1 Introduction

In this work, we propose a mechanism to direct the distribution of contended cells that will reveal the priority of a secondary cognitive radio user (CR). Additionally, we model a learning process unique to the fluid and distributed nature of the IEEE 802.22 network (WRAN). We model a cognitive radio network (CRN) as a game among CRs, and maximize the utility of a set of overlapping base stations (BS). According to the 802.22 draft, the self-coexistence quiet periods are used for the specific purpose to detect overlapping WRANs, and are designed to support dynamic resource sharing between the overlapping base stations (BSs), targeting fair and efficient scheduling. We build our formulation on the specific case where the BS cells must resort to adaptive on demand channel allocation. We examine a worst-case scenario, maximizing the exploitation of uncertainties inherent to a spectrum sensing period. Our focus is on the behavior of the cognitive radio during the coexistence window, and their use of the coexistence beacon protocol. We provide an alternative iteration of the process so that determines channels which are acquired to satisfy the QoS requirements of the given workloads. We determine that intra-channel sharing is optimized by the cooperation, and the resulting inter-system demand alliviates the channel contention processes with coexisting CRN. As network use in sparse areas becomes more widespread, it is likely that a central dispatcher will not be able to provide the desired QoS across the distributed CRN.

We consider the division of contended frames during the contention beacon protocol. We propose a priority network policy, determined by a queuing network for frame allocation. The CRs will choose the queue with the best priority match. We focus on three main issues in WRAN planning: (1) Spectrum allocation. A network at equilibrium should maximize the throughput of the CRN through intelligent use of base stations, (2) Quality of service (QoS). We consider QoS to be a guarantee of a minimum rate of service while adhering to a priority protocol. (3) Truthfulness in spectrum priority claims. We show that we are able to uphold the desired property of self-coexistence, and model a defense strategy as a Stackelberg game between the CRs and an adversary type CR node with arrival rate and strategy as a directed mutations of player type coefficients. Our goal is to maintain network coherence with the desired QoS converging to a dynamic, stable equilibrium.

2 Related Work

A branch-and-bound (B/&B) algorithm for IEEE 802.22-based LTE networks has been developed in a queue-based control (QBC), where resource allocation is controlled by the queue size of nodes following their packet arrival probabilities. They achieve optimal power and resource block assignment for each mobile user, trading execution time and end-to-end packet delay. (CITE)

CR nodes are advancing technologically (Huawei), and will have the (communications) resources to collaborate on the allocation of spectra in addition to determining open channels, as in cooperative spectrum sensing (CITE).

Sakin and Razzaque solve the issue of self-coexistence as a large MIP problem, which is known to be NP-hard, and several other approximations taking the approach of nonlinear optimization (CITE).

With the 802.22 draft and the advent of orthogonal frequency modulation (OFM), higher level algorithms began to form making use of the network capabilities. Sengupta and Brahma (CITE) addressed the issues of efficiency and utilization using a graph-theoretical approach.

The mean field model was shown for general i.i.d random process, In [CITE] it was proven that there exists a Nash equilibrium in the steady-state using Martingales.

3 The Model

3.1 The Evolutionary Game

In order to clarify our model, we use the CRN self-coexistence window header frame as an update window to a type of central limit order book, which is owned and updated by the BS. The priority assignments from the eCRs waiting for contended frames from the BS are shared within the CRN via the eCRs within overlapping areas. We form a cooperative game where the eCRs are responsible for forming queues in order to optimize frame utilization of in the case where multiple BS must exist on the same channel, defined in the 802.22 standard draft as on demand frame contention. Our intention is to construct a queuing network at equilibrium, where the eCRs form queues to wait for frames from a single, or multiple BS. We use a supermarket game, which is relevant in scenarios where (1) the players choose which queue to join without directions from a central dispatcher; (2) global workload or queue length information is not available and customers randomly choose a finite number of queues based on knowledge from their nearest neighbors; (3) there is a cost associated with finding and then waiting in queues.

We have a set of CRN, each containing a set of queues, one assigned to each BS. The queues formed at BSs that overlap cooperate intelligently. The 802.22 IEEE draft makes use of overlapping regions via the contention beacon protocol. As in (CITE), we may assume that the BS randomly selects a cluster head for each queue, who may aggragate or transmit data with the BS and the other eCRs. Thus the importance of trust among queue members; queues are natural targets for an adversary. The eCRs participate in a selection scheme that encourages cooperation by a shared fitness function. The queues are self-organizing and self-enforcing; a player's utility function influences their decision process, and players may apply peer pressure to affect queue fitness.

Following the lead of Lal and Rao (1997), we define two types of eCR nodes: cherry pickers and time-constrained. Cherry pickers are known to have a lower opportunity cost, and prefer to maximize their utility in terms of sustainable bandwidth. Time-constrained players make up the majority, attributing a higher weight to the cost of selecting a queue. In order to construct a viable model for our final goal, we must have (1) self-configuration, and (2) automatic neighbor relations. We address our CRs as a finite set of actions, and formalize the game play as sphere-of-influence (SIG)(CITE) graph. To begin, we propose a set of mixed strategies defined by a probability distribution over the finite set of feasible strategies, and define a supermarket game played by the eCRs.

3.1.1 The Supermarket Game

Consider a single CRN containing M heterogenous FIFO queues with unit exponential service rate and global Poisson arrival rate λ. A CR completing a transmission at queue i will either move to some new queue j with (fixed) probability P_{ij} or leave the system with probability $1-\sum_{j=1}^{m} P_{ij}$, which is non-zero for some subset of the queues. In order to arrive at an advanced decision model, we first address three main ways to describe the choice L_i, which serves to characterize the supermarket game as a dynamic interaction of intelligent players: (1) Choice structure, (2) Preference maximization, and (3) Utility maximization. Player i's choice of L_i queues forms a preferred set, the interactions between the eCRs in forming their preferred set of queues, and the interactions between eCRs belonging to a specific queue form a decision profile. We define each queue of of eCRs as a coalition within the CRN network including one or more BS, where the decision is based on the expected probabilty distribution of the mixed strategies the eCRs. We model the arrival rate of the eCRs i as a Poisson binomial distribution, bounded above by λ.

The rules of the queue are as follows: A queue cannot refuse a new member, however, we consider that any queue member can submit a participation request to the BS. Players joining the game choose a number of queues to be sampled uniformly at random and joins the best queue with respect to the strategies of the other players.

Denote a function $L_i(\cdot)$, from $[0, c_{MAX}]$ to $\mathcal{L}$ as the strategy of player i. Following standard game theoretic notation, all other players choose L_{-i}. The expected total cost of player i playing strategy L_i, with all other strategies fixed, is given by,

$$\mathbb{E}\big(C(L_i, L_{-i})\big) = c\mathbb{E}[W(L_i, L_{-i})] + \hat{c}\mathbb{E}\big(\theta(L_i, L_{-i})\big),$$

where $W(L_i, L_{-i})$ is the wait time, and c is the cost per unit waiting time, and $\hat{c}$ is the cost of sampling one queue. We define the fitness function, $\theta : \mathcal{S} \mapsto \mathbb{R}^n$, of player i playing strategy L_i.

Now, $\mathcal{P}(\mathcal{L})$ is the family of sets containing all probability distributions over all subsets of $\mathcal{L}$. The mixed strategy $s_i(L_i) \in \mathcal{S} = \mathcal{P}(\mathcal{L})$ is the probability that eCR i samples $L_i(\cdot)$ queues, the expected total cost for playing strategy s_i is given by,

$$C(s_i, s_{-i}) = \sum_{L_i \in s_i} C(L_i, s_{-i}) s(L_i).$$

There exists a bijection from $\mathcal{L}$ to $\mathcal{S} = \mathcal{P}(\mathcal{L})$. Let there be an arrangement such that $s_i(L_i)$ be a non-decreasing step function in $\mathbb{R}$, and normalizing factor $\pi \in [0,1]^n$ such that $\sum_{\mathcal{L} \in s_i} \pi_i = 1$. In other words, given m queues, define the indexed strategy set $\{L_1 \leq L_2 \leq \cdots \leq L_m\} \subset \mathcal{L}$ where the sum of the probabilities $\mathbb{P}(L_i)$ equals 1.

We adopt the mean field theorem from Xu and Hajek (CITE). Fix $s_i(\cdot) \in \mathcal{S}$, and suppose all other players use mixed strategy $s_{-i}(\cdot)$. The cost bmatrix is given as,

$$C(s_i, s_{-i}) = \sum_{L_i \in s_i} C(L_i, s_{-i}) s(L_i).$$

Define (t) to be the number of queues of length at least k at time t. From (CITE), if $s_1, s_2 \in \mathcal{S}$, with $s_1 <_{st} s_2$, then $\mathbb{E}[W(L, s_1)] > \mathbb{E}[W(L, s_2)]$. The mean field equation describes a separable and complete metric space $(\mathcal{S}, \mathbb{R}^n, \mathcal{D})$, where $\mathcal{D}$ is the space of right-continuous, left-limited (cadlag) functions.

Let Ω denote the class of strictly increasing continuous mappings of $[0,1]$ onto itself. For $\pi \in \Omega$, $\pi \cdot \mathcal{L} \in [0,1]$, and we have a complete and separable metric space $\mathcal{L}, \mathbb{N}, \mathcal{D}(\mathbb{R}, \mathbb{N})$. In the mean field model, $[s] = [s_1, s_2, \cdots, s_n]$ is composed of the strategies where $s_1 \leq_{st} s_2 \implies \sum_{i=k,\cdots,n} s_1(L) \leq s_2(L) \ \forall \ \kappa \in \mathcal{L}$. That is, s_1 is first order stochastically dominated by s_2. Fix $s_{-i} \in \mathcal{S}$, and let the expected *marginal value* of L_i be,

$$V(L_i, s_{-i}) = \sum_{L_i \in s_{-i} \in \mathcal{S}} \mathbb{E}[C(L_i, L_{-i})],$$

where we define $\mathbb{E}[C(L_i, L_{-i})]$ as a stochastic process on a projective representation of the rotation group $SO(3)$. The process κ is determined by tensor flow. Let κ be a tensor, for cost $c, \hat{c}$, define,

$$\theta_i = \arctan\left(\frac{c}{\hat{c}}\right).$$

The mapping exists if and only if there exists a strategy vector $[s]$, and a function ϕ such that $\phi(s_i) = C((-\infty, s_i])$, $i = 1, 2, \cdots, N$ and $\phi(s_1) < \phi(s_2) < \cdots < \phi(s_N)$, where phi is a skew-distribution, i.e. a factorization of the skew-symmetric bmatrix of the 4-vector, $[L_i]_{\times s_{-i}}$ that is also

left-tailed.
Proof: TODO: **Proof:** Let $\phi : \mathcal{S} \times \mathcal{S} \to \mathcal{S}$ be a function such that for cost C,

We have a rotation g_ϕ about the z-axis through ϕ,

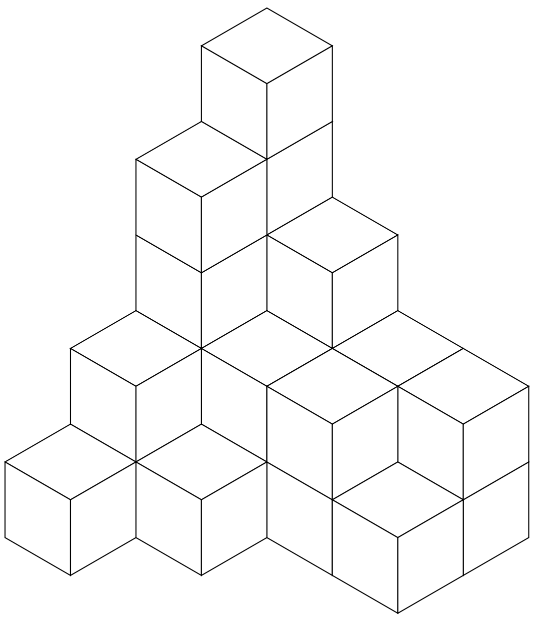
$$\zeta - = e^{i\phi}\zeta$$

and $\omega' = e^{i\theta}\omega$, a rotation of θ through the x-axis.

$$\phi^{-1}(S_i) = \sup_{s_{L_i} \in S_i} [s]_\times = \begin{bmatrix} 0 & -t & \hat{c} \\ t & 0 & -c \\ -\hat{c} & c & 0 \end{bmatrix}$$

To illustrate, let $n = 2$ and consider $[s] = [s_1, s_2]$. Since $\pi \in [0,1]^n$ such that $\sum_{\mathcal{L} \in s_i} \pi_i = 1$ holds for all $s_i \in [s]$, and $\mathbb{P}\big(\gamma([s])\big) = 1$, we have that $[s]_\times$ forms a basis for S_i, and determines the mean field equation describing the number of queues of length at least t with respect to the wait time, where,

$$\int_{t=0}^{\infty} \kappa_{s_{-i}}^{L_i}(t) = \mathbb{E}[W(L_i, s_{-i})].$$



We construct a bilinear form. Let $c, \hat{c} \leq 1$, and define a measure,

$$\mu_i = \min\{\rho(s_i, s_j) : j \neq i\}, \qquad (j = 1, \cdots n),$$

and a labeling, $(1,2) \mapsto (c, \hat{c})$, so that,

$$(\rho)^{-1}(S_i) = \sup_{s_{L_i} \in S_i} -\omega, \quad \begin{cases} (c, \hat{c}) = (1,2) \\ -1, \quad (c, \hat{c}) = (2,1) \\ 0 \quad c = \hat{c} \end{cases} .$$

defines a mapping of a player's actions to a bifurcation, determined by the quadratic form $\pm[\omega^2 t^2 - c^2 - \hat{c}^2 - \phi^2]$ surface $\rho(L_i) = c + \hat{c}i$, where $\phi(L_i) = r(t)e^{i\theta}$, where

$$r(t) = \left\{ \begin{bmatrix} c & -\overline{\hat{c}} \\ \hat{c} & \overline{c} \end{bmatrix} : \int_{-\infty}^{t} |c| = 1 \right\}.$$

Suppose ρ_i defines neighbors in $\mathcal{S}$, and uses a bilinear form,

$$c \cdot \hat{c} = \pm[\omega_1\omega_2 t_1 t_2 - r_1 r_2],$$

where $\omega = 1/\lambda$, and orthonormal basis $-\eta(e_0, e_0) = \eta(e_1, e_1) = \eta(e_2, e_2) = \eta(e_3, e_3) = 1$, where the function η is the skew-symmetric operator in the Minkowski space. We may extend the arrangement $s_1 < s_2 < \cdots s_N$ to the trihedral plane for fixed time τ. We build from the geometric form, and extend the cross-product $[s_1]_\times s_2$, which is characterized by the transition $L_i = k$ to $L_{i+1} = k+1$, forming a tensor flow of similarity groups by intent. Let there be a distribution of K queues, with compound distribution described as

$$\theta_\zeta(t) = \int_{\hat{C}} r(t) d\tau = e^{\lambda(\phi_\zeta(t)-1)}.$$

We examine the skew-Hermetian assignment, and the resulting tangent vector, or four-velocity. The four coordinate functions $\theta^a(\tau),\ a = 0, 1, 2, 3$ are real functions of a real variable τ. Without the existence of a metric one can speak of the difference between a point p on the curve at the parameter value τ_0 and a point on the curve $(\tau_0 + \Delta\tau)$ farther away. In the limit $\Delta\tau \to 0$, let there be a point such that,

$$\begin{bmatrix} n \\ K \end{bmatrix} = \begin{bmatrix} 1 \\ K \end{bmatrix} = \begin{bmatrix} 0 \\ K \end{bmatrix} = 1.$$

We define our rotary group as the cotangent vector field $[\pi]$ as n approaches N, the size of the queues of the tangent neighbor set, $n \geq \max_t\{N\}_\phi$, which define a field that follows the similarities of the extended cost function.

TODO:
1. form the mobius negative binomial stitching
2. attach to geodesics 3. axis-align to random process with cone to inft Consider a mapping $\mathcal{S} \mapsto \mathcal{D}$ that preserves the quadratic form $(t, r, \theta) \mapsto (\omega^2 t^2 - c^2 - \hat{c}^2)$, and the general orthogonal group $O(1, 3)$. The jump-diffusion process was constructed to have ergodic properties so that after initially flowing away from its initial condition it would generate samples from the posterior probability model. We have that $\phi \cdot \mathcal{S} \subset \mathcal{S}$ is the subset of skew-Hermetian matrices known as signed permutation matrices. We extend our mean-field model to include this representation by setting $\left[0//k\right] = \left[1//k\right]$ at the stop time τ. We introduce a transform $\gamma : s_i(\mathcal{L}) \to [0, 1]^2$ such that,

$$\gamma(s_i \in \mathcal{S}) = \gamma\left(\sum_{s_1}^{s_i} \pi_{L_i}\right) - \gamma\left(\sum_{s_1}^{s_{i-1}} \pi_{L_i}\right) = 1,$$

Let $\mathcal{D}$ be the space of all cadlag ("continu a droite, limites a gauches") functions; the space of right-continuous functions on $[0, 1]$ with left limits, and define a norm on $\mathcal{S}$,

$$||s_i|| = \sup_{c \in s_{\mathcal{L}_i}} |L_i(c)|.$$

Define the variation process,

$$\langle s_i \rangle = \sum_{L_i \in s_{L_i}}$$

TODO: We proceed to determine the skew-distribution. The map projection onto the complex plane is given by the Lorentz translation, which gives the displacement $L_i = k$ to $L_{i+1} = k+1$ as a function of time. The mapping of π divided by $\Delta\tau$ defines a vector, the tangent vector of the world line at the point s.

$$\vec{s} = (t, c, \hat{c}) = \left(\frac{dr}{d\tau}\ , \hat{c}\frac{dt}{d\tau} \right),$$

and

$$\phi(t) = -\epsilon \cdot \vec{s},$$

with derivative,

$$\frac{\partial \phi}{\partial s} = -\epsilon \cdot \frac{\partial s}{\partial s} = -\epsilon \cdot I = -\epsilon.$$

TODO: Explain. We take the variation process of S_i as,

$$\langle S_i \rangle = \big\{ s \in S_{-i} | \sup_{s_{L_{-i}} \times L_j} ||s_i - s_j|| < \infty \big\}.$$

Now, let $\phi : \mathcal{S} \times \mathbb{N} \to S$ be a stochastic process defining dS_t. Suppose $L(\cdot) \in \mathcal{S}$ is given, the solution to $s_i(k) = \int_{c_i - k}^{c_i} \pi_i(c)\ dc$ gives a set of unique jumping points $0 = c_0 < c_1 < \cdots < c_{L_{MAX}} = c_{MAX}$. Any cadlag finite variation process S has quadratic variation (TODO: don't forget to use) equal to the sum of the squares of the jumps $0 = c_0 < c_1 < \cdots c_n$. Let $W_t : [0, +\infty) \times \Omega \to \mathcal{S}$ be a one-dimensional geometric (exponential) Weiner process. We define right-continuity as a *stopping time* $\tau : \mathcal{S} \to [0, +\infty]$, which occurs at a random jumping point. This is the first hitting time for the region $\{\hat{c} \in [0, \hat{c}_{MAX}] | c \geq c_{max}\}$. We have the following stochastic differential equation, with stopping time $\tau(\omega)$,

$$dS_t = \theta S_t dt + \phi S_t dW_t,$$

with Ito drift-diffusion process,

$$\int_0^t \frac{dS_t}{S_t} = \theta t + \phi W_t$$

and $\mathbb{E}[S_t] = S_0 e^{\omega t}$. Denote the mapping $(c, \hat{c}) \mapsto (r, t, \phi)$ by the open ball,

$$S = \{s_i \in \mathcal{S} : \phi(s_i, s_j) < r_i,\ j \neq i\}, \qquad (j = 1, \cdots N),$$

where $\mathbb{E}[W(s_i, S_{-i}])] = r_i = \sqrt{c^2 + \hat{c}^2} \in [0,1] \times [0,1]$, with transition given by the pairity of the arrival process, $\operatorname{sgn}(S_t) = (-1)^m$, so that $c_1 < c_2$ implies $s_{L_1} >_{st} s_{L_2}$. Now, $s_{L_1} \leq_{st} s_{L_2}$, indicates that s_{L_1} is second-order stochastically dominated by s_{L_2}, that is,

$$\int_{-\infty}^{s} |s_{L_1}(t) - s_{L_2}|\ dt \geq 0.$$

is the sphere of influence $S_i \in \mathcal{S} \times \mathcal{S}$ over the strategy space of player i, $s_i \circ \mathcal{L} = S_i \in \mathcal{S} \times \mathcal{S}$. A player's preference is thus modeled as cost coefficients assigned to the wait time, c, and a cost $\hat{c}$, associated with the fitness of the queue.

3.2 Enhanced Cognitive Structure

The ability of the CRN to reconfigure based on cognition of the environment motivates us to construct our model as a study of momentum in a network between eCRs. We can think of this quantity as a measurement of flux passing through the boundary of the ball B_i. Which we model as a time-stopped Weiner process. TODO: FINISH, although it is a positive quantity, the exterior of the ball is a field with the same measure, and so although the field may have velocity, there is no motivating factor as equilibrium is achieved at stasis.

The stereographic coordinates of the upper sheet of a two-sheeted hyperboloid in $\mathbb{R}^3$ are,

$$L = \begin{cases} \phi = \frac{x}{1+z}, \\ \psi = \frac{y}{1+z} \end{cases}$$

1. projection onto big sphere (exp) 2. limits being the map projection onto (sphere, hyperboloid)

Lemma 3.1. *The Hyperbolic Interior*
TODO: 1. show gradient in each sphere from type distribution, 2. parameterize the hyperboloid - use tangent space 3. complete the exterior, must be convex 4. define the incidents on the SIG

TODO (Model the queue):
1. model the (fair?) cooperation 2. show equilibrium 3.