CS 5786: A1

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Problem 1

2a

Find the relationship between the PCA projection matrices W and W' given:

$$\mathbf{x}_t' = R\mathbf{x}_t' + \mathbf{v} \tag{1}$$

We define a matrix $\mathbf{V}$ to be a $n \times d$ matrix whose rows are all $\mathbf{v}$ ($V_{ij} = v_j$)

$$X' = \left(RX^T\right)^T + V = XR^T + V \tag{2}$$

Rotations for data already centered remains center, so $\mathbf{v}$ is the mean for for $\mathbf{x_t}$. The covariance matrix for X and X' is:

$$\Sigma = (X - E[X])^T (X - E[X]) = X^T X \tag{3}$$

$$\Sigma' = \left(XR^T\right)^T XR^T = RX^T XR^T = R\Sigma R^T \tag{4}$$

For all $\mathbf{w}_j$ that are eigenvectors of Σ and are columns of W, right multiply Σ' by $R\mathbf{w}_j$:

$$\Sigma' R\mathbf{w}_j = R\Sigma\mathbf{w}_j \tag{5}$$

Since $\mathbf{w}_j$ is an eigenvector of Σ,

$$\Sigma' R\mathbf{w}_j = R\lambda'\mathbf{w}_j = \lambda' R\mathbf{w}_j \tag{6}$$

Since $\Sigma\mathbf{w}_j = \lambda\mathbf{w}_j$,

$$\mathbf{w}' = R\mathbf{w} \tag{7}$$

2b

The compressed data can be found from:

$$\mathbf{y}_t = (\mathbf{x}_t - E[\mathbf{x}_t])^T W = \mathbf{x}_t^T W \tag{8}$$

$$\mathbf{y}_t' = \left(\mathbf{x}_t{}'^T - \mathbf{v}^T\right) W' = (R\mathbf{x}_t)^T RW = \mathbf{x}_t^T R^T RW = \mathbf{x}_t^T W = \mathbf{y}_t \tag{9}$$

This suggests that the rotation and translation does not affect the resulting compressed data. However, the choice of the eigenvectors in W is not unique and can either be $\mathbf{w}_j$ or $-\mathbf{w}_j$. The method used to determine the eigenvectors will depend on the values in the covariant matrix, so the sign of the compressed data will vary based on the transformation.

Problem 2