CS204: Algorithms

End Semester, Spring 2015,

IIT Patna

Time: 3 Hrs

Full marks: 50

1. State true or false. No marks will be awarded without valid reasoning. Please try to answer these in the first two pages of your answer script. (1×10)

   (a) The following array is a max heap: [10, 3, 5, 1, 4, 2].

   (b) If $f(n) = O(n^2)$ and $g(n) = O(n^2)$, then $f(n) = O(g(n))$.

   (c) Every problem in NP can be solved in exponential time.

   (d) Given an array of n integers, each belonging to $\{-1, 0, 1\}$, we can sort the array in $O(n)$ time in the worst case.

   (e) Every directed acyclic graph has exactly one topological ordering.

   (f) Consider a weighted directed graph $G = (V, E, w)$ and let X be a shortest $s - t$ path for $s, t \in V$. If we double the weight of every edge in the graph, setting $w(e) = 2w(e)$ for each $e \in E$, then X will still be a shortest $s - t$ path in (V, E, w).

   (g) If a depth-first search on a directed graph $G = (V, E)$ produces exactly one back edge, then it is possible to choose an edge $e \in E$ such that the graph $G = (V, E - \{e\})$ is acyclic.

   (h) Let G = (V, E) be a directed graph where every vertex has at most three outgoing edges. Then every vertex has at most three incoming edges.

   (i) If all of the edge capacities in a graph are an integer multiple of 7, then the value of the maximum flow will be a multiple of 7.

   (j) Suppose we run DFS on a directed graph, and we discover a vertex v with $pre(v) = 1$ and $post(v) = 2|V|$. Then the graph must be strongly connected. (pre[i] - start time, post[i] - finish time for node i).

2. Answer briefly. (2.5×4)

   (a) Solve this recurrence relation:
   $T(x, c) = \Theta(x)$ for $c \leq 2$,
   $T(x, y) = \Theta(x) + S(x, y/2)$
   $S(c, y) = \Theta(y)$ for $c \leq 2$,
   $S(x, y) = \Theta(y) + S(x/2, y)$. Find $T(n, n)$

   (b) Perform a depth-first search on the graph (Fig - 1) starting at A. Label every edge in the graph with T if it is a tree edge, B if it is a back edge, F if it is a forward edge, and C if it is a cross edge. Whenever faced with a decision of which node to pick from a set of nodes, pick the node whose label occurs earliest in the alphabet.

   (c) Run Dijkstra's algorithm on the directed graph (Fig - 2), starting at vertex S. What is the order in which vertices get removed from the priority queue? What is the resulting shortest-path tree?

   (d) Given a directed acyclic graph in which there is exactly one source node s and one sink node t. Give an efficient algorithm to find out the number of paths between s and t.

(40)

3. Describe Kruskal's algorithms to find a minimum spanning tree of a given undirected graph. Analyze the time complexity of the algorithm. Present a working example using Fig - 3. $(4 + 3 + 3)$

Answer any 4 from the following. (5×4)

4. Given a sequence of n matrices, present an efficient algorithm to find the best way to multiply these matrices. Analyze time complexity of your algorithm.

5. Given a graph $G = (V, E)$ and an integer K, target is to find a subset $V' \subseteq V$ such that $|V'| \leq K$ and for each edge $(u, v) \in E$ at least one of u or v belongs to V'. Proof that this problem is NP-Complete.

6. Given a text $T[1, ..., n]$ (n characters) and a pattern $P[1, ..., m]$ (both of which are strings over the same alphabet), present a linear time algorithm to find all occurrences of P in T. Analyze the time complexity of your algorithm.

7. Consider an array $A[1 \cdots n]$ constructed by the following process: we start with n distinct elements, sort them, and then rotate the array k steps to the right. For example, we might start with the sorted array $[1, 4, 5, 9, 10]$, and rotate it right by $k = 3$ steps to get $[5, 9, 10, 1, 4]$. Give an $O(\lg n)$-time algorithm that finds and returns the position of a given element x in array A, or returns None if x is not in A. Your algorithm is given the array $A[1 \cdots n]$ but does not know k.

8. A graph $G = (V, E)$ is a near-tree if it is connected and has at most $(n + 6)$ edges, where $n = |V|$. Give an algorithm that runs in $O(n)$ time, has as input a weighted near-tree G, and returns an minimum spanning tree of G. All edge-costs can be assumed to be distinct.

9. Present an algorithm to find the articulation points in an undirected connected graph. Your algorithm should run in $O(V + E)$ time.

10. Proof that the expected number of comparisons needed to insert n random elements into an initially empty binary search tree is $O(n \lg n)$, $n \geq 1$.

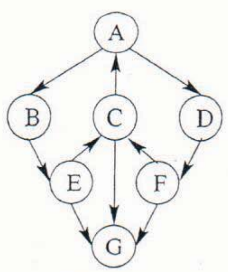

Fig - 1

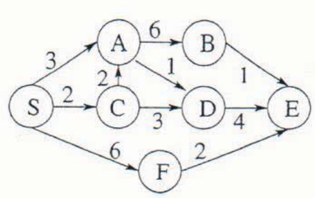

Fig - 2

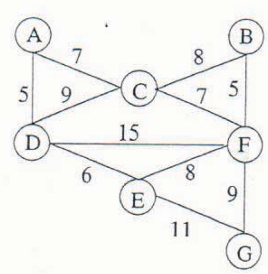

Fig - 3