14. 01. 19

Суть$^{(TM)}$

$F(x) = ?$

$F'(x) = f(x)$

$\frac{1}{2\sqrt{x}} = f \quad F = \sqrt{x}$

$\sqrt[3]{x} = f \quad F = \frac{3}{4} \cdot x^{\frac{4}{3}}$

$f = x^k \quad F = \frac{x^{k+1}}{k+1} \quad (k \neq -1)$

$f = x^{-1} \quad F = \begin{cases} \ln x, & x > 0 \\ \ln -x, & x < 0 \end{cases}$

$\int f(x)\,dx = F(x) + c$

неопр. интеграл

функция

указатель на переменную интегр.

некая конкрет. константа

константа

$\underbrace{f(x)\,dx}_{dF(x)}$

$\int dF(x) = F(x) + c$

$\int (x^2 + \sin x)\,dx = \int x^2\,dx + \int \sin x\,dx = \frac{x^3}{3} - \cos x + c$

$\int (x+1)^3 4\,dx = (x+1)^4 + c$

$\int \frac{(x+2)^2}{x} = \int \frac{x^2 + 4x + 4}{x}\,dx = \int x\,dx + 4x + \int \frac{4}{x}\,dx =$

$= \frac{x^2}{2} + 4x + 4\ln|x| + C$

$$\int 2\sin\cos x\,dx = \int 2\sin x(\sin x)' = \int (\sin^2 x)'dx = \sin^2 x + C$$

Замена переменных

$$\int f(g(x))\,g'(x)dx = \int f(y)dy \qquad (1)$$

$$\int 2\sin\cos x\,dx = \begin{bmatrix} y = \sin(x) \\ dy = \cos(x)dx \end{bmatrix} = \int 2y\,dy = 2\cdot\frac{y^2}{2} + C = y^2 + C =$$

$$= \sin^2(x) + C$$

$$\int \sin(2x)\,dx = \begin{bmatrix} 2x = y \\ dy = 2dx \end{bmatrix} = \int \sin(y)\,\frac{dy}{2} = -\frac{\cos y}{2} + C =$$

$$= -\frac{\cos 2x}{2} + C$$

$$\int \sqrt{4x+1}\,dx = \begin{bmatrix} y = 4x+1 \\ dy = 4dx \end{bmatrix} = \int \sqrt{y}\cdot\frac{dy}{4} = \frac{1}{4}\cdot\frac{y^{\frac{3}{2}}}{3/2} + C =$$

$$= \frac{1}{6}(4x+1)^{\frac{3}{2}} + C$$

$$\int f(x)dx = \int f(g(t))\cdot g'(t)\,dt \qquad (2)$$

$x = g(t)$

$dx = g'(t)dt$

$t = \varphi(x)$ и $g(\varphi(x)) = x \quad \forall x \in I.$

$$\int \frac{1}{\sqrt{4-8x^2}}\,dx \;=$$

1) $x = \frac{1}{\sqrt{2}}\sin t$

2) $t = \arcsin(\sqrt{2}x)$

3) $dx = \frac{1}{\sqrt{2}} \cdot \cos t\, dt$

$4 - 8x^2 > 0$

$x \in \left(-\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}\right)$

$$= \frac{1}{\sqrt{2}} \int \frac{\cos t\, dt}{\sqrt{4-8\left(\frac{1}{\sqrt{2}}\sin t\right)^2}} = \ldots = \frac{1}{\sqrt{2}} \int \frac{\cos t\, dt}{2|\cos t|} =$$

$t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

$$= \frac{1}{2\sqrt{2}} \int dt = \frac{t}{2\sqrt{2}} + C =$$

$$= \frac{\arcsin(\sqrt{2}x)}{2\sqrt{2}} + C$$

$$\int \frac{x}{1+(x+1)^2}\,dx = \begin{bmatrix} y = x+1 \\ dy = dx \end{bmatrix} = \int \frac{y-1}{1+y^2}\,dy = \int \frac{y}{1+y^2}\,dy -$$

$$- \int \frac{1}{1+y^2}\,dy = \int \frac{y}{1+y^2}\,dy - \operatorname{arctg}(y) = \begin{bmatrix} t = y^2 \\ dt = 2dy \end{bmatrix} =$$

$$= \int \frac{1}{1+t}\,\frac{dt}{2} - \operatorname{arctg}(y) = \frac{1}{2}\ln|1+t| - \operatorname{arctg}(y) + C =$$

$$= \frac{1}{2}\ln\left(1+(x+1)^2\right) - \operatorname{arctg}(x+1) + C$$

$F(x) = ?$

$F'(x) = \frac{x}{1+(1+x)^2}$

$F(-1) = 8$

$F(x) = \int \frac{x}{1+(x+1)^2} dx =$

$= \frac{1}{2} \ln(1+(1+x)^2) - \operatorname{arctg}(x+1) + C$

$C = 8$

Формула интегрирования по частям

$u(x),\ v(x) \Rightarrow$

$$\int u'(x) v(x) + u(x) \cdot v'(x)\, dx = \int (u v)' dx = u(x) \cdot v(x) + C$$

$$\int u(x) \cdot v'(x) dx = u(x) v(x) - \int v(x) u'(x) dx$$

$$\int u\, dv = uv - \int v\, du$$

$$\int (x^2+x) e^x dx = \begin{bmatrix} u(x) = x^2+x \\ v'(x) = e^x \\ v(x) = e^x \end{bmatrix} = (x^2+x)e^x - \int e^x \cdot (2x+1) dx =$$

$$= (x^2+x)e^x - \int e^x dx - 2\int x e^x dx = \begin{bmatrix} u(x) = x \\ v(x) = e^x \end{bmatrix} =$$

$$= (x^2+x)e^x - e^x - 2xe^x + 2e^x + C = e^x(x^2 - x + 1) + C$$

21.01.19

$$\int tg\,x\,dx = \int \frac{\sin x}{\cos x}dx = -\int \frac{d\cos x}{\cos x} = -\ln|\cos x| + C$$

$$\int \frac{\sqrt{x}}{1+x}dx = \int \frac{x}{1+x}\cdot\frac{dx}{\sqrt{x}} = \begin{bmatrix} \sqrt{x} = t \\ d\sqrt{x} = dt \\ \frac{x^{-\frac{1}{2}}}{2}dx = dt \end{bmatrix} =$$

$$= 2\int \frac{t^2}{1+t^2}dt$$

$$\int arctg\,x\,dx = \begin{bmatrix} u = arctg\,x \\ V = x \\ dV = dx \end{bmatrix} = x\cdot arctg\,x - \int \frac{x}{1+x^2}dx =$$

$$= x\cdot arctg\,x - \frac{1}{2}\int \frac{dx^2}{1+x^2} = x\cdot arctg\,x - \frac{1}{2}\ln(1+x^2) + C$$

$$\overset{F}{\overset{\shortparallel}{\int e^x \sin x\,dx}} = \begin{bmatrix} e^x = u \\ \sin x = V' \end{bmatrix} = -e^x\cdot\cos x - \int(-\cos x)e^x dx =$$

$$= \begin{bmatrix} u = e^x \\ \cos x = V' \end{bmatrix} = -e^x\cos x + e^x\sin x - \int \sin x\, e^x dx$$

$$F = \frac{1}{2}e^x(\sin x - \cos x) + C$$

$$\int \frac{P(x)}{Q(x)}\,dx$$

$$P(x) = B_n x^n + \ldots B_1 x + B_0$$

$$Q(x) = A_m x^m + \ldots + A_1 x_1 + A_0$$

если $n \geq m$, то $\exists\, S(x)$ и $R(x)$:

$$P(x) = S(x) \cdot Q(x) + R(x)$$

$$\frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)}$$

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\left(\ln|x+1| - \ln|x-1|\right) + C = \underline{\frac{1}{2}\ln\frac{|1+x|}{|1-x|} + C}$$

$$\frac{1}{1-x^2} = \frac{1}{(x-1)(x+1)} = \frac{(1-x+1+x)\cdot\frac{1}{2}}{(1-x)(1+x)} = \frac{1}{2}\,\frac{1}{1-x} +$$

$$\frac{1}{2}\,\frac{1}{1+x} = \frac{a}{x+1} + \frac{b}{x-1} \qquad a = \frac{1}{2}, \quad b = -\frac{1}{2}$$

Фото

28.01.19

1

$$\int \frac{x^3+x}{(x+1)^2}\,dx = \int \frac{x^3+x}{x^2+2x+1} = \int\left(x-2 + \frac{4x+2}{x^2+2x+1}\right)dx =$$

$$\frac{4x+2}{x^2+2x+1} = \frac{a}{x+1} + \frac{b}{(x+1)^2} = \frac{ax+a+b}{(x+1)^2} = \begin{bmatrix} a=4 \\ b=-2 \end{bmatrix} =$$

$$= \int (x-2)\,dx + \int \frac{4}{x+1}\,dx + \int \frac{-2}{(x+1)^2}\,dx = \frac{x^2}{2} - 2x + 4\ln|x+1| + \frac{2}{x+1} + C$$

$$\frac{?}{(x-a)^k(x-b)^k\dots\underbrace{(x^2+cx+d)^m}_{D<0}} = \frac{a_1}{x-a} + \frac{a_2}{(x-a)^2} + \dots + \frac{a_k}{(x-a)^k} + \dots + \frac{c_1x+d_1}{x^2+cx+d} + \dots + \frac{c_mx+d_m}{(x^2+cx+d)^m}$$

2

$$\int \frac{x^2}{x^4-2x^2+1}\,dx = \int \frac{x^2}{(x^2-1)^2}\,dx = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$$

?

$$\frac{d_1x+c_1}{x^2-1} + \frac{d_2x+c_2}{(x^2-1)^2}$$

$$(d_1x+c_1)(x^2-1) + d_2x + c_2$$

$$d_1x^3 - d_1x + c_1x^2 + d_2x + c_2 = x^2$$

$$\frac{x^2}{(x^4-2x^2+1)} = \frac{x^2}{(x-1)^2(x+1)^2} = \frac{a}{x-1} + \frac{b}{(x-1)^2} + \frac{c}{x+1} + \frac{d}{(x+1)^2} =$$

$$a(x-1)(x+1)^2 + b(x+1) + c(x-1)^2(x+1) + dx(x-1)^2 = x^2$$

$$ax^3 + ax^2 - ax - 1 + bx^2 + 2bx + b + cx^3 - cx^2 - cx + c +$$
$$dx^2 - 2dx + d = x^2$$

$$\begin{cases} a + c = 0 \\ a + b - c + d = 1 \\ -a + 2b - c - 2d = 0 \\ -a + b + c + d = 0 \end{cases} \qquad \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 1 & 1 \\ -1 & 2 & -1 & -2 & 0 \\ -1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{cases} b = \frac{1}{4} \\ c = -\frac{1}{4} \\ d = \frac{1}{4} \\ a = \frac{1}{4} \end{cases}$$

$$= \frac{1}{4}\left(\frac{1}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{x+1} + \frac{1}{(x+1)^2}\right)$$

$$\int \bigcirc = \ldots\ldots$$

$$\int \frac{1}{x^4+1}\,dx$$

I) Неприводимый сомн (x^4+1)

1) $x^4+1>0$

– нет корней

$$x^4+1=(ax^2+bx+c)(\alpha x^2+\beta x+\gamma)$$

$D<0$ $\qquad$ $D<0$

$$a\alpha x^4+(a\beta+b\alpha)x^3+(a\gamma+c\alpha+b\beta)x^2+(b\gamma+c\beta)x+$$
$$+c\gamma=(x^4+1)$$

$$\begin{cases} a\alpha=1 \\ a\beta+b\alpha=0 \\ a\gamma+c\alpha+b\beta=0 \\ b\gamma+c\beta=0 \\ c\gamma=1 \end{cases}$$

#Фото

04.02.19

$$\int \frac{1x}{(x^2+1)^n}\,dx = \left[\begin{matrix} u = x^2 \\ x\,dx = \frac{du}{2} \end{matrix}\right] = \frac{1}{2}\int \frac{du}{(u+1)^n} = \ldots$$

$$\int \frac{1}{(x^2+1)^n}\,dx = I_n$$

$$I_2 = \int \frac{dx}{(x^2+1)^2} = \int \frac{(x^2+1)}{(x^2+1)^2}\,dx - \int \frac{x^2\,dx}{(x^2+1)^2} =$$

$$= \underbrace{\int \frac{1}{(x^2+1)}\,dx}_{\operatorname{arctg} x + C} - \underbrace{\int \frac{x^2\,dx}{(x^2+1)^2}}_{(1)}$$

$$(1) \int \underbrace{x}_{u(x)} \underbrace{\frac{x}{(x^2+1)^2}}_{v'(x)}\,dx = \left[\begin{matrix} u = x \\ v' = \frac{x}{(x^2+1)^2} \\ v = \frac{-1}{2(x^2+1)^2} \end{matrix}\right] = x \cdot \frac{-1}{2(x^2+1)} -$$

$$-\int (x)' \frac{-1}{2(x^2+1)}\,dx = \frac{1}{2}\left(\frac{-x}{x^2+1} + \underline{\int \frac{1}{x^2+1}\,dx}\right)$$

$$I_n = \int \frac{1}{(x^2+1)^n} = \int \frac{x^2+1}{(x^2+1)^n}\,dx - \int \frac{x^2}{(x^2+1)^n}\,dx$$

$$\int \frac{1}{(x^2+1)^{n-1}}\,dx = I_{n-1}$$

$$\int \frac{x^2}{(x^2+1)^n}\,dx = \begin{bmatrix} u = x \\ v' = \frac{x}{(x^2+1)^n} \\ V(x) = \frac{-1}{2(n-1)(x^2+1)^{n-1}} \end{bmatrix} = \dots + \dots I_{n-1}$$

$$\int \frac{1}{(x^2+2x+3)^2}\,dx = \frac{S(x)}{(x^2+2x+3)^{2-1}} + \int \frac{R(x)}{(x^2+2x+3)}\,dx$$

$$\deg R(x) = \deg(S(x)) < \deg(x^2+2x+3)$$

$$S(x) = Ax+B, \quad R(x) = Cx+D$$

$$\left(\int \frac{1}{(x^2+2x+3)^2}\,dx\right)' = \left(\frac{Ax+B}{(x^2+2x+3)^2}\right)' + \left(\int \frac{Cx+D}{(x^2+2x+3)^2}\right)$$

11.02.19

1) $\int \frac{e^x+e^{3x}}{1-e^{2x}+e^{4x}}\,dx = \left[\begin{array}{l} t=e^x \\ dt=e^x dx \end{array}\right] = \int \frac{1+t^2}{1-t^2+t^4}\,dt$

2) $\int \frac{dx}{2-e^x+e^{2x}} = \int \frac{dt}{t(2-t-t^2)}$

3) $\int \frac{dx}{\cos^3 x} = \int \frac{\cos x}{\cos^4 x}\,dx = \left[\begin{array}{l} \sin x = y \\ \cos x^2 = 1-y^2 \end{array}\right] = \int \frac{dy}{(1-y^2)^2}$

Подстановка Эйлера

$\int \frac{1-\sqrt{1+x+x^2}}{x\sqrt{1+x+x^2}} =$

$\sqrt{1+x+x^2} = tx+1 \qquad t = \frac{\sqrt{1+x+x^2}-1}{x}$

$1+x = t^2x+2t$

$x = \frac{2t-1}{1-t^2}$.

18.02.19

$f(x)$; $[a, b]$

$$S = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$$

$$S = \sum_{k=1}^{n} S_{\square nk}$$

$$S_{\square nk} = f(x_k) \cdot (x_k - x_{k-1}) = f\left(\frac{k}{n}\right)\left(\frac{k}{n} - \frac{k-1}{n}\right) = \frac{k}{n^2}$$

$f(x) = x^2$; $[a, b] = [1; 3]$

$a = 1 \qquad b = 3$

$x_n = 1 \qquad x_{n+1} = 1 + \frac{1}{n} \ldots \; x_{3n} = 1 + \frac{2n}{n}$

$2n+1$ точка, $2n$ прямоугольников.

$$S_{\square n k} = \left(\frac{k}{n}\right)^2 \left(\frac{k}{n} - \frac{k-1}{n}\right) = \frac{k^2}{n^3}$$

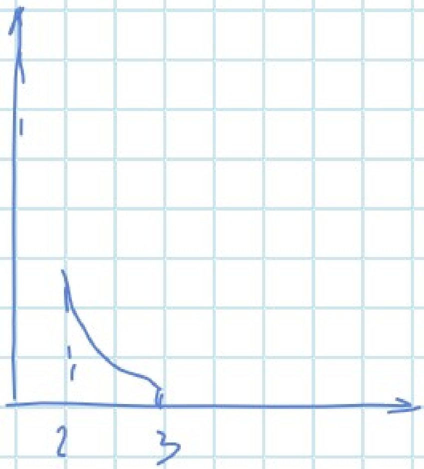
$$S_{\square} = \sum_{k=n+1}^{3n} \frac{k^2}{n^3} = \frac{1}{n^3} \underbrace{\sum_{k=n+1}^{3n} k^2}_{A} =$$

$(k+1)^3 = k^3 + 3k^2 + 3k + 1$

$3\sum k^2 = \sum\left((k+1)^3 - k^3\right) - 3\sum k - \sum 1$

$3k^2 = (k+1)^3 - k^3 - 3k - 1$

$f(x)=\frac{1}{x}$, $[a,b]=[2,3]$



~~$y_0=2$~~

~~$y_n=3$~~

~~$y_k=2+\frac{k}{n}$~~

~~$S_{[]n.k} f(y_k)(y_{k+1}-y_k)=\frac{1}{2n+k}$~~

— плохое разбиение

$x_0=2$, $x_n=3$, $x_k=2\sqrt[n]{\frac{3}{2}}$

Теорема: Формула Ньютона–Лейбница.

1) $f(x)$ — определена на $[a,b]$

2) $\int f(x)\,dx = F(x)+C$

Тогда:

$$\int_a^b f(x)\,dx = F(b)-F(a)$$

25.02.19

Теорема: если $f_1(x)$ – инт. интегрируем. на $[a,b]$ и

$f_2(x) = f_1(x) \quad \forall x \in [a, b]\ ,\ x \neq x_0$

то интегралы равны

$\lim \frac{1}{n}\left(\sqrt{1+\frac{1}{n}} + \ldots + \sqrt{1+\frac{n}{n}}\right) = \frac{1}{n}\int_1^2 \sqrt{u}\, du$