

ST102/ST109 Exercise 9

In this exercise you will practise approximating distributions and topics related to multivariate random variables. Question 1 involves an unseen (as far as lectures are concerned) application of approximating distributions. Question 2 deals with random sample surveys of people. Finally, Questions 3 and 4 cover discrete bivariate distributions.

Your answers to this problem set should be submitted as a pdf file upload to Moodle. It will be covered by your class teacher in your ninth class, which will take place in the week commencing Monday 28 November 2022.

1. We have found that the Poisson distribution can be used to approximate a binomial distribution, and a normal distribution can be used to approximate a binomial distribution. It should not be surprising that a normal distribution can be used to approximate a Poisson distribution. It can be shown that the approximation is suitable for large values of the Poisson parameter λ , and should be adequate for practical purposes when $\lambda \geq 10$.

- (a) Suppose X is a Poisson random variable with parameter λ . If we approximate X by a normal variable which $\sim N(\mu, \sigma^2)$, what are the values which should be used for μ and σ^2 ?

Hint: What are the mean and variance of a Poisson distribution?

- (b) Use this approach to estimate $P(X > 10)$ for a Poisson random variable with $\lambda = 14$. Use a continuity correction.

Note: The exact value of this probability, from the Poisson distribution, is 0.8243188.

- 2.* This question is about calculations based on random sample surveys of people.

- (a) It is believed that 40% of the 20 adults in a village are supporters of ST102 FC. If this belief is correct, and four *different* people are picked at random and asked whether they support ST102 FC, what is the probability that exactly three will be supporters?
 - (b) It is believed that 40% of the many thousands of adults in Holborn are supporters of ST102 FC. If this belief is correct, and 40 people are picked at random and asked about their allegiance, what is the probability that exactly twenty will be supporters?
 - (c) It is believed that 40% of the many thousands of adults in Holborn are supporters of ST102 FC. If this belief is correct, and 100 people are picked at random and asked about their allegiance, what is the probability that at least thirty will be supporters?
 - (d) If you have used a suitable approximation in any of the previous parts, explain why it is appropriate in each case.
 - (e) Comment on the differences, if any, in the assumptions and methods you have used when calculating the probabilities obtained above.

3. X and Y are discrete random variables with the following joint probability function:

		$X = x$		
		-1	0	1
$Y = y$	0	0.30	0.25	0.10
	1	0.15	0.05	0.15

- (a) Obtain the marginal distributions of X and Y , respectively.
 - (b) Calculate $E(X)$, $\text{Var}(X)$, $E(Y)$ and $\text{Var}(Y)$.
 - (c) Obtain the conditional distributions of Y given $X = -1$, and of X given $Y = 0$.
 - (d) Calculate $E_{Y|X}(Y | X = -1)$ and $E_{X|Y}(X | Y = 0)$.
 - (e) Calculate $E(XY)$, $\text{Cov}(X, Y)$ and $\text{Corr}(X, Y)$.
 - (f) Find $P(X > Y)$ and $P(X^2 > Y^2)$.
 - (g) Are X and Y independent? Explain why or why not.
- 4.* A box contains 4 red balls, 3 green balls and 3 blue balls. Two balls are selected at random without replacement. Let X represent the number of red balls in the sample and Y the number of green balls in the sample.
- (a) Arrange the different pairs of values of (X, Y) as the cells in a table, each cell being filled with the probability of that pair of values occurring, i.e. provide the joint probability distribution.
 - (b) What does the random variable $Z = 2 - X - Y$ represent?
 - (c) Calculate $\text{Cov}(X, Y)$.
 - (d) Calculate $P(X = 1 | -2 < X - Y < 2)$.