ST202/ST206 – Autumn Term

Problem set 5

Due: 12 noon, Wednesday AT Week 7

1. For each of the following, derive the moment-generating function and cumulant-generating function:

   (a) Bernoulli(p),
   (b) Bin(n, p),
   (c) Geometric(p),
   (d) NegBin(r, p).

   Can you spot any associations between these MGFs?

2. Show that for a random variable X, the third cumulant is equal to the third central moment, that is, $\kappa_3 = \mu_3$.

3. Work out the moment-generating function of $Z \sim \mathrm{N}(0, 1)$, then set $X = \mu + \sigma Z$ to find the MGF of the general normal, $X \sim \mathrm{N}(\mu, \sigma^2)$. Now use this MGF to find the mean and variance of $Y = e^X$.
   [Y has the log-normal distribution, popular as a skew distribution for positive variables.]

4. Find the moment-generating function of the Laplace distribution (also known as the double exponential), which has density function

   $$f_X(x) = \frac{1}{2}e^{-|x|} \quad -\infty < x < \infty.$$

   Use this expression to obtain its first four cumulants.

5. Let Z be a random variable with density

   $$f_Z(z) = \frac{1}{k}, \quad \text{for} \quad -b < z < b.$$

   (a) Find k.
   (b) Find the moment-generating function of Z.
   (c) Is the moment-generating function you have specified well-defined in an open interval around the origin? If not, how might you resolve this problem? Explain your answer clearly.