ST102 Exercise 16

In this exercise you will practise aspects of interval estimation and hypothesis testing. Question 1 requires a confidence interval for σ^2. Question 2 ensures you can easily navigate Murdoch and Barnes' *Statistical Tables*. Question 3 involves testing the value of a probability parameter. Question 4 covers hypothesis testing of the mean of a normal population. Finally, Question 5 involves testing a parameter of a Poisson distribution.

Your answers to this problem set should be submitted as a pdf file upload to Moodle, *as directed by your class teacher*. It will be covered by your class teacher in your sixteenth class, which will take place in the week commencing Monday 27 February 2023.

1. A random sample of size $n = 18$ drawn from a normal distribution had a sample variance of $s^2 = 27.64$. Construct a 98% confidence interval for σ^2.

2. Let $Z \sim N(0,1)$, $X_n \sim \chi^2_n$ and $T_n \sim t_n$ (for example, $X_2 \sim \chi^2_2$ and $T_3 \sim t_3$). Find the constants k in each of the following cases. Use the closest tabulated values where appropriate.

   (a) $P(Z < k) = 0.0681$.

   (b) $P(Z < -1.72) = k$.

   (c) $P(-0.23 < Z < 2.04) = k$.

   (d) $P(X_6 < k) = 0.20$.

   (e) $P(X_{26} > k) = 0.025$.

   (f) $P(18.939 < X_{28} < 50.993) = k$.

   (g) $P(T_7 > k) = 0.99$.

   (h) $P(T_{19} < 2.861) = k$.

   (i) $P(|T_{60}| < k) = 0.98$.

3. Let $\{X_1, X_2, \ldots, X_{12}\}$, taking values either 1 or 0, be the outcomes of an experiment of tossing a coin 12 times, where:

   $$P(X_i = 1) = \pi = 1 - P(X_i = 0) \quad \text{for } \pi \in (0,1).$$

   We are interested in testing:

   $$\mathrm{H}_0 : \pi = 0.50 \quad \text{vs.} \quad \mathrm{H}_1 : \pi \neq 0.50.$$

   Suppose there are 3 X_is taking the value 1, and 9 X_is taking the value 0. Will you reject the null hypothesis at the 10% significance level?

4. A random sample of 16 observations from the population $N(\mu, \sigma^2)$ yields the sample mean $\bar{x} = 9.31$ and the sample variance $s^2 = 0.375$. At the 5% significance level, test the following hypotheses by obtaining critical values:

   (a) $\mathrm{H}_0 : \mu = 9$ vs. $\mathrm{H}_1 : \mu > 9$.

   (b) $\mathrm{H}_0 : \mu = 9$ vs. $\mathrm{H}_1 : \mu < 9$.

   (c) $\mathrm{H}_0 : \mu = 9$ vs. $\mathrm{H}_1 : \mu \neq 9$.

   Repeat the above exercise with the additional assumption that $\sigma^2 = 0.375$. Compare the results with those derived without this assumption and comment.

5. In a given city, it is assumed that the number of car accidents in a given week follows a Poisson distribution. In past weeks, the average number of accidents per week was 8, and this week there were 3 accidents. Is it justified to claim that the accident rate has dropped? Calculate the p-value of the test, and assume a 5% significance level.