

# ST202/ST206 – Autumn Term

## Problem set 3

Due: 12 noon, Wednesday AT Week 4

Please submit your answers to the questions marked †.

- 1.† Suppose that  $X \sim \text{Geometric}(p)$ , with probability mass function (PMF)

$$f_X(x) = (1-p)^{x-1}p \quad \text{for } x = 1, 2, \dots$$

Work out the cumulative distribution function (CDF) of  $X$ . Find the **survival function** of  $X$ , defined as  $S_X(x) = P(X > x)$ . What do you observe? How could you have arrived at this answer directly?

- 2.† The claims received by an insurance company have CDF  $F_X(x)$ . Now suppose that the company decides not to pay out claims for amounts less than or equal to  $c$ . Work out the CDF of the claims paid in terms of  $F_X(x)$ . What is the CDF for the claims *not* paid? [*Hint: Work out the probability that a claim is for an amount  $\leq x$ , conditional on it being  $> c$ .*]

3. Let  $Y \sim \text{NegBin}(r, p)$ , which has PMF

$$f_Y(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r} \quad \text{for } y = r, r+1, \dots$$

Show that  $f_Y(y)$  is a valid PMF. You may use the negative binomial expansion,

$$(1-a)^{-n} = \sum_{j=0}^{\infty} \binom{j+n-1}{n-1} a^j, \quad \text{provided } |a| < 1.$$

- 4.† The **Gamma function** is defined as

$$\Gamma(t) = \int_0^{\infty} u^{t-1} e^{-u} du.$$

- (a) Work out  $\Gamma(1)$  and show that  $\Gamma(t) = (t-1)\Gamma(t-1)$  for  $t > 1$ .  
(b) If  $k \in \mathbb{Z}^+$ , a positive integer, what is  $\Gamma(k)$  equal to?

A random variable  $X$  has the **Polya** distribution if its PMF is

$$f_X(x) = \frac{\Gamma(r+x)}{x!\Gamma(r)} p^r (1-p)^x \quad \text{for } x = 0, 1, \dots$$

where  $0 < p < 1$  and  $r > 0$  are parameters. Explain why the negative binomial distribution is a special case of the Polya distribution. [*Hint: Use the form of the NegBin where we count the number of failures.*]