ST102/ST109 Outline solutions to Exercise 2

1. There is more than one way to answer this question, because the sets can be expressed in different, but logically equivalent, forms. One way to do so is the following.

   (a) $A \cap B^c \cap C^c$, i.e. A and not B and not C.

   (b) $A^c \cap B^c \cap C^c$, i.e. not A and not B and not C.

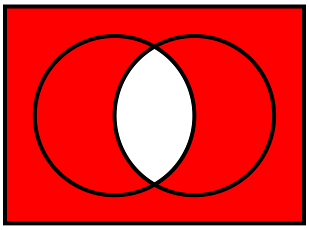
   (c) $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$, i.e. only A or only B or only C.

   (d) $(A \cap B) \cup (A \cap C) \cup (B \cap C)$, i.e. A and B, or A and C, or B and C.

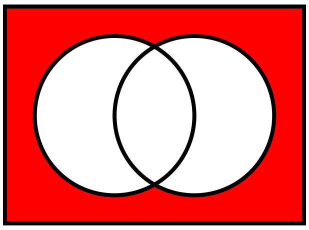
   Note that this includes $A \cap B \cap C$ as a subset, so we do not need to write $(A \cap B) \cup (A \cap C) \cup (B \cap C) \cup (A \cap B \cap C)$ separately.

   – $((A \cap C) \cup (A \cap B) \cup (B \cap C)) \cap (A \cap B \cap C)^c$, i.e. A and B, or A and C, or B and C, but not A and B and C.

2. For $(A \cap B)^c = A^c \cup B^c$ we have:



   For $(A \cup B)^c = A^c \cap B^c$ we have:



3. Using De Morgan's laws, we obtain:

$$(A \cup B)^c = A^c \cap B^c = \{a, b, c\} \cap \{b, c, d\} = \{b, c\}.$$

4. (a) These are the points not in A, hence they must be either below 1 or above 5, so:

$$A^c = \{x \,|\, x < 1 \text{ or } x > 5\}.$$

(b) These are the points in either A or B or both, so they must be between 1 and 5 or between 3 and 7. Hence:

$$A \cup B = \{x \mid 1 \leq x \leq 7\}.$$

(c) These are the points in B but not in C. Noting that $B \subset C^c$, then:

$$B \cap C^c = B = \{x \mid 3 < x \leq 7\}.$$

(d) These are the points in none of the three sets, hence:

$$A^c \cap B^c \cap C^c = \{x \mid 0 < x < 1 \text{ or } x > 7\}.$$

(e) These are the points common to $A \cup B$ and C. There are no such values, hence:

$$(A \cup B) \cap C = \emptyset.$$

5.* (a) We have:

$$(A \cap B) \cap (A \cap B^c) = (A \cap A) \cap (B \cap B^c) = A \cap \emptyset = \emptyset.$$

This uses the results of commutativity, associativity, $A \cap A = A$, $A \cap A^c = \emptyset$ and $A \cap \emptyset = \emptyset$ on page 29.

Similarly:

$$(A \cap B) \cup (A \cap B^c) = A \cap (B \cup B^c) = A \cap S = A$$

using the results of the distributive laws, $A \cup A^c = S$ and $A \cap S = A$, again given on page 29.

(b) Since (3)–(5) in the question are all partitions:

$$\begin{aligned} P(A \cup B) &= P(A \cap B^c) + P(A \cap B) + P(A^c \cap B) \\ P(A) &= P(A \cap B^c) + P(A \cap B) \\ P(B) &= P(A^c \cap B) + P(A \cap B) \end{aligned}$$

we have that:

$$\begin{aligned} P(A \cup B) &= (P(A) - P(A \cap B)) + P(A \cap B) + (P(B) - P(A \cap B)) \\ &= P(A) + P(B) - P(A \cap B). \end{aligned}$$