ST107 Exercise 3

In this exercise you will practise discrete probability distributions. Question 1 requires you to determine probabilities of events for a new random variable, T, derived from the original random variable, X. Question 2 requires you to work with the binomial and Poisson distributions, and you may find it convenient to use the result that $P(A) = 1 - P(A^c)$. Question 3 asks you to work with a binomial random variable, and a transformation of it. Finally, Question 4 deals with the Poisson distribution, and in (c) remember the definition of a conditional probability (note that (c) is challenging!).

You should attempt these questions ahead of the corresponding class. It will be covered by your class teacher in your third class, which will take place in the week commencing Monday 10 February 2025.

1. A random variable X can take either the value 3 or the value 6. The respective probabilities of its taking these values are:

$$P(X = 3) = 0.75 \quad \text{and} \quad P(X = 6) = 0.25.$$

   (a) Determine the mean and the variance of X.

   (b) If twelve independent values of X are sampled, and the total of the twelve values is T, find:

      i. the probability that $T = 40$

      ii. the probability that $T = 45$.

2. It is known that 8% of the people living inside the M25 believe that mobile telephone use should *not* be made available throughout the London Underground system.

   It is proposed to conduct a survey of these people as part of an effort to encourage the provision of such a service.

   (a) Find the probability that, in a pilot survey of 17 people, at least three of them do *not* favour the new service.

   (b) Using a suitable approximation, find the probability of at least three people not in favour in a larger random sample of 110.

   In both parts, explain any assumptions you are making in order to justify the methods of calculation which you use.

3. A random variable X has a binomial distribution with $n = 6$ and $\pi = 0.3$.

   (a) Write out the probability distributions of X and $(X - 2)^2$.

   (b) Find the mean and the variance of X, and the mean of $(X - 2)^2$ using:

      i. the distributions derived in (a)

      ii. the properties of the expectation operator, $\mathrm{E}(\cdot)$, and noting that the mean and variance of a binomial random variable are $n\pi$ and $n\pi(1 - \pi)$, respectively.

4. (a) What are the assumptions which a process needs to satisfy to qualify as having a Poisson distribution?

   (b) Arrivals at a post office may be modelled as following a Poisson distribution with a rate parameter of 78 arrivals per hour. Find:

      i. the probability of exactly six arrivals in a period of two minutes

      ii. the probability of more than two arrivals in 90 seconds

      iii. the probability that the time to arrival of the next customer is less than one minute.

   (c) If D is the time to arrival of the next customer (in minutes), calculate:

$$P(D > 1.9 \,|\, D > 1).$$