ST202/ST206 – Autumn Term

Problem set 3

Due: 12 noon, Wednesday AT Week 4

1. Suppose that $X \sim \text{Geometric}(p)$, with probability mass function (PMF)
$$f_X(x) = (1-p)^{x-1}p \quad \text{for} \quad x = 1, 2, \ldots$$
Work out the cumulative distribution function (CDF) of X. Find the **survival function** of X, defined as $S_X(x) = P(X > x)$. What do you observe? How could you have arrived at this answer directly?

2. The claims received by an insurance company have CDF $F_X(x)$. Now suppose that the company decides not to pay out claims for amounts less than or equal to c. Work out the CDF of the claims paid in terms of $F_X(x)$. What is the CDF for the claims *not* paid? *[Hint: Work out the probability that a claim is for an amount $\leq x$, conditional on it being $> c$.]*

3. Let $Y \sim \text{NegBin}(r, p)$, which has PMF
$$f_Y(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r} \quad \text{for} \quad y = r, r+1, \ldots$$
Show that $f_Y(y)$ is a valid PMF. You may use the negative binomial expansion,
$$(1-a)^{-n} = \sum_{j=0}^{\infty} \binom{j+n-1}{n-1} a^j, \quad \text{provided } |a| < 1.$$

4. The **Gamma function** is defined as
$$\Gamma(t) = \int_0^{\infty} u^{t-1} e^{-u} du.$$
   (a) Work out $\Gamma(1)$ and show that $\Gamma(t) = (t-1)\Gamma(t-1)$ for $t > 1$.
   (b) If $k \in \mathbb{Z}^+$, a positive integer, what is $\Gamma(k)$ equal to?

   A random variable X has the **Polya** distribution if its PMF is
$$f_X(x) = \frac{\Gamma(r+x)}{x!\Gamma(r)} p^r (1-p)^x \quad \text{for} \quad x = 0, 1, \ldots$$
   where $0 < p < 1$ and $r > 0$ are parameters. Explain why the negative binomial distribution is a special case of the Polya distribution. *[Hint: Use the form of the* NegBin *where we count the number of* failures.*]*