THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

January 2020 Exam

ST102

Elementary Statistical Theory

Suitable for all candidates

Instructions to candidates

This paper contains two questions. Answer **BOTH** questions.

This examination counts for 25% of the final grade.

The marks in brackets reflect marks for each question. Full working must be shown to gain all marks for each question.

Time allowed -	**Reading Time:**	*10 minutes*
	Writing Time:	*1 hour*
You are supplied with:		*Murdoch & Barnes Statistical Tables, 4th edition*
You may also use:		*No additional materials*
Calculators:		*Calculators are allowed in this examination*

1. Suppose a die is thrown and the probability of getting i, for $i = 1, 2, \ldots, 6$, is given by:

$$P(D = i) = ai$$

where a is a constant, and D is the random variable denoting the outcome of the throw.

(a) Show that $a = 1/21$.

(5 marks)

(b) If $D \geq 3$, then the same die is thrown again once. Otherwise, an independent fair die is thrown twice. The final outcome is recorded in the random variable S, which is the sum of all values of dice thrown.

i. Show that:

$$P(S = 4) = \frac{16}{1{,}323}.$$

(20 marks)

ii. If $S = 4$, what is the probability that $D \leq 2$?

(10 marks)

iii. Find $\mathrm{E}(D \mid S = 4)$.

(15 marks)

2. (a) Consider two random variables X and Y, where X can take the values 0, 1 and 2 and Y can take the values 0 and 1. The joint probabilities for each pair are given by the following table.

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	0.1	0.1	0.4
$Y = 1$	0.2	0.1	0.1

Calculate:

i. $\mathrm{E}(Y)$

(6 marks)

ii. $P(Y < X \mid X < 2)$

(7 marks)

iii. $P(Y < X \mid X + Y = 2)$.

(7 marks)

(b) The random variable X has the cumulative distribution function given by:

$$F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ 0.4x^{1.5} + 0.6x & \text{for } 0 < x \leq 1 \\ 1 & \text{for } x > 1. \end{cases}$$

i. Determine:

$$\mathrm{Cov}\left(\frac{1}{\sqrt{X}}, X\right).$$

(20 marks)

ii. Calculate:

$$P\left(X < \frac{9}{16} \,\middle|\, X > \frac{1}{4}\right).$$

(10 marks)

[END OF PAPER]