ST202/ST206 – Autumn Term

Problem set 1

Due: 12 noon, Wednesday AT Week 2

Please submit your answers to the questions marked †.

1.† Suppose that $(\Omega, \mathcal{F})$ is a measurable space, and $A \in \mathcal{F}$. Show that the following functions satisfy the properties of probability measures:

(a) $\mathrm{P}(A) = |A|/|\Omega|$, where $|A|$ is the number of outcomes in A. (You may assume that $|\Omega| < \infty$.)

(b) $\mathrm{P}(A) = \mathbf{1}_A(\omega)$ for some $\omega \in \Omega$.

2.† Consider a set Ψ and two subsets $A, B \subset \Psi$, where $A \cap B \neq \varnothing$ but $A \cap B \neq A$ and $A \cap B \neq B$. What is the size of the smallest σ-algebra containing both A and B? What about if $A \subset B$ or $A \cap B = \varnothing$? Generalise your results to 3 sets, then to N sets.

3. Suppose that $\{A_i : i = 1, 2, \ldots\}$ is a decreasing sequence of sets, i.e. $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots$ Prove that $\lim_{n\to\infty} \mathrm{P}(A_n) = \mathrm{P}\left(\bigcap_{i=1}^{\infty} A_i\right)$.

4.† An insurance claimant is trying to hide three fraudulent claims among seven genuine claims. The claimant knows that the insurance company processes claims in batches of five or in batches of 10. For batches of five, the insurance company will investigate one claim at random to check for fraud; for batches of 10, two of the claims are randomly selected for investigation. The claimant has three possible strategies:

(a) submit all 10 claims in a single batch,

(b) submit two batches of five, one containing two fraudulent claims, the other containing one,

(c) submit two batches of five, one containing three fraudulent claims, the other containing 0.

What is the probability that all three fraudulent claims will go undetected in each case? What is the optimal strategy for the fraudster?