MH4320 Assignment 5

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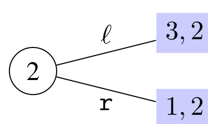
1 Subgame Perfect Equilibria

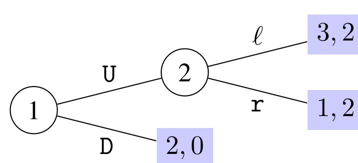
1.1 Subgame Perfect Equilibria in Pure Strategies

For the subgame:



There is no difference for player 2 to play ℓ or `r`.

For the subgame:



We have:

	ℓ	`r`
`U`	$3, 2$	$1, 2$
`D`	$2, 0$	$2, 0$

The pure NEs are $(\mathtt{U}, \ell)$ and $(\mathtt{D}, \mathtt{r})$.

1.2 Subgame Perfect Equilibria in Mixed Strategies

For player 1 and 2 both play pure strategy, the NEs are $(\mathtt{U}, \ell)$ and $(\mathtt{D}, \mathtt{r})$.

There is no difference for play 2 to choose between ℓ and `r`. So let player 2 plays $(p_2, 1 - p_2)$.

When $p_2 > \frac{1}{2}$, `U` is the best response, as $3p_2 + (1 - p_2) = 1 + 2p_2 > 2$. So player 1 should play `U`.

When $p_2 < \frac{1}{2}$, `D` is the best response, as $3p_2 + (1 - p_2) = 1 + 2p_2 < 2$. So player 1 should play `D`.

When $p_2 = \frac{1}{2}$, there is no difference for player 1 to choose between `U` and `D`. So player 1 can play $(p_1, 1 - p_1)$ for any $p_1 \in [0, 1]$.

So all SPEs in mixed strategies are:

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$$\left\{\left[(p_1, 1-p_1), \left(\frac{1}{2}, \frac{1}{2}\right)\right] : p_1 \in [0,1]\right\} \cup \left\{[(1,0),(p_2, 1-p_2)] : p_2 > \frac{1}{2}\right\} \left\{[(0,1),(p_2, 1-p_2)] : p_2 < \frac{1}{2}\right\}$$

2 Incomplete Information Extensive-Form Games

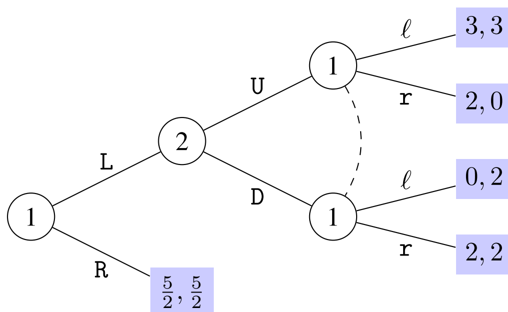
2.1 Strategic-Form Representation

	U	D
L, ℓ	$3, 3$	$0, 2$
L, r	$2, 0$	$2, 2$
R, $*$	$\frac{5}{2}, \frac{5}{2}$	$\frac{5}{2}, \frac{5}{2}$

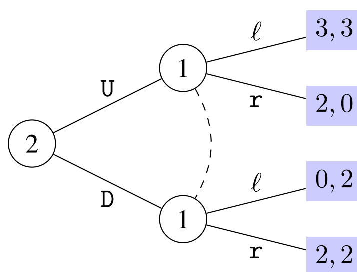
2.2 Number of Subgames

There are 2 subgames:

1. The first subgame is the whole game, the starting node is 1.

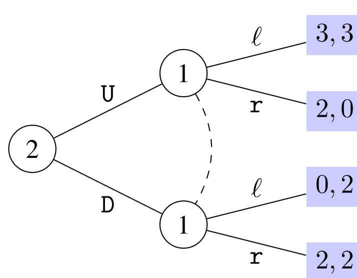


2. The second subgame is the game after player 1 playing L, the starting node is 2.



2.3 Subgame Perfect Equilibria

Let's see the subgame first:

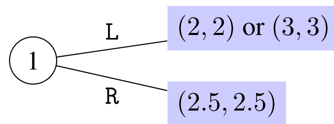


We can convert it to strategic form:

	U	D
ℓ	$3,3$	$0,2$
r	$2,0$	$2,2$

The pure NEs for the subgame are: $(\ell, \mathtt{U}), (\mathtt{r}, \mathtt{D})$, the utilities for each NEs are $(3,3),(2,2)$. And there is a mixed NE $\left[\left(\frac{2}{3}, \frac{1}{3}\right), \left(\frac{2}{3}, \frac{1}{3}\right)\right]$, the utilities are $(2,2)$.

So when we perform backward induction to the whole game:



For the NE $2,2$, player 1 will change to R, and for $3,3$, player 1 will choose L.

So the SPEs are:

$$\left\{[\mathtt{L}, \ell, \mathtt{U}], [\mathtt{R}, \mathtt{D}, \mathtt{r}], \left[\mathtt{R}, \frac{2}{3}\mathtt{U} + \frac{1}{3}\mathtt{D}, \frac{2}{3}\ell + \frac{1}{3}\mathtt{r}\right]\right\}$$

3 War of Attrition

For the second stage, we have the subgame:

$-c+$

	$\mathtt{f}_2$	$\mathtt{q}_2$
$\mathtt{F}_2$	$-c, -c$	$v, 0$
$\mathtt{Q}_2$	$0, v$	$0, 0$

For the table, there are 2 pure NEs: $(\mathtt{F}_2, \mathtt{q}_2), (\mathtt{Q}_2, \mathtt{f}_2)$, with payoff $(v, 0), (0, v)$ separately.

For mixed strategy, let P_1 plays $(p_1, 1-p_1)$, P_2 plays $(q_2, 1-q_2)$.

We have:

$$\begin{cases} -cq_1 + v(1-q_1) = 0 \\ -cq_2 + v(1-q_2) = 0 \end{cases}$$

So

$$q_1 = q_2 = \frac{v}{c+v}$$

So the mixed strategy is

$$\left[\left(\frac{v}{c+v}\mathtt{F}_2, \frac{c}{c+v}\mathtt{Q}_2\right), \left(\frac{v}{c+v}\mathtt{f}_2, \frac{c}{c+v}\mathtt{q}_2\right)\right]$$

Now, let's consider the first stage.

If we take $(\mathtt{F}_2, \mathtt{q}_2)$ as NE, We have:

	$\mathtt{f}_1$	$\mathtt{q}_1$
$\mathtt{F}_1$	$v-c, -c$	$v, 0$
$\mathtt{Q}_1$	$0, v$	$0, 0$

It is easy to find the pure NEs $(\mathtt{F}_1, \mathtt{q}_1), (\mathtt{Q}_1, \mathtt{f}_1)$ and mixed NE $\left(\frac{v}{c+v}\mathtt{F}_1 + \frac{c}{c+v}\mathtt{Q}_1, \frac{v}{c}\mathtt{f}_1 + \frac{c-v}{c}\mathtt{q}_1\right)$.

So the NEs are:

$$\left\{[(\mathtt{F}_1, \mathtt{F}_2), (\mathtt{q}_1, \mathtt{q}_2)], [(\mathtt{Q}_1, \mathtt{F}_2), (\mathtt{f}_1, \mathtt{q}_2)], \left[\left(\frac{v}{c+v}\mathtt{F}_1 + \frac{c}{c+v}\mathtt{Q}_1, \mathtt{F}_2\right), \left(\frac{v}{c}\mathtt{f}_1 + \frac{c-v}{c}\mathtt{q}_1, \mathtt{q}_2\right)\right]\right\}$$

If we take $(\mathtt{Q}_2, \mathtt{f}_2)$ as NE, We have:

	$\mathtt{f}_1$	$\mathtt{q}_1$
$\mathtt{F}_1$	$-c, v-c$	$v, 0$
$\mathtt{Q}_1$	$0, v$	$0, 0$

It is easy to find the pure NEs $(\mathtt{F}_1, \mathtt{q}_1)$, $(\mathtt{Q}_1, \mathtt{f}_1)$ and mixed NE $\left(\frac{v}{c}\mathtt{F}_1 + \frac{c-v}{c}\mathtt{Q}_1, \frac{v}{c+v}\mathtt{f}_1 + \frac{c}{c+v}\mathtt{q}_1\right)$.

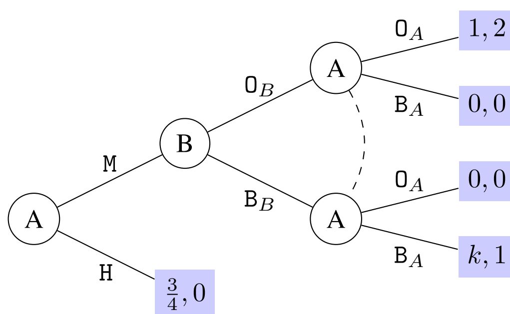
So the NEs are:

$$\left\{[(\mathtt{F}_1, \mathtt{Q}_2), (\mathtt{q}_1, \mathtt{f}_2)], [(\mathtt{Q}_1, \mathtt{Q}_2), (\mathtt{f}_1, \mathtt{f}_2)], \left[\left(\frac{v}{c}\mathtt{F}_1 + \frac{c-v}{c}\mathtt{Q}_1, \mathtt{Q}_2\right), \left(\frac{v}{c+v}\mathtt{f}_1 + \frac{c}{c+v}\mathtt{q}_1, \mathtt{f}_2\right)\right]\right\}$$

4 Alice and Bob

4.1 Extensive and Strategic Form of the Game

4.1.1 Extensive Form

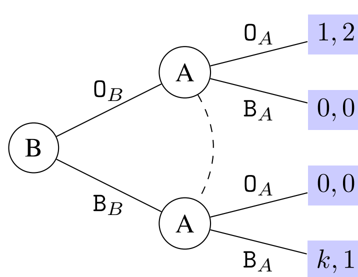


4.1.2 Strategic Form

	$\mathtt{O}_B$	$\mathtt{B}_B$
$\mathtt{M}, \mathtt{O}_A$	$1, 2$	$0, 0$
$\mathtt{M}, \mathtt{B}_A$	$0, 0$	$k, 1$
$\mathtt{H}, \mathtt{O}_A$	$\frac{3}{4}, 0$	$\frac{3}{4}, 0$
$\mathtt{H}, \mathtt{B}_A$	$\frac{3}{4}, 0$	$\frac{3}{4}, 0$

4.2 Values of k

For the subgame:



We can write it in strategic form:

	$\mathtt{O}_B$	$\mathtt{B}_B$
$\mathtt{O}_A$	$1, 2$	$0, 0$
$\mathtt{B}_A$	$0, 0$	$k, 1$

There are 2 pure NEs $(\mathtt{O}_A, \mathtt{O}_B), (\mathtt{B}_A, \mathtt{B}_B)$ with outcome $(1, 2)$ and $(k, 1)$.

For mixed strategy, let A plays $(p_A, 1 - p_A)$, B plays $(p_B, 1 - p_B)$.

We have:

$$\begin{cases} 2p_A = 1 - p_A \\ p_B = k(1 - p_B) \end{cases}$$

So

$$\begin{cases} p_A = \frac{1}{3} \\ p_B = \frac{k}{1+k} \end{cases}$$

So the utility for Alice is:

$$\frac{1}{3} \cdot \frac{k}{1+k} + \frac{2}{3} \cdot k \cdot \frac{1}{1+k} = \frac{k}{1+k}$$

To make all the subgame-perfect equilibria involves Alice's going to the movies, we should have:

$$\begin{cases} 1 > \frac{3}{4} \\ k > \frac{3}{4} \\ \frac{k}{1+k} > \frac{3}{4} \end{cases}$$

So $k > 3$.