MH 4320 Computational Economics

Assignment 2 August 26, 2024

Question 1: Football Penalty Kick

The *football penalty kick* game models the situation in football where a player is about to kick a penalty: usually the goalie stands at the middle of the goal line and the player choose to aim either to the left or to the right of the goal. Because the player stands very close to the goal, the goalie has to decide on a direction in advance and starts diving in this direction even before the player hits the ball. If the goalie dives in the direction chosen by the player, he saves the shot, otherwise he fails.

1. The penalty kick can be modeled by a 2×2 strategic-form game between the kicker and the goalie matchup.

Kicker/Goalie	Left	Right
Left	$0, 1$	$1, 0$
Right	$1, 0$	$0, 1$

   Find a mixed Nash equilibrium of this simple game.

2. Now consider a scenario where the kicker kicks worse to the right than left. In particular, when the kicker kicks to the right and the goalie still goes to the left, the kicker will now only score 75% of the time and misses completely 25% of the time. So the new payoff matrix is as following

Kicker/Goalie	Left	Right
Left	$0, 1$	$1, 0$
Right	$0.75, 0.25$	$0, 1$

   Find a Nash equilibrium for this new game, and try to think about how is the equilibrium going to adjust to the skills of the players.

Question 2: Chicken Game

Use the indifference principle to *find* a *mixed strategy Nash equilibrium* in the following "game of chicken"

	D	S
D	0,0	-1,1
S	1,-1	-5,-5

Question 3: Who Doesn't Like Rock-Paper-Scissors

Anna is three and likes to play Rock-Paper-Scissors. However, she faces a difficulty — three year olds find it hard to make "scissors" with their fingers.

Suppose that we capture this problem by treating her playing Rock-Paper-Scissors against her older sister Elsa (who has no problem making scissors) using the asymmetric 3×3 game shown in below (with Anna as player 1 and Elsa as player 2).

	R	P	S
R	$0, 0$	$-1, 1$	$1, -1$
P	$1, -1$	$0, 0$	$-1, 1$
S	$-1-c, 1$	$1-c, -1$	$-c, 0$

(a) Consider first the version of this game where $c > 1$. (You can think of this as a model for the extreme situation where Anna is physically incapable of playing scissors.) Find a mixed-strategy Nash equilibrium of this game.

(b) Compute Anna's expected payoff in the equilibrium you found in (a).

(c) Consider a new version of this game with $0 < c < 1$. Find a mixed-strategy Nash equilibrium of this game in which both players play every strategy with positive probability.

Question 4

Consider the following 3×3 game.

	L	C	R
T	$0, 4$	$1, 2$	$2, 0$
M	$-1, 2$	$1, 3$	$3, 1$
B	$1, 3$	$-2, 1$	$2, 8$

1. *Find* all pure Nash equilibria of the game.
2. *Find* all mixed Nash equilibria of the game with the following supports: both players play *every* pure strategy with a positive probability.
3. *Find* all mixed Nash equilibria of the game with the following supports: player 1 plays a mix of T and M, and player 2 plays the pure strategy C.