ST202/ST206 – Autumn Term

Problem set 2

Due: 12 noon, Wednesday AT Week 3

1. Six people stand in a circle. We choose two at random, say A and B. What is the probability that there are exactly r people between A and B in the clockwise direction? What if the six people stand in a line instead of a circle? Try replacing 6 by n for general problems of the same sort.

2. There are n urns, of which the k^{th} contains $k-1$ red balls and $n-k$ black balls, for $k = 1, \ldots, n$. You pick an urn at random and remove two balls at random without replacement. Find the probability that:

   (a) the first ball is black;

   (b) the second ball is black;

   (c) the second ball is black given that the first is black.

3. You toss a fair coin twice. Consider the following events:
   A : *'The first toss is heads'*
   B : *'The second toss is heads'*
   C : *'First and second toss show the same side'*.
   Show that A, B, C are pairwise independent events, but not (mutually) independent events.

4. Suppose that the genders of all children in a family are independent and that boys and girls are equally probable, that is, both have probability 1/2. Let A be the event *'There are children of both genders'*, and B be the event *'Not more than one child is a girl'*.

   (a) For families of three children, calculate the probabilities of the events A, B, and $A \cap B$.

   (b) Do the same for families of four children.

   (c) Are A and B independent events in (a) and (b)?

   (d) * Let C_j be the event *'The family has exactly j children'*, for $j = 0, 1, 2, \ldots$ If $P(C_j) = 1/2^{j+1}$, write down an expression for the probability that a family has exactly k boys, for $k = 0, 1, 2, \ldots$ [To evaluate this expression, look up the negative binomial series.]