On commutativity, total orders, and sorting

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Motivation

- The goal is to study free monoids and free commutative monoids.
- We created a framework to formalize different algebraic structures, free algebras and their universal properties.
- Univalent type theory gives us higher inductive types, which allows us to reason with commutativity and equations of algebras. (No setoid hell!)
- Using the framework, we study the relationship between sorting and total orders.

Homotopy Type Theory

Homotopy Type Theory extends intensional MLTT and allows us to reason with equivalences more powerfully.

- Function extensionality
- Quotient types (via higher inductive types)
- Equalities between types (via univalence)
- Mere propositions