January 2022 Exam

THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

Assessment paper and instructions to candidates:

ST102/ST109
Elementary Statistical Theory (Part I)

Suitable for 2021/22 syllabus only – not for resit candidates

Instructions to candidates

This paper contains four questions. Answer **ALL FOUR** questions. All questions will be given equal weight (25%).

Full working must be shown to gain all marks for each question.

This examination counts for 50% of the final grade for ST102, and for 100% of the final grade for ST109.

Make sure your five-digit candidate number and course code are written or included at the top of every page of your answers.

If you think there is an error in the paper, you should clearly state your assumptions and proceed with the question(s). In addition, you should note the suspected error clearly at the beginning of your submission under the heading 'NOTE'. This approach will enable examiners to make any necessary adjustments equitably across all students when looking at individual scripts ex post.

Time allowed to complete assessment: 2 hours 10 minutes

Additional time added for assessment upload: 1 hour

Answer **all four** questions.

1. (a) A box contains 12 light bulbs, of which two are defective. If a person selects 5 bulbs at random, without replacement, what is the probability that both defective bulbs will be selected?

(7 marks)

(b) A and B are independent events such that:

$$P((A \cup B)^c) = \pi_1 \quad \text{and} \quad P(A) = \pi_2.$$

Determine $P(B)$ as a function of π_1 and π_2.

(7 marks)

(c) A county is made up of three (mutually exclusive) communities A, B and C, with proportions of people living in them given by the following table:

Community	A	B	C
Proportion	0.20	0.50	0.30

Given a person belongs to a certain community, the probability of that person being vaccinated is given by the following table:

Community given	A	B	C
Probability of being vaccinated	0.80	0.70	0.60

i. We choose a person from the county at random. What is the probability that the person *is not* vaccinated?

(5 marks)

ii. We choose a person from the county at random. Find the probability that the person is in community A, given the person *is* vaccinated.

(4 marks)

iii. In words, *briefly* explain how the ‘probability of being vaccinated’ for each community would be known in practice.

(2 marks)

2. (a) *Use the total probability formula* to answer each of the following. A, B and C are events, with complementary events denoted by '^{c}' as per usual course notation.

i. If $P(A \mid B) = P(A \mid B^c)$, show that A and B are independent.

(7 marks)

ii. If $P(A \mid C) > P(B \mid C)$ and $P(A \mid C^c) > P(B \mid C^c)$, show that $P(A) > P(B)$.

(7 marks)

(b) The random variable X has a Poisson distribution with parameter $\lambda = 3$. Calculate $P(X \geq 2 \mid X > 0)$.

(5 marks)

(c) Surviving contestants on *Squid Game* reach the 'Glass Stepping Stones' round, where players must cross a two-panel wide bridge. One panel is made of tempered glass (supporting a player's weight) and the other panel is made of regular glass (which cannot support their weight). The bridge consists of 18 sets of two panels (so to cross the bridge a player must safely navigate the correct 'path' of 18 tempered panels). The second player follows the first player.

i. What is the probability that the *first* player crosses the bridge successfully? State any assumptions you make.

(3 marks)

ii. Would the second player be at an advantage (relative to the first player)? Briefly explain why or why not.

(3 marks)

3. (a) A random variable, X, has the following probability density function:

$$f(x) = \begin{cases} x/5 & \text{for } 0 \leq x < 2 \\ (20 - 4x)/30 & \text{for } 2 \leq x \leq 5 \\ 0 & \text{otherwise.} \end{cases}$$

i. Sketch the graph of $f(x)$. (The sketch can be drawn on ordinary paper – no graph paper needed.)

(3 marks)

ii. Derive the cumulative distribution function of X.

(7 marks)

iii. Find the mean and the standard deviation of X.

(7 marks)

(b) i. A coffee machine can be calibrated to produce an average of μ millilitres (ml) per cup. Suppose the quantity produced is normally distributed with a standard deviation of 9 ml per cup. Determine the value of μ such that 250 ml cups will overflow only 1% of the time.

(4 marks)

ii. Suppose now that the standard deviation, σ, can be fixed at specified levels. What is the largest value of σ that will allow the amount of coffee dispensed to fall *within* 30 ml of the mean with a probability of at least 95%?

(4 marks)

4. (a) Suppose that X is a discrete random variable for which the moment generating function is:

$$M_X(t) = \frac{1}{4}(\mathrm{e}^{3t} + \mathrm{e}^{6t} + \mathrm{e}^{9t}) + \frac{1}{8}(\mathrm{e}^{2t} + \mathrm{e}^{4t})$$

for $-\infty < t < \infty$. Write down the probability distribution of X.

(6 marks)

(b) Consider two random variables, X and Y. They both take the values -1, 0 and 1. The joint probabilities for each pair of values, (x, y), are given in the following table.

	$X = -1$	$X = 0$	$X = 1$
$Y = -1$	0.09	0.16	0.15
$Y = 0$	0.09	0.08	0.03
$Y = 1$	0.12	0.16	0.12

i. Determine the marginal distributions and calculate the expected values of X and Y, respectively.

(4 marks)

ii. Calculate the covariance of the random variables X and Y.

(4 marks)

iii. Calculate $\mathrm{E}(X \mid Y = 0)$ and $\mathrm{E}(X \mid X + Y = 1)$.

(6 marks)

iv. Define $U = |X|$ and $V = Y$. Calculate $\mathrm{E}(U)$ and the covariance of U and V. Are U and V correlated?

(5 marks)

[END OF PAPER]