ST202/ST206 – Michaelmas Term

Problem set 9

Due: 12 noon, Wednesday MT Week 11

1. Suppose that the random variables $X_1, \ldots, X_n$ are independent and Poisson distributed, with $\mathbb{E}(X_i) = \lambda_i$. What is the probability distribution of their sum, $Y = X_1 + \ldots + X_n$?

2. Let X and Y be random variables with joint density

$$f_{X,Y}(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Find the density function of their sum, $Z = X + Y$.

3. Suppose that Y is the number of insurance claims made by an individual policyholder in one year. We assume that $Y|X = x \sim \text{Pois}(x)$ where $X \sim \text{Gamma}(\alpha, \theta)$.

   (a) Work out the probability that the policyholder makes exactly y claims in a year.

   (b) What is the expected number of claims in a year?

4.* Suppose that U and V are independent standard normal random variables, and let $X = U$ and $Y = \rho U + \sqrt{1-\rho^2}V$, where $-1 < \rho < 1$.

   (a) Show that the joint density of X and Y is

$$f_{X,Y}(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right\},$$

   for $x, y \in \mathbb{R}$.

   (b) Now let $X^* = \mu_X + \sigma_X X$ and $Y^* = \mu_Y + \sigma_Y Y$, where μ_X, μ_Y, σ_X, σ_Y are constants, and $\sigma_X, \sigma_Y > 0$. What is the joint density of X^* and Y^*?

   (c) Write down the conditional density of Y given X. What probability distribution is this?