From Scramble to Solution

Developing an Optimal Solver for
the Kilominx, a Rubik's Cube Variant

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Abstract

The Rubik's Cube is the most well-known combination puzzle of all time, and has inspired numerous variants of the puzzle. One such puzzle is the Kilominx, a 2x2 dodecahedron-shaped puzzle with 12 faces and 20 cubies (individual cube elements of the puzzle). While the Rubik's Cube has already been extensively researched, the Kilominx has not.

This report details my work in creating an optimal solver for the Kilominx, guaranteeing solutions in as few moves as possible. A separate program I wrote precomputes several pattern databases, each containing the number of moves required to solve different sets of cubies in all possible states. The solver uses an iterative-deepening A* (IDA*) algorithm to find the optimal solution, using the pattern databases as a lower-bound heuristic. In order to efficiently index into the pattern databases, the solver computes a Lehmer code for the puzzle state, which is then used to calculate the state's sequential index.

In theory, the solver can optimally solve any state given enough time, but in practice it can only find solutions up to a depth of 14 within a reasonable timeframe. As solutions require longer sequences of moves to solve, the solver takes exponentially more time to find the optimal solution. With further optimisation, the solver could be used to help tighten the bounds on the Kilominx's God's Number - the maximum number of moves needed to solve the puzzle from any given state.

Declaration

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Introduction

When the Rubik's Cube was globally released in 1980, it became an instant success, selling around 200 million units by the end of 1983 [1]. The Rubik's Cube also quickly became a popular subject of research for computer scientists, in part due to the massive 43 quintillion possible states [2] that the cube can be in. Many different algorithms were designed to try and solve the cube in as few a number of moves as possible, but it wasn't until 1997 that Richard E. Korf published a paper [3] describing a method to solve the Rubik's Cube optimally (the shortest possible number of moves to solve any given cube state) by using large lookup tables called pattern databases[4] as a heuristic function to guide an IDA* search algorithm.

The widespread success of the Rubik's Cube also led to the creation of a number of different variants of the puzzle, each working similarly to the Rubik's Cube but of different shapes and sizes. The Kilominx is one of these variants, part of the larger "minx" family of dodecahedron-shaped puzzles. The Kilominx is a 2x2 dodecahedral puzzle with 12 faces and 20 cubies (the individual cube-shaped elements of the puzzle). Although some research has been done on tightening the bounds of its God's Number, no optimal solver has previously been created for the puzzle.

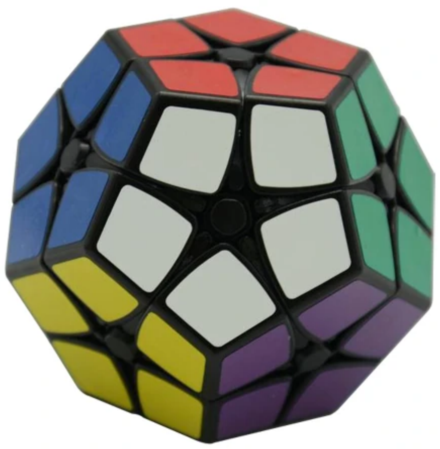

Figure 1: An image of a Kilominx.

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