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- Resonating Property : amplified x signals in the past second, then it would try to amplify the same number (x) in this turn as well.

→ due to bizzare electrical framework, it may actually transmit more or less than that number.

$\Downarrow$

~~Assuming~~ "tries".

(1) Model with Markov Chain with countably infinite states.

Reaching '0' is ABSORBING STATE.
Markov chain is absorbing if:
(1) at least one absorbing state
(2) possible to get to A.S. in finite # of steps.

- Our problem is an absorbing markov chain.

let transit^n matrix $= P$, with t transient states, r absorbing states.

{canonical form}.

$$\Rightarrow P = \begin{bmatrix} Q & R \\ 0 & I_r \end{bmatrix}$$

$\hookrightarrow I_1$

$W_1, \ldots W_n \to$ IID exponential RVs.
$\hookrightarrow$ inter-arrival times.
$\hookrightarrow$ Poisson process.

$\Rightarrow T_1 = W_1, \quad T_2 = W_0 + W_1 \ldots \quad T_n = T_{n-1} + W_n$

$N_t = \max \{ n \mid W_1 + \ldots + W_n \leq t \}.$
$T_n = \sup \{ t \mid N_t \leq n-1 \}.$

$W_n = T_n - T_{n-1}$
for Markov chains,
for Poisson process, we know,

$$P[N_t = n] = \frac{e^{-\mu t}(\underbrace{\mu t}_{\mu t \Rightarrow \lambda})^n}{n!}$$

Transition probability,

$$P_{ij}(t) = \sum_{n=0}^{\infty} \frac{e^{-\mu t}(\mu t)^n}{n!} \underbrace{K_{ij}^{(n)}}_{\text{transition matrix}}.$$

where $K_{ij}^{(n)} = P[X_{s+t} = j \mid X_s = i, N_{t+s} - N_s = n]$
$= P[X_s = j \mid X_0 = i, N - N_0 = n]$

Problem has countably infinite Markov states,

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fundamental matrix,

$$N = (I_t - Q)^{-1}$$

$\hookrightarrow t =$ number of transient states.

$\to \infty$.

Absorbing probability,

$B = NR$ $\longrightarrow$ non-zero $t \times r$ matrix describing $P($~~transient~~ transient to absorbing state$)$.

Also,

$$\left(\lim_{n \to \infty} P^n\right)_{ij} = B_{ij}.$$

$(i = 1$ to $j = 0.)$

$\Rightarrow$ more, less or same. $\Rightarrow$ Assuming all have equal probability,

$$K = \begin{bmatrix} 1/3 & 1/3 & \cdots \\ 1/3 & 1/3 & \cdots \\ \vdots & & \ddots \\ \vdots & & 1/3 \end{bmatrix}$$

$$\boxed{P_{ij}(t) = \frac{1}{3} \sum_{n}^{\infty} e^{-\mu t} (\mu t)^n / n!}$$ required transition matrix.

for Q, drop last row and last column of P.

As ∞, no difference.

(2) $t = N\mathbb{1}$

↳ column vector.

$$t = \underbrace{(I_t - Q)^{-1}}_{= N} \begin{bmatrix} 1 \\ 1 \\ \vdots \end{bmatrix}$$

$$\Rightarrow \boxed{t_{\text{hat}} = \sum_{j=1}^{\infty} N_{1j}} \quad [\text{ sum of row } 1 \text{ }].$$

$$N = \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \vdots & & \ddots & \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & \cdots & \cdots \\ \vdots & \vdots & & \ddots \end{bmatrix} \frac{1}{3} s(n) \right\}^{-1}$$

$$N = \left\{ \begin{bmatrix} 1 - \frac{1}{3}s(n) & -\frac{1}{3}s(n) & -\frac{1}{3}s(n) & \cdots \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \\ \vdots & & \cdots & 1 - \frac{1}{3}s(n) \end{bmatrix} \right\}^{-1}$$

↳ symmetric matrix, all elements except diagonals same.

~~One sol^n can be,~~

$$\cancel{\Rightarrow N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ -1 & 1 & 0 & 0 & 0 & 0 & \cdots \\ \vdots & -1 & & & & & \end{bmatrix}}$$

$$M = \left(1 - \cancel{\tfrac{1}{3}(S(n))} + \tfrac{1}{3}S(n)\right) I + \tfrac{1}{3}S(n)\, J$$

↳ all 1 matrix

$$= I + \tfrac{1}{3}S(n)\, J$$

$$Z = xI + yJ$$

Let

$$I = MZ$$

$$= \left(I + \tfrac{1}{3}S(n)J\right)(xI + yJ)$$

$$I = xI + yJ + y\left(\tfrac{n}{3}S(n)\right)J + \tfrac{1}{3}S(n)J.$$

$$\boxed{x = 1}$$

$$y + \frac{yn}{3}S(n) + \frac{1}{3}S(n) = 0.$$

$$y\left(\frac{3 + nS(n)}{\cancel{3}}\right) = -\frac{1}{\cancel{3}}(S(n))$$

$$\boxed{y = \frac{-S(n)}{nS(n) + 3}}$$

inverse is $= I + \left(\dfrac{-S(n)}{nS(n)+3}\right) J.$

$$\Rightarrow N = \begin{bmatrix} 1 - \frac{S(n)}{n(S(n))+3} & -\frac{S(n)}{nS(n)+3} & + \cdots \\ \vdots & & \\ \vdots & & \end{bmatrix}$$

$$t_{halt} = \sum_{j=1}^{n} N_{ij}$$

$$= 1 - \sum_{j=1}^{n} \frac{S(n)}{nS(n)+3}$$

$$= 1 - \frac{\cancel{n}\, S(n)}{\cancel{n}\, S(n) + 3/n}$$

$$\boxed{t_{halt} = 1 - \frac{nS(n)}{nS(n)+3}}$$

(3) $E[|Z_n|] < \infty \quad \forall \; n \geq 1.$

and $E[Z_n \mid Z_{n-1}, \ldots, Z_1] = Z_{n-1} \quad \forall n \geq 2$

Martingale of the product form,

$E[Z_n] = 1$ for $n \to \infty$.

Also,

$Z_n = \exp\{\eta S_n - n\gamma(\eta)\}$

$a \to$ region required.

$$Z_n = \exp\{\lambda X_n - \gamma(\lambda)\} Z_{n-1}.$$

$$r X_n = \gamma(r)$$

is semi-invariant generating function.

$$\frac{\gamma(r)}{r} = X_n.$$

for a stopped process,

$$E[Z_1] = E[Z_n^{*}] = E[Z_n].$$

$$\hookrightarrow E[Z_j] = E[\exp\{\lambda S_j - J\gamma(\lambda)\}] = 1$$

$$\Rightarrow \Pr\{S_{J_{n+}} \geq \alpha\} \leq \exp[-r\alpha + n\gamma(r)]; \; 0 \leq r < r^*$$