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Inner Approximation Method for a Reverse Convex Programming Problem

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Abstract. In this paper, we consider a reverse convex programming problem constrained by a convex set and a reverse convex set, which is defined by the complement of the interior of a compact convex set X. We propose an inner approximation method to solve the problem in the case where X is not necessarily a polytope. The algorithm utilizes an inner approximation of X by a sequence of polytopes to generate relaxed problems. It is shown that every accumulation point of the sequence of optimal solutions of the relaxed problems is an optimal solution of the original problem.

Key Words. Global optimization, reverse convex programming problem, dual problem, inner approximation method, penalty function method.

1. Introduction

In this paper, we consider a reverse convex programming problem constrained by a convex set and a reverse convex set, which is defined by the complement of the interior of a compact convex set X. In the case where X is a polytope in the problem, a solution method using duality has been proposed (Refs. 1–4). Duality is one of the most powerful tools in dealing with a global optimization problem like the problem described above. The dual problem to the problem is a quasiconvex maximization problem over a convex set; solving one of the two problems, the original problem and the

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dual problem, is equivalent to solving the other (Refs. 3–4). Since the feasible set of the dual problem is a polytope, there exists a vertex which solves the dual problem. Moreover, since the objective function of the dual problem is the quasiconjugate function of the objective function of the original problem, for every vertex, the objective function value is obtained by solving a constrained convex minimization problem. Consequently, an optimal solution of the original problem is obtained by solving a finite number of constrained convex minimization problems.

We propose an inner approximation method to solve the reverse convex programming problem in the case where X is not necessarily a polytope. The algorithm utilizes an inner approximation of X by a sequence of polytopes. That is, at every iteration of the algorithm, a relaxed problem, in which X is replaced by a polytope contained in X, is solved. Then, it is shown that every accumulation point of the sequence of optimal solutions of the relaxed problems is an optimal solution of the original problem. Every relaxed problem can be solved through a finite number of constrained convex minimization problems. By using penalty functions, such constrained problems can be transformed into unconstrained convex minimization problems. Thus, the minimum of the optimal values of such unconstrained problems underestimates the optimal value of the relaxed problem.

The organization of this paper is as follows. In Section 2, we describe a reverse convex programming problem. Moreover, we describe an equivalent problem to the original problem and its dual problem, where equivalence is understood in the sense that the sets of optimal solutions coincide. In Section 3, we formulate an inner approximation algorithm for the problem and establish the convergence of the algorithm. In Section 4, we propose another inner approximation algorithm for the problem which incorporates a penalty function method. In Section 5, for the sake of computational efficiency, we propose a procedure identifying redundant constraints for the subproblem. In Section 6, we describe a numerical example and computational experiments of the algorithm proposed in Section 4.

Throughout this paper, we use the following notation: int X, bd X, co X denote the interior set of $X \subset R^n$, the boundary set of X, and the convex hull of X, respectively. Let

$$\bar{R} = R \cup \{-\infty\} \cup \{+\infty\}.$$

For $a, b \in R^n$, let

$$]a, b[= \{x \in R^n: x = a + \delta(b-a), 0 < \delta < 1, \delta \in R\},$$

$$]a, b[= \{x \in R^n: x = a + \delta(b-a), 0 < \delta \leq 1, \delta \in R\}.$$