January 2019 Exam

ST102

Elementary Statistical Theory

Suitable for all candidates

Instructions to candidates

This paper contains two questions. Answer **BOTH** questions.

This examination counts for 25% of the final grade.

The marks in brackets reflect marks for each question. Full working must be shown to gain all marks for each question.

Time allowed -	**Reading Time:**	*10 minutes*
	Writing Time:	*1 hour*
You are supplied with:		*Murdoch & Barnes Statistical Tables, 4th edition*
You may also use:		*No additional materials*
Calculators:		*Calculators are allowed in this examination*

1. (a) Suppose that $A \subset B$. Show that A and $B \cap A^c$ form a partition of B. You may use any properties of set operators, as defined in the course, and clearly show where these are applied.

(10 marks)

(b) There are three cards: one is black on both sides, one is red on both sides, and one is black on one side and red on the other side. We choose a card at random and we see one side (also chosen at random). If the side we see is black, what is the probability that the other side is also black?

(10 marks)

(c) Suppose X is a continuous random variable with the following probability density function:

$$f(x) = \begin{cases} \dfrac{k}{\lambda}\left(\dfrac{x}{\lambda}\right)^{k-1} \mathrm{e}^{-(x/\lambda)^k} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $k > 0$ and $\lambda > 0$ are parameters of this distribution.

i. Derive the cumulative distribution of X.

(15 marks)

ii. Calculate $P(2 < X \leq 6)$ when $k = 1$ and $\lambda = 2$.

(4 marks)

iii. State the probability density function which results when $k = 1$ and $\lambda = 1$, and identify which specific distribution this is.

(4 marks)

iv. When $k = 1$ and $\lambda = 1$, *derive* the moment generating function of X.

Hint: You may use any known properties of the common probability distributions seen in the course without proof.

(7 marks)

[TURN TO NEXT PAGE]

2. (a) Suppose we have a biased coin which comes up heads with probability π. An experiment is carried out so that X is the number of independent flips of the coin required until r heads show up, where $r \geq 1$ is known. Explain why the probability function of X is:

$$p(x) = \begin{cases} \binom{x-1}{r-1}\pi^r(1-\pi)^{x-r} & \text{for } x = r, r+1, r+2, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

(10 marks)

(b) Consider two random variables X and Y. They both take the values 0, 1 and 2. The joint probabilities for each pair are given by the following table, where $\theta \in \mathbb{R}$ is a parameter.

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	$1 - \frac{8\theta}{10}$	0	$\frac{\theta}{10}$
$Y = 1$	$\frac{\theta}{10}$	$\frac{\theta}{10}$	$\frac{2\theta}{10}$
$Y = 2$	$\frac{\theta}{10}$	$\frac{\theta}{10}$	$\frac{\theta}{10}$

i. What is the range of values the parameter θ can take?

(10 marks)

ii. Calculate $P(X = 2 \mid X + Y = 3)$ and state the possible values of θ for which this probability is defined. (Be precise in your use of limits in any inequalities.)

(10 marks)

iii. Calculate Cov(X, Y). Your answer should be expressed as a function of θ.

(20 marks)

[END OF PAPER]