Article

Title

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1. Introduction

1.1. Notations and writing conventions

All manifolds are assumed to be smooth. Thorough this document, the next writing conventions are applied: M is a smooth manifold. For a vector bundle $E \xrightarrow{\pi} M$, the notation $\Gamma(U;E)$ with $U \subset M$ an open subset of the manifold M stands for the $C^\infty(M)$-module of smooth sections over U. The functor $U \mapsto \Gamma(U;E)$ defines a sheaf denoted by Γ_E. Finally, $\Gamma(E)$ is a shorthand notation for $\Gamma(M;E)$. Lowercase letters are used for sections, uppercase ones for tangent vectors.

2. The gauge equation

Let $E \xrightarrow{\pi} M$ be a vector bundle. An affine connection ∇ is a $\mathbb{R}$-linear mapping:

$$\nabla \colon \Gamma(E) \to \Gamma(T^\star M \otimes E) \tag{1}$$

such that for any $f \in C^\infty(M)$, any $s \in \Gamma(E)$ and any tangent vector X, $\nabla_X fs = X(f)s + f\nabla_X s$. Let $E^\star \xrightarrow{\pi^\star} M$ be the bundle obtained by dualizing E fiberwise. A section $\theta \in \Gamma(E^* \otimes E)$ defines two bundle morphisms:

$$\begin{tikzcd} E \arrow[r, "\theta"] \arrow[dr, "\pi"'] & E \arrow[d, "\pi"] \\ & M \end{tikzcd} \qquad \begin{tikzcd} E^\star \arrow[r, "\theta^t"] \arrow[dr, "\pi^\star"'] & E^\star \arrow[d, "\pi^\star"] \\ & M \end{tikzcd} \tag{2}$$

where θ^t is such that for any $p \in M$, $X \in T_pM, \xi \in T_p^\star M$:

$$\left(\theta_p^t \xi\right)(X) = \xi(\theta_p X) \tag{3}$$

Definition 2.1. Let (∇_1, ∇_2) be a couple of affine connections. A $(1,1)$-tensor θ is said to be a solution of the gauge equation.

3. KV cohomology

4. A spectral sequence

5. Application to statistical manifolds

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References