Fast MiniMax Polynomial Approximations of Sine and Cosine

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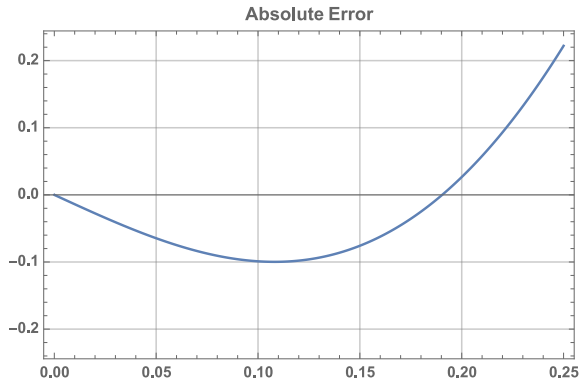
Abstract

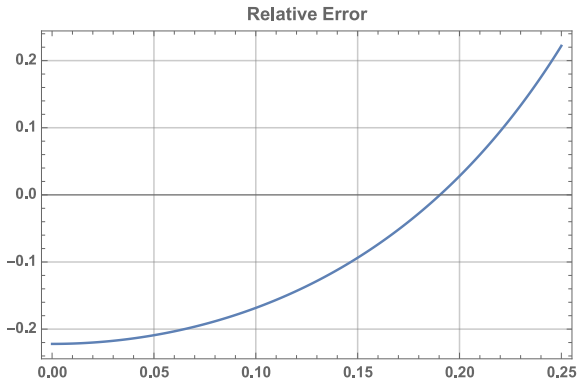
Polynomial approximations of the sine and cosine functions are given. The *polynomials approximating the sine function use only odd powers, all coefficients for even powers are zero. Likewise, the cosine approximations use only even coefficients. The error function which was minimized to derive the coefficients is the maximum of the relative or absolute error over the "first" quarter of one period of the sine function. The polynomials can be evaluated quickly using Horner's scheme.*

Approximations

Sin, rel. error minimized, degree 1

Maximum relative error: 0.22203094070331456367337575787965714468
Maximum absolute error: 0.22203094070331460246107513832856943 5





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` first):

```
4.8881237628132584098443005533142777 6*x1
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` first):

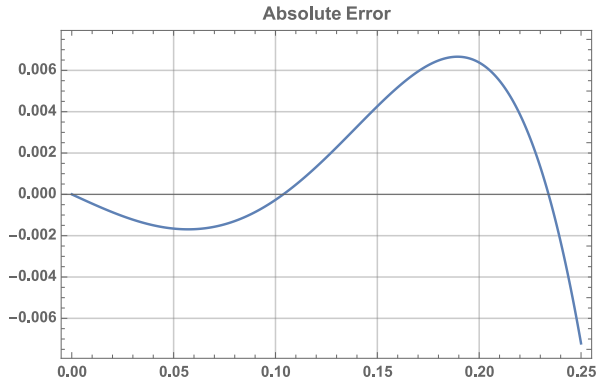
```
0.7779690592966854363266242120342855 32*x1
```

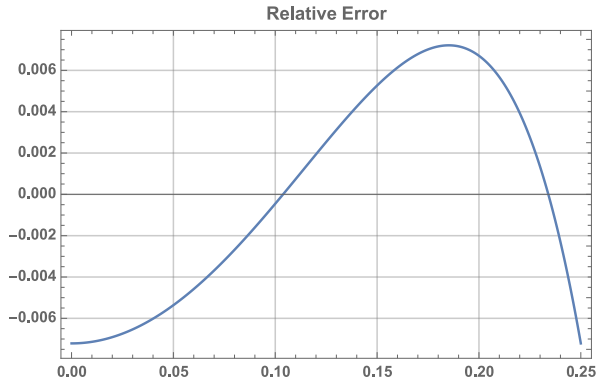
WL-style coefficient list:

```
{0.7779690592966854363266242120342855324239961813175798762 95`36.}
```

Sin, rel. error minimized, degree 3

Maximum relative error: 0.007212271016835766940189492226143009280
Maximum absolute error: 0.007212271016835771401977585615614191170





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.23786927189520681011206791022365216 - 36.267493695400798331519652042977742 8*x2)
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

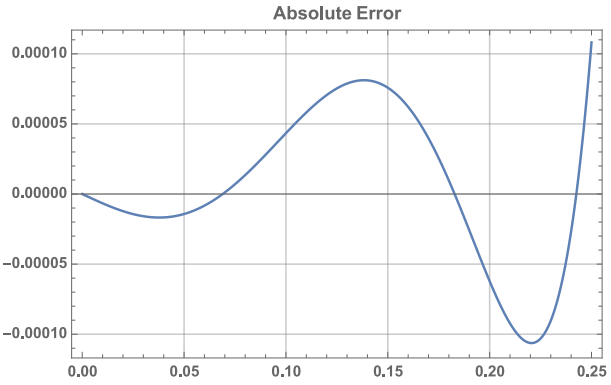
```
x1*(0.99278772898316423305981050777385699072036924440533142211 61 -
0.14621029021538302923287780626424867716432840683271688674 68*x2)
```

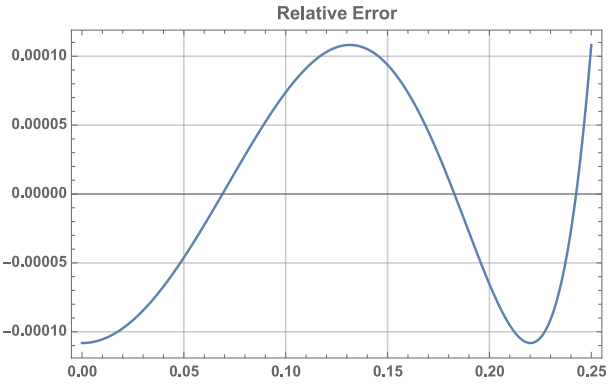
WL-style coefficient list:

```
{0.99278772898316423305981050777385699072036924440533142211 61`36.,
-0.14621029021538302923287780626424867716432840683271688674 68`36.}
```

Sin, rel. error minimized, degree 5

Maximum relative error: 0.000108178744189107114435292843058434688
Maximum absolute error: 0.000106306760359058556267427386435330041





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

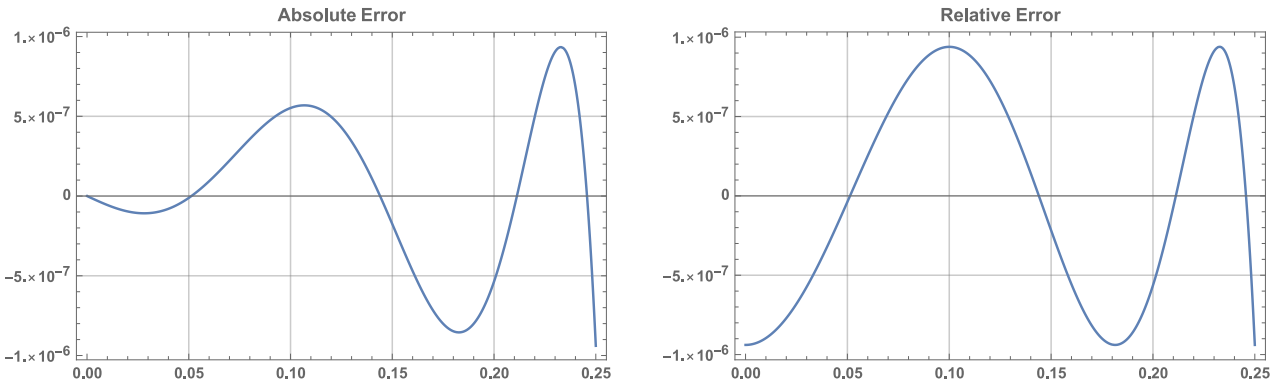
```
x1*(6.28250560008354834003487064338056964 + x2*(-41.166442330890373252441407700088139
+ 74.45241870690724282114195436417961 88*x2))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(0.99989182125581089288556470715694156531183565924600519983 23`36.,
0.16596011654087898906318538099654040737986438003099260784 91`36.,
0.00760290334336935120704015646842617914764720041155497339 78`36.}
```

Sin, rel. error minimized, degree 7

Maximum relative error: 9.391010236635250730174039564370516 15e-7
Maximum absolute error: 9.336090733372493244494421520753125 54e-7



C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.28317940663383263695539184084148414 + x2*(-41.338942498443847521160084500464897 7
+ x2*(81.39535863002157904789488711656431 36 -
71.47469428207048354830555278835274 35*x2)))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

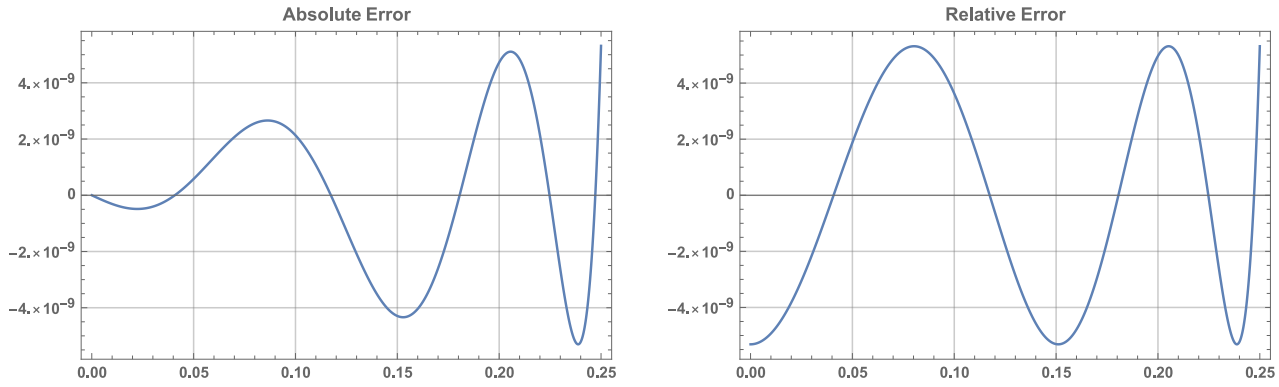
```
x1*(0.99999906089897633647492698259604356 3 + x2*(-
0.16665554092757693364619760720094973 2 + x2*(0.00831189980138987918776159520367912155
- 0.00018488140288607191103313968000519799 2*x2)))
```

WL-style coefficient list:

```
{0.99999906089897633647492698259604356294838532652082170054 46`36.,
-0.16665554092757693364619760720094973158655554875976148478 11`36.,
0.00831189980138987918776159520367912154944646171009610815 73`36.,
-0.00018488140288607191103313968000519799205314913045216128 5`36.}
```

Sin, rel. error minimized, degree 9

Maximum relative error: 5.313992663247683879740359682295910 15e-9
Maximum absolute error: 5.300722786203392515499308892934082 98e-9



C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185273790785852747319290794149 49 + x2*(-41.341677478391525285564024402764 3612
+ x2*(81.602231242727422642146513407621290 9 + x2*(-
76.574992181999212819200093402081709 4 + 39.710918143805847145300486089341623 3*x2))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

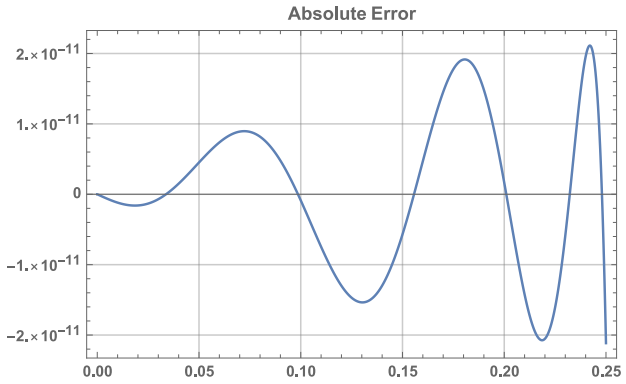
```
x1*(0.99999999468600733675231612025964 0318 + x2*(-
0.16666656684007151359069525 9999128453 + x2*(0.00833302513896936729848481553136180314
+ x2*(-0.00019807418727426970874574114 1088641071 +
2.60190306765146018582500885337773154e-6*x2))))
```

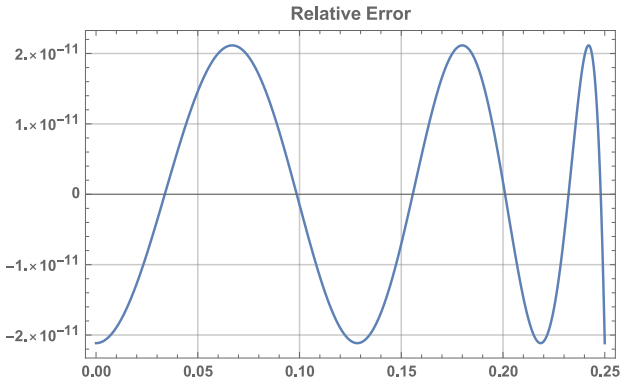
WL-style coefficient list:

```
{0.99999999468600733675231612025964031770408984965016112167 68`36.,
-0.16666656684007151359069526999912845336686031758293219743 75`36.,
0.00833302513896936729848481553136180314350792121665303685 07`36.,
-0.00019807418727426970874574114108864107080224037782968963 64`36.,
2.60190306765146018582500885337773153525519090092682 67`36.*^-6}
```

Sin, rel. error minimized, degree 11

Maximum relative error: 2.11510139959757396623100216943462049e-11
Maximum absolute error: 2.11510139959758439372658049257632678e-11





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185307046690736553822205897387 58 + x2*(-41.34170209692603585622951543991611 89
+ x2*(81.60522369013058790704879967455275 55 + x2*(-
76.70417025222345364431411736788083 29 + x2*(42.00779713610879654774146210695429 74 -
14.38139074330718527194844466788838 83*x2)))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

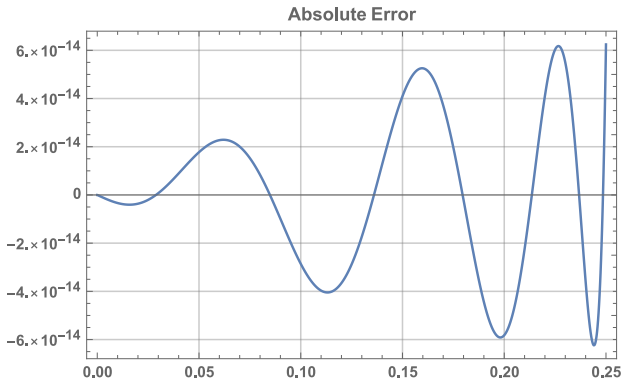
```
x1*(0.99999999997884898600402426033768998 + x2*(-
0.16666666608826069641316426188531006 7 + x2*(0.00833333072055773645376566203656709979
+ x2*(-0.00019840832823261955290156010801025724 + x2*(2.75239710746326498401791551303359689e-6 - 2.38683465210310276398300017947222950415355599042654`36.*^-8)))))
```

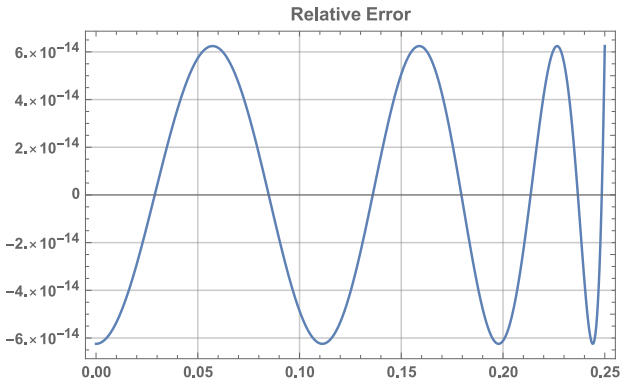
WL-style coefficient list:

```
{0.99999999997884898600402426033768997830565379511431838709 12`36.,
-0.16666666608826069641316426188531006700676253352048799257 44`36.,
0.00833333072055773645376566203656709979016117551827171135 25`36.,
-0.00019840832823261955290156010801025724215383627252516127 85`36.,
2.75239710746326498401791551303359688544577333333978 91`36.*^-6,
-2.38683465210310276398300017947222950415355599042654`36.*^-8}
```

Sin, rel. error minimized, degree 13

Maximum relative error: 6.24400681813329809580032940385023694e-14
Maximum absolute error: 6.17666638106202002061075213966124135e-14





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.28318530717919415440631052356951227 + x2*(-41.34170223981849124915045865633090009
+ x2*(81.605249133417790978917872994215311 4 + x2*(-
76.705846494128015850565131216445423 5 + x2*(42.058102841594004620961308093810776 9 +
x2*(-15.08103171730178007748914181651420 71 +
3.663464722294328726533521430984945 56*x2))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

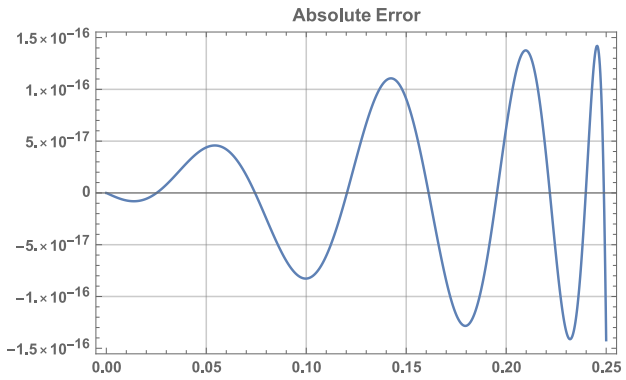
```
x1*(0.99999999999999375599318186670190 42 + x2*(-0.16666666666432331458181574238248 9749
+ x2*(0.00833333331876551401513171161164153048 + x2*(-
0.00019841266411622150098365220068523596 + x2*(2.75569319265949080406017672747752072e-
6 + x2*(-2.50295188656032073490380438676828409e-8 +
1.54011703714146442508663314577356389e-10*x2))))))
```

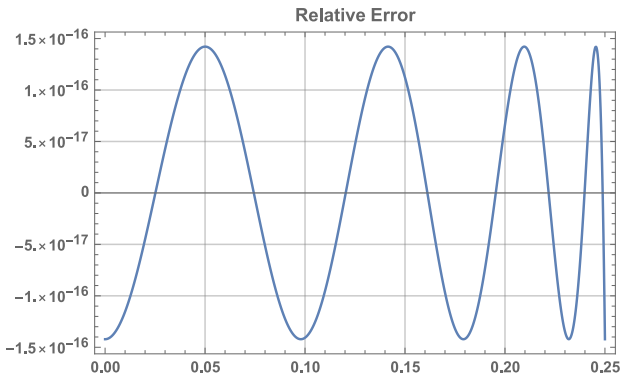
WL-style coefficient list:

```
{0.99999999999999375599318186670190419967059614976305799594333`36.,
-0.16666666666432331458181574238248974862642569859835931417 45`36.,
0.00833333331876551401513171161164153048451251611936122442 96`36.,
-0.00019841266411622150098365220068523596046745651060318265 92`36.,
2.75569319265949080406017672747752072439156342581731 67`36.*^-6,
-2.50295188656032073490380438676828409194568629418124`36.*^-8,
1.54011703714146442508663314577356388660980948457`36.*^-10}
```

Sin, rel. error minimized, degree 15

Maximum relative error: 1.42161430527196489869592847801671319e-16
Maximum absolute error: 1.41245476968494368607577626036076699e-16





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.2831853071795855836986752304464862 5 + x2*(-41.341702240398020533328485721507442 4
+ x2*(81.605249275512840355944627940733515 + x2*(-
76.705859683290845912260288897651644 1 + x2*(42.058689667355158274657513813964066 4 +
x2*(-15.094499474813222906393285223716816 7 + x2*(3.817288638728443261982459887616798 5
- 0.692156923645456821010024918081277 04*x2)))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

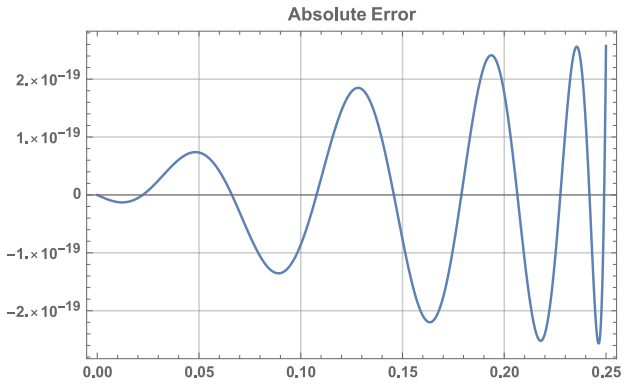
```
x1*(0.99999999999999985783856947280351013 + x2*(-
0.16666666666665965316478012885983157377652352270157145394 + x2*(0.008333333333275921396760575381067456588174757253550368042 +
x2*(-0.00019841269823222509368910723738080263684093235335042782 15 + x2*(2.75573164212929639596445204076151988260358467513068 48 +
x2*(-2.50518708834909025185900171656489449572835030666892 - 7.37066278281678179293546235379257047617831110441655000938005607 78*x2)))))))
```

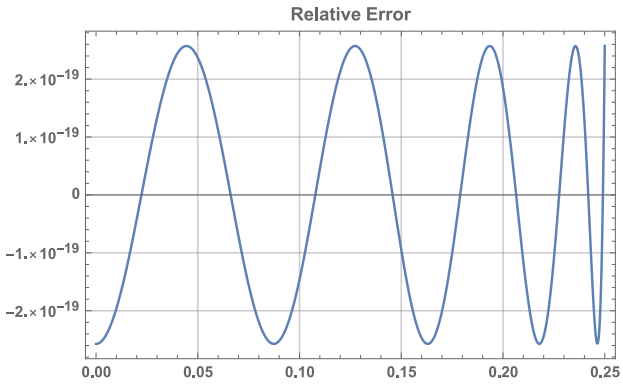
WL-style coefficient list:

```
{0.99999999999999985783856947280351013040715219832868115900 35`36.,
-0.16666666666665965316478012885983931573776523522701571453 94`36.,
0.00833333333327592139676057538106745658817475725355036804 2`36.,
-0.00019841269823222509368910723738080263684093235335042782 15`36.,
2.75573164212929639596445204076151988260358467513068 48`36.*^-6,
-2.50518708834909025185900171656489449572835030666892`36.*^-8,
1.60478446330181144282630647088443870156679737868 5`36.*^-10,
-7.37066278281678179293546235379257047617831110441655000938005607 78`36.*^-13}
```

Sin, rel. error minimized, degree 17

Maximum relative error: 2.57116077148351358053428669077559189e-19
Maximum absolute error: 2.57158863905688068506495893266591 46e-19





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.2831853071795864753090938026242340 1 + x2*(-41.341702240399756229470742188124051 4
+ x2*(81.605249276073406084943784865080224 1 + x2*(-
76.705859752798496385369729419590543 3 + x2*(42.058693923785165621251935818064615 6 +
x2*(-15.094641620507466998448752130490302 9 + x2*(3.819927050429970918220663342427153 5
+ x2*(-0.717723579656548249620955921341133889 +
0.100856328715300930892577112029351 3*x2))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

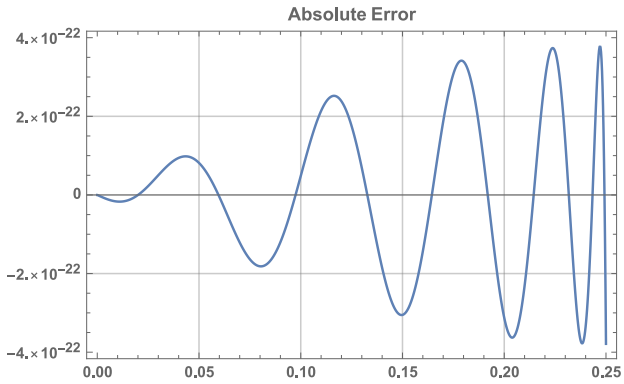
```
x1*(0.99999999999999999974277490079943975 + x2*(-
0.16666666666666665052276732335384060 4 + x2*(0.00833333333333331650314094866886116 3 + x2*(-0.00019841269841201840459252750531485886 +
x2*(2.75573192101527564362114785169078252e-6 + x2*(-
2.50521067982746148969440582709985054e-8 + x2*(1.60589364903732230834314189302038183e-
10 + x2*(-7.64291780693694318128770390349958602e-13 +
2.72047909631134875287705126898888084e-15*x2))))))))
```

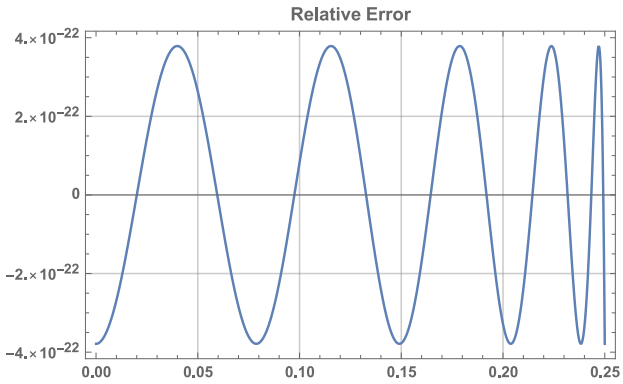
WL-style coefficient list:

```
{0.99999999999999999974277490079943974921931893770708123582 9`36.,
-0.16666666666666666505227673233538406035633332849207383550534`36.,
0.00833333333333331650314094866886116346181290699911270280004`36.,
-0.00019841269841201840459252750531485886042201325098977013 26`36.,
2.75573192101527564362114785169078251581322431724010 19`36.*^-6,
-2.50521067982746148969440582709985053875861023873028`36.*^-8,
1.60589364903732230834314189302038182635258434323 6`36.*^-10,
-7.64291780693694318128770390349958601528148130282634171742053251 56`36.*^-13,
2.72047909631134875287705126898888083963734654333622328608986302`36.*^-15}
```

Sin, rel. error minimized, degree 19

Maximum relative error: 3.787731259622796161601867600476215 36e-22
Maximum absolute error: 3.787084678424713752818118718121959 44e-22





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185307179586476925290686481920509 + x2*(-41.341702240399760226657760062901697 6
+ x2*(81.605249276075050470380568826307087 2 + x2*(-
76.705859753060642322364621132278755 4 + x2*(42.058693944822122106511392903853964 7 +
x2*(-15.094642572400446632829783453854418 5 + x2*(3.819952426739470338514814835545421 73
+ x2*(-0.718118794094021935415314539973412925 +
x2*(0.104182006356927195351417794664316173 -
0.011679245621112652287086852060663885 9*x2)))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

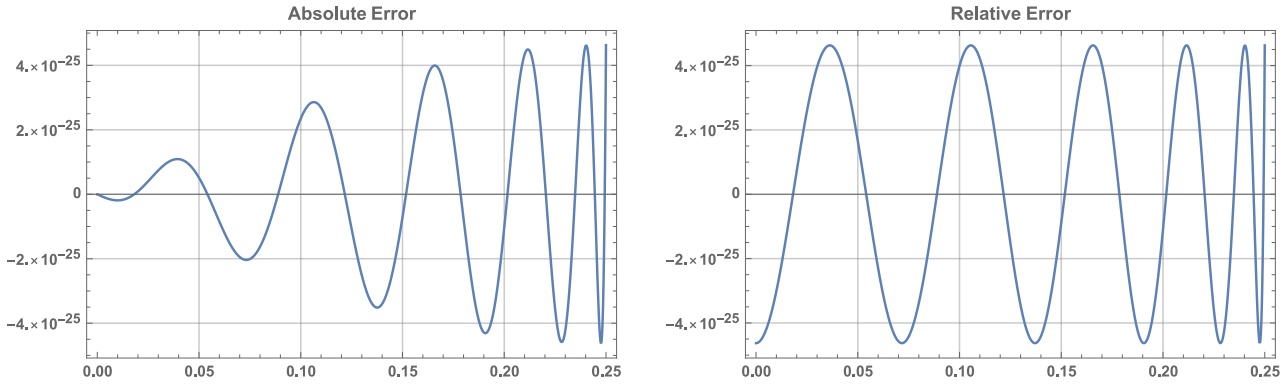
```
x1*(0.999999999999999999999962122687403772 + x2*(-
0.166666666666666666663719416621963726822821738821081688670 51 + x2*(0.008333333333333333295212653322266277182
+ x2*(-0.000198412698412696489459896530659927773 +
x2*(2.755731922393640188475789092053992620471071288236991 8 + x2*(-
2.505210837810176057293702312804117116403003552778 09 + x2*(1.605904317213369423566600577967820206290107021337 +
x2*(-7.647126379077169703808598988356805868941068026233140450558080437 7 + x2*(2.810185281538986226361949764996562739386698510535321307472583 02 +
-7.979897136484996428897391086791149372588097505978416781270 43 * x2))))))))
```

WL-style coefficient list:

```
{0.999999999999999999999962122687403772038383981323995237829 84`36.,
-0.166666666666666666663719416621963726822821738821081688670 51`36.,
0.008333333333333333295212653322266277182458955615635781001 73`36.,
-0.000198412698412696489459896530659927773316283914658123048 3`36.,
2.755731922393640188475789092053992620471071288236991 8`36.*^-6,
-2.505210837810176057293702312804117116403003552778 09`36.*^-8,
1.605904317213369423566600577967820206290107021337`36.*^-10,
-7.647126379077169703808598988356805868941068026233140450558080437 7`36.*^-13,
2.810185281538986226361949764996562739386698510535321307472583 02`36.*^-15,
-7.979897136484996428897391086791149372588097505978416781270 43`36.*^-18}
```

Sin, rel. error minimized, degree 21

Maximum relative error: 4.627645448215620473087483953786193 4e-25
Maximum absolute error: 4.618996488374575394412893719513041 04e-25



C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185307179586476925286385892361706 + x2*(-41.341702240399760233957574760745318 4
+ x2*(81.605249276075054196669803070606195 5 + x2*(-
76.705859753061384206102101286340774 4 + x2*(42.058693944897448474758086305086516 1 +
x2*(-15.094642576808003470197473512907406 + x2*(3.819952584162403551003618178229229 64
+ x2*(-0.718122281575531016590944853973730878 +
x2*(0.104228779138823154519786966406945894 + x2*(-
0.012027065638624884851118647592908215 4 +
0.001100678895526036629452117987215239 05*x2))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

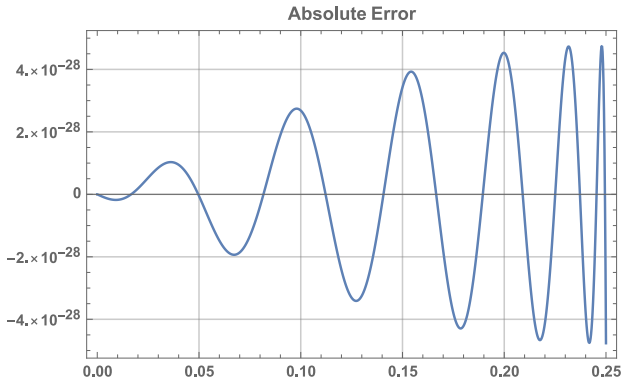
```
x1*(0.999999999999999999999999953723545518 + x2*(-
0.166666666666666666666662294435507217 6 + x2*(0.008333333333333333333326462983215615 5052
+ x2*(-0.000198412698412698408467841110285163174 +
x2*(2.755731922398575655037059877917726 69e-6 + x2*(-
2.505210838541684544777015647379315 48e-8 + x2*(1.605904383393818767568249890241401 85e-
10 + x2*(-7.647163516681788845366210164718202 2e-13 +
x2*(2.811446921507891643655353965805977 81e-15 + x2*(-
8.217546728915916174083142347947081 82e-18 + 1.904949222717455287526245176006782 6e-
20*x2))))))))))
```

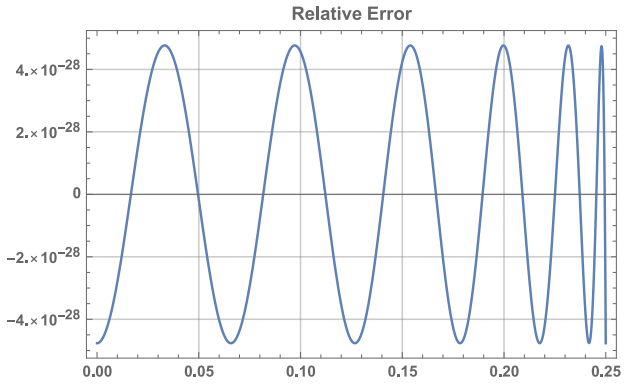
WL-style coefficient list:

```
{0.99999999999999999999999953723545517843795269125160462124 7`36.,
-0.16666666666666666666662294435507217560691557323875430383 46`36.,
0.00833333333333333333326462983215615505152625316475209067 416`36.,
-0.00019841269841269840846784111028516317353841047600352510 31`36.,
2.75573192239857565503705987791772668621433790336050 18`36.*^-6,
-2.50521083854168454477701564737931547607794389387424`36.*^-8,
1.60590438339381876756824989024140184690415663085 5`36.*^-10,
-7.64716351668178884536621016471820219963341039008440104597892048 44`36.*^-13,
2.81144692150789164365535396580597781135144226330978109730188007`36.*^-15,
-8.21754672891591617408314234794708182468121816329375332577331`36.*^-18,
1.90494922271745528752624517060067825957342389015851393219 5`36.*^-20}
```

Sin, rel. error minimized, degree 23

Maximum relative error: 4.766480755274531860880893756651121 57e-28
Maximum absolute error: 4.766082164104511671143345267150211 3e-28





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.28318530717958647692528676356413758 + x2*(-41.34170224039976023396840675608368 8
+ x2*(81.60524927607505420338780620080528 47 + x2*(-
76.70585975306138583875381313480633 73 + x2*(42.05869394489765271066146406630828 94 +
x2*(-15.09464257682295160074187977960106 84 + x2*(3.81995258484608093627454902397668405
+ x2*(-0.71812230169611904703341424780048 1873 +
x2*(0.10422916014779405372632190983088619 + x2*(-
0.01203155184950185675447822607417930 98 + x2*(0.00113056530917082146808609764898537513
- 0.00008606400804827516736083238865692858 95*x2))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

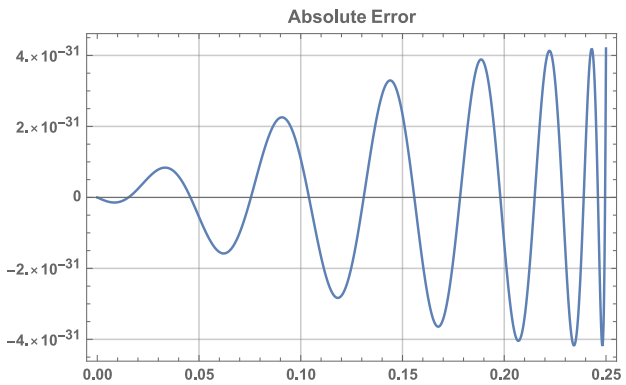
```
x1*(0.99999999999999999999999999952335192 + x2*(-
0.16666666666666666666666666612913901636 + x2*(0.008333333333333333333333332324771023532 48
+ x2*(-0.00019841269841269841269097129562979401177965545916653324 4 +
x2*(2.7557319223985890367982571080758617424449984241237545 6 + x2*(-
2.5052108385441654394696345698033097386174810019522 6 6 + x2*(1.6059043836812360811720396069695295656170714138 52 6 6 -
10 + x2*(-7.6471637309425500948423673170157637264577070186224572413934724798 6 6 -13 +
x2*(2.8114571987701521441081095082232726625240385310759474376034347 6 6 -15 +
x2*(-8.2206119526891109883544174225358531181554347466127168395724 4 6 6 -18 +
x2*(1.9566737544349787982165880283092434046749347825604792770 72 6 6 -20 -
3.7729808013177213874697295479755214285683977393186202 56 6 6 -23*x2))))))))))
```

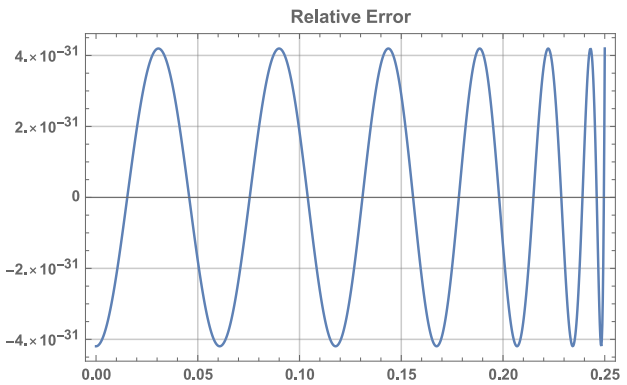
WL-style coefficient list:

```
{0.99999999999999999999999999952335192447254681391191062416576 36.,
-0.16666666666666666666666666612913901636088469857873112526244 1 6 36.,
0.00833333333333333333333332324771023532477241742740572853056 4 6 36.,
-0.00019841269841269841269097129562979401177965545916653324 4 6 36.,
2.7557319223985890367982571080758617424449984241237545 6 36.*^-6,
-2.5052108385441654394696345698033097386174810019522 6 6 36.*^-8,
1.6059043836812360811720396069695295656170714138 52 6 6 36.*^-10,
-7.6471637309425500948423673170157637264577070186224572413934724798 6 6 36.*^-13,
2.8114571987701521441081095082232726625240385310759474376034347 6 6 36.*^-15,
-8.2206119526891109883544174225358531181554347466127168395724 4 6 6 36.*^-18,
1.9566737544349787982165880283092434046749347825604792770 72 6 6 36.*^-20,
-3.7729808013177213874697295479755214285683977393186202 56 6 6 36.*^-23}
```

Sin, rel. error minimized, degree 25

Maximum relative error: 4.19504943676691951565111532789986982e-31
Maximum absolute error: 4.19479319145296338424017042763622346e-31





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185307179586476925286766556369 94 + x2*(-41.341702240399760233968420075661488
+ x2*(81.605249276075054203397670641523 9 + x2*(-76.705859753061385841626503758965214 6
+ x2*(42.058693944897653144245864994863449 1 + x2*(-
15.094642576822990312678998345023189 6 + x2*(3.819952584848276691073886590394371 57 +
x2*(-0.718122301778249903012936806567606579 +
x2*(0.104229162200223206350206379193020344 + x2*(-
0.012031585770243912813348189981422053 9 + x2*(0.001130921226186785287132423266986583 3
+ x2*(-0.000088211428362806586304877880608116432 5 +
5.673346764733132348249911485458095 67e-6*x2))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

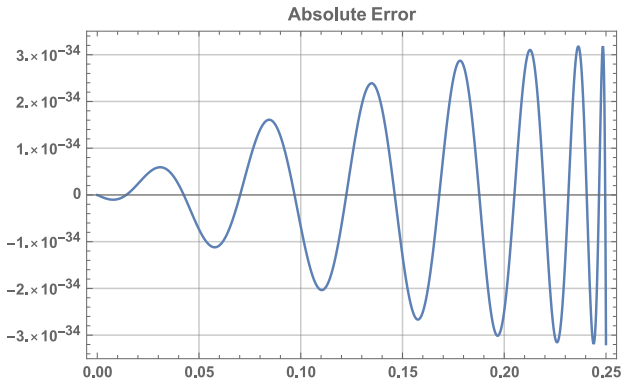
```
x1*(0.999999999999999999999999999995805 + x2*(-0.16666666666666666666666666611004393
+ x2*(0.008333333333333333333333333210417163705 + x2*(-
0.000198412698412698412698401996495140527 +
x2*(2.755731922398589065207184314857091 25e-6 + x2*(-
2.505210838544171864369327085908057976673 23e-8 + x2*(1.605904383682159174372655182788935 58e-
10 + x2*(-7.647163731817147780437772427237 27502e-13 +
x2*(2.811457254131978987267878614935560 15e-15 + x2*(-
8.220635129188791008182723719956806 4e-18 +
x2*(1.957289741391454566306153869768773 0e-20 + x2*(-
3.867122078290808911438479217308254 49e-23 + 6.300030039642659660935726587150464 94e-
26*x2))))))))))))
```

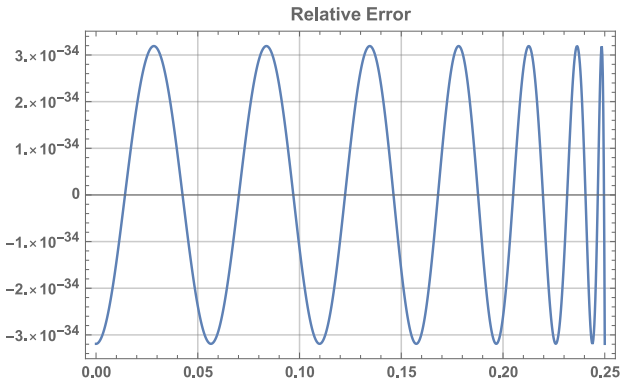
WL-style coefficient list:

```
{0.999999999999999999999999999958049505632330804843488833 55`36.,
-0.166666666666666666666666666611004392868353581689516388022 6`36.,
0.008333333333333333333333332104171637049662811756862941271 2`36.,
-0.000198412698412698412698401996495140527260627738466370743 9`36.,
2.755731922398589065207184314857091251932172772506293 9`36.*^-6,
-2.505210838544171864369327085908057976673234731475 48`36.*^-8,
1.605904383682159174372655182788935581823815330571`36.*^-10,
-7.647163731817147780437772427237275018529304554893125642295742222 4`36.*^-13,
2.811457254131978987267878614935560148132733690302528899082054 39`36.*^-15,
-8.220635129188791008182723719956806364190538144074705439422 5`36.*^-18,
1.957289741391454566306153869768773015467566771738370972438`36.*^-20,
-3.867122078290808911438479217308254492129186490305787767`36.*^-23,
6.300030039642659660935726587150464941264404235698381`36.*^-26}
```

Sin, rel. error minimized, degree 27

Maximum relative error: 3.191293471507349986383645862212493 32e-34
Maximum absolute error: 3.185626532841541822386118310632200 52e-34





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.28318530717958647692528676655900376 + x2*(-41.341702240399760233968420089456318 9
+ x2*(81.605249276075054203397682665876656 2 + x2*(-
76.705859753061385841630636139391244 3 + x2*(42.058693944897653144985773262651676 3 +
x2*(-15.094642576822990391696108173349149 + x2*(3.819952584848282117084109111151998 52
+ x2*(-0.718122301778499921527007241276411483 +
x2*(0.104229162208116980728820985024673378 + x2*(-
0.012031585941496737905835606121450330 3 + x2*(0.001130923736292058071792875565540852 77
+ x2*(-0.000088235178379996691986223885714774444 9 +
x2*(5.804292221940452601472298626704057 11e-6 - 3.195645206340327739954725176362541 26e-
7*x2))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

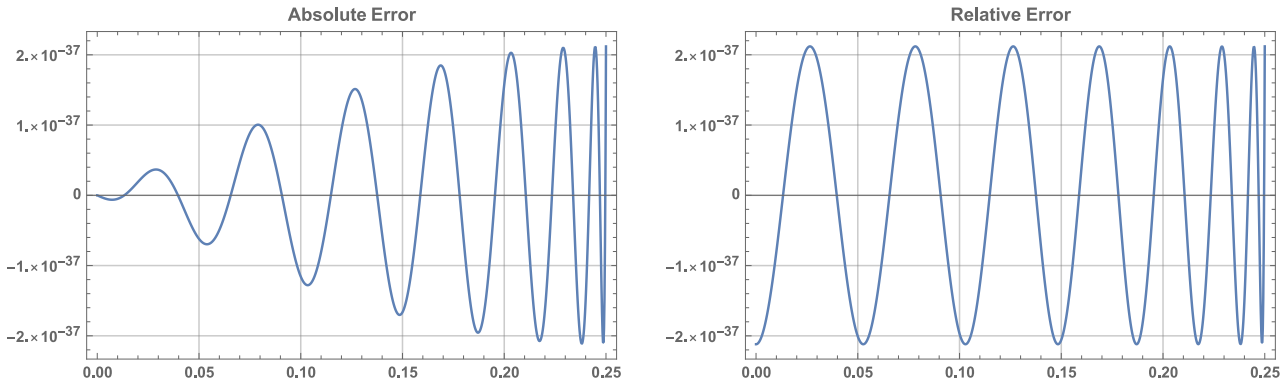
```
x1*(0.99999999999999999999999999999968 + x2*(-0.16666666666666666666666666666617 45
+ x2*(0.00833333333333333333333333333206984855 + x2*(-
0.000198412698412698412698412685597040 12 + x2*(2.75573192239858906525566391899938823e-
6 + x2*(-2.505210838544171877483550822235931 78e-8 +
x2*(1.605904383682161455462121270072078 36e-10 + x2*(-
7.647163731819810185585680829277384 56e-13 +
x2*(2.811457254344904130008685008369124 17e-15 + x2*(-
8.220635246198054526036169730180362 72e-18 +
x2*(1.957294085640361096753392121607949 22e-20 + x2*(-
3.868163261021213139425747703280114 29e-23 +
x2*(6.445439856488160587316528371618576 83e-26 -
8.988808133494882025041025459715443 94e-29*x2))))))))))))
```

WL-style coefficient list:

```
{0.99999999999999999999999999999999968087065284927700227292 79`36.,
-0.16666666666666666666666666666666617450479123256107519266592 9`36.,
0.00833333333333333333333333333320698485483130870158581679744`36.,
-0.00019841269841269841269841268559704012034441499963221530 58`36.,
2.75573192239858906525566391899938823266682095845948 66`36.*^-6,
-2.50521083854417187748355082223593178248609991800212`36.*^-8,
1.60590438368216145546212127007207835754860954063 7`36.*^-10,
-7.64716373181981018558568082927738455532962107590188435360974320 65`36.*^-13,
2.81145725434490413000868500836912416607960180340656880251047 34`36.*^-15,
-8.22063524619805452603616973018036272193424537305514986488987`36.*^-18,
1.95729408564036109675339212160794922223131647910887797001 9`36.*^-20,
-3.86816326102121313942574770328011428915826793094216718 4`36.*^-23,
6.44543985648816058731652837161857682977904251432993 8`36.*^-26,
-8.98880813349488202504102545971544393905535465468 1`36.*^-29}
```

Sin, rel. error minimized, degree 29

Maximum relative error: 2.11865278589589790150771723488302663e-37
Maximum absolute error: 2.11931467978181502621186468604035581e-37



C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

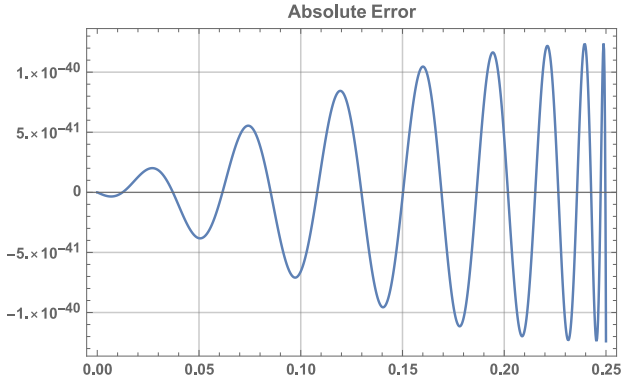
```
x1*(6.28318530717958647692528676655900577 + x2*(-41.34170224039976023396842008946851 76
+ x2*(81.60524927607505420339768267823862 38 + x2*(-
76.70585975306138584163064108887705 17 + x2*(42.05869394489765314498680993343595 78 +
x2*(-15.09464257682299039182643878028839 82 + x2*(3.81995258484828212771684891854446925
+ x2*(-0.71812230177850051112131489263567 7992 +
x2*(0.10422916220813978971317694457073 9309 + x2*(-
0.01203158594211891424000749819179271 94 + x2*(0.00113092374821068230148408235972999528
+ x2*(-0.00008823533528715220863219569017674866 28 +
x2*(5.80564396908612819199696524959602588e-6 + x2*(-
3.26426039768770010216497856951684167e-7 + 1.55585170384795037267005983349015 02e-
8*x2))))))))))))
```

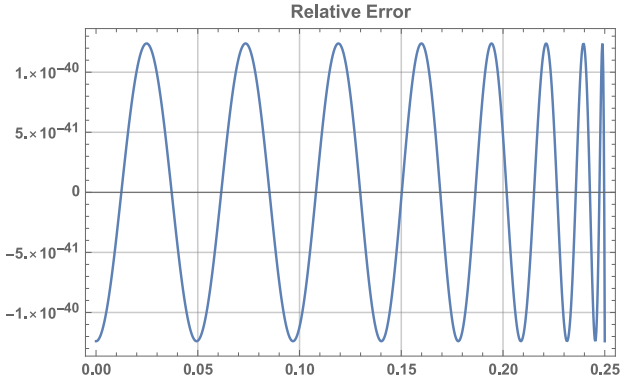
C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(1. + x2*(-0.16666666666666666666666666666666666629 +
x2*(0.008333333333333333333333333333333222318 + x2*(-
0.00019841269841269841269841269841269399723487 +
x2*(2.7557319223985890652557318428102656105098213550152136 8`36.*^-6,
-2.505210838544171877505181387347793332835840032271178`36.*^-8,
1.605904383682161459932114833597309330771365730808`36.*^-10,
-7.647163731819816464076398625661852174934254609497913440791946261 5`36.*^-13,
2.811457254345519375139445619663315310349043376319217427947656 65`36.*^-15,
-8.220635246623159306645623164866113942914748013980341222418 22`36.*^-18,
1.957294106267970161030776772136444602071745118832180868973`36.*^-20,
-3.868170139711838628414193582189201702190090331493296849`36.*^-23,
6.446940918908087959342749040847196041906010499895226`36.*^-26,
-9.181811032828283212983372673653141166820179978293`36.*^-29,
1.108541656130669366807638850080113924587353420 8`36.*^-31}
```

Sin, rel. error minimized, degree 31

Maximum relative error: 1.23931564680083903547907147997002462e-40
Maximum absolute error: 1.23877067991372097417082601537393501e-40





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.2831853071795864769252867665590057 7 + x2*(-41.341702240399760233968420089468526 9
+ x2*(81.605249276075054203397682678249486 8 + x2*(-
76.705859753061385841630641093888776 6 + x2*(42.058693944897653144986811146928281 9 +
x2*(-15.094642576822990391826616030226374 4 + x2*(3.819952584848282127733769747924786 99
+ x2*(-0.718122301778500512230037066912561024 +
x2*(0.104229162208139841079859116740720578 + x2*(-
0.012031585942120623546842337989959902 4 + x2*(0.001130923748251688663047603830276846 52
+ x2*(-0.000088235335990128373725340249723119086 6 +
x2*(5.805652367232453119211483338114168 53e-6 + x2*(-
3.264924439243192937197912662361338 37e-7 + x2*(1.587081028012417951500085384683419 07e-
8 - 6.612670493037454633517002650446795 04e-10*x2))))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

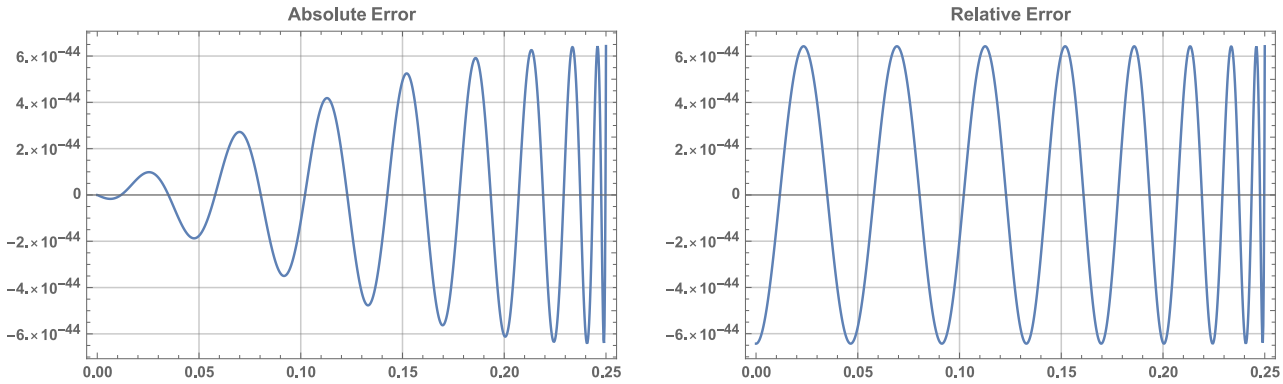
```
x1*(1. + x2*(-0.16666666666666666666666666666667 +
x2*(0.008333333333333333333333333333333249 + x2*(-
0.000198412698412698412698412698412687163 + x2*(2.75573192239858906525573192231961 84e-
6 + x2*(-2.505210838544171877505210804968216 56e-8 +
x2*(1.605904383682161459939228333954662 44e-10 + x2*(-
7.647163731819816475882994770631243 16e-13 +
x2*(2.811457254345520760694391712570016 11e-15 + x2*(-
8.220635246624327198233762646162277 98e-18 +
x2*(1.957294106338940029631344796431285 95e-20 + x2*(-
3.868170170529780237555265347287045 83e-23 +
x2*(6.446950244721636542039647630711462 47e-26 + x2*(-
9.183678869133499416039113076350312 1e-29 + x2*(1.130792495747003170096895243360358 82e-
31 - 1.193441026646244252264641506557405 98e-34*x2))))))))))))))
```

WL-style coefficient list:

```
{0.99999999999999999999999999999999999999876068435319916097 9`36.,
-0.16666666666666666666666666666666666666641613489805345332 5466`36.,
0.00833333333333333333333333333333333324899959534506163042 4`36.,
-0.00019841269841269841269841269841268716262470716588786410 04`36.,
2.75573192239858906525573192231961840488063548920616 54`36.*^-6,
-2.50521083854417187750521080496821655859791471476632`36.*^-8,
1.60590438368216145993922833395466243655265730301 7`36.*^-10,
-7.64716373181981647588299477063124316054772227778763841951716960 61`36.*^-13,
2.81145725434552076069439171257001611249247415654989502803636 87`36.*^-15,
-8.22063524662432719823376264616227797820442372812654382365 61`36.*^-18,
1.95729410633894002963134479643128595082630241569383882473 3`36.*^-20,
-3.86817017052978023755526534728704582999943145237508117`36.*^-23,
6.44695024472163654203964763071146246911778444394319 1`36.*^-26,
-9.18367886913349941603911307635031209682695613006 3`36.*^-29,
1.13079249574700317009689524336035881524359887 07`36.*^-31,
-1.19344102664624425226464150655740597507269935865182923480685298`36.*^-34}
```

Sin, rel. error minimized, degree 33

Maximum relative error: 6.438543145073709597710257920488864 63e-44
Maximum absolute error: 6.430342937256410434981681473703738 65e-44



C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.28318530717958647692528676655900577 + x2*(-41.34170224039976023396842008946852 69
+ x2*(81.60524927607505420339768267824949 51 + x2*(-
76.70585975306138584163064109389312 26 + x2*(42.05869394489765314498681114813252 6 +
x2*(-15.09464257682299039182661623233541 02 + x2*(3.81995258484828212773379204260342464
+ x2*(-0.71812230177850051223173810707767794 4 +
x2*(0.10422916220813984117257316433382557 6 + x2*(-
0.01203158594212062722681442320911297 35 + x2*(0.00113092374825179596628759690774300626
+ x2*(-0.00008823533599242430744901998647088163 04 +
x2*(5.80565240283899742598987165915956159e-6 + x2*(-
3.26492833228813238903435564038289 41e-7 + x2*(1.58736546929074325449663367029125159e-8
+ x2*(-6.73728155422516796504139324592253817e-10 +
2.47481846249220105437501580433199674e-11*x2))))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

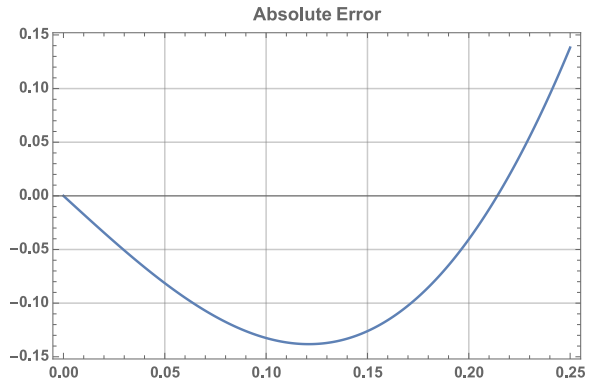
```
x1*(1. + x2*(-0.1666666666666666666666666666666666667 +
x2*(0.00833333333333333333333333333333333333 + x2*(-
0.000198412698412698412698412698412698404 + x2*(2.7557319223985890652557319223985218e-
6 + x2*(-2.50521083854417187750521083851162406e-8 +
x2*(1.60590438368216145993923770661606664e-10 + x2*(-
7.64716373181981647590110886153864847e-13 +
x2*(2.81145725434552076319524249332218442e-15 + x2*(-
8.22063524662432971259125266249393488e-18 +
x2*(1.95729410633912573976392818456761357e-20 + x2*(-
3.86817017063043222968282883823100715e-23 +
x2*(6.44695028426131356918907627824313965e-26 + x2*(-
9.18368981960883578196465536976398661e-29 + x2*(1.13099515966732849116990337581046170e-
31 + x2*(-1.21593057197470527850970994082426868e-34 +
1.13137790086518396610373370175684751e-37*x2)))))))))))))))
```

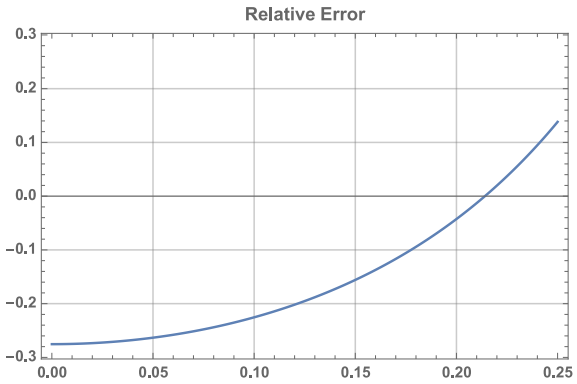
WL-style coefficient list:

```
{0.9999999999999999999999999999999999999999999356965706274384`36.,
-0.166666666666666666666666666666666666666666519700006184307058`36.,
0.0083333333333333333333333333333333333327739485882032634114`36.,
-0.0001984126984126984126984126984126984042514871664405923237`36.,
2.7557319223985890652557319223985218027201394619072219`36.*^-6,
-2.5052108385441718775052108385116240617924912504513 3`36.*^-8,
1.6059043836821614599392377066160664037573161675 76`36.*^-10,
-7.647163731819816475901108861538648469985032145922999842200969 7539`36.*^-13,
2.811457254345520763195242493322184415015774609868230394243460 01`36.*^-15,
-8.2206352466243297125912526624939348832141279927437321178999 3`36.*^-18,
1.957294106339125739763928184567613567301471721630668480 58`36.*^-20,
-3.868170170630432229682828838231007147041914821155425765`36.*^-23,
6.44695028426131356918907627824313965063761537956948`36.*^-26,
-9.18368981960883578196465536976398661056738378534 3`36.*^-29,
1.13099515966732849116990337581046169899632062 43`36.*^-31,
-1.2159305719747052785097099408242686842588946448412617064784749 1`36.*^-34,
1.131377900865183966103733701756847505493801701690120194319 48`36.*^-37}
```

Sin, abs. error minimized, degree 1

Maximum relative error: 0.275388646223291508763148658158904588
Maximum absolute error: 0.138216852866331001654175858019012891





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` first):

```
4.552867411465324151126344273367702 45*x1
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` first):

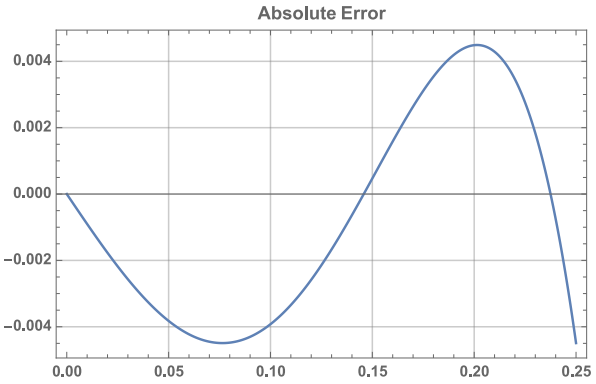
```
0.724611353776708491236851341841095412*x1
```

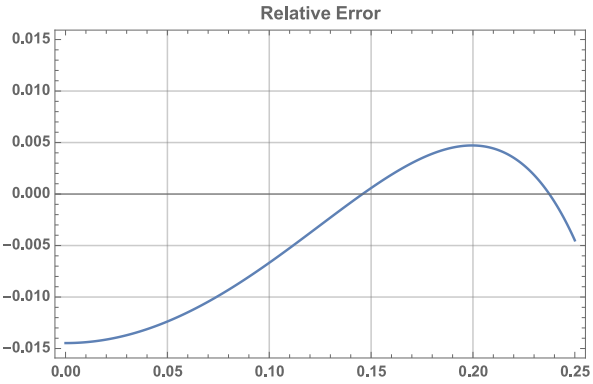
WL-style coefficient list:

```
{0.724611353776708491236851341841095411890482883916831463236 4`36.}
```

Sin, abs. error minimized, degree 3

Maximum relative error: 0.014470457021196464311146410020167892 3
Maximum absolute error: 0.004491734879217209487163683101290322 9





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.19226474423583116640920631407741 68 - 35.363706940043200292380734073423 7452*x2)
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

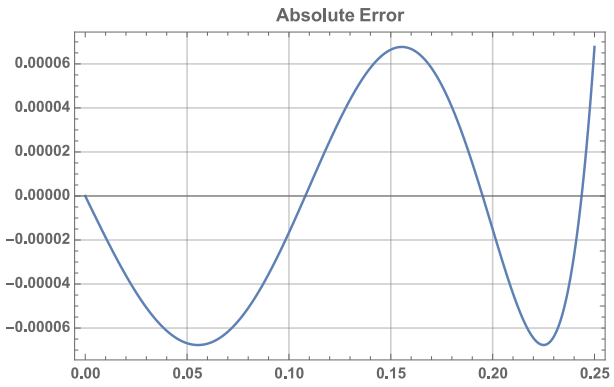
```
x1*(0.985529542978803535688853589979832108 -
0.142566726507797377224105326882192683*x2)
```

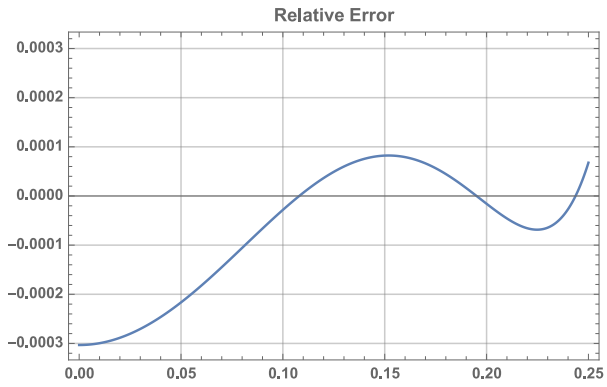
WL-style coefficient list:

```
{0.985529542978803535688853589979832107671918809422681757940 9`36.,
-0.142566726507797377224105326882192683476356048097815520818 3`36.}
```

Sin, abs. error minimized, degree 5

Maximum relative error: 0.000303226860956541311622126708083403337
Maximum absolute error: 0.000067706402415861164447484494669721288 4





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.2812800766220821491468958126456729 + x2*(-41.095242687120897021132333252580018 7
+ 73.585514734755164095668867279642332 3*x2))
```

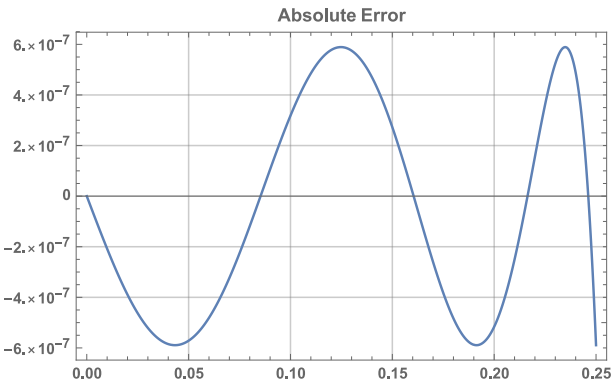
C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

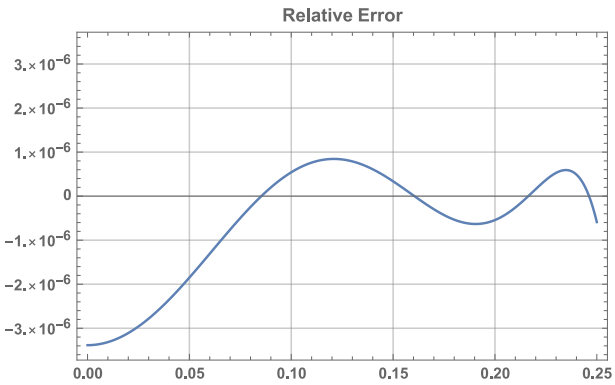
```
x1*(0.99969677313904345868837787329191659666287888105630234774 1`36.,
-0.16567307932054613904477208090807321445813000322154633815 99`36.,
0.00751437717830006597565730091774665237371359654169147909 72`36.}
```

Sin, abs. error minimized, degree 7

Maximum relative error: 3.384091997226920674153086779616822 01e-6

Maximum absolute error: 5.89148446885004117552807336268428l4e-7





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283164044302471356715402700309485 33 + x2*(-41.337142371100102923631190025063304 8
+ x2*(81.340768887664067654209653573769347 2 -
70.993433272075175056213268939606112 3*x2)))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

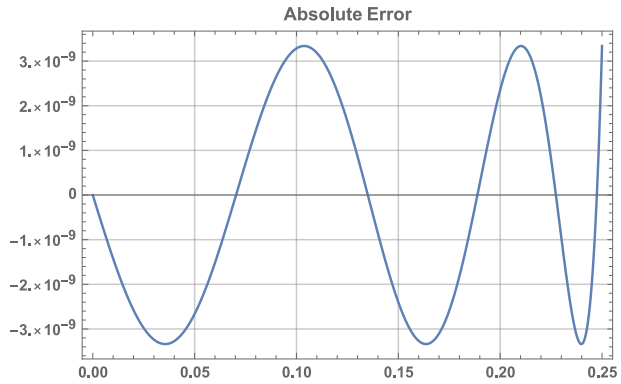
```
x1*(0.999996615908002773079325846913220383 + x2*(-
0.166648283818950568293660541409488659581060378179437246821 6 + x2*(0.008306325227159893964654117826159010794567004297748722301 6 -
0.000183636539769467852972802241586834839733575780525463001 6*x2)))
```

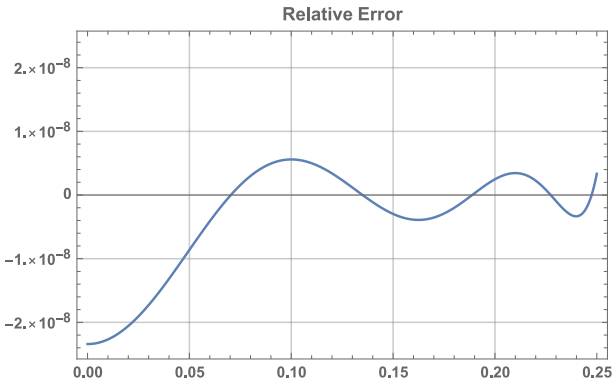
WL-style coefficient list:

```
{0.99999661590800277307932584691322038317799488920881454930356 36.,
-0.16664828381895056829366054140948865958106037817943724682166 36.,
0.00830632522715989396465411782615901079456700429774872230166 36.,
-0.00018363653976946785297280224158683483973357578052546300l 36.}
```

Sin, abs. error minimized, degree 9

Maximum relative error: 2.34101179326720584279701250915594214e-8
Maximum absolute error: 3.33811237735309799387214422296813557e-9





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185160089477443018853398557545 39 + x2*(-41.341655031416278077164972474139774 5
+ x2*(81.601004073261773524288948414194246 1 + x2*(-
76.549782293595742685664884070895689 1 + 39.536706065730207990689836742155331 6*x2))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

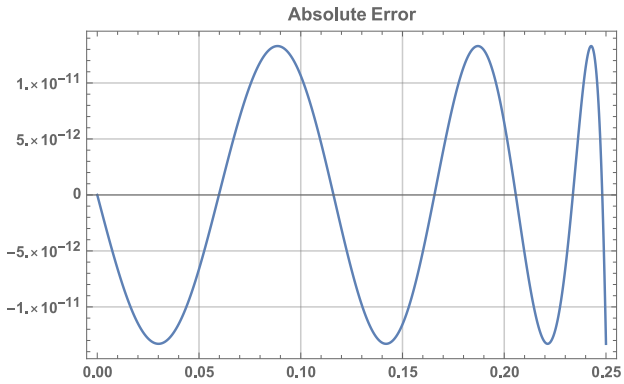
```
x1*(0.999999976589882067327941572029874908 + x2*(-
0.166666476346397125275908028138103582 + x2*(0.008332899823351751253555068919714072 98
+ x2*(-0.000198008977627954312733279528442081684 +
2.590488500536052284318054761189006498852245201199509 4*x2))))
```

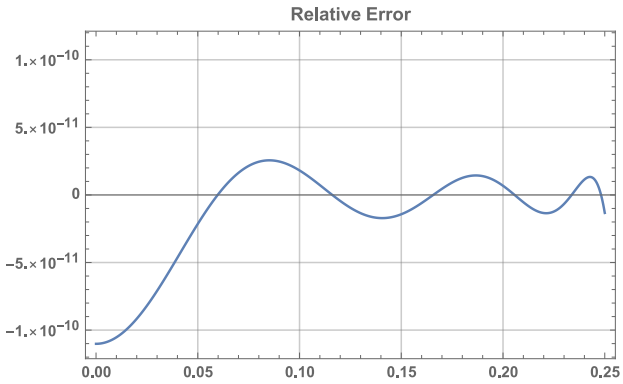
WL-style coefficient list:

```
{0.99999997658988206732794157202987490844057860566408943177 82`36.,
-0.16666647634639712527590802813810358171131324386982597983 82`36.,
0.00833289982335175125355506891971407297521482140589828593 09`36.,
-0.00019800897762795431273327952844208168361980020326138194 55`36.,
2.59048850053605228431805476118900649885224520119950 94`36.*^-6}
```

Sin, abs. error minimized, degree 11

Maximum relative error: 1.10148099345850673176490062657393315e-10
Maximum absolute error: 1.32971443901248371442745936559819048e-11





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185306487505557501680396338886 58 + x2*(-41.34170192977267827526420814992286 41
+ x2*(81.60520943107645645508267103541319 2 + x2*(-
76.70366782753266608479991943535639 35 + x2*(41.99998982534864350263029290165403 41 -
14.33702467903133759696876370668494 94*x2)))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

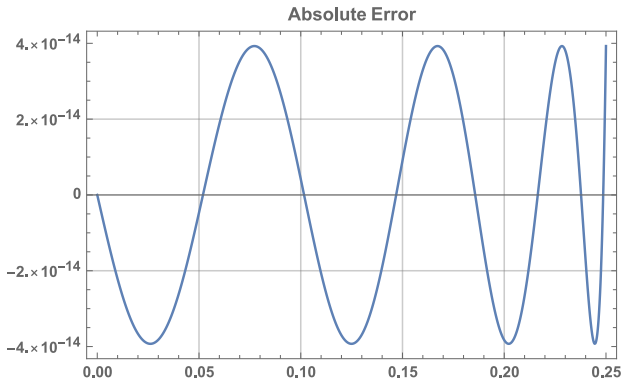
```
x1*(0.99999999988985190065414932682350994 + x2*(-
0.16666666541439166295723807683295033163323824634945504603 59 + x2*(0.00833332926445715285723741015926085083
+ x2*(-0.00019840702862605795189293170629136909542488708468270840 64 +
x2*(2.75188556386854406868696924998396177e-6 - 2.37947135452770603348051628038825 47e-
8*x2)))))
```

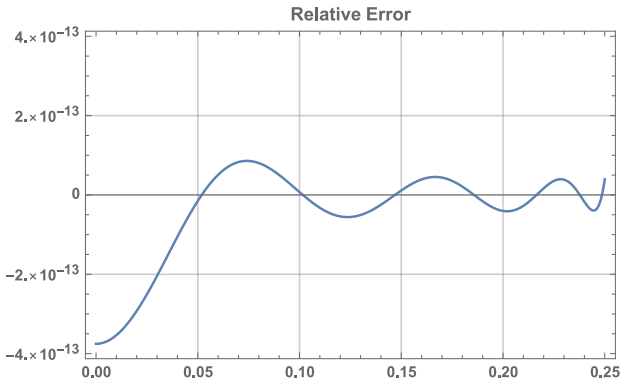
WL-style coefficient list:

```
{0.99999999988985190065414932682350993734260668512778603665 64`36.,
-0.16666666541439166295723807683295033163323824634945504603 59`36.,
0.00833332926445715285723741015926085082578326506027774781 61`36.,
-0.00019840702862605795189293170629136909542488708468270840 64`36.,
2.75188556386854406868696924998396177108624078376926 01`36.*^-6,
-2.37947135452770603348051628038825470328452008685123`36.*^-8}
```

Sin, abs. error minimized, degree 13

Maximum relative error: 3.752786255504396218534104873551898 18e-13
Maximum absolute error: 3.927519960919690098229483927349881 82e-14





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185307177228531779129694573357 82 + x2*(-41.34170223898954427853128557798468 42
+ x2*(81.60524903205595395523872531303582 73 + x2*(-
76.70584112975559272204887090153005 81 + x2*(42.05796387497369216798509903590110 08 +
x2*(-15.07929491438386442879162095779289 6 +
3.65508391952242788813000654371816106*x2))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

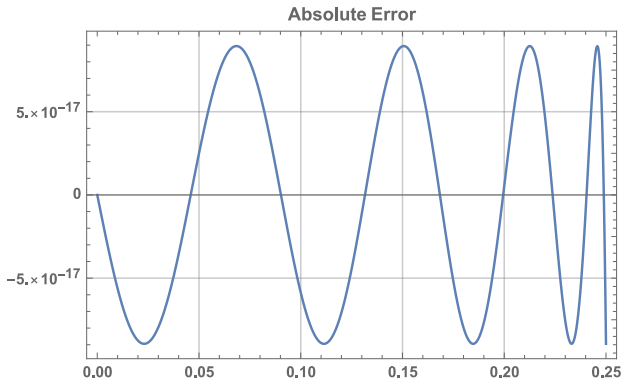
```
x1*(0.99999999999962472137444956037814659 + x2*(-
0.16666666666098146311103815915481880316573488120087194233 82 + x2*(0.00833333330841468508328542103458644311
+ x2*(-0.00019841265024036363054090364301488522642093977652529568 71 + x2*(2.75568408741356365778988994429302032e-6 + x2*(-
2.50266363478673728875335064390623419610463588904002e-8 + 1.536593755736466119055019561001252930645643666126e-
10*x2))))))
```

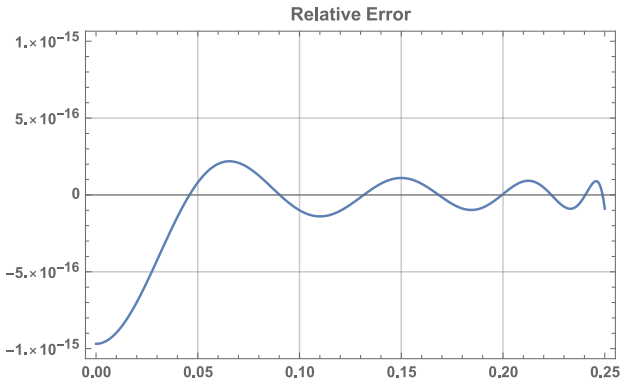
WL-style coefficient list:

```
{0.99999999999962472137444956037814658951264481018163510926 99`36.,
-0.16666666666098146311103815915481880316573488120087194233 82`36.,
0.00833333330841468508328542103458644310924415968019566601 67`36.,
-0.00019841265024036363054090364301488522642093977652529568 71`36.,
2.75568408741356365778988994429302032030522006441470 14`36.*^-6,
-2.50266363478673728875335064390623419610463588904002`36.*^-8,
1.53659375573646611905501956100125293064564366612 6`36.*^-10}
```

Sin, abs. error minimized, degree 15

Maximum relative error: 9.685199158162140458365214211028952 28e-16
Maximum absolute error: 8.945289417098071319854822388052073 29e-17





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185307179580391535181999312921 09 + x2*(-41.34170224039508262691375485758683 93
+ x2*(81.605249275026290026468388062757998 5 + x2*(-
76.705859647469529848428062047703158 9 + x2*(42.05868830538994865298005835456775 6 +
x2*(-15.094471616631987517183050926678201 6 + x2*(3.816997428325180432257114596001556 25
- 0.690935882396552049219204056252025327*x2)))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

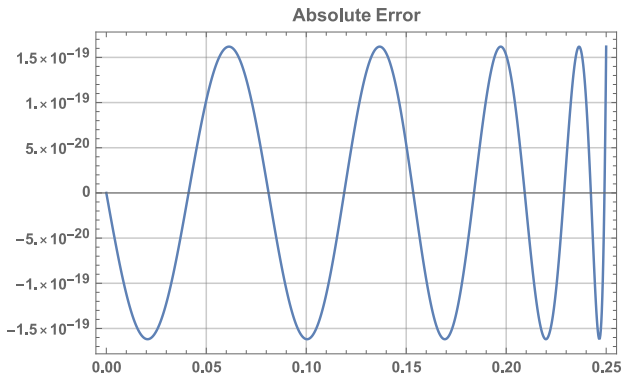
```
x1*(0.999999999999999903148008418378595416 + x2*(-
0.166666666666647809166043102595249597 + x2*(0.008333333333322623603809875967403959193
+ x2*(-0.000198412698139567192404904096148800949 +
x2*(2.755731552891836710032787042186293314e-6 + x2*(-
2.505182464813473702405434497810336528103302772492 64e-8 + x2*(1.604662038729048793662050782941579626582130210373e-
10 - 7.357660119718093309004786444861642139779707121691926130044559802 5e-13*x2)))))))
```

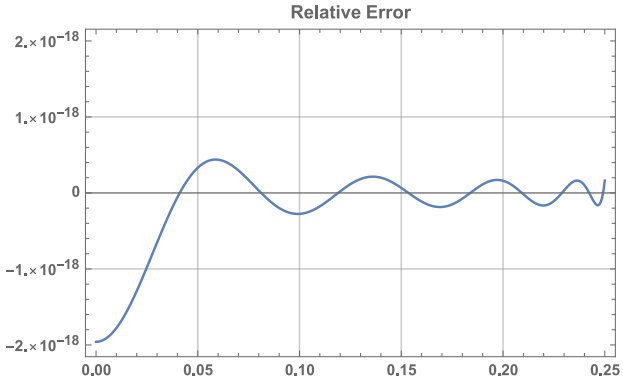
WL-style coefficient list:

```
{0.999999999999999031480084183785954163478578897104772391685 9`36.,
-0.166666666666647809166043102595249596591178972321058612104 6`36.,
0.008333333333322623603809875967403959193472065530348059248 08`36.,
-0.000198412698139567192404904096148800948836181522052644 01`36.,
2.755731552891836710032787042186293137556301440330155 4`36.*^-6,
-2.505182464813473702405434497810336528103302772492 64`36.*^-8,
1.604662038729048793662050782941579626582130210373`36.*^-10,
-7.357660119718093309004786444861642139779707121691926130044559802 5`36.*^-13}
```

Sin, abs. error minimized, degree 17

Maximum relative error: 1.9588806162806512600608897509311 6177e-18
Maximum absolute error: 1.6189725085765451457185636769151 7451e-19





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185307179586464617276859825524 31 + x2*(-41.34170224039974841088209029727240 14
+ x2*(81.60524927607172259609198946171641 19 + x2*(-
76.70585975263461508169094248777497 98 + x2*(42.05869391528073067283027530761642 38 +
x2*(-15.09464136913723492496629091420806 76 + x2*(3.81992280531953245260843650545316673
+ x2*(-0.71768544121039314091993575704711 5907 +
0.10071494377836113524243016871392 4538*x2)))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

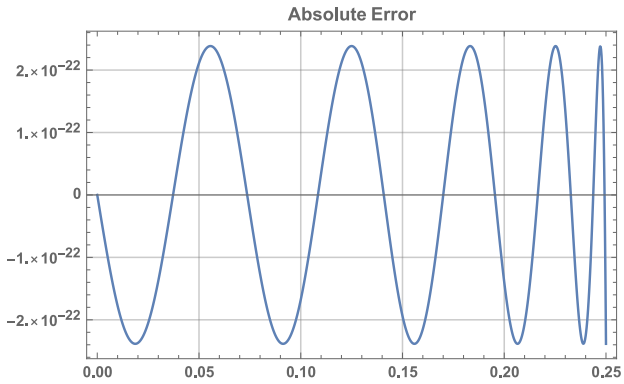
```
x1*(0.99999999999999999804111938371934874 + x2*(-
0.16666666666666661900258218266876855 1 + x2*(0.00833333333333329931175450059789792 9703
+ x2*(-0.00019841269841159449784293916956471444588849626190607854 57 +
x2*(2.75573192045805570474164069181139307e-6 + x2*(-
2.50521063810832659313336530795770274e-8 + x2*(1.60589186439698687926057420595617862e-
10 + x2*(-7.64251167703190749594387242108845966e-13 +
2.71666540637857550793519752037048238e-15*x2)))))))
```

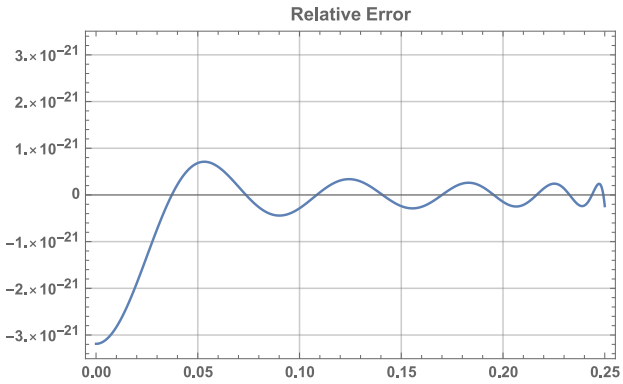
WL-style coefficient list:

```
{0.99999999999999999804111938371934873993911024906883822900 23`36.,
-0.16666666666666661900258218266876855077712583019716311787 42`36.,
0.00833333333333329931175450059789792970313039590502723860866`36.,
-0.00019841269841159449784293916956471444588849626190607854 57`36.,
2.75573192045805570474164069181139306683613504611549 22`36.*^-6,
-2.50521063810832659313336530795770273775241470514717`36.*^-8,
1.60589186439698687926057420595617862037015091283`36.*^-10,
-7.64251167703190749594387242108845966110574759542134426850838210 98`36.*^-13,
2.71666540637857550793519752037048237546999699142578124831683858`36.*^-15}
```

Sin, abs. error minimized, degree 19

Maximum relative error: 3.188528834388194324513860837177427 82e-21
Maximum absolute error: 2.384457284012414794096410824295304 53e-22





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.28318530717958647690525264903525941 + x2*(-41.341702240399760210450694673963193 3
+ x2*(81.605249276075046071647225861154004 5 + x2*(-
76.705859753060096442321510330160891 2 + x2*(42.058693944785259337330439767575543 8 +
x2*(-15.094642570932888638807077073736136 + x2*(3.819952391324174049192188469225034 93
+ x2*(-0.718118284976825145517253087459866405 +
x2*(0.104177995925885648161541781733250451 -
0.011665922067321720785680012313122839 1*x2))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

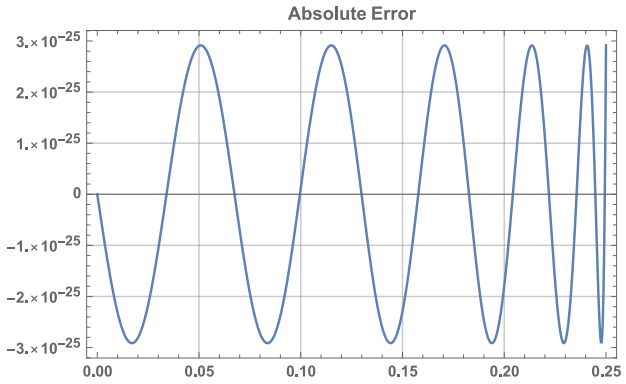
```
x1*(0.99999999999999999999681147116561181 + x2*(-
0.16666666666666666657185632528807903755782866054891953024 92 + x2*(0.008333333333333332502938380296781349383147902718763166884 7
+ x2*(-0.000198412698412695077448705835001537767447894619324340346 99 +
x2*(2.7557319223912248992632174522240604616598546783321574 e-6 + x2*(-
2.505210837566610025563811650262194771907637850885 63 e-8 + x2*(1.605904302324813009015017314276228710341600713799 e-
10 + x2*(-7.647120957573683604944907209347269692309786021889674791387799473 1 e-13 +
x2*(2.810077104948040455819379288463465588137952074564543312548706 06 e-15 - 7.970793758390793011296801848868938e-
18*x2)))))))))
```

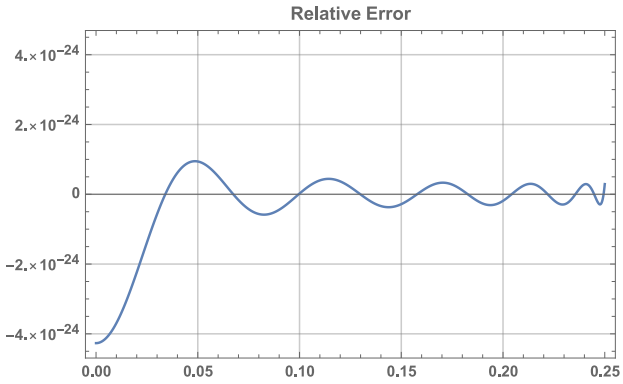
WL-style coefficient list:

```
{0.99999999999999999999681147116561180567548613916282257206 14`36.,
-0.16666666666666666657185632528807903755782866054891953024 92`36.,
0.008333333333333333250293838029678134938314790271876316688 47`36.,
-0.000198412698412695077448705835001537767448946193243403469 9`36.,
2.755731922391224899263217452224060461659854678332157 4`36.*^-6,
-2.505210837566610025563811650262194771907637850885 63`36.*^-8,
1.605904302324813009015017314276228710341600713799`36.*^-10,
-7.647120957573683604944907209347269692309786021889674791387799473 1`36.*^-13,
2.810077104948040455819379288463465588137952074564543312548706 06`36.*^-15,
-7.970793758390793011296801848868938000708645837363692391171 66`36.*^-18}
```

Sin, abs. error minimized, degree 21

Maximum relative error: 4.266984737587786907829565297784509 74e-24
Maximum absolute error: 2.913582809066737442530039925282989 63e-25





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185307179586476925259956303196 6 + x2*(-41.341702240399760233930657635659924 2
+ x2*(81.605249276075054187682575264253068 4 + x2*(-
76.705859753061382823084815906296277 4 + x2*(42.058693944897331014692536249173951 8 +
x2*(-15.094642576801987311540558194385367 3 + x2*(3.819952583968753783592547974642557 21
+ x2*(-0.718122277631009308859760818550070395 +
x2*(0.104228729786475701384518698659704212 + x2*(-
0.012026719514360864496950769249616083 2 +
0.001099637156573654081628831545517374 62*x2))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

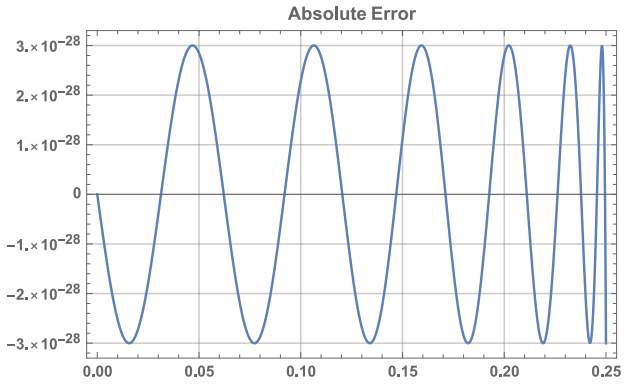
```
x1*(0.999999999999999999999995733015262 41 + x2*(-
0.166666666666666666666514429531755384 + x2*(0.008333333333333333333172854406289157612
+ x2*(-0.000198412698412698404890432651178389459 +
x2*(2.755731922398567958924273603723357 85e-6 + x2*(-
2.505210838540686061653152386322924 9e-8 + x2*(1.605904383312408592055084459334817 69e-
10 + x2*(-7.647163474677239943274233630231018 21e-13 +
x2*(2.811445590287220513076977855353897 4e-15 + x2*(-
8.217310237954690814658508558959265 86e-18 + 1.903146281082359052396624286110786 29e-
20*x2))))))))))
```

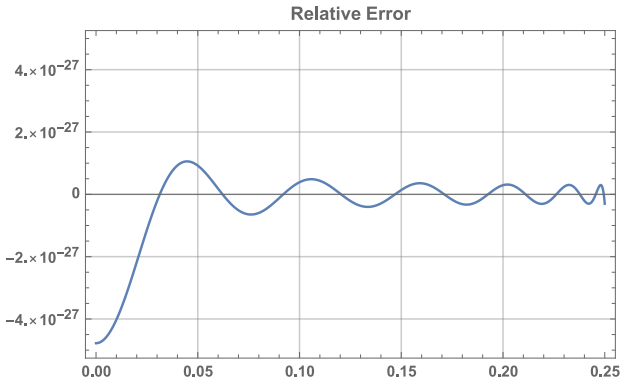
WL-style coefficient list:

```
{0.99999999999999999999999573301526241221309217043470221532283`36.,
-0.16666666666666666666665144295317553844763842771223800035615`36.,
0.00833333333333333333317285440628915761216847194073139223086`36.,
-0.00019841269841269840489043265117838945914487969137818629 41`36.,
2.75573192239856795892427360372335784717782094439601 23`36.*^-6,
-2.50521083854068606165315238632292490247507560260129`36.*^-8,
1.60590438331240859205508445933481768978146383269 3`36.*^-10,
-7.64716347467723994327423363023101821326803649014255111878906062 1`36.*^-13,
2.81144559028722051307697785535389739668667191152879786637946201`36.*^-15,
-8.21731023795469081465850855895926586425298298692905836507771`36.*^-18,
1.90314628108235905239662428611078629446114548557878833184 7`36.*^-20}
```

Sin, abs. error minimized, degree 23

Maximum relative error: 4.777545623747964662025950251570225 06e-27
Maximum absolute error: 3.001397484558428765040709117789514 5e-28





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.28318530717958647692528673654080 13 + x2*(-41.34170224039976023396837012502937 +
x2*(81.60524927607505420337305295748090 35 + x2*(-76.70585975306138583599920319656914 25
+ x2*(42.05869394489765242390618872767831 78 + x2*(-
15.09464257682293331099382240594766 42 + x2*(3.81995258484532999611569776961421019 +
x2*(-0.71812230167586873667409496336971244 6 +
x2*(0.10422915979115762671358246999961600 2 + x2*(-
0.01203154790004076709992539893802253 02 + x2*(0.00113054034746971435186969004818869744
- 0.00008599533925970412239632955811260631 89*x2))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

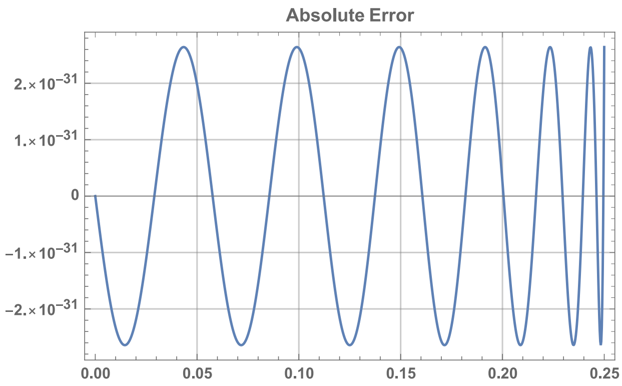
```
x1*(0.9999999999999999999999999999522245438 + x2*(-
0.1666666666666666666666666465237937929 + x2*(0.008333333333333333333333308182050187 6582
+ x2*(-0.0001984126984126984126838460304586 63913 +
x2*(2.7557319223985890180097358286906 5055e-6 + x2*(-
2.505210838544162403977085847362618 72e-8 + x2*(1.605904383680920386653273713816 03984e-
10 + x2*(-7.647163730726907942308792090229 44105e-13 +
x2*(2.811457189150310166614701687584 19682e-15 + x2*(-
8.220609254202045777476349221157 71996e-18 +
x2*(1.956630553122302605759344536154 64494e-20 -
3.769970413737549993940115881662 65576e-23*x2))))))))))))
```

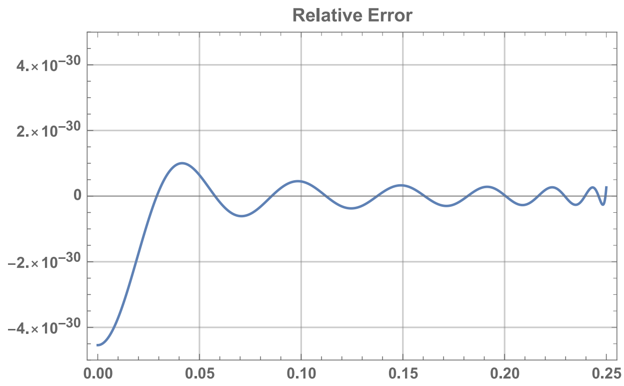
WL-style coefficient list:

```
{0.99999999999999999999999999995222454376252035337974049748 2937`36.,
-0.16666666666666666666666664652379379286488643115678779792951`36.,
0.0083333333333333333333333081820501876582399977495068271349 81`36.,
-0.00019841269841269841268384603045866391251226025860268939 04`36.,
2.75573192239858901800973582869065054542880465846580 83`36.*^-6,
-2.50521083854416240397708584736261872369779495664064`36.*^-8,
1.605904383680920386653273713816039840492673476821`36.*^-10,
-7.6471637307269079423087920902294410475937314471523551814066774086`36.*^-13,
2.811457189150310166614701687584196822280947242470219893703537 94`36.*^-15,
-8.22060925420204577747634922115771996253975618769297273363978`36.*^-18,
1.956630553122302605759344536154644944715977548038046433445`36.*^-20,
-3.769970413737549993940115881662655764946730707344189387`36.*^-23}
```

Sin, abs. error minimized, degree 25

Maximum relative error: 4.541463305547925712818854671890578 09e-30
Maximum absolute error: 2.641818325705518078154366768689645 57e-31





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185307179586476925286766530470911 + x2*(-41.341702240399760233968420034060314
3 + x2*(81.605249276075054203397650755649697 + x2*(-
76.705859753061385841622077617797510 6 + x2*(42.058693944897653143692418175503868 2 +
x2*(-15.094642576822990269785840943170177 7 + x2*(3.819952584848274514567173110447396 86
+ x2*(-0.718122301778175535228077214535034256 +
x2*(0.104229162198499551619812117313211425 + x2*(-
0.012031585743517746452024826661176431 + x2*(0.001130920960735826888965665414157877 96
+ x2*(-0.000088209901504660549040505709491135 8628 +
5.669478957249326239874812308159062 45e-6*x2))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

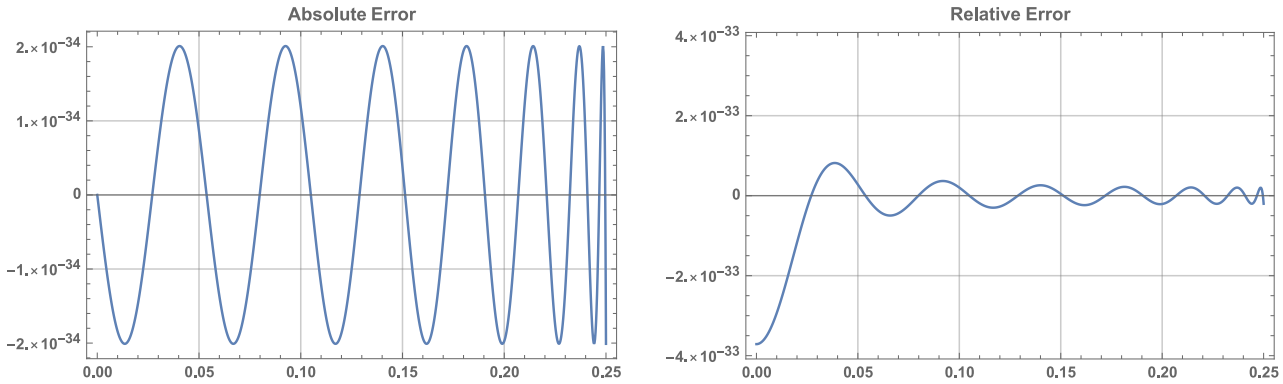
```
x1*(0.999999999999999999999999999545854 + x2*(-
0.166666666666666666666666666443291682 + x2*(0.008333333333333333333333333007347361421
+ x2*(-0.000198412698412698412698390547531326 46 +
x2*(2.755731922398589065170921870058940641 e-6 + x2*(-
2.505210838544171857250483279198991016331 e-8 + x2*(1.605904383682158259371331303811238 66e-
10 + x2*(-7.647163731816355850392172112273520 33e-13 +
x2*(2.811457254085485458076146606590564700 e-15 + x2*(-
8.220635110928017552882444486385882042905 e-18 +
x2*(1.957289281974453908252149172496632442 e-20 + x2*(-
3.867055141988375694442671718790215432710 e-23 + 6.295734990468686001606949192410224 18e-
26*x2))))))))))))
```

WL-style coefficient list:

```
{0.999999999999999999999999999954585366944520742871811451827`36.,
-0.166666666666666666666666666443291681756492865819291130433 4`36.,
0.008333333333333333333333330073473614208742344969942774953 5`36.,
-0.000198412698412698412698390547531326460415192968583618470 7`36.,
2.755731922398589065170921870058940641663988152581778 2`36.*^-6,
-2.505210838544171857250483279198991016331266528061 97`36.*^-8,
1.605904383682158259371331303811238660187285594815`36.*^-10,
-7.647163731816355850392172112273520328011874956355745386340900650 4`36.*^-13,
2.811457254085485458076146606590564700724024633075245684437748 22`36.*^-15,
-8.220635110928017552882444486385882042905661818429423089788 66`36.*^-18,
1.957289281974453908252149172496632442040228084168532083033`36.*^-20,
-3.867055141988375694442671718790215432710784210135955 9`36.*^-23,
6.295734990468686001606949192410224184991591510072555`36.*^-26}
```

Sin, abs. error minimized, degree 27

Maximum relative error: 3.710932568160163222920288767320977 09e-33
Maximum absolute error: 2.009856914379731188858930087255907 14e-34



C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.28318530717958647692528676655898245 + x2*(-41.341702240399760233968420089416288 4
+ x2*(81.605249276075054203397682643472957 4 + x2*(-
76.705859753061385841630630281192581 5 + x2*(42.058693944897653144984907697820040 3 +
x2*(-15.094642576822990391616147018240379 + x2*(3.819952584848282112187059889663008 51
+ x2*(-0.718122301778499715970493510273790408 +
x2*(0.104229162208110976342636405287321068 + x2*(-
0.012031585941374801564785625614276035 1 + x2*(0.001130923734603714145075944889099235 47
+ x2*(-0.000088235163179284101134147567619937553 4 +
x2*(5.804212001432939386164260891878405 39e-6 - 3.193762149834639359634179481999677 31e-
7*x2))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

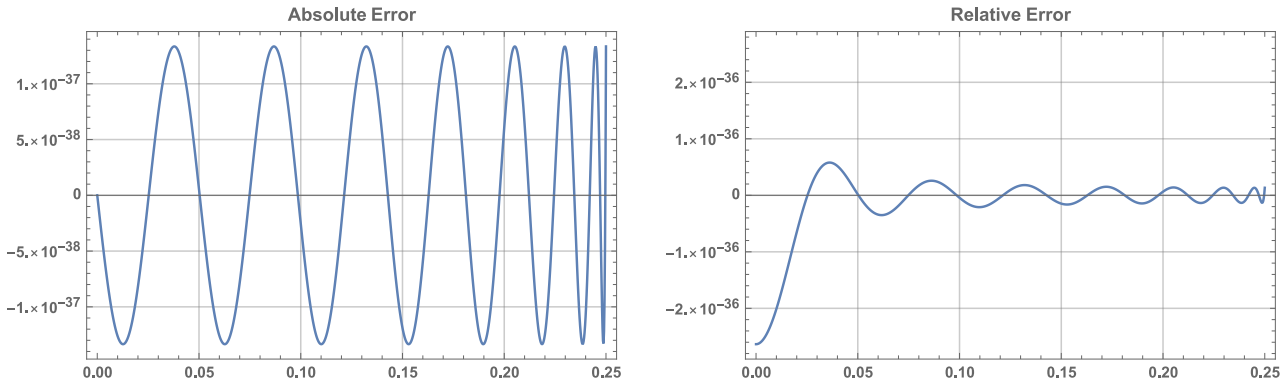
```
x1*(0.99999999999999999999999999999629 + x2*(-0.16666666666666666666666666666456 07
+ x2*(0.00833333333333333333333333332978203629 + x2*(-
0.000198412698412698412698412670443817038 +
x2*(2.75573192239858906525560720623886909e-6 + x2*(-
2.505210838544171877470279918243240 88e-8 + x2*(1.605904383682161453403406498823463 79e-
10 + x2*(-7.647163731819807996648903973830 6564e-13 +
x2*(2.811457254344742168857516795414123 13e-15 + x2*(-
8.220635246114740972203615059085011 98e-18 + x2*(1.957294082718337731035943654297292e-
20 + x2*(-3.868162594633508543289977365076 6175e-23 +
x2*(6.445350774747155389100108972262190 76e-26 -
8.983511414822677979273488889290960 93e-29*x2))))))))))))
```

WL-style coefficient list:

```
{0.99999999999999999999999999999999962890674318398378469392544`36.,
-0.16666666666666666666666666666666456070126353432891956614180 8`36.,
0.00833333333333333333333333332978203628511774457139924920 39`36.,
-0.00019841269841269841269841267044381703844410980704205000 5`36.,
2.75573192239858906525560720623886909291247805886848 28`36.*^-6,
-2.50521083854417187747027991824324087511113096517861`36.*^-8,
1.60590438368216145340340649882346378749646855876 4`36.*^-10,
-7.64716373181980799664890397383065639993753238850112586873117539 63`36.*^-13,
2.81145725434474216885751679541412312629008978586189567944795211`36.*^-15,
-8.22063524611474097220361505908501198251161289442159899001078`36.*^-18,
1.95729408271833773103594365429729199508254530535587306039 8`36.*^-20,
-3.86816259463350854328997736507661749606328416040111985 6`36.*^-23,
6.44535077474715538910010897226219076116363164290412 3`36.*^-26,
-8.98351141482267797927348888929096092795914892125 7`36.*^-29}
```

Sin, abs. error minimized, degree 29

Maximum relative error: 2.6344814436387452977882246711202076 4e-36
Maximum absolute error: 1.3348133524543095429239138787618073 9e-37



C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.28318530717958647692528676655900575 + x2*(-41.34170224039976023396842008946848 46
+ x2*(81.60524927607505420339768267821721 2 + x2*(-
76.70585975306138584163064108238001 94 + x2*(42.05869394489765314498680881440044 08 +
x2*(-15.09464257682299039182631745667009 49 + x2*(3.81995258484828212770804570578395629
+ x2*(-0.71812230177850051067773182724237765 3 +
x2*(0.10422916220813977387246692592539148 4 + x2*(-
0.01203158594211851059144346052141889 18 + x2*(0.00113092374820339150006009187157072642
+ x2*(-0.00008823533519597738946576445319216054 64 +
x2*(5.80564321844408342010290676383443878e-6 + x2*(-
3.26422380203998240640952562297723349e-7 + 1.55505138528488273803732379221574841e-
8*x2)))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

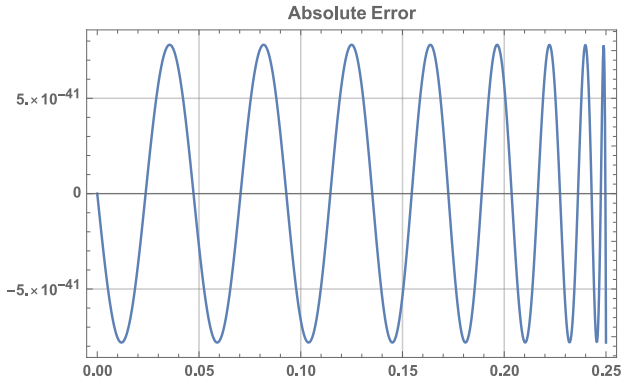
```
x1*(1. + x2*(-0.16666666666666666666666666666666666496 +
x2*(0.008333333333333333333333333333333003666 + x2*(-
0.00019841269841269841269841269382917813 +
x2*(2.7557319223985890652557317694898267 8e-6 + x2*(-
2.505210838544171877505161251644546 43e-8 + x2*(1.605904383682161459928413971440362 95e-
10 + x2*(-7.647163731819816459352757093677529409543617140789829257811512 2158e-13 +
x2*(2.811457254345518947855198558659819043672838109889415434425723 21e-15 + x2*(-
8.220635246622883511947506042561541621975816351910486069442 69e-18 +
x2*(1.957294106255351942800127086063738487337227226183867887934e-20 + x2*(-
3.868170135714803746125158855990949566188123040639976 29e-23 +
x2*(6.446940085349409600613396728354058162371543383828437e-26 + x2*(-
9.181708095476163309161208081453804191976682353325e-29 + 1.107971430534527450800813573949680712668221778 1e-
31*x2)))))))))))))
```

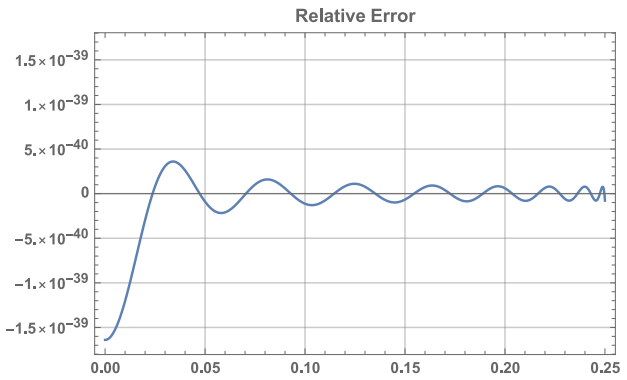
WL-style coefficient list:

```
{0.99999999999999999999999999999999999973655185564230600780122`36.,
-0.16666666666666666666666666666666666495806373574789945461718 9`36.,
0.00833333333333333333333333333333330036661041654228588672654 6`36.,
-0.00019841269841269841269841269838291781259049914143954776 73`36.,
2.75573192239858906525573176948982677914300341679515 84`36.*^-6,
-2.50521083854417187750516125164454643100996241655573`36.*^-8,
1.60590438368216145992841397144036295344953640186 7`36.*^-10,
-7.64716373181981645935275709367752940954361714078982925781151221 58`36.*^-13,
2.81145725434551894785519855865981904367283810988941543442572321`36.*^-15,
-8.22063524662288351194750604256154162197581635191048606944269`36.*^-18,
1.95729410625535194280012708606373848733722722618386788 7934`36.*^-20,
-3.86817013571480374612515885599094956618812304063997629`36.*^-23,
6.44694008534940960061339672835405816237154338382843 7`36.*^-26,
-9.18170809547616330916120808145380419197668235332 5`36.*^-29,
1.10797143053452745080081357394968071266822177 81`36.*^-31}
```

Sin, abs. error minimized, degree 31

Maximum relative error: 1.64002557798425265331386526431095701e-39
Maximum absolute error: 7.80601195686311420504782780641951757e-41





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.283185307179586476925286766559005770 + x2*(-41.341702240399760233968420089468526
+ x2*(81.605249276075054203397682678249469 2 + x2*(-
76.705859753061385841630641093882651 6 + x2*(42.058693944897653144986811145712485 4 +
x2*(-15.094642576822990391826615877503260 8 + x2*(3.819952584848282127733756813215870 93
+ x2*(-0.718122301778500512229268635441459545 +
x2*(0.104229162208139841047063127709829973 + x2*(-
0.012031585942120622529267043190991338 2 + x2*(0.001130923748251665685183863308193918 35
+ x2*(-0.000088235335989754785476883718992656724 7 +
x2*(5.805652362973486954649313410613116 03e-6 + x2*(-
3.264924116247615991578271615826725 59e-7 + x2*(1.587066396160577797584371784513340 06e-
8 - 6.609675321111034224695068438276072 77e-10*x2))))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

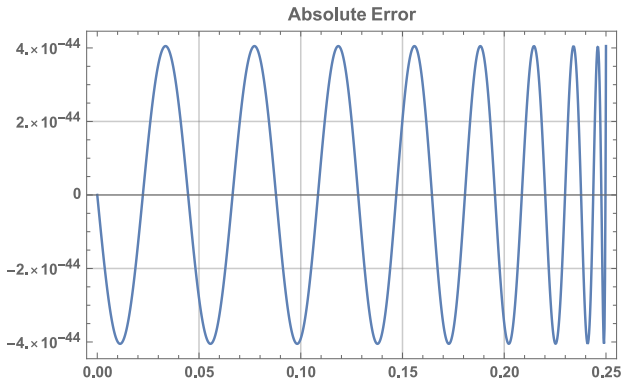
```
x1*(1. + x2*(-0.16666666666666666666666666666667 +
x2*(0.008333333333333333333333333333333 3069 + x2*(-
0.000198412698412698412698412698412671319 +
x2*(2.755731922398589065255731922239958 08e-6 + x2*(-
2.505210838544171877505210779621236 82e-8 + x2*(1.605904383682161459939222896215505 06e-
10 + x2*(-7.647163731819816475874811873008313 35e-13 +
x2*(2.811457254345520759809759050069993 19e-15 + x2*(-
8.220635246624326502970862579627032 93e-18 +
x2*(1.957294106338900261753673901523053 83e-20 + x2*(-
3.868170170513402412248387203196347 97e-23 +
x2*(6.446950239992220927722710735937271 41e-26 + x2*(-
9.183677960601706408708855159547432 1e-29 + x2*(1.130782070577797758507791922718732 38e-
31 - 1.192900464242202969379711013732035 67e-34*x2))))))))))))))
```

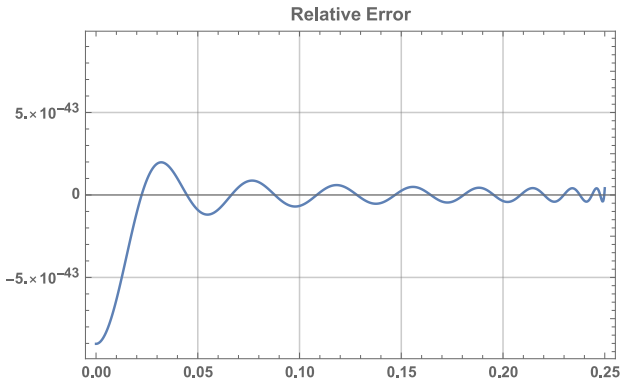
WL-style coefficient list:

```
{0.99999999999999999999999999999999999999835997450161551476 71`36.,
-0.16666666666666666666666666666666666666546123306699188675 1738`36.,
0.00833333333333333333333333333333333306948563055744330311 194`36.,
-0.00019841269841269841269841269841267131924100023666626334 77`36.,
2.75573192239858906525573192223995807942567183413047 7`36.*^-6,
-2.50521083854417187750521077962123682401042055739648`36.*^-8,
1.60590438368216145993922289621550505906415123174 9`36.*^-10,
-7.64716373181981647587481187300831334528706612188496137952714956 75`36.*^-13,
2.81145725434552075980975905006999319274992857658877472801886566`36.*^-15,
-8.22063524662432650297086257962703293359428698085145324427873`36.*^-18,
1.95729410633890026175367390152305382698755222952929604679 6`36.*^-20,
-3.86817017051340241224838720319634796646899065906232863 1`36.*^-23,
6.44695023999222092772271073593727141160836338729945 4`36.*^-26,
-9.18367796060170640870885515954743210270282334957 5`36.*^-29,
1.13078207057779775850779192271873237541489504 22`36.*^-31,
-1.19290046424220296937971101373203567177141867580122112873616 08`36.*^-34}
```

Sin, abs. error minimized, degree 33

Maximum relative error: 9.026298748117456456274547237281524 09e-43
Maximum absolute error: 4.050418132811158145511600410345714 79e-44





C-style Horner evaluation for sin(2πx) approximation (set `x1 = x` and `x2 = x * x` first):

```
x1*(6.28318530717958647692528676655900577 + x2*(-41.34170224039976023396842008946852 69
+ x2*(81.60524927607505420339768267824949 5 + x2*(-
76.70585975306138584163064109389311 77 + x2*(42.05869394489765314498681114813140 04 +
x2*(-15.09464257682299039182661623217336 23 + x2*(3.81995258484828212773379202678015315
+ x2*(-0.71812230177850051223173701471566 2347 +
x2*(0.10422916220813984117251841981674 7688 + x2*(-
0.01203158594212062722479186954722041 75 + x2*(0.00113092374825179591086433399148871892
+ x2*(-0.00008823533599242318440577718497314448 41 +
x2*(5.80565240282239984379553819712014009e-6 + x2*(-
3.26492833054978809891389086260956241e-7 + x2*(1.58736534709332145815783307969688033e-
8 + x2*(-6.73722985845646172199141106651886326e-10 +
2.47382386455741853500280877434932012e-11*x2)))))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x1 = x` and `x2 = x * x` first):

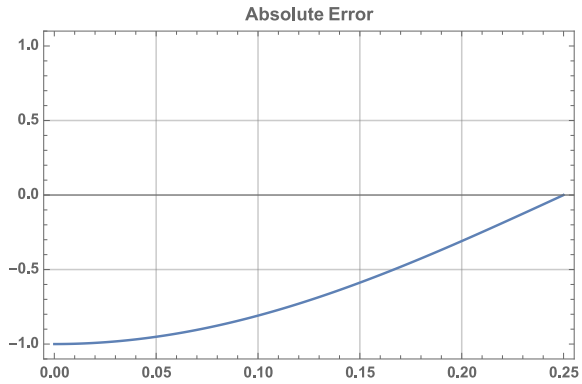
```
x1*(1. + x2*(-0.16666666666666666666666666666667 +
x2*(0.0083333333333333333333333333333333 + x2*(-
0.00019841269841269841269841269841269841 +
x2*(2.7557319223985890652557319223984480 6e-6 + x2*(-
2.50521083854417187750521083848472947e-8 + x2*(1.60590438368216145993923769996397832e-
10 + x2*(-7.64716373181981647590109722915908442e-13 +
x2*(2.81145725434552076319376582531091405e-15 + x2*(-
8.22063524662432971120933376991172232e-18 +
x2*(1.95729410633912564384267263771198416e-20 + x2*(-
3.86817017063038299631930987571173923e-23 + x2*(6.44695028424288260229317958932788 41e-
26 + x2*(-9.18368981471916803573077288615983239e-29 +
x2*(1.13099507260187617461053670403005966e-31 + x2*(-
1.21592124202981493418420109807105932e-34 + 1.13092321453537296083130424961430560 23851e-
37*x2))))))))))))))
```

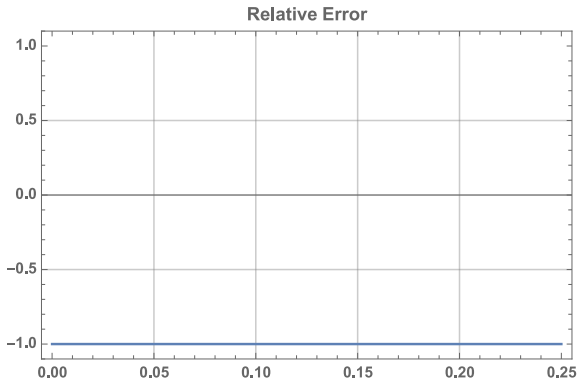
WL-style coefficient list:

```
{0.99999999999999999999999999999999999999999909745009041883 27`36.,
-0.16666666666666666666666666666666666666666592037639331048 6732`36.,
0.00833333333333333333333333333333333333314941769534174164 696`36.,
-0.00019841269841269841269841269841269839139938484810051031 47`36.,
2.75573192239858906525573192239844805629224857908013 86`36.*^-6,
-2.50521083854417187750521083848472946967502287375286`36.*^-8,
1.60590438368216145993923769996397832476719653363 1`36.*^-10,
-7.64716373181981647590109722915908442143543944987923115152653295 24`36.*^-13,
2.81145725434552076319376582531091404628433537330520565428188616`36.*^-15,
-8.22063524662432971120933376991172232436731314677095939547517`36.*^-18,
1.95729410633912564384267263771198416166100019596910747570 6`36.*^-20,
-3.86817017063038299631930987571173922629141266722321668 1`36.*^-23,
6.44695028424288260229317958932788409760675899324014 8`36.*^-26,
-9.18368981471916803573077288615983238763537275900 8`36.*^-29,
1.13099507260187617461053670403005966310283494 19`36.*^-31,
-1.21592124202981493418420109807105931793761843316881812093955282`36.*^-34,
1.13092321453537296083130424961430560238518099837345109582762`36.*^-37}
```

Cos, rel. error minimized, degree 0

Maximum relative error: 0.99999999999999990028472498526237759
Maximum absolute error: 0.99999999999999990028472498526237759





C-style Horner evaluation for sin(2πx) approximation:

```
9.971527501473762240958349911284860751341224492349366537271189`36.*^-17
```

C-style Horner evaluation for sin(x) approximation:

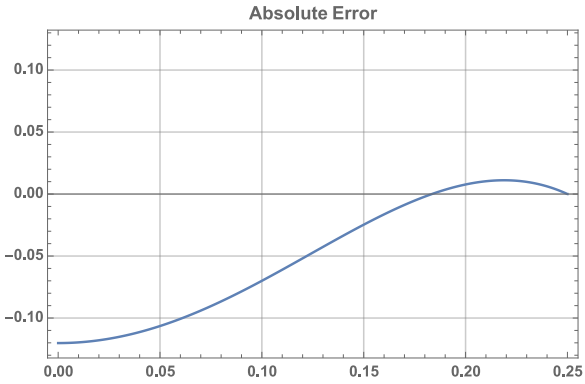
```
9.971527501473762240958349911284860751341224492349366537271189`36.*^-17
```

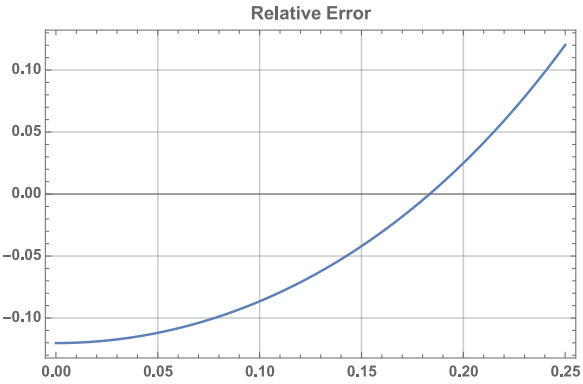
WL-style coefficient list:

```
{9.971527501473762240958349911284860751341224492349366537271189`36.*^-17}
```

Cos, rel. error minimized, degree 2

Maximum relative error: 0.12019830520430475459625115867870539 9
Maximum absolute error: 0.12019830520430475459625115867870539 9





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.879801694795695245403748841321294601 - 14.076827116731124118229697715437471 1*x2
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

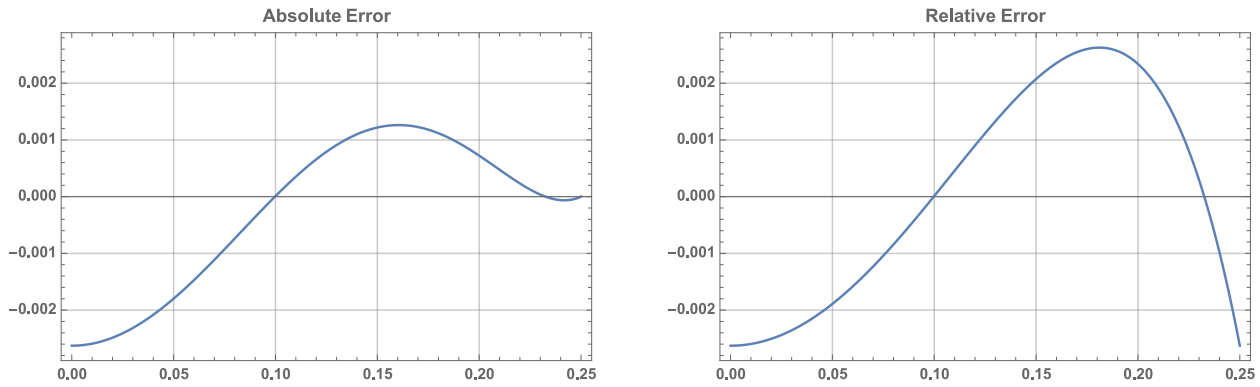
```
0.879801694795695245403748841321294601 - 0.3565701963489385869351765937398189597686514213566533980325*x2
```

WL-style coefficient list:

```
{0.879801694795695245403748841321294601025514325070465250062536.,
-0.3565701963489385869351765937398189597686514213566533980325`36.}
```

Cos, rel. error minimized, degree 4

Maximum relative error: 0.0026273549595220093009723413016528138 7
Maximum absolute error: 0.0026273549595220093009723413016528138 7



C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.9973726450404779906990276586983471861311355716720078400587 + x2*(-19.382570345302876702491766424192573 9 +
54.793728394483661687985964526525408 3*x2)
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

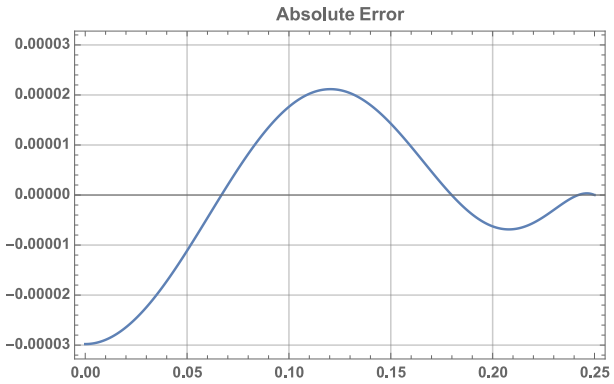
```
0.9973726450404779906990276586983471861311355716720078400587 + x2*(-0.490966242354240750313919970830772248 +
0.035156965210360153679189300303172928795410532429239404777*x2)
```

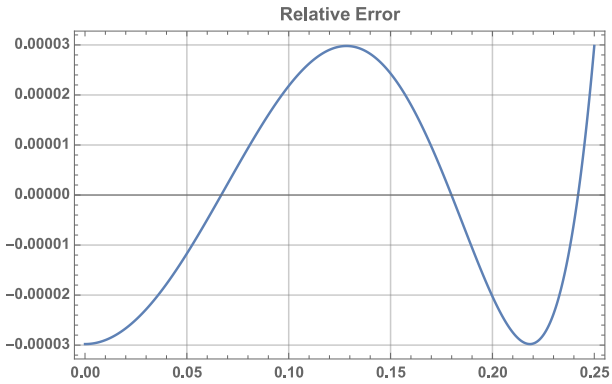
WL-style coefficient list:

```
{0.997372645040477990699027658698347186131135571672007840058 7`36.,
-0.490966242354240750313919970830772247960323681801174453409 3`36.,
0.035156965210360153679189300303172928795410532429239404777`36.}
```

Cos, rel. error minimized, degree 6

Maximum relative error: 0.0000297893100469313736764129442719222356
Maximum absolute error: 0.0000297893100469313736764129442719222356



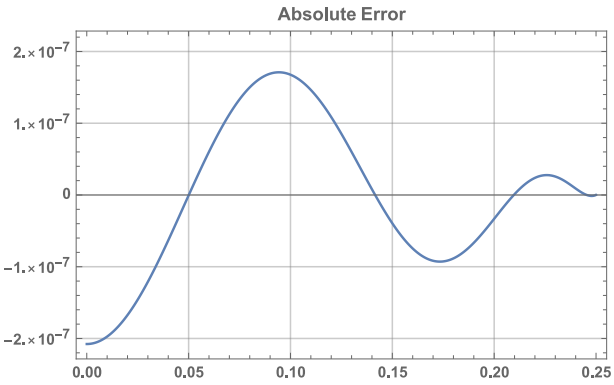


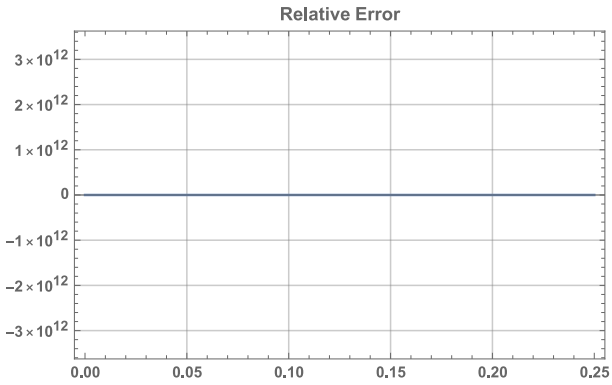
C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99997021068995306862632358705572807776440381287507087178888`36.,
-0.49978270670468880914046661772633345496262171815331727506 4`36.,
0.04136611496384822525693838725764599434026744816775686400 56`36.,
-0.00124123975823986007021296049447201019821261277604017390 03`36.}
```

Cos, rel. error minimized, degree 8

Maximum relative error: 3.2966932019124183402978459364213512 5e12
Maximum absolute error: 2.0772878506073244619477794608897801 1e-7





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999979227121493926755380522205391l + x2*(-19.739104078078984423570235200493203 8 +
x2*(64.92986441734314183359306437757045 93 + x2*(-85.13011694765188346138889531683921 7
+ 55.42049784776376508903254188936099 91*x2)))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

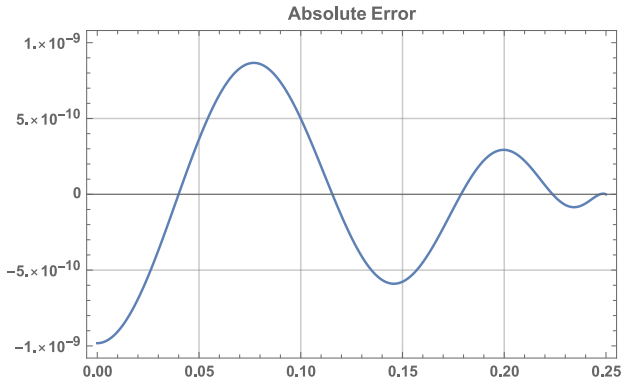
```
0.99999979227121493926755380522205391l + x2*(-0.49999734730756479826168926327517092 6 +
x2*(0.04166055224421903343374865691383869 6 + x2*(-
0.00138357918540809043059034096681139650069252787361019990 3 +
0.00002281557802268525783918013034280062 1l*x2)))
```

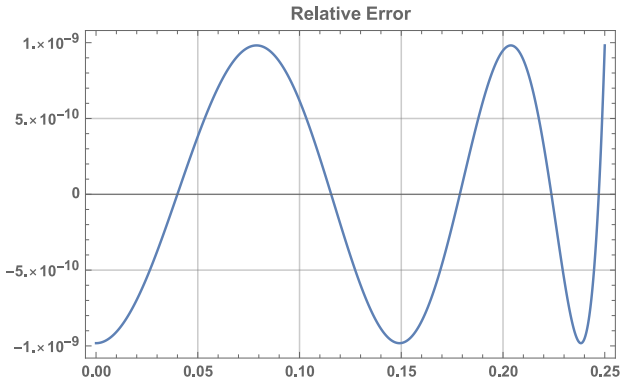
WL-style coefficient list:

```
{0.99999979227121493926755380522205391102198931153846153953 02`36.,
-0.49999734730756479826168926327517092596935904636338141377 61`36.,
0.04166055224421903343374865691383869615286460440120440958 5`36.,
-0.00138357918540809043059034096681139650069252787361019990 33`36.,
0.00002281557802268525783918013034280062166606426286324514 37`36.}
```

Cos, rel. error minimized, degree 10

Maximum relative error: 9.818993236778140784758532407656166 57e-10
Maximum absolute error: 9.818993236778140784758532407656166 57e-10





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

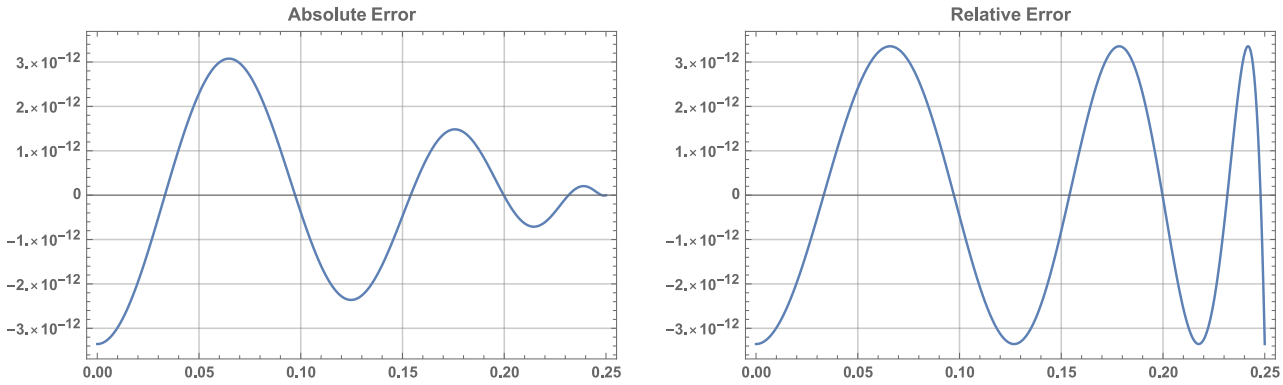
```
0.99999999901810067632218592152414676 + x2*(-19.739208032054899568259911126682762 4 +
x2*(64.93928782457875385841940038108542 51 + x2*(-85.45116163081640449674656542621073 41
+ x2*(60.10292880124621596797249652514367 92 -
24.73379445955237813729507250361243 94*x2))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

```
0.99999999901810067632218592152414676 + x2*(-0.49999998049253581064488831264724178 +
x2*(0.041666598527435249497052983107926881792434768262528390376 9`36.,
0.0013887969715117499354050093607473354568100294068286064696`36.,
0.000024743246897989778467719953143233170004301641985987533 8`36.,
-2.5792418318252055980398115457876350835965698949425 31`36.*^-7}
```

Cos, rel. error minimized, degree 12

Maximum relative error: 3.3550223770591169654965648793721355 3e-12
Maximum absolute error: 3.3550223770591169654965648793721355 3e-12



C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

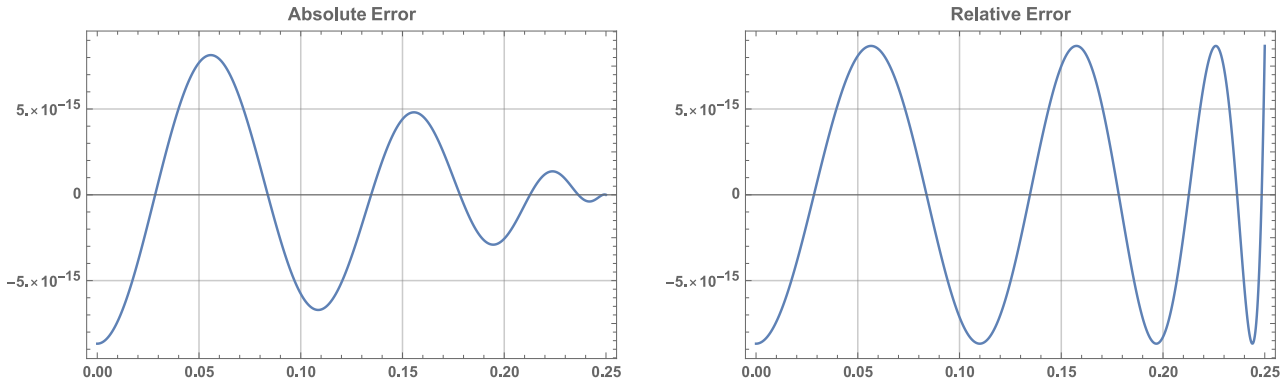
```
0.99999999999664497762294088303450344 + x2*(-19.739208798392478282001076336809467 +
x2*(64.939393282862889478723091451855669 9 + x2*(-85.456760688627053551670913896517608
+ x2*(60.242519229947729662723197057445707 7 + x2*(-
26.384294683341718978801286862966783 7 + 7.482426460177312366100732915862326 63*x2)))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

```
0.9999999999966449776229408830345034351206278644747810789636`36.,
-0.4999999990409344686474973754012715337305225209508672691796`36.,
0.0416666661919898461055893453767336908690840881385524373943`36.,
-0.0013888879703277092068138435556020346755801164008593851926`36.,
0.0000248007136556145113256051130495176343944360042516099796`36.,
-2.7513561116457137114195920891056951642233106148417176`36.*^-7,
1.9764418299584177279944484831045178095358117934665`36.*^-9}
```

Cos, rel. error minimized, degree 14

Maximum relative error: 8.675326437280207907499206194714854 27e-15
Maximum absolute error: 8.675326437280207907499206194714854 27e-15



C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

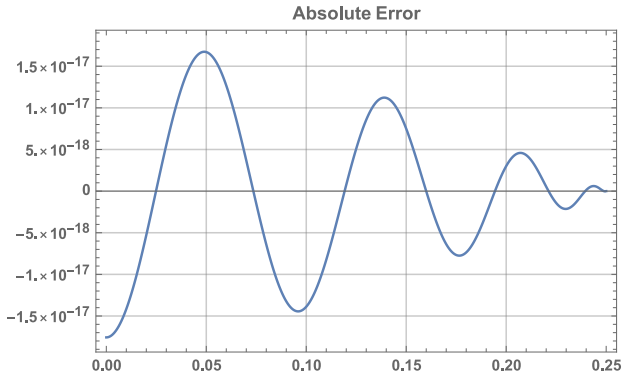
```
0.9999999999999913246735627197920925 + x2*(-19.739208802165385317528125439523807 +
x2*(64.939394019155072705556239499759341 4 + x2*(-85.456816842207821078877367186140396 2
+ x2*(60.244622320021700380473555489092483 + x2*(-
26.425704632433149624494248688812120 7 + x2*(7.894512103646625824560190333632142 16 -
1.636256916138937687630690020298606 95*x2))))))
```

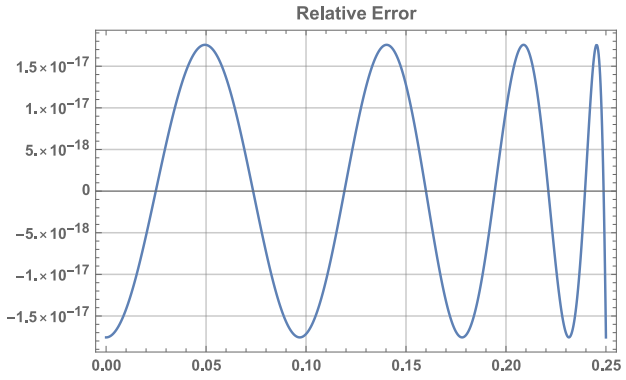
C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

```
0.9999999999999913246735627197920925 + x2*(-0.4999999999996622985177758227029981347820510428448621068418 +
x2*(0.041666666664412501559930358186670219 + x2*(-
0.0013888888829650733003878347394756424069146582848425073730 +
x2*(0.000024801579458296458001032167371448607257627878828741773 8 + x2*(-
2.7556743440594677886032696595971833769161596347843880 * ^-7 +
2.0852919880739418602929719762765470683432821021175 * ^-9 +
-1.0947963434564337359304337635706349951535970831 2 * ^-11}
```

Cos, rel. error minimized, degree 16

Maximum relative error: 1.756995551992551337530037563556998 76e-17
Maximum absolute error: 1.756995551992551337530037563556998 76e-17





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

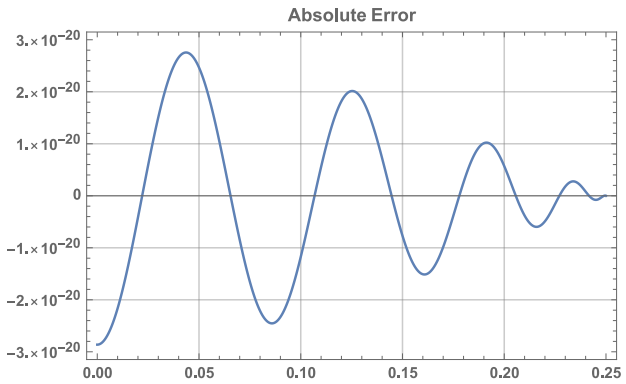
```
0.9999999999999999982430044480074486 62 + x2*(-19.739208802178681939276481858775061 2 +
x2*(64.939394022656209143797722089541625 3 + x2*(-85.456817205058621783650728774792583 7
+ x2*(60.244641258656183397655589644431288 + x2*(-
26.426252302086855764217434459983774 5 + x2*(7.903430628150753983919198681167382 23 +
x2*(-1.712917152107789996352635346800164 25 +
0.270783473417456507078234525883083018*x2)))))))
```

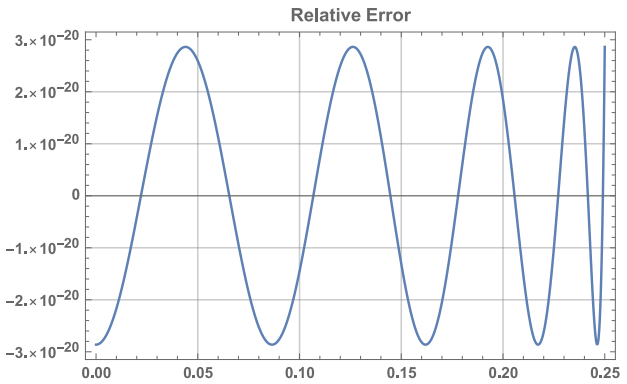
C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

```
0.9999999999999999982430044480074486624699624364430012387391 2`36.,
-0.4999999999999991058812728034744362675252233271368901237966`36.,
0.0416666666666589143440688443179245049794077100058356260936`36.,
-0.0013888888888623142917574713089718510686343043428500349693`36.,
0.0000248015872549765577961155967511699095293017759361650482`36.,
-2.7557314550896079518997256563558464173004311820463840`36.*^-7,
2.0876477673101671021960972328849059583671196948314`36.*^-9,
-1.1460886223152144048083015396436919068417012412 4`36.*^-11,
4.5892768875448174777617890429148314415406223516404297038602340 4`36.*^-14}
```

Cos, rel. error minimized, degree 18

Maximum relative error: 2.8627205479174578745311809588915696 9e-20
Maximum absolute error: 2.8627205479174578745311809588915696 9e-20





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.9999999999999999999971372794520825 42 + x2*(-19.739208802178717164804262724590098 9 +
x2*(64.939394022668260033916914508888914 1 + x2*(-85.456817206688343627604294168635522 8
+ x2*(60.244641371400282186566445498898851 7 + x2*(-
26.426256758803318347357235588179824 8 + x2*(7.90353558996711987941677721380101198 +
x2*(-1.714375161807754458843143390316851 28 +
x2*(0.281817053201561500323407638868350504 -
0.035098382958841707876037353787129572 6*x2))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

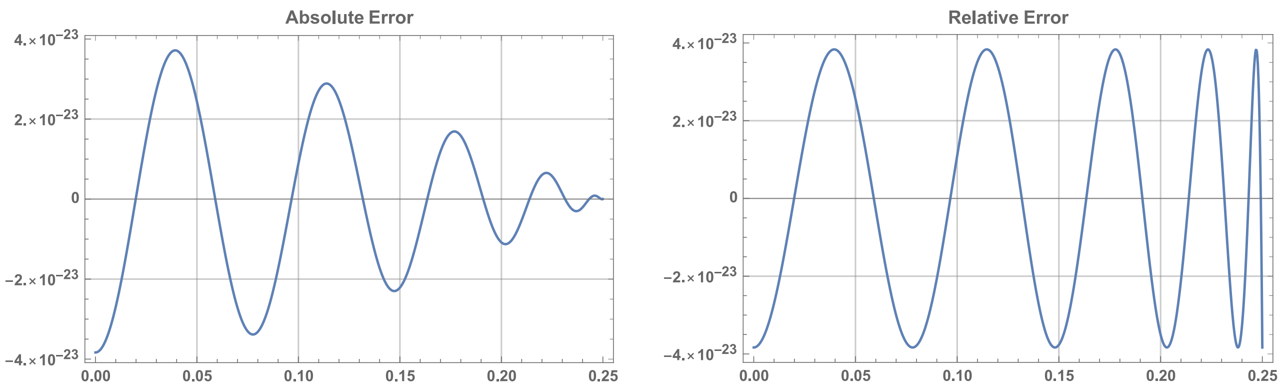
```
0.99999999999999999997137279452082542 + x2*(-0.499999999999999998154315099318475409 +
x2*(0.04166666666666666464830761430785494906 + x2*(-
0.001388888888888880138354292714761533424 +
x2*(0.000024801587301391185639801128147609099 + x2*(-
2.75573191983631520203018748182144674e-7 + x2*(2.08767549239714497727103009851893486e-
9 + x2*(-1.1470641559691907631205172698119569e-11 +
x2*(4.77627556974286641810975958891813886e-14 -
1.50677871898184264862447669562686491e-16*x2))))))))
```

WL-style coefficient list:

```
{0.999999999999999999971372794520825421254688190411084303105`36.,
-0.4999999999999999981543150993184754092229377420536693426857`36.,
0.0416666666666666646483076143078549490646989405601871543181`36.,
-0.00138888888888888013835429271476153342409790381664992857501`36.,
0.0000248015873013911856398011281476090989997999054664564914`36.,
-2.755731919836315202030187481821446739864646629753236`36.*^-7,
2.0876754923971449772710300985189348556318360528203`36.*^-9,
-1.1470641559691907631205172698119568957183338975`36.*^-11,
4.776275569742866418109759588918138864877800612332857230548545722`36.*^-14,
-1.506778718981842648624476695626864908924873401656320824178316`36.*^-16}
```

Cos, rel. error minimized, degree 20

Maximum relative error: 3.83379747773817394469035669860874374e-23
Maximum absolute error: 3.71671348650100186455052283486671174e-23



C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999999999996166202522262 + x2*(-19.739208802178717237548386510809448 5 +
x2*(64.939394022668291426827055667599344 8 + x2*(-85.456817206693714188774079273142609 5
+ x2*(60.244641371875170399062785545968485 3 + x2*(-
26.426256783277573670644588740963126 4 + x2*(7.90353636735277623649870100249206 01 +
x2*(-1.714390605773830786213338865286244 99 +
x2*(0.282004156316146650381968681921656903 + x2*(-
0.036363321180458379679822726612982405 4 +
0.00365995253718846099230289486666213 71*x2)))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

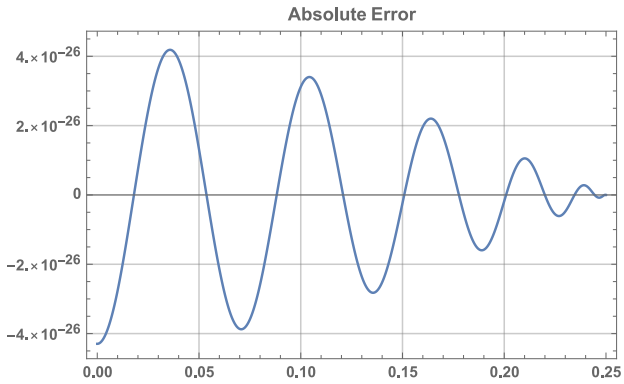
```
0.99999999999999999999996166202522262 + x2*(-0.49999999999999999999694528057959 59 +
x2*(0.04166666666666666662551729508075132679500131196778141161 56 +
0.00138888888888888866871197072840050 +
x2*(0.00002480158730158668819724885003316416265091445442910187 15 + x2*(-
2.75573192238849227440874853664844263e-7 + x2*(2.08767569773929399952910743265163937e-
9 + x2*(-1.14707448930830276887881891315116143e-11 +
x2*(4.77944661998650334511553845226073297e-14 + x2*(-
1.56108270202840630711319117658432415e-16 + 3.97995430894436507218477438332260387e-
19*x2)))))))))
```

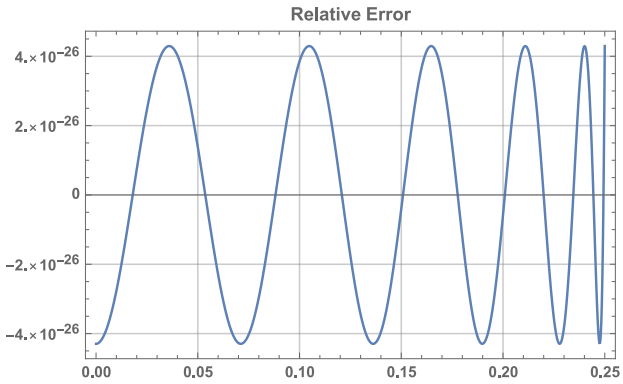
WL-style coefficient list:

```
{0.99999999999999999999996166202522261826055309643301391256 24`36.,
-0.49999999999999999999694528057959590007574851430121377652 48`36.,
0.04166666666666666662551729508075132679500131196778141161 56`36.,
-0.00138888888888888866871197072840050489909317981184585821 87`36.,
0.00002480158730158668819724885003316416265091445442910187 15`36.,
-2.75573192238849227440874853664844263444632882945129 5`36.*^-7,
2.08767569773929399952910743265163936862832302692 28`36.*^-9,
-1.14707448930830276887881891315116143490006820471`36.*^-11,
4.77944661998650334511553845226073296759836807050035621736112584 3`36.*^-14,
-1.56108270202840630711319117658432415382790761212955032593784 94`36.*^-16,
3.97995430894436507218477438332260386907883164039905034710 78`36.*^-19}
```

Cos, rel. error minimized, degree 22

Maximum relative error: 4.2949554259621960981052729653649860 9e-26
Maximum absolute error: 4.2949554259621960981052729653649860 9e-26





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999999999999995705044574 + x2*(-19.739208802178717237668683665147091 +
x2*(64.939394022668291490855079079550856 8 + x2*(-85.456817206693727709148103081288 2183
+ x2*(60.244641371876656768384184253041035 + x2*(-26.426256783374110626189036418304 4783
+ x2*(7.903536371303819187173113041578116 46 + x2*(-
1.714390710591963678483699941442727 87 + x2*(0.282005957136193996718322627667697367 +
x2*(-0.036382669575364847529461164135969 5308 +
x2*(0.003778174075246863064172787814790266 12 -
0.000313693618892545548127138801407448319*x2))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

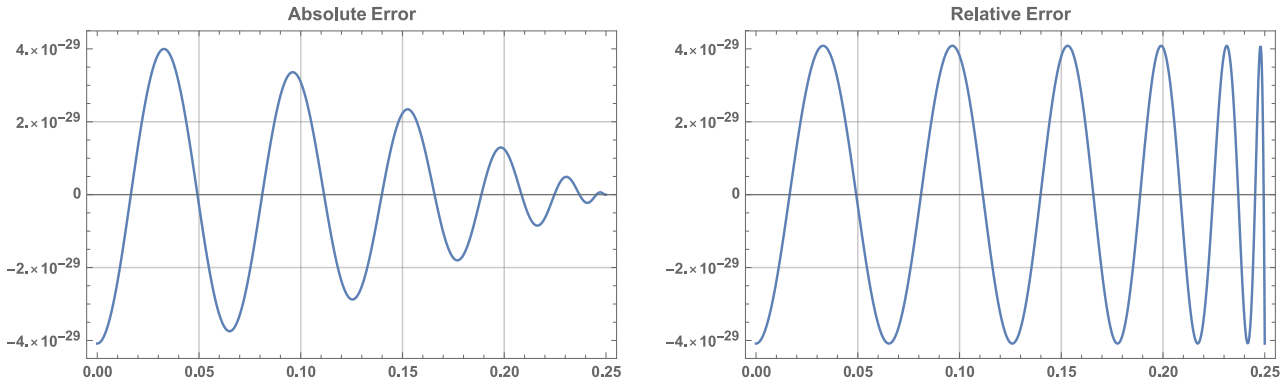
```
0.99999999999999999999999995705044574 + x2*(-0.49999999999999999999999955855121670957 +
x2*(0.041666666666666666666659920459010178 82 + x2*(-
0.001388888888888888888845216079270130966 +
x2*(0.000024801587301587300107579620159247 9227 + x2*(-
2.755731922398559154946001768967100 69e-7 + x2*(2.087675698782940282086783239102823 78e-
9 + x2*(-1.147074559440631919959462915349974 47e-11 +
x2*(4.779477140541238873236840764978336 64e-14 + x2*(-
1.561913331454431709031311828899792 27e-16 +
x2*(4.108512347614181679514400092821596 62e-19 -
8.640693108210331196223998885726106 61e-22*x2))))))))))
```

WL-style coefficient list:

```
{0.99999999999999999999999995705044574037803901894727034635`36.,
-0.49999999999999999999999585512167095696503084422648130894 06`36.,
0.04166666666666666666665992045901017882124040251110735423723`36.,
-0.00138888888888888888845216079270130966148984208110365956 21`36.,
0.00002480158730158730010757962015924792270837514728271198 69`36.,
-2.75573192239855915494600176896710069372729682675574 3`36.*^-7,
2.08767569878294028208678323910282377885059280264 76`36.*^-9,
-1.14707455944063191995946291534997447341038821848`36.*^-11,
4.77947714054123887323684076497833664356529174903849339935367079`36.*^-14,
-1.56191333145443170903131182889979227400820622182942583449173 66`36.*^-16,
4.10851234761418167951440009282159662338218589989996641620 37`36.*^-19,
-8.64069310821033119622399888572610660892617089391795280 33`36.*^-22}
```

Cos, rel. error minimized, degree 24

Maximum relative error: 4.0845347276700114734949208961162755 5e-29
Maximum absolute error: 4.0845356096706675936031957333105612e-29



C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999999999999999995915464 + x2*(-19.739208802178717237668981814387 4965 +
x2*(64.939394022668291490960080180944787 4 + x2*(-85.456817206693727735976432955291 5735
+ x2*(60.244641371876660355837353595504306 9 + x2*(-
26.426256783374396791863008770745967 6 + x2*(7.90353637131842771866677103924451885 +
x2*(-1.714390711086948490220165783300652 67 +
x2*(0.28200596840598336913179954273674240 4 + x2*(-
0.03638284014867963835311100008940515 27 + x2*(0.00377982073991600318926934396234629 96
+ x2*(-0.00032287282823405367698653655509664426 2 +
0.00002249133215128104365241869852223314 91*x2)))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

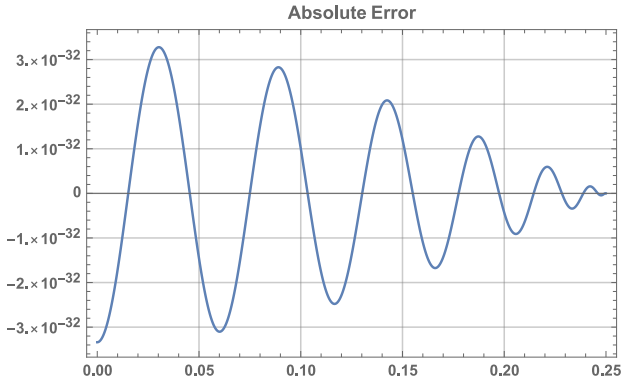
```
0.99999999999999999999999999995915464 + x2*(-0.49999999999999999999999999995304654619 +
x2*(0.04166666666666666666666665758053288133 + x2*(-
0.00138888888888888888888818884157334806 +
x2*(0.00002480158730158730158446767231016406840490345108873036 + x2*(-
2.75573192239858899632267969671451625e-7 + x2*(2.08767569878679904539999412282442082e-
9 + x2*(-1.14707455977181925161291069715764596e-11 +
x2*(4.77947733154323482806513694443407265e-14 + x2*(-
1.56192065419180836809895430039312807e-16 +
x2*(4.11030298562894815972674061988163591e-19 + x2*(-
8.89353449904256905090989216944095986e-22 + 1.56927240766816931826135305111096059e-
24*x2))))))))))
```

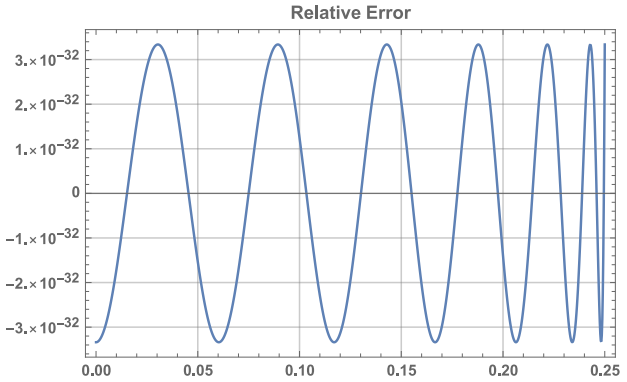
WL-style coefficient list:

```
{0.99999999999999999999999999995915464390329332406396804266667`36.,
-0.49999999999999999999999999953046546194256316498901872446491`36.,
0.04166666666666666666666665758053288133419675553748473400086`36.,
-0.00138888888888888888888818884157334805676759827069639251015`36.,
0.00002480158730158730158446767231016406842049034510887303 6`36.,
-2.75573192239858899632267969671451625242712519379923 9`36.*^-7,
2.08767569878679904539999412282442082032153778626 8`36.*^-9,
-1.14707455977181925161291069715764595837566109368`36.*^-11,
4.77947733154323482806513694443407265351407009118969144838554352 6`36.*^-14,
-1.56192065419180836809895430039312807309030968313972612550068 16`36.*^-16,
4.11030298562894815972674061988163590842635063387877591937 4`36.*^-19,
-8.89353449904256905090989216944095986432430279004475452 21`36.*^-22,
1.56927240766816931826135305111096058569545085372221 55`36.*^-24}
```

Cos, rel. error minimized, degree 26

Maximum relative error: 3.33862465029656795822935765352100898e-32
Maximum absolute error: 3.33875354262687897026003767929157552e-32





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999999999999999999996661 + x2*(-19.739208802178717237668981999574328 8 +
x2*(64.939394022668291490960221632999542 2 + x2*(-85.456817206693727736019449168540211 9
+ x2*(60.244641371876660362710402231016141 3 + x2*(-
26.426256783374397451682978897168743 8 + x2*(7.90353637131846871386720292089942776 +
x2*(-1.714390711088667485558536788199706 18 +
x2*(0.28200596845562847197479399292904144 2 + x2*(-
0.03638284113845029231576307312738790 45 + x2*(0.00377983412801100302703133708895885354
+ x2*(-0.00032299017760043942744384670692125762 9 +
x2*(0.00002309280180291588104749068577275829 79 -
1.36892355366832187250371434753876765e-6*x2)))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

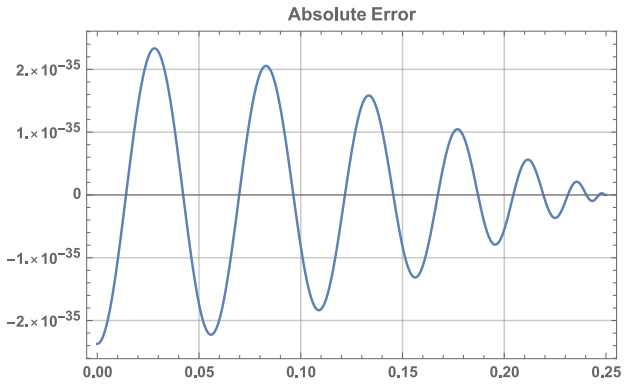
```
0.99999999999999999999999999999996661 + x2*(-0.49999999999999999999999999999549188 +
x2*(0.04166666666666666666666666665643465635 + x2*(-
0.00138888888888888888888888796356941544 +
x2*(0.00002480158730158730158729717733965665615291093301459457 94 + x2*(-
2.75573192239858906512875274817328617e-7 + x2*(2.08767569878680987405690005120423929e-
9 + x2*(-1.14707455977296940706362564543678305e-11 +
x2*(4.77947733238462710426914020405770261e-14 + x2*(-
1.56192069668280744952451466714923773e-16 +
x2*(4.11031754428943398478372408474600821e-19 + x2*(-
8.89676688822843112509732501470500729555896413859035169 39 + x2*(1.61123834023328757158702786214322464e-
24 - 2.41937182488060032250287418487354979e-27*x2)))))))))))
```

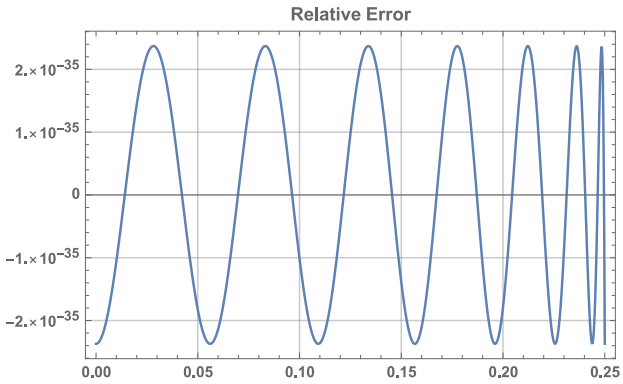
WL-style coefficient list:

```
{0.99999999999999999999999999999996661246457373121029739962 3`36.,
-0.49999999999999999999999999995491879943352738350875183721 9`36.,
0.04166666666666666666666666656434656346490963213952160471 45`36.,
-0.00138888888888888888888888796356941543631699875894440825 03`36.,
0.00002480158730158730158729717733965665615291093301459457 94`36.,
-2.75573192239858906512875274817328617277157275404448 2`36.*^-7,
2.08767569878680987405690005120423928974367402453 24`36.*^-9,
-1.14707455977296940706362564543678305385024695939`36.*^-11,
4.77947733238462710426914020405770261468570721862552520860847436`36.*^-14,
-1.56192069668280744952451466714923772638212335818295721345639 55`36.*^-16,
4.11031754428943398478372408474600820576951104692918579389 24`36.*^-19,
-8.89676688822843112509732501470500729555896413859035169 39`36.*^-22,
1.61123834023328757158702786214322464113453365416920515`36.*^-24,
-2.41937182488060032250287418487354978994261463873037`36.*^-27}
```

Cos, rel. error minimized, degree 28

Maximum relative error: 2.37094722065961068544199718420836137e-35
Maximum absolute error: 2.37094722065961068544199718420836137e-35





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999999999999999999999998 + x2*(-19.739208802178717237668981999752155 6 +
x2*(64.939394022668291490960221792317699 + x2*(-85.456817206693727736019506039333498 9
+ x2*(60.244641371876660362721100505691104 2 + x2*(-
26.426256783374397452898820939352068 1 + x2*(7.903536371318468804052318955307759 79 +
x2*(-1.714390711088672055813145471494158 01 +
x2*(0.282005968455790804791695935304804337 + x2*(-
0.036382841142533038490523417648852478 9 + x2*(0.003779834200398621863665422608780726 49
+ x2*(-0.000322991062711766531658738805143944 69 +
x2*(0.000023099906737724246170791969311392348 4 + x2*(-
1.402627180048152246329903748793950 83e-6 + 7.16144822225800304819861827511239386e-
8*x2))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

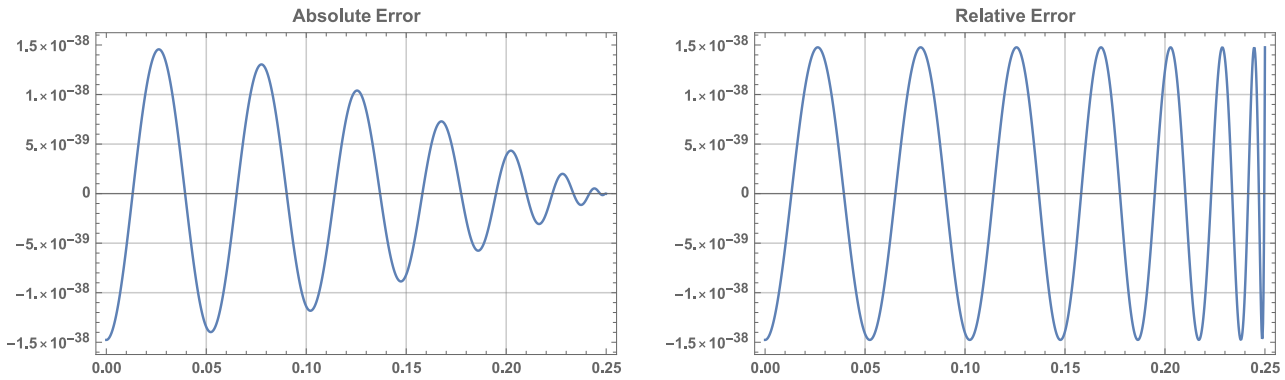
```
0.99999999999999999999999999999999998 + x2*(-0.49999999999999999999999999999996284 +
x2*(0.04166666666666666666666666665688991 + x2*(-
0.00138888888888888888888888786329251 +
x2*(0.00002480158730158730158730158161839888 66 + x2*(-
2.75573192239858906525554084213999 61e-7 + x2*(2.08767569878680989787880284152372166e-9
+ x2*(-1.14707455977297246495629818374412263e-11 +
x2*(4.77947733238737834397847654869924 77e-14 + x2*(-
1.56192069685808034053910479920657012e-16 + x2*(4.11031762300614207750648139470658 79e-
19 + x2*(-8.89679126862663976086835125404000055e-22 +
x2*(1.61173406801312979277352877800301076e-24 + x2*(-
2.47893804670588492529166036962942878e-27 + 3.20600756109725541928196721886212037e-
30*x2))))))))))))
```

WL-style coefficient list:

```
{0.99999999999999999999999999999999976290527793403893145579 9`36.,
-0.49999999999999999999999999999999628436482377572766217621366`36.,
0.04166666666666666666666666666656889912748853482991006951 46`36.,
-0.00138888888888888888888888887863292513626021226520636435`36.,
0.00002480158730158730158730158161839888659007306201442456 7`36.,
-2.75573192239858906525554084213999609588109322832168 5`36.*^-7,
2.08767569878680989787880284152372166067943240757 37`36.*^-9,
-1.14707455977297246495629818374412262516059266131`36.*^-11,
4.77947733238737834397847654869924770262806489424198137176209044 2`36.*^-14,
-1.56192069685808034053910479920657012369535916338575681308183 94`36.*^-16,
4.11031762300614207750648139470658790284976487521795179829 65`36.*^-19,
-8.89679126862663976086835125404000055159244170870736924 06`36.*^-22,
1.61173406801312979277352877800301076153501479167442 5`36.*^-24,
-2.47893804670588492529166036962942877886197186308255`36.*^-27,
3.20600756109725541928196721886212037201474393281`36.*^-30}
```

Cos, rel. error minimized, degree 30

Maximum relative error: 1.476299423798796255236278349349564 42e-38
Maximum absolute error: 1.476320421936952749321505807819606 41e-38



C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
1. + x2*(-19.739208802178717237668981999752302 2 +
x2*(64.939394022668291490960221792469949 1 + x2*(-85.456817206693727736019506102377842 5
+ x2*(60.244641371876660362721114295549962 8 + x2*(-
26.426256783374397452900651021482757 6 + x2*(7.90353637131846880421187139140169693 +
x2*(-1.714390711088672065405279924865279 71 +
x2*(0.282005968455791214248525283406958707 + x2*(-
0.036382841142545640507515788420439251 3 + x2*(0.00377983420067921786099683766019805811
+ x2*(-0.000322991067190683705522220368542132537 +
x2*(0.000023099956706669067412275237656107374 8 + x2*(-
1.402997162506302776210399323745242 37e-6 + x2*(7.32479698171313153116454350681126065e-
8 - 3.254755444749110648985293727092442 27e-9*x2))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

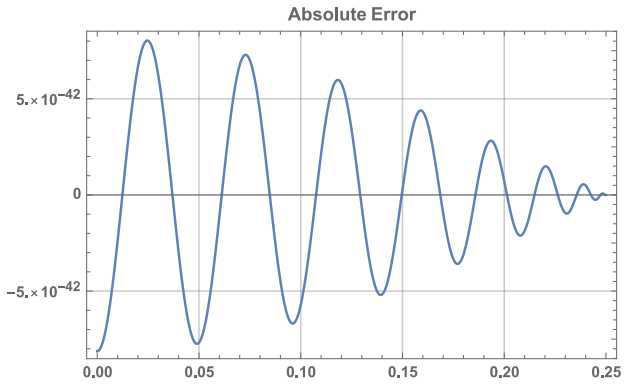
```
1. + x2*(-0.49999999999999999999999999999999997 +
x2*(0.04166666666666666666666666666666665864 + x2*(-
0.00138888888888888888888888888888792216 +
x2*(0.00002480158730158730158730158729542476 92 + x2*(-
2.75573192239858906525573168323374212e-7 + x2*(2.08767569878680989792094774070130099e-
9 + x2*(-1.14707455977297247137425958205150 5e-11 +
x2*(4.77947733238738528351115798998742201e-14 + x2*(-
1.56192069685862134697357839719554288e-16 +
x2*(4.11031762331127151219297331433042247e-19 + x2*(-
8.89679139199844723682446508751816549646662479004285776 3e-22 + x2*(1.61173755446230144849643763129805917e-
24 + x2*(-2.47959193649582299714152529144837331e-27 +
x2*(3.27913485904815466449434894827885198e-30 -
3.69081529290172836989264646507263976e-33*x2)))))))))))))
```

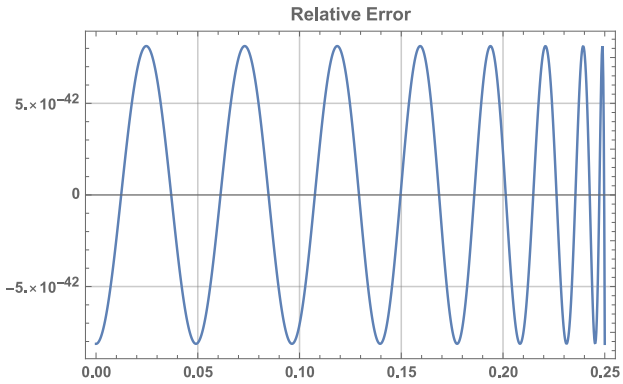
WL-style coefficient list:

```
{0.99999999999999999999999999999999999985236795780630472506 6`36.,
-0.49999999999999999999999999999999997342214229933327205167 3`36.,
0.04166666666666666666666666666666665864061862251237986210721`36.,
-0.00138888888888888888888888888888879221584297992849798038592`36.,
0.00002480158730158730158730158729542476921748062781577324 59`36.,
-2.75573192239858906525573168323374211652755740358163 2`36.*^-7,
2.08767569878680989792094774070130098772633560579 56`36.*^-9,
-1.14707455977297247137425958205150500339470449215`36.*^-11,
4.77947733238738528351115798998742200974216981652905597614838963`36.*^-14,
-1.56192069685862134697357839719554287879591048038545939075646 38`36.*^-16,
4.11031762331127151219297331433042247288009231911297351082 31`36.*^-19,
-8.89679139199844723682446508751816549646662479004285776 23`36.*^-22,
1.61173755446230144849643763129805917221851012715981 49`36.*^-24,
-2.47959193649582299714152529144837331207731297760 73`36.*^-27,
3.27913485904815466449434894827885198338459055474`36.*^-30,
-3.69081529290172836989264646507263976295027324681405855698945822 8`36.*^-33}
```

Cos, rel. error minimized, degree 32

Maximum relative error: 8.126196242428032788590514446773988 84e-42
Maximum absolute error: 7.74314831575100009029431429191882 99e-42





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
1. + x2*(-19.739208802178717237668981999752302 3 +
x2*(64.939394022668291490960221792470074 1 + x2*(-85.456817206693727736019506102437276 1
+ x2*(60.244641371876660362721114310505305 3 + x2*(-
26.426256783374397452900653312544409 3 + x2*(7.903536371318468804212103147404479 33 +
x2*(-1.714390711088672065421563247547905 12 +
x2*(0.282005968455791215068931892569073801 + x2*(-
0.036382841142545670713824470742842222 1 + x2*(0.003779834200680037505508939854539991 07
+ x2*(-0.000322991067207052330456610766908529837 +
x2*(0.000023099956944247218614130783730669856 4 + x2*(-
1.402999599684846785740039565417549 26e-6 + x2*(7.326471662936655669588682271786344 87e-
8 + x2*(-3.323904914431884180293245621106216 09e-9 +
1.297051088071684885293308975904143 18e-10*x2))))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

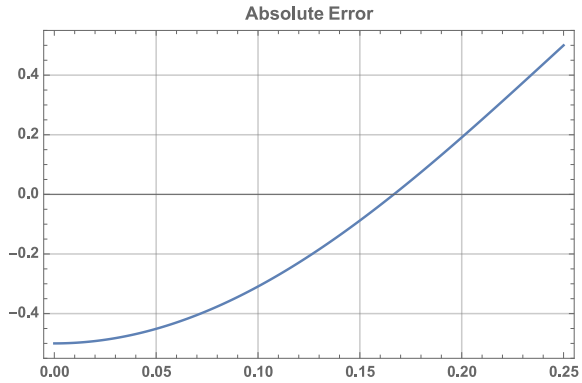
```
1. + x2*(-0.5 + x2*(0.041666666666666666666666666666666666666 + x2*(-
0.001388888888888888888888888888888888881 +
x2*(0.000024801587301587301587301587301587301 5816028 + x2*(-
2.755731922398589065255731922145812 84e-7 + x2*(2.087675698786809897921008957775826 33e-
9 + x2*(-1.147074559772972471385154522788761 32e-11 +
x2*(4.779477332387385297415526090075421 69e-14 + x2*(-
1.561920696858622643734832466716074 43e-16 +
x2*(4.110317623312162820939356588323539 54e-19 + x2*(-
8.896791392449321134612979897440203 52e-22 +
x2*(1.611737571038680065743133815514591 63e-24 + x2*(-
2.479596243851836983029876894857133 55e-27 +
x2*(3.279884573421339524768103361101826 05e-30 + x2*(-
3.769229147501147644538403393031724 07e-33 + 3.725643493034930304069667120135333 41e-
36*x2))))))))))))))
```

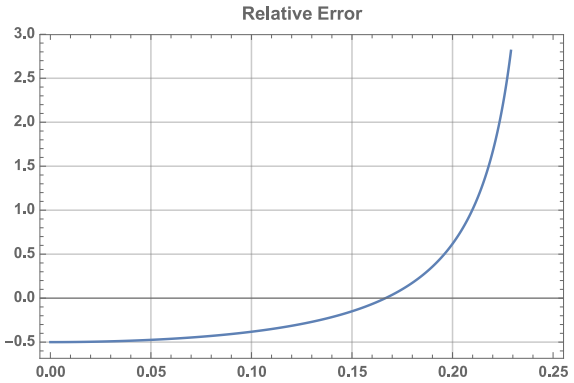
WL-style coefficient list:

```
{0.99999999999999999999999999999999999999999918738037575719686`36.,
-0.49999999999999999999999999999999999999983344428619194852 51`36.,
0.04166666666666666666666666666666666666609447791771624088 2855`36.,
-0.00138888888888888888888888888888888881045509885184309388778`36.,
0.00002480158730158730158730158730158160283155320487836677 51`36.,
-2.75573192239858906525573192214581284233592259546661 5`36.*^-7,
2.08767569878680989792100895777582633428366776885 44`36.*^-9,
-1.14707455977297247138515452278876132047257648017`36.*^-11,
4.77947733238738529741552609007542169005422786870159146661988327 3`36.*^-14,
-1.56192069685862264373483246671607443196868994163037693841677 53`36.*^-16,
4.11031762331216282093935658832353953628678667504141400560 07`36.*^-19,
-8.89679139244932113461297989744020351895746920701251089 04`36.*^-22,
1.61173757103868006574313381551459163407549767008758544`36.*^-24,
-2.47959624385183698302987689485713355289089989867847`36.*^-27,
3.27988457342133952476810336110182604708142146 65`36.*^-30,
-3.76922914750114764453840339303172406660033845078238033821589061 2`36.*^-33,
3.72564349303493030406966712013533341181468190722150957374561 8`36.*^-36}
```

Cos, abs. error minimized, degree 0

Maximum relative error: ∞
Maximum absolute error: 0.49999999999999997507118124631559315 5





C-style Horner evaluation for sin(2πx) approximation:

```
0.50000000000000002492881875368440684528788368630685672760 73`36.
```

C-style Horner evaluation for sin(x) approximation:

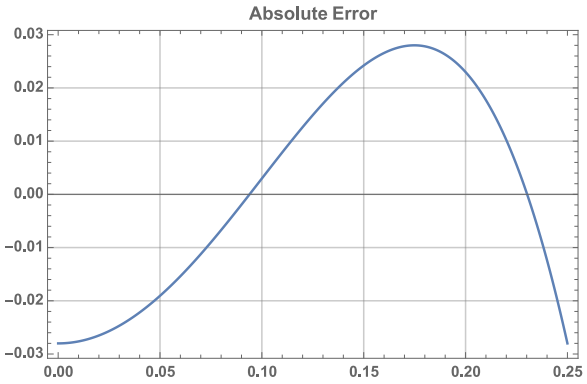
```
0.50000000000000002492881875368440684528788368630685672760 73`36.
```

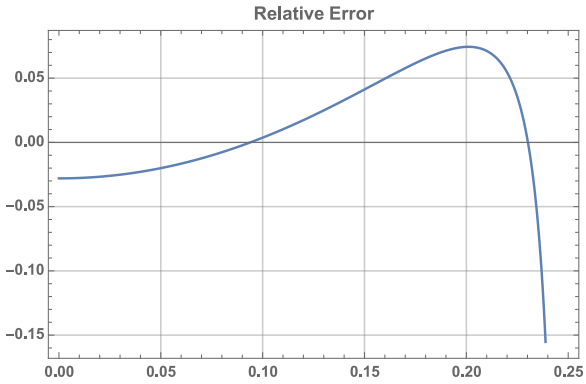
WL-style coefficient list:

```
{0.50000000000000002492881875368440684528788368630685672760 73`36.}
```

Cos, abs. error minimized, degree 2

Maximum relative error: ∞
Maximum absolute error: 0.028004797977063875929106055289124147 5





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.971995202022936124070893944710875852 - 16.000000000000000217969250785347613 8*x2
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

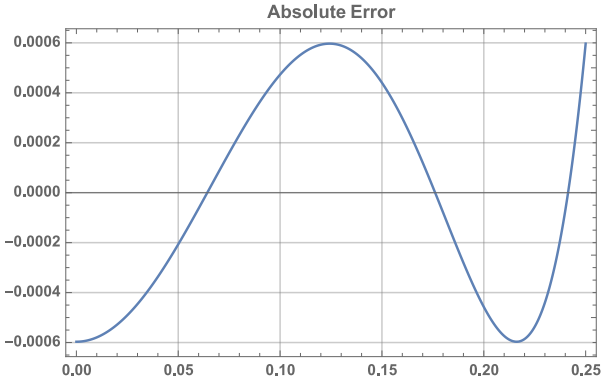
```
0.971995202022936124070893944710875852 - 0.405284734569351091296743474640156079*x2
```

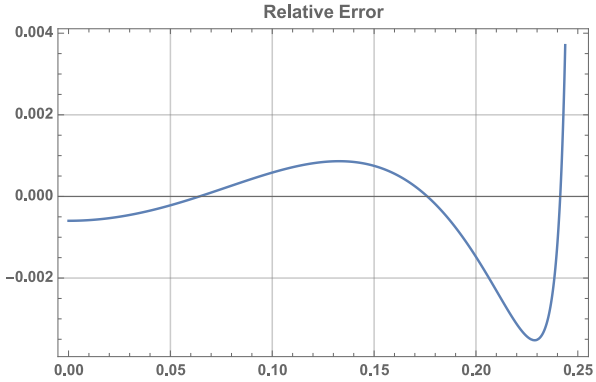
WL-style coefficient list:

```
{0.971995202022936124070893944710875852475453119558120286446 4`36.,
-0.405284734569351091296743474640156078949723693243123416952 5`36.}
```

Cos, abs. error minimized, degree 4

Maximum relative error: ∞
Maximum absolute error: 0.000596770526309982297457348580684014551





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

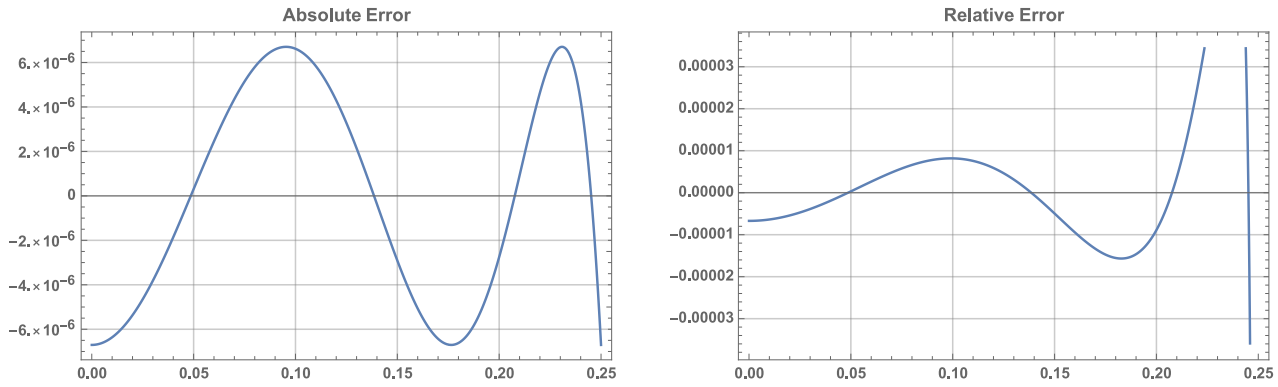
```
0.99940322947369001770254265141931598 5 + x2*(-19.564747722254987834884137817353040 6 +
57.341510065550516466412763296957574 8*x2)
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

```
0.99940322947369001770254265141931598 5 + x2*(-0.49558084922065181146401380196728165596772610364556081218 06`36.,
0.03679168279935904949510893458563084246684855846560898624 5`36.}
```

Cos, abs. error minimized, degree 6

Maximum relative error: ∞
Maximum absolute error: 6.704717832578335600338712977330548 32e-6



C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999329528216742166439966128702266 9 + x2*(-19.735752060573199792313114453181377 6 +
x2*(64.660541218447534625985758233091652 9 - 78.216131988421407239016179329701802*x2))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

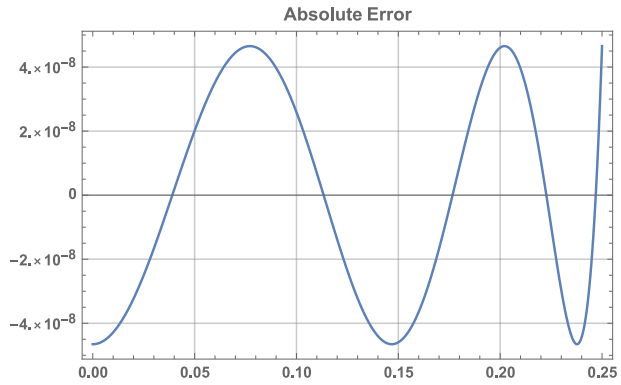
```
0.99999329528216742166439966128702266 9 + x2*(-0.49991243971224581435251505760757806 +
x2*(0.041487748045429213225366747119595544 7 -
0.00127120948569655081466419067530634131*x2))
```

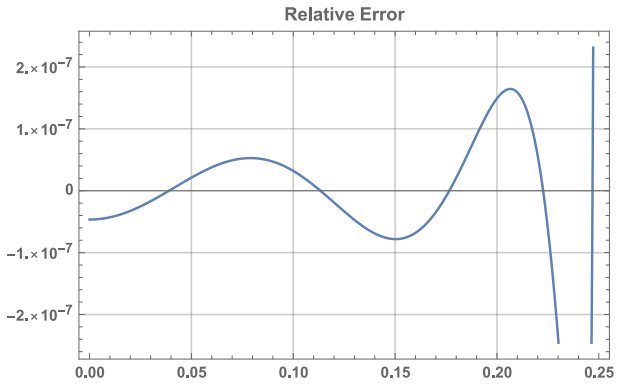
WL-style coefficient list:

```
{0.99999329528216742166439966128702266945167860481173577390 48`36.,
-0.49991243971224581435251505760757805968976176815141509965 32`36.,
0.04148774804542921322536674711959554467851274222025566584 17`36.,
-0.00127120948569655081466419067530634130934178405218507701 9`36.}
```

Cos, abs. error minimized, degree 8

Maximum relative error: ∞
Maximum absolute error: 4.653332986369358756907553664884749 53e-8





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

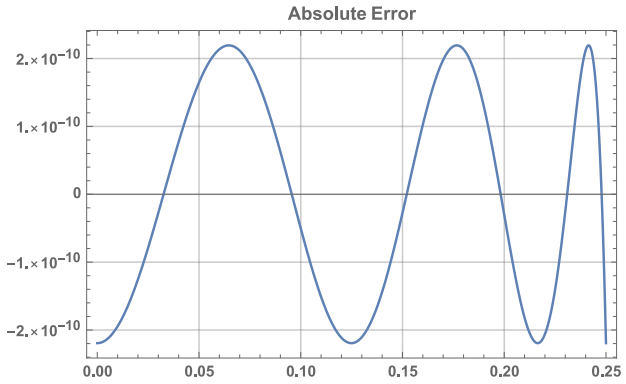
```
0.99999995346667013630641243092446335115250470051501982175 8`36.,
-0.49999905347076729097546897993796763981502099843648660589 01`36.,
0.04166358469310783866539471960407575670792806337108083630 54`36.,
-0.00138537043082318983893723662479142647881297116471084384 25`36.,
0.00002315393165905387621757424415885234666560293066439825 68`36.}
```

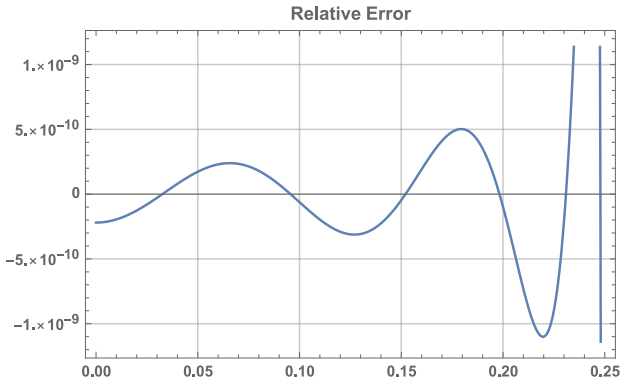
WL-style coefficient list:

```
0.99999995346667013630641243092446335115250470051501982175 8`36.,
-0.49999905347076729097546897993796763981502099843648660589 01`36.,
0.04166358469310783866539471960407575670792806337108083630 54`36.,
-0.00138537043082318983893723662479142647881297116471084384 25`36.,
0.00002315393165905387621757424415885234666560293066439825 68`36.}
```

Cos, abs. error minimized, degree 10

Maximum relative error: ∞
Maximum absolute error: 2.193483178155763772450217138197521l9e-10





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999978065168218442462303648381 + x2*(-19.739208548913522979242130820260143 +
x2*(64.939346629488580483420803415602572 6 + x2*(-85.453571626835256521523334135409 6818
+ x2*(60.144015092965607314363320149065 4729 -
24.982237806207740000213808639512345 6*x2))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

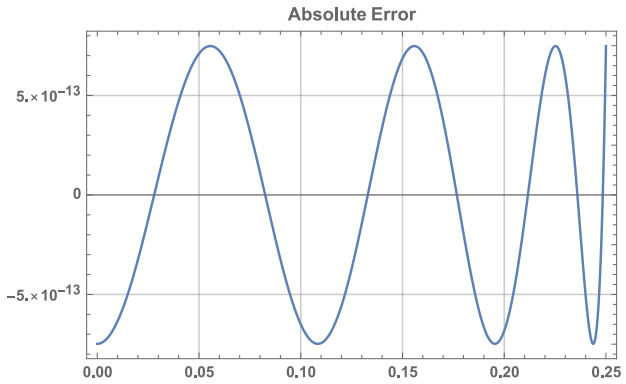
```
0.99999999978065168218442462303648381 + x2*(-0.499999993584717685582395813817447263 +
x2*(0.041666636258070297524849870957504294 2 + x2*(-
0.001388836140027525033228705938495735899892368080534149932 8 +
x2*(0.000024760161352583123575883333369950123077937035654647809 7 -
2.605149521548271120193829720290563411060759747697501e-7*x2))))
```

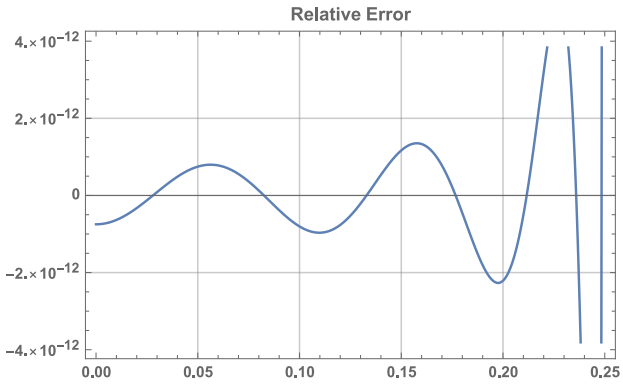
WL-style coefficient list:

```
{0.99999999978065168218442462303648381011825182749479113629 28`36.,
-0.49999999358471768558239581381744726275684902818793994113 97`36.,
0.04166663625807029752484987095750429422933987771030528740 96`36.,
-0.00138883614002752503322870593849573589989236808053414993 28`36.,
0.00002476016135258312357588333336995012307793703565464780 97`36.,
-2.60514952154827112019382972029056341106075974769750 1`36.*^-7}
```

Cos, abs. error minimized, degree 12

Maximum relative error: ∞
Maximum absolute error: 7.481698606943581708220700994127968 24e-13





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999925183013930564182917793 + x2*(-19.739208801003852370515885285902990 7 +
x2*(64.939393721430336173776154168200481 7 + x2*(-85.456788233486083649846368635182413 5
+ x2*(60.243313517298031569844698094382343 1 + x2*(-
26.394972920605901156180763162383900 6 + 7.536729521803462978657412093963214 06*x2)))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

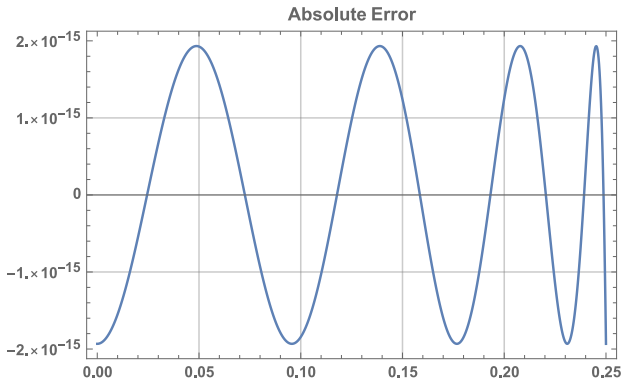
```
0.99999999999925183013930564182917793 + x2*(-0.49999999997024032526006257958113640443953102744303961808 13 +
x2*(0.04166666647338519768394139712827476612893130470085811180 86 + x2*(-
0.00138888841800116468296195769839668651 +
x2*(0.00002480104064879784622291859112066251612409870955524098 75 + x2*(-
2.75246963898123729813292275409227504e-7 + 1.99078568526577620551890608596753138e-
9*x2)))))
```

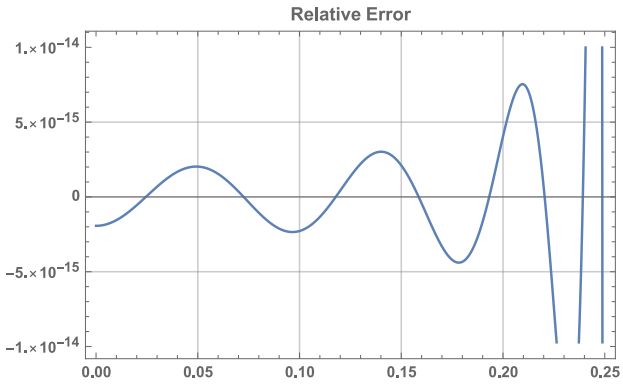
WL-style coefficient list:

```
{0.99999999999925183013930564182917792990058720317482364595 14`36.,
-0.49999999997024032526006257958113640443953102744303961808 13`36.,
0.04166666647338519768394139712827476612893130470085811180 86`36.,
-0.00138888841800116468296195769839668650583804148554449162 87`36.,
0.00002480104064879784622291859112066251612409870955524098 75`36.,
-2.75246963898123729813292275409227503583652845284924 4`36.*^-7,
1.99078568526577620551890608596753138200320127043 24`36.*^-9}
```

Cos, abs. error minimized, degree 14

Maximum relative error: ∞
Maximum absolute error: 1.93230575844198160047566973503160827e-15





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999806769424155801839952 + x2*(-19.739208802174755865492738517855831 6 +
x2*(64.939394021335857369643849089438327 2 + x2*(-85.456817035934295593319821903276522 3
+ x2*(60.244630621952328309898151613101427 9 + x2*(-
26.425889066660665413717089975822946 5 + x2*(7.896559497889739079948193402987570 11 -
1.645240546920825950738244960884237 06*x2))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

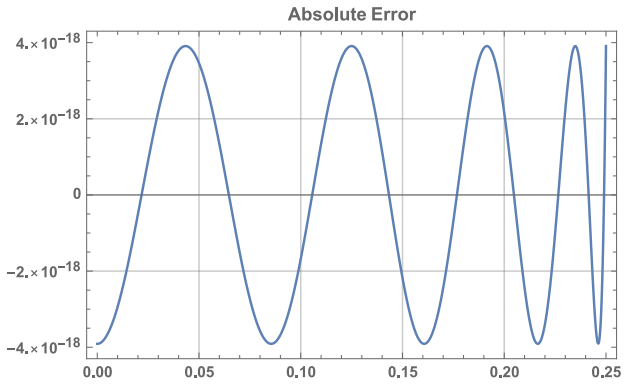
```
0.99999999999999806769424155801839952 + x2*(-0.499999999999899657270563796735665619 +
x2*(0.041666666665811745101356054363828004 9 + x2*(-
0.001388888886113617287065238579644381 8 +
x2*(0.000024801582876045365647972254355030392281050305378937401 - x2*(-
2.755693576873729547450346789874027233624171165487519 - 1.100807163660746200871494928005552315192534542 95 - 1.100807163660746200871494928005552315192534542 95*x2))))))
```

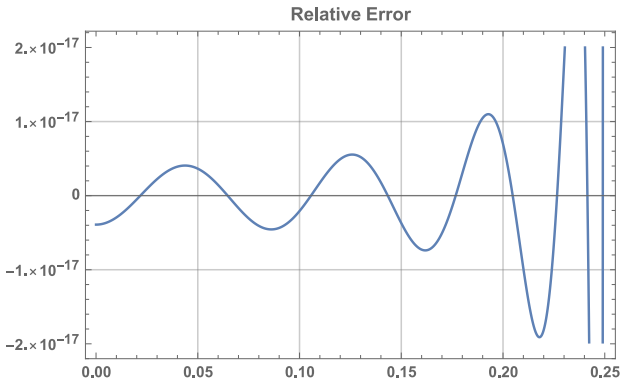
WL-style coefficient list:

```
{0.99999999999999806769424155801839952433026496839173298809 15`36.,
-0.49999999999989965727056379673566561886315489813163904273 5`36.,
0.04166666666581174510135605436382800486388340998808344025 88`36.,
-0.00138888888611361728706523857964438185845100289045525692 92`36.,
0.00002480158287604536564797225435503039228105030537893740 1`36.,
-2.75569357687372954745034678987402723362417116548751 9`36.*^-7,
2.08583279601185364835442459941203767911232615022 27`36.*^-9,
-1.10080716366074620087149492800555231519253454295`36.*^-11}
```

Cos, abs. error minimized, degree 16

Maximum relative error: ∞
Maximum absolute error: 3.9101804892769845364885285091867615 2e-18





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999999608981951072301546 + x2*(-19.739208802178707095137280041061959 6 +
x2*(64.939394022663960757319625987229563 + x2*(-85.456817205981735856122646007246913 5
+ x2*(60.244641313230924425507687663195767 1 + x2*(-
26.426254069097002482358880592835091 3 + x2*(7.903462537592473794261002496844957 36 +
x2*(-1.713218858756278322848985870563724 72 +
0.271947641663980014582272096782358561*x2)))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

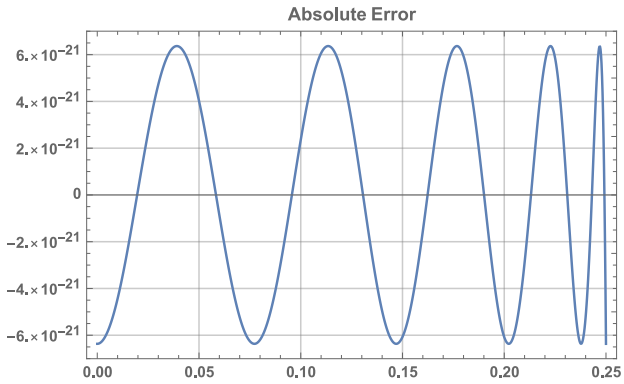
```
0.99999999999999999608981951072301546 + x2*(-0.499999999999999743086670706902716 78 +
x2*(0.041666666666663887964497659102665977 + x2*(-
0.001388888888877317224470635740183373 51 +
x2*(0.000024801587277443953621558261218996467 + x2*(-
2.755731639353550758089659311148635192907 4 + x2*(2.087656196013839808779401380851063 96e-
9 + x2*(-1.146290489963444695554933024702436 8e-11 +
4.609007376852587339879060598944168310662693008458895617026741406e-14*x2)))))))
```

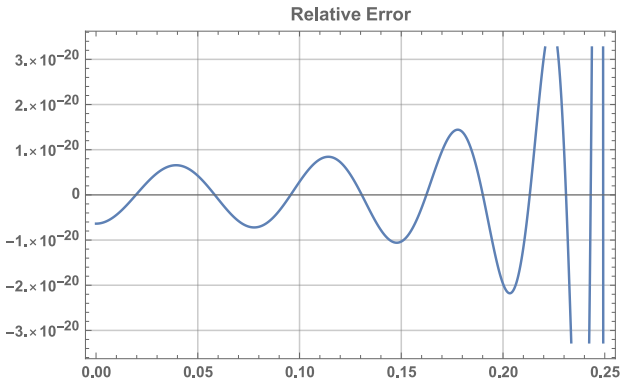
WL-style coefficient list:

```
{0.99999999999999999608981951072301546351147149081323847653 47`36.,
-0.49999999999999974308667070690271677998029806251317539862 39`36.,
0.04166666666666388796449765910266597696096498142850249265 92`36.,
-0.00138888888887731722447063574018337351481413970779995682 8`36.,
0.00002480158727744395362155826121899646740402306263548968 27`36.,
-2.75573163935355075808996593111486351929074749244764`36.*^-7,
2.08765619601383980877940138085106396159794587561 2`36.*^-9,
-1.14629048996344469555493302470243688036777579 11`36.*^-11,
4.60900737685258733987906059894416831066269300845889561702674140 6`36.*^-14}
```

Cos, abs. error minimized, degree 18

Maximum relative error: ∞
Maximum absolute error: 6.367051524534698806613960388109077 71e-21





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999999999936329484754653 + x2*(-19.739208802178717217283496511244860 4 +
x2*(64.939394022668280721479619485341442 3 + x2*(-85.456817206691520815089326857519872
+ x2*(60.244641371646979947499734658594106 5 + x2*(-
26.426256769642974452621242189860153 4 + x2*(7.903535871423751317815517075837994 87 +
x2*(-1.714379442198890292299154803022125 45 + x2*(0.281852302567082628235004107298904 42
- 0.035219758172904970519731315086247829 2*x2))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

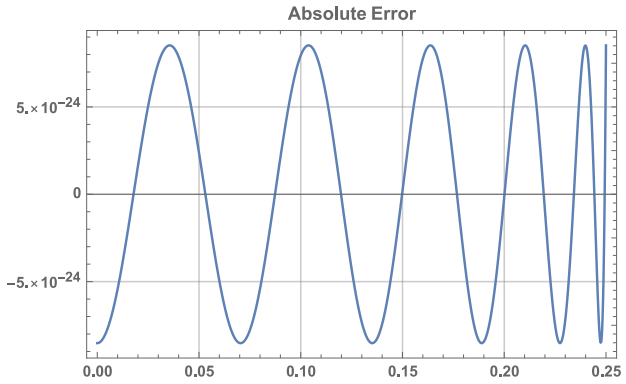
```
0.99999999999999999999936329484754653 + x2*(-0.49999999999999999948362962029518 1441 +
x2*(0.04166666666666665975671061945090 3831 + x2*(-
0.00138888888888885302085566728426120377 +
x2*(0.00002480158730149274647435112538868325 + x2*(-
2.75573192096667530569389294627340694e-7 + x2*(2.08767556674236743347120826564072266e-
9 + x2*(-1.14706701991831626578598421821623353e-11 +
x2*(4.77687298101189663061322965166463478e-14 -
1.51198937469144760417667489952621371e-16*x2))))))))
```

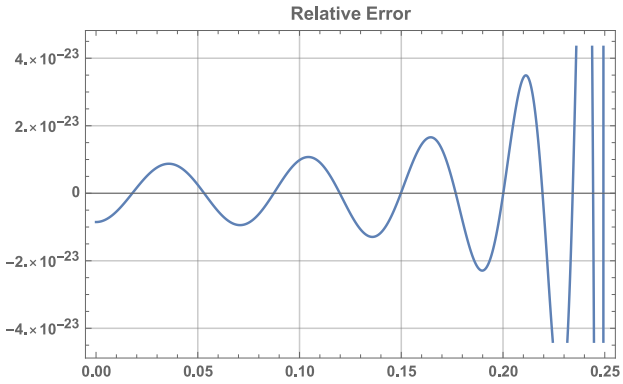
WL-style coefficient list:

```
{0.99999999999999999999936329484754653011933860396118909222948`36.,
-0.49999999999999999948362962029518144119300564717340598814 36`36.,
0.04166666666666665975671061945090738307834170455991492464 05`36.,
-0.00138888888888885302085566728426120377491643064837872459 39`36.,
0.00002480158730149274647435112538868325002001681523613375 65`36.,
-2.75573192096667530569389294627340694238020931716298 6`36.*^-7,
2.08767556674236743347120826564072265844377723283 95`36.*^-9,
-1.14706701991831626578598421821623352721410933706`36.*^-11,
4.77687298101189663061322965166463478496439817892604834881257762 5`36.*^-14,
-1.51198937469144760417667489952621370787833642408683608257115 67`36.*^-16}
```

Cos, abs. error minimized, degree 20

Maximum relative error: ∞
Maximum absolute error: 8.52289673067732074183853338428631923e-24





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999999999999147710326932 + x2*(-19.739208802178717237635967892831953 2 +
x2*(64.939394022668291469822365763071932 7 + x2*(-85.456817206693722457637942412024073 6
+ x2*(60.244641371875984386439254384918520 6 + x2*(-
26.426256783323894051166182056626992 9 + x2*(7.90353636896598002129119927354601458 +
x2*(-1.714390640700997190112166976907107 45 +
x2*(0.28200461501466951542912896864137438 2 + x2*(-
0.03636666710460504443802880684381784 29 +
0.00367035423724807832643664170925383103*x2))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

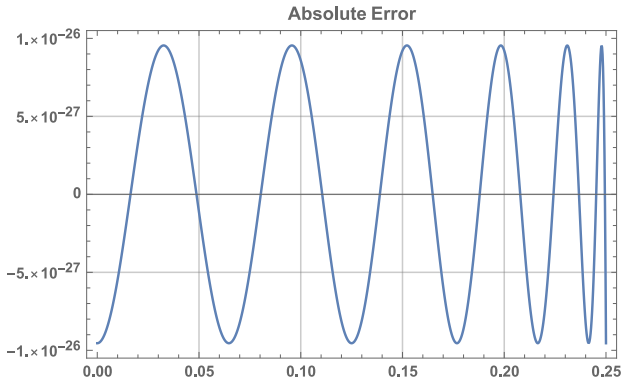
```
0.99999999999999999999999147710326932 + x2*(-0.49999999999999999999916374290248388 3 +
x2*(0.04166666666666666666531041132354245666 + x2*(-
0.00138888888888888880310187101535943782 +
x2*(0.00002480158730158702330056294007508313 38 + x2*(-
2.75573192239332256695859278971613616e-7 + x2*(2.08767569816541291395884906777039919e-
9 + x2*(-1.14707451267757546018273288630548098e-11 +
x2*(4.77945439407442225649362245140933143e-14 + x2*(-
1.56122634304188074433057975000970292e-16 + 3.99126546408967813735306959579906681e-
19*x2)))))))))
```

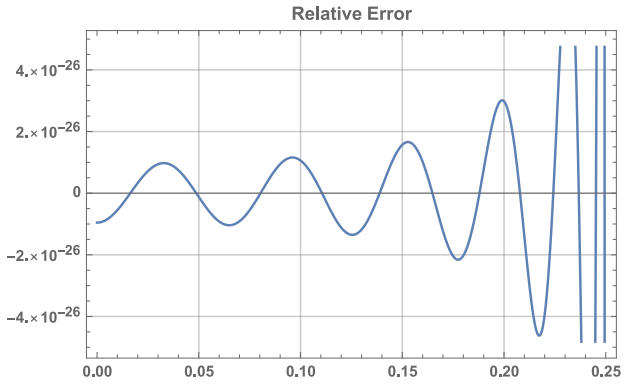
WL-style coefficient list:

```
{0.99999999999999999999999914771032693226792581614666157136806`36.,
-0.49999999999999999999991637429024838834426704193701566430571`36.,
0.04166666666666666666531041132354245665858258251481027830567`36.,
-0.00138888888888888880310187101535943782469528098133443878 92`36.,
0.00002480158730158702330056294007508313379944314695449761 71`36.,
-2.75573192239332256695859278971613615990085599050942 7`36.*^-7,
2.08767569816541291395884906777039918905196843826 04`36.*^-9,
-1.14707451267757546018273288630548098229779060 31`36.*^-11,
4.77945439407442225649362245140933143144871762909900447261362434 4`36.*^-14,
-1.56122634304188074433057975000970291545952255588294882748103 68`36.*^-16,
3.99126546408967813735306959579906680849278273860035296290 66`36.*^-19}
```

Cos, abs. error minimized, degree 22

Maximum relative error: ∞
Maximum absolute error: 9.544671661523474687736538666375555 33e-27





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999999999999999045532834 + x2*(-19.739208802178717237666893800395623 18 +
x2*(64.939394022668291490926656799955374 3 + x2*(-85.456817206693727725991307991640 7808
+ x2*(60.244641371876658814922487331903735 4 + x2*(-
26.426256783374256506608901974470633 + x2*(7.90353637131033228597146895668324877 +
x2*(-1.714390710779453177654881590699011 87 +
x2*(0.282005960615049890840257815427711553 + x2*(-
0.036382709774842889125763896445491231 8 + x2*(0.00377843725478991383698755314008036108
- 0.000314439329951950534507742809671497557*x2))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

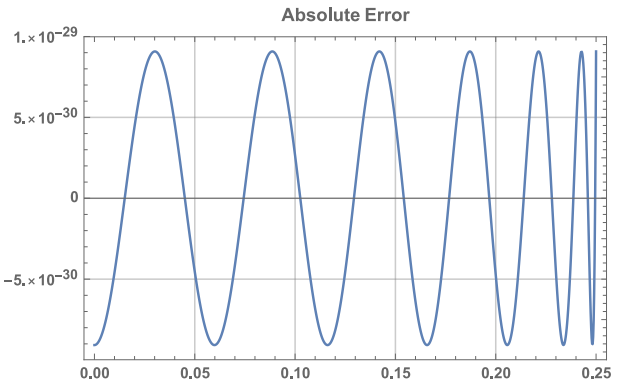
```
0.99999999999999999999999999045532834 + x2*(-0.49999999999999999999999988855734667139160760530235885247321 +
x2*(0.04166666666666666666666451305655707591 + x2*(-
0.00138888888888888888887259053682415636492569729980402381653 +
x2*(0.00002480158730158730095010196426246219826847507634209413650 +
x2*(-2.75573192239857436736648083348872503e-7 + x2*(2.08767569878466068134178874547613160480569581128430 +
+ x2*(-1.14707455956607849017989306493981385e-11 +
x2*(4.77947719950138421925491727313196747e-14 + x2*(-
1.56191505722390941644917208508280764e-16 +
x2*(4.10879853781630806206359136865815304e-19 -
8.66123372498941866442018293026718247e-22*x2)))))))))
```

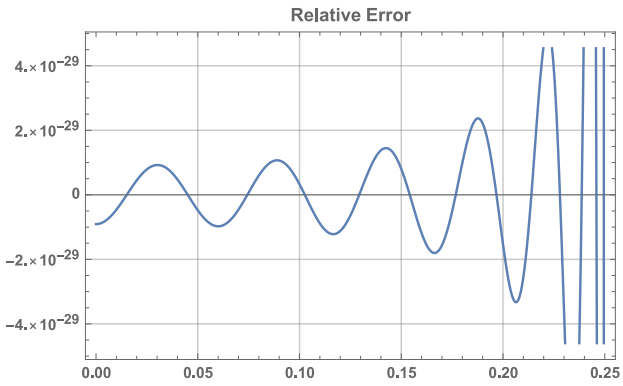
WL-style coefficient list:

```
{0.99999999999999999999999999045532833847652531226346133362430`36.,
-0.49999999999999999999999988855734667139160760530235885247321`36.,
0.04166666666666666666666451305655707591114433939470283412393`36.,
-0.00138888888888888888887259053682415636492569729980402381653`36.,
0.00002480158730158730095010196426246219826847507634209413650`36.,
-2.75573192239857436736648083348872503220096695086652900`36.*^-7,
2.08767569878466068134178874547613160480569581128430`36.*^-9,
-1.14707455956607849017989306493981385366066528616`36.*^-11,
4.77947719950138421925491727313196747442094623060838814630540641700`36.*^-14,
-1.56191505722390941644917208508280763654839358400681267545610020`36.*^-16,
4.10879853781630806206359136865815303788956234624018393368840`36.*^-19,
-8.66123372498941866442018293026718246921489552383623559150`36.*^-22}
```

Cos, abs. error minimized, degree 24

Maximum relative error: ∞
Maximum absolute error: 9.074012464863397295685465218088400710e-30





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999999999999999999092551 + x2*(-19.739208802178717237666898195066512666 +
x2*(64.939394022668291490960177799095977 4 + x2*(-85.456817206693727736004016253259417 5
+ x2*(60.244641371876660359887849614797255 8 + x2*(-
26.426256783374397144538594535592790 3 + x2*(7.90353637131844726649219966859504084 +
x2*(-1.714390711087664279226960790594890 97 + x2*(0.28200596842352218031128201216497598
+ x2*(-0.036382840433298158651524700606635768 4 +
x2*(0.00377982367593955383830539328888570653 + x2*(-
0.00032289026362925187386766589190180395 1 +
0.00002253671591150551310202822430790448 97*x2))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

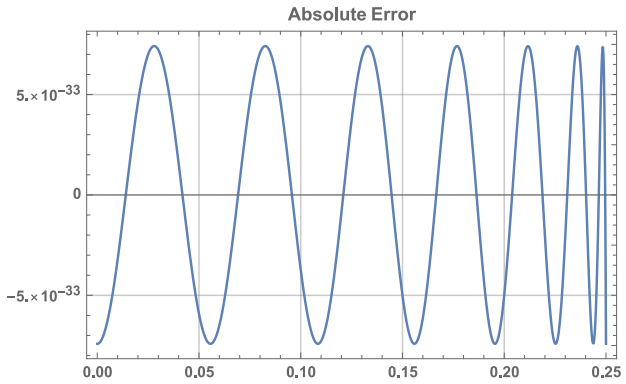
```
0.99999999999999999999999999999092551 + x2*(-0.49999999999999999999999998756607315 +
x2*(0.04166666666666666666666666384394668041 + x2*(-
0.00138888888888888888888888863713975854479 +
x2*(0.00002480158730158730158613518544244610 33 + x2*(-
2.75573192239858903309971467028860 84 + x2*(2.08767569878680420885065648353959571e-9
+ x2*(-1.14707455977229817590627572868734584e-11 +
x2*(4.77947733184048509878946027377473279e-14 + x2*(-
1.56192066641052311392222501358600492e-16 +
x2*(4.11030617835876168464466883764666513e-19 + x2*(-
8.89401475713535349215938946493873708e-22 + 1.57243893787622689684247777925479598e-
24*x2)))))))))))
```

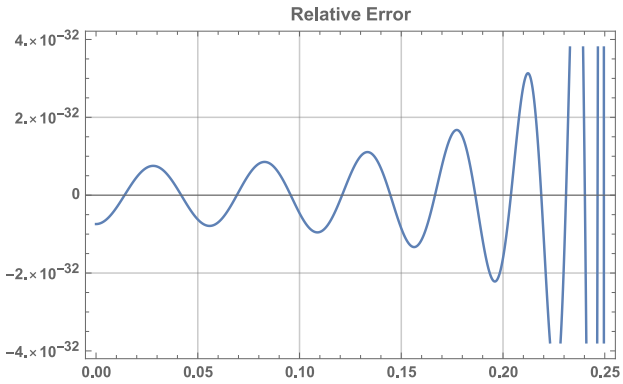
WL-style coefficient list:

```
{0.99999999999999999999999999999092551269428581308572529363326`36.,
-0.49999999999999999999999998756607315132448502454735282786 2`36.,
0.04166666666666666666666666384394668040875449368957279721938`36.,
-0.00138888888888888888888886371397585447932599973866703341059`36.,
0.00002480158730158730158613518544244610326883105783031748 44`36.,
-2.75573192239858903309971467028860840109903392707848 4`36.*^-7,
2.08767569878680420885065648353959570987305353339 14`36.*^-9,
-1.14707455977229817590627572868734584144462729737`36.*^-11,
4.77947733184048509878946027377473278921953559451475044637944085 2`36.*^-14,
-1.56192066641052311392222501358600492453270203383749843863145 44`36.*^-16,
4.11030617835876168464466883764666513363529686060875235587 51`36.*^-19,
-8.89401475713535349215938946493873707926015267625303994 74`36.*^-22,
1.57243893787622689684247777925479598408442305672511397`36.*^-24}
```

Cos, abs. error minimized, degree 26

Maximum relative error: ∞
Maximum absolute error: 7.41592015742532335431430174500802618e-33





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
0.99999999999999999999999999999258 + x2*(-19.739208802178717237668981999705780 2 +
x2*(64.939394022668291490960221744077126 5 + x2*(-85.456817206693727736019486276409975 8
+ x2*(60.244641371876660362716876219119872 + x2*(-
26.426256783374397452358037150975055 6 + x2*(7.903536371318468759210909196228168 27 +
x2*(-1.714390711088669532590908754877820 83 +
x2*(0.282005968455691884731877286121954956 + x2*(-
0.036382841139801826027873931851891 86 + x2*(0.003779834147495079036952968407549 1542 +
x2*(-0.000322990359111596258898986185104 70773 +
x2*(0.000023093788063462385021601938759532 0672 -
1.371297630287145863037598347931948 41e-6*x2)))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

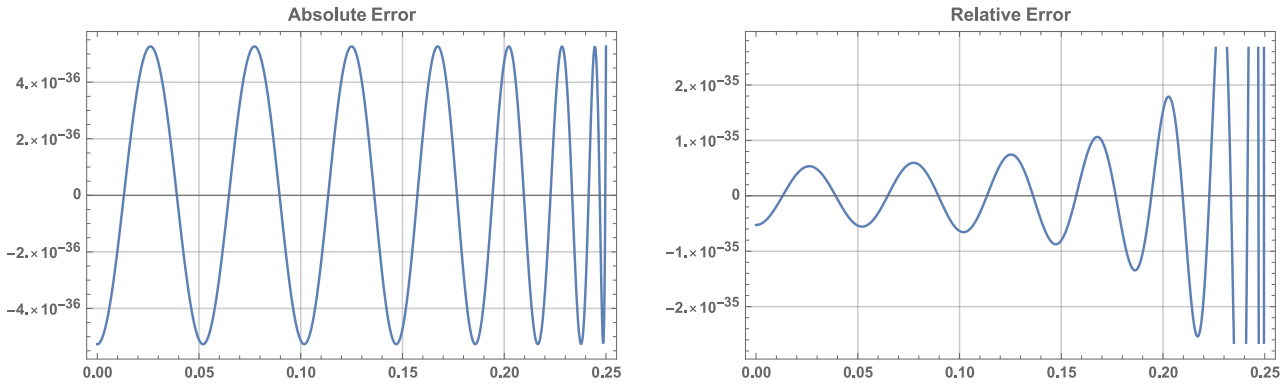
```
0.99999999999999999999999999999258 + x2*(-0.49999999999999999999999999998821582 +
x2*(0.041666666666666666666666666356165953 + x2*(-
0.001388888888888888888888888856666592301 +
x2*(0.000024801587301587301587299842558960 6102 + x2*(-
2.755731922398589065199147870522322 89e-7 + x2*(2.087675698786809886034190878568281 79e-
9 + x2*(-1.147074559772970776704012592944042 75e-11 +
x2*(4.779477332385701832710965008413737 48e-14 + x2*(-
1.561920696740828989655957238751568 15e-16 +
x2*(4.110317565477067289738270209553544 79e-19 + x2*(-
8.896771887954620754991700602081772 69e-22 +
x2*(1.611307153919021711731781717233528 63e-24 -
2.423567657486664334497492679741413 91e-27*x2)))))))))))
```

WL-style coefficient list:

```
{0.99999999999999999999999999999992584079842574676645685 7`36.,
-0.49999999999999999999999999999988215816393807958149253644462`36.,
0.04166666666666666666666666663561659531728081953134199887 09`36.,
-0.00138888888888888888888888856666592300627116913383756199 97`36.,
0.00002480158730158730158729984255896061020580070916025357 34`36.,
-2.75573192239858906519914787052232288587018082721140 3`36.*^-7,
2.08767569878680988603419087856828178782507101481 77`36.*^-9,
-1.14707455977297077670401259294404275038579175573`36.*^-11,
4.77947733238570183271096500841373747566207762963885382409882356 5`36.*^-14,
-1.56192069674082898965595723875156815411312470382156333818518 03`36.*^-16,
4.11031756547706728973827020955354479151647996593021024367 48`36.*^-19,
-8.89677188795462075499170060208177269488198032108175015 9`36.*^-22,
1.61130715391902171173178171723352862891439967446495 95`36.*^-24,
-2.42356765748666433449749267974141390605043897285044`36.*^-27}
```

Cos, abs. error minimized, degree 28

Maximum relative error: ∞
Maximum absolute error: 5.265287349466932458521035494549942 95e-36



C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

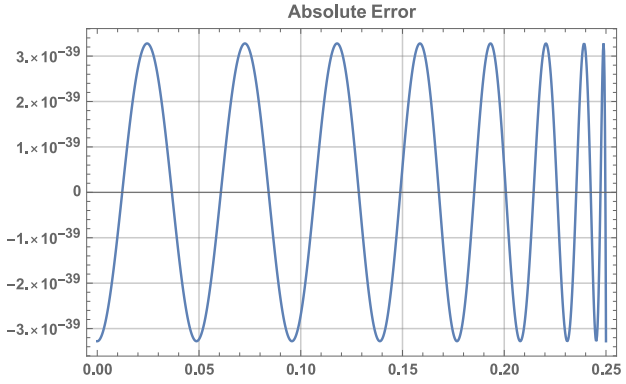
```
0.99999999999999999999999999999999 + x2*(-19.739208802178717237668981999752264 4 +
x2*(64.939394022668291490960221792424769 1 + x2*(-85.456817206693727736019506081073764 9
+ x2*(60.244641371876660362721109035784934 4 + x2*(-
26.426256783374397452899869211246883 1 + x2*(7.90353637131846880413605151304712308 +
x2*(-1.714390711088672060365698545216547 68 +
x2*(0.282005968455790977704032856389483551 + x2*(-
0.036382841142537674681925810321252468 6 + x2*(0.00377983420048601204073094498943433597
+ x2*(-0.000322991063844913148302400612547208 02 +
x2*(0.000023099916361081480959435182132219 8123 + x2*(-
1.402675373412530720975446813874594 82e-6 + 7.17223743843259587983341565680992535e-
8*x2))))))))))))
```

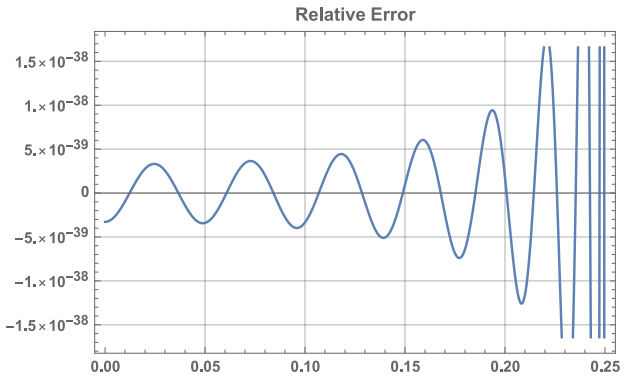
C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

```
0.99999999999999999999999999999999999 + x2*(-0.49999999999999999999999999999999904 +
x2*(0.04166666666666666666666666666666375979 + x2*(-
0.00138888888888888888888888888854167715 +
x2*(0.00002480158730158730158730158513007829913841059542167188 2 +
x2*(-2.75573192239858906525565015600716900017280483060972 6 *^-7,
2.08767569878680989790092033650028630203513390639 04 *^-9,
-1.14707455977297246800234700978276497799421836094 *^-11,
4.77947733238738127452139849402089782756424347637962641759937265 9 *^-14,
-1.56192069685827937291476847853211619283296252459010397711085 98 *^-16,
4.11031762310117306718115082701454485407264939463620292375 29 *^-19,
-8.89679129983917189840815976028648599744817903566140325 85 *^-22,
1.61173473945708322829830267621096394641158694516010423 *^-24,
-2.47902322141678170083969783303284804576427794822448 *^-27,
3.21083763283142640339634627057560775022901951197 *^-30}
```

Cos, abs. error minimized, degree 30

Maximum relative error: ∞
Maximum absolute error: 3.27804061650188319204074676993384692e-39





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
1. + x2*(-19.739208802178717237668981999752302 2 +
x2*(64.939394022668291490960221792470037 6 + x2*(-85.456817206693727736019506102417682 4
+ x2*(60.244641371876660362721114304973648 9 + x2*(-
26.426256783374397452900652368379589 4 + x2*(7.90353637131846880421199739135869544 +
x2*(-1.714390711088672065413381012284952 64 +
x2*(0.282005968455791214617239056200980108 + x2*(-
0.036382841142545652574034217580716121 6 + x2*(0.00377983420067950282609963830941211 05
+ x2*(-0.000322991067195497046314497636683891633 +
x2*(0.000023099956763372475275245892721409973 9 + x2*(-
1.402997604957764626692617318227688 3e-6 + x2*(7.325002465025295555854380338945440 95e-8
- 3.259054867608312675036291027529536 63e-9*x2))))))))))))))
```

C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

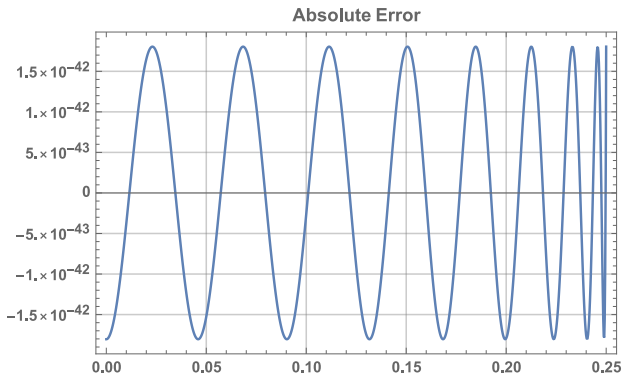
```
1. + x2*(-0.49999999999999999999999999999999999 +
x2*(0.04166666666666666666666666666666666 432 + x2*(-
0.00138888888888888888888888888888856 966 +
x2*(0.00002480158730158730158730158729930432 38 + x2*(-
2.75573192239858906525573182368823 88e-7 + x2*(2.08767569878680989792098102289751236e-9
+ x2*(-1.14707455977297247137967990498343 616e-11 +
x2*(4.77947733238738528976017154405452 036e-14 + x2*(-
1.56192069685862186499098115280481 984e-16 +
x2*(4.11031762331158139273849029500580 264e-19 + x2*(-
8.89679139213103074540244650455765 911e-22 +
x2*(1.61173755841862972561211457857514 863e-24 + x2*(-
2.47959271846395979322459305387477 117e-27 +
x2*(3.27922684896863327088335713153287 456e-30 -
3.69569073620576969980558269093164 118e-33*x2))))))))))))))
```

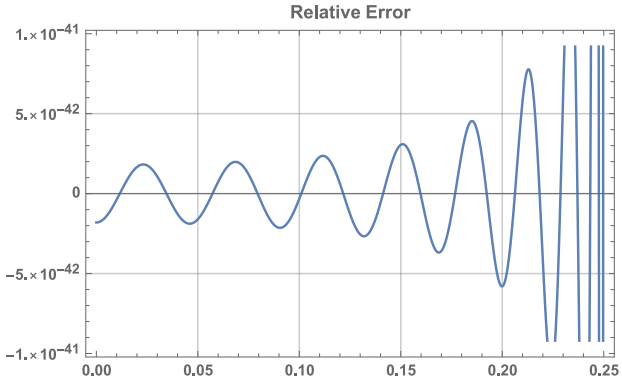
WL-style coefficient list:

```
{0.99999999999999999999999999999999999999967219593834981168078`36.,
-0.49999999999999999999999999999999999999931969397676806491784 64`36.,
0.04166666666666666666666666666666666664322740268027355186039 99`36.,
-0.00138888888888888888888888888888885696571015335165201114657`36.,
0.00002480158730158730158730158729930432381438993698296523 66`36.,
-2.75573192239858906525573182368823879632602622295043 7`36.*^-7,
2.08767569878680989792098102289751236494390944135 37`36.*^-9,
-1.14707455977297247137967990498343616208174553 21`36.*^-11,
4.77947733238738528976017154405452035973556908021241069714184056 8`36.*^-14,
-1.56192069685862186499098115280481984210576229190443959597577 81`36.*^-16,
4.11031762331158139273849029500580263770546602206801386224 83`36.*^-19,
-8.89679139213103074540244650455765911405620610913468438 99`36.*^-22,
1.61173755841862972561211457857514862674896127252646804`36.*^-24,
-2.47959271846395979322459305387477117204123871735524`36.*^-27,
3.27922684896863327088335713153287455501491955828`36.*^-30,
-3.69569073620576969980558269093164118008932637005790439999126744`36.*^-33}
```

Cos, abs. error minimized, degree 32

Maximum relative error: ∞
Maximum absolute error: 1.804065345948063343602814041854056 66e-42





C-style Horner evaluation for sin(2πx) approximation (set `x2 = x * x` first):

```
1. + x2*(-19.739208802178717237668981999752302 3 +
x2*(64.939394022668291490960221792470074 1 + x2*(-85.456817206693727736019506102437308 8
+ x2*(60.244641371876660362721114310514159 5 + x2*(-
26.426256783374397452900653313999865 1 + x2*(7.90353637131846880421210330486386 94 +
x2*(-1.714390711088672065421575044110734 99 +
x2*(0.28200596845579121506956391324526299 1 + x2*(-
0.03638284114254567073850782086033904 93 + x2*(0.00377983420068003821433238732024539201
+ x2*(-0.00032299106720706727935546053012172844 6 +
x2*(0.00002309956944475901712622691173646145 6 + x2*(-
1.40299960215288451000857092116244391e-6 + x2*(7.32647344405838402329703796332643925e-
8 + x2*(-3.32398203242913388247207307411141944e-9 +
1.29856563034244219466942373289485008e-10*x2))))))))))))))
```

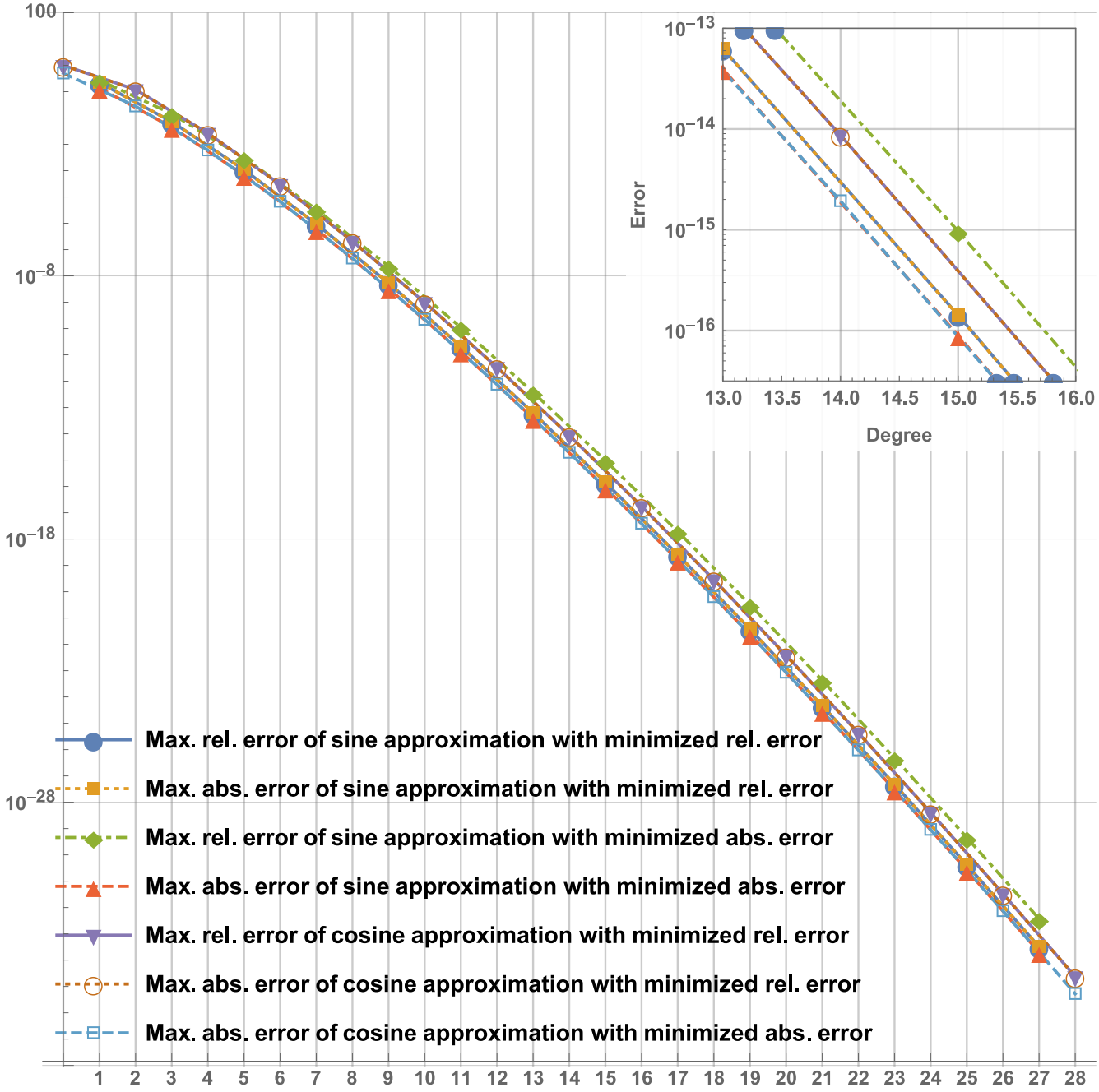
C-style Horner evaluation for sin(x) approximation (set `x2 = x * x` first):

```
1. + x2*(-0.5 + x2*(0.041666666666666666666666666666666666667 + x2*(-
0.0013888888888888888888888888888888888864 +
x2*(0.00002480158730158730158730158730158852479 + x2*(-
2.7557319223985890652557319222975878e-7 + x2*(2.087675698786809897921008999367858891e-
9 + x2*(-1.1470745597729724713851624157019854e-11 +
x2*(4.779477332387385297426237666543066294771374723240742407138232996e-14 + x2*(-
1.561920696858622644794492301012029703830306121643695844793605 5e-16 + x2*(4.110317623312163591737585624665985348969827541249436704736 4e-19 + x2*(-8.896791392449732902145014805035656764098425222011316309 2e-22 +
x2*(1.611737571054635815902777991176114534863092445051349 74e-24 + x2*(-
2.479596248213732077052392183545394186136939296913 26e-27 +
x2*(3.279885370786558278824488445877258321762776608 36e-30 + x2*(-
3.769316597476557116354369024245674966440083291633780001954584 76e-33 + 3.729993857186246957470587356807689871034471818406125197383 43e-36*x2)))))))))))))
```

WL-style coefficient list:

```
{0.99999999999999999999999999999999999999981958798622600974`36.,
-0.49999999999999999999999999999999999995773278516675801501`36.,
0.04166666666666666666666666666666666650219703299293281931 2`36.,
-0.00138888888888888888888888888888886355617159998388643421`36.,
0.00002480158730158730158730158730158524793464405427576130 49`36.,
-2.75573192239858906525573192229758788112246636647774 3`36.*^-7,
2.08767569878680989792100899936785890839272139504 62`36.*^-9,
-1.14707455977297247138516241570198546513244099493`36.*^-11,
4.77947733238738529742623766654306629477137472324074240713823299 6`36.*^-14,
-1.56192069685862264479449230101202970383030612164369584479360 55`36.*^-16,
4.11031762331216359173758562466598534896982754124943670473 64`36.*^-19,
-8.89679139244973290214501480503565676409842522201131630 92`36.*^-22,
1.61173757105463581590277799117611453486309244505134974`36.*^-24,
-2.47959624821373207705239218354539418613693929691326`36.*^-27,
3.27988537078655827882448844587725832176277660836`36.*^-30,
-3.76931659747655711635436902424567496644400832916337800019545847 6`36.*^-33,
3.72999385718624695747058735680768987103447181840612519738343`36.*^-36}
```

Appendix A: Summary of Polynomial Degree vs. Max. Error



Appendix B: Why approximate sin(2πx) et al. this way?

Context

The approximations are provided with DSP applications in mind. Calculating sines and cosines with low memory footprint, only floating-point multiply-adds and no more precision than needed can improve performance signifacantly in some cases.

The interval [-1/2, 1/2] can be folded with the function f(x) = max(min(x, 1/2 - x), -1/2 - x)) such that a polynomial p(x), which approximates sin(2πx) over [-1/4, 1/4] (which is true for odd polynomials fitted over [0, 1/4]) can be evaluated over [-1/2, 1/2] as p(f(x)). To approximate sin(2πx) over [-∞, ∞], one can evaluate p(f(x - x + 1/2)). Similar folding functions can be used for cosine

approximations and obviously, both the polynomials approximating sine as well as those approximating cosine can be used as building block for either sine or cosine over [-∞, ∞].

What sorts of intervals and sine function variations to optimize over

I chose to approximate sin(2πx) for 0 < x < 1/4 and cos(2πx) for 0 < x < 1/4. sin(2πx a) for 0 < x < 1/(4a) and any a ∈ is already covered then, since it is trivial to transform the coefficients accordingly and the optimum approximation coefficients would be the same, except for that transormation. Similarly, (a sin(2πx)) for any a ∈ does not gain anything. This includes -sin and -cos in particular. sin(2πx + a) for 0 < x < 1/4 and any a ∈ is not generally sufficient since one needs to approximate at least a complete quarter of one period from an extremum to a zero of the sine function to be able to calculate all sine output values from that approximation. At the same time, approximating over a bigger interval than a quarter of a period is either redundant or would (probably) need unnecessarily many coefficients (although I have not tested or proven that last assumption).

What sorts of polynomial coefficient sets to use

After having found a number of optimal approximations for given sets of coefficients between a_0 and a_7 and by minimizing the p-Norm of the error for p ∈ {2, 4, 6, 8, ∞}, it became empirically clear that the most efficient method of achieving the lowest error using the fewest coefficients is to only use coefficients with an odd index (a_1, a_3, a_5, a_7, ...) for sine and only coefficients with an even index (a_0, a_2, a_4, a_6, ...) for cosine. This is somewhat expected since sine and cosine are odd and even functions, respectively. Other coefficients tend to improve the approximation less than using the next higher-order odd (or even) coefficient while needing the same number of multiplications and additions. Generally, the results always showed the least error for a given number of nonzero coefficients when using only odd or even coefficients. Interestingly, the approximations of the sine function for a given number of odd coefficients were usually marginally or even substantially better than the approximations of the cosine function for an equal number of even coefficients. I have not attempted to prove these results or found someting like a proof, so they may not generally hold true for higher degree polynomials, other values of p, or other norms in general, but the trend was definitely visible in the data. Evaluating a polynomial with n coefficients a_i where i = 2 j + 1 and j ∈ {0, ..., n-1} (having a sine approximation in mind) takes merely one multiplication more than evaluating such a polynomial where i = 2 j (having a cosine approximation in mind) when using Horner's scheme.

Some of the discussed findings are reflected by [Garrett2012] and [samhocevar2018].

More design variables

Other things to consider when finding such sine approximations include choosing wether the points sin(0) == 0 and sin(2π 1/4) == 1 should be matched perfectly by the polynomial, whether one wants to minimize relative, absolute, or some other error, choosing the norm one wants to minimize and choosing the numerical precision of the coefficients one wants to derive.

Having "exact" output values at the zeros and extrema of the sine function might come in handy in

certain situations, but it comes at the cost of a higher maximum error over the interval.

When trying to find approximations whose errors are near a floating-point epsilon, one should obviously minimize the relative error. For an approximation that does not focus on reaching some floating-point precision, minimizing the absolute error may be more appropriate, depending on the application.

Of the p-Norms, $p \to \infty$ (the Max-Norm) does not increase the mean error significantly compared to an approximation using, say, the 2-norm, while the maximum error decreases substantially, which is often desirable.

Methods to find optimal approximations

One can of course formulate the problem as an optimization problem and do an exhaustive (or "smart") search of the parameter space. I was able to do this fairly successfully with polynomials of low degrees. An advantage of this method was that I did not need to implement any fancy algorithms or use any algebra to get special polynomials where some coefficients $a_i == 0$. That being said, this method is of course very naive. It can reach exponential runtime and I would highly discourage it.

The Remez algorithm [Remez1934] can be used to minimize the Max-Norm.

[Garrett2012] describes an algorithm for minimizing the 2-Norm of the absolute error. I did not check wether this can be generalized to other p-Norms, but I would assume so.

Summary

So basically, my goal was to find polynomials, whose even coefficients are equal to zero, with a minimum error on the interval [0, 1/4] when approximating $\sin(2\pi x)$. Similarly, I found approximations for $\cos(2\pi x)$ using only even coefficients. I chose the Max-Norm to measure the error over the approximation interval. I found approximations both for minimal relative error and for minimal absolute error. The value at $x == 0$ is then fixed by definition for the sine approximations, but I left other extrema (and zeros of the cosine approximations) open for optimization. The coefficients were found with extremely high precision and then rounded to 36 digits before evaluation.

I am sure something like this or quite similar approximations have been done thousands of times before, but I had a hard time finding a collection like in this document, so I chose to make one myself.

Implementation

Although there were a couple of options, I ended up using the Julia language to do the actual approximation finding. I had tried with the Wolfram Language's Function Approximation Package and the C++ Boost Libraries among others, but nothing seemed to work quite the way I needed it to. I'm sure that I can blame myself in part. It's always kind of funny and sad how much time one can waste on such little stupid things, always thinking that the solution is basically right around the next corner while not anticipating the next problem awaiting you instead. Anyhow… There is a

Julia package called Remez.jl [simonbyrne2018], which only worked in Julia 0.6 at the time of writing. Here's the code I finally wrote to get my approximations:

```
using Remez
setprecision (2000)

for sine in[true, false]
  for relative in[true, false]
    for degree in 0 : 16
      filename = "outfiles/"*(sine?"" : "co")*"sine-"*lpad(degree, 2,
        0)*(relative?"-relative.txt" : "-absolute.txt")
      println ("Computing: "*(sine?"S" : "Cos")*"ine, "*(relative?"relative" :
        "absolute")*" error, degree $degree. -> $filename")
      if sine
        out = ratfn_minimax ((x) -> sin (sqrt (x))/(sqrt (x)), (BigFloat (1. e -
          318), BigFloat (pi*pi/4.)), degree, 0, (relative?(x, y) -> 1./y
          : (x, y) -> sqrt (x)))
      else
        out = ratfn_minimax ((x) -> cos (sqrt (x)), (BigFloat (1. e - 318),
          BigFloat (pi*pi/4.)), degree, 0, (relative?(x, y) -> 1./y : (x, y)
          -> 1.))
      end

      open (filename, "w") do f
        write (f, string (out))
      end
    end
  end
end
```

Bibliography

- [Garrett2012] Charles Kristopher Garrett, "Fast Polynomial Approximations to Sine and Cosine", Technical Report (2012)
- [samhocevar2018] https://github.com/samhocevar/lolremez/wiki/Tutorial-3-of-5:-changing-variables-for-simpler-polynomials (accessed 2018-09-12)
- [simonbyrne2018] https://github.com/simonbyrne/Remez.jl (accessed 2018-09-12)
- [Remez1934] E. Ya. Remez, "Sur la détermination des polynômes d'approximation de degré donnée", Comm. Soc. Math. Kharkov 10, 41 (1934); "Sur un procédé convergent d'approximations successives pour déterminer les polynômes d'approximation, Compt. Rend. Acad. Sc. 198, 2063 (1934)