ST102/ST109 Exercise 4

In this exercise you will practise conditional probability, the total probability formula and Bayes' theorem. Question 1 is a variation of the Monty Hall problem covered in the lecture. Question 2 requires you to derive a conditional probability – think about the information you are given and apply the standard definition of conditional probability. Question 3 is a variation of the 'boy or girl paradox'. Finally, Question 4 makes use of Bayes' theorem, although the challenge is to think about how the various events can occur.

Your answers to this problem set should be submitted as a pdf file upload to Moodle. It will be covered by your class teacher in your fourth class, which will take place in the week commencing Monday 23 October 2023.

1. Consider a modified version of the Monty Hall problem discussed in Example 2.29. In this version, there are n boxes ($n \geq 3$), of which 1 box contains the prize and the other $n-1$ boxes are empty. You again select one box at first. Monty, who knows where the prize is, then opens $n-2$ of the remaining $n-1$ boxes, all of which are shown to be empty. If Monty has a choice of which boxes to open (i.e. if the prize is in the box you chose at first), he will choose at random which one of the boxes to leave unopened.

   Suppose that you have chosen Box 1, and then Monty opens Boxes 3 to n, leaving Box 2 unopened. After we have observed this, what is the probability that the prize is in Box 1, and what is the probability that it is in Box 2?

2.* James places k objects into n boxes at random (i.e. with equal probability). Given that the first box is empty, what is the probability that the second box is also empty?

   Hint: Each of the k objects can be placed into any of the n boxes.

3. After many years of happy marriage, Mrs Abdey announces 'I have three children'.

   (a) Mrs Abdey then says at least two of them are boys. *Use Bayes' theorem* to determine the probability that all three of her children are boys.

   (b) Mrs Abdey then says the two oldest children are boys. *Use Bayes' theorem* to determine the probability that all three of her children are boys.

   You may assume that the probability of a birth resulting in a boy is 0.5, and hence also for a girl. You may also assume that the sex of each child is independent.

   Note you are explicitly asked to use Bayes' theorem to solve this!

4.* A person tried by a three-judge panel is declared guilty if at least two judges cast votes of guilty (i.e. a majority verdict).

   Suppose that when the defendant is in fact guilty, each judge will independently vote guilty with probability 0.85, whereas when the defendant is in fact not guilty (i.e. innocent), this probability drops to 0.25.

If 70% of defendants are guilty, compute the conditional probability that judge number 3 votes guilty given that:

(a) judges 1 and 2 vote guilty

(b) judges 1 and 2 cast 1 guilty and 1 not guilty vote each (in either order)

(c) judges 1 and 2 both cast not guilty votes.