Contents

Problem Set 1

Problem 1. [24 points]

Translate the following sentences from English to predicate logic. The domain that you are working over is X, the set of people. You may use the functions $S(x)$, meaning that "x has been a student of 6.042," $A(x)$, meaning that "x has gotten an 'A' in 6.042," $T(x)$, meaning that "x is a TA of 6.042," and $E(x, y)$, meaning that "x and y are the same person."

(a) [6 pts] There are people who have taken 6.042 and have gotten A's in 6.042

$$\exists x \in X, S(x) \text{ and } A(x).$$

(b) [6 pts] All people who are 6.042 TA's and have taken 6.042 got A's in 6.042

$$\forall x \in X, T(x) \text{ and } S(x) \implies A(x).$$

(c) [6 pts] There are no people who are 6.042 TA's who did not get A's in 6.042.

$$\forall x \in X, T(x) \iff A(x).$$

(d) [6 pts] There are at least three people who are TA's in 6.042 and have not taken 6.042

$$\exists x, y, z \in X, x \neq y \text{ and } y \neq z \text{ and } x \neq z \text{ and } T(x) \text{ and } T(y) \text{ and } T(z) \text{ and } \neg S(x) \text{ and } \neg S(y) \text{ and } \neg S(z)$$

Problem 2. [24 points]

Use a truth table to prove or disprove the following statements:

(a) [12 pts]

$$\neg(P \vee (Q \wedge R)) = (\neg P) \wedge (\neg Q \vee \neg R)$$

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$\neg(P \vee (Q \wedge R))$	$\neg P$	$\neg Q$	$\neg R$	$\neg Q \vee \neg R$	$(\neg P) \wedge (\neg Q \vee \neg R)$
T	T	T	T	T	F	F	F	F	F	F
T	T	F	F	T	F	F	F	T	T	F
T	F	T	F	T	F	F	T	F	T	F
T	F	F	F	T	F	F	T	T	T	F
F	T	T	T	T	F	T	F	F	F	F
F	T	F	F	F	T	T	F	T	T	T
F	F	T	F	F	T	T	T	F	T	T
F	F	F	F	F	T	T	T	T	T	T

According to the truth table above

$$\neg(P \vee (Q \wedge R)) = (\neg P) \vee (\neg Q \vee \neg R)$$

(b) [12 pts]

$$\neg(P \wedge (Q \vee R)) = \neg P \vee (\neg Q \vee \neg R)$$

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$\neg(P \vee (Q \wedge R))$	$\neg P$	$\neg Q$	$\neg R$	$\neg Q \vee \neg R$	$\neg P \vee (\neg Q \vee \neg R)$
T	T	T	T	T	F	F	F	F	F	F
T	T	F	F	T	F	F	F	T	T	T
T	F	T	F	T	F	F	T	F	T	T
T	F	F	F	T	F	F	T	T	T	T
F	T	T	T	T	F	T	F	F	F	T
F	T	F	F	F	T	T	F	T	T	T
F	F	T	F	F	T	T	T	F	T	T
F	F	F	F	F	T	T	T	T	T	T

According to the truth table above

$$\neg(P \vee (Q \wedge R)) \neq \neg P \vee (\neg Q \vee \neg R)$$