

Spectral Efficiency of MU-Massive MIMO System for Perfect and Imperfect CSI Condition

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Abstract— Multi User Massive MIMO (MU-Massive MIMO) is a form of multi-user large-scale antennas technology, in which hundreds numbers of antennas serve a significantly smaller number of users. This system is assumed operates over a frequency-selective and uses Orthogonal Frequency Division Multiplexing (OFDM). The system performance is observed in two conditions, the first condition is that the BTS is assumed to know channel information (perfect CSI), the second condition is that the BTS performs the channel estimation process at a certain coherence interval (imperfect CSI). BTS estimates channel information from the pilot sent by user using Minimum Mean Square Error (MMSE) estimator. Several precoding techniques are used to overcome Multiuser Interference (MUI) at the receiver, namely Maximum Ratio (MRT), Zero Forcing (ZF) and MMSE. The parameters observed in this study is spectral efficiency. From the simulation results, it can be seen that the use of a large number of antenna arrays can significantly increase the spectral efficiency. ZF and MMSE precoding works better than MRT precoding in minimizing MUI. Spectral efficiency is also affected by the channel model, where the spectral efficiency on the UR-LOS channel is higher than Rayleigh channel. In the imperfect CSI condition, the channel estimation error is getting bigger, so that the spectral efficiency will decrease and become lower than the perfect CSI condition.

Keywords— MU-Massive MIMO, Rayleigh, UR-LOS, Perfect CSI, Imperfect CSI, Spectral Efficiency, MRT, ZF, MMSE.

I. INTRODUCTION

In recent years, Multiple Input Multiple Output (MIMO) technology has been developed to support the development of high-speed wireless communication system. This technology has better performance than Single Input Single Output (SISO). This concept becomes the background for the development of Massive MIMO system. Massive MIMO system is a system that uses a very large number of antennas on the BS, the antennas can be hundreds or even more [1]. By using massive antenna elements, the spectral efficiency and energy efficiency will be significantly increased, compared to the small-scale MIMO system.

Multi User Massive MIMO (MU-Massive MIMO) is used to serve multiuser simultaneously. Hundreds of antennas on a BS serve tens of users at same time and frequency resources, where each user uses single antenna. The design and analysis of the Massive MIMO system is an interesting topic to study [2]-[5]. The Channel State Information (CSI) on the BS or user

side is required, because CSI is not only useful for obtaining high SNR on the user side, but also useful for reducing interference generated by other users in a cell. However, channel estimation will be very complex because it is proportional to the large number of BS antennas, hence some previous research on Massive MIMO system assumed perfect CSI condition on both the BS and the user side [7]-[9]. Meanwhile, there is no perfect CSI condition in practical terms, because the BS needs to estimate the channel response at certain coherence interval. Therefore, this study investigate the spectral efficiency of Massive MIMO system when the BS performs channel estimation. Imperfection of the channel estimation process due to the channel estimation error is known as imperfect CSI condition, because BS only know the noisy version of the channel.

We assume that the system works on TDD operation because uplink and downlink channel are reciprocal. BS can obtain CSI from uplink pilots training sent by users. The number of transmitted pilots is proportional to the number of users. Then BS uses CSI to precode the transmitted signal in order to reduce Multi User Interference (MUI). Form [1,12], it is shown that the use of large-scale antennas on the BS will increase spectral efficiency without bound. Hence, the use of a massive number of antenna elements will significantly increase the spectral efficiency.

This paper analyses single cell MU-Massive MIMO communication system with a downlink scheme at Rayleigh and Uniformly Random Line of Sight (UR-LOS) channel. Because these channels represent very different propagation conditions. The Rayleigh channel has rich scatterers, while the UR-LOS channel has no scatterers at all. In practical terms, we want a channel model that includes these conditions. By knowing performance of Massive MIMO in these two channel conditions, we can estimate Massive MIMO performance on other channel models.

We also proposed several linier precoding methods in order to overcome multiuser interference, such as Maximum Ratio (MRT), Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) linier precoding. Linier precoding has low computational complexity and nearly optimal to use since the number of BS antennas is large. By using the proposed precoding method, interference between users can be minimized properly, so that spectral efficiency increases.

This system is assumed to be operated over a frequency-selective channel and uses Orthogonal Frequency Division Multiplexing (OFDM). We investigated the performance in specific conditions. First, we assume that the BS knows the channel information state (perfect CSI condition). And second, BS estimates the channel at a certain coherence interval (Imperfect CSI condition) using MMSE estimator. The parameters that will be observed are Bit Error Rate (BER) and Spectral Efficiency (SE).

Notations: Uppercase and lowercase bold letters denote matrices and vectors. $\mathbb{E}\{\cdot\}$ denotes expectation operator. For a matrix \mathbf{H} , we denote the superscripts $(\cdot)^T$, $(\cdot)^H$ as transpose, and Hermitian matrix operation, respectively. While $\text{tr}(\mathbf{H})$ is trace of matrix \mathbf{H} and \mathbf{I}_K is $(K \times K)$ identity matrix. The entries of k -th row and m -th column of matrix \mathbf{H} is denoted by $[\mathbf{H}]_{k,m}$. \mathbb{C} represents complex number, while \mathcal{CN} represents complex normal random variable. The accent $|\cdot|$ denote magnitude of a complex number.

II. SYSTEM MODEL

The downlink system of single cell MU-Massive MIMO is shown in Fig. 1. This system is assumed work on a frequency-selective channel, therefore OFDM technique is used to overcome Inter-symbol Interference (ISI). BS is equipped with M number of antennas and simultaneously serves K number of users, each user uses a single antenna, where M is much larger than K .

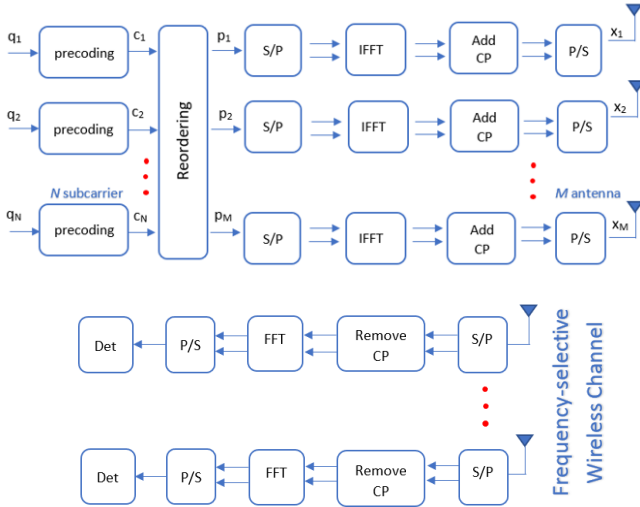


Fig. 1 System model of MU-Massive MIMO downlink scheme.

The total number of OFDM subcarriers is denoted by N where N consists of subcarriers designed for data transmission (N_d) and subcarriers for guard band (N_g) in order to reduce out-of-band radiation. Signal vector $\mathbf{q}_n \in \mathbb{C}^{K \times 1}$ contains QAM modulated symbols at n -th subcarrier for $n \in N_d$, and no signal transmitted at guard band $\mathbf{q}_n \in 0^{K \times 1}$ for $n \in N_g$. In perfect CSI condition, we assumed that BS already knows the CSI and uses it to precode the transmitted signal. We focus on analyzing the system performance under Rayleigh and UR-LOS channel. The first channel considered is Rayleigh channel where there is no direct line of sight path. The matrix $\mathbf{H}_{RL} \in \mathbb{C}^{K \times M}$ is a time-domain channel response in Rayleigh

channel associated with L number of channel taps, where $l = 1, 2, \dots, L$. This matrix contains complex normal random variables $[\mathbf{H}_{RL}]_{k,m} \sim \mathcal{CN}(0, \sigma_l^2)$, here σ_l^2 is power delay profile at l -th path. The second channel condition considered is UR-LOS where there is no local scatterer between BS and users. And all users are in line of sight to the BS antennas. The UR-LOS time domain channel response between k -th user and BS is described below [10]:

$$\mathbf{h}_k = \sqrt{\beta} [1 \ e^{2\pi j d_H \sin \theta_k} \ \dots \ e^{2\pi j d_H (M-1) \sin \theta_k}]^T \quad (1)$$

Here \mathbf{h}_k is channel response associated with k -th user, β is large-scale fading coefficient, d_H is array spacing and θ_k is angular position of each user which is measured relative to array boresight. Then, \mathbf{h}_k can be expressed as a matrix.

$$\mathbf{H}_{UR-LOS} = \begin{bmatrix} h_1^1 & h_1^2 & \dots & h_1^M \\ h_2^1 & h_2^2 & \dots & h_2^M \\ \dots & \dots & \ddots & \dots \\ h_K^1 & h_K^2 & \dots & h_K^M \end{bmatrix} \quad (2)$$

BS will only transmit the precoded signal at the specified angle of users. User position is random and $\sin(\theta_k)$ is uniformly distributed at interval $[-1, 1]$. BS uses Uniform Linear Array (ULA) antennas. Where ULA can only detect the user position uniquely at intervals $[-\pi/2, \pi/2]$.

Channel matrix is assumed to be constant at certain coherence interval. In the imperfect CSI condition, BS needs to estimate the channel response. The channel estimation can be obtained from training pilots which is transmitted by all users. At each coherence interval, each user transmits orthogonal pilot sequences at length τ_p . The transmitted pilot is a unitary matrix Φ which is known at both ends of the link [6, 10]. The number of transmitted pilots must be greater than the number of users ($\tau_p \geq K$). Collectively, all users transmit $K \times \tau_p$ pilots [6].

$$\mathbf{X}_p = \sqrt{\tau_p} \Phi^H \quad (3)$$

Here $\mathbf{X}_p \in \mathbb{C}^{K \times \tau_p}$ is pilot signal which is transmitted by all user, and τ_p is length of pilot sequence. Then, BS will receive pilot signal and noise [6].

$$\mathbf{Y}_p = \sqrt{\rho_{ul}} \mathbf{H}_{ul} \mathbf{X}_p + \mathbf{W}_p \quad (4)$$

Where $\mathbf{Y}_p \in \mathbb{C}^{M \times \tau_p}$ is received pilot signal, ρ_{ul} is uplink SNR, \mathbf{H}_{ul} is channel response to be estimated and \mathbf{W}_p is AWGN noise. Then, BS will multiply the received pilot signal with the unitary matrix Φ in order to estimate channel response [6].

$$\mathbf{Y}_p' = \mathbf{Y}_p \Phi \quad (5)$$

The channel estimation using MMSE estimator is described as:

$$\hat{h}_k^m = \frac{\sqrt{\tau_P \rho_{ul} \beta}}{1 + \tau_P \rho_{ul} \beta} [\mathbf{Y}_p']_{mk} \quad (6)$$

Where \hat{h}_k^m is estimation channel of k -th user to m -th BS antenna. $[\mathbf{Y}_p']_{mk}$ is k -th row and m -th column of matrix \mathbf{Y}_p' . The channel estimation error can be calculated as $\tilde{h}_k^m = \hat{h}_k^m - h_k^m$. We can calculate the Mean Square Error (MSE) of the channel estimation using $\mathbb{E}\{|\tilde{h}_k^m|^2\}$. After BS knows channel information, then BS will use this channel matrix to precode the transmitted signals. There are several simple linear precoding techniques which have low computational complexity and nearly optimal to use since the number of BS antennas is large. In this paper we use MRT, ZF and MMSE linear precoding technique and described as follows [8]:

$$\mathbf{A}_n^{\text{MRT}} = \propto \mathbf{H}_n^H \quad (7)$$

$$\mathbf{A}_n^{\text{ZF}} = \propto \mathbf{H}_n^H (\mathbf{H}_n \mathbf{H}_n^H)^{-1} \quad (8)$$

$$\mathbf{A}_n^{\text{MMSE}} = \propto \mathbf{H}_n^H \left(\mathbf{H}_n \mathbf{H}_n^H + \frac{K}{p_d} \mathbf{I}_K \right)^{-1} \quad (9)$$

Where $\mathbf{A}_n \in \{\text{MRT}, \text{ZF}, \text{MMSE}\}$ is precoding matrix for n -th subcarrier, $\propto = \sqrt{1/\mathbb{E}\{\text{tr}(\mathbf{A}_n \mathbf{A}_n^H)\}}$ is a scaled factor to satisfied the total transmit power at BS, p_d is downlink SNR, and \mathbf{H}_n is channel response at n -th subcarrier which is described as:

$$\mathbf{H}_n = \sum_{l=0}^{L-1} \mathbf{H}_l \exp\left(-jn \frac{2\pi}{N} l\right), \quad n = 0, 1, \dots, N-1 \quad (10)$$

Here $\mathbf{H}_l \in \{\mathbf{H}_{RL}, \mathbf{H}_{UR-LOS}\}$ is channel response at l -th path. Next, the symbols at each subcarrier \mathbf{q}_n are multiplied by each subcarrier precoding matrix \mathbf{A}_n and resulting frequency-domain precoded vector $\mathbf{c}_n \in \mathbb{C}^{M \times 1}$ which contains the symbols that will be transmitted over n -th subcarrier via M BS antennas. Then precoded vector \mathbf{c}_n is reordered in order to be transmitted to the M BS antennas by using reordering process as described below.

$$|\mathbf{p}_1, \dots, \mathbf{p}_M| = |\mathbf{c}_1, \dots, \mathbf{c}_N|^T \quad (11)$$

Hence, each BS antenna transmits the signal from all subcarriers. Time-domain signal \mathbf{x}_m is obtained by performing an Inverse Fast Fourier transform (IFFT) of \mathbf{p}_m . Cyclic prefix is inserted to the time-domain signals in order to overcome ISI between OFDM symbols. After passing the wireless channel, all users receive time-domain baseband signal at discrete-time n which in represented in (7):

$$\mathbf{Y}(n) = \sum_{l=0}^{L-1} \mathbf{H}_l \mathbf{X}(n-l) + \mathbf{W}(n) \quad (12)$$

Where $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_K] \in \mathbb{C}^K$ is received baseband vector at time sample n for all users, \mathbf{H}_l is time-domain channel response at l -path, $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_M] \in \mathbb{C}^M$ is transmitted signal at M BS antenna at time sample n , and $\mathbf{w}_n \in \mathcal{CN}(0, N_0 \mathbf{I}_K)$ is Additive White Gaussian Noise (AWGN) at time sample n . Here N_0 is noise power spectral density (PSD) and \mathbf{I}_K is $K \times K$ identity matrix. After that, each user removes cyclic prefix from time-domain received signal. Then, frequency-domain signal $\hat{\mathbf{y}}_k$ is retrieved using Fast Fourier Transform (FFT) of \mathbf{y}_k . Moreover, the received signal on n -th subcarrier and at k -th user is described as follow:

$$\hat{\mathbf{y}}_{k,n} = [\mathbf{H}_n \mathbf{A}_n]_{k,k} \mathbf{q}_{k,n} + \sum_{\substack{v=1 \\ v \neq k}}^K [\mathbf{H}_n \mathbf{A}_n]_{k,v} \mathbf{q}_{v,n} + \mathbf{w}_{k,n} \quad (13)$$

Here, $\mathbf{A} \in \{\text{MRT}, \text{ZF}, \text{MMSE}\}$, \mathbf{H}_n is frequency-domain channel matrix at n -th subcarrier, $\mathbf{q}_{k,n}$ is symbol to be transmitted over n -th subcarrier for k -th user, while $\mathbf{q}_{v,n}$ is symbol to be transmitted over n -th subcarrier for all user except k -th user, and $\mathbf{w}_{k,n}$ is AWGN on n -th subcarrier at k -th user. The first term on the right-hand side of (13) stands for desired signal of k -th user, the second term represents multi user interference, and the third term is AWGN.

We assumed that there is only intra-cell interference and no inter-cell interference. By using (13) and assuming that $\mathbf{q}_n \in \mathbb{C}^{K \times 1}$ for $n \in N_d$, then signal to Interference Noise Ratio (SINR) at k -th user over n -th subcarrier is defined below [1]:

$$\text{SINR}_{k,n}^A = \frac{[\rho_{dl} |\mathbf{H}_n \mathbf{A}_n|^2]_{k,k}}{\rho_{dl} \sum_{\substack{v=1 \\ v \neq k}}^K [\mathbf{H}_n \mathbf{A}_n]_{k,v} + \mathbf{w}_{kn}} \quad (14)$$

Where $\text{SINR}_{k,n}^A$ is effective SINR for k -user at n -th subcarrier using precoding \mathbf{A} , and ρ_{dl} is downlink SNR. Then the sum spectral efficiency of all users can be obtained from the following approach [10]:

$$\text{SE} = K \log_2(1 + \text{SINR}_{k,n}^A) \quad (15)$$

The SE formula in (15) can be calculated for different channel types and precoding schemes. The downlink SE for the k -th user in a cell really depends on the used of precoding. Therefore, choosing the most appropriate precoding technique is very important. By using precoding, each signal of precoded vector is sent from all BS antennas but with a difference amplitude and phase, so that the signal will be sent directly to users.

III. NUMERICAL RESULTS

A. Single Cell MU-Massive MIMO System

We simulated downlink scheme of single cell MU-Massive MIMO system which operates at a frequency of 3.4 GHz. We assumed that there is no inter-cell interference. BS uses ULA antennas with spacing between antenna elements is $\lambda/2$. The total number of subcarriers is equal to the N-point

FFT size. To support the development of the 5G system, the OFDM modulation scheme refers to 5G system numerology [13] as shown in Table 1. We consider a Rayleigh fading frequency-selective channel with $L = 22$ taps and has exponential distribution of power delay profile. Tapped delay lines can be used to represent sparse multipath channel for wireless communication system.

TABLE 1. SYSTEM PARAMETERS

OFDM Parameters	Parameter Value
Total bandwidth (B)	5 MHz
Number of subcarriers (N)	512
Number of used subcarriers (N_d)	300
Subcarrier spacing (Δ_f)	15 kHz
Useful symbol duration (T_u)	66.77 μ s
Cyclic prefix duration (T_{cp})	4.9 μ s

B. Bit Error Rate (BER)

To evaluate BER of the system, we plot the BER as a function of SNR with the fixed number of BS antennas and users. BS is equipped with 100 antennas (M) and serves 10 users (K) simultaneously.

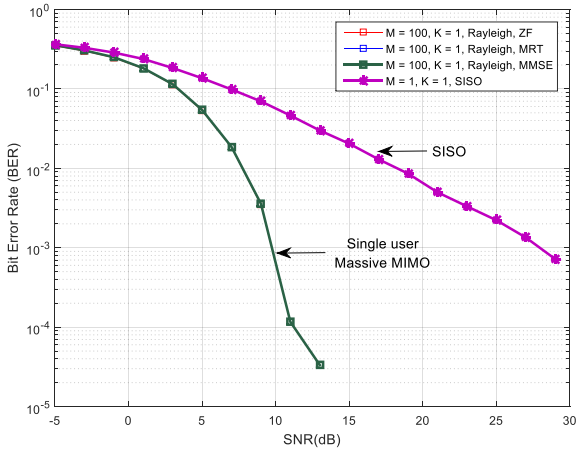


Fig. 2 BER MU-Massive MIMO at Rayleigh channel. In this example there are $M = 100$ antennas, $K = 1$ user for Massive MIMO system.

In order to work properly, the number of BS antennas should be at least four times the number of users ($K \leq 4M$) [10]. All user positions are random and uniformly distributed at interval $[-\pi/2, \pi/2]$. Fig. 2 shows BER of single user Massive MIMO system and SISO at Rayleigh channel. Based on the result of this simulation, it can be seen that BER of Massive MIMO system is lower than SISO. All of precoding schemes yield the same BER because there is no multiuser interference. The BER depends on the strength of desired signal which represents as SNR. Increasing SNR means increasing the transmit power, which results smaller BER. Fig. 3 shows BER of multiuser Massive MIMO system and SISO. For MU-Massive MIMO system, $K = 10$ users. Several users which are served simultaneously will cause multiuser interference. All precoding schemes have different performance due to the effects of multiuser interference. The

simulation result shows that ZF and MMSE yield the lower BER than MRT.

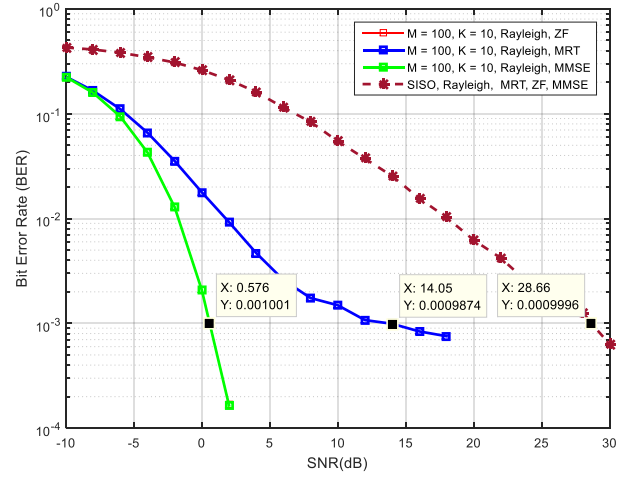


Fig. 3 BER MU-Massive MIMO at Rayleigh channel. In this example there are $M = 100$ antennas, $K = 10$ users for Massive MIMO system.

Then, we investigate BER of MU-Massive MIMO system at UR-LOS channel as shown in Fig. 4. The user position is in the line of sight to the BS. All precoding scheme provide same BER under the UR-LOS channel. Overall, BER in UR-LOS channel is lower than Rayleigh channel.

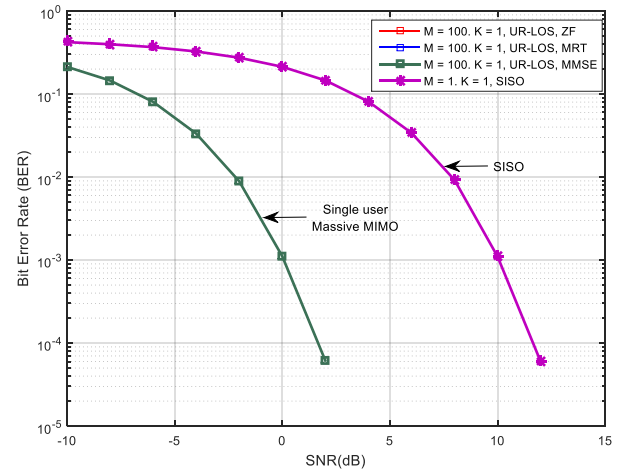


Fig. 4 BER MU-Massive MIMO at UR-LOS channel. In this example there are $M = 100$ antennas, $K = 1$ users for Massive MIMO system.

C. Spectral Efficiency (SE)

Spectral efficiency is a deterministic number that can be measured in bits per time unit per bandwidth or commonly known as bit/s/Hz. Where the sum of SE is obtained from the sum of SE from all users in a cell. A high SE value is the main key objective in designing a wireless communication system. By increasing the number of antennas on the BS, the SE will be higher. In this simulation, SE is observed by increasing the number of BS antennas with a constant number of users.

BS is equipped with a varying number of antennas, ranging from 50 to 300 and simultaneously serves 30 users. The first condition is assumed to be perfect CSI condition, so that there is no channel estimation on the BS. This system is

simulated in two different channel conditions, which are Rayleigh and UR-LOS.

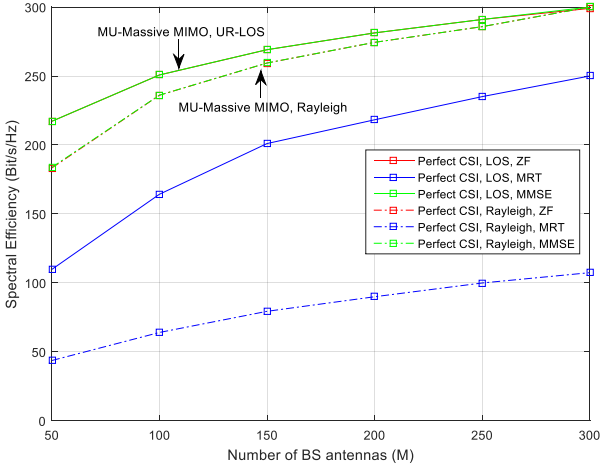


Fig. 5. Spectral Efficiency of MU-Massive MIMO at perfect CSI condition. In this example there are $K = 30$ users and the downlink SNR = 10 dB.

The SE as a function of antenna variation in perfect CSI conditions is shown in Fig. 5. Due to the fact that there is no channel estimation process in BS, so that BS is fully knowing the channel information without any noise and pilot contamination effects. This system works at downlink SNR of 10 dB. At high SNR, ZF and MMSE precoding work optimally and yield the same spectral efficiency both at Rayleigh and UR-LOS channel condition. The spectral efficiency of ZF and MMSE precoding also higher than MRT precoding, because ZF and MMSE precoding is better at minimizing multiuser interference than MRT. In the UR-LOS channel, the strong interference is caused by users with similar angle, so that spectral efficiency at UR-LOS channel is higher than Rayleigh.

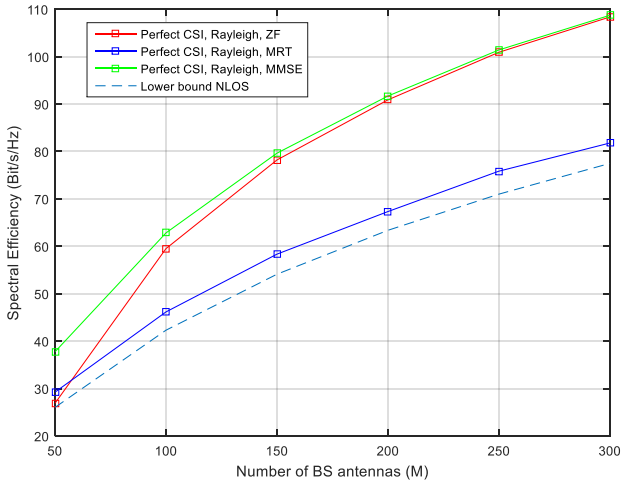


Fig. 6. Spectral Efficiency of MU-Massive MIMO at perfect CSI condition. In this example there are $K = 30$ users and the downlink SNR = 0 dB.

Then, we simulate the system performance at downlink SNR of 0 dB. The overall system performance will decrease if the downlink SNR decreases as shown in Fig. 6. At low downlink SNR, MMSE precoding works better than ZF and MRT. This is because MMSE not only minimizes multiuser

interference properly, but also increase SINR on the user's side, therefore the spectral efficiency will increase. In practical, there is no perfect CSI condition, because the BS needs to estimate the channel at a certain coherence interval.

Then, we focus to investigate the system performance under imperfect CSI condition. BS estimates the channel response from pilot signal transmitted by users through uplink channel. The total number of pilots transmitted by each user must be greater or equal to the number of active users in a cell, therefore each user transmits 30 symbols of pilot sequences ($\tau_p = K$). This is the minimum number of pilots that can be sent by each user. The number of pilots limit how many users that can be served simultaneously. After despreading pilot signals, BS can obtain a noisy version of the channel.

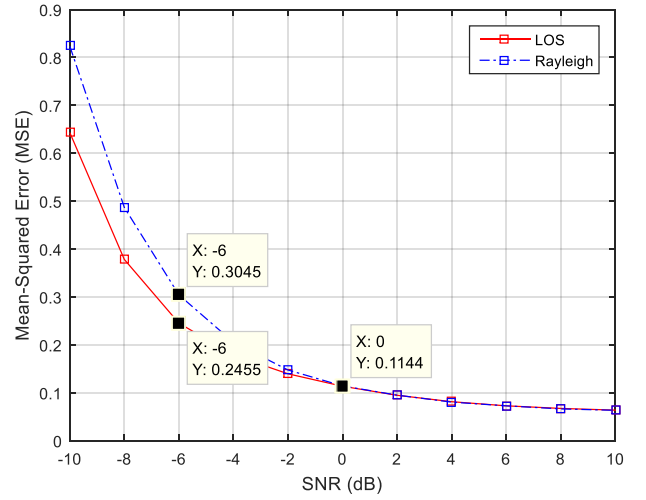


Fig. 7. Minimum Mean-Square Error (MSE) of estimated channel.

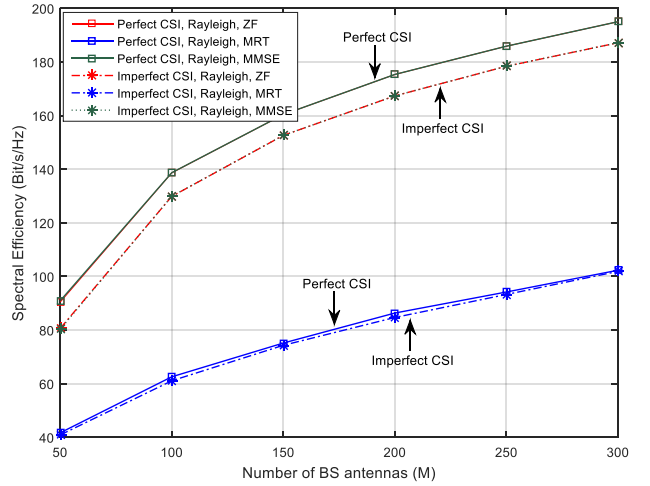


Fig. 8. Spectral Efficiency of MU-Massive MIMO at perfect and imperfect CSI condition. In this example there are $K = 30$ users, the downlink SNR = 10 dB, and the uplink SNR = -5 dB.

Then, MSE of channel estimation can be calculated and shown in Fig. 7. Channel estimation error really depends on uplink SNR at the BS, the higher the uplink SNR, the channel estimation error will be smaller. At low uplink SNR region, MSE of UR-LOS channel is lower than Rayleigh channel.

MSE of these two channel conditions has the same value when the uplink SNR is above 0 dB.

Spectral efficiency of perfect and imperfect CSI condition under Rayleigh channel is shown in Fig. 8. Downlink SNR is set to 10 dB, and uplink SNR is set to -5 dB. The spectral efficiency of imperfect CSI condition is lower than perfect CSI condition due to the channel estimation error. The SE as a function of the number of users is shown in Fig. 9. We can see that the sum SE will increase as the number of users increases on condition that the number of users is still smaller than the number of BS antennas ($K < M$). At 100 fixed number of BS antennas, spectral efficiency continues to increase because the number of users is still smaller than the number of BTS antenna. This condition will be different when the number of BS antennas is 30 and serves exceeds 30 users, then the SE started to decrease due to the high interference between users, and precoding is not able to overcome it. As in [10], the number of BS antennas should be much larger than the number of users, leading to an antenna-users ratio ($M/K \geq 1$). This ratio makes linear precoding nearly optimal since multiuser interference is small.

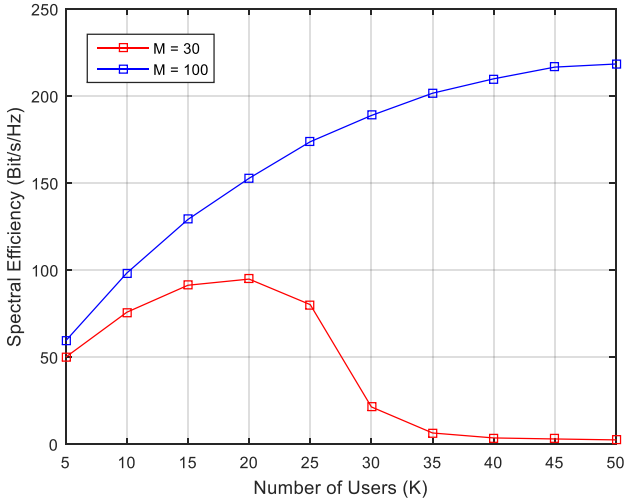


Fig. 9 Spectral Efficiency of MU-Massive with different number of users

IV. CONCLUSION

In this study, we investigated the performance of downlink scheme single cell MU-Massive MIMO system under perfect CSI and imperfect CSI condition. MRT, ZF and MMSE linear precoding techniques are implemented to reduce multiuser interference. We consider linear precoding because their low computational complexity and nearly optimal to use since the number of BS antennas is large. When SNR is more than 0 dB, ZF and MMSE precoding work equally well and yield nearly the same spectral efficiency. Meanwhile, when SNR is less than 0 dB, MMSE precoding works better than ZF.

The spectral efficiency increases significantly as the number of BS antennas increases. When the BS is equipped with 50 antennas and serves 30 users simultaneously, the total spectral efficiency at Rayleigh channel under perfect CSI condition is 180 b/s/Hz, and when the number of BS antennas is increased to 300 antennas and serving the same number of users, the spectral efficiency increases to 300 b/s/Hz. Under imperfect CSI condition, channel estimation error is getting bigger at low uplink SNR, so that the spectral efficiency will decrease. When the uplink SNR is 10 dB, spectral efficiency of 100 BS antennas at perfect and imperfect CSI is nearly same, which is 140 b/s/Hz. However, when the uplink SNR decreases to -5 dB, the spectral efficiency at the imperfect CSI condition decreases to 130 b/s/Hz.

- [1] H. Q. Ngo and T. L. Marzetta, "Energy and Spectral Efficiency of Very Large Multiuser MIMO Systems," in *IEEE transaction on communication*, vol. 61, no. 4, 2013.
- [2] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of BS antennas" in *IEEE transaction on wireless communication* vol. 9, no. 11, pp. 3590–3600, Nov. 2010.
- [3] Hong Yang, T.L. Marzetta, "Performance of Conjugate and Zero-Forcing Beamforming in Large-Scale Antenna Systems" in *EEE Journal on Selected Areas in Communications* VOL. 31, NO. 2, February 2013.
- [4] H.Q.Ngo, E.G.Larson, T.L.Marzetta, "Massive MU-MIMO Downlink TDD Systems with Linear Precoding and Downlink Pilots", in *Fifty first Annual Allerton Conference Allerton House, UIUC, Illinois, USA* October 2 - 3, 2013.
- [5] A.Nasser, M. Elsabrouty, "Frequency-Selective Massive MIMO Channel Estimation and Feedback in Angle-Time Domain", in *IEEE Symposium on Computers and Communication (ISCC)*, 2016.
- [6] T. L. Marzetta and E. G. Larsson, "Fundamentals of Massive MIMO", *United Kingdom: Cambridge University Press*, 2016.
- [7] R. Zayani, H. Syaiek, D. Roviras, "PAPR-aware Massive MIMO-OFDM Downlink", in *IEEE Acces*, vol.7, Page 25474 – 25484, 2019.
- [8] Sven. Jacobson, Giuseppe Durisi, Christoph Studer, "Massive MU-MIMO-OFDM Downlink with One-Bit DACs and Linear Precoding". in *IEEE Global Communications Conference*. 2018.
- [9] Sven Jacobsson, Giuseppe Durisi, Mikael Coldrey, and Christoph Studer, "Linear Precoding With Low-Resolution DACs for Massive MU-MIMO-OFDM Downlink", in *IEEE Transaction on Wireless Communication*, vol. 18, no. 3, 2019.
- [10] Emil Björnson, Jakob Hoydis and Luca Sanguinetti, "Massive MIMO Networks: Spectral, Energy, and Hardware Efficiency", *Foundations and Trends® in Signal Processing*: Vol. 11, No. 3-4, pp 154–655. DOI: 10.1561/20000000093, 2017.
- [11] Y.S.Cho, J.Kim, W.Y.Yang, C.G.Kang, "MIMO-OFDM Wireless Communication with Matlab", *John Wiley & Sons, Ltd.* 2010.
- [12] Emil Björnson, Jakob Hoydis and Luca Sanguinetti, "Massive MIMO Has Unlimited Capacity."
- [13] 5G; NR; Base Station (BS) conformance testing Part 1: Conducted conformance testing TS 38.141-1 version 15.5.0 Release 15, 3GPP, 2020.