1 Proposed Topic

In order to construct a viable model of a game, we must have (1) self-configuration, and (2) automatic neighbor relations. We address our players as a finite set of actions, and attempt to formalize the game play as sphere-of- influence (SIG) graph. To begin, we propose a set of mixed strategies defined by a probability distribution over the finite set of feasible strategies, and define a basic game played by intelligent players. We make use of mean field theorem to prove the existence of a dense strategy space, and define a decision model in the extended (compacted) strategy space determined by a drift diffusion process.

Given a stochastic arrival process, we define a subset of the field of right-continuous, left-limited *cadlag* functions, and show there exists a mapping of player types to a time-dependent arrangement, where we may then apply our statistical game theoretic approach. We conjecture that the choice of arrangement and random process results in a proximity/incidence graph with nice properties.

2 Proposed Theorem

Let π be a normalizing function acting on an angle $\theta \in \mathbb{C}$. The mapping $\mathbb{N} \mapsto \mathcal{K} \times \mathcal{S}$ exists if and only if there is the product space $\{\pi(k) \mapsto r(\tau)\}$, where $k = 1$ defines the mapping

$$\theta(\cdot, k) \mapsto \mathcal{P}^k \times \mathcal{K}.$$

$\mathcal{K}$ is the space of *cadlag* functions restricted to $\mathbb{A}^{\mathbb{R}} \times [0, 1]^{\mathbb{R}}$, and $\mathcal{P}$ is the projective space of the player's strategy distribution $\mathcal{S}$. We define right-continuity as a *stopping time* $\tau : \mathcal{S} \to [0, +\infty]^{\mathbb{R}}$.

Let $\mathcal{I}$ be an arrangement on $\mathcal{K}$ where $\theta(s_1, r(t)) > \theta(s_1, r(t + 1))$ implies that $\pi(k) < \pi(k + 1)$ for all $t \in \tau$. Then, fixing $t \in \tau$, let $\theta' \neq \theta$ be given by

$$\theta' = \theta \cdot \sqrt{\frac{1 + \beta^2}{1 - \beta^2}},$$

where $\beta = \tan \theta$.

We claim that there exists a geodesic such that there is a mapping from the origin, $0_{\mathcal{S} \times \mathcal{S}}$, is defined by the ball

$$B(\tau, \cdot) = \{[s_i, s_{-i}] : i \in \mathcal{I} \subset \mathcal{K} \times \tau\},$$

where $[\cdot, \cdot] : \mathcal{S} \times \mathcal{S} \mapsto \mathcal{S}$ is the Lie bracket operator.

We have a bijection from $\pi(\mathcal{K}) \in \mathcal{S} \times \mathcal{S}$ to $r(\tau) \in \mathcal{S}$ such that $B_{s_{-i}}(\tau) < \pi(\mathcal{K})$ reveals a subalgebra extending the strategy space with respect to the real variable τ, and is a homeomorphy with respect to the ???. Thus, the resulting subspace topology is simply connected.

Now, given a function ϕ, we examine the extended strategy space where s_1 stochastically dominating s_2 implies that $\mathbb{E}[\phi(\cdot, s_2)] > \mathbb{E}[\phi(\cdot, s_1)]$. Suppose ϕ^{-1} preserves the quadratic form $(t^2 - \langle s \rangle) \mapsto (t, r, \theta)$, so that

$$\int_t^{t+1} \phi^{-1}(\theta(\mathcal{K}))dt = \theta(\cdot, k + 1) - \theta(\cdot, k).$$

We have that $2\pi\delta\mathcal{K} \leq \delta\mathcal{S}$?

The players arrive to the game at a rate determined by arrival process $\phi(\tau, \cdot)$.

$$d\mathcal{S}_t = \theta(\cdot, t)\phi(\mathcal{S}_t)dt + \phi_t(\mathcal{S}_t)dW_t,$$

where $W_t : [0, +\infty) \times \mathcal{S} \to \mathcal{S}$ is a one-dimensional stop-time Brownian motion. As any cadlag finite variation process has quadratic variation equal to the sum of the squares of the jumps $0 = s_0 < s_1 < \cdots s_n$, the solution to

$$s_i(\tau) = \int_{\theta_i(s_i,t)}^{\theta_i(s_i,t+1)} \pi(\tau)\ d\tau$$

for $t \in \tau$ gives a set of unique jumping points $0 = s_0 < s_1 < \cdots < s_{\overline{\mathcal{K}}} = s_{MAX}$. Let each player's stop time τ occur at a random jumping point within their strategy space, thereby fixing $\overline{\mathcal{K}}$ for that player.

We consider a ball B, where a nonzero flux at the boundary represents an uncertainty state, and take a random measure on $(\mathcal{S} \times \mathcal{S}, \mathcal{K})$, i.e. the $\sigma(\phi)$-algebra of the arrival process. Each ball B represents a distribution of possible strategies, and as subset of the measure space we are able to compute its density function. Define the density operator ρ on $\mathcal{S} \times \mathcal{S}$ as

$$\rho = \sum |s_i\rangle\langle s_{-i}|$$

where $|s_i\rangle\langle s_{-i}|$ is the outer product. The expectation value of a state $[s]$ is given by $\langle [s] \rangle = \mathrm{tr}\rho[s]$, and is pure imaginary. The density matrix used here is defined to be the statistical state of a system in quantum mechanics, and is particularly useful in dealing with mixed states. Define a graph $\mathcal{G}$ as the set subsets of $\mathcal{S} \times \mathcal{S}$ of fully connected nodes. We claim that there exists an additional, induced metric, on $\mathcal{G}$. The closed sphere of influence graph covers the intersections of the closed balls $\{\overline{B}\} \subset \mathcal{S} \times \mathcal{S}$, where

$$\overline{B} = \{B_i \in \mathcal{S} : \min \rho(s_i, s_{-i}) \leq r_{\tau\times\tau}\}.$$

We finally claim to have an immersion in the surrounding dynamical complex field $\mathbb{C}^{\tau\times\tau}$. The density matrix compresses the space $\mathcal{S} \times \mathcal{S}$ to its canonical form. Thus, by Schur's lemma, the intertwining map $phi^{-1} \mapsto StimesS$, is either 0, or an isomorphism. Thus,$\mathcal{G}$ is a unitary structure that can be seen as an orthogonal structure, a complex structure, and a symplectic structure.

TODO: FINISH! The complex field encases the distributions of the player strategies; that is, the gradient of the distribution across the boundary of $\mathcal{G}$ determines the orientation of exterior. We proceed to determine the skew-distribution of the closed manifold. We examine the skew-Hermetian assignment, and the resulting tangent vector, or four-velocity. The four coordinate functions $\theta^c(\tau), c = 0, 1, 2, 3$ are real functions of a real variable τ. We have that $\phi \cdot \mathcal{S} \subset \mathcal{S}$ is the subset of skew-Hermetian matrices known as signed permutation matrices. We extend our mean-field model to include this representation by setting $\begin{bmatrix} 0 \\ k \end{bmatrix} = \begin{bmatrix} 1 \\ k \end{bmatrix}$ at the stop time τ.

Define the the player's initial utility function as a hyperbolic absolute risk aversion (HARA), and so must adhere to, for utility μ,

$$\frac{\mu''(\langle s_i, s_{-i}\rangle)}{\mu'(\langle s_i, s_{-i})}.$$

3 Research Plan

- Step 1: Formulate the relevant graph models using appropriate abstraction.
- Step 2: Establish the existence/uniqueness of differential tensor/flow .
- Step 3: Implement a python/ruby module/scaffolding to model to aid in prototyping conditions to define (possible) deterministic systems.
- Step 4: Based on the analytical results, implement various systems' tensorflow.

4 References

[1] ”Sphere of Influence Graphs in General Metric Spaces”, T.S. Michael, T. Quint
[2] ”The Supermarket Game”, Jiaming Xu, Bruce Hajek
[3] ”Modeling Population Dynamics in Changing Environments”, Michelle Shen
[4] ”Self-coexistence among interference-aware IEEE 802.22 networks with enhanced air-interface”, S. Sengupta, S. Brahma, M. Chatterjee, N. Sai Shankar

5 Literature

In addition to related work, the sourced literature must include historical papers to give perspective and formalization, and determine the foundation of the proposed theory.

[1] M. E. Bratman. Intention, Plans, and Practical Reason. CSLI Publications, Stanford University, 1987.
[2] I. Gilboa and E. Zemel. Nash and correlated equilibria: Some complexity considerations. Games and Economic Behavior, 1:80-93, 1989.
[3] E. Kalai. Games, computers, and O.R. In ACM/SIAM Symposium on Discrete Algorithms, 1995.
[4] Noam Nisan and Amir Ronen. Algorithmic mechanism design. In Proc. 31st ACM Symp. on Theory of Computing, pages 129-140, 1999.
[5] C. H. Papadimitriou. On the complexity of the parity argument and other inefficient proofs of existence. Journal of Computer and System Sciences, 48(3):498-532, 1994.
[6] J. von Neumann and O. Morgenstern. Theory of Games and Economic Behavior, Second Edition. Princeton University Press, second edition, 1947.