Engineering Physics

UNIT-I: WAVE OPTICS

Interference

Principle of Superposition:

When two or more waves overlap, the resultant displacement at a point equal to the algebraic sum of the individual displacement at that point. This is the principle of superposition.

Thus, if $y_1(x, t)$ and $y_2(x, t)$ are the wave functions characterizing two waves traveling in space, their resultant is given by

$$Y(x, t) = y_1(x, t) + y_2(x, t)$$

Coherent waves:

If two waves possess the same frequency and phase, then they are known as coherent waves.

Examples:

(1) Two pin holes illuminated by the same light, and

(2) Two point sources on the surface of an electric bulb.

Coherence is of two types: Namely
Temporal coherence
Spatial coherence

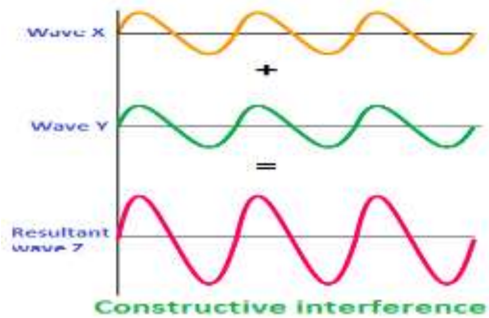
<u>Temporal Coherence</u>:

"If it is possible to predict the phase relation at a point on the wave W.R.T another point on the same wave then the wave has temporal coherence." It is also called longitudinal coherence.

<u>Spatial Coherence</u>:

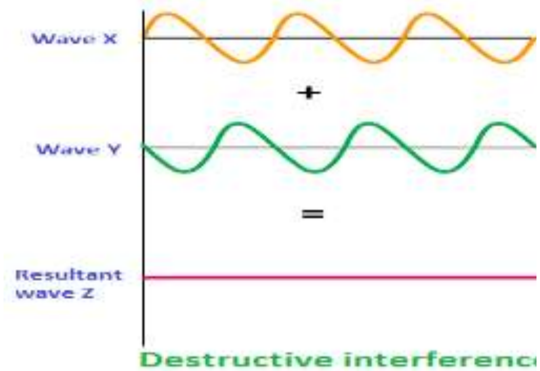
"If it is possible to predict the phase relation at a point on the wave W.R.T another point on another wave then the wave has spatial coherence." It is also called transfer coherence.

Note: The constructive and destructive interferences represent the maximum and minimum intensity positions, respectively



- The bright fringes are more intense and they are due to **constructive interference**, where the resultant amplitude is equal to the sum of the amplitudes due to individual waves.

The Resultant **constructive interference** $Y=Y_1+Y_2$



- The dark fringes are completely dark and they are due to **destructive interference**, where the resultant amplitude is equal to the difference of the amplitudes due to individual waves.

The Resultant **destructive interference** $Y=Y_1-Y_2$

Conditions for the interference of light:

The two light sources should be coherent.
The two sources must emit continuous waves of same wavelength and frequency

To observe the fringes :

The separation between the two sources should be small
The distance between the two sources and the screen should be large.
The background should be dark.

For good contrast:

The amplitude of the light waves should be equal or nearly equal
The sources should be narrow i.e. They must extremely small.
The sources should be monochromatic.

1.2. Interference of light:

The modification in light intensity in the region of superposition of two or more coherent waves is known as "interference of light".

**Types of Interference:

The phenomenon of interference requires two wave fronts to interact. These wavefronts can be obtained in two different ways, resulting two different type of interference, namely

i) Division of wavefront type of interference:

In this type of interference, the incident wave front is divided into two parts using the phenomenon of reflection, refraction or diffraction. The two parts of the wavefronts are then made to travel unequal distances before reuniting at same angle leading to the produce interference pattern.

Examples: 1) Young's double slit experiment and

2) Fresnel's Biprism.

ii) Division of amplitude type of interference:

In this type of interference, the amplitude of the incoming wave divided into number of parts through the process of reflection (or) refraction. These two parts are then travel along different optical paths and finally superimpose to produce interference pattern.

To produce this type of interference pattern, point (or) narrow line source is not essential. However, the broad light sources can be employed to yield bright interference bands.

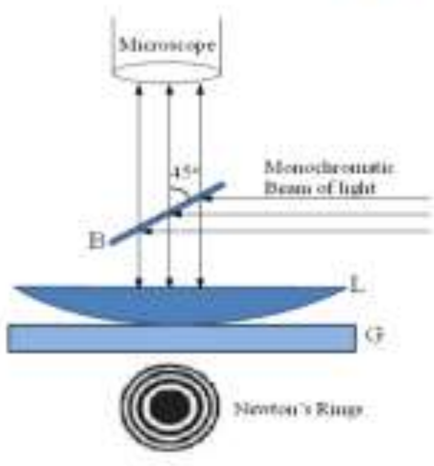
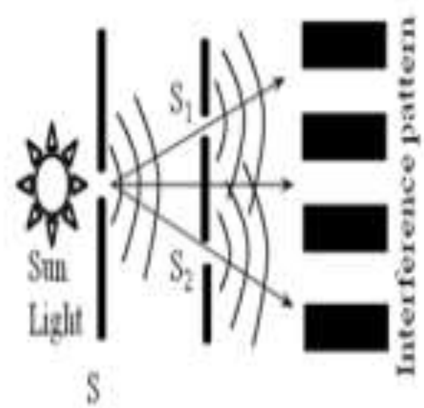
Examples: 1) Thin films interference and

2) Newton's Rings.

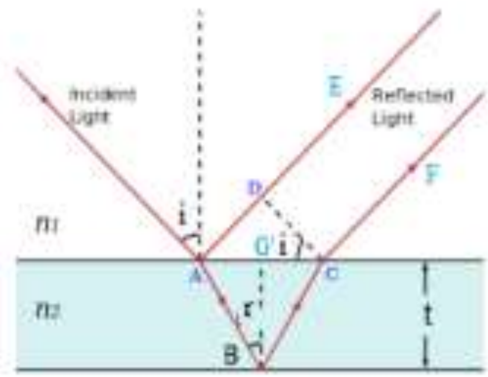
Q1: Can Young's double slit experiment prove the particle nature of light?

- From the interference phenomenon, it is clear that the combination of dark and bright bands is the evidence for interference and it is due to the wave nature of light.
- Suppose the light has particle nature, when passing through the slits, the particles gather in clumps on the screen without producing any intensity modification.
- We can't understand the interference phenomenon.
- The **interference pattern** was the evidence Young needed to determine that light was a wave and not a particle as Newton had suggested.
- Therefore, Young's double slit experiment disproves the particle nature of light.

1.3. Difference between interference due to amplitude division and wavefront division:

Interference due to amplitude division		Interference due to wavefront division	
1	Two coherent sources have been obtained by dividing the amplitude of a beam originated from a common source. So, coherent sources are produced by division of amplitude.	1	Two coherent sources have been obtained by dividing the wavefront of a beam originated from a common source. So, coherent sources are produced by division of wavefront.
2	Coherent sources are obtained either partial reflection or refraction.	2	Coherent sources are obtained due to lenses or mirrors
3	Light beams travel along different paths and finally brought together to produce interference.	3	Light beams travel along the same path and they are brought together to produce interference.
4	Light beams travel in different media.	4	Light beams travel in one medium only.
5		5	
6	Example: Interference in Thin films, Newton Rings, and Michelson's interferometer	6	Example: Young's double slit experiment, Fresnel's Biprism, Lloyd's mirror method, etc.

1.4. Interference in Uniform Thin films by Reflection



Consider a thin film of thickness 't' and refractive index μ. Let a ray of light is incident at a point A with angle of incidence 'i'. This ray partly reflected along AE and partly refracts along AB and incident at B with an angle 'r'. Finally, this ray of light along CF.

The reflected rays AE and CF are parallel to each other and constitute reflected system to produce interference pattern. To study the interference phenomenon, we have to calculate the effective path difference between AE and CF. From the figure,

Path difference = Path (AB+BC) in film – Path AD in air

$= \mu_{film}.\ (AB+BC) - \mu_{air}.\ AD$

$= \mu\ (AB+BC) - AD$ (since $\mu_{film} = \mu$ and $\mu_{air} = 1$)

From Figure we have, $AB = BC = \dfrac{t}{\cos r}$ and $AD = 2\,\mu t \dfrac{\sin^2 r}{\cos r}$

Substituting the values of AB, BC and AD, we get

Path difference = $2\ \mu\ t\ \mathrm{Cos}\ r$

Since the ray of light reflects from the denser medium, it undergoes a phase difference of 'π'

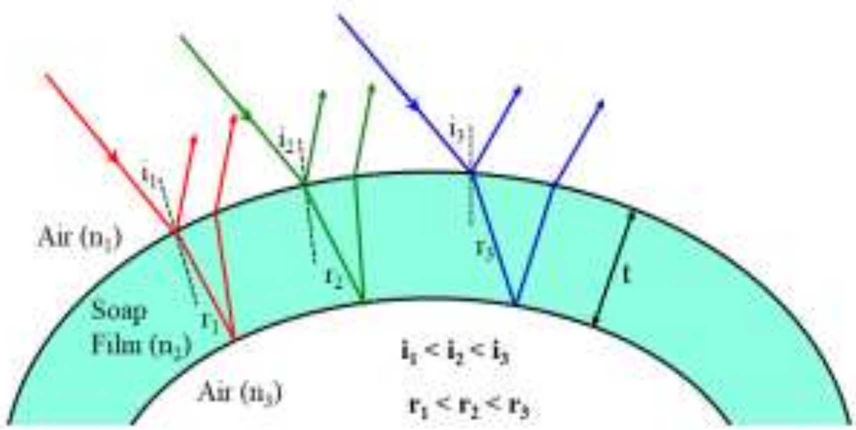
Therefore, **effective path difference = $2\ \mu\ t\ \mathrm{Cos}\ r + \lambda/2$**

Condition for bright fringe, $2\ \mu\ t\ \mathrm{Cos}\ r + \lambda/2 = n\lambda$

(or) $2\ \mu\ t\ \mathrm{Cos}\ r = (2n-1)\lambda/2$, n = 1,2,3, …..

1.4.1. Multi-colour in soap bubble

- In case of soap bubble, the thickness (t) of film is constant.
- And the refractive index (μ) and the angle of refraction (r) are variables.
- Since, the Sun light is composed of different colours or different wavelength (λ) values, the refractive index (μ) of the film also varies with λ.
- Due to the curved nature of bubble, even if parallel rays are incident, the angle of incidence (i) varies for different points on the bubble and hence the angle of refraction (r) varies as shown in figure.



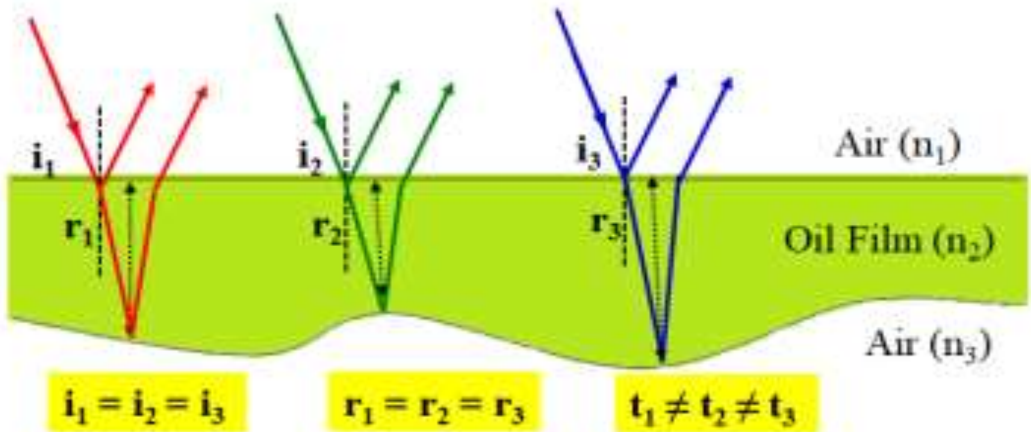
- The condition for bright band is given as

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2} \text{ where n} = 0,1,2,3,\ldots$$

- The varying values of μ and r can satisfy the condition of constructive interference for a particular wavelength (say λ_{red}) only. Accordingly, that point will appear in that colour (red).
- For other neighbouring wavelengths, the condition of destructive interference is satisfied.
- In a similar way, different points different points satisfy the condition of constructive interference for different wavelengths (or) colours producing multi-colours.

1.4.2. Multi-colour in Oil film

- Let us consider a thin layer of oil film floating on road/water.
- Though the film is perfectly flat on upper surface, its lower surface is not uniform as shown in figure.



- In this case, the thickness 't' of the film is not constant throughout it.
- When a beam of parallel rays such as Sun light is incident, the angle of incidence (i) and hence angle of refraction (r) will remain constant.
- Since the refractive index is a wavelength dependent, it varies for different values of λ.
- Hence, different points on the film satisfy the condition of constructive interference for different colours (or wavelengths) depending up on the values of μ and t.
- Thus, the oil film appears with multi-colours.

1.4.3. Applications of Thin Film technology or thin films

- Thin film technology is used for defrosting aircraft windshields.
- Thin film technology is used in cellular functions in the human body.
- Thin films are used for protecting the surface of many optical elements.
- Thin-film cells convert solar energy into electricity through the photovoltaic effect.

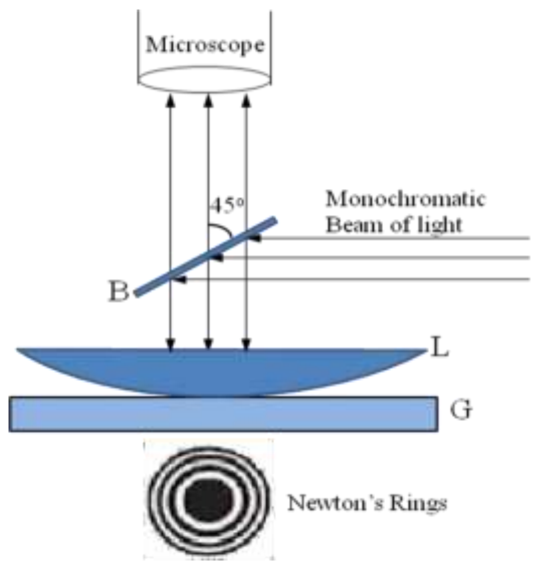
Newton's Rings (Interference in Non-uniform Thin films)

Definition: When Plano-convex lens with its convex surface placed on a glass plate is illuminated normally with a monochromatic light a circular concentric rings pattern is observed. These rings are known as Newton's rings.

Experimental arrangement of Newton's Rings

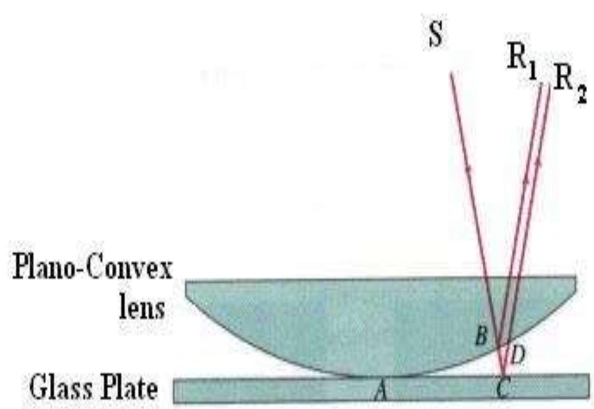
The experimental arrangement to observer the Newton's rings is shown in figure.

In figure, 'L' is a Plano–convex lens of large radius of curvature and is placed on an optically that glass plate G. A beam of monochromatic light is allowed to fall on a partially silvered glass plate B which is inclined at an angle of 45^0 with the vertical. The glass plate B reflects a part of incident light towards the air film enclosed by the lens – glass plate system. The reflected beam from the air film can be viewed with microscope M. Due to the interference of the light rays reflected from the lower surface of the lens and the upper surface of the glass plate, a circular concentric dark and bright fringes are produced as shown in figure.



Formation of Newton's Rings – Theoretical explanation:

The formation of Newton's Rings can be explained using the following figure.
Consider a Plano – convex lens (L) of radius of curvature R, placed on a glass plate P. Let a monochromatic ray of light AB is Incident on the lens. Glass plate system with angle of incidence 'i '



A part of the light is reflected at 'C' (glass-air boundary), which goes out in the form of ray-1 without any phase change.

→ The other part is reflected along CD.

→ At point D, the light ray again reflected and goes out in the form ray-2 with a phase change of π ((or) path difference $\frac{\lambda}{2}$)

→ The reflected rays 1& 2 are superposed to form interference pattern

→ As the interference pattern is observed in the reflected light,

Effective path difference = $2\mu t\ Cos$ (i+r) + $\frac{\lambda}{2}$ ------------------→ (1)

Where i is the angle of incidence, r is the angle of reflection, t is the thickness of the air film and μ is the refractive index of air.

→ We know that, for air film, μ = 1 and for normal incidence r = 0. The angle of incidence i is very small, hence, Cos i can be neglected.

∴ Effective path difference = $2t + \frac{\lambda}{2}$ ------------------→ (2)

At the point of contact of lens and glass plate:

At this point, t = 0

∴ Effective path difference = $\frac{\lambda}{2}$

This is the condition for minimum intensity. Thus a dark fringe is formed at the centre with a monochromatic light waves.

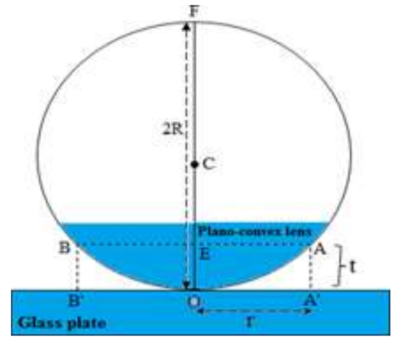
Condition for nth minimum: $2t + \frac{\lambda}{2}$ = (2n+1) $\frac{\lambda}{2}$, n= 0, 1, 2, 3…

∴ $2t$ = n λ , n = 0, 1, 2, 3… ------------------→ (3a)

Condition for nth maximum: $2t + \frac{\lambda}{2}$ = n λ, n = 1, 2, 3……………

∴ $2t$ = (2n-1) $\frac{\lambda}{2}$, n = 1, 2, 3... ------------------→ (3b)

Theory of Newton's Rings (OR) Expression for diameter of Dark and Bright Rings:



Consider a plano-convex lens of radius of curvature R placed on a plane glass plate as shown in figure.

Let us imagine a dark ring of radius 'r'. Let the thickness of air film is t (say) for the dark ring which is under consideration.

According to principle of circles,

AE×BE = OE×EF

$\Rightarrow r \times r = t(2R - t)$ $\Rightarrow r2 = t(2R - t)$

$\Rightarrow r^2 \approx 2Rt$ (Since t is small, t^2 can be neglected)

$$\Rightarrow 2t = \frac{r^2}{R} \longrightarrow \quad (1)$$

This is the path difference b/n. the light rays reflected from the top & bottom surfaces of air film.

Case-I: Condition for a Dark Ring:

$$\Rightarrow 2t = n\lambda \longrightarrow \quad (2)$$

From Eqs. 1& 2, we obtain, $r^2/R = n\lambda$

$$\Rightarrow r = \sqrt{n\lambda R}$$

But radius $r = \frac{Diameter(D)}{2}$

$$D = \sqrt{4n\lambda R} \quad \text{(or)} \quad D_n = \sqrt{4n\lambda R} \longrightarrow (3)$$

This is the expression for diameter of n^{th} dark ring.

Case-II: Condition for a Bright Ring:

$$2t = (2n-1)\,\lambda/2 \longrightarrow \quad (4)$$

$$\Rightarrow 2t = \frac{r^2}{R}$$

From. Eqs. 1& 4, we obtain, $\frac{r2}{R} = (2n-1)\frac{\lambda}{2}$

$$r^2 = (2n-1)\frac{\lambda R}{2} \text{ (or) } D^2 = (2n-1)2\lambda R$$

$$D = \sqrt{(2n-1)2\lambda R} \quad \text{(Here } \sqrt{2\lambda R} \text{ constant)}$$

$$D \alpha \sqrt{(2n-1)} \longrightarrow (5)$$

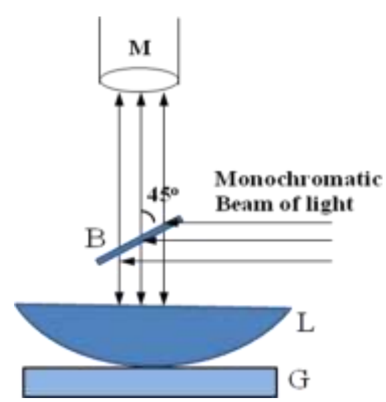
This is the expression for diameter of nth bright ring.
With increase in the order (n), the rings get closer and the fringe width decreases

Applications of Newton's Rings:

1. Wavelength (λ) of a monochromatic light can be determined.
2. Radius of curvature (R) of a curved surface can be determined.
3. Refractive index (μ) of a transparent liquid can be determined.

To determine the wavelength (λ) of a monochromatic light:

The wavelength of given monochromatic light source can be determined by measuring the diameters of various rings formed. For this purpose, Newton's Rings have been formed as shown in figure using a plano-convex lens of known radius of curvature (R) with the given monochromatic source of light of wavelength (λ).



Let Dm and Dn are the diameters of mth and nth dark rings, respectively.

We know that the $D_n = \sqrt{4n\lambda R}$

For nth ring Diameter

$$D_n^2 = 4n\lambda R \longrightarrow (1)$$

For mth ring Diameter

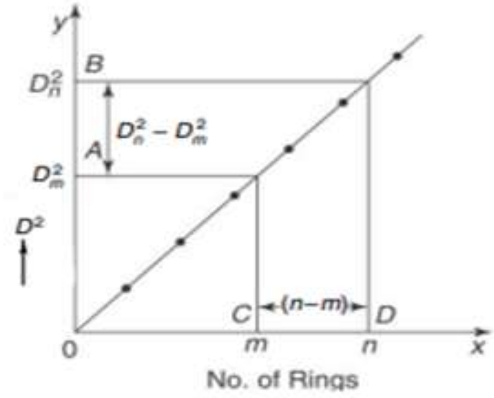
$$D_m^2 = 4m\lambda R \longrightarrow (2)$$

From equations (1)-(2) will get

$$D_n^2 - D_m^2 = 4(n-m)\lambda R \longrightarrow (3)$$

$$\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}$$

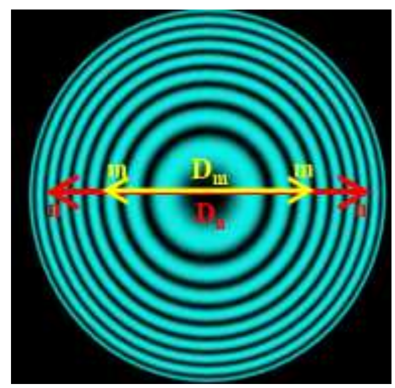
Using the above formula, the wavelength (λ) of a monochromatic light can be calculated.



Plot of D^2 w.r.t. the number of rings

Experimental

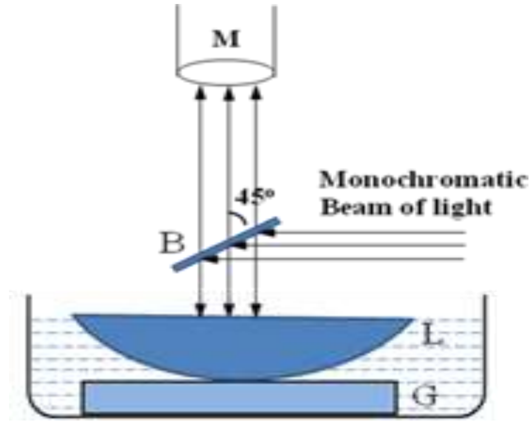
Experimental arrangements are made as given in Figure. After forming the Newton's rings, the microscope is adjusted so that the centre of the crosswire is adjusted at the central dark spot of the ring pattern. By counting the number of rings, the microscope is moved to the extreme left of the pattern and the cross wire is adjusted tangentially in the middle of the nth (21st) dark ring. The reading of the microscope is noted. Now the microscope is moved to the right and its readings are noted successively at $(n-3)^{th}$ (18^{th}), $(n-6)^{th}$ (15^{th}) … rings, etc., with a difference of three rings up to the central dark spot. Again crossing the central dark spot in the same direction, the readings corresponding to $(n-6)^{th}$ (15^{th}), $(n-3)^{th}$ (18^{th}) and n^{th} (21^{st}) rings are noted. The difference between the left and right reading gives the diameter of the particular ring. A graph is plotted between the number of the rings and the square of the corresponding diameter.



From the graph,

$$\frac{D_n^2 - D_m^2}{n-m} = \frac{AB}{CD}$$

The radius R of the plano convex lens can be obtained with the help of a spherometer. Substituting these values in the formula, the wavelength of the light source can be known



To determine the radius of curvature (R) of a curved surface:

The radius of curvature (R) of a curved surface can also be determined by measuring the diameters of various rings formed. For this purpose, Newton's Rings have been formed as shown in figure using the given curved surface of radius of curvature (R) with the monochromatic source of light of wavelength (λ). The value of R can be determined using the following formula.

We know that the $D_n = \sqrt{4n\lambda R}$

For n[th] ring Diameter

$$D_n^2 = 4n\lambda R \longrightarrow \quad (1)$$

For m[th] ring Diameter

$$D_m^2 = 4m\lambda R \longrightarrow \quad (2)$$

From equations (1)-(2) will get

$$D_n^2 - D_m^2 = 4(n-m)\lambda R \longrightarrow \quad (3)$$

$$R = \frac{D_n^2 - D_m^2}{4(n-m)\lambda}\ \text{Cm}$$

Using the above formula, the wavelength (R) radius of a curvature can be calculated.

To determine the refractive index (μ) of a transparent liquid:

The refractive index of a transparent liquid can be obtained by measuring the diameters of any two dark rings, in air and in the given liquid.

Newton's Rings have been formed by illuminating the lens and glass plate system placed in air with monochromatic source of light. Let Dm and Dn are the diameters of m[th] and n[th] dark rings, respectively.

We know that the

$$D_n = \sqrt{4n\lambda R}$$

For n^th ring Diameter

$$D_n^2 = 4n\lambda R \longrightarrow \quad (1)$$

For m^th ring Diameter

$$D_m^2 = 4m\lambda R \longrightarrow \quad (2)$$

From equations (1)-(2) will get

$$D_n^2 - D_m^2 = 4(n-m)\lambda R \longrightarrow \quad (3)$$

Now, the air film has been replaced with the liquid whose μ has to be determined without disturbing the lens-glass plate arrangement as shown in the figure.

Then we have $$\Rightarrow D_n^{2'} - D_m^{2'} = \frac{4(n-m)\lambda R}{\mu} \longrightarrow \quad (4)$$

From Eqs. (3) and (4), we obtain

$$\mu = \frac{D_n^2 - D_m^2}{D_n^{2'} - D_m^{2'}}$$

Using the above formula μ can be calculated.

1. Newton's rings are observed in the reflected light of wavelength 5900 Å. The diameter of 10th dark ring is 0.5 cm. Find the radius of curvature of the lens.

Given:

Wavelength of light used, $\lambda = 5900$ Å $= 5900\times10^{-8}$ cm

Let Number of dark ring, n = 10

Diameter of 10th dark ring, $D_n = 0.5$ cm

Radius of curvature of the lens, R =?

Diameter of nth dark ring, $D_n = \sqrt{4n\lambda R}$

$$\Rightarrow D_n^2 = 4n\lambda R$$

$$\Rightarrow R = \frac{D_n^2}{4n\lambda}$$

$$\Rightarrow R = \frac{0.5^2}{4\times10\times5900\times10^{-8}} \qquad \therefore R = 105.9\ cm$$

2. In a Newton's rings experiment, the diameter of 15th ring was found to be 0.59 cm and that of 5th ring is 0.336 cm. If the radius of curvature of lens is 100 cm, find the wave length of light.

Given:

Let Number of dark rings, n = 15 and m = 5

Diameter of 15th dark ring, $D_n = 0.59$ cm

Diameter of 5th dark ring, $D_m = 0.336$ cm

Radius of curvature of the lens, R = 100 cm

Wavelength of light used, λ =?

Wavelength of light, $\lambda = \dfrac{D_n^2 - D_m^2}{4(n-m)R}$

$$\Rightarrow \lambda = \frac{0.59^2 - 0.336^2}{4\times(15-5)\times100} \qquad \therefore \lambda = 5.88\times10^{-5}\ cm$$

3. Calculate the thickness of air film at 10th dark ring in a Newton's rings system viewed normally by a reflected light of wavelength 500 nm. The diameter of 10th dark ring is 2mm.

Given:

Let Number of dark ring, n = 10

Wavelength of light used, $\lambda = 500$ nm

Diameter of 10th dark ring, $D_n = 2$ mm

In case of Newton's rings, the condition for dark ring is given as

$$2t = n\lambda \qquad \Rightarrow t = \frac{n\lambda}{2}$$

$$\Rightarrow t = \frac{10\times500\ nm}{2} \qquad \therefore t = 2500\ nm$$

4. If the diameter of 20th dark ring has changed from 1.55 cm to 1.33 cm when a transparent liquid is introduced between the plano-convex lens and plane glass plate, find the refractive index of the liquid.

Given:

Let Number of dark ring, n = 20

Diameter of the ring in air, $D_1 = 1.55$ cm

Diameter of the ring in liquid, $D_2 = 1.33$ cm

Refractive index of liquid, μ =?

Diameter of dark ring, $D_n^2 = \dfrac{4n\lambda R}{\mu}$

With air medium, $D_1^2 = \dfrac{4n\lambda R}{1}$ $(\because\ for\ air\ \mu=1)$

With liquid medium, $D_2^2 = \dfrac{4n\lambda R}{\mu}$

$$\mu = \frac{D_1^2}{D_2^2} = \frac{1.55^2}{1.33^2} \qquad \therefore \mu = 1.358$$

General questions:

1. What happens' the ring system, when air film is replaced with liquid film? Since $\mu_{air} < \mu_{liquid}$, the diameter of rings decreases.
2. Can we use polychromatic (white) light in Newton's Rings experiment?

   No. Because, a ring system with multiple colours is formed and there is distinction between bright and dark rings.
3. What happens' the ring system, when the lens is lifted upwards slowly? The ring system disappears due to increased thickness of air film.
4. What will happen to Newton's Rings if a plane glass plate is replaced by a plane mirror?

   No rings are formed. But uniform illumination will result. If the plane glass plate is replaced by a plane mirror, the transmitted light rays before will be reflected now. Due to the superposition between reflected and transmitted (now reflected) rays, uniform illumination will result.
5. What will happen to Newton's Rings if a plane glass plate is replaced by another plano- convex lens?

   More compressed rings will form. If the glass plate is replaced with another plano-convex lens, the optical paths changes more rapidly resulting rings to form more compressed rings.
6. Newton's Rings are circular. Why?

   Since the lens is symmetric along its axis, the thickness is constant along the circumference of a ring of given radius. Thus, Newton's rings are circular.
7. Why an extended source of light is required for the formation of Newton's rings?

   An extended source of light is a collection of large number of point sources and each point gives a beam of parallel rays. All the parallel rays from one point correspond to one Newton ring. Thus, an extended source of light is required for the formation of Newton's rings.
8. Newton's rings are not evenly spaced. Why?

   Since the diameter of the ring does not increase in the same proportion as the order of ring, they do not spaced evenly.

Diffraction

Diffraction: The phenomenon of bending of light near the edges of an obstacle or spreading of light within the geometrical shadow region of an obstacle is called the diffraction of light. The diffraction effects were first observed by Grimaldi in 1665.

Examples:

- Bending of light around the corners of door and windows.
- Sun rays coming from the clouds.
- The rainbow pattern found on the hologram of your credit/debit card.
- The rainbow pattern found on a CD/DVD.
- The setting sun appears to be red because of the diffraction of light from dust particles in the atmosphere
- Glittering of dust particles in weather.
- Shining/glittering of leaves.
- Light modification around the edges of razor blade.

Applications of Diffraction of light: The diffraction of light is used

1. To transform the light into its spectrum using grating

2. In discovering the structures of materials

3. in discovering the medicines and drugs

4. In identifying the dust particles in atmosphere

5. in holography

Conditions for Diffraction:

Diffraction effect depends upon the size of the obstacle and the wavelength of light.

For diffraction to be more effective, the size of the obstacle must be in the order of wavelength of the light used.

The diffraction effect has been observed only when a portion of the wave front is cutoff by an obstacle.

The condition to observe the diffraction due to an object (or narrow slit) on a screen:

$$\left(\frac{b^2}{l}\right) \approx 4\lambda$$

where l is the distance between the screen and the object,

b is the size of the object and λ is the wavelength of light used.

The alternate bright and dark fringes in diffraction are due to the interference of secondary wavelets.

The intensity of diffraction fringes decrease with increase of diffraction order.

Since the intensity of secondary wavelets decrease with increase of distance from the source, the intensity of higher order diffraction maxima decreases.

Distinction between interference and diffraction

Interference	Diffraction
1. It is due to superposition of two wave fronts from two coherent sources.	1. It is due to superposition of secondary wavelets from two different parts of the same wave front.
2. Interference bands are of equal width.	2. The width of diffraction bands decreases with the increase of order.
3. The intensity of bright fringes remains the same.	3. The intensity of bright fringes decreases with the increase of order.
4. All dark fringes have zero intensity.	4. The intensity of dark fringes is not zero.

Types of diffraction:

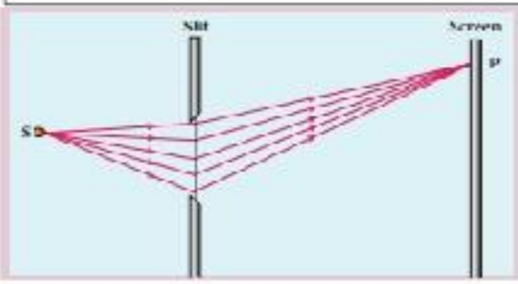
The phenomenon of diffraction has been explained by Fresnel and Fraunhofer. Hence, the diffraction of light is classified into two categories.

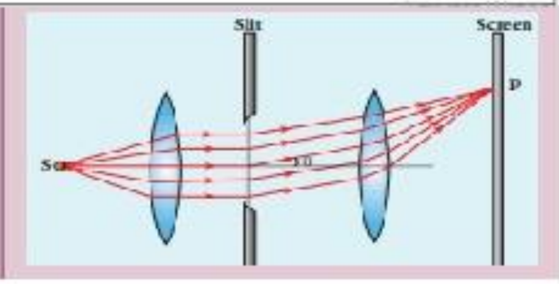
1. Fresnel's diffraction

2. Fraunhofer diffraction

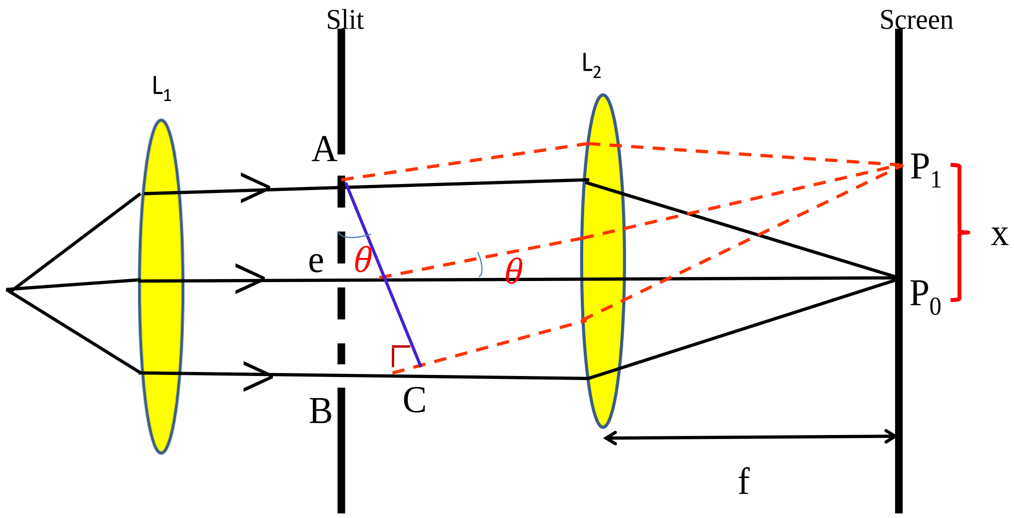
Distinction between Fresnel's and Fraunhofer's

Fresnel diffraction	Fraunhofer diffraction
• In this case, the source of light and the screen are effectively at finite distances from the obstacle.	• In this case, the source of the light and screen are effectively at infinite distances from the obstacle.
• Incident wave front is a spherical wave front.	• Incident wave front is a Plane wave front.
• No lenses are required.	• Lenses are required.
• Phase of secondary wavelets is not same at all points in the plane of obstacle.	• Phase of secondary wavelets is same at all points in the plane of obstacle.
• Theoretical treatment is complex.	• Theoretical treatment is simple.
• The condition to observer the Fresnel diffraction is $\frac{b^2}{l\lambda} \approx 1$.	• The condition to observe the Fraunhofer diffraction is $\frac{b^2}{l\lambda} << 1$.





Fraunhofer's single slit diffraction:



To study the Fraunhofer's diffraction due to single slit, consider a slit of width 'e'. Let a plane wave front of monochromatic light of wavelength 'λ' propagates normally and illuminates the slit. The diffracted light is focused by means of a convex lens 'L_2' on a screen placed at distance equal to the focal length of convex lens. Each Point on the slit (AB) act as sources of secondary wave lets and sends out secondary waves in all directions. The diffracted rays after passing through lens L_2 are brought to focus on the screen.

The waves travelling from A and B reaching P_0 are in phase. Thus the path difference between AP_0 and BP_0 is zero. The waves superpose constructively resulting in central maximum at P_0 (Bright region). Now consider secondary wavelets travelling in the direction AP_1 inclined at an angle of 'θ'. All the secondary wavelets travelling in this direction reach the point P1 on the screen.

A perpendicular is drawn from A to the line BP_1. BC is the path difference between the waves travelling from A and B reaching P_1.

Since, from right angled triangle ABC,

$$\sin\theta = \frac{BC}{AB}$$

The Path difference= $BC = e\sin\theta$

Now consider the Phase difference= $\delta = \frac{2\pi}{\lambda} \times pathdifference$

Phase difference $= \delta = \frac{2\pi}{\lambda} \times e\sin\theta$

Now slit can be divided into "n" equal parts

$$\frac{1}{n} \times \frac{2\pi}{\lambda} \times e\sin\theta = \text{(assuming as “d’)}$$

Using the method of vector addition of amplitudes, the resultant amplitude (R) is given by

$$R = \frac{a\sin\frac{nd}{2}}{\sin\frac{d}{2}} \qquad \text{(Here d=}\frac{1}{n} \times \frac{2\pi}{\lambda} \times e\sin\theta\text{)}$$

$$R = \frac{a\sin\dfrac{n\times\frac{1}{n}\times\frac{2\pi}{\lambda}\times e\sin\theta}{2}}{\sin\dfrac{\frac{1}{n}\times\frac{2\pi}{\lambda}\times e\sin\theta}{2}}$$ Since $$R = \frac{a\sin\frac{\pi}{\lambda}e\sin\theta}{\sin\dfrac{\frac{\pi}{\lambda}ein\theta}{n}}$$

We are assuming that $\alpha = \frac{\pi}{\lambda}e\sin\theta$

Since $R = \dfrac{a\sin\alpha}{\sin\frac{\alpha}{n}}$ (Here $\sin\frac{\alpha}{n} = \frac{\alpha}{n}$)

$$R = \frac{a\sin\alpha}{\frac{\alpha}{n}} = R = \frac{na\sin\alpha}{\alpha} = R = A\left(\frac{\sin\alpha}{\alpha}\right)$$ (Where A = na)

The resultant intensity can be written as $I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 = I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$ where $I_0 = A^2$

The above equation represents the resultant intensity due to single slit.

Condition for principal maximum:

$$R = A\left(\frac{\sin\alpha}{\alpha}\right) = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + - - - - -\right]$$

If the negative terms vanish, the value of R is maximum.

i.e., α =0 or $\alpha = \frac{\pi}{\lambda}e\sin\theta = 0$

Since $\sin\theta = 0$

Thus, the secondary wavelets travelling normal to the slit (R=A) that corresponds to principal maximum.

Positions of Secondary Maxima in Fraunhofer's diffraction at single slit:

The Position of secondary maxima can be obtained by differentiating the expression of resultant intensity (I) w.r.t to phase difference α and equating to zero.

$$\therefore \frac{dI}{d\alpha} = 0 \quad \Rightarrow \frac{d}{d\alpha}\left[I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$I_0 \cdot \frac{2\sin\alpha}{\alpha} . \frac{(\alpha.\cos\alpha - \sin\alpha)}{\alpha^2} = 0$$

$$\because d(uv) = u\,dv + v\,du$$
$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

This equation implies that

$\sin\alpha = 0$ (or) $(\alpha.\cos\alpha - \sin\alpha) = 0$

But $\sin\alpha = 0$ reveals the position of intensity

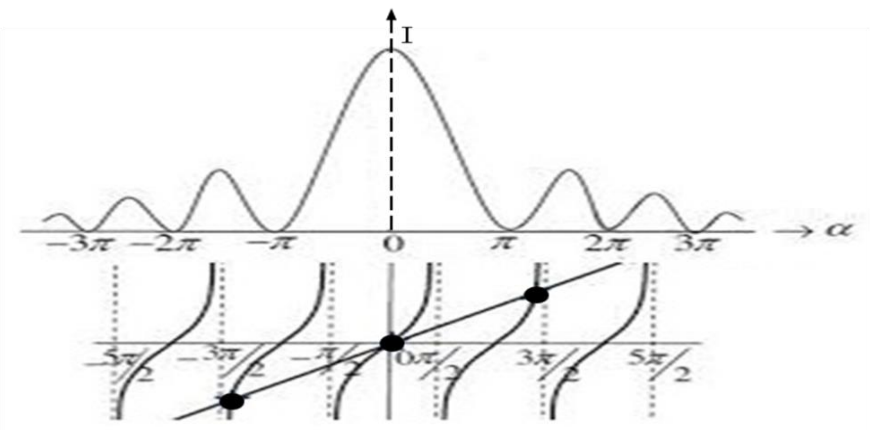
Thus the position of secondary maxima following to the

$(\alpha.\cos\alpha - \sin\alpha) = 0$

The values of α satisfying $\alpha = tan\alpha$ are obtained graphically by plotting the curves, $y = \alpha$ and y = tanα on the same graph.

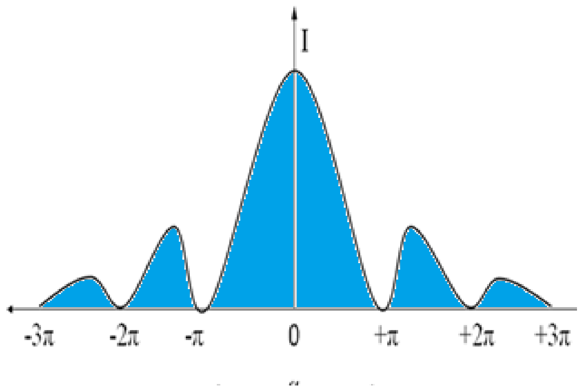
The points of intersection of the two curves gives the values of α which satisfy y = tanα.

$$\alpha = 0 \pm \frac{\pi}{2} \pm \frac{3\pi}{2} \pm \frac{5\pi}{2} \pm \ldots\ldots\ldots..$$



Intensity Distribution on the screen due to Fraunhofer's diffraction at single slit:

A schematic representation of the variation of the intensity (I) as a function of net phase difference (α) is shown in figure.

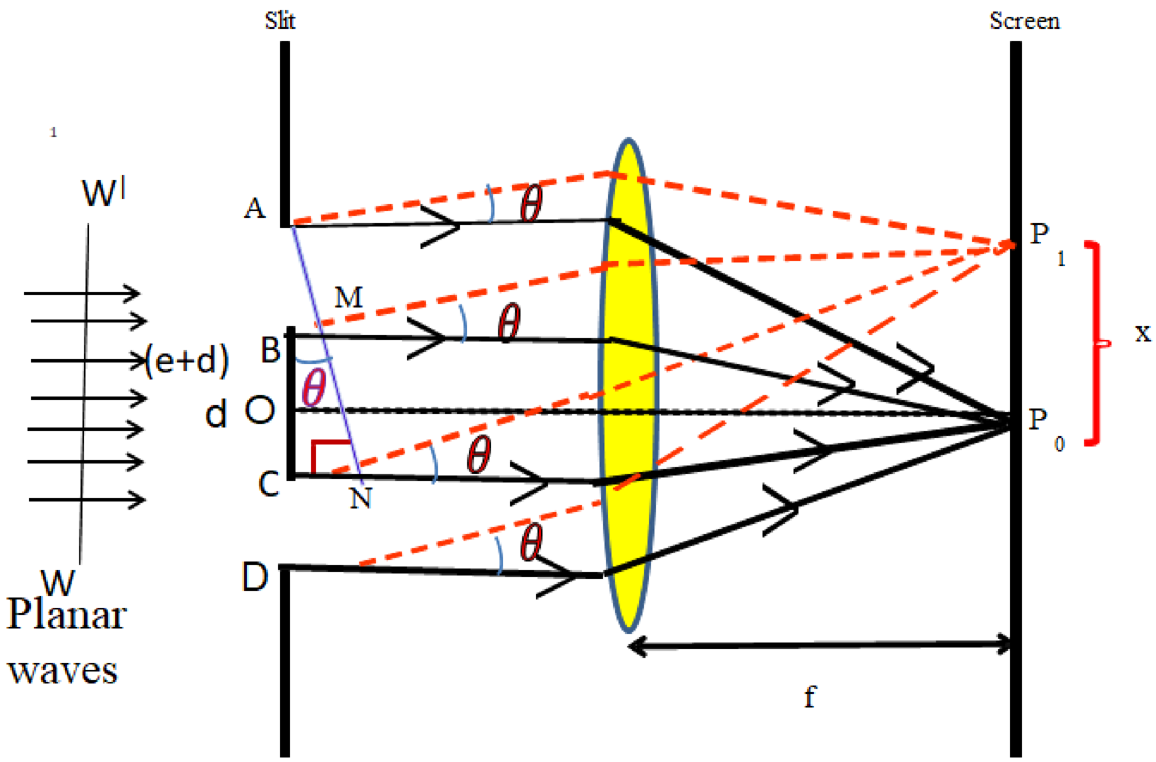


From the figure, it is clear that the diffraction pattern due to single slit consist of central bright maximum followed by secondary maxima on both sides.

Intensity of higher diffraction orders decrease with increase of order of diffraction.

Secondary maxima are not peaked exactly midway between two minima i.e., the positions of the secondary maxima are slightly displaced towards the central maximum.

FRAUNHOFER DIFFRACTION AT DOUBLE SLIT



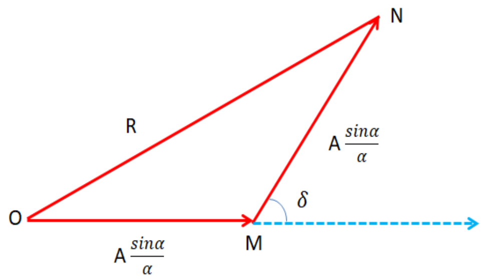
$WW^{|}$ is the planar waves of monochromatic wave length λ incident on two rectangular slits waves parallel to one another and perpendicular to the plane of the paper. The width of each slit (open portion) is e and the width of the opaque portion is d. Each and every point on the screen acts secondary wavelets. These wavelets are focused on a point P_0 and placed on equal focal length (f) of the screen to obstacle. Therefore, P corresponds to the position of the central bright maximum.

In this case, the diffraction pattern has to be considered in two parts (i) the interference phenomenon due to the secondary waves emanating from the corresponding points of the two slits and (ii) the diffraction pattern due to the secondary waves from the two slits individually. For calculating the positions of interference or maxima and minima, the angle is denoted as θ, where θ refers to the angle between the direction of the secondary waves and the initial direction of the incident light.

Expression for Resultant Intensity

Let us now find an expression for the resultant intensity and its variation with the angle θ. By Huygens' principle, every point in the slits AB and CD sends out secondary wavelets in all directions. From the theory of diffraction at a single slit, the resultant amplitude due to wavelets diffracted from each slit in a direction θ is

$$A = A\left(\frac{\sin\alpha}{\alpha}\right)$$



In the case of double slit, we have interference of two waves of amplitude A each, having a phase difference

$$\delta = \frac{2\pi}{\lambda}\times(e+d)\sin\theta$$

from the $Sin\theta = \dfrac{DC}{AB} \Rightarrow DC = (e+d)Sin\theta$

$$DC = Pathdifference = (e+d)Sin\theta$$

$$ON^2 = (OM)^2 + (MN)^2 + 2OMON\cos\delta$$

$$R^2 = \left(A\frac{\sin\alpha}{\alpha}\right)^2 + \left(A\frac{\sin\alpha}{\alpha}\right)^2 + 2\left(A\frac{\sin\alpha}{\alpha}\right)\left(A\frac{\sin\alpha}{\alpha}\right)\cos\delta$$

$$R^2 = 2\left(A\frac{\sin\alpha}{\alpha}\right)^2(1+2\cos\delta)$$

$$R^2 = 2\left(A\frac{\sin\alpha}{\alpha}\right)^2\left(1+2\cos^2\frac{\delta}{2}-1\right)$$

$$R^2 = 4\left(A\frac{\sin\alpha}{\alpha}\right)^2\left(\cos^2\frac{\delta}{2}\right)$$

$$\frac{\delta}{2} = \beta = \frac{\pi}{\lambda}(e+d)\sin\theta$$

$$R^2 = 4\left(A^2\frac{\sin^2\alpha}{\alpha^2}\right)\left(\cos^2\beta\right)$$

$$I = 4\left(I_0\frac{\sin^2\alpha}{\alpha^2}\right)\left(\cos^2\beta\right)$$ **(Here=** $A^2 = I_0$ **)**

Thus, the resultant intensity at any point depends on two factors –

(i) The factor $\left(I_0\frac{\sin^2\alpha}{\alpha^2}\right)$ is the same as that derived for a single slit Fraunhofer diffraction.

It represents the intensity variation in the diffraction pattern due to any individual slit.

(ii) The factor $\cos^2\beta$ gives the interference pattern due to waves overlapping from the two slits. The resultant intensity at any point on the screen is given by the product of these two factors and will be zero when either of these factors is zero

Maxima and Minima

The diffraction term $\left(I_0\frac{\sin^2\alpha}{\alpha}\right)$ gives the central maximum in the direction $\theta = 0$ having alternate minima and secondary maxima of decreasing intensity on either side.

The angular positions of minima are given by,

$$\sin\alpha = 0$$

$$\alpha = \pm n\pi$$

i.e., $\frac{\pi}{\lambda} e \sin\theta = \pm n\pi$ n=1,2,3,……….. but not zero

$$\sin\theta = \pm\frac{n\lambda}{e}$$

The angular positions of secondary maxima approach to

$$\alpha = \pm\frac{3\pi}{2} \pm \frac{5\pi}{2} \pm \frac{7\pi}{2} + ---------$$

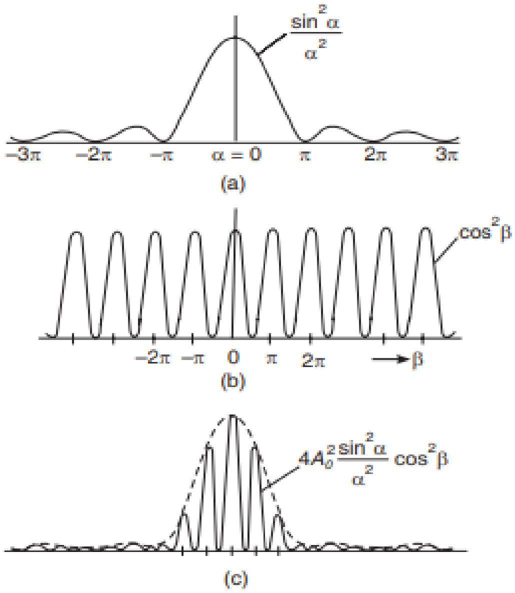
According to the second factor $(\cos^2\beta)$ the intensity will be maximum when

$$\cos^2\beta = 1$$

i.e., $\cos^2\beta = 1$ when n=0,1,2,3,……………

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi = 1 \qquad \text{where } n = 0, \theta = 0$$

Thus, the central maximum of interference pattern lies along the direction of incident light. This is called the principal maximum of zero order. The central maximum of diffraction pattern also lies along this direction. At this point all the waves arrive with the same phase. Hence, the intensity of central maximum is the highest.



The intensity will be maximum when

$$\cos^2\beta = 0$$

i.e., $\frac{\delta}{2} = \pm(2n+1)\frac{\pi}{2} \Rightarrow$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm(2n+1)\frac{\pi}{2} \Rightarrow (e+d)\sin\theta = \pm(2n+1)\frac{\lambda}{2}$$

It can be shown that for small values of θ the maxima and minima are equally spaced.

If e is kept constant and d is varied, the positions of maxima and minima due to diffraction remain unchanged while those due to interference undergo a change. Above figure represent the intensity distribution determined by the factor of $\cos^2\beta$ and $\left(I_0\frac{\sin^2\alpha}{\alpha^2}\right)$.respectively.

The resultant of these curves is shown in above figure. The resultant is obtained by multiplying the ordinates of first two curves at every point.

The entire pattern may be considered as consisting of interference fringes due to interference of light from both the slits; their intensity is being modulated by the diffraction occurring at individual slits.

Diffraction Grating:

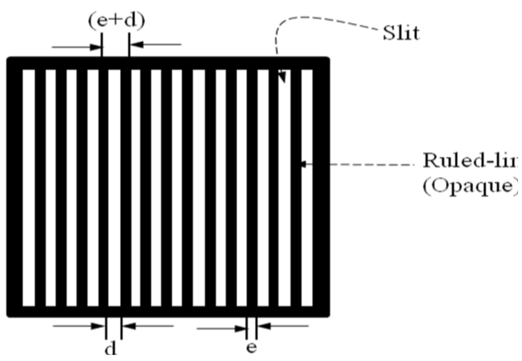
An optical device consisting of large number of parallel slits of the same width and separated by equal opaque space is known as diffraction grating.

Grating is based on diffraction at multi slits (or)N-slits.

There are two kinds of diffraction.

1. Transmission Grating (Plane)

2. Reflective Grating (Plane / Concave)



Fraunhofer used the first diffraction grating consisting of large number of parallel wires placed side by side very closely at regular intervals.

Now-a-days, grating are constructed by ruling equidistant parallel lines on a transparent material like glass with a fine diamond point.

The ruled lines are opaque to light while the space between the lines is transparent to light and act as a slit.

This grating is known as transmission grating.

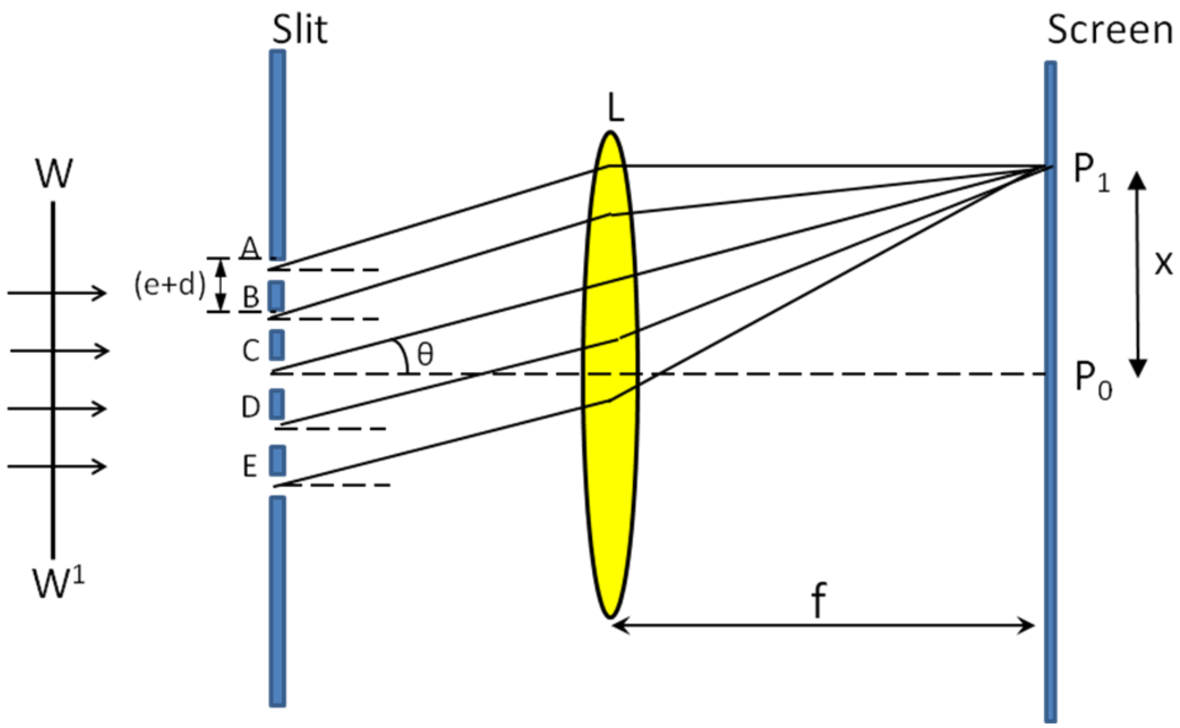
The following figure shows the schematic diagram of a transmission diffraction grating.

Commercial gratings are produced by taking the cost of an actual grating on a transparent film such as Cellulose acetate.

The solution of cellulose acetate is poured on the ruled surface and allowed to dry to form a thin film, detachable from the surface.

These impressions of grating are preserved by mounting the film in between two glass plates.

Expression for Grating Equation:



The following figure represents the section of a plane transmission grating placed perpendicular to the plane of the paper.

The following figure represents the section of a plane transmission grating placed perpendicular to the plane of the paper.

Let d and e be the width of each slit and opaque space, respectively.

Then (e+d) is known as grating element.

Let a plane wave front,   of monochromatic light of wavelength, λ is incident on normally on the grating as shown in figure.

The secondary wavelets travelling along the direction of incident wave are focused at a point P_0 on the screen placed at a distance of 'f' from the lens.

Since all the secondary wavelets travelled equal distance to reach P_0 they reinforce constructively at the point P_0 and it is known as the central maximum.

Now, consider the secondary wavelets travelling at an angle θ with the direction of incident wave and comes to focus at P_1.

The diffraction effect at P_1 can be studied applying the theory of Fraunhofer diffraction at single slit.

The amplitude of the each secondary wave is given as

$$a = A \bullet \left(\frac{\sin\alpha}{\alpha}\right), where \alpha = \frac{\pi}{\lambda} e \sin\theta \text{ --------------------- } (1)$$

Let N be the number of slits on grating.

Then, the path difference between the wavelets coming out from two consecutive slits is given as

$$\Delta = (e+d)\sin\theta \text{ --- } \mathbf{(2)}$$

Corresponding phase difference is given by

$$\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta(say) \text{ ------------------------ } \mathbf{(3)}$$

$$Here, \beta = \frac{\pi}{\lambda}(e+d)Sin\theta \text{ --------------------------------- } (4)$$

This is known as the phase difference for N slits

According to the method of vector addition of amplitudes, the resultant amplitude is given by

$$R = \frac{a\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}}$$

In the present case,

$$a = A \bullet \left(\frac{\sin\alpha}{\alpha}\right), n = N and \delta = 2\beta$$

Thus, the resultant amplitude at point P_1 is given by

$$a = A \bullet \left(\frac{\sin\alpha}{\alpha}\right) \bullet \left(\frac{SinN\beta}{Sin\beta}\right) \text{------------------}(5)$$

$$I = R^2 = A^2 \bullet \left(\frac{\sin\alpha}{\alpha}\right)^2 \bullet \left(\frac{SinN\beta}{Sin\beta}\right)^2$$

(or)

$$I = I_0 \bullet \left(\frac{\sin\alpha}{\alpha}\right)^2 \bullet \left(\frac{SinN\beta}{Sin\beta}\right)^2 \text{--------------------} (6)$$

Here, the factor $I_0 \bullet \left(\frac{\sin\alpha}{\alpha}\right)^2$ gives the distribution of intensity due to single slit

While the factor $\left(\frac{SinN\beta}{Sin\beta}\right)^2$ gives the distribution of intensity as combined effect of all the slits

Condition for maximum intensity Position:

The locations of maximum intensities are defined from the following condition

$$\beta = \pm n\pi \qquad n=0,1,2,3 \text{ --------}$$

$$\Rightarrow \frac{\pi}{\lambda}(e+d)\sin\theta = n\pi$$

$$\Rightarrow (e+d)\sin\theta = n\lambda$$

$$\therefore Sin\theta = Nn\lambda$$

This equation is known as grating equation.

Where $N = \frac{1}{(e+d)}$ is call the grating order (or) Number of lines per unit width of grating and (e+d) is known as grating element.

Intensity Distribution due to Diffraction Grating:

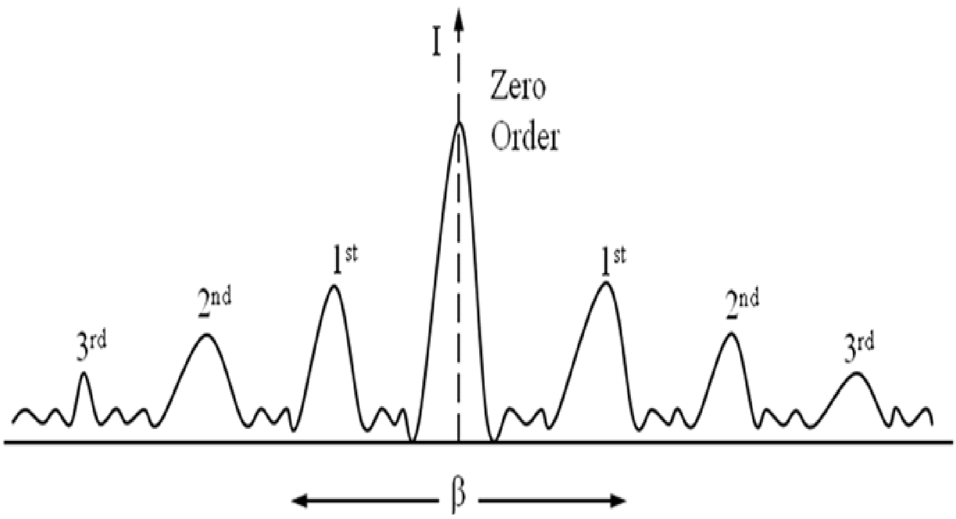
The grating equation is given as $Sin\theta = Nn\lambda$

Here n=0, corresponds to zero order maximum.

n =$\pm$1 corresponds to first order maximum

n =$\pm$2 corresponds to second order maximum

The $\pm$ sign shows that there are two maxima of the same order lying on either side of zero order maximum as shown in figure.



Grating Spectrum consist of central maximum followed by secondary maxima on both sides.

Further, the intensity of secondary maxima decreases with increase of diffraction order.

Additionally, between the two adjacent maxima there are several intensity positions of interference of secondary wave lets present. Thus the dark regions are not completely dark.

Maximum Possible Diffraction Orders with a Grating:

The grating equation is given as $Sin\theta = Nn\lambda$, n=0,1,2,3,.........

Where N is called the Number of lines per unit width on grating, n is the diffraction order and λ is the wavelength of incident light.

The maximum value that θ_{max} can have is 90° only

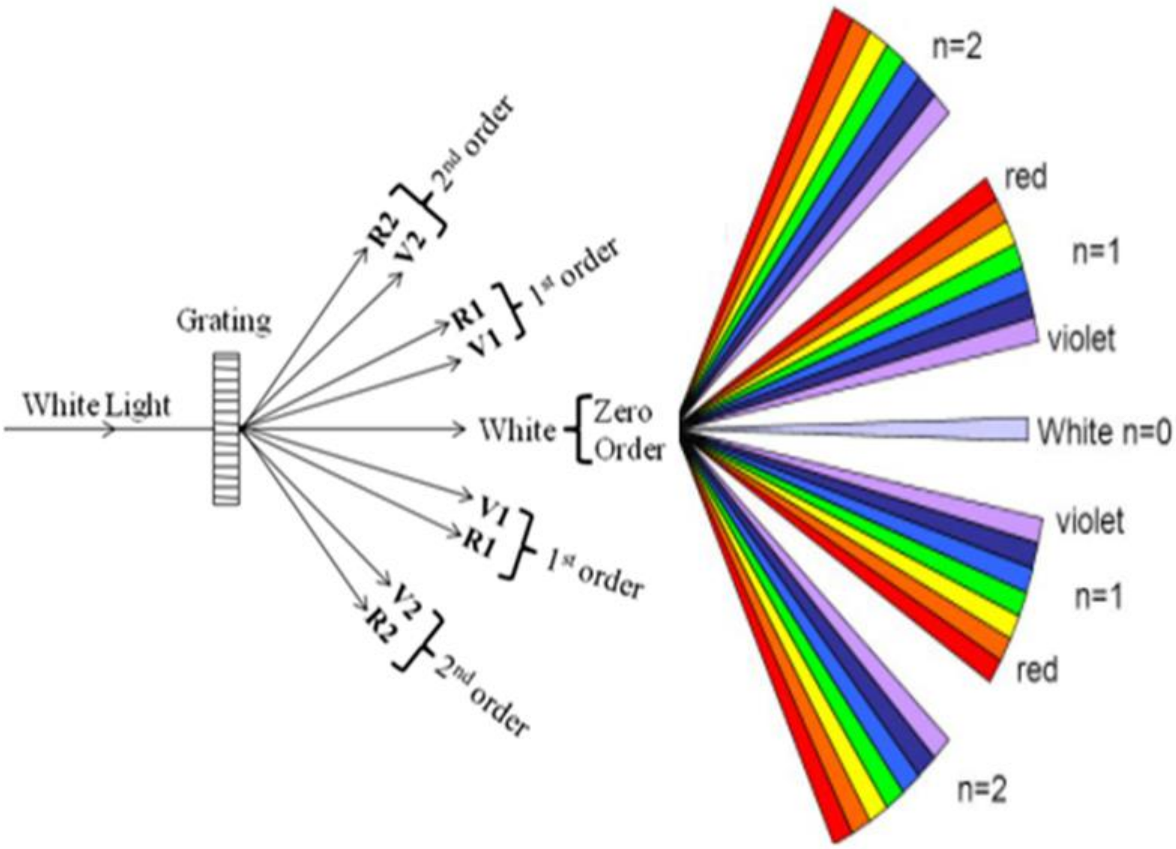
Hence, the maximum value of $\sin\theta = 1$.

From the above equation, we have $N.n_{max}.\lambda \leq 1 (or) n_{max} \leq \frac{1}{N.\lambda}$

From the above equation gives the maximum no. of possible diffraction orders.

Dispersive power of grating:

It is the property of grating to create separation between successive colours of white light.



Dispersive power can be defined as the ratio of change in diffraction angle to the change in wavelength of light

(OR)

- Dispersive power of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in wavelength between the two spectral lines.
- It can also be defined as the difference in the angle of diffraction per unit change in wavelength.
- The diffraction of the nth order principal maximum for a wavelength, l, is given by the equation

$$\Rightarrow (e+d)\sin\theta = n\lambda$$

Differentiate this equation with respect to θ and λ

We get $\Rightarrow (e+d)Cos\theta d\theta = nd\lambda$

'(or) $\Rightarrow \frac{d\theta}{d\lambda} = \frac{nN}{\cos\theta}$ (Here $\frac{1}{(e+d))} = N$)

$\frac{d\theta}{d\lambda} \propto n$ Where "**n**" is the order of the spectrum. So as higher is the order of spectral lines then the wavelength will be greater.

$\frac{d\theta}{d\lambda} \propto N$ N is number of lines per on grating. That means any two spectral lines can be separated more in same order. If more lines are drawn on the diffracted

$\frac{d\theta}{d\lambda} \propto \frac{1}{\cos\theta}$ If $\theta = 0$, $\cos\theta = 1$, the angular dispersion is less

If $\theta = 0, \cos\theta = 1$, the angular dispersion of any two spectral lines is directly proportional to the difference in wavelength of the spectral lines. A spectrum of this type is called a normal spectrum.

If the linear spacing of two spectral lines of wavelengths λ and $\lambda + d\lambda$ is dx in the focal plane of the telescope objective or photographic plate, then

$$dx = f d\theta$$

Where f is the focal length of the objective. The linear dispersion is

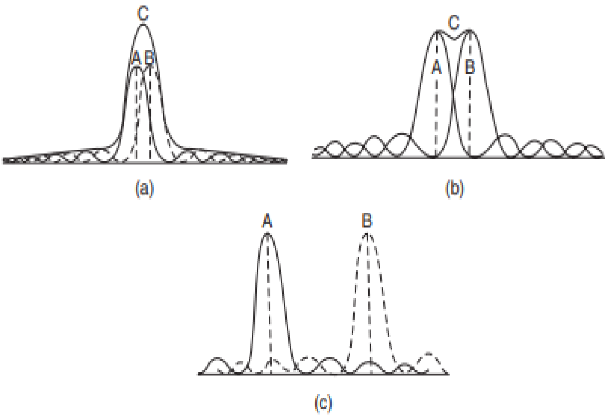
$$\frac{dx}{d\lambda} = \frac{f d\theta}{d\lambda} = \frac{f n N^{|}}{\cos\theta}$$

$$dx = \frac{f n N^{|}}{\cos\theta} d\lambda$$

The linear dispersion is useful in studying the photographs of a spectrum.

RESOLVING POWER:

The ability of an optical instrument to produce distinctly separate images of two objects located very close to each other is called its resolving power.



The images of two sources are discernable when they satisfy Rayleigh criterion as seen in the third image at the top

Rayleigh's Criterion:

Rayleigh suggested that the two images of such point-objects lying close to each other may be regarded as separated if the central maximum of one falls on the first minimum of the other.

i.e., the equivalent to the condition that the distance between the centers of the patterns shall be equal to the radius of the central disc. This is called the Rayleigh criterion for resolution and is also known as Rayleigh's limit of resolution.

$\theta = \frac{1.22\lambda}{b}$, $\theta =$ angle of resolution (Rad), b = diameter of circular opening, $\lambda =$ wave length

Application of Rayleigh's Criterion:

RESOLVING POWER OF A PLANE TRANSMISSION GRATING: (R.P)

It is the ratio of wavelength of any spectral line (λ) to the wavelength of difference between the adjacent lines $(d\lambda)$. So that there two lines appears to be just resolved.

$$R..P = \frac{\lambda}{d\lambda}$$

The m[th] principal maxima of $(\lambda + d\lambda)$ the $(\theta_m + d\theta_m)$ is given by

We know that from grating formula

$$\Rightarrow (e+d)\sin\theta = m\lambda$$

$$\Rightarrow (e+d)\sin(\theta_m + d\theta_m) = m(\lambda + d\lambda) \text{---------------------}(1)$$

Grating formula for mN+1 minima condition

$$N(e+d)\sin(\theta_m + d\theta_m) = (mN+1)\lambda$$

$$(e+d)\sin(\theta_m + d\theta_m) = m(\lambda + d\lambda) \qquad \text{(From equation 1)}$$

$$Nm(\lambda + d\lambda) = (mN+1)\lambda$$

$$\lambda = m.Nd\lambda \qquad \therefore \frac{\lambda}{d\lambda} = m.N$$

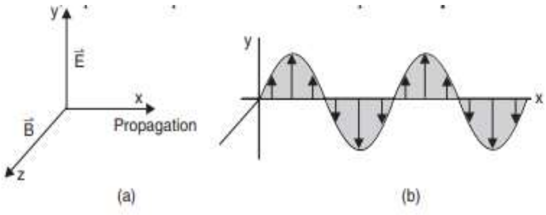
The above equation is known as Resolving power of grating

Diffraction Completed

POLARIZATION

Polarization: "The process of restricting the vibrations of the electric field vector (E) in a particular plane is called polarization"

According to electromagnetic theory, light is an electromagnetic disturbance in which the electric and magnetic fields are in the transfer plane the two fields varying continuously and rapidly with time. It has been found experimentally that it is the electric field E-designated briefly as the electric vector that produces the optical polarization.



A light wave is described by the electric field vector, E

Polarization of light waves:

When ordinary light is passed through a pair of tourmaline plates P_1 and P_2 with their planes at right angles to the direction of light as shown in fig (a) the intensity is maximum in this position. But when the plane of P_2 is rotated through 90°, the plane of P_2 is perpendicular to the plane of P_1 as shown in fig (b). The intensity is minimum in this case, this shows that

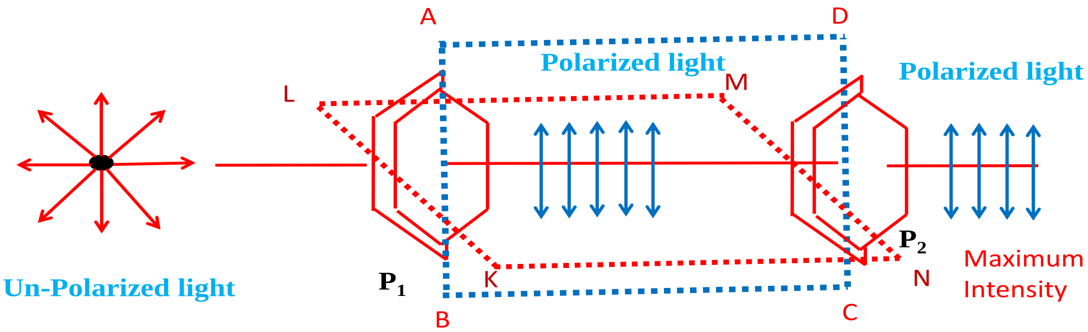


Fig: (a)

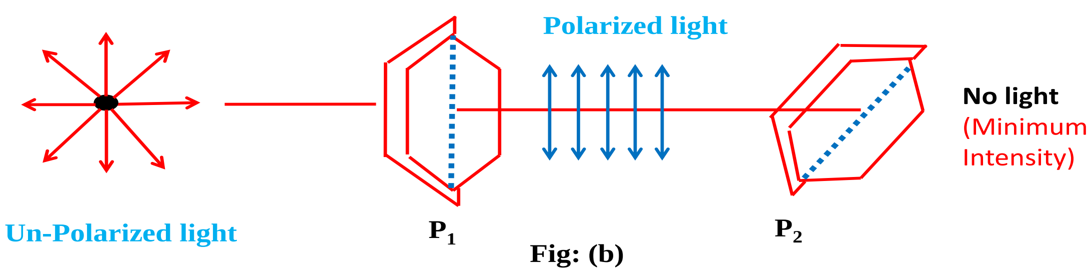


Fig: (b)

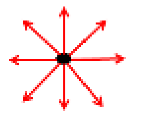
light is a transvers wave motion. It is clear that after passing through the crystal P_1 the light vibrates only in one direction. "The light which has required the property of one sidedness is called the polarized light.

Any instrument which produces polarized light is known as **polarizer**.

Some instrument is required to detect polarization and also the direction of vibration in the polarized light such a device is known as **analyzer**.

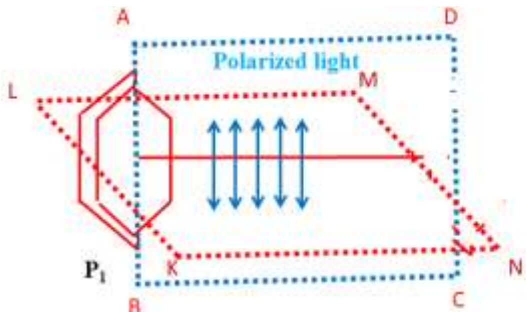
Natural or Un polarized light

Light consist of vibrations of electric vector (E) taking place in all transverse direction with random orientation.



Plane Polarized light

Light in which the vibrations are confined in one particular transverse plane only.



The plane in which vibrations takes place is known as plane of vibration i.e., Plane ABCD

The Plane at right angles to the plane of vibration is called the plane of polarization i.e., Plane LMNK

Diagrammatically representation of polarized light

The diagrammatically representation of natural unpolrised light consisting of linear vibrations in all transverse direction as shown in fig (**a**)

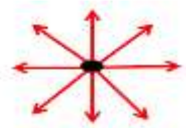


Fig: (a)

In fig (**b**) shows the double headed arrows represent vibrations of electric vector in the plane of the paper and "**dots**" are the top views of such arrows that are oriented **normal (or) perpendicular** to the plane of the paper.

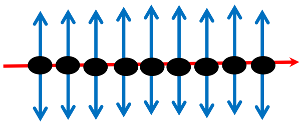


Fig: (b)

In fig (**c**) represented the plane polarized light travelling to the right and consisting of vibrations which are **normal (or) perpendicular** to the plane of paper.

Fig: (c)

In fig (**d**) represented the plane polarized light travelling to the right and consisting of vibrations to the plane of paper.

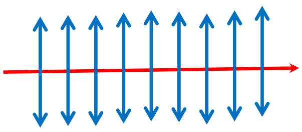


Fig: (d)

	Unpolarized light	*Polarized light*
1.	Consists of waves with planes of vibration equally distributed in all directions about the ray direction.	Consists of waves having their electric vector vibrating in a single plane normal to ray direction.
2.	Symmetrical about the ray direction	Asymmetrical about the ray direction
3.	Produced by conventional light sources.	Is to be obtained from unpolarized light with the help of polarizers.
4.	May be regarded as the resultant of two *incoherent* waves of equal intensity but polarized in mutually perpendicular planes.	May be regarded as the resultant of two mutually perpendicular *coherent* waves having zero phase difference.

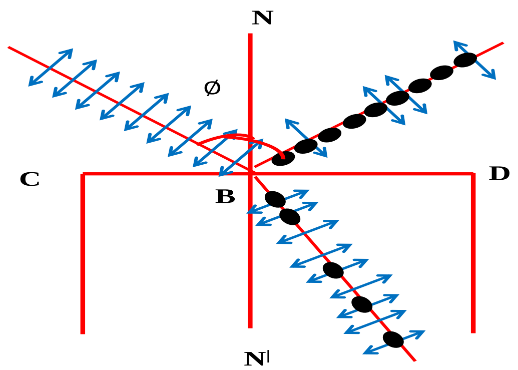
Methods of producing Plane polarized light

There are three important methods for producing Plane polarized light

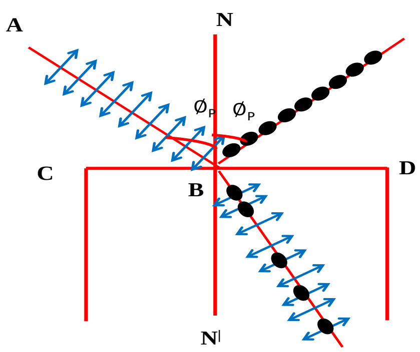
(1) By Reflection (2) By Refraction (3) By double refraction

Polarization by Reflection

It was discovered by E.L. Malus in the year of 1808. In Fig (a) a beam of natural light is incident in air on the glass surface CD. This beam consists of two vibrations at right angles to each other. It is found that the reflected and refracted the reflected beam is partially polarized for all angle of incidence. The reflected light mostly consists of dot components along with a few arrow components. The transmitted light mostly consists of arrow components with some dot components as shown in beside figure



But when the light is incident at the polarizing angle then none of the arrow components is reflected, they are all transmitted. Allowed 15% of dots components are reflected from the glass surface. Hence the reflected light is weak and completely polarized. The refracted light is a mixture arrow components all of which are refracted and the remaining 85% of the dot components. Hence it is strong but only partially polarized.



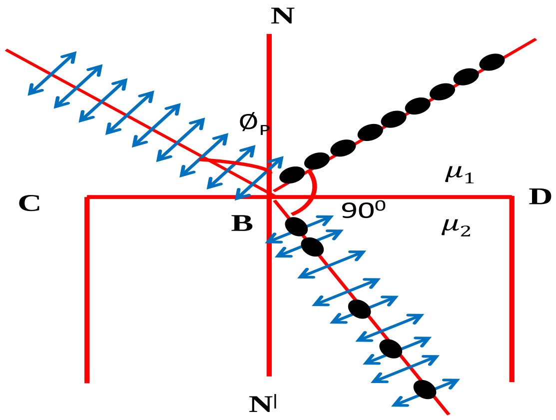
Brewster's LAW

Brewster, found that the polarizing angle depends upon refractive indices of the reflecting material and of the medium in which it is located. According to Brewster, the tangent of the polarizing angle is equal to the refractive index of the reflecting material with respect to its surroundings.

$$\tan\phi_P = \frac{\mu_2}{\mu_1}$$

μ_1 = absolute refractive index of the surrounding media

μ_2 = absolute refractive index of the reflecting material



$$\therefore \tan\phi_p = \mu_{12}$$

According to smell's Law

$$\mu_{12} = \frac{\sin\phi_p}{\sin r_0}$$

$$\therefore \tan\phi_p = \mu_{12}$$

$$\frac{\sin\phi_p}{\cos\phi_p} = \frac{\sin\phi_p}{\sin r_0}$$

$$Cos\phi p = Sin r_0$$

$$Sin(90^\circ - \phi_p) = Sin r_0$$

$$\Rightarrow 90^\circ - \phi_p = r_0$$

$$\Rightarrow \phi_p + r_0 = 90^\circ$$

When the light is incident at the polarising angle the reflected and refracted rays are perpendicular to each other.

Application of Brewster's law:

(i) Brewster's law can be used to determine the refractive indices of opaque materials.

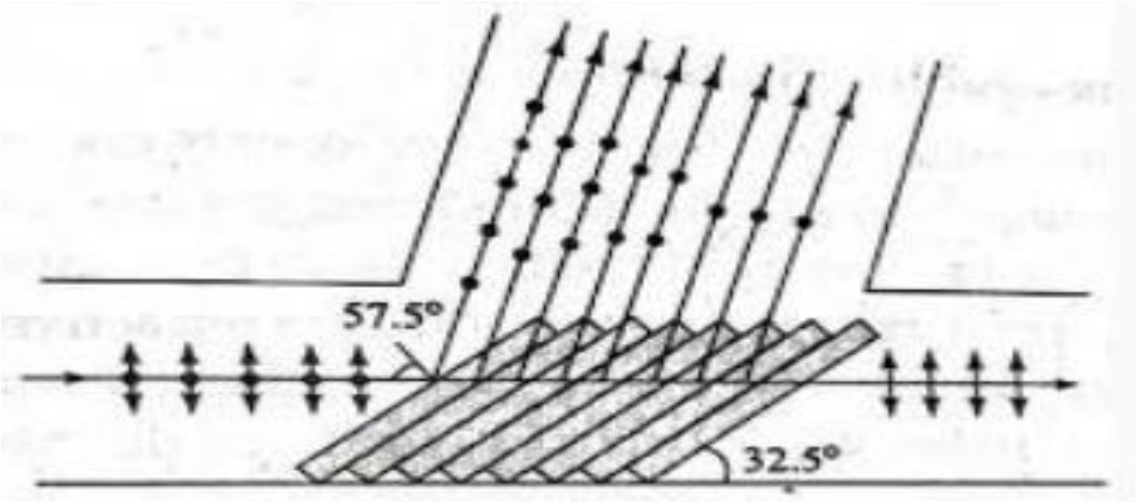
(ii) It helps us in calculating the polarizing angle necessary for total polarization of reflected light for any material if its refractive index is known. However, the law is not applicable for metallic surfaces.

(iii) In gas lasers are arranged at Brewster angle to the axis of the laser tube for linearly polarized light.

(iv) Another application utilises the Brewster angle for transmitting a light beam into or out of an optical fibre without reflection losses.

Polarization by Refraction - Pile of Plates

When un-polarized light is incident at Brewster angle on a smooth glass surface, the reflected

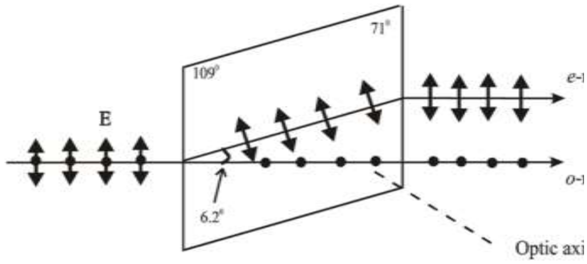


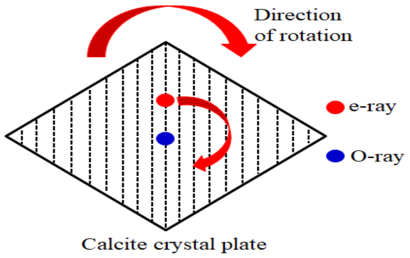
light is totally polarized, while the refracted light is partially polarized. If natural light is transmitted through a single plate, the transmitted beam is only partially polarized. If a stack of glass plates is used instead of a single plate, reflections from successive surfaces occur leading to the filtering of the **dots** -component in the transmitted ray. Ultimately, the transmitted ray consists of **arrows**-component alone. It is found that a stack of about 15 glass plates is required for this purpose. The glass plates are supported in a tube of suitable size and inclined at an angle of about 33° to the axis of the tube, as shown in the above figure. Such an arrangement is called a pile of plates. Un-polarized light enters the tube and is incident on the plates at Brewster angle and the transmitted light will be totally polarized parallel to the plane of incidence.

Polarization by double Refraction

Anisotropic media are those homogeneous media in which velocity of light waves are not same in all direction. Crystal having this property is known as double refracting or bi-refringent, because two type of Huygens wave fronts are produced in such crystals.

The phenomenon of double refraction by calcite was first observed by Bartholinus in 1669. Many of the crystalline surfaces like quartz, mica, sugar, topaz, aragonite and ice. Etc., are found to exhibit double refraction.





Natural calcite crystal has a rhombohedral form having angles of 71° and 109°. If a rhombohedral crystal has placed over a black dot on a piece of paper, two dots are seen.

If a beam of natural light falls on one of the faces of the crystal, two refracted beams are produced. One of these two rays i.e., O-ray passes through the crystal without deviation. It is called ordinary ray because it obeys the Snell's Law of refraction. The E-ray deviates from its perpendicular incidence, it is called extra ordinary ray. It does not obey Snell's Law of refraction. The velocity of E-ray in different in different direction, whereas the o-ray travels with same velocity in all directions. If the calcite crystal is rotated around the o-ray as on axis, then E-ray rotates around the stationary O-ray.

In this case the O-ray consist of dot components i.e., vibrations of the o-ray are perpendicular to the optic axis, the E-ray consist of arrow components arrow components i.e., its vibrations takes place along the optic axis.

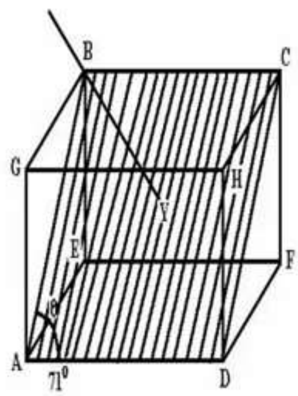
The refractive index of the crystal with respect to the ordinary light is known as ordinary refractive index μ_0 and it is 1.66.

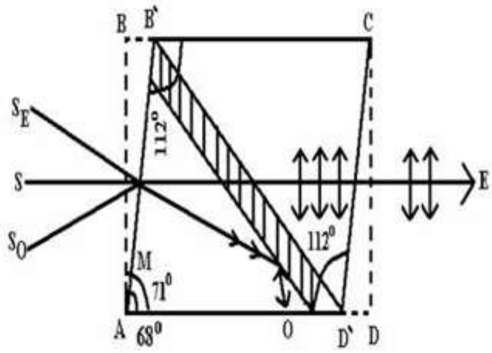
The refractive index of the crystal with respect to the ordinary light is known as ordinary refractive index μ_0 and it is 1.458.

Nicol Prism:

It is an optical device made from calcite and is used in optical instruments both for producing and analysing plane polarised light. It is based on the phenomenon of double refraction and was invented by Nicol.

Construction:





A Nicol prism is made from calcite crystal. It was designed by William Nicol in 1820. A calcite crystal whose length is three times as its width is taken.

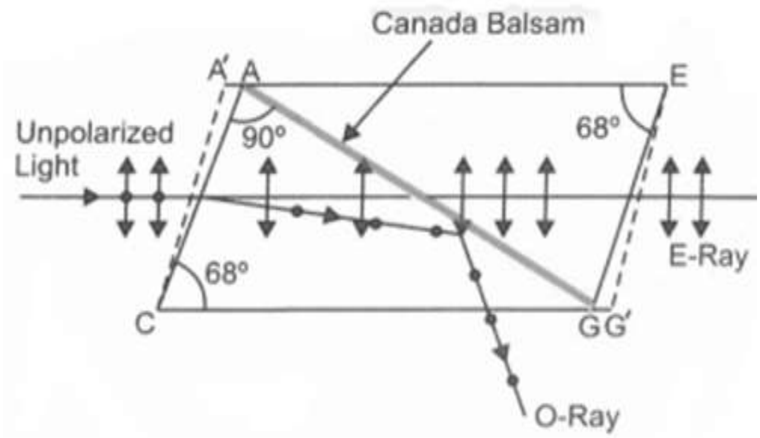
The end faces of this crystals are grounded in such a way that the angles in the principal section becomes 68 and 112 instead of

71 and 109 the crystal is cut in two pieces by a plane perpendicular **to** the principal section as well as the new end faces.

The two parts of the crystal are then cemented together with Canada balsam. The refractive index of Canada balsam lies between the refractive indices for the ordinary and extra-ordinary rays for calcite**.(μ_O=1.66, μe =1.486 and μCanada balms = 1.55)**

Working Principle:

The position of optic axis AB is as shown in figure. Unpolarized light is made to fall on the crystal as shown in figure.



The ray after entering the crystal suffers double refraction and splits up into O-ray and e-ray.

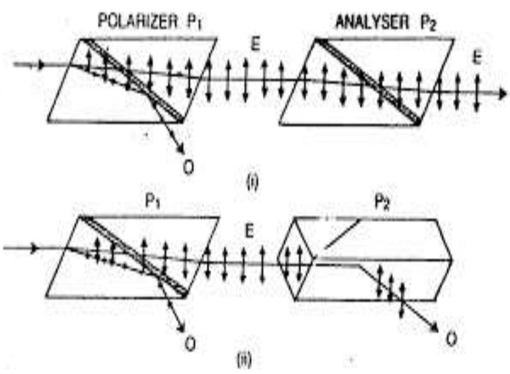
The values of the refractive indices and the angles of incidence at the Canada balsam layer are such that are e-ray is transmitted while the o-ray is internally reflected.

The ordinary o-ray strikes the black surface of the crystal at an angle greater than the critical angle.It travelling optically in denser medium to optically rarer medium. It suffers total internal reflection.

The totally reflected o-ray strikes the black surface of the crystal at bottom so that the o-ray is completely absorbed.

Thus we get only the linearly polarized e-ray coming out of the Nicol prism with the direction of vibration as shown in figure.

Thus, the Nicol Prism works as a polarizer.



The Nicol prism is the most widely used polarizing device. **It is good polarizer but expressive. It has a limited field of view of about 28 degrees.**

Negative crystals

Negative crystals are crystals in which refractive index corresponding to E-Ray (nE) is less than the refractive index corresponding to O-Ray (nO) in all directions except for Optic axis. Ex: Calcite , Tourmaline ,Ruby

Positive crystals

Positive crystals are crystals in which refractive for O-Ray is less than that for E-Ray($n0<nE$). Example : Quartz (SiO_2), Sellaite (MgF_2),Rutile (TiO_2)

Optic axis

Optic axis of a crystal is the direction along which a ray of transmitted light suffers no birefringence (double refraction).

Light propagates along optic axis with a speed independent of its polarization.

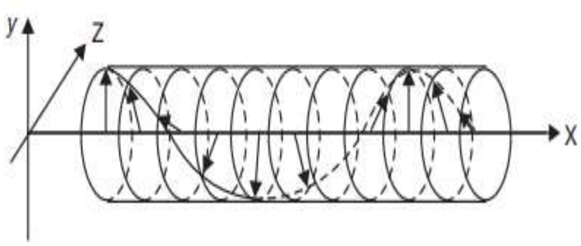
Different types of polarized light:

(1) Production of Plane polarized light: (Reference Diagram of first diagram)

When a un polarized light is passed through a pair of tourmaline crystal plates P_1 and $P_2$with their planes at right angles to the direction of propagation of light (fig 1), the intensity is maximum in this position. But when the plane P_2 is rotated through 90° i.e the plane of P_2 is perpendicular to plane of P_1 (fig 2), the intensity is maximum in this position. This show that light is a transverse wave motion. It is also clear that after passing through crystal P_1, the light vibrates only in one direction i.e it is said to be polarized because it has acquired the property of one sidedness.

(2) Circularly Polarized light :

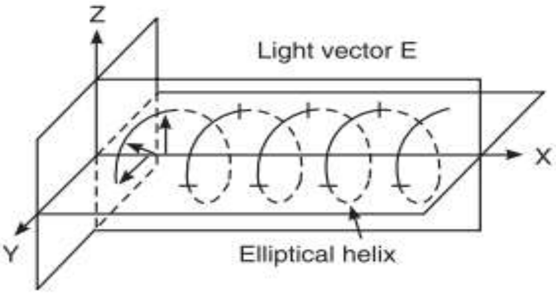
In a circular polarization, the electrical vector of constant amplitude no longer oscillates but rotates while proceeding in the form of helix. The projection of a wave on a plane intercepting the axis of propagation gives a circle with the amplitude of the vector remaining constant. If the vector rotates in the clockwise direction with respect to the direction of propagation, it results in right circularly polarized light while the rotation in the anti-clock wise direction results in left circularly polarized light.



Space variation of E; E vector sweeps a circular helix in space

(3) **Elliptically Polarized light :**

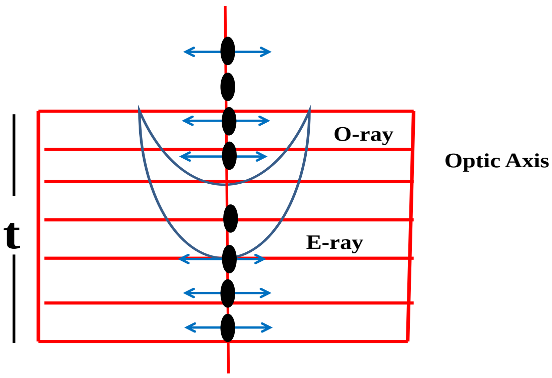
If the amplitude of the electric vector is not a constant but varies periodically then it results in elliptically polarized light. For example, if the electrical vector has minimum amplitude while oscillating vertically and rotates while propagating to have maximum amplitude when oscillating horizontally as shown in figure, than it results elliptically polarized light.



Space variation of E; E vector sweeps a flattened helix in space

Quarter wave Plates

A wave plate is an optical device made from doubly refracting crystals like calcite or quartz. Let us consider a calcite crystal plate cut with optic axis parallel to the surface. When a plane polarized light of wavelength λ falls normally on the crystal surface, the light split up into o – ray and e – ray as shown in figure.



Quarter wave plate of calcite crystal

Both ordinary and extra-ordinary travels along the same path with different velocities. We know that in the case of a calcite crystal, the velocities of e – ray is greater than the o – ray. As a result, a phase difference is introduced between them.

Let μ_0 and μ_e be the refractive indices of ordinary and extraordinary rays respectively.

If 't' is the thickness of the plate then

Optical path of o – ray = μ_0 t

Optical path of e – ray = μ_e t

$\therefore$ Optical path difference between o – ray and e – ray = $(\mu_0 - \mu_e)t$

If the thickness of the plate is cut in such way that it can produce a path difference of $\frac{\lambda}{4}$ or phase difference of $\frac{\pi}{2}$ between o – ray and e – ray, then the plate is called quarter wave plate (QWP).

i.e $(\mu_0 - \mu_e)t = \frac{\lambda}{4}$ (or) $t = \frac{\lambda}{4(\mu_0 - \mu_e)}$

N**ote:**

When a calcite crystal plate is cut with a thickness so that it can produce path difference of $\frac{\lambda}{4}$ between o – ray and e – ray, then it is called QWP

Applications:

(1) Wave plates are very useful in production and analysis of polarized light of different kinds.

(2) A QWP is used to produce circularly polarized light, if the incident light makes an angle of 45° with the optic axis and elliptically polarized light for other angles of incidence

(3) The emergent light from a HWP is a plane polarized light.

Limitations:

As the wave plates are designed for a particular wavelength (λ), they are not useful for the other wavelengths.

Half Wave Plate:

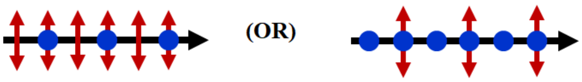
A half wave plate is a thin plate of birefringent crystal having the optic axis parallel to its refracting faces and its thickness chosen such that it introduces a half-wave $\left(\frac{\lambda}{2}\right)$ path difference (or a phase difference of 180°) between e-ray and o-ray. When a plane polarized light wave is incident on a bi-refringent crystal having the optic axis parallel to its refracting faces, it splits into two waves: o- and e-waves. The two waves travel along the same direction inside the crystal but with different velocities. As a result, when they emerge from the rear face of the crystal, an optical path difference would be developed between them.

$$(\mu_0 - \mu_e)t = \frac{\lambda}{4} \text{ (or) } t = \frac{\lambda}{2(\mu_0 - \mu_e)}$$

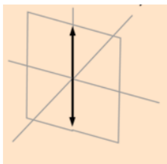
1. When the optical path difference is 0 or an even or odd multiple of $\frac{\lambda}{2}$, the resultant light wave is linearly polarized.

2. When the optical path difference is $\frac{\lambda}{4}$, the resultant light wave is elliptically polarized.

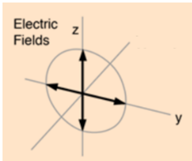
3. In the particular instance when the wave amplitudes are equal and the optical path difference is $\frac{\lambda}{4}$, the resultant light wave is circularly polarized



4. Linear polarized Light: The light in which the electric vector oscillates in a given constant orientation is known as linearly polarized light.



5. Circularly polarized Light: If light is composed of two plane waves of equal amplitude but differing in phase by 90°, then the light is said to be circularly polarized.



6. Elliptically polarized Light: If light is composed of two plane waves of unequal amplitude but differing in phase by 90°, then the light is said to be circularly polarized.