Extended Finite Element Method (XFEM)

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October 26, 2023

Content

Extended Finite Element Method (XFEM)

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Motivation

nfitted Metho

Level Set Function

Extended Finite

Motivation

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Extended Finite Elements

Motivation



Figure: Microstructure of a composite material

Applications:

- Crack propagation
- Microstructured problems
- Composite materials
- ► Time-depending domains

Advantages:

- Discontinuities within elements possible
- Avoiding complex mesh generation

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Interface Problems



Figure: Composite material

$$-\nabla \cdot (\mu_i \nabla u_i) = f$$
 in Ω_i (1)
 $u_i = g$ on $\partial \Omega$ (2)

$$[u] = g_s$$
 on Γ (3)

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Elements

Unfitted Method

- background mesh
- cut cells

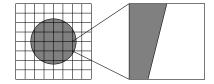


Figure: Unfitted mesh

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► Tracking the interface

$$\phi: \Omega \to \mathbb{R} \quad , \quad \phi(x) \begin{cases} = 0 & , x \in \Gamma \\ < 0 & , x \in \Omega_1 \\ > 0 & , x \in \Omega_2 \end{cases}$$

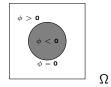


Figure: Level set function

▶ i.e. signed distance function

Implementation in deal.II



Figure: Cut cells

- loop over all cells
- using the level set function to find the cut cells
- active_fe_index()

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Enriched Elements



Figure: Standard degrees of freedom



Figure: Enriched degrees of freedom

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► Standard finite element space

$$V_h = \{ \varphi \in V : \phi |_{\mathcal{K}} \in Q_p \}$$

► Enriched finite element space

$$V_h^s = \{ \varphi \in V : \varphi|_K \in Q, \varphi|_{K_i} \in Q, i = 1, 2 \}$$

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Extended Finite Elements

Standard finite Element solution

$$u_h = \sum_{i \in I} u_i N_i \tag{4}$$

where N_i are standard shape functions.

Extended finite element solution

$$u_h = \sum_{i \in I} u_i N_i + \sum_{j \in J} a_j M_j \tag{5}$$

where M_j are enriched shape functions.

$$M_j(x) = N_j(x)\Psi(x) \tag{6}$$

with Ψ the Heaviside function.