1 Proposed Topic

In order to construct a viable model of a game, we must have (1) self-configuration, and (2) automatic neighbor relations. We address our players as a finite set of actions, and attempt to formalize the game play as sphere-of- influence (SIG) graph. To begin, we propose a set of mixed strategies defined by a probability distribution over the finite set of feasible strategies, and define a basic game played by intelligent players. We make use of mean field theorem to prove the existence of a dense strategy space, and define a decision model in the extended (compacted) strategy space determined by an Itô drift diffusion process.

Given a Poisson arrival process, we define a subset of the field of right-continuous, left-limited *cadlag* functions, and show there exists a mapping of player types to a time-dependent arrangement, where we may then apply our statistical game theoretic approach. We conjecture that the choice of arrangement and random process results in a proximity/incidence graph with nice properties.

2 Proposed Theorem

Let there be a mapping $\mathbb{N} \mapsto \mathcal{D} \times \mathcal{S}$ if and only if there is the product space $\{k\pi \mapsto \mathcal{K}r(\tau)$, where $k = 1$ defines the mapping

$$\theta(\cdot, k) \mapsto \mathcal{P} \times \mathcal{D},$$

where $\mathcal{D}$ is the space of *cadlag* functions restricted to $\mathbb{A}^k \times [0,1]^k$, i.e. $\mathcal{K}$, and let $\mathcal{P}$ be the projective space of the player's strategy distribution $\mathcal{S}$, and π is a normalizing function. We define right-continuity as a *stopping time* $\tau : \mathcal{S} \to [0, +\infty]$, which occurs at a random jumping point.

Let $\mathcal{I}$ be an arrangement on $\mathcal{K}$ where $\theta(s_1, r(t)) > \theta(s_1, r(t+1))$ implies that $\pi k < \pi(k+1)$ for all $t \in \tau$. We claim that there exists a geodesic such that the mapping from the origin, $0_{\mathcal{S} \times \mathcal{S}}$, defined by the ball

$$B(\tau, \cdot) = \{[s_i, s_{-i}] : i \in \mathcal{I} \subset \mathcal{K} \times \tau\},$$

is a bijection from $\pi(\mathcal{K})$ to $r(\tau)$ and we have that $B_{s_{-i}} < \pi(\mathcal{K})$ reveals a sub-algebra extending the strategy space in the real variable τ.

Now, given a function ϕ, we examine the extended strategy space where s_1 stochastically dominating s_2 implies that $\mathbb{E}[\phi(\cdot, s_2)] > \mathbb{E}[\phi(\cdot, s_1)]$. Suppose ϕ_{-1} preserves the quadratic form $(t^2 - s^2) \mapsto (t, r, \theta)$, so that

$$\int_t^{t+1} \phi^{-1}(\theta(\mathcal{K}))dt = \theta(\cdot, k+1) - \theta(\cdot, k).$$

The players arrive to the game at a rate determined by arrival process $\phi(\tau, \cdot)$.

$$d\mathcal{S}_t = \theta(\cdot, t)\phi(\mathcal{S}_t)dt + \phi_t(\mathcal{S}_t)dW_t,$$

where $W_t : [0, +\infty) \times \mathcal{S} \to \mathcal{S}$ is a one-dimensional noise process. As any cadlag finite variation process has quadratic variation equal to the sum of the squares of the jumps $0 = s_0 < s_1 < \cdots s_n$, the solution to

$$s_i(\tau) = \int_{\theta_i(s_i, t)}^{\theta_i(s_i, t+1)} \pi(\tau) \, d\tau$$

for $t \in \tau$ gives a set of unique jumping points $0 = s_0 < s_1 < \cdots < s_{\overline{\mathcal{K}}} = s_{MAX}$.

Consider ball B, the nonzero flux at the boundary represents an uncertainty state. Each ball B represents a distribution of possible strategies, and as subset of the measure space we are able to

compute its density function. Define the density operator ρ on $\mathcal{S} \times \mathcal{S}$ as

$$\rho = \sum |s_i\rangle\langle s_{-i}|$$

where $|s_i\rangle\langle s_{-i}|$ is the outer product. The expectation value of a state $[s]$ is given by $\langle[s]\rangle = \operatorname{tr}\rho[s]$, The density matrix used here is defined to be the statistical state of a system in quantum mechanics, and is particularly useful in dealing with mixed states. For a random measure on $(\mathcal{S} \times \mathcal{S}, \mathcal{D})$, i.e. the σ-algebra of the arrival process, we define a graph $\mathcal{G}$ as the set subsets of $\mathcal{S} \times \mathcal{S}$ of fully connected nodes. We claim that there exists an additional, induced metric, on G. The the closed sphere of influence graph covers the intersections of the closed balls $\{\overline{B}\} \subset \mathcal{S} \times \mathcal{S}$, where

$$\overline{B} = \{B_i \in \mathcal{S} : \min \rho(s_i, s_{-i}) \leq r(\tau)\}.$$

We claim to have an immersion in the surrounding dynamical complex field $\mathbb{C}^\tau$. The complex field encases the distributions of the player strategies; that is, the gradient of the distribution across the boundary of $\mathcal{G}$ determines the orientation of exterior.

We examine the skew-Hermetian assignment, and the resulting tangent vector, or four-velocity. The four coordinate functions $\theta^a(\tau), a = 0, 1, 2, 3$ are real functions of a real variable τ. We have that $\phi \cdot \mathcal{S} \subset \mathcal{S}$ is the subset of skew-Hermitian matrices known as signed permutation matrices. We extend our mean-field model to include this representation by setting $\begin{bmatrix} 0 \\ k \end{bmatrix} = \begin{bmatrix} 1 \\ k \end{bmatrix}$ at the stop time τ.

We proceed to determine the skew-distribution of the closed manifold Define the the player's initial utility function as a hyperbolic absolute risk aversion (HARA), and so must adhere to, for utility μ,

$$\frac{\mu''(\langle S_i, S_{-i}\rangle)}{\mu'(\langle S_i, S_{-i}\rangle)}.$$

3   Research Plan

- Step 1: Formulate the relevant graph models using appropriate abstraction.

- Step 2: Establish the existence/uniqueness of differential tensor/flow .

- Step 3: Implement a python/ruby module/scaffolding to model to aid in prototyping conditions to define (possible) deterministic systems.

- Step 4: Based on the analytical results, implement various systems' tensorflow.

4   References

[1] "Sphere of Influence Graphs in General Metric Spaces", T.S. Michael, T. Quint
[2] "The Supermarket Game", Jiaming Xu, Bruce Hajek
[3] "Modeling Population Dynamics in Changing Environments", Michelle Shen
[4] "Self-coexistence among interference-aware IEEE 802.22 networks with enhanced air-interface", S. Sengupta, S. Brahma, M. Chatterjee, N. Sai Shankar

5 Literature

In addition to related work, the sourced literature must include historical papers to give perspective and formalization, and determine the foundation of the proposed theory. [1] M. E. Bratman. Intention, Plans, and Practical Reason. CSLI Publications, Stanford University, 1987.

[2] I. Gilboa and E. Zemel. Nash and correlated equilibria: Some complexity considerations. Games and Economic Behavior, 1:80-93, 1989.

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[4] Noam Nisan and Amir Ronen. Algorithmic mechanism design. In Proc. 31st ACM Symp. on Theory of Computing, pages 129-140, 1999.

[5] C. H. Papadimitriou. On the complexity of the parity argument and other inefficient proofs of existence. Journal of Computer and System Sciences, 48(3):498-532, 1994.

[6] J. von Neumann and O. Morgenstern. Theory of Games and Economic Behavior, Second Edition. Princeton University Press, second edition, 1947.