ST202/ST206 – Autumn Term

Problem set 7

1. Suppose that the random variables X and Y have joint density function

$$f_{X,Y}(x,y) = \begin{cases} kxy & \text{if } 0 < x < 1,\ 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the value of k.
   (b) Work out the marginal densities, $f_X(x)$ and $f_Y(y)$.
   (c) Compute $\mathbb{E}(X)$ and $\text{Var}(Y)$.
   (d) Evaluate $\mathbb{E}[9(X-1)Y^2]$.

2. Consider the function

$$f_{X,Y}(x,y) = \begin{cases} k(x^2+y^2) & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Under which condition is $f_{X,Y}$ a valid joint density for X and Y?
   (b) Compute $P(X < Y^2)$.

3. Consider random variables X and Y with joint density

$$f_{X,Y}(x,y) = \begin{cases} 8xy & \text{if } 0 < y < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Evaluate $\text{Cov}(X,Y)$ and $\text{Corr}(X,Y)$.

4.* Suppose that F_X and F_Y are cumulative distribution functions. By checking the elementary properties of joint CDFs, identify the cases in which G **cannot** be a joint CDF:

   (a) $G(x,y) = F_X(x) + F_Y(y)$
   (b) $G(x,y) = F_X(x)F_Y(y)$
   (c) $G(x,y) = \max[F_X(x), F_Y(y)]$
   (d) $G(x,y) = \min[F_X(x), F_X(y)]$