# Extended Finite Element Method (XFEM)

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# Content

# Motivation

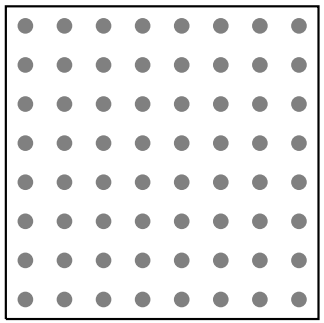

Figure: Microstructure of a composite material

Applications:

- Crack propagation
- Microstructured problems
- Composite materials
- Time-depending domains

Advantages:

- Discontinuities within elements possible
- Avoiding complex mesh generation

# Interface Problems



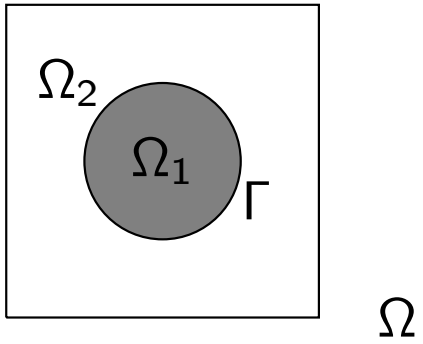


Figure: Composite material

$$-\nabla \cdot (\mu_i \nabla u_i) = f \quad \text{in } \Omega_i \tag{1}$$

$$u_i = g \quad \text{on } \partial\Omega \tag{2}$$

$$[u] = g_s \quad \text{on } \Gamma \tag{3}$$

# Unfitted Method

- background mesh
- cut cells

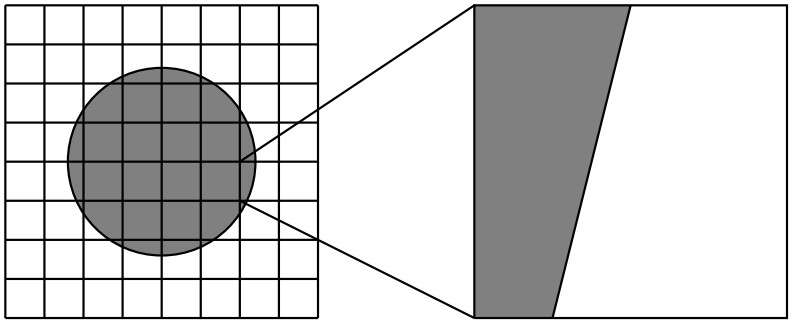

Figure: Unfitted mesh

# Level Set Function

- Tracking the interface

$$\phi : \Omega \to \mathbb{R}$$

$$\begin{aligned} \phi &= 0 && \text{on } \Gamma \\ \phi &< 0 && \text{in } \Omega_1 \\ \phi &> 0 && \text{in } \Omega_2 \end{aligned}$$

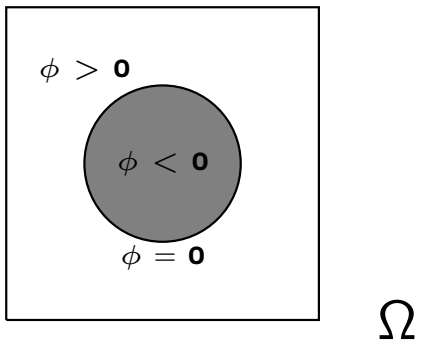


Figure: Level set function

- i.e. signed distance function

# Enriched Elements



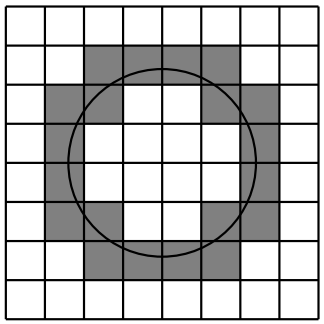

Figure: Cut cells

# Enriched Elements



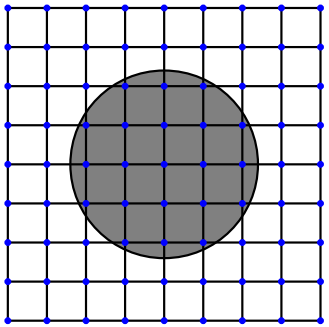

Figure: Standard degrees of freedom

# Enriched Elements



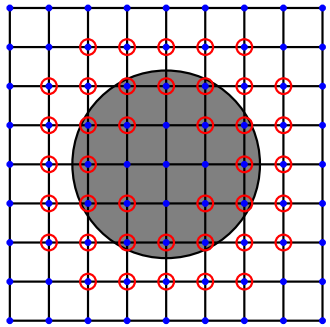

Figure: Enriched degrees of freedom

# Extended FEM

$$u_h = \sum_{i \in I} u_i \varphi_i + \sum_{j \in J} a_j M_j \tag{4}$$

# Boundary and Interface condition

# Implementation

# XFEValues Class