ST102 Class 18 – Solutions to Additional exercises

1. If $Y \sim t_8$, then $X = Y^2 \sim F_{1,8}$. Also:

$$0.20 = P(X > c) = P(Y > \sqrt{c}) + P(Y < -\sqrt{c}) = 2 \times P(Y > \sqrt{c}).$$

Therefore, $P(Y > \sqrt{c}) = 0.10$. Using Table 7 of Murdoch and Barnes' *Statistical Tables*, we have:

$$P(Y > 1.397) = 0.10.$$

Hence $\sqrt{c} = 1.397$ and so $c = 1.952$.

2. X can be represented as $X = W/V$, where $W \sim \chi^2_n$ and $V \sim \chi^2_n$ with W and V independent of each other. Therefore:

$$P(W > V) = P(W < V) = 0.50.$$

Equivalently:

$$P(X > 1) = P(X < 1) = 0.50.$$

Therefore, the median of the distribution of X is 1.

3. We have:

Source	DF	SS	MS	F
Treatment	4	271.36	67.84	6.40
Error	10	106.00	10.60	
Total	14	377.36		

4. (a) The completed table is:

Source	DF	SS	MS	F
Category	5	62.10	12.42	3.96
Year	3	31.17	10.39	3.31
Residual	15	47.10	3.14	
Total	23	140.37		

(b) We test the null hypothesis:

H_0 : There is no difference between the profit margins of different categories

against the alternative hypothesis:

H_1 : There is a difference between the profit margins of different categories.

The test statistic value is 3.96 and at a 5% significance level, $F_{0.05,\,5,\,15} \approx 2.90$, hence we reject H_0 and conclude that there is evidence of a difference between the profit margins of different categories.

We also test the null hypothesis:

H_0 : There is no difference between the profit margins in different years

against the alternative hypothesis:

H_1 : There is a difference between the profit margins in different years.

The test statistic value is 3.31 and at a 5% significance level, $F_{0.05,\,3,\,15} \approx 3.29$, hence we again (just) reject H_0 and conclude that there is evidence of a difference between the profit margins in different years.

(c) The sample means of these two categories are 1.1 and 1.4. Since $t_{0.025,\,15} = 2.131$ and $\widehat{\sigma}^2 = 3.14$, a 95% confidence interval is:

$$1.4 - 1.1 \pm 2.131 \times \sqrt{3.14 \times \left(\frac{1}{4} + \frac{1}{4}\right)} = 0.3 \pm 2.67 \quad \Rightarrow \quad (-2.37, 2.97).$$

Since it contains 0 there is no evidence of a difference between the means.