

# 1 Proposed Topic

In order to construct a viable model of a game, we must have (1) self-configuration, and (2) automatic neighbor relations. We address our players as a finite set of actions, and attempt to formalize the game play as sphere-of- influence (SIG) graph. To begin, we propose a set of mixed strategies defined by a probability distribution over the finite set of feasible strategies, and define a basic game played by intelligent players. We make use of mean field theorem to prove the existence of a dense strategy space, and define a decision model in the extended (compactified) strategy space determined by a drift diffusion process.

Given a stochastic arrival process, we define a subset of the field of right-continuous, left-limited *cadlag* functions, and show there exists a mapping of player types to a time-dependent arrangement, where we may then apply our statistical game theoretic approach. We conjecture that the choice of arrangement and random process results in a proximity/incidence graph with nice properties.

# 2 Proposed Theorem

Let  $\pi$  be a normalizing function acting on an angle  $\theta$ . The mapping  $\mathbb{N} \mapsto \mathcal{D} \times \mathcal{S}$  exists if and only if there is the product space  $\{\pi(k) \mapsto r(\tau)\}$ , where  $k = 1$  defines the mapping

$$\theta(\cdot, k) \mapsto \mathcal{P} \times \mathcal{D}.$$

$\mathcal{D}$  is the space of *cadlag* functions restricted to  $\mathbb{R} \times [0, 1]^{\mathbb{R}}$ , which we will denote  $\mathcal{K}$ , and  $\mathcal{P}$  is the projective space of the player's strategy distribution  $\mathcal{S}$ . We define right-continuity as a *stopping time*  $\tau : \mathcal{S} \rightarrow [0, +\infty]$ .

Let  $\mathcal{I}$  be an arrangement on  $\mathcal{K}$  where  $\theta(s_1, r(t)) > \theta(s_1, r(t+1))$  implies that  $\pi(k) < \pi(k+1)$  for all  $t \in \tau$ . Then, fixing  $t \in \tau$ , let  $\theta' \neq \theta$  be given by

$$\theta' = \theta \cdot \sqrt{\frac{1 + \beta^2}{1 - \beta^2}},$$

where  $\beta = \tan \theta$ .

We claim that there exists a geodesic such that there a mapping from the origin,  $0_{\mathcal{S} \times \mathcal{S}}$ , is defined by the ball

$$B(\tau, \cdot) = \{[s_i, s_{-i}] : i \in \mathcal{I} \subset \mathcal{K} \times \tau\}.$$

We have a bijection from  $\pi(\mathcal{K})$  to  $r(\tau)$  and we have that  $B_{s_{-i}}(\tau) < \pi(\mathcal{K})$  reveals a sub-algebra extending the strategy space in the real variable  $\tau$ .

Now, given a function  $\phi$ , we examine the extended strategy space where  $s_1$  stochastically dominating  $s_2$  implies that  $\mathbb{E}[\phi(\cdot, s_2)] > \mathbb{E}[\phi(\cdot, s_1)]$ . Suppose  $\phi^{-1}$  preserves the quadratic form  $(t^2 - s^2) \mapsto (t, r, \theta)$ , so that

$$\int_t^{t+1} \phi^{-1}(\theta(\mathcal{K})) dt = \theta(\cdot, k+1) - \theta(\cdot, k).$$

The players arrive to the game at a rate determined by arrival process  $\phi(\tau, \cdot)$ .

$$d\mathcal{S}_t = \theta(\cdot, t)\phi(\mathcal{S}_t)dt + \phi_t(\mathcal{S}_t)dW_t,$$

where  $W_t : [0, +\infty) \times \mathcal{S} \rightarrow \mathcal{S}$  is a one-dimensional noise process. As any *cadlag* finite variation process has quadratic variation equal to the sum of the squares of the jumps  $0 = s_0 < s_1 < \dots s_n$ ,

the solution to

$$s_i(\tau) = \int_{\theta_i(s_i, t)}^{\theta_i(s_i, t+1)} \pi(\tau) d\tau$$

for  $t \in \tau$  gives a set of unique jumping points  $0 = s_0 < s_1 < \dots < s_{\bar{\mathcal{K}}} = s_{MAX}$ . Let each player's stop time  $\tau$  occur at a random jumping point within their strategy space, thereby fixing  $\bar{\mathcal{K}}$  for that player.

We consider a ball  $B$ , where a nonzero flux at the boundary represents an uncertainty state. Each ball  $B$  represents a distribution of possible strategies, and as subset of the measure space we are able to compute its density function. Define the density operator  $\rho$  on  $\mathcal{S} \times \mathcal{S}$  as

$$\rho = \sum |s_i\rangle\langle s_{-i}|$$

where  $|s_i\rangle\langle s_{-i}|$  is the outer product. The expectation value of a state  $[s]$  is given by  $\langle[s]\rangle = \text{tr}[\rho[s]]$ . The density matrix used here is defined to be the statistical state of a system in quantum mechanics, and is particularly useful in dealing with mixed states. For a random measure on  $(\mathcal{S} \times \mathcal{S}, \mathcal{D})$ , i.e. the  $\sigma$ -algebra of the arrival process, we define a graph  $\mathcal{G}$  as the set subsets of  $\mathcal{S} \times \mathcal{S}$  of fully connected nodes. We claim that there exists an additional, induced metric, on  $\mathcal{G}$ . The closed sphere of influence graph covers the intersections of the closed balls  $\{\bar{B}\} \subset \mathcal{S} \times \mathcal{S}$ , where

$$\bar{B} = \{B_i \in \mathcal{S} : \min \rho(s_i, s_{-i}) \leq r(\tau)\}.$$

We finally claim to have an immersion in the surrounding dynamical complex field  $\mathbb{C}^{\tau \times \tau}$ . The complex field encases the distributions of the player strategies; that is, the gradient of the distribution across the boundary of  $\mathcal{G}$  determines the orientation of exterior.

TODO: FINISH! We proceed to determine the skew-distribution of the closed manifold. We examine the skew-Hermetian assignment, and the resulting tangent vector, or four-velocity. The four coordinate functions  $\theta^c(\tau)$ ,  $c = 0, 1, 2, 3$  are real functions of a real variable  $\tau$ . We have that  $\phi \cdot \mathcal{S} \subset \mathcal{S}$  is the subset of skew-Hermetian matrices known as signed permutation matrices. We extend our mean-field model to include this representation by setting  $\begin{bmatrix} 0 \\ k \end{bmatrix} = \begin{bmatrix} 1 \\ k \end{bmatrix}$  at the stop time  $\tau$ .

Define the the player's initial utility function as a hyperbolic absolute risk aversion (HARA), and so must adhere to, for utility  $\mu$ ,

$$\frac{\mu''(\langle s_i, s_{-i} \rangle)}{\mu'(\langle s_i, s_{-i} \rangle)}.$$

### 3 Research Plan

- Step 1: Formulate the relevant graph models using appropriate abstraction.
- Step 2: Establish the existence/uniqueness of differential tensor/flow .
- Step 3: Implement a python/ruby module/scaffolding to model to aid in prototyping conditions to define (possible) deterministic systems.
- Step 4: Based on the analytical results, implement various systems' tensorflow.

## 4 References

- [1] "Sphere of Influence Graphs in General Metric Spaces", T.S. Michael, T. Quint
- [2] "The Supermarket Game", Jiaming Xu, Bruce Hajek
- [3] "Modeling Population Dynamics in Changing Environments", Michelle Shen
- [4] "Self-coexistence among interference-aware IEEE 802.22 networks with enhanced air-interface", S. Sengupta, S. Brahma, M. Chatterjee, N. Sai Shankar

## 5 Literature

In addition to related work, the sourced literature must include historical papers to give perspective and formalization, and determine the foundation of the proposed theory.

- [1] M. E. Bratman. Intention, Plans, and Practical Reason. CSLI Publications, Stanford University, 1987.
- [2] I. Gilboa and E. Zemel. Nash and correlated equilibria: Some complexity considerations. *Games and Economic Behavior*, 1:80-93, 1989.
- [3] E. Kalai. Games, computers, and O.R. In *ACM/SIAM Symposium on Discrete Algorithms*, 1995.
- [4] Noam Nisan and Amir Ronen. Algorithmic mechanism design. In *Proc. 31st ACM Symp. on Theory of Computing*, pages 129-140, 1999.
- [5] C. H. Papadimitriou. On the complexity of the parity argument and other inefficient proofs of existence. *Journal of Computer and System Sciences*, 48(3):498-532, 1994.
- [6] J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior*, Second Edition. Princeton University Press, second edition, 1947.