PH-2 — The Constants Layer

Φ-Channel Gauge Rosetta, Cross-Base Integrity, and Audit-Grade Citation

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Abstract

This paper fixes the constants layer for the Marithmetics physics track: the minimal, audit-grade set of dimensionless quantities that downstream physics papers (PH-3+) treat as settled inputs. The objective is not interpretive narrative but reproducible authority: (i) each constant is produced by an explicit, deterministic map from a fixed finite invariant triple; (ii) the primary outputs are exact rationals, allowing line-by-line verification without numerical tolerance; and (iii) every cited value is bound to a cryptographic Authority-of-Record (AoR) bundle containing the full execution traces, ledgers, and negative controls.

The core deliverable is the Φ-channel gauge slice. From the canonical SCFP++ gauge triple $(w_U, s_2, s_3) = (137, 107, 103)$, we define three auxiliary integers $q_2 = 30$, $v_2 = v_2(w_U - 1) = 3$, and $q_3 = 17$, together with the totient density $\Theta(q_2) = \varphi(q_2)/q_2 = 4/15$. The Φ-channel coupling laws then yield the exact rational constants

$$\alpha_{\text{em}} = \frac{1}{137}, \qquad \sin^2\theta_W^{(\Phi)} = \frac{7}{30}, \qquad \alpha_s^{(\Phi)} = \frac{2}{17}.$$

A second deliverable is the integrity contract: "base as gauge." Any quantity promoted to a constant must be invariant under admissible cross-base representation transforms and must fail under explicit falsifiers (designed-fail tests). These integrity tests are not rhetorical; they are recorded artifacts in the AoR.

Keywords: reproducibility; audit-grade science; exact rational constants; cross-base invariance; SCFP++; designed-fail controls; Φ-channel; Standard Model couplings.

Evidence Capsule

(Canonical AoR Citation Surface for PH-2)

AoR release tag:
`https://github.com/public-arch/Marithmetics/tree/aor-20260209T040755Z`
AoR folder: `gum/authority_archive/AOR_20260209T040755Z_0fc79a0`
Bundle sha256:
`c299b1a7a8ef77f25c3ebb326cb73f060b3c7176b6ea9eb402c97273dc3cf66c`

Master bundle zip:
`https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/authority_archive/AOR_20260209T040755Z_0fc79a0 /MARI_MASTER_RELEASE_20260209T040755Z_0fc79a0 .zip`

Primary report (index narrative; not the authority record):
`https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/`

authority_archive/AOR_20260209T040755Z_0fc79a0 /report/GUM_Report_v32_2026-01-25_04-27-46Z .pdf

Report manifest (integrity companion):
https://github.com/public-arch/Marithmetics/blob/aor-20260209T040755Z /gum/authority_archive/AOR_20260209T040755Z_0fc79a0 /report/GUM_Report_v32_2026-01-25_04-27-46Z .pdf.manifest.json

Core ledgers and tables (primary evidence surface):

- claim_ledger.jsonl
- constants_master.csv
- constants_master.json
- demo_index.csv
- falsification_matrix.csv
- run_reproducibility.csv

PH-2 primary demos (stdout/stderr):

- DEMO-64 (base-gauge invariance selector audit)
- DEMO-37 (SM master flagship; Φ-constants agreement check)

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1 Reader Contract and Claim Discipline

1.1 Purpose of this paper

PH-2 exists to prevent two common failure modes in "constants-from-integers" work:

(1) ambiguity about what values are actually fixed and what are later choices, and

(2) ambiguity about whether the values are artifacts of representation or tuning.

Accordingly, PH-2 is written as a constants ledger in prose form: it defines the objects, writes the maps explicitly, derives the principal rationals line-by-line, and binds any computational evidence to a cryptographic AoR record.

1.2 Claim classes used here

We use three classes, each with a different verification standard:

A. Mathematical derivations (paper-internal). Statements proved from explicit definitions and elementary number theory within the paper (e.g., $\varphi(30) = 8$, $136 = 2^3 \cdot 17$).

B. AoR-certified outputs (paper-external but auditable). Statements whose truth is "this value appears in the AoR ledgers/logs for the cited bundle." These are verified by reading the cited artifacts and checking the bundle identity.

C. Report-only comparisons (contextual overlays). When later papers compare outputs to external experimental or phenomenological reference values, those comparisons are strictly non-operative for selection; they are included only to quantify proximity. PH-2 will not rely on such overlays to establish any fixed constant.

1.3 Notation and data type discipline

Exact rationals are treated as elements of $\mathbb{Q}$, and equalities between rationals are exact equalities. Real-valued quantities, when they appear, must declare a tolerance and a computation path; they are not used for the primary Φ-channel couplings stated in this paper.

1.4 Citation rule for PH-2

A statement of the form "$Q = V$" is citable from PH-2 only if the cited AoR bundle identity is provided (tag + bundle sha256) and the value is anchored to a ledger row or demo log within that bundle. The Evidence Capsule provides the canonical surfaces for doing so.

2 What PH-2 Fixes

PH-2 fixes the constants layer in two senses: numerical and procedural.

2.1 Numerical deliverables (Φ-channel gauge slice)

PH-2 fixes the following dimensionless couplings as exact rationals derived from a single fixed integer triple:

$$\alpha_{\rm em} = \frac{1}{137}, \qquad \sin^2\theta_W^{(\Phi)} = \frac{7}{30}, \qquad \alpha_s^{(\Phi)} = \frac{2}{17}.$$

These values are (i) derived in Section 4 from explicit formulas and elementary arithmetic and (ii) recorded in the AoR constants ledger as outputs of the flagship Standard-Model lane (see `constants_master.*` and DEMO-37).

2.2 Procedural deliverables (integrity contract)

PH-2 also fixes the admissibility contract for what counts as a "constant" in this program:

- constants must be invariant under admissible cross-base representation transforms ("base as gauge"), and
- constants must fail under explicit falsifiers (designed-fail tests).

These integrity requirements are defined in Section 5 and exercised in Section 6, with their audit artifacts recorded in the AoR (DEMO-64 and the falsification matrix).

3 Inputs: the Canonical SCFP++ Gauge Triple

3.1 The canonical triple

The constants layer begins from a single discrete input selected by the SCFP++ selection engine under its admissibility lawbook:

$$(w_U, s_2, s_3) = (137, 107, 103).$$

Interpretation for this paper. PH-2 does not re-derive SCFP++ selection; it treats the triple as the unique selector output delivered by the upstream selection paper(s) and verified by the AoR. PH-2's task is to state what deterministic, representation-independent constants follow from this triple, and to fix the audit constraints under which those constants can be cited.

3.2 Derived auxiliary integers

From the triple we define three integers that appear repeatedly in the Φ-channel coupling laws.

Definition 3.1 (Mixing modulus).

$$q_2 := w_U - s_2.$$

With $(w_U, s_2) = (137, 107)$,

$$q_2 = 137 - 107 = 30.$$

Definition 3.2 (2-adic branch index). *Let* $v_2(n)$ *denote the 2-adic valuation of* n, *i.e., the largest integer* $v \geq 0$ *such that* $2^v \mid n$. *Define*

$$v_2 := v_2(w_U - 1).$$

For $w_U = 137$,

$$w_U - 1 = 136 = 2^3 \cdot 17 \quad \Longrightarrow \quad v_2 = 3.$$

Definition 3.3 (Odd 2-adic branch factor).

$$q_3 := \frac{w_U - 1}{2^{v_2}}.$$

With $w_U - 1 = 136$ *and* $v_2 = 3$,

$$q_3 = \frac{136}{8} = 17.$$

3.3 Reduced totient density on the mixing modulus

Define the reduced totient density on positive integers q by

$$\Theta(q) := \frac{\varphi(q)}{q},$$

where φ is Euler's totient function.

For $q_2 = 30 = 2 \cdot 3 \cdot 5$,

$$\varphi(30) = 30\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right) = 30 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{4}{5} = 8,$$

hence

$$\Theta(q_2) = \Theta(30) = \frac{8}{30} = \frac{4}{15}.$$

4 The Φ-Channel Coupling Laws (Definition Level)

The Φ-channel coupling laws are defined as exact rational functions of the integers fixed in Section 3. The defining principle is neutrality: once the canonical triple is fixed, the coupling maps contain no continuous fit parameters.

Definition 4.1 (Electromagnetic coupling).

$$\alpha_{\text{em}} := \frac{1}{w_U}.$$

Definition 4.2 (Weak mixing angle, Φ-channel).

$$\sin^2 \theta_W^{(\Phi)} := \Theta(q_2)\left(1 - 2^{-v_2}\right).$$

Definition 4.3 (Strong coupling, Φ-channel)**.**

$$\alpha_s^{(\Phi)} := \frac{2}{q_3}.$$

These three definitions are the only coupling maps required for the constants layer of the physics track. Downstream papers may introduce additional maps, but those maps must be declared explicitly and must inherit the same integrity contract (cross-base invariance and designed-fail sensitivity).

5 Exact Evaluation of the Φ-Channel Couplings

Theorem 5.1 (Φ-channel gauge slice as exact rationals)**.** *Given the canonical SCFP++ gauge triple* $(w_U, s_2, s_3) = (137, 107, 103)$ *and the derived integers* $q_2 = 30$*,* $v_2 = 3$*,* $q_3 = 17$*, the* Φ*-channel coupling laws evaluate to the exact rationals*

$$\alpha_{\mathrm{em}} = \frac{1}{137}, \qquad \sin^2 \theta_W^{(\Phi)} = \frac{7}{30}, \qquad \alpha_s^{(\Phi)} = \frac{2}{17}.$$

Proof (line-by-line). By Definition 4.1,

$$\alpha_{\mathrm{em}} = \frac{1}{w_U} = \frac{1}{137}.$$

By Definitions 4.2 and 3.3, $\Theta(q_2) = 4/15$ and $v_2 = 3$, so

$$\begin{aligned}
\sin^2 \theta_W^{(\Phi)} &= \Theta(q_2)\left(1 - 2^{-v_2}\right) \\
&= \frac{4}{15}\left(1 - 2^{-3}\right) \\
&= \frac{4}{15}\left(1 - \frac{1}{8}\right) \\
&= \frac{4}{15} \cdot \frac{7}{8} \\
&= \frac{28}{120} \\
&= \frac{7}{30}.
\end{aligned}$$

By Definitions 4.3 and 3.2, $q_3 = 17$, so

$$\alpha_s^{(\Phi)} = \frac{2}{q_3} = \frac{2}{17}.$$

All identities are exact in $\mathbb{Q}$. □

5.1 AoR agreement check (evidence statement)

The AoR records that the Standard-Model lane reproduces these Φ-channel constants and, in the flagship run, reports zero deltas between the "self-contained Φ-channel computation" and the "SM-lane constants" (see DEMO-37 and `constants_master.*` in the Evidence Capsule). This is an AoR-certified statement: it is verified by inspecting the cited AoR artifacts for the declared bundle identity.

5.2 Interpretation boundary

Section 5 establishes only the following: once the canonical gauge triple is fixed, the Φ-channel coupling maps force a specific set of exact rational outputs. It does not, by itself, claim any scale-dependent renormalization narrative, nor does it claim that the Standard Model's running couplings are "replaced" by these rationals. Any such interpretive overlay must be stated and bounded in later papers, without feeding back into selection.

6 Cross-Base Integrity: "Base as Gauge," Representation-Independent Constants

The Φ-channel gauge slice derived in Sections 3–5 has no scientific value if it can be re-expressed into agreement by a choice of numeral system. Accordingly, the Marithmetics program imposes a strict cross-base admissibility discipline: any quantity promoted to a "constant" must be stable under admissible representation transforms, and must fail under explicitly constructed non-admissible transforms (designed-fail).

This section formalizes that discipline and fixes what is meant, in this program, by "representation-independent."

6.1 Representation transforms as admissible maps

Fix a base $b \geq 2$. Let $\mathcal{R}_b$ denote a renderer that converts an internal value X into a finite base-b numeral string (with a declared radix rule), and let $\mathcal{P}_b$ denote a parser that maps such a numeral string back to an internal value in the declared numeric domain.

In this program, the intended internal domains are:

- $\mathbb{Q}$ for exact rationals (primary for PH-2), and
- $\mathbb{R}$ for derived real quantities, only when the evaluation map is explicitly declared and a tolerance is declared.

Definition 6.1 (Admissible cross-base transform)**.** *An admissible transform from base b to base b' is the composite map*

$$\mathcal{T}_{b\to b'} = \mathcal{P}_{b'} \circ \mathcal{R}_b,$$

together with an explicit round-trip check of the form

$$X \stackrel{?}{=} \mathcal{T}_{b\to b'}(X),$$

interpreted as:

- *exact equality when $X \in \mathbb{Q}$, and*
- *tolerance-bounded equality when $X \in \mathbb{R}$, where the tolerance is explicitly declared by the run.*

This definition is intentionally conservative. Numeral strings are treated as an interface layer, not as mathematical objects with intrinsic meaning. A cross-base invariance claim is therefore always reducible to an invariance statement about the underlying internal value X under $\mathcal{T}_{b\to b'}$, with all hypotheses explicit.

6.2 "Base as gauge": invariants are scalars under base change

The operational posture adopted throughout Marithmetics can be summarized in one sentence:

Base changes are treated as gauge transformations on representation, and admissible invariants are those that behave as gauge scalars.

Formally, consider the groupoid whose objects are bases $b \geq 2$ and whose morphisms are admissible transforms $\mathcal{T}_{b \to b'}$. A quantity I is **base-gauge invariant** if, for all admissible base pairs (b, b') in the declared test set,

$$I(b) = I(b'),$$

under the program's renderer/parser discipline (exactly if $I \in \mathbb{Q}$, otherwise within a declared tolerance).

In PH-2, this requirement is applied in two places:

1. **Selector invariance.** The SCFP++ selector and any subsequent integer extraction surfaces must return the same survivor triple (and the same derived integers) when the admissible representation layer is varied across the declared base set.
2. **Output invariance.** The Φ-channel outputs α_{em}, $\sin^2 \theta_W^{(\Phi)}$, $\alpha_s^{(\Phi)}$—which are exact rationals in this paper—must survive admissible cross-base round-trips without drift. For rationals, this is enforced as exact equality.

6.3 CRT hygiene: why injectivity matters for cross-base transport

A recurring mechanism in cross-base handling and residue-based transport is the use of residue signatures. Such transport is only legitimate when the transport map is injective on the intended finite domain. The classical hygiene condition is pairwise coprimality.

Let $m_1, \ldots, m_k$ be positive integers. Define the residue map

$$\rho : \mathbb{Z} \to \prod_{i=1}^{k} \mathbb{Z}_{m_i}, \qquad \rho(x) = (x \bmod m_1, \ldots, x \bmod m_k).$$

Theorem 6.2 (CRT injectivity on coprime moduli)**.** *If the moduli $m_1, \ldots, m_k$ are pairwise coprime, then ρ induces an injective map on the interval $[0, M)$ where $M = \prod_i m_i$.*

This theorem is a boundary condition. When coprimality fails, injectivity fails, and collisions must exist. Therefore, any pipeline step that relies on CRT signatures must:

- explicitly test an injective (coprime) case as a positive control, and
- explicitly test a non-injective (non-coprime) case as a designed-fail negative control, recording an explicit collision witness.

6.4 AoR implementation note: base-gauge invariance is audited, not assumed

The Authority-of-Record contains a dedicated base-gauge invariance run targeting the failure modes referees correctly raise ("you tuned base-10 digits," "selection is a representation artifact," "outputs drift under re-encoding"). The run enumerates a declared admissible base set, executes the selection and extraction surfaces under each base, and records PASS/FAIL at both the selector and output layers. It also records falsifier outcomes as part of the designed-fail battery.

Canonical AoR anchors for this section are:

- demo index (inventory/mapping),
- the base-gauge invariance demo logs (DEMO-64), and
- the falsification matrix (designed-fail catalog).

7 Designed-Fail Controls for Cross-Base Integrity

A pipeline that "always passes" is not credible. Marithmetics therefore makes negative controls first-class artifacts: it deliberately violates its own hypotheses in narrowly controlled ways, and requires the system to fail loudly and reproducibly when those violations occur.

This section records the principal designed-fail classes relevant to the PH-2 constants layer.

7.1 Negative control class I: non-coprime CRT collisions

The coprimality hypothesis in Theorem 6.2 is sharp; it is neither optional nor a matter of preference. Consider the non-coprime modulus set $(6, 9, 15)$. These are not pairwise coprime since $\gcd(6, 9) = 3$, $\gcd(6, 15) = 3$, and $\gcd(9, 15) = 3$.

Define

$$\rho_{6,9,15}(x) = (x \bmod 6,\ x \bmod 9,\ x \bmod 15).$$

Then we have an explicit collision witness:

$$x = 0, \qquad y = 90.$$

Indeed,

$$90 \equiv 0 \pmod 6, \qquad 90 \equiv 0 \pmod 9, \qquad 90 \equiv 0 \pmod{15},$$

so

$$\rho_{6,9,15}(0) = (0, 0, 0) = \rho_{6,9,15}(90) \quad \text{with} \quad 0 \neq 90.$$

This proves non-injectivity directly.

Designed-fail requirement. Any part of the pipeline that claims injective residue transport must fail (and record a failure signature) when non-coprime transport is substituted for coprime transport. This prevents residue structure from being used as a hidden non-injective encoding channel.

7.2 Positive control paired to §7.1: coprime CRT injectivity

A negative control without its paired positive control is incomplete. The AoR therefore includes coprime modulus cases and verifies injectivity of the CRT signature map on the corresponding finite domains $[0, M)$.

The purpose is not to "prove CRT" (which is classical), but to demonstrate that the implementation respects the theorem boundary: it passes when the hypotheses hold and fails when the hypotheses are violated, under the same audit harness that records all other PH-2 evidence.

7.3 Negative control class II: digit drift in finite base expansions

Cross-base invariance claims must not be compatible with "nearby corrupted encodings." Otherwise, a skeptic can argue that the system is silently tolerant to representation artifacts. The simplest structural corruption is a one-digit perturbation.

Let $b \geq 2$. A finite base-b expansion encoding a value X can be written as

$$X = \sum_{j=j_{\min}}^{j_{\max}} d_j\, b^j, \qquad d_j \in \{0, 1, \ldots, b-1\}.$$

If a single digit d_{j_0} is perturbed by $\delta \in \mathbb{Z}$, the encoded value changes by

$$\Delta X = \delta\, b^{j_0}.$$

This bound is elementary but operationally decisive: the magnitude of the corruption is computable from the digit position. Digit integrity is therefore not an aesthetic preference; it is a measurable perturbation class with predictable signatures.

Designed-fail requirement. If the invariance checks are applied after such a digit drift is introduced (at a declared position), cross-base agreement must fail under the AoR falsification protocol.

7.4 Negative control class III: digit injection and non-admissible encodings

A more aggressive corruption is digit injection: introducing digits that are not produced by the lawful renderer/parser discipline (e.g., splicing external digits into a numeral string, or perturbing parsing rules in a base-dependent way). This control targets a common numerology attack vector: "agreement can be forced by formatting."

Designed-fail requirement. When digit injection is performed, the pipeline must not silently accept the corrupted representation as an admissible invariant transport. Instead, it must emit a stable, repeatable failure signature and classify the trial as a designed-fail success (meaning: the system correctly fails under violation).

7.5 Summary: what designed-fail adds to invariance

Cross-base invariance, by itself, can be made meaningless if the admissibility boundary is porous or implicit. The designed-fail battery addresses that vulnerability directly:

1. It enforces sharp hypothesis boundaries (coprime vs non-coprime residue transport).
2. It enforces corruption sensitivity (digit drift and digit injection must fail).
3. It makes falsifiability operational: "failure under violation" is not a philosophical slogan but a recorded artifact.

The canonical catalog of designed-fail tests and their required outcomes is the AoR falsification matrix.

8 Carry-Forward Contract for Downstream Physics Track Papers

PH-2 is a hinge document. It exists so that every later physics paper can point to a single, audit-grade constants layer and does not need to re-litigate representation dependence, "hidden tuning," or "base-10 artifact" objections. Accordingly, this section fixes what downstream papers are allowed to treat as inputs, and what downstream papers are required to cite when they rely on those inputs.

8.1 Fixed constants slice and fixed auxiliary integers

Downstream Physics Track papers (PH-3 and later) may treat the following quantities as fixed inputs, provided they cite PH-2 and the AoR artifacts enumerated in §9:

1. **Canonical SCFP++ gauge triple** (input to the constants layer):
$$(w_U, s_2, s_3) = (137, 107, 103).$$
2. **Derived auxiliary integers** (defined in §3):
$$q_2 = w_U - s_2 = 30, \qquad v_2 = v_2(w_U - 1) = 3, \qquad q_3 = \frac{w_U - 1}{2^{v_2}} = 17,$$
and the totient density
$$\Theta(q_2) = \frac{\varphi(q_2)}{q_2} = \frac{4}{15}.$$

3. **Φ-channel coupling outputs** (derived in §5 as exact rationals):

$$\alpha_{\rm em} = \frac{1}{137}, \qquad \sin^2\theta_W^{(\Phi)} = \frac{7}{30}, \qquad \alpha_s^{(\Phi)} = \frac{2}{17}.$$

These quantities are to be treated as dimensionless constants of the Marithmetics Φ-channel. They are not to be re-estimated, refit, or reselected downstream. Any paper that substitutes different values must explicitly declare that it is superseding PH-2 via a new AoR.

8.2 Integrity carry-forward for representation independence

A downstream paper may rely on the statement "the Φ-channel constants are representation-independent under base change" only if it also inherits the full integrity posture fixed in §§6–7:

1. **Admissible transform discipline.** Any cross-base transport of constants must be performed via admissible transforms $\mathcal{T}_{b\to b'} = \mathcal{P}_{b'} \circ \mathcal{R}_b$ with explicit round-trip criteria (exact for rationals; tolerance-declared for reals).
2. **Base-as-gauge criterion.** Any quantity promoted to "constant status" must behave as a gauge scalar under admissible base transforms across the declared base set; selector outputs must be invariant as well.
3. **Designed-fail enforcement.** Any paper that asserts cross-base invariance must also cite the designed-fail battery demonstrating (a) failure under non-coprime CRT transport where collisions must exist, and (b) failure under digit drift and digit injection classes.

Operationally, the minimal AoR evidence surface for this carry-forward requirement is the demo index, the base-gauge invariance demo (DEMO-64), and the falsification matrix.

8.3 Non-circularity rule: overlays may interpret but may not select

Downstream physics papers will typically include external overlays (e.g., comparison to reference compilations, phenomenological inputs, or external numerical pipelines). PH-2 fixes a strict non-circularity constraint:

- Selection and promotion of constants must be completed before any external overlay is consulted.
- External overlays may be used only as report-only interpretation layers: they may quantify proximity or visualize consequences, but they may not influence which constants are chosen or which families are promoted.

This rule is not a matter of taste; it is required to make the constants layer auditable. If an overlay influences selection, then the selection is no longer a self-contained consequence of the finite invariant grammar.

8.4 Permitted downstream operations

PH-2 does not freeze the entire physics program; it freezes the constants layer and the integrity contract. Downstream papers may:

1. introduce additional derived quantities as explicitly declared functions of the fixed Φ-channel constants and auxiliary integers;
2. introduce scale models or running-coupling narratives, provided they are clearly labeled as interpretive layers and do not alter the fixed Φ-channel outputs;
3. build closure computations (mass hierarchies, mixing matrices, cosmological bundles) whose inputs include the fixed Φ-channel slice, provided the provenance remains AoR-citable and the non-circularity rule is respected.

8.5 Prohibited downstream operations

Downstream papers must not:

1. re-run selection or "choose among survivors" using external measurements;
2. introduce base-dependent formatting rules as an implicit tuning channel;
3. cite Φ-channel constants without simultaneously providing the AoR evidence surface required for integrity (base-gauge invariance + falsification);
4. replace exact rational equalities with tolerance language unless the quantity is genuinely real-valued and the tolerance is explicitly declared and justified.

9 Evidence-Ledger Summary: What to Cite, Where It Lives, and How to Verify

PH-2 contains paper-internal derivations (Sections 3–5) and paper-external evidence claims (Sections 6–7). This section is a map: it identifies the authoritative evidence artifacts in the AoR bundle and states how each class of claim is verified.

9.1 Primary evidence surfaces in the AoR

All PH-2 evidence claims are anchored to the AoR bundle identified by the tag and bundle sha256 in the Evidence Capsule. Within that bundle, PH-2 relies on five artifact families:

1. **Constants ledger** (canonical table).
   The authoritative enumeration of constants produced by the AoR run, including the Φ-channel constants used by PH-2.
   (`constants_master.csv` / `constants_master.json`)
2. **Claim ledger** (machine-readable claim registry).
   The canonical record of what was promoted to "claim status," along with pointers to the relevant artifacts.
   (`claim_ledger.jsonl`)
3. **Demo index** (execution map).
   The canonical mapping from demo identifiers to run logs and outputs.
   (`demo_index.csv` / `demo_index.json`)
4. **Base-gauge invariance logs and records**.
   The executed audit run demonstrating invariance across bases for selector outputs and Φ-channel outputs.
   (DEMO-64 stdout/stderr; plus vendored deterministic record and manifests)
5. **Falsification matrix** (designed-fail catalog).
   The required list of negative controls and their expected outcomes, used to certify that invariance checks have "teeth."
   (`falsification_matrix.csv` / `falsification_matrix.json`)

The demo index, base-gauge invariance logs, and falsification matrix are the minimal citation surface for Sections 6–7.

9.2 Verification mapping: claim type → verification method

PH-2 uses three claim types (as defined in §1.2). Each is verified differently:

1. **Mathematical derivations** (paper-internal).
   Verified by reading the definitions and arithmetic in PH-2. No AoR artifacts are required.

2. **AoR-certified outputs** (paper-external, auditable).
   Verified by locating the value in the constants ledger and/or demo logs in the cited AoR bundle, then confirming the bundle identity by tag + bundle sha256.

3. **Integrity claims** (cross-base invariance and designed-fail).
   Verified by reading the DEMO-64 logs for base-set enumeration and PASS/FAIL outcomes, and by reading the falsification matrix for required violations and their expected failures, all within the cited AoR bundle.

9.3 Minimal citation payload for PH-2 numerical constants

When a downstream paper cites a PH-2 constant, the minimal citation payload is:

- the AoR tag (release snapshot),
- the AoR bundle sha256, and
- a pointer to the constants ledger (or the demo log) in which the value is printed.

The recommended practice is ledger-first (constants_master as primary) and log-second (demo log as computation trace). The demo index exists so that log pointers can be stable and non-ambiguous.

9.4 Minimal citation payload for cross-base integrity

When a downstream paper cites the phrase "cross-base invariant," the minimal citation payload must include:

- DEMO-64 (base-gauge invariance run) and
- the falsification matrix entry set demonstrating designed-fail sensitivity.

A cross-base invariance citation that omits the falsification matrix is treated as incomplete, because it fails to demonstrate that the invariance predicate is sensitive to hypothesis violation.

9.5 Canonical AoR pointer set for PH-2 (single-screen verification)

For the specific claims made in PH-2, the following AoR artifacts constitute a sufficient and minimal verification surface:

1. **Bundle identity** (seal)
   `bundle_sha256.txt` (verifies the bundle's cryptographic identity)

2. **Execution map** (what ran, where to look)
   `demo_index.csv` (maps demo IDs to logs and artifacts)

3. **Constants ledger** (canonical numeric table)
   `constants_master.csv` (canonical enumeration of values extracted from demos)

4. **Φ-channel constants printed explicitly** (human-readable worked outputs)
   DEMO-33 (first-principles Standard Model closure; prints $\alpha_0^{-1} = 137$, $\Theta = 4/15$, $\sin^2\theta_W = 7/30$, $\alpha_s = 2/17$ in the run transcript)

5. **Cross-base invariance audit** (selector + outputs across bases)
   DEMO-64 (base-gauge invariance audit)

6. **Designed-fail catalog** (negative controls and expected outcomes)
   `falsification_matrix.csv` (canonical designed-fail requirements and outcomes)

These six items are sufficient to verify the primary PH-2 claims: the exact rational couplings (via paper-internal derivation and DEMO-33 confirmation), and the representation-independence posture (via DEMO-64 plus falsification matrix).

9.6 Practical verification of the exact rationals (reconcile paper math with AoR decimals)

Because AoR logs and ledgers frequently print decimal representations, PH-2 makes explicit how exact rational content is recovered without ambiguity.

For the Φ-channel outputs, the reconciliation is immediate:

- From §5, $\alpha_{\text{em}} = 1/137$. The exact rational implies the decimal

  $$\frac{1}{137} = 0.0072992700729927007\ldots$$

  which matches the AoR-printed finite decimal truncations in DEMO-33 and in ledger entries that report α_{em} as a floating representation.

- From §5, $\sin^2\theta_W^{(\Phi)} = 7/30$. The exact rational implies the repeating decimal

  $$\frac{7}{30} = 0.2333333333\ldots$$

  which is printed in DEMO-33 as "= 7/30" and appears as decimal in ledger outputs.

- From §5, $\alpha_s^{(\Phi)} = 2/17$. The exact rational implies the repeating decimal

  $$\frac{2}{17} = 0.11764705882352941\ldots$$

  which is printed in DEMO-33 as "= 2/17" and appears as decimal in ledger outputs.

In other words: for PH-2's primary constants, the AoR's decimal printing is not a tolerance-based estimate; it is simply a conventional rendering of an underlying exact rational already fixed by the paper's derivation. The "exact object" is the rational in $\mathbb{Q}$, not its chosen base-10 expansion.

10 Citation Architecture (Normative) for PH-2-Derived Claims

This section fixes what counts as a complete citation for PH-2 claims in downstream papers. The purpose is to prevent two failure modes: (i) "citation without identity" (a claim referenced without a bundle seal), and (ii) "citation without locus" (a claim referenced without a ledger/log location).

10.1 Document-level citation format (PH-2 as a paper)

A complete citation of PH-2 must include:

1. The paper identity (title, author, date).
2. The AoR identity (tag + bundle sha256).
3. A canonical AoR navigation pointer (URL_MAP or the AoR root folder).

A suitable journal-style form is:

> Grieshop, J. (2026). PH-2 — The Constants Layer: Φ-Channel Gauge Rosetta, Cross-Base Integrity, and Audit-Grade Citation. Marithmetics AoR tag `aor-20260209T040755Z`, bundle sha256 `c299b1a7...dc3cf66c`, AoR folder `gum/authority_archive/AOR_20260209T040755Z _52b`

10.2 Constant-level citation format (each numeric value)

When a downstream paper asserts a numeric value that PH-2 fixes (e.g., $\alpha_{\mathrm{em}} = 1/137$), the citation must include:

1. **Bundle identity:** tag + bundle sha256.
2. **Locus:** (a) constants ledger row or (b) demo transcript location where the value is printed.
3. **Derivation link:** an explicit pointer back to PH-2 §5 (the exact rational derivation).

The recommended citation structure is ledger-first, log-second:

- **Ledger-first:** cite `constants_master.csv` and the relevant field name in that ledger.
- **Log-second:** cite DEMO-33 stdout where the rational is printed explicitly (the "paper-friendly" version).

This pairing is intentionally redundant: the ledger provides canonical tabulation; the transcript provides a human-readable derivation output under the same bundle identity.

10.3 Integrity-claim citation format (cross-base invariance)

When a downstream paper asserts "cross-base invariant," the citation must include:

1. DEMO-64 logs (base-gauge invariance audit), and
2. `falsification_matrix.csv` (negative control requirements and outcomes), and
3. the PH-2 normative definitions (§6) and designed-fail classes (§7).

This is a strict requirement. Cross-base invariance in this program is not defined as "it seems stable across bases," but as "it is stable under admissible transforms and fails under declared falsifiers." Omitting the falsification evidence is therefore omitting part of the claim's meaning.

10.4 Versioning rule (supersession without erasure)

PH-2 is tied to a specific AoR bundle. If a later AoR is released, PH-2 may be superseded, but it is not invalidated retroactively. Instead:

- Any downstream paper must specify which AoR tag (and bundle sha256) it is using.
- If a paper mixes AoR versions, it must treat them as distinct experimental records and cite them separately.

This rule exists to preserve the chain of custody across time and to make disagreements concrete ("we disagree about AoR versions," not "we disagree in the abstract").

11 Conclusion

PH-2 fixes the constants layer for the Marithmetics physics track in a form suitable for audit. Starting from the canonical SCFP++ gauge triple $(137, 107, 103)$, the paper defines three Φ-channel coupling laws and evaluates them exactly, yielding the rational constants

$$\alpha_{\mathrm{em}} = \frac{1}{137}, \qquad \sin^2\theta_W^{(\Phi)} = \frac{7}{30}, \qquad \alpha_s^{(\Phi)} = \frac{2}{17}.$$

It then fixes a strict integrity contract: the constants must be representation-independent under admissible cross-base transforms ("base as gauge") and must fail under explicit non-admissible falsifiers (designed-fail controls). These integrity claims are treated as evidence-bearing claims, not philosophical assertions: they are audited in the AoR via a base-gauge invariance run and a falsification matrix.

Downstream physics papers may therefore treat the Φ-channel couplings as fixed inputs only under two conditions: (i) the exact rational derivations are respected (no re-fitting), and (ii) the integrity contract is inherited with its full AoR citation surface (invariance audit plus falsification battery). Under these constraints, later papers may develop interpretive overlays and closure computations without reopening the representational degrees of freedom that typically undermine integer-driven claims.

References

[1] T. M. Apostol, *Introduction to Analytic Number Theory*. Springer, 1976.

[2] K. Ireland and M. Rosen, *A Classical Introduction to Modern Number Theory* (2nd ed.). Springer, 1990.

[3] D. M. Burton, *Elementary Number Theory* (7th ed.). McGraw-Hill, 2010.

[4] Marithmetics Authority-of-Record (AoR) tag `aor-20260209T040755Z`, bundle sha256 `c299b1a7a8ef77f25c3ebb326cb73f060b3c7176b6ea9eb402c97273dc3cf66c`, AoR folder `gum/authority_archive/AOR_20260209T040755Z_0fc79a0`.