Extended Finite Element Method (XFEM)

Janna Puderbach

October 26, 2023

Content

Motivation

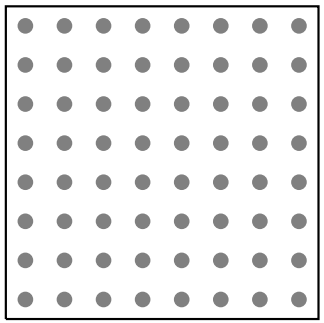

Figure: Microstructure of a composite material

Applications:

- Crack propagation
- Microstructured problems
- Composite materials
- Time-depending domains

Advantages:

- Discontinuities within elements possible
- Avoiding complex mesh generation

Interface Problems

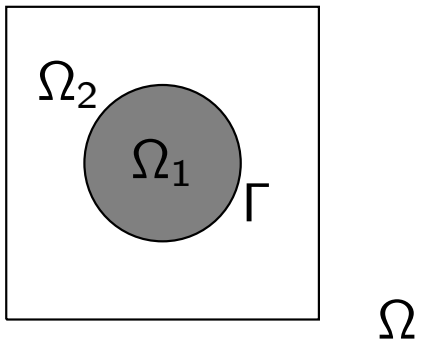

Figure: Composite material

$$-\nabla \cdot (\mu_i \nabla u_i) = f \quad \text{in } \Omega_i \tag{1}$$

$$u_i = g \quad \text{on } \partial\Omega \tag{2}$$

$$[u] = g_s \quad \text{on } \Gamma \tag{3}$$

Unfitted Method

- background mesh
- cut cells

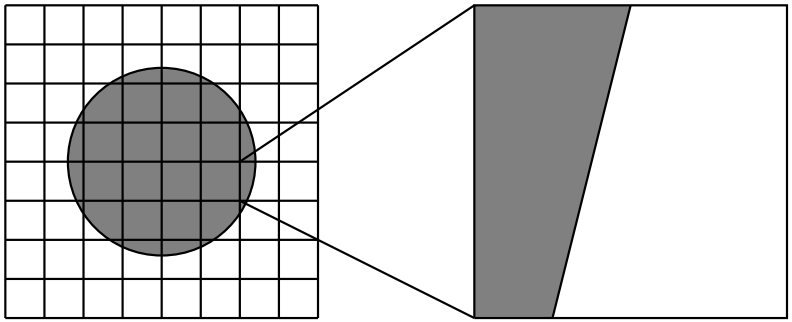

Figure: Unfitted mesh

Level Set Function

- Tracking the interface

$$\phi : \Omega \to \mathbb{R} \quad , \quad \phi(x) \begin{cases} = 0 & , x \in \Gamma \\ < 0 & , x \in \Omega_1 \\ > 0 & , x \in \Omega_2 \end{cases}$$

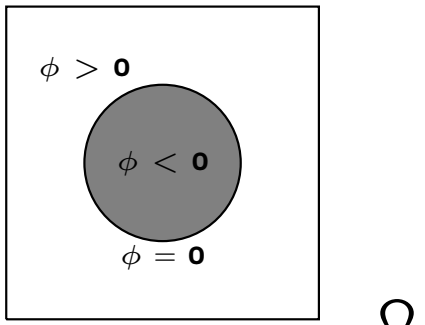

Figure: Level set function

- i.e. signed distance function

Implementation in deal.II

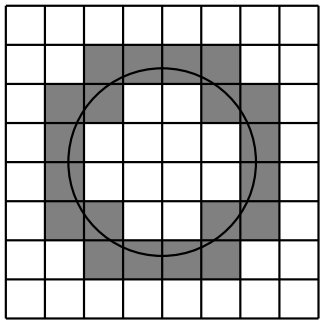

Figure: Cut cells

- loop over all cells
- using the level set function to find the cut cells
- `active_fe_index()`

Enriched Elements

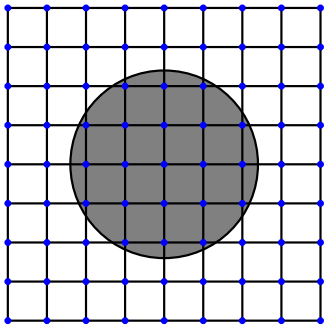

Figure: Standard degrees of freedom

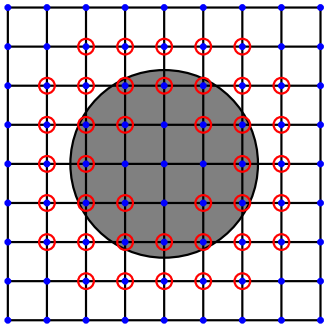

Figure: Enriched degrees of freedom

Standard Finite Elements

- Standard finite element space

$$V_h = \{\varphi \in V : \phi|_K \in Q_p\}$$

- Enriched finite element space

$$V_h^s = \{\varphi \in V : \varphi|_K \in Q, \varphi|_{K_i} \in Q, i = 1, 2\}$$

Extended Finite Elements

- Standard finite Element solution

$$u_h = \sum_{i \in I} u_i N_i \tag{4}$$

where N_i are standard shape functions.

- Extended finite element solution

$$u_h = \sum_{i \in I} u_i N_i + \sum_{j \in J} a_j M_j \tag{5}$$

where M_j are enriched shape functions.

$$M_j(x) = N_j(x)\Psi(x) \tag{6}$$

with Ψ the Heaviside function.