Question 1:

A. Convert to decimal.

1. 10011011_2
   $= 1 * 2^0 + 1 * 2^1 + 0 * 2^2 + 1 * 2^3 + 1 * 2^4 + 0 * 2^5 + 0 * 2^6 + 1 * 2^7$
   $= 1 + 2 + 8 + 16 + 128$
   $= 155_{10}$

2. 456_7
   $= 6 * 7^0 + 5 * 7^1 + 4 * 7^2$
   $= 6 + 35 + 196$
   $= 237_{10}$

3. $38A_{16}$
   $A_{16} = 10$
   $= 10 * 16^0 + 8 * 16^1 + 3 * 16^2$
   $= 10 + 128 + 768$
   $= 906_{10}$

4. 2214_5
   $= 4 * 5^0 + 1 * 5^1 + 2 * 5^2 + 2 * 5^3$
   $= 4 + 5 + 50 + 250$
   $= 309_{10}$

B. Convert to binary.

1. 69_{10}
   69/2 = 34 R 1
   34/2 = 17 R 0
   17/2 = 8 R 1
   8/2 = 4 R 0
   4/2 = 2 R 0
   2/2 = 1 R 0
   ½ = 0 R 1

   $= 1000101_2$

2. $485_{10} =$
   485/2 = 242 R 1
   242/2 = 121 R 0
   121/2 = 60 R 1
   60/2 = 30 R 0
   30/2 = 15 R 0
   15/2 = 7 R 1
   7/2 = 3 R 1
   3/2 = 1 R 1
   ½ = 0 R 1

   $= 111100101_2$

3. $6D1A_{16}$ Use the 4-bit hexadecimal to binary table as 16 is a power of 2
   6 = 0110, D = 1101, 1 = 0001, A = 1010
   $= 110110100011010_2$

C. Convert to hexadecimal.

1. 1101011_2 Use the 4-bit hexadecimal to binary table as 16 is a power of 2
   0110 1011 0110 = 6, 1011 = B
   $= 6B_{16}$

2. 895_{10}
   895/16 = 55 R 15 15 = F
   55/16 = 3 R 7
   3/16 = 0 R 3

   $= 37F_{16}$

Question 2:

1. $7566_8 + 4515_8 = 14303_8$

$$
\begin{array}{r}
\scriptstyle 1\ 1\ 1 \quad \text{carry} \\
7566 \\
+\ 4515 \\
\hline
14303
\end{array}
$$

2. $10110011_2 + 1101_2 = 11000000_2$

$$
\begin{array}{r}
\scriptstyle 1\ 1\ 1\ 1\ 1\ 1 \quad \text{carry} \\
10110011 \\
+ \qquad 1101 \\
\hline
11000000
\end{array}
$$

3. $7A66_{16} + 45C5_{16} = C02B_{16}$ A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

$$
\begin{array}{r}
\scriptstyle 1\ 1 \quad \text{carry} \\
7A66 \\
+\ 45C5 \\
\hline
C02B
\end{array}
$$

6 + C = 6 + 12 = 18 18 – 16 = 2

A + 5 + 1 = 10 + 5 + 1 = 16 7 + 4 + 1 = 12

4. $3022_5 - 2433_5 = 34_5$

$$
\begin{array}{r}
\scriptstyle 2\ 4\ 1 \quad \text{borrow} \\
\cancel{3}\,\cancel{0}\,\cancel{2}\,2 \\
-\ 2433 \\
\hline
0034
\end{array}
$$

(5 + 2) – 3 = 4 (5 + 1) – 3 = 3

Question 3:

A. Convert to 8-bits two's complement.

1. 124_{10}

   124/2 = 62 R | 0
   62/2 = 31 R | 0
   31/2 = 15 R | 1
   15/2 = 7 R | 1
   7/2 = 3 R | 1
   3/2 = 1 R | 1
   ½ = 0 R | 1

   = $01111100_{\text{8-bit 2's comp}}$

2. -124_{10} = $10000100_{\text{8-bit 2's comp}}$

   1 1 1 1 1 1 carry
   0 1 1 1 1 1 0 0 = 124
   + 1 0 0 0 0 1 0 0 = -124
   1 0 0 0 0 0 0 0 0 = 2^8

3. 109_{10}

   109/2 = 54 R | 1
   54/2 = 27 R | 0
   27/2 = 13 R | 1
   13/2 = 6 R | 1
   6/2 = 3 R | 0
   3/2 = 1 R | 1
   ½ = 0 R | 1

   = $01101101_{\text{8-bit 2's comp}}$

4. -79_{10}

   79/2 = 39 remainder | 1
   39/2 = 19 remainder | 1
   19/2 = 9 remainder | 1
   9/2 = 4 remainder | 1
   4/2 = 2 remainder | 0
   2/2 = 1 remainder | 0
   ½ = 0 remainder | 1

   = 01001111 = 79

   1 1 1 1 1 1 1 1 carry
   0 1 0 0 1 1 1 1 = 79
   + 1 0 1 1 0 0 0 1 = -79
   1 0 0 0 0 0 0 0 0 = 2^8

   = $10110001_{\text{8-bit 2's comp}}$

B. Convert to decimal.

1. $00011110_{\text{8-bit 2's comp}}$ positive number

   = $0 * 2^0 + 1 * 2^1 + 1 * 2^2 + 1 * 2^3 + 1 * 2^4 + 0 * 2^5 + 0 * 2^6$
   = 2 + 4 + 8 + 16
   = 30_{10}

2. $11100110_{\text{8-bit 2's comp}}$ negative number

   1 1 1 1 1 1 carry
   1 1 1 0 0 1 1 0
   + 0 0 0 1 1 0 1 0
   1 0 0 0 0 0 0 0 0 = 2^8

   00011010
   = $0 * 2^0 + 1 * 2^1 + 0 * 2^2 + 1 * 2^3 + 1 * 2^4 + 0 * 2^5 + 0 * 2^6$
   = 2 + 8 + 16
   = 26 ➔ = $(-26)_{10}$

3. $00101101_{\text{8-bit 2's comp}}$ positive number
$= 1 * 2^0 + 0 * 2^1 + 1 * 2^2 + 1 * 2^3 + 0 * 2^4 + 1 * 2^5 + 0 * 2^6$
$= 1 + 4 + 8 + 32$
$= 45_{10}$

4. $10011110_{\text{8-bit 2's comp}} =$ negative number

```
1 1 1 1 1 1 1        carry
  1 0 0 1 1 1 1 0
+ 0 1 1 0 0 0 1 0
1 0 0 0 0 0 0 0 0     = 2^8
```

01100010
$= 0 * 2^0 + 1 * 2^1 + 0 * 2^2 + 0 * 2^3 + 0 * 2^4 + 1 * 2^5 + 1 * 2^6$
$= 2 + 32 + 64$
$= 98$ ➔ $= (-98)_{10}$

Question 4:

1. Exercise 1.2.4: Writing truth tables, sections b, c

b. $\neg(p \vee q)$

p	**q**	**$(p \vee q)$**	**$\neg(p \vee q)$**
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

c. $r \vee (p \wedge \neg q)$

p	**q**	**r**	**$\neg q$**	**$p \wedge \neg q$**	**$r \vee (p \wedge \neg q)$**
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

2. Exercise 1.3.4: Truth tables, sections b, d

b. $(p \to q) \to (q \to p)$

p	**q**	**$(p \to q)$**	**$(q \to p)$**	**$(p \to q) \to (q \to p)$**
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

d. $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	**q**	**$\neg q$**	**$(p \leftrightarrow q)$**	**$(p \leftrightarrow \neg q)$**	**$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$**
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Question 5:

1. Exercise 1.2.7, sections b, c

   b. The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

   $(B \wedge D) \vee (D \wedge M) \vee (B \wedge M)$

   c. Applicant must present either a birth certificate or both a driver's license and a marriage license.

   $B \vee (D \wedge M)$

2. Exercise 1.3.7, sections b – e

   b. A person can park in the school parking lot if they are a senior or at least seventeen years of age.

   $(s \vee y) \rightarrow p$

   c. Being 17 years of age is a necessary condition for being able to park in the school parking lot.

   $p \rightarrow y$

   d. A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

   $p \leftrightarrow (s \wedge y)$

   e. Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

   $p \rightarrow (s \vee y)$

3. Exercise 1.3.9, sections c, d

   c. The applicant can enroll in the course only if the applicant has parental permission.

   $c \rightarrow p$

   d. Having parental permission is a necessary condition for enrolling in the course.

   $c \rightarrow p$

Questions 6:

1. Exercise 1.3.6, sections b – d

   b. Maintaining a B average is necessary for Joe to be eligible for the honors program.
   If Joe is to be eligible for the honors program, then he must maintain a B average.

   c. Rajiv can go on the roller coaster only if he is at least four feet tall.
   If Rajiv can go on the roller coaster, then he is at least four feet tall.

   d. Rajiv can go on the roller coaster if he is at least four feet tall.
   If he is at least four feet tall, then Rajiv can go on the roller coaster.

2. Exercise 1.3.10, sections c – f

   c. $(p \lor r) \leftrightarrow (q \land r)$
   False. Whether r is T or F, $(p \lor r)$ will evaluate true, and $(q \land r)$ will evaluate false. In a biconditional statement, the proposition is true only when both have the same truth values (both are T or both are F).

   d. $(p \land r) \leftrightarrow (q \land r)$
   Unknown. If r is T, then the expression is false. If r is F, then the expression is true.

   e. $p \rightarrow (r \lor q)$
   Unknown. If r is T, then the expression is true. If r is F, then the expression is false.

   f. $(p \land q) \rightarrow r$
   True. A conditional is only false if the hypothesis is T, and the conclusion is F. Whether r is T or F, the expression evaluates to true.

Questions 7:

Exercise 1.4.5, sections b – d

b. If Sally did not get the job, then she was late for interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

$\neg j \rightarrow (l \vee \neg r)$
$(r \wedge \neg l) \rightarrow j$

j	l	r	$(l \vee \neg r)$	$(r \wedge \neg l)$	$\neg j \rightarrow (l \vee \neg r)$	$(r \wedge \neg l) \rightarrow j$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	T	F	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	F	T	F	F
F	F	F	T	F	T	T

Logically equivalent.

c. If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

$j \rightarrow \neg l$
$\neg j \rightarrow l$

j	l	$j \rightarrow \neg l$	$\neg j \rightarrow l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

Not logically equivalent.

d. If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

$(r \vee \neg l) \rightarrow j$
$j \rightarrow (r \wedge \neg l)$

j	l	r	$(r \vee \neg l)$	$(r \wedge \neg l)$	$(r \vee \neg l) \rightarrow j$	$j \rightarrow (r \wedge \neg l)$
T	T	T	T	F	T	F
T	T	F	F	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	F	T
F	T	F	F	F	T	T
F	F	T	T	T	F	T
F	F	F	T	F	F	T

Not logically equivalent.

Questions 8:

1. Exercise 1.5.2, sections c, f, i

c. $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(p \to q) \land (p \to r)$	Starting condition
$(\neg p \lor q) \land (\neg p \lor r)$	Conditional identities
$\neg p \lor (q \land r)$	Distributive laws
$p \to (q \land r)$	Conditional identities

f. $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg p \land q))$	Starting condition
$\neg(p \lor (q \land \neg p))$	Commutative laws
$\neg p \land \neg(q \land \neg p))$	De Morgan's laws
$\neg p \land (\neg q \lor \neg\neg p))$	De Morgan's laws
$\neg p \land (\neg q \lor p))$	Double negation law
$(\neg p \land \neg q) \lor (\neg p \land p)$	Distributive laws
$(\neg p \land \neg q) \lor (p \land \neg p)$	Commutative laws
$(\neg p \land \neg q) \lor F$	Complement laws
$\neg p \land \neg q$	Identity laws

i. $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$(p \land q) \to r$	Starting condition
$\neg(p \land q) \lor r$	Conditional identities
$(\neg p \lor \neg q) \lor r$	De Morgan's laws
$\neg p \lor (\neg q \lor r)$	Associative laws
$\neg p \lor (r \lor \neg q)$	Commutative laws
$\neg p \lor (\neg\neg r \lor \neg q)$	Double negation law
$(\neg p \lor \neg\neg r) \lor \neg q$	Associative laws
$\neg (p \land \neg r) \lor \neg q$	De Morgan's laws
$(p \land \neg r) \to \neg q$	Conditional identities

2. Exercise 1.5.3, sections c, d

c. $\neg r \lor (\neg r \to p)$

$\neg r \lor (\neg r \to p)$	Starting condition
$\neg r \lor (\neg\neg r \lor p)$	Conditional identities
$\neg r \lor (r \lor p)$	Double negation law
$(\neg r \lor r) \lor p$	Associative laws
$(r \lor \neg r) \lor p$	Commutative laws
$T \lor p$	Complement laws
$p \lor T$	Commutative laws
T	Domination laws

Tautology.

d. $\neg(p \rightarrow q) \rightarrow \neg q$

$\neg(p \rightarrow q) \rightarrow \neg q$	Starting condition
$\neg(\neg p \vee q) \rightarrow \neg q$	Conditional identities
$(\neg\neg p \wedge \neg q) \rightarrow \neg q$	De Morgan's laws
$(p \wedge \neg q) \rightarrow \neg q$	Double negation law
$\neg(p \wedge \neg q) \vee \neg q$	Conditional identities
$(\neg p \vee \neg\neg q) \vee \neg q$	De Morgan's laws
$(\neg p \vee q) \vee \neg q$	Double negation law
$\neg p \vee (q \vee \neg q)$	Associative laws
$\neg p \vee T$	Complement laws
T	Domination laws

Tautology.

Questions 9:

1. Exercise 1.6.3, sections c, d

   c. There is a number that is equal to its square.

   $\exists x\ (x = x^2)$

   d. Every number is less than or equal to its square.

   $\forall x\ (x \leq x^2)$

2. Exercise 1.7.4, sections b – d

   b. Everyone was well and went to work yesterday.

   $\forall x\ (\neg S(x) \wedge W(x))$

   c. Everyone who was sick yesterday did not go to work.

   $\forall x\ (S(x) \rightarrow \neg W(x))$

   d. Yesterday someone was sick and went to work.

   $\exists x\ (S(x) \wedge W(x))$

Questions 10:

1. Exercise 1.7.9, sections c – i

   c. $\exists x((x = c) \rightarrow P(x))$

   True. P(x) is T in a.

   d. $\exists x(Q(x) \wedge R(x))$

   True. In e, both are T.

   e. $Q(a) \wedge P(d)$

   True. $T \wedge T$ equals T.

   f. $\forall x ((x \neq b) \rightarrow Q(x))$

   True. When in all other instances where x does not equal b, Q(x) is T.

   g. $\forall x (P(x) \vee R(x))$

   False. In e, $P(x) \vee R(x)$ is T.

   h. $\forall x (R(x) \rightarrow P(x))$

   True. In all instances, $R(x) \rightarrow P(x))$ is T.

   i. $\exists x(Q(x) \vee R(x))$

   True. In a, $Q(x) \vee R(x))$ is T.

2. Exercise 1.9.2, sections b – i

   b. $\exists x \forall y Q(x, y)$

   True. When x = 2, all values in the row are T.

   c. $\exists x \forall y P(y, x)$

   True. When y = 1, all values are T.

   d. $\exists x \exists y S(x, y)$

   False. None of the values are true.

   e. $\forall x \exists y Q(x, y)$

   False. There does not exist a column where all values are T.

   f. $\forall x \exists y P(x, y)$

   True. When y = 1, all values are T.

   g. $\forall x \forall y P(x, y)$

   False. Not all x and all y are T.

   h. $\exists x \exists y Q(x, y)$

   True. When x = 2 and y = 2, the value is T.

   i. $\forall x \forall y \neg S(x, y)$

   True. All the values are false, which means they are all true because of negation.

Questions 11:

1. Exercise 1.10.4, sections c – g

   c. There are two number whose sum is equal to their product.

   $\exists x \, \exists y \, ((x + y) = xy)$

   d. The ratio of every two positive numbers is also positive.

   $\forall x \, \forall y \, ((x > 0 \wedge y > 0) \rightarrow x/y > 0)$

   e. The reciprocal of every positive number less than one is greater than one.

   $\forall x \, ((0 < x < 1) \rightarrow (1/x > 1))$

   f. There is no smallest number.

   $\neg\exists x \, \forall y \, (x <= y)$

   g. Every number besides 0 has a multiplicative inverse.

   $\forall x \, \exists y \, ((x \neq 0) \rightarrow xy = 1)$

2. Exercise 1.10.7, sections c – f

   c. There is at least one new employee who missed the deadline.

   $\exists x \, (N(x) \wedge D(x))$

   d. Sam knows the phone number of everyone who missed the deadline.

   $\forall x \, (D(x) \rightarrow P(Sam, x))$

   e. There is a new employee who knows everyone's phone number.

   $\exists x \, \forall y \, (N(x) \wedge P(x, y))$

   f. Exactly one new employee missed the deadline.

   $\exists x \, \forall y \, (N(x) \wedge D(x) \wedge ((x \neq y) \wedge N(y)) \rightarrow \neg D(y)))$

3. Exercise 1.10.10, sections c – f

   c. Every student has taken at least one class besides Math 101.

   $\forall x \, \exists y \, ((y \neq \text{Math } 101) \wedge T(x, y))$

   d. There is a student who has taken every math class besides Math 101.

   $\exists x \, \forall y \, ((y \neq \text{Math } 101) \rightarrow T(x, y))$

   e. Everyone besides Sam has taken at least two different math classes.

   $\forall x \, \exists y \, \exists z \, ((x \neq Sam) \rightarrow ((y \neq z) \wedge T(x, y) \wedge T(x, z))$

   f. Sam has taken exactly two math classes.

   $\exists x \, \exists y \, \forall z \, ((x \neq y) \wedge T(Sam, x) \wedge T(Sam, y) \wedge ((z \neq x \wedge z \neq y) \rightarrow \neg T(Sam, z)))$

Questions 12:

1. Exercise 1.8.2, sections b – e

   b. Every patient was given the medication or the placebo or both.

   $\forall x\ (D(x) \lor P(x))$

   Negation: $\neg\forall x\ (D(x) \lor P(x))$

   De Morgan's law: $\exists x\ (\neg D(x) \land \neg P(x))$

   English: There exists a patient who was not given the medication and the placebo.

   c. There is a patient who took the medication and had migraines.

   $\exists x\ (D(x) \land M(x))$

   Negation: $\neg\exists x\ (D(x) \land M(x))$

   De Morgan's law: $\forall x\ (\neg D(x) \lor \neg M(x))$

   English: Every patient either did not take the medication or did not have migraines (or both).

   d. Every patient who took the placebo had migraines.

   $\forall x\ (P(x) \rightarrow M(x))$

   Negation: $\neg\forall x\ (P(x) \rightarrow M(x))$

   De Morgan's law: $\exists x\ (P(x) \land \neg M(x))$

   English: There exists a patient who took the placebo and did not have migraines.

   e. There is a patient who had migraines and was given the placebo.

   $\exists x\ (M(x) \land (P(x))$

   Negation: $\neg\exists x\ (M(x) \land (P(x))$

   De Morgan's law: $\forall x\ (\neg M(x) \lor \neg P(x))$

   English: Every patient either did not have migraines or was not given the placebo (or both).

2. Exercise 1.9.4, sections c – e

   c. $\exists x\ \forall y\ (P(x, y) \rightarrow Q(x, y))$

$\neg\exists x\ \forall y\ (P(x, y) \rightarrow Q(x, y))$	Negation
$\forall x\ \exists y\ \neg(\neg P(x, y) \lor Q(x, y))$	
$\forall x\ \exists y\ (P(x, y) \land \neg Q(x, y))$	

   d. $\exists x\ \forall y\ (P(x, y) \leftrightarrow P(y, x))$

$\neg\exists x\ \forall y\ (P(x, y) \leftrightarrow P(y, x))$	Negation
$\forall x\ \exists y\ \neg((P(x, y) \rightarrow P(y, x)) \land ((P(y, x) \rightarrow P(x, y)))$	
$\forall x\ \exists y\ \neg((\neg P(x, y) \lor P(y, x)) \land (\neg P(y, x) \lor P(x, y)))$	
$\forall x\ \exists y\ (\neg\ (\neg P(x, y) \lor P(y, x)) \lor \neg\ (\neg P(y, x) \lor P(x, y)))$	
$\forall x\ \exists y\ ((P(x, y) \land \neg P(y, x)) \lor (P(y, x) \land \neg P(x, y)))$	

   e. $\exists x\ \exists y\ P(x, y) \land \forall x\ \forall y\ Q(x, y)$

$\neg(\exists x\ \exists y\ P(x, y) \land \forall x\ \forall y\ Q(x, y))$	Negation
$\neg\exists x\ \exists y\ P(x, y) \lor \neg\forall x\ \forall y\ Q(x, y)$	
$\forall x\ \forall y\ \neg P(x, y) \lor \exists x\ \exists y\ \neg Q(x, y))$	