ST102/ST109 Exercise 8

In this exercise you will practise working with some common probability distributions. Question 1 involves the continuous uniform distribution, where you need to determine the expectation of a function of the random variable. Questions 2 to 4 relate to the Poisson distribution and/or the exponential distribution. Think about whether counts (discrete) or times (continuous) are required. Question 5 deals with the normal distribution.

Your answers to this problem set should be submitted as a pdf file upload to Moodle. It will be covered by your class teacher in your eighth class, which will take place in the week commencing Monday 21 November 2022.

1.* A newsagent, James, has n newspapers to sell and makes £1.00 profit on each sale. Suppose the number of customers of these newspapers is a random variable with a distribution which can be approximated by:

$$f(x) = \begin{cases} 1/200 & \text{for } 0 \leq x \leq 200 \\ 0 & \text{otherwise.} \end{cases}$$

If James does not have enough newspapers to sell to all customers, he figures he loses £5.00 in goodwill from each unhappy (non-served) customer. However, if he has surplus newspapers (which only have commercial value on the day of print), he loses £0.50 on each unsold newspaper. What should n be (to the nearest integer) to maximise profit?

Hint: If $X \leq n$, James' profit (in £) is $X - 0.5(n - X)$. If $X > n$, James' profit is $n - 5(X - n)$. Find the expected value of profit as a function of n, and then select n to maximise this function. (There is no need to verify it is a maximum.)

2. Each morning telephone calls arrive at a reception desk according to a Poisson process with on average three calls every 5 minutes. Let T denote the waiting time, in minutes, until the first call is received after 09:00.

   (a) What is the distribution of T?

   (b) Calculate $P(T > 4)$.

3. (a) Arrivals at a post office may be modelled as following a Poisson distribution with a rate parameter of 96 arrivals per hour. Find:

      i. the probability of exactly five arrivals in a period of two minutes

      ii. the probability of more than two arrivals in 45 seconds

      iii. the probability that the time to arrival of the next customer is less than one minute.

   (b) If T is the time to arrival of the next customer (in minutes), calculate:

$$P(T > 2.4 \,|\, T > 1).$$

4. Suppose that commercial aeroplane crashes in a certain country occur at the rate of 2.75 per year.

   (a) Is it reasonable to assume that such crashes are Poisson events? Briefly explain.

   (b) What is the probability that two or more crashes will occur next year?

   (c) What is the probability that the next two crashes will occur within three months of one another?

5. The life, in hours, of a light bulb is assumed to be normally distributed with a mean of 185 hours. If a consumer requires at least 95% of the light bulbs to have lives exceeding 160 hours, what is the largest value that the standard deviation can have?