ST102 Class 16 – Additional exercises

1. A random sample of size 1 is taken from the probability density function:

$$f(x) = \begin{cases} (\theta + 1)x^{\theta} & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

   The null hypothesis $\mathrm{H}_0 : \theta = 1$ is to be rejected in favour of $\mathrm{H}_1 : \theta > 1$ if $x \geq 0.90$. What is the significance level of the test?

2. You wish to test whether a coin is fair. In 400 tosses of a coin, 217 heads and 183 tails appear. Is it reasonable to assume that the coin is fair? Justify your answer with an appropriate hypothesis test. Calculate the p-value of the test, and assume a 5% significance level.

3. Suppose we test $\mathrm{H}_0 : \pi = 0.75$ vs. $\mathrm{H}_1 : \pi < 0.75$ using a random sample of size $n = 7$ with the decision rule of rejecting H_0 if $X \leq 3$, where X has a binomial distribution.

   (a) What is the significance level of the test?

   (b) Determine the power of the test for some different values of π.