kappa_{\mathrm{A39}}(\mathrm{edge})$ controlled by the explicit secondary terms.

6.5 What NT-3 contributes downstream

At this point the mathematics-track architecture has all required components to support the remainder of the suite:

- a finitary invariant alphabet $\mathcal{S}_{\text{struct}}$,
- an admissible fixed-point selection mechanism (SCFP/SCFP++) that produces canonical generators,
- universal envelopes (A-39" area-law form) that prevent numerological overfitting by forcing every selection/closure to pass through explicit, representation-stable bounds.

7 Reproducibility Surface for NT-3 Claims (AoR)

All evidence-backed claims in the Marithmetics suite are cited to the Authority-of-Record (AoR) bundle and its ledgers. The canonical AoR citation surface for this release is the URL map committed under the release tag:

- **AoR folder**: `gum/authority_archive/AOR_20260209T040755Z_0fc79a0`
- **Bundle SHA-256**: `c299b1a7a8ef77f25c3ebb326cb73f060b3c7176b6ea9eb402c97273dc3cf66c`

Canonical ledger artifacts (stable URLs under the release tag):

claim_ledger.jsonl
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z/gum/authority_archive/AOR_20260209T040755Z_0fc79a0/claim_ledger.jsonl

SUMMARY.md
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z/gum/authority_archive/AOR_20260209T040755Z_0fc79a0/SUMMARY.md

demo_index.csv
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z/gum/authority_archive/AOR_20260209T040755Z_0fc79a0/GUM_BUNDLE_v30_20260209T040755Z/tables/demo_index.csv

falsification_matrix.csv
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z/gum/authority_archive/AOR_20260209T040755Z_0fc79a0/GUM_BUNDLE_v30_20260209T040755Z/tables/falsification_matrix.csv

The contract used throughout this suite is:

- Mathematics claims (definitions, finitary determinism, bounded constructions) are proved in-text.
- Computed outputs (selected integers, fixed-point certificates, cross-base sweeps, falsification runs) are cited to the AoR artifacts above.

References

[1] H. L. Montgomery and R. C. Vaughan, *Multiplicative Number Theory I: Classical Theory*, Cambridge Studies in Advanced Mathematics, Cambridge University Press.

[2] H. Iwaniec and E. Kowalski, *Analytic Number Theory*, American Mathematical Society Colloquium Publications.

[3] G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers*, Oxford University Press.

[4] Public-Arch / Marithmetics, Authority-of-Record bundle `AOR_20260209T040755Z_0fc79a0` (tag `aor-20260209T040755Z`), cited via Appendix C artifacts and ledgers.

A SCFP/SCFP++ Certificate Specification (NT-3)

A.1 Scope

This appendix provides a referee-readable specification of:

1. what must be declared to define an SCFP channel unambiguously;
2. what must be computed to certify that a fixed point is forced (or falsified);
3. how "uniqueness" is operationally verified as a finite statement.

A.2 Canonical objects and encodings

Every SCFP channel operates on explicitly finite objects. To make the selection deterministic and auditable, each object is assigned a canonical encoding $\mathrm{enc}(\cdot)$ into a finite bitstring.

The encoding scheme is required to satisfy:

(E1) Injectivity: $\mathrm{enc}(X) = \mathrm{enc}(Y) \Rightarrow X = Y$.

(E2) Canonical ordering: finite sets are encoded via sorted lists in lexicographic order of their element encodings.

(E3) Declared endianness and integer encoding: integers are encoded in a fixed signed or unsigned representation.

(E4) Declared base-set ordering: base sets B and stress sets B_{stress} are ordered lists, not multisets.

A.3 Channel specification (what must be declared before selection)

An SCFP channel declaration is a finite data record Spec containing:

(S1) Base sets. Proof base list $B = [b_1, \ldots, b_s]$; optional stress base list B_{stress}; for each base b: modulus $d = b - 1$ and its factorization.

(S2) Rosetta mapping rule. A declared mapping rule that specifies how structural objects are compared across bases, explicitly forbidding literal-digit transport.

(S3) Candidate libraries and bounds. Finite libraries with canonical enumerations: $\mathcal{P}$, $\mathcal{M}$, $\mathcal{W}_\Psi$, $\mathcal{K}$, $\mathcal{F}$.

(S4) Normalization gauges. Declared reference objects $M_\star, \Psi_\star, \kappa_\star$ and any additional reference constants needed to define $(\hat{\chi}, \hat{\theta}, \hat{\Psi}, \hat{\kappa})$.

(S5) Constraint configuration. Whether C4 extremality is a maximization or minimization; whether SCFP++ tie-break is enabled.

(S6) Designed-FAIL catalog. A declared list of negative controls and the expected falsifier type.

A.4 Deterministic evaluation pipeline

Given Spec, the evaluation pipeline is:

Stage 0. Candidate enumeration. Enumerate the finite candidate set $\mathcal{C} = \mathcal{P} \times \mathcal{M} \times \mathcal{W}_\Psi \times \mathcal{K} \times \mathcal{F}$ in the lexicographic order induced by $\mathrm{enc}(\cdot)$.

Stage 1. Constraint filtering. Filter $\mathcal{C}$ to $\mathcal{A} \subseteq \mathcal{C}$ by evaluating predicates C1–C6 on each candidate.

Stage 2. Cross-base signature evaluation. For every survivor $\mathfrak{c} \in \mathcal{A}$ and every base $b \in B$, compute $\mathcal{R}_b(\mathfrak{c})$ and $\Phi_b(\mathfrak{c})$.

Stage 3. SCFP++ stress test (optional). If B_{stress} is declared, verify C2 across B_{stress} for each survivor.

Stage 4. Tie-break closure (optional). If Spec declares equivalence quotient, compute $\mathfrak{c}^\star = \arg\min_{\mathfrak{c}\in\mathcal{A}} \mathsf{DL}(\mathrm{enc}(\mathfrak{c}))$.

A.5 Predicate checklist (C1–C6) in checkable form

C1. Principal-character survival. PASS if and only if every DOC-admissible kernel used by the candidate is strictly contractive on non-principal frequencies.
C2. Base invariance. PASS if and only if $\mathcal{R}_b(\mathfrak{c}) = \mathcal{R}_{b_1}(\mathfrak{c})$ for every base $b \in B$.
C3. Inverse symmetry. PASS if and only if $P = P^{-1}$.
C4. Maximal period with extremal circulation. PASS if and only if the candidate's package attains maximal normalized cycle index and extremal circulation.
C5. Minimal wheel exactness. PASS if and only if the wheel M satisfies the declared exactness predicate and no strictly smaller wheel does.
C6. Universal finite envelope. PASS if and only if the candidate uses the declared universal bound $\kappa_\star$.

A.6 Falsifier types (designed-FAIL map)

A channel run is declared FAIL if any of the following occurs:

F1 (Base drift): C2 fails on B or on B_{stress}.

F2 (Multiplicity): $|\mathcal{A}| > 1$ after C1–C6.

F3 (Empty survivor): $|\mathcal{A}| = 0$.

F4 (Non-principal dependence): C1 fails.

F5 (Wheel non-minimality): C5 fails.

F6 (Non-universal envelope): C6 fails.

A.7 Minimal certificate payload

The SCFP/SCFP++ certificate emitted for a channel must contain:

1. The full channel specification Spec.
2. The candidate count $|\mathcal{C}|$.
3. The survivor counts after each constraint stage C1–C6.
4. The full encoded survivor list $\{\mathrm{enc}(\mathfrak{c}) : \mathfrak{c} \in \mathcal{A}\}$.
5. For each survivor and each base: $\mathcal{R}_b(\mathfrak{c})$ and $\Phi_b(\mathfrak{c})$.
6. Hashes (SHA-256) of all intermediate tables.
7. The negative control outcomes and their expected/observed falsifier labels.

B Symbol and Object Index (NT-3)

B.1 Base and residue objects

Symbol	Description
b	A base (integer, $b \geq 2$) used for cross-base evaluation.
$d = b - 1$	The base-adjacent modulus. Many Marithmetics invariants are defined canonically on U_d.
$U_d = (\mathbb{Z}/d\mathbb{Z})^\times$	The unit group modulo d. Elements are residue classes coprime to d.
$\lambda(d)$	The Carmichael exponent of U_d; the least positive integer such that $u^{\lambda(d)} \equiv 1 \pmod d$ for all $u \in U_d$.
$\mathrm{ord}_d(u)$	The multiplicative order of $u \in U_d$.

B.2 Characters and normalization

Symbol	Description
χ	A multiplicative character on a finite group (in applications, on U_d or a related finite group).
$\hat{\chi}$	A normalized "cycle index" invariant derived from order data; the canonical prototype is $\hat{\chi}(u) = \frac{\mathrm{ord}_d(u)}{\lambda(d)} \in (0, 1]$.
$M_\star, \Psi_\star, \kappa_\star$	Declared normalization gauges for the wheel modulus, circulation, and envelope respectively.
$\hat{\theta}, \hat{\Psi}, \hat{\kappa}$	Normalized invariants computed relative to the declared gauges; each must be representation-stable and base-portable under the Rosetta discipline.

B.3 SCFP objects

Symbol	Description
$P \subseteq U_d$	A unit package. In admissible channels it is typically inverse-symmetric.
P^{-1}	The inverse package $\{u^{-1} : u \in P\}$.
M	A CRT wheel modulus used for exactness and for specifying admissible congruence structure.
(C, w)	A circulation channel consisting of a cycle extraction rule C and a weight vector w.
$\mathcal{G}$	An envelope class specifying a universal admissible bound on an analytic or combinatorial residual.
F	An admissible template map (finite grammar) producing Φ from normalized invariants.
Φ	A dimensionless scalar assembled from normalized invariants: $\Phi = F(\hat{\chi}, \hat{\theta}, \hat{\Psi}, \hat{\kappa})$.
$\mathcal{C}$	The finite candidate space $\mathcal{P} \times \mathcal{M} \times \mathcal{W}_\Psi \times \mathcal{K} \times \mathcal{F}$.
$\mathcal{A}$	The SCFP survivor set after applying C1–C6 (and stress/tie-break if declared).
SCFP / SCFP++	The fixed-point selection mechanism (SCFP) and its referee-grade extension (SCFP++) with stress bases, designed-FAIL suite, and auditable tie-break closure on declared equivalences.

B.4 A-39 (area-law envelope) objects

Symbol	Description
$\mathbb{T} = \mathbb{R}/\mathbb{Z}$	The additive torus used to express fractional-part constraints.
$\mathcal{U}_i(a_i/q_i) \subset \mathbb{T}$	A neighborhood of the rational a_i/q_i of width $\asymp (q_i r)^{-1}$.
W_i	A nonnegative admissible majorant window on $\mathbb{T}$ used to bound indicator functions of $\mathcal{U}_i(a_i/q_i)$.
$W = W_1 \otimes W_2$	The tensor window on $\mathbb{T}^2$.

B.5 Evidence and reproducibility objects

Symbol	Description
AoR	Authority-of-Record: A hash-locked bundle containing the full run artifacts, ledgers, and commit-pinned provenance for computed claims.
SHA-256$(\cdot)$	The cryptographic hash function used to seal artifacts and intermediate tables.
Spec	The declared channel specification record required to define an SCFP run unambiguously.
enc$(\cdot)$	A canonical encoding of finite objects into bitstrings, required for deterministic enumeration and tie-breaks.

C Canonical AoR Citation Surface (Release Tag)

The sole authoritative citation surface for computed results in this release is the AoR bundle under the tag:

release tag:	aor-20260209T040755Z
AoR folder:	gum/authority_archive/AOR_20260209T040755Z_0fc79a0
Bundle SHA-256:	c299b1a7a8ef77f25c3ebb326cb73f060b3c7176b6ea9eb402c97273dc3cf66c

Canonical artifacts:

Master zip (complete bundle)
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/
authority_archive/AOR_20260209T040755Z_0fc79a0 /MARI_MASTER_RELEASE_20260209T040755Z _
0fc79a0 .zip

Report (GUM, v32)
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/
authority_archive/AOR_20260209T040755Z_0fc79a0 /report/GUM_Report_v32_2026-01-25_
04-42-51Z.pdf

Core ledgers:

claim_ledger.jsonl
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/
authority_archive/AOR_20260209T040755Z_0fc79a0 /claim_ledger.jsonl

SUMMARY.md
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/
authority_archive/AOR_20260209T040755Z_0fc79a0 /SUMMARY.md

runner_transcript.txt
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/
authority_archive/AOR_20260209T040755Z_0fc79a0 /runner_transcript.txt

run_metadata.json
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/
authority_archive/AOR_20260209T040755Z_0fc79a0 /run_metadata.json

Indices and master tables:

constants_master.csv
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/
authority_archive/AOR_20260209T040755Z_0fc79a0 /GUM_BUNDLE_v30_20260209T040755Z /tables/
constants_master.csv

demo_index.csv
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/
authority_archive/AOR_20260209T040755Z_0fc79a0 /GUM_BUNDLE_v30_20260209T040755Z /tables/
demo_index.csv

falsification_matrix.csv
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/
authority_archive/AOR_20260209T040755Z_0fc79a0 /GUM_BUNDLE_v30_20260209T040755Z /tables/
falsification_matrix.csv

run_reproducibility.csv
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/
authority_archive/AOR_20260209T040755Z_0fc79a0 /GUM_BUNDLE_v30_20260209T040755Z /tables/
run_reproducibility.csv

When citing a computed result, the required minimum citation triple is:

1. the AoR bundle identity (folder + SHA-256),
2. the ledger entry in `claim_ledger.jsonl`, and
3. the demo log and/or vendored artifact referenced by `demo_index.csv`.