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**Abstract:** To be completed.

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## 1. Introduction

### 1.1. Notations and writing conventions

All manifolds are assumed to be smooth. Thorough this document, the next writing conventions are applied:  $M$  is a smooth manifold. For a vector bundle  $E \xrightarrow{\pi} M$ , the notation  $\Gamma(U; E)$  with  $U \subset M$  an open subset of the manifold  $M$  stands for the  $C^\infty(M)$ -module of smooth sections over  $U$ . The functor  $U \mapsto \Gamma(U; E)$  defines a sheaf denoted by  $\Gamma_E$ . Finally,  $\Gamma(E)$  is a shorthand notation for  $\Gamma(M; E)$ . Lowercase letters are used for sections, uppercase ones for tangent vectors.

## 2. The gauge equation

Let  $E \xrightarrow{\pi} M$  be a vector bundle. An affine connection  $\nabla$  is a  $\mathbb{R}$ -linear mapping:

$$\nabla: \Gamma(E) \rightarrow \Gamma(T^*M \otimes E) \quad (1)$$

such that for any  $f \in C^\infty(M)$ , any  $s \in \Gamma(E)$  and any tangent vector  $X$ ,  $\nabla_X f s = X(f)s + f \nabla_X s$ . Let  $E^* \xrightarrow{\pi^*} M$  be the bundle obtained by dualizing fiberwise  $E$ . A section  $\theta \in \Gamma(E^* \otimes E)$  defines two bundle morphisms:

$$\begin{array}{ccc} E & \xrightarrow{\theta} & E \\ & \searrow \pi & \downarrow \pi \\ & & M \end{array} \quad \begin{array}{ccc} E^* & \xrightarrow{\theta^*} & E^* \\ & \searrow \pi^* & \downarrow \pi^* \\ & & M \end{array} \quad (2)$$

where  $\theta^*$  is such that for any  $p \in M$ ,  $X \in T_p M$ ,  $\xi \in T_p^* M$ :

$$(\theta_p^* \xi)(X) = \xi(\theta_p X) \quad (3)$$

## 3. KV cohomology

## 4. A spectral sequence

## 5. Application to statistical manifolds

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## References

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