Graph algorithms and Competitive programming

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[AlgoMada]

A brief history of AlgoMada

2021	2022	2023	PRESENT
Association creation	4 Contests organized, Newbie, Rookie and Pro	Clean Code Hackathon +Webinar, Bot Programming Contest, Participation to Indabax	AlgoGames

About me

- Telecommunications, ESPA Alumni
- Computer Science, University of Reunion Island Alumni
- FaceDev Admin since 2012
- Founder member of AlgoMada
- Clojure dev
- Computer Science Enthusiast
- Current interests: Cryptocurrency, Clojure programming language
- Side project: BetaX Community github.com/puchka

Motivation: Why I do this?

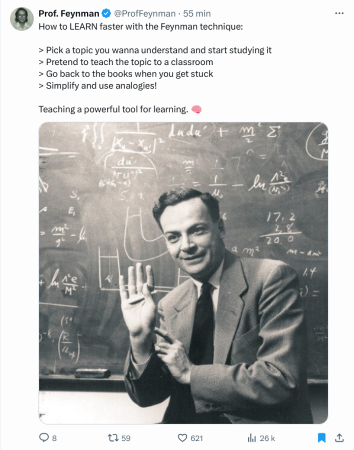


Figure 1: Feynman technique for studying

Definition of Computer Science

"We are about to study the idea of a computational process. Computational processes are abstract beings that inhabit computers. As they evolve, processes manipulate other abstract things called data. The evolution of a process is directed by a pattern of rules called a program. People create programs to direct processes. In effect, we conjure the spirits of the computer with our spells." Structure and Interpretation of Computer Programs, Harold Abelson and Gerald J. Sussman

Definition of Competitive Programming

'Competitive Programming' in summary, is this: "Given well-known Computer Science (CS) problems, solve them as quickly as possible!".

Tips to be competitive

- Type Code Faster
- Quickly identify problem type
- Do Algorithm Analysis
- Master Programming Languages
- Master the Art of Testing Code
- Practice and More Practice

Data Structures

Data structure is ‘a way to store and organize data’ in order to support efficient insertions, queries, searches, updates, and deletions.

What is a graph?

A data structure to represent link between objects. A graph is defined by a set of nodes V and a set of edges E.

We can summarize this definition by the following formula:

$$G = (E, V)$$

Example: https://www.redblobgames.com/pathfinding/grids/graphs.html#properties

What's the difference between a graph and a tree?

A graph can contain cycles (a node can be visited twice).

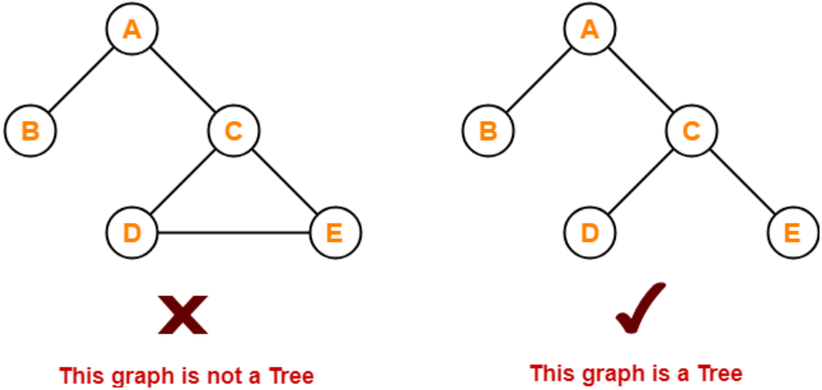


Figure 2: Tree vs Graph

Different type of graphs

- Acyclic Graph

A graph that has no cycle.

- Cyclic Graph

A graph that has at least one cycle.

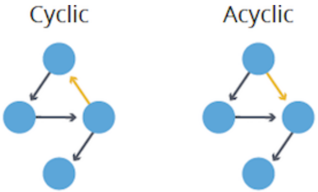


Figure 3: Acyclic vs Cyclic graph

Different type of graphs

- Directed Graph

A graph in which edge has direction. That is the nodes are ordered pairs in the definition of every edge.

- Undirected Graph

A graph in which edge are not directed. Meaning, the edges are defined by an unordered pair of nodes.

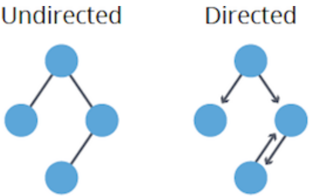


Figure 4: Undirected vs Directed Graph

Different type of graphs

- Directed Acyclic Graph

A graph that is both directed and acyclic.

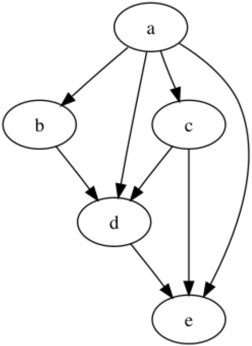


Figure 5: Directed Acyclic Graph

Different type of graphs

- Connected graph

Every pair of nodes has a path linking them. Put in another way, there are no inaccessible node.

- Disconnected graph

A graph in which there is at least one inaccessible node.

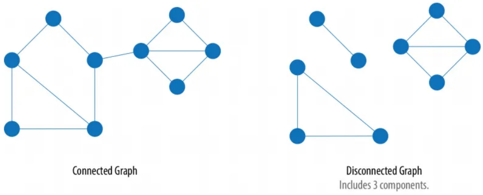


Figure 6: Disconnected vs Connected Graph

Different type of graphs

- A multigraph

A graph that can have multiple edges between the same nodes.

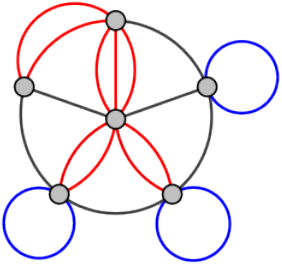


Figure 7: A multigraph with multiple edges (red) and several loops (blue). By 0x24a537r9 - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=12247695

Different way to represent a graph

There are 2 ways to represent a graph:

- adjacency list For each node, provide a list of other nodes that are adjacent to it.
- adjacency matrix A matrix construct by aligning the nodes in the row and the columns and putting a value if the nodes are linked by an edge.

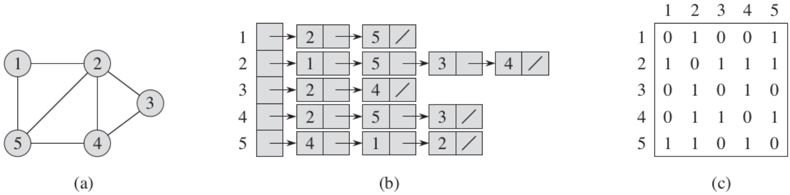


Figure 8: (a) Undirected graph with 5 vertices and 7 edges (b) Adjacency-list representation (c) Adjacency-matrix representation

Adjacency Matrix

In contest problems involving graph, usually `V` is known, thus we can build a 'connectivity table' by setting up a 2-D, `O(V^2)` static array: `int AdjMat[V][V]`.

For an unweighted graph, we set `AdjMat[i][j] = 1` if there is an edge between vertex `i-j` and set `0` otherwise.

For a weighted graph, we set `AdjMat[i][j] = weight(i, j)` if there is an edge between vertex `i-j` with `weight(i, j)` and set `0` otherwise.

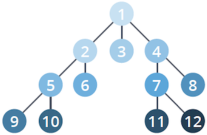
Graph traversal algorithms

BFS (Breadth-First Search)

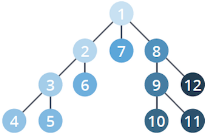
A graph traversal algorithm in which one explore every possible node in the current depth level before going to the next. Usually used to find shortest path distance from the start to a given vertex and the associated predecessor subgraph.

DFS (Depth-First Search)

A graph traversal algorithm in which one start with a root node (arbitrarily chosen) then explore as far as possible along each branch before backtracking. Usually used as a subroutine in another algorithm.



Breadth-first search



Depth-first search

BFS (Breadth-First Search)

```
BFS(G, s)
for each vertex u ∈ G.V − {s}
    u.color = WHITE
    u.d = ∞
    u.π = NIL
s.color = GRAY
s.d = 0
s.π = NIL
Q = ∅
ENQUEUE(Q, s)
while Q ≠ ∅
    u = DEQUEUE(Q)
    for each v ∈ G.Adj[u]
        if v.color == WHITE
            v.color = GRAY
            v.d = u.d + 1
            v.π = u
            ENQUEUE(Q, v)
    u.color = BLACK
```

Figure 10: Breadth-first search pseudo-code

BFS (Breadth-First Search)

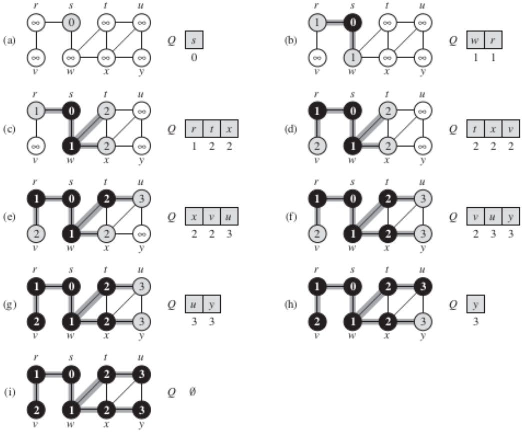


Figure 11: Operation of BFS on an undirected graph

Predecessor subgraph

The procedure BFS builds a breadth-first tree as it searches the graph, as Fig-11 illustrates. The tree corresponds to the π attributes.

More formally, for a graph $G = (V, E)$ with source s, we define the **predecessor subgraph** of G as $G_\pi = (V_\pi, E_\pi)$

$$V_\pi = v \in V : v.\pi \neq NIL$$

and

$$E_\pi = (v, \pi, v) : v \in V_\pi - s$$

Breadth-first trees

The predecessor subgraph G_π is a **breadth-first tree** if V_π consists of the vertices reachable from s and, for all $v \in V$, the subgraph G_π contains a unique simple path from s to v that is also a shortest path from s to v in G.

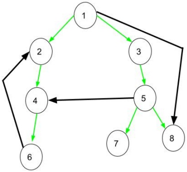
Depth-First Search

```
DFS(G)

for each vertex u ∈ G.V
    u.color = WHITE
    u.π = NIL
time = 0
for each vertex u ∈ G.V
    if u.color == WHITE
        DFS-VISIT(G, u)

DFS-VISIT(G, u)

time = time + 1               // white vertex u has just been discovered
u.d = time
u.color = GRAY
for each v ∈ G.Adj[u]         // explore edge (u, v)
    if v.color == WHITE
        v.π = u
        DFS-VISIT(G, v)
u.color = BLACK               // blacken u; it is finished
time = time + 1
u.f = time
```

Figure 12: Depth-First Search Pseudocode

Tree, Forward, Back and Cross Edges in Depth-First Search

- Tree Edge: It is an edge which is present in the tree obtained after applying DFS on the graph.
- Forward Edge: It is an edge (u, v) such that v is a descendant but not part of the DFS tree.
- Back edge: It is an edge (u, v) such that v is the ancestor of node u but is not part of the DFS tree.
- Cross Edge: It is an edge that connects two nodes such that they do not have any ancestor and a descendant relationship between them.



Depth-First Search

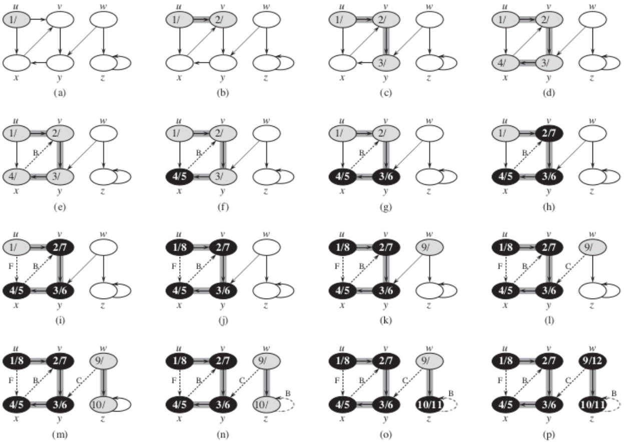


Figure 14: Depth-First Search progress on a directed graph

Path-finding

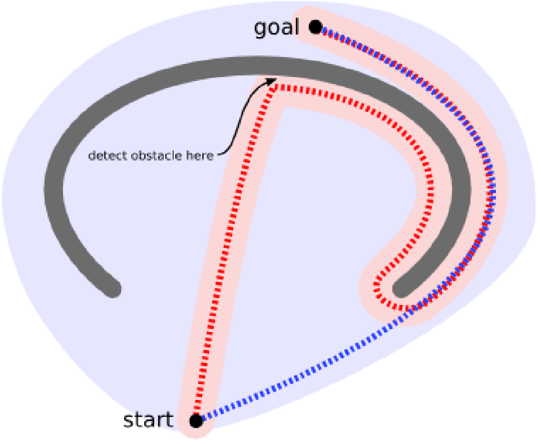


Figure 15: Example path-finding situation

Path-finding

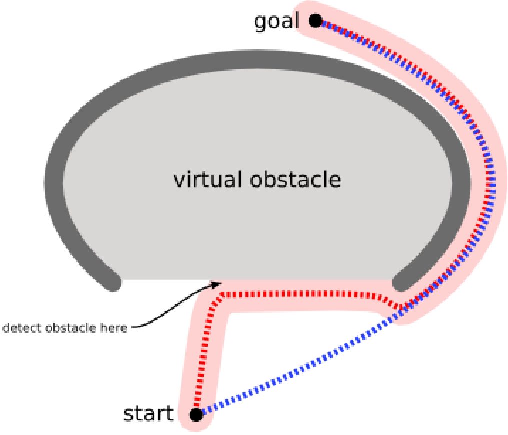


Figure 16: Example path-finding situation

Path finding algorithms

A* algorithm

A* (pronounced “A-Star”) is a graph traversal and path-finding algorithm. Given a source and a goal node, the algorithm find the shortest-path (with respect to given weights) from source to goal.

Dijkstra algorithm

Dijkstra algorithm solves the single-source shortest-paths problem on a weighted directed graph for the case in which all weights are non-negative.

Shortest-path estimate

The algorithms that follow use the technique of relaxation. For each vertex $v \in V$, we maintain an attribute $v.d$, which is an upper bound on the weight of a shortest path from source s to v. We call $v.d$ **shortest-path estimate**.

```
INITIALIZE-SINGLE-SOURCE(G, s)
for each vertex v ∈ G.V
    v.d = ∞
    v.π = NIL
s.d = 0
```

Figure 17: INITIALIZE-SINGLE-SOURCE procedure

After initialization, we have $v.\pi = NIL$ for all $v \in V$, $s.d = 0$, and $v.d = \infty$ for $v \in V - s$.

Relaxation

The process of relaxing an edge (u, v) consists of testing whether we can improve the shortest path to found so far by going through u and, if so, updating $v.d$ and $v.\pi$.

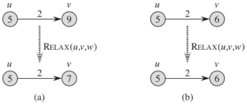


Figure 18: Relaxing an edge (u, v). The shortest-path estimate of each vertex appears within the vertex.

Relaxation

RELAX(u, v, w)

```
1  if v.d > u.d + w(u, v)
2      v.d = u.d + w(u, v)
3      v.π = u
```

Figure 19: relax procedure

A* Algorithm
History

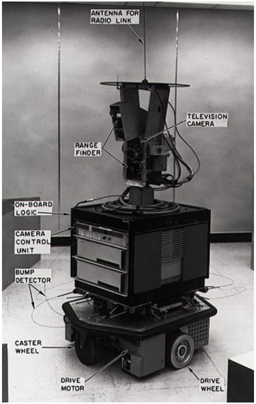


Figure 20: A* was invented by researchers working on Shakey the Robot's path planning.

Application in Video Games

For 2D video games, a tile map can be transformed into a graph. Each cell of the grid will be a node in the graph and the edges are going to be the four directions: east, north, west, south.

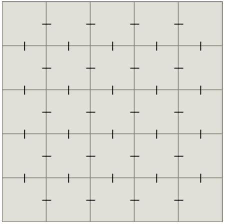


Figure 21: Map as graph

Example: https://www.redblobgames.com/pathfinding/grids/graphs.html#grids

Dijkstra's Algorithm

Dijkstra's Algorithm works by visiting vertices in the graph starting with the object's starting point. It then repeatedly examines the closest not-yet-examined vertex, adding its vertices to the set of vertices to be examined.

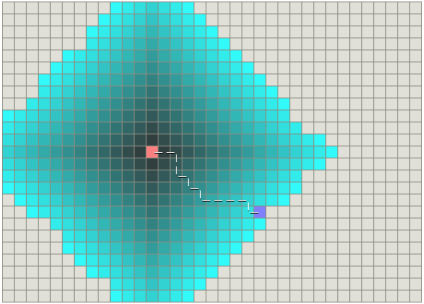


Figure 22: Dijkstra algorithm

Dijkstra's Algorithm

```
DIJKSTRA(G, w, s)
INITIALIZE-SINGLE-SOURCE(G, s)
S = ∅
Q = G.V
while Q ≠ ∅
    u = EXTRACT-MIN(Q)
    S = S ∪ {u}
    for each vertex v ∈ G.Adj[u]
        RELAX(u, v, w)
```

Figure 23: Dijkstra's Algorithm pseudocode

Gready Best-First search

The Greedy Best-First-Search algorithm works in a similar way, except that it has some estimate (called a heuristic) of how far from the goal any vertex is. Instead of selecting the vertex closest to the starting point, it selects the vertex closest to the goal. Greedy Best-First-Search is not guaranteed to find a shortest path.

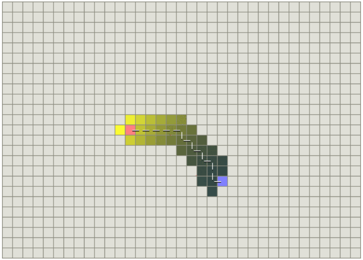


Figure 24: Greedy Best-first search

Dijkstra's Algorithm and Best-First-Search

Let's consider the concave obstacle as described in the previous section. Dijkstra's Algorithm works harder but is guaranteed to find a shortest path:

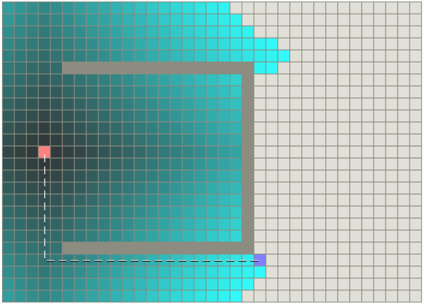


Figure 25: Dijkstra's Algorithm with obstacle

Dijkstra's Algorithm and Best-First-Search

Greedy Best-First-Search on the other hand does less work but its path is clearly not as good:

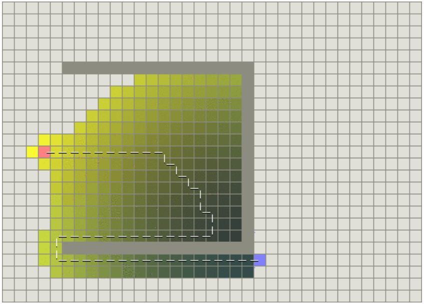


Figure 26: Greedy Best-First Search with trap

The A* Algorithm

A* is like Dijkstra's Algorithm in that it can be used to find a shortest path. A* is like Greedy Best-First-Search in that it can use a heuristic to guide itself. In the simple case, it is as fast as Greedy Best-First-Search:

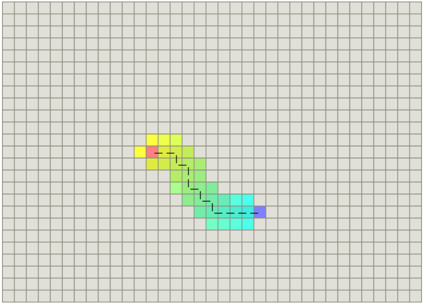
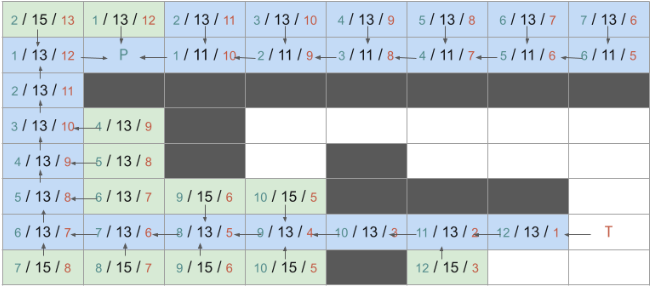
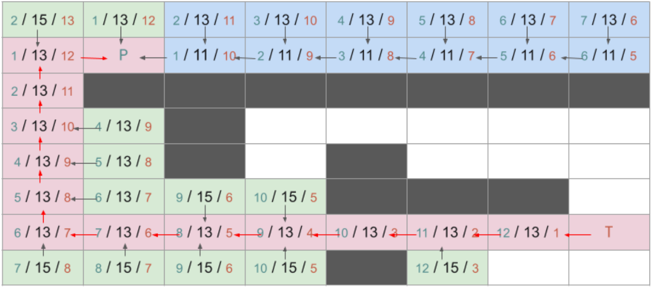


Figure 27: A* algorithm

Path finding using A*



Path finding using A*



References

Web

- https://en.wikipedia.org/wiki/A*_search_algorithm
- https://www.redblobgames.com/pathfinding/a-star/introduction.html
- http://theory.stanford.edu/~amitp/GameProgramming/AStarComparison.html
- https://www.geeksforgeeks.org/tree-back-edge-and-cross-edges-in-dfs-of-graph/
- https://github.com/bradtraversy/traversy-js-challenges/tree/main/08-binary-trees-graphs/11-adjacency-matrix-adjacency-list
- https://github.com/npretto/pathfinding

References
Book

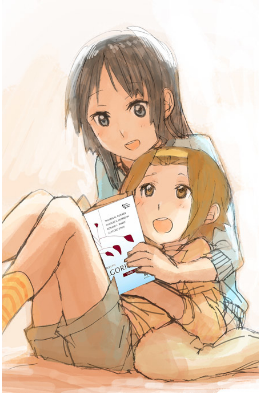


Figure 28: Introduction to Algorithms, 3rd edition by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein