27.01.19

$$\int \frac{dx}{x(x^{10}+2)} = \int \frac{dx}{x^{11}\left(1+\frac{2}{x^{10}}\right)} = -\frac{1}{10}\int \frac{d\left(\frac{1}{x^{10}}\right)}{1+2\left(\frac{1}{x^{10}}\right)} =$$

$$= -\frac{1}{10}\int \frac{dy}{1+2y} = -\frac{1}{20}\int \frac{d(2y+1)}{2y+1} = -\frac{1}{20}\ln\left(1+\frac{2}{x^{10}}\right)+C$$

$$\int \frac{2^{\cos x}\cdot \sin x}{\sqrt{1-4^{\cos x}}}dx = -\int \frac{2^{\cos x}\, d\cos x}{\sqrt{1-2^{2\cos x}}} = \frac{1}{\ln 2}\int \frac{dy}{\sqrt{1-y^2}} = \ldots$$

$$\int \frac{dx}{\sin x} = \int \frac{\sin x}{\sin^2 x}dx = -\int \frac{d\cos x}{1-\cos^2 x} = -\int \frac{dy}{1-y^2} = \ldots$$

$$\frac{P_1(x)}{(x-a)^k} = \frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + \ldots +$$

$$A_1 \ln|x-a| - A_2(x-a)^{-1}$$

$$\frac{A_k}{1-k}(x-a)^{1-k}$$

05.02.19

$$\int x \ln\frac{x-1}{x+1}\,dx = \begin{bmatrix} u = \ln\frac{x-1}{x+1} \\ dV = x\,dx \\ V = x^2/2 \end{bmatrix} = \frac{x^2}{2}\cdot\ln\frac{x-1}{x+1} -$$

$$-\frac{1}{2}\int x^2 \frac{2}{x^2-1}\,dx = x - \frac{1}{2}\ln\frac{x-1}{x+1} + C$$

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$$\int \frac{x^2 dx}{\sqrt{1+x+x^2}} \qquad \sqrt{x^2+x+1} = x+t = \frac{-t^2+t-1}{1-2t}$$

$$\cancel{x^2}+x+1 = \cancel{x^2}+2tx+t^2$$

$$x = \frac{t^2-1}{1-2t}$$

$$dx = \frac{2t(1-2t)+2t^2-2}{(1-2t)^2}\,dt$$

$$I = \int \frac{\frac{(t^2-1)^2}{(1-2t)^2}\cdot\frac{2\cancel{(-t^2+t-1)}}{(1-2t)^2}}{\cancel{\frac{-t^2+t-1}{1-2t}}} = 2\int\frac{(t^2-1)^2}{(1-2t)^3}\,dt$$

$$\int \cos x \cdot \cos 2x \cdot \cos 3x \, dx$$

$$\boxed{e^{ix} = \cos x + i \sin x}$$

$$\frac{e^{ix}+e^{-ix}}{2} \cdot \frac{e^{2ix}+e^{-2ix}}{2} \cdot \frac{e^{3ix}+e^{-3ix}}{2}$$

$$\int e^{aix} = \frac{e^{aix}}{ai}$$

---

$$\int \sin^n x \, dx = I_n$$

$$\int \sin^n x \, dx = \begin{bmatrix} u = \sin^{n-1} x \\ dv = \sin dx \\ v = -\cos x \end{bmatrix} = -\cos x \cdot \sin^{n-1} x + \int (n-1) \cos x^2 \cdot \sin^{n-2} x$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \cdot \int (1-\sin^2 x)(\sin^{n-2} x)$$

$$I_n = -\cos x \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

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$$\int \frac{dx}{1+\sin x - 3\cos x} = 2\int \frac{dt}{1+t^2+2t-3-3t^2} = -\int \frac{dt}{t^2-t+1} =$$

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad = \frac{-1}{\sqrt{3}} \operatorname{arctg}\left(\frac{tg\frac{x}{2}-\frac{1}{2}}{\sqrt{3}}\right)+C$$

$$dx = \frac{2dt}{1+t^2}$$

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26.02.19

## Несобственный интеграл

$$\int_a^{+\infty} f(x)\,dx = \lim_{B\to+\infty} \int_a^{B} f(x)\,dx \quad \text{— I-ый род}$$

$f(x) \in C[a, +\infty]$

$$\int_0^1 \frac{1}{x}\,dx \quad \text{— II-ой род}$$

Необх. усл.: $\exists \int_a^{\infty} f(x)\,dx \Rightarrow f(x) \to 0$

$$\int_0^{+\infty} \frac{1}{x(x-1)}\,dx = \int_0^{\frac{1}{2}} + \int_{\frac{1}{2}}^{1} + \int_1^2 + \int_2^{+\infty}$$

$$\int_{-\infty}^{+\infty} \sin x\,dx \text{ — } \not\exists$$

$$\int_0^{+\infty} \frac{dx}{x} = \int_0^1 \frac{dx}{x} + \int_1^{+\infty} \frac{dx}{x} = \ln x\Big|_0^1 + \ln x\Big|_1^{+\infty}$$

$$\int_1^{+\infty} \frac{dx}{x^{\alpha}} = \frac{1}{(1-\alpha)(x^{1-a})}\Big|_1^{+\infty} \quad a > 1$$

$$\int_0^{+\infty} \frac{x^2}{x^6 - x^3 + 1}\,dx$$

$$\frac{x^2}{x^6 - x^3 + 1} < \frac{2}{x^4} \longrightarrow$$

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$f(x) \sim g(x)$ на $+\infty$

$$\Downarrow$$

$$\int_a^{+\infty} f(x) \longmapsto \quad \sim \quad \int_a^{+\infty} g(x) \longmapsto$$

$$\int_e^{+\infty} \frac{dx}{(\ln x)^p x} = \int_1^{+\infty} \frac{dt}{t^p}$$

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$$\int_{-2}^{2} (x^2 + 2 \cdot 4^x)\,dx$$

$$\int_{-2}^{2} x^2\,dx \Leftarrow \sum_{k=1}^{n} \frac{4}{n}\left(-2 + \frac{4k}{n}\right)^2 \quad \# \text{фото}$$

$-2 \qquad 2 \qquad 2 + \frac{4k}{n} = x_k$