ST102/ST109 Exercise 3

In this exercise you will practise classical probability (Questions 1, 2 and 3), the definition of independent events (Question 4), and determine the probability of winning a particular game (Question 5). Remember a '*' indicates a more challenging question!

Your answers to this problem set should be submitted as a pdf file upload to Moodle. It will be covered by your class teacher in your third class, which will take place in the week commencing Monday 16 October 2023.

1. A box contains 15 light bulbs, of which two are defective. If a person selects 6 bulbs at random, without replacement, what is the probability that both defective bulbs will be selected?

2. A box contains 20 equal-sized balls, numbered 1 to 20. Two balls are drawn at random simultaneously. What is the probability that the numbers on the two balls will differ by more than 2?

3.* If n people are seated in a random manner in a row containing $2n$ seats, what is the probability that no two people will occupy adjacent seats?

4. Suppose A and B are independent events, i.e. $P(A \cap B) = P(A)\,P(B)$. Prove that:

   (a) A and B^c are independent events

   (b) A^c and B^c are independent events.

   Hint: Question 5 of Exercise 2 is useful for (a).

5.* Consider a game where two fair dice are rolled. On the first roll if the total score is 7 or 11, the player wins. If the total score is 2, 3 or 12, the player loses. If any other total score occurs, the total score shown is called the 'point' and the game continues (the value of the point is then fixed). On each subsequent roll of the two fair dice, if the total score is the same as the point in the first round the player wins; if the total score is 7 the player loses; any other total score means the game continues. What is the probability that the player wins?

   Hint: Consider defining the probability of the 'point' to be π.