Вариант 1

№1 $\begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 3 \\ 1 & 2 & -1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} e_1 \\ e_2 \\ \\ \end{matrix}$

e

e_1, e_2 - б. в $\ker \varphi$

$\begin{pmatrix} 1 & 0 & 2 \\ 0 & -2 & 1 \end{pmatrix} \begin{matrix} \varphi(e_3) \\ \varphi(e_4) \end{matrix}$
$\begin{matrix} 0 & 0 & 1 & f_3 \end{matrix}$

$$A \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \rightarrow$$

$$\rightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right) \rightarrow$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ -2 & 1 & 0 & 1 \end{pmatrix}$$

$$A(\varphi, e, f) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ -2 & 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & 1 & 0 \\ 4 & -2 & 0 & -2 \\ -4 & 3 & 2 & 1 \end{pmatrix}$$

№2 $Q(x,y,z) = x^2 + ay^2 + z^2 + 4xy - 4xz - 8yz$

$$\begin{pmatrix} 1 & 2 & -2 \\ 2 & a & -4 \\ -2 & -4 & 1 \end{pmatrix}$$

$\delta_1 = 1$ $\qquad \frac{\delta_1}{\delta_0} \quad \frac{\delta_2}{\delta_1} \quad \frac{\delta_3}{\delta_2}$

$\delta_2 = a - 4$

$\delta_3 = a + 16 + 16 - 4a - 16 - 4 =$

$= -3a + 12$

1) $a \in (-\infty; 4)$ $\quad \delta_2 < 0 \quad \delta_3 > 0$

$Q = \tilde{x}^2 - \tilde{y}^2 - \tilde{z}^2$

2) $a \in (4, +\infty)$ $\quad \delta_2 > 0 \quad \delta_3 < 0$

$Q = \tilde{x}^2 + \tilde{y}^2 - \tilde{z}^2$

3) $a = 4$

$x^2 + 4y^2 + z^2 + 4xy - 4xz - 8yz =$

$= \underline{(x + 2y - 2z)}^2 - (2y - 2z)^2 + 4y^2 + z^2 - 8yz =$

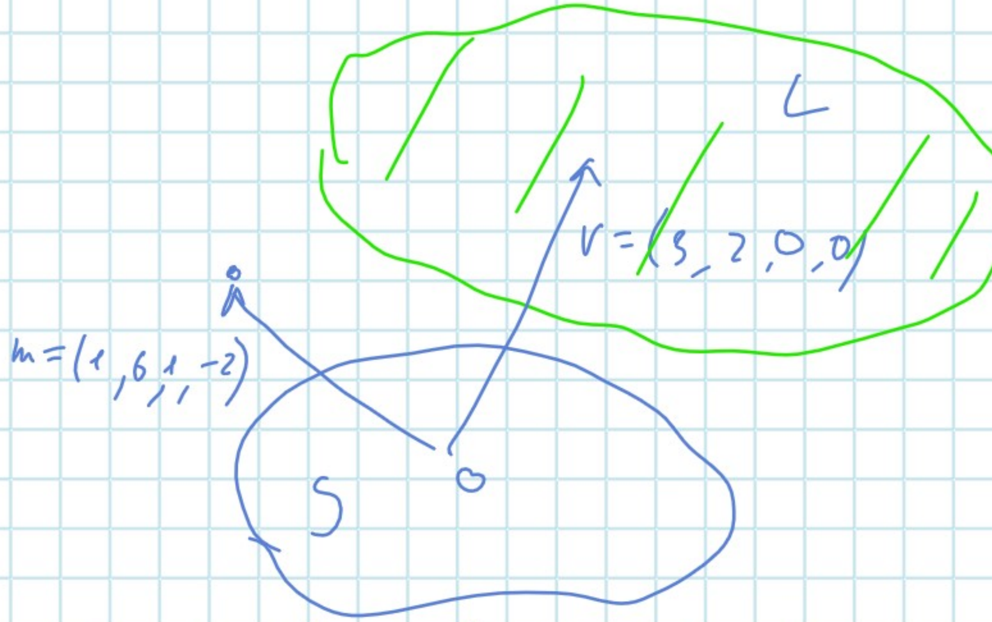
$= \tilde{x}^2 - 4y^2 + 8yz - 4z^2 - 8yz = \tilde{x}^2 - 4y^2 - 4z^2 =$

$= \tilde{x}^2 - \tilde{y}^2 - \tilde{z}^2$

№3

$$\begin{cases} 2x_1 + x_2 + x_3 - 2x_4 = 3 \\ 3x_1 - x_2 + 3x_3 - x_4 = 2 \end{cases}$$

$$\begin{pmatrix} 2 & 1 & 1 & -2 \\ 3 & -1 & 3 & -1 \end{pmatrix} \to$$



$$\begin{pmatrix} 2 & 1 & 1 & -2 \\ 1 & -2 & 2 & 1 \end{pmatrix} \cdot(-2) \to \begin{pmatrix} 0 & 5 & -3 & -4 \\ 1 & -2 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 2 & 1 \\ 0 & 5 & -3 & -4 \end{pmatrix}$$

Частное решение: $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ ФСР $\begin{pmatrix} 3 \\ 4 \\ 0 \\ 5 \end{pmatrix}$ $\begin{pmatrix} 4 \\ 3 \\ 5 \\ 0 \end{pmatrix}$

$X - X_0 =$ дальше не интересно

✓4 $\varphi(x) = [v, x] + (v, x)w$ $\qquad v = (1, -1, 2)$

$w = (2, -1, -1)$

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & -1 & 2 \\ 1 & 0 & 0 \end{vmatrix} = (0, 2, 1) \qquad \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & -1 & 2 \\ 0 & 1 & 0 \end{vmatrix} = (-2, 0, 1)$$

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & -1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = (-1, -1, 0)$$

$$\varphi(e_1) = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \qquad \varphi(e_2) = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix}$$

$$\varphi(e_3) = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -2 \end{pmatrix}$$

$$-\ \lambda^3 - \lambda^2 + 6\lambda - 6 \ \Big|\ \begin{array}{l} \lambda - 1 \\ \hline \lambda^2 + 6 \end{array}$$

$$A(\varphi, e) = \begin{pmatrix} 2 & -4 & 3 \\ 1 & 1 & -3 \\ 0 & 2 & -2 \end{pmatrix}$$

$$\chi_\varphi(\lambda) = \begin{vmatrix} 2-\lambda & -4 & 3 \\ 1 & 1-\lambda & -3 \\ 0 & 2 & -2-\lambda \end{vmatrix} = (\lambda^2 - 4)(1 - \lambda) + 6 + 6(2 - \lambda) -$$

$$- 4(2 + \lambda) = \lambda^2 - \lambda^3 - 4 + \underline{4\lambda} + 6 + 12 - \underline{6}\lambda - 8 \underline{- 4}\lambda =$$

$$= -\lambda^3 + \lambda^2 - 6\lambda + 6 = -(\lambda - 1)(\lambda^2 + 6)$$

№5

$$A = \begin{pmatrix} 2/3 & 1/3 & 2/3 \\ -1/3 & -2/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \end{pmatrix}$$

$$\chi_\varphi(\lambda) = \frac{1}{81}\begin{vmatrix} 2-3\lambda & 1 & 2 \\ -1 & -2-3\lambda & 2 \\ -2 & 2 & 1-3\lambda \end{vmatrix} = (9\lambda^2 - 4)\quad(1-3\lambda) -$$

$$-4 - 4 \quad - 4(2+3\lambda) \quad - 4(2-3\lambda) + 1 - 3\lambda =$$

$$= -27\lambda^3 + 9\lambda^2 + 9\lambda - 27 = \frac{1}{9}(-3\lambda^3 + \lambda^2 + \lambda - 3) =$$

$$= \frac{1}{9}(\lambda+1)(-3\lambda^2 + 4\lambda - 3)$$

$$\begin{array}{l|l} -3\lambda^3 + \lambda^2 + \lambda - 3 & \lambda + 1 \\ \cline{2-2} \underline{-3\lambda^3 - 3\lambda^2} & -3\lambda^2 + 4\lambda - 3 \\ \quad 4\lambda^2 & \\ \quad \underline{4\lambda^2 + 4\lambda} & \\ \quad\quad -3\lambda \to & \end{array}$$

$$\lambda_1 = -1$$

$$D = 16 - 4 \cdot 3\ 3 = i2\sqrt{5}$$

$$\lambda_{2,3} = \frac{-4 \pm i2\sqrt{5}}{-6} = \frac{2}{3} \pm i\frac{\sqrt{5}}{3}$$

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2/3 & \sqrt{5}/3 \\ 0 & +\sqrt{5}/3 & 2/3 \end{pmatrix}$$

№7 $24x^2 + 7y^2 - 32z^2 + 52yz + 96x - 94y - 92z - 77 = 0$

$$A = \begin{pmatrix} 24 & 0 & 0 \\ 0 & 7 & 26 \\ 0 & 26 & -32 \end{pmatrix} \qquad a = \begin{pmatrix} 96 \\ -92 \\ -77 \end{pmatrix}$$

$$\begin{array}{r} 26 \\ 26 \\ \hline 156 \\ 52 \\ \hline 676 \end{array}$$

$$\chi(\lambda) = \begin{vmatrix} 24-\lambda & 0 & 0 \\ 0 & 7-\lambda & 26 \\ 0 & 26 & -32-\lambda \end{vmatrix} = (24-\lambda)\big((7-\lambda)(-32-\lambda) - 676\big)$$

$$= (\lambda - 24)(-224 - 7\lambda + 32\lambda + \lambda^2 - 676) =$$

$$= (\lambda - 24)(\lambda^2 + 25\lambda - 900) =$$

$$\sqrt{D} = 65 \qquad \lambda_2 = \frac{-25-65}{2} = -45 \qquad \lambda_3 = \frac{-25+65}{2} = 20$$

$$= (\lambda - 24)(\lambda - 20)(\lambda + 45)$$

1. $\lambda = 24$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -17 & 26 \\ 0 & 26 & -56 \end{pmatrix}$$

$0 \quad 0$

w8 $\begin{pmatrix} 6 & 0 & 4 & 0 \\ 5 & 4 & 4 & 2 \\ -1 & 0 & 2 & 0 \\ -3 & 0 & -6 & 4 \end{pmatrix}$

$$\chi_\varphi(\lambda) = \begin{vmatrix} 6-\lambda & 0 & 4 & 0 \\ 5 & 4-\lambda & 4 & 2 \\ -1 & 0 & 2-\lambda & 0 \\ -3 & 0 & -6 & 4-\lambda \end{vmatrix} =$$

$$= (4-\lambda)\begin{vmatrix} 6-\lambda & 4 & 0 \\ -1 & 2-\lambda & 0 \\ -3 & -6 & 4-\lambda \end{vmatrix} = (4-\lambda)^2 \begin{vmatrix} 6-\lambda & 4 \\ -1 & 2-\lambda \end{vmatrix} =$$

$$= (4-\lambda)^2 \left((6-\lambda)(2-\lambda)+4\right) = (4-\lambda)^2 (12-6\lambda-2\lambda+\lambda^2+4) =$$

$$= (4-\lambda)^2(\lambda^2-8\lambda+16) = (\lambda-4)^4$$

$$A-4E = \begin{pmatrix} 2 & 0 & 4 & 0 \\ 5 & 0 & 4 & 2 \\ -1 & 0 & -2 & 0 \\ -3 & 0 & -6 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x_1 + x_3 = 0 \\ -6x_3 + 2x_4 = 0 \end{cases}$$

$e_2 = (0\ 0\ 0\ 1)$

$e_1 = (f - 4\cdot \mathrm{Id})e_2 = (0\ 2\ 0\ 0)$

$$(A-4E)^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow A = J_4(4)$$

$$\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad N_1 = \left\langle \begin{pmatrix} -2 \\ 0 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\rangle$$

$e_1 = Bf_1$

$f_2 \quad f_1$

$Bf_2 \quad Bf_1$

$$f_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad f_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad Bf_1 = \begin{pmatrix} 2 \\ 5 \\ -1 \\ -3 \end{pmatrix} \qquad Bf_2 = \begin{pmatrix} 4 \\ 4 \\ -2 \\ -6 \end{pmatrix}$$

№5 $\quad A = \begin{pmatrix} 3 & 2 & 4 \\ 1 & 4 & -2 \end{pmatrix}$

$$\begin{pmatrix} 3 & 1 \\ 2 & 4 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} 3 & 2 & 4 \\ 1 & 4 & -2 \end{pmatrix} = \begin{pmatrix} 10 & 10 & 10 \\ 10 & 20 & 0 \\ 10 & 0 & 20 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 10 & 10 & 10 \\ 10 & 20 & 0 \\ 10 & 0 & 20 \end{pmatrix}$$

$$\chi = \begin{vmatrix} 10-\lambda & 10 & 10 \\ 10 & 20-\lambda & 0 \\ 10 & 0 & 20-\lambda \end{vmatrix} = (20-\lambda)^2(10-\lambda) - 200(20-\lambda) =$$

$$= (20-\lambda)\left((20-\lambda)(10-\lambda) - 200\right) = (20-\lambda)(\lambda^2 - 30\lambda) =$$

$$= \lambda(20-\lambda)(30-\lambda)$$

1) $\lambda = 0$

$$\begin{pmatrix} 10 & 10 & 10 \\ 10 & 20 & 0 \\ 10 & 0 & 20 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = v_1$$

$\lambda = 30$

$$\begin{pmatrix} -20 & 10 & 10 \\ 10 & -10 & 0 \\ 10 & 0 & -10 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = v_2$$

$\lambda = 30$

$$\begin{pmatrix} -10 & 10 & 10 \\ 10 & 0 & 0 \\ 10 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$\sigma_2 = \sqrt{20}$

$\sigma_1 = \sqrt{30}$

$e_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad e_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} = P$$

$$f_i = \frac{A \cdot e_i}{\sigma_i} \qquad \begin{pmatrix} \sqrt{30} & 0 \\ 0 & \sqrt{20} \end{pmatrix}$$

$$Q\,(f_1, f_2 \ldots)$$

$$Q \cdot \Sigma \cdot P^T = A$$