# ST102 Class 19 – Solutions to Additional exercises

1. (a) From the summary statisitcs, we have $\bar{x} = 65.846$ and $\bar{y} = 80.077$. Therefore, the sample correlation coefficient is:

$$\widehat{\rho} = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sqrt{(\sum x_i^2 - n\bar{x}^2)(\sum y_i^2 - n\bar{y}^2)}} = \frac{1{,}657.154}{\sqrt{2{,}037.692 \times 1{,}440.923}} = 0.9671.$$

   (b) The point estimate of the slope coefficient is:

$$\widehat{\beta}_1 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} = \frac{1{,}657.154}{2{,}037.692} = 0.8133$$

   and for the intercept we have:

$$\widehat{\beta}_0 = \bar{y} - \widehat{\beta}_1 \bar{x} = 80.077 - 0.8133 \times 65.846 = 26.5245.$$

   Hence the line of best fit is $\widehat{y} = 26.5245 + 0.8133x$.

2. (a) In this example, $n = 6$, and:

$$\sum_{i=1}^{n} x_i = 21, \quad \sum_{i=1}^{n} x_i^2 = 91, \quad \sum_{i=1}^{n} y_i = 311, \quad \sum y_i^2 = 19{,}855 \quad \text{and} \quad \sum_{i=1}^{n} x_i y_i = 1{,}342.$$

   We have, noting $\bar{x} = 21/6 = 3.5$ and $\bar{y} = 311/6 = 51.8\dot{3}$, that:

$$\widehat{\beta}_1 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} = \frac{1{,}342 - 6 \times 3.5 \times 51.8\dot{3}}{91 - 6 \times (3.5)^2} = 14.49$$

   and:

$$\widehat{\beta}_0 = \bar{y} - \widehat{\beta}_1 \bar{x} = 51.83 - 14.49 \times 3.5 = 1.13.$$

   Hence the equation of the least squares line is:

$$\widehat{y} = 1.13 + 14.49x.$$

   (b) We have:

$$\begin{aligned}\widehat{\sigma}^2 = \frac{\text{Total SS} - \text{Regression SS}}{n-2} &= \frac{(\sum y_i^2 - n\bar{y}^2) - \widehat{\beta}_1^2(\sum x_i^2 - n\bar{x}^2)}{n-2} \\ &= \frac{(19{,}855 - 6 \times (51.8\dot{3})^2) - (14.49)^2 \times (91 - 6 \times (3.5)^2)}{6-2} \\ &= 15.68.\end{aligned}$$

   Hence:

$$\text{E.S.E.}(\widehat{\beta}_1) = \sqrt{\frac{\widehat{\sigma}^2}{\sum x_i^2 - n\bar{x}^2}} = \sqrt{\frac{15.68}{91 - 6 \times (3.5)^2}} = 0.9466.$$

The test statistic value for testing $\mathrm{H}_0 : \beta_1 = 16$ is:

$$t = \frac{\widehat{\beta}_1 - 16}{\mathrm{E.S.E.}(\widehat{\beta}_1)} = \frac{14.49 - 16}{0.9466} = -1.60.$$

With $\mathrm{H}_1 : \beta_1 < 16$, we reject $\mathrm{H}_0$ if $t < t_{0.10,\,4} = -1.533$, hence we reject $\mathrm{H}_0$ at the 10% significance level.

(c) We have:

$$\mathrm{E.S.E.}(\widehat{\beta}_0) = \sqrt{\frac{\widehat{\sigma}^2}{n} \frac{\sum x_i^2}{\sum x_i^2 - n\bar{x}^2}} = \sqrt{\frac{15.68}{6} \times \frac{91}{91 - 6 \times (3.5)^2}} = 3.6864.$$

The test statistic value for testing $\mathrm{H}_0 : \beta_0 = 0$ is:

$$t = \frac{\widehat{\beta}_0}{\mathrm{E.S.E.}(\widehat{\beta}_0)} = \frac{1.13}{3.6864} = 0.31.$$

With $\mathrm{H}_1 : \beta_0 \neq 0$, we reject $\mathrm{H}_0$ if $|t| > t_{0.05,\,4} = 2.132$, hence we do not reject $\mathrm{H}_0$ at the 10% significance level.