Graph traversal and Path-finding Algorithms used in Video Games

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What is a graph?

A data structure to represent link between objects. A graph is defined by a set of nodes V and a set of edges E.

We can summarize this definition by the following formula:

$$G = (E, V)$$

What's the difference between a graph and a tree?

A graph can contain cycles (a node can be visited twice).

Different type of graphs

▶ Acyclic Graph A graph that has no cycle.
▶ Cyclic Graph A graph that has at least one cycle.
▶ Directed Graph A graph in which edge has direction. That is the nodes are ordered pairs in the definition of every edge.
▶ Undirected Graph A graph in which edge are not directed. Meaning, the edges are defined by an unordered pair of nodes.
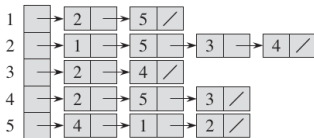▶ Directed Acyclic Graph A graph that is both directed and acyclic.

Different way to represent a graph

There are 2 ways to represent a graph:

▶ adjacency list For each node, provide a list of other nodes that are adjacent to it.
▶ adjacency matrix A matrix construct by aligning the nodes in the row and the columns and putting a value if the nodes are linked by an edge.
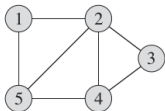
Figure 1: (a) Undirected graph with 5 vertices and 7 edges (b) Adjacency-list representation (c) Adjacency-matrix representation

Graph traversal algorithms

DFS (Depth-First Search)

A graph traversal algorithm in which one start with a root node (arbitrarily chosen) then explore as far as possible along each branch before backtracking.

BFS (Breadth-First Search)

A graph traversal algorithm in which one explore every possible node in the current depth level before going to the next.

BFS (Breadth-First Search)

```
BFS(G, s)
 1   for each vertex u ∈ G.V − {s}
 2       u.color = WHITE
 3       u.d = ∞
 4       u.π = NIL
 5   s.color = GRAY
 6   s.d = 0
 7   s.π = NIL
 8   Q = ∅
 9   ENQUEUE(Q, s)
10   while Q ≠ ∅
11       u = DEQUEUE(Q)
12       for each v ∈ G.Adj[u]
13           if v.color == WHITE
14               v.color = GRAY
15               v.d = u.d + 1
16               v.π = u
17               ENQUEUE(Q, v)
18       u.color = BLACK
```

Figure 2: Breadth-first search pseudo-code

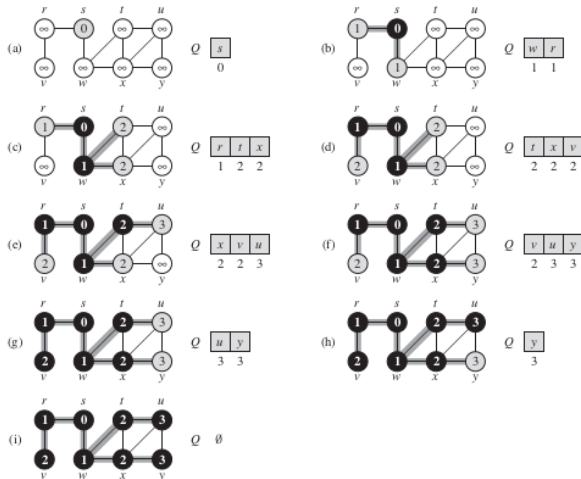# BFS (Breadth-First Search)

Figure 3: Operation of BFS on an undirected graph

Depth-First Search

```
DFS(G)
1   for each vertex u ∈ G.V
2       u.color = WHITE
3       u.π = NIL
4   time = 0
5   for each vertex u ∈ G.V
6       if u.color == WHITE
7           DFS-VISIT(G, u)

DFS-VISIT(G, u)
 1   time = time + 1              // white vertex u has just been discovered
 2   u.d = time
 3   u.color = GRAY
 4   for each v ∈ G.Adj[u]        // explore edge (u, v)
 5       if v.color == WHITE
 6           v.π = u
 7           DFS-VISIT(G, v)
 8   u.color = BLACK              // blacken u; it is finished
 9   time = time + 1
10   u.f = time
```

Figure 4: Depth-First Search Pseudocode
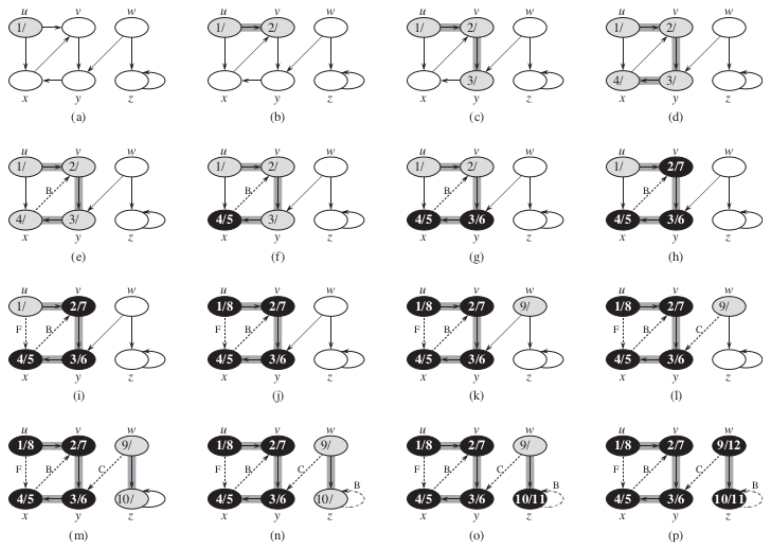
Depth-First Search

Figure 5: Depth-First Search progress on a directed graph

Path finding algorithms

A* algorithm

A* (pronounced "A-Star") is a graph traversal and path-finding algorithm. Given a source and a goal node, the algorithm find the shortest-path (with respect to given weights) from source to goal.

Dijkstra algorithm

Dijkstra algorithm solves the single-source shortest-paths problem on a weighted directed graph for the case in which all weights are non-negative.
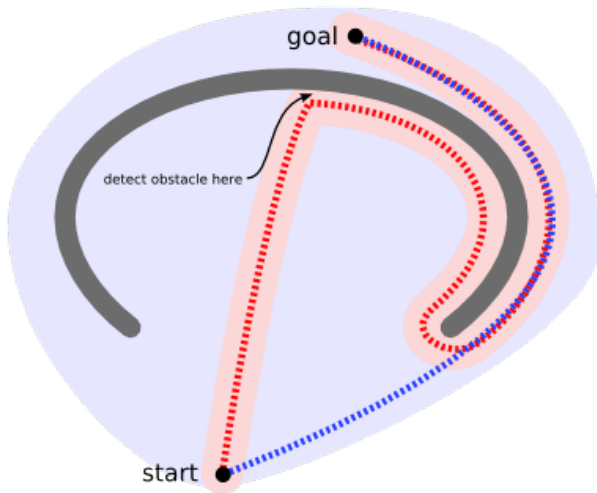
Path-finding

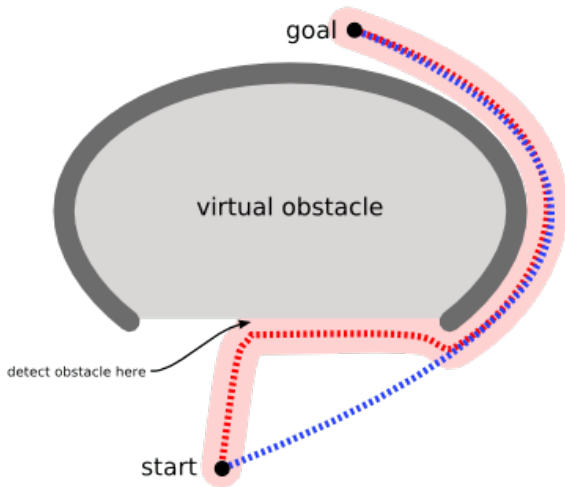Figure 6: Example path-finding situation

Path-finding

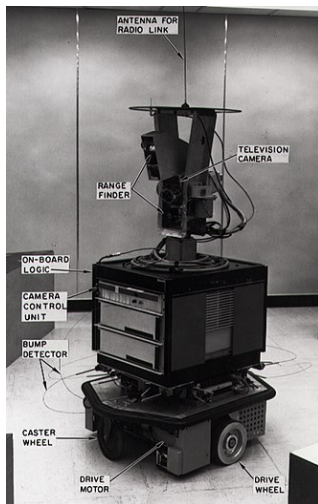Figure 7: Example path-finding situation

Figure 8: A* was invented by researchers working on Shakey the Robot's path planning.

References

▶ https://en.wikipedia.org/wiki/A*_search_algorithm
▶
  http://theory.stanford.edu/~amitp/GameProgramming/AStarCompa