# EE387 : BASIC SIGNAL REPRESENTATION AND CONVOLUTION

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# PART 1: Basic Signal Representation in MATLAB

1. Write a MATLAB program and necessary functions to generate the following signal

$$y(t) = r(t+3) - 2r(t+1) + 3r(t) - u(t-3)$$

Then plot it and verify analytically that the obtained figure is correct.

**Code for ramp function**

```
function y = ramp(t,m,ad)
% t: length of time
% m: slope of the ramp function
% ad: advance (positive), delay (negative) factor
y=[];
count=1;
p=-(ad/m);
for i=t
    if(m>0)
        if i< p
            y(count)=0;
        else
            y(count)=m*i + ad;
        end


    else
        if i< p
            y(count)=m*i + ad;
        else
            y(count)=0;
        end
    end
    count=count+1;
end
```

**Code for unit-step function**

```
function y = ustep(t,ad)
% ad: advance (positive), delay (negative) factor
% t: length of time
y=[];
count=1;
for i =t
    if i< (-1*ad)
        y(count)=0;
    else
        y(count)=1;
    end
    count=count+1;
end
```

Execute following programme,

```
clear all;
Ts=0.01;
t= -5:Ts:5;

y1 = ramp(t,1,3);
y2 = ramp(t,1,1);
y3 = ramp(t,1,0);
y4 = ustep(t,-3);
y = y1-2*y2+3*y3-y4;

plot(t,y,'k');
xlabel( 'time' ) ;
ylabel( 'y(t)' ) ;
```

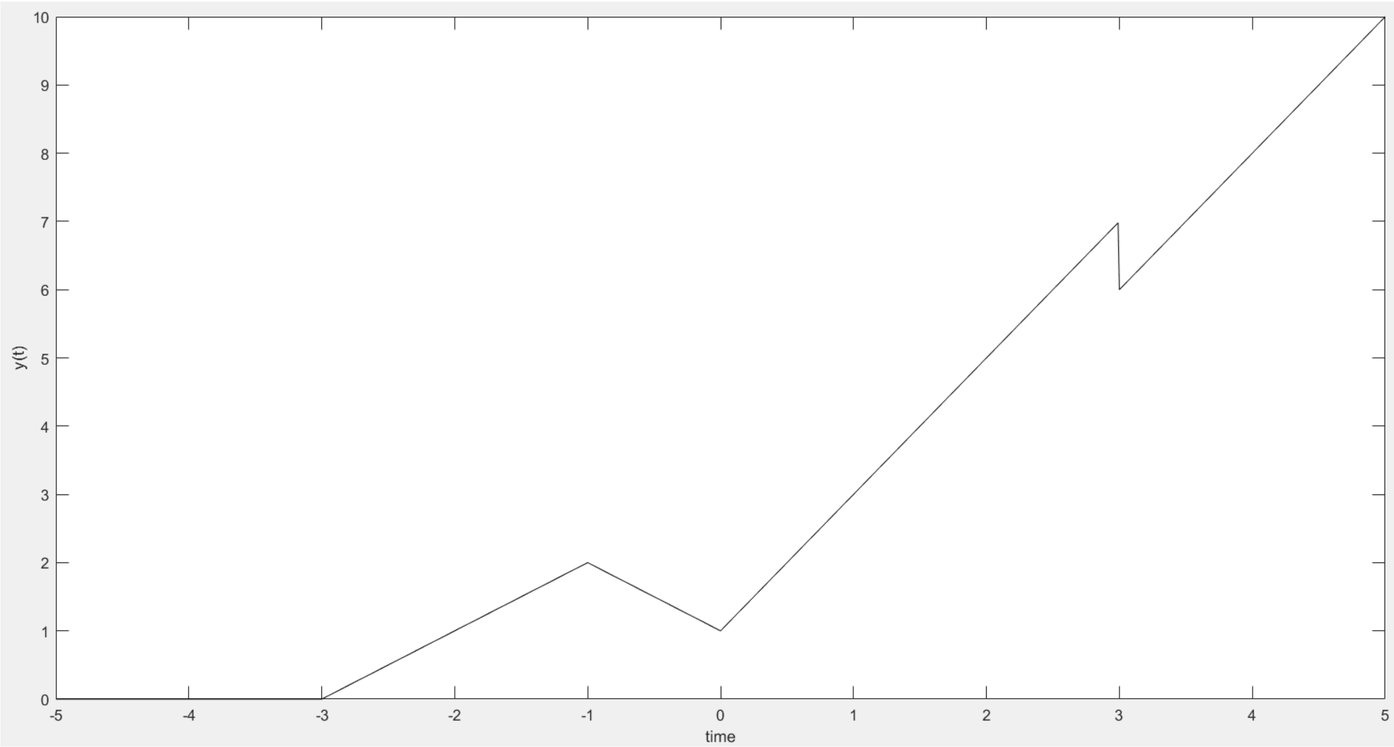


Figure 01: Variation of y(t) with time(t)

Analytical Explanation

The analysis of the graph can be divided into 5 regions in on the time axis.

**Region 1 (t < -3),**

In this region since there is no function involved, y = 0

Therefore, y(t) = 0

**Region 2 (-3 <= t < -1),**

The function y = t + 3 , involved in this region.

Therefore, y(t) = t + 3

As displayed on the graph, from -3 to -1 it has behaved according to y(t) = t + 3

**Region 3 (-1 <= t < 0),**

Two functions are involved in this region. They are y = (t+3) , y = (t+1)

Therefore, y(t) = (t + 3) – 2(t + 1) = (-t + 1)
As displayed on the graph, from -1 to 0 it has behaved according to y(t) = -t + 1 with negative gradient.

**Region 3 (0 <= t < 3),**

Three functions are involved in this region. They are y = (t+3) , y = (t+1), y = t

Therefore, y(t) = (t + 3) – 2(t + 1) + 3(t) = (2t + 1)
As displayed on the graph, from 0 to 3 it has behaved according to y(t) = 2t + 1 with positive increased gradient than in region 2.

**Region 4 (3 <= t),**

Four functions are involved in this region. They are y = (t+3) , y = (t+1), y = t , y = -1

(from the unit step)

Therefore, y(t) = (t + 3) – 2(t + 1) + 3(t) - 1 = (2t)
As displayed on the graph, above 3 it has behaved according to y(t) = 2t with the same gradient in region 1. Since the y intercept has decreased to 0, the graph has came down at t = 3.

2. For the damped sinusoidal signal $x(t) = 3e^{-t} \cos(4\pi t)$ write a MATLAB program to generate x(t) and its envelope, then plot

**Code**

```
clear all;
Ts=0.01;
t= -5:Ts:5;
y=[];
count=1;

for i=t
    y(count)=3*exp(-i)*cos(4*pi*i);
    count=count+1;
end

[yupper,ylower] = envelope(y,30,'peak')
figure(1)
plot(t,y); hold on;
plot(t,yupper);
plot(t,ylower);
grid;
xlabel('t'); ylabel('x(t)')
```

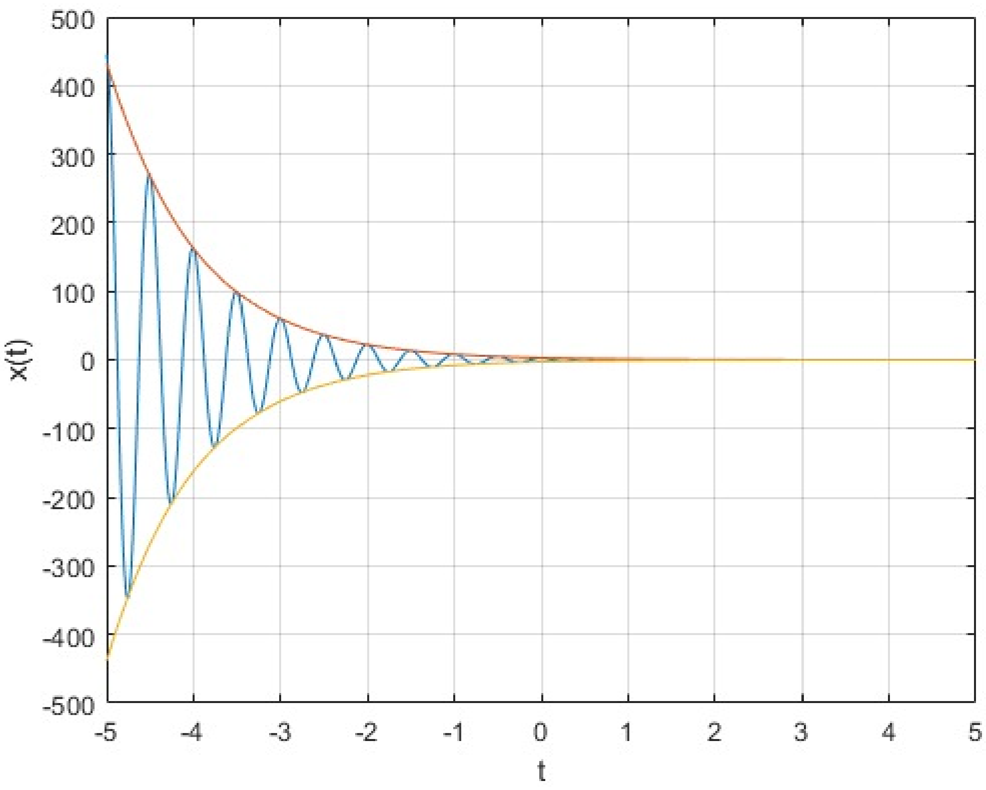


Figure 02: Variation of y(t) with time(t)

# PART 2: Time-Domain Convolution

## Elementary signal operations

### Code for rectangular pulse

```
function x = rect(t)
x=[];
count=1;
for i = t
    if (i> -0.5 & i<0.5)
        x(count)=1;
    else
        x(count)=0;
    end
    count=count+1;
end
```

Execute following programme,

```
clear all;
f_s=100; % sampling frequency
T_s=1/f_s;
t =[-5:T_s:5];

x1 = rect(t);
x2 = rect(t-1);
x3 = rect(t/2);
x4 = rect(t)+(1/2)*rect(t-1);
x5 = rect(-t)+(1/2)*rect(-t-1);
x6 = rect(1-t)+(1/2)*rect(-t);

subplot(3,2,1);
plot(t,x1);
axis( [-2 2 -1 2]);
xlabel( 'time (sec)' )
ylabel('x_1(t)')
title ('Plot 1: A rectangular pulse');

subplot(3,2,2);
plot(t,x2);
axis( [-2 2 -1 2]);
xlabel( 'time (sec)' )
ylabel('x_2(t)')
title ('Plot 2: A time shifted pulse');

subplot(3,2,3);
plot(t,x3);
axis( [-2 2 -1 2]);
xlabel( 'time (sec)' )
ylabel('x_3(t)')
title ('Plot 3: A time scaled pulse');

subplot(3,2,4);
plot(t,x4);
axis( [-2 2 -1 2]);
xlabel( 'time (sec)' )
ylabel('x_4(t)')
title ('Plot 4: x4 = rect(t)+(1/2)*rect(t-1)');

subplot(3,2,5);
plot(t,x5);
axis( [-2 2 -1 2]);
xlabel( 'time (sec)' )
```

```
ylabel('x_5(t)')
title ('Plot 5: x5 = rect(-t)+(1/2)*rect(-t-1)');

subplot(3,2,6);
plot(t,x6);
axis( [-2 2 -1 2]);
xlabel( 'time (sec)' )
ylabel('x_6(t)')
title ('Plot 5: x6 = rect(1-t)+(1/2)*rect(-t)');
```

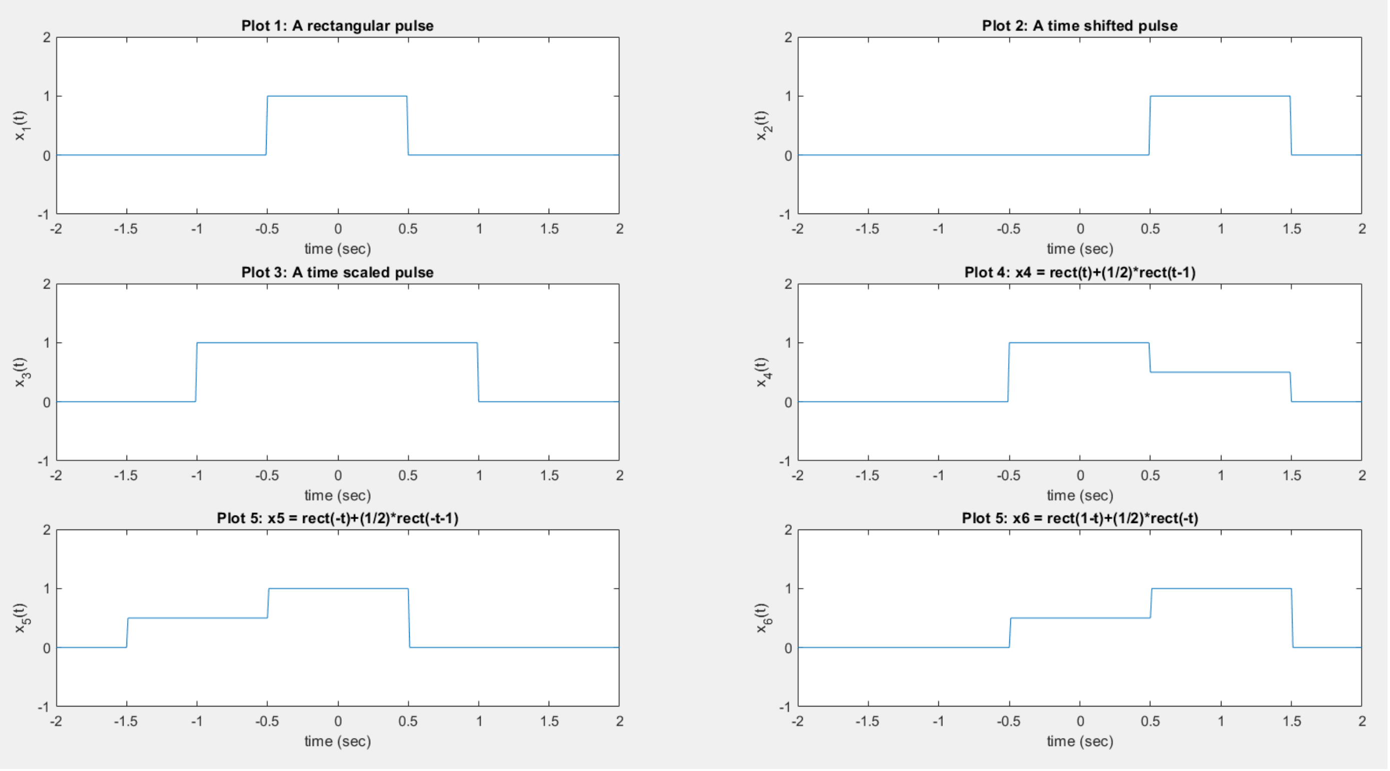


Figure 3: Variations of the function with time

# Convolution

### Code for convolution

```
clear all;
f_s=100; % sampling frequency
T_s=1/f_s;
t =[-5:T_s:5];

x1 = rect(t);
x2 = rect(t-1);

y = conv(x1,x1); %convolution of x1 and x2
t_y = -10:T_s:10; % seperate time axis for signal y
y1 = T_s*conv(x1,x1);

plot(t_y, y1);
axis( [-2 2 -1 2] ) ;
xlabel( 'time (sec)');
ylabel('y_1(t)');
title('Figure : y_1(t) = x_1(t)*x_2(t)');
```

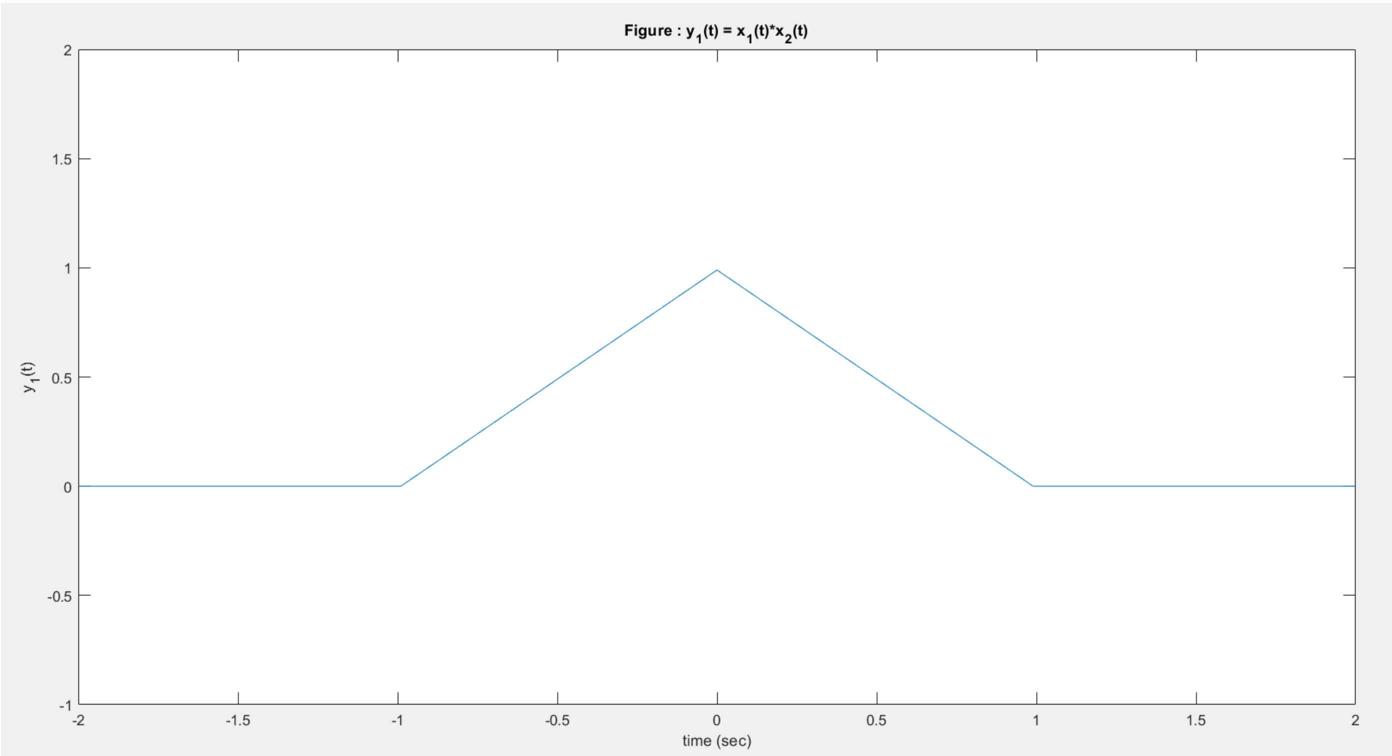


Figure 04: Variation of y_1(t) with time(t)

# EXERCISE

1. 1) x(n) = { 1,2,4 }, h(n) = {1,1,1,1,1}

**Code**

```
clear all;
x=[1,2,3];
h=[1,1,1,1,1];
y=conv(x,h);

n1=1:length(x);
n2=1:length(h);
n3=1:length(y);

subplot(2,2,1);
stem(n1,x)
axis( [0 8 0 7]);
xlabel( 'n');
ylabel('x(n)');

subplot(2,2,2);
stem(n2,h)
axis( [0 8 0 7]);
xlabel( 'n');
ylabel('h(n)');

subplot(2,2,3);
stem(n3,y)
axis( [0 8 0 7]);
xlabel( 'n');
ylabel('y(n)');
```

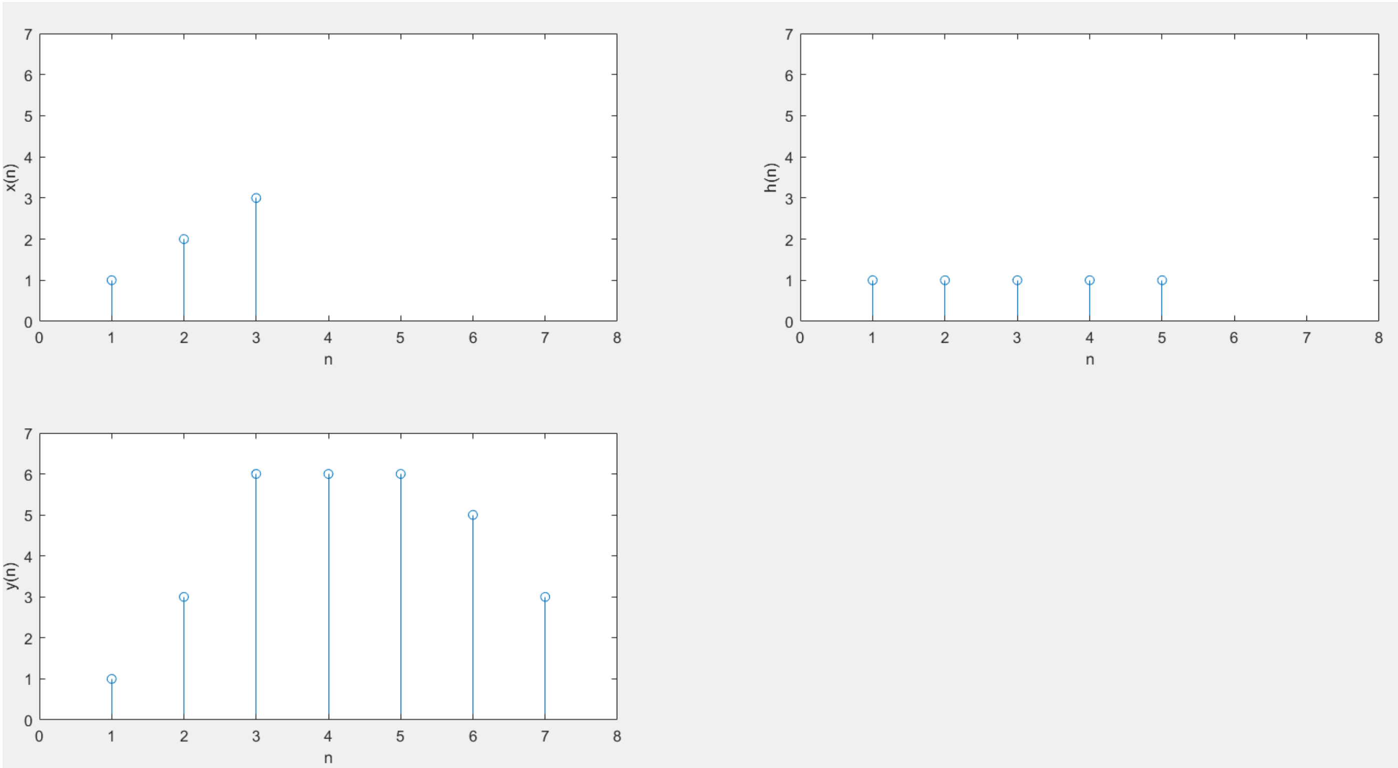


Figure 05: Variation of x(n), h(n) and y(n) with n

2) x(n) = { 1,2,3,4,5 }, h(n) = {1}

**Code**

```
clear all;
x=[1,2,3,4,5];
h=[1];
y=conv(x,h);

n1=1:length(x);
n2=1:length(h);
n3=1:length(y);

subplot(2,2,1);
stem(n1,x)
axis( [0 6 0 6]);
xlabel( 'n');
ylabel('x(n)');

subplot(2,2,2);
stem(n2,h)
axis( [0 6 0 6]);
xlabel( 'n');
ylabel('h(n)');

subplot(2,2,3);
stem(n3,y)
axis( [0 6 0 6]);
xlabel( 'n');
ylabel('y(n)');
```

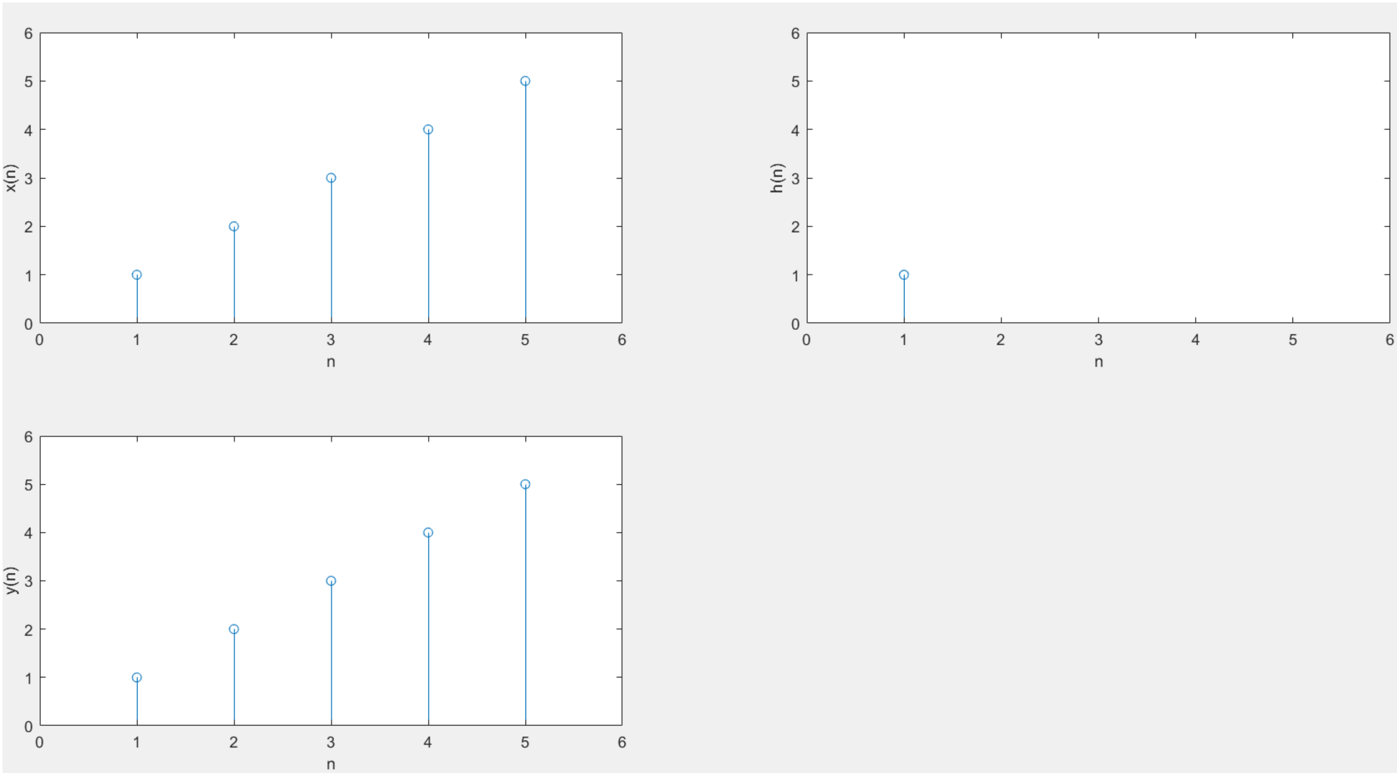


Figure 06: Variation of x(n), h(n) and y(n) with n

3) x(n) = h(n) ={ 1,2,0,2,1}

**Code**

```
clear all;
x=[1,2,0,2,1];
h=[1,2,0,2,1];
y=conv(x,h);

n1=1:length(x);
n2=1:length(h);
n3=1:length(y);

subplot(2,2,1);
stem(n1,x)
axis( [0 10 0 10]);
xlabel( 'n');
ylabel('x(n)');

subplot(2,2,2);
stem(n2,h)
axis( [0 10 0 10])
xlabel( 'n');
ylabel('h(n)');


subplot(2,2,3);
stem(n3,y)
axis([0 10 0 10])
xlabel( 'n');
ylabel('y(n)');
```

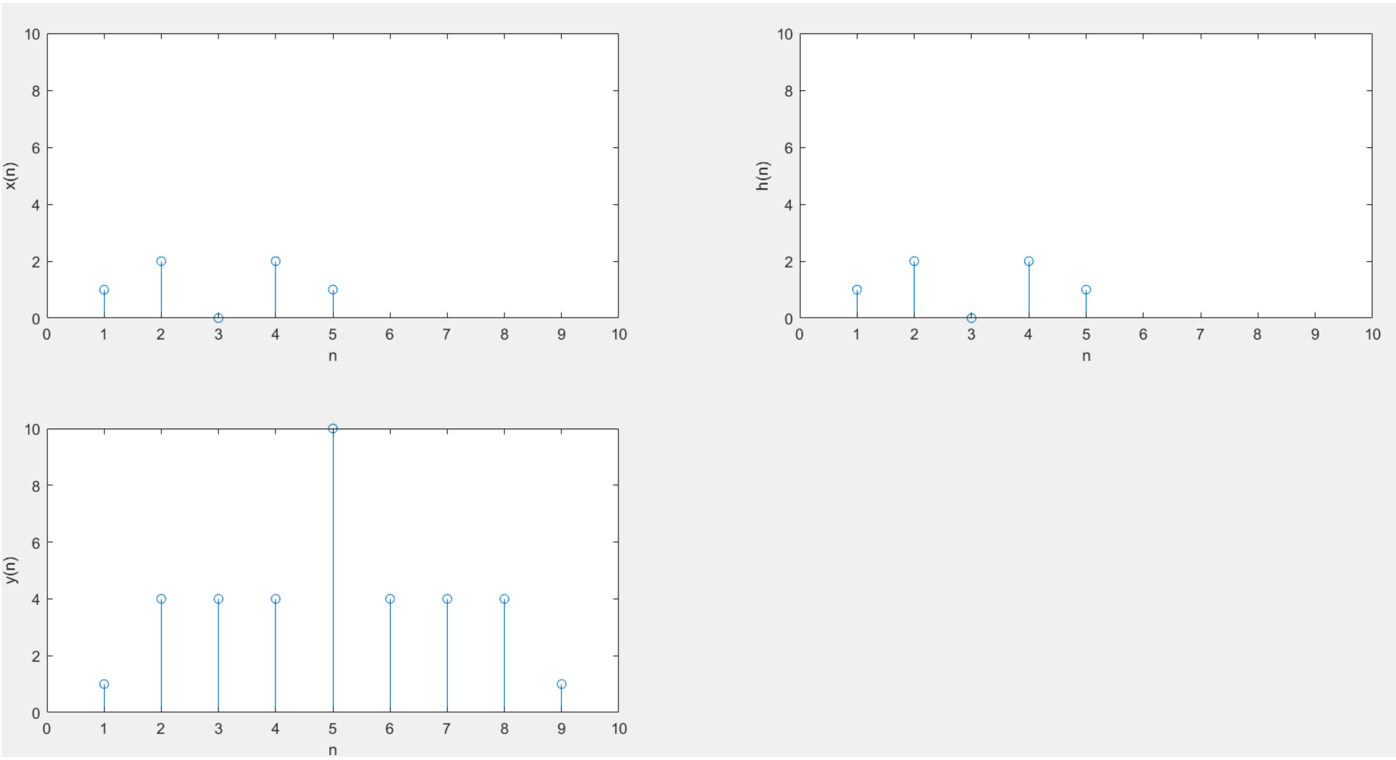


Figure 06: Variation of x(n), h(n) and y(n) with n

2.

Method 1 (Using MATLAB)

```
function r=h(n)
    r=[];
    count=1;
    for i=n
        if i>=0 && i<4
            r(count)=0.5^i;
        else
            r(count)=0;
        end
    count = count+1;
    end
```

By executing following program using above function,

```
clear all;
n=0:1:4;
h=h(n);
y=[1,2,2.5,3,3,3,2,1];
[x,r] = deconv(y,h);
disp(x);
```

Therefore, x(n) = 1.0000 , 1.5000 , 1.5000, 1.7500, 1.5625

Method 2 (Manual Calculation)

h(n) = 1, 1/2 , 1/4, 1/8

y(n) = 1, 2, 2.5, 3, 3, 3, 2, 1,0

Consider, x(n) = a, b, c, d, e

| | 1 | 1/2 | 1/4 | 1/8 |
|---|---|---|---|---|
| a | a | a/2 | a/4 | a/8 |
| b | b | b/2 | b/4 | b/8 |
| c | c | c/2 | c/4 | c/8 |
| d | d | d/2 | d/4 | d/8 |
| e | e | e/2 | e/4 | e/8 |

Equating diagonal sum with y values,

a = y(0)
a = 1

a/2 + b = 2
1/2 + b = 2
b = 1.5

a/4 + b/2 + c = 2.5
1/2 + 1.5/2 + c = 2.5
c = 1.5

a/8 + b/4 + c/2 + d = 3
1/8 + 1.5/4 + 1.5/2 + d = 3
d = 1.75

b/8 + c/4 + d/2 + e = 3
1.5/8 + 1.5/4 + 1.75/2 + e = 3
e = 1.5625

Therefore, x(n) ={ 1, 1.5, 1.5, 1.75, 1.5625 }

```
clear all;
n=0:1:4;
h=h(n);
y=[1,2,2.5,3,3,3,2,1,0];
[x,r] = deconv(y,h);

n1=1:length(x);
n2=1:length(h);
n3=1:length(y);

subplot(2,2,1);
stem(n1,x)
axis( [0 9 0 4]);
xlabel( 'n');
ylabel('x(n)');

subplot(2,2,2);
stem(n2,h)
axis( [0 9 0 4]);
xlabel( 'n');
ylabel('h(n)');

subplot(2,2,3);
stem(n3,y)
axis( [0 9 0 4]);
xlabel( 'n');
ylabel('y(n)');
```

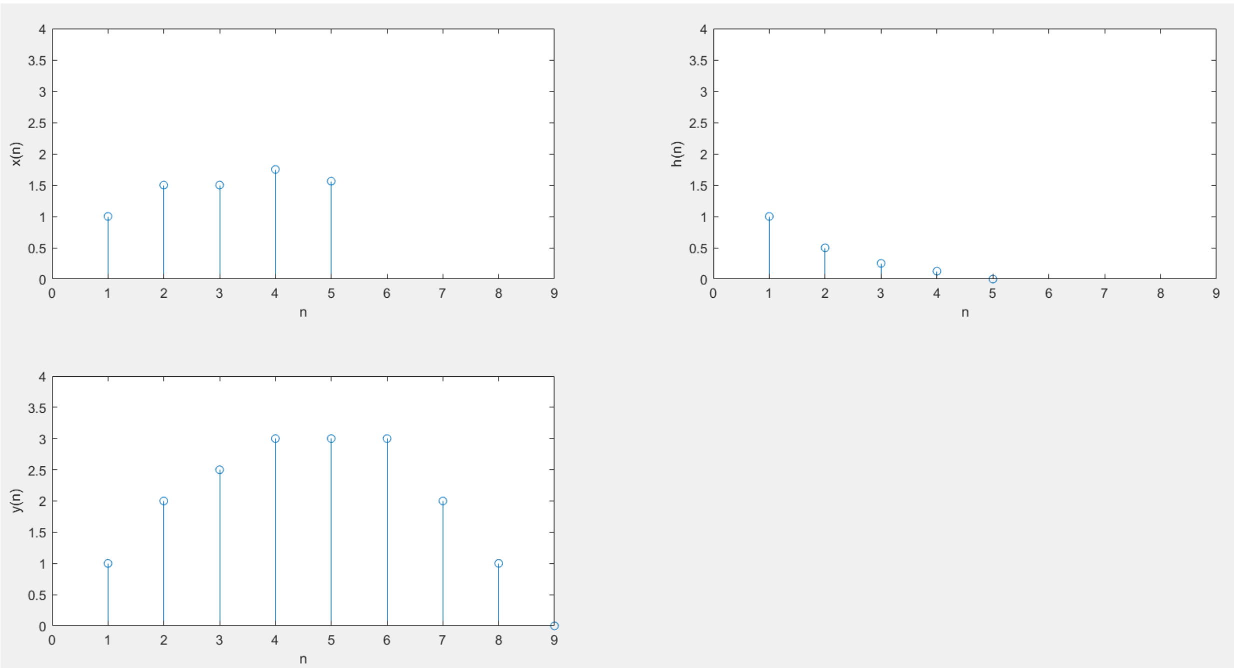


Figure 07: Variation of x(n), h(n) and y(n) with n