ST102 Exercise 15

In this exercise you will practise aspects of asymptotic distributions and interval estimation. Question 1 requires you to obtain the asymptotic sampling distribution of the maximum likelihood estimator of the Bernoulli parameter, π. Question 2 uses the sample standard deviation for determining a confidence interval for the population mean. In Question 3, think about using a two-point (i.e. Bernoulli) distribution. Finally, Question 4 requires you to evaluate whether 'σ' or the sample standard deviation is appropriate to calculate a confidence interval.

Your answers to this problem set should be submitted as a pdf file upload to Moodle, *as directed by your class teacher*. It will be covered by your class teacher in your fifteenth class, which will take place in the week commencing Monday 20 February 2023.

1.* Let $\{X_1, X_2, \ldots, X_n\}$ be a random sample from a Bernoulli distribution with parameter π, where $0 < \pi < 1$. By computing Fisher information, determine the asymptotic distribution of the maximum likelihood estimator of π, which is $\widehat{\pi} = \bar{X}$.

2. In a survey of students, the number of hours per week of private study is recorded. For a random sample of 22 students, the sample mean is 19.1 hours and the sample standard deviation is 3.8 hours. Treat the data as a random sample from a normal distribution.

   (a) Find a 90% confidence interval for the mean number of hours per week of private study in the student population.

   (b) Recompute your confidence interval in the case that the sample size is, in fact, 121, but the sample mean and sample standard deviation values are unchanged. Comment on the two intervals.

3.* In a study of consumers' views on guarantees for new products, 410 out of a random sample of 475 consumers agreed with the statement: *'Product guarantees are worded more for lawyers to understand than to be easily understood by consumers.'*

   (a) Find an *approximate* 95% confidence interval for the population proportion of consumers agreeing with this statement.

   (b) Would a 99% condidence interval for the population proportion be wider or narrower than that found in (a)? Explain your answer.

4. Suppose a random survey of 400 first-time home buyers finds that the sample mean of annual household income is £36,000 and the sample standard deviation is £17,000.

   (a) An economist believes that the 'true' standard deviation is $\sigma = $ £12,000. Based on this assumption, find an *approximate* 90% confidence interval for μ, i.e. for the average annual household income of all first-time home buyers.

(b) Without the assumption that σ is known, find an *approximate* 90% confidence interval for μ.

(c) Are the two confidence intervals very different? Which one would you trust more, and why?