Large Language Model Guided Graph Clustering

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Abstract

Graph clustering on text-attributed graphs (TAGS), i.e., graphs that include natural language text as additional node information, is typically performed using graph neural networks (GNNs), which forego the text in lieu of embeddings. While GNN methods ensure scalability and effectively leverage graph topology, text attributes contain rich information that can be leveraged using large language models (LLMs). However, many real-world applications have limited hardware resources or LLM API call budgets that prevent their naive use. To reconcile these constraints when performing clustering on TAGs, we propose an active learning framework that performs graph clustering using LLM refinment (GCLR) by selectively prompting an imperfect LLM oracle for feedback and, subsequently, finetuning the GNN-based clustering solution to incorporate the feedback. GCLR uses different prompting strategies to improve the LLM's reliability as an oracle and uses noise-controlling finetuning to handle this imperfect, but useful feedback. Extensive experiments demonstrate that GCLR can significantly improve clustering performance over state-of-the-art GNN methods.

1 Introduction

Graph clustering seeks to perform an unsupervised assignment of nodes to different clusters such that the resulting assignments capture salient topology and uncover useful concepts. Notably, many real-world problems can naturally be formulated as graph clustering, including recommending groups of items in an e-commerce shopping graph or identifying groups of friends in social networks (Newman, 2006; Newman and Reinert, 2016; Yang and Leskovec, 2012). Most modern, performative clustering methods utilize graph neural network (GNN) encoders due to their expressivity (Xu et al., 2019),

scalability, and ability to effectively handle vectorvalued node attributes (Kipf and Welling, 2017; Veličković et al., 2018).

Recently, however, there has been growing interest in text-attributed graphs (TAGs) (Yang et al., 2021; Yan et al., 2023), where natural language text is available as an additional node attribute. Unfortunately, GNNs are not able to directly handle this information rich text and instead utilize semantic embeddings, potentially limiting overall performance. To this end, a variety of (pre/co/joint) trainingbased (Chien et al., 2022; Zhao et al., 2023a; Ioannidis et al., 2022; Mayromatis et al., 2023; Xie et al., 2023) and graph specific prompting-based strategies (He et al., 2023; Zhao et al., 2023c; Fatemi et al., 2023; Guo et al., 2023; Tang et al., 2023) have been recently proposed for using large language models (LLMs) (Touvron et al., 2023; Bai et al., 2022) in conjunction with GNNs on supervised tasks, e.g., link prediction, node classification, and graph classification, to directly handle this text and take advantage of the LLM's impressive worldknowledge.

While clustering on TAGs could also benefit from joint LLM+GNN methods, it not only remains unclear how to adapt existing supervised approaches for unsupervised graph clustering, but also is prohibitively expensive in many real-world applications due to significant hardware requirements, incurred through training or hosting LLMs, or API expenditure, incurred by prompting over large sets of nodes. Given that GNN clustering methods are scalable to large graphs by design and have much lighter hardware requirements (Fettal et al., 2022; Devvrit et al., 2022; Liu et al., 2023; Bianchi et al., 2020; Ying et al., 2018; Tsitsulin et al., 2023), it is more cost effective to selectively use the LLM to improve the GNN's initial clustering assignment; thereby limiting the overall expenditure. While a natural framework for such a resource constrained setting is active learning

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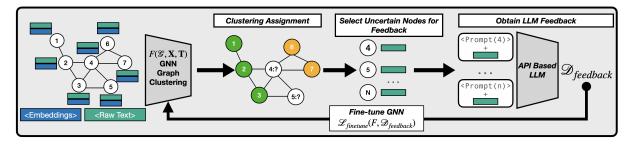


Figure 1: **Overview of GCLR.** Graph clustering is a fundamental graph machine learning task (Liu et al., 2022; Tsitsulin et al., 2023) where nodes must be assigned to different clusters to uncover interesting semantic concepts. This unsupervised task has many practical applications, including identifying communities in social networks (Girvan and Newman, 2002), analyzing protein-protein interaction networks (Rives and Galitski, 2003) and making targeted marketing recommendations (Tang and Liu, 2010). Our proposed method, GCLR, is designed to improve the quality of a GNN-based graph clustering solution using active learning. In particular, given a starting GNN clustering, F, GCLR identifies uncertain nodes, obtains LLM guidance through prompting and then fine-tunes the GNN.

(AL) (Ren et al., 2020; Sener and Savarese, 2018; Ma et al., 2023; Kazemi et al., 2022), which selectively queries an expensive oracle for labels to maximize performance under a fixed budget, there are several differences arising from an LLM oracle and the unsupervised nature of graph clustering that must be addressed. Namely, that (i) it is unclear how to select, query, and incorporate LLM feedback to improve GNN clustering solutions, and (ii) the LLM is an *imperfect* oracle, complicating how the model should be updated.

Our Work. To this end, we propose GCLR (Graph Clustering with LLM Refinement), a flexible active learning framework specifically designed for clustering on TAGS. It uses carefully designed prompting strategies to elicit more reliable and useful feedback for clustering from the LLM and uses simple strategies when fine-tuning to improve tolerance to noisy labels, overall outperforming GNN-based clustering methods. Our contributions are summarized as follows:

- Eliciting Graph Clustering Feedback from LLMs (Sec. 3.1.) We rigorously study how to obtain feedback from LLMs that is both amenable to clustering and a useful signal for fine-tuning.
- Incorporating Noisy Feedback from LLMs (Sec. 3.2.) Given the feedback provided by the LLM, we propose training protocols that support fine-tuning deep graph clustering algorithms with imperfect feedback.
- Extensive Experiments Refining Clustering with GCLR (Sec.4) Across three text-attributed graphs with four different graph clustering algorithms, we demonstrate that GCLR can improve the graph clustering performance.

Due to space constraints, we have included re-

lated work on deep attributed clustering and methods that combine LLMs and GNNs to perform learning tasks in App. B.

2 Problem Formulation

In this section, we formally introduce our problem setting, as well as assumptions and constraints.

Notations. Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{T},\mathcal{X},[\mathcal{Y}])$ represent a graph with its respective node set, edge set, raw node-based text information, embedded node attribute information (e.g., some embedding of a node's text), and optional ground-truth cluster assignment. Further, let N be the number of the nodes, M be the number of edges, K be the desired (or ground-truth) number of clusters, K0 the dimension of the hidden representation, K1 the dimension of the hidden representation of K2.

Problem statement. Let $\mathbf{F}: (\mathbf{A}, \mathbf{X}) \to \mathbf{Z}^{N \times d}$ be a GNN-based encoder that outputs d-dimensional node representations, and $C: (\mathbf{Z}, K) \to [0, K]^N$ be an embedding-based clustering algorithm, e.g., k-means, pooling layer, where C may optionally be parameterized and optimized end-to-end with the encoder. Then, the clustering assignments, $\mathbf{K}^{N \times K}$ can be obtained as: $\mathbf{K} = \mathbf{C}(\mathbf{F}(\mathbf{A}, \mathbf{X}), K)$. K is assumed to be an imperfect assignment, i.e., there exist samples that are mis-assigned to clusters and/or cluster topics are noisy. We seek to use the LLM's world-knowledge and natural language understanding to improve K. Given that K is already topology-aware due to the GNN encoder and semantics-aware since pre-trained sentence transformers are used to encode the nodelevel raw text attributes, the LLM provides an complementary source of information. Indeed, the performance of LLMs in zero-shot node classification

suggests that their world knowledge is well-suited for graph tasks. We assume pre/co/joint-training is prohibitively expensive and only prompting is available to obtain LLM feedback, and further make the reasonable assumption that there is a limited budget, \mathbf{B} , for API calls/prompting. Thus, our objective is to induce the best refined assignment, \mathbf{K}_{refine} , while remaining under budget. This problem setting is amenable to active learning, which we introduce conceptually here but note that subsequent sections will discuss how GCLR instantiates AL for clustering.

Active Learning. While much of deep learning is data-intensive and requires large labeled datasets for strong performance, deep active learning seeks to maximize performance in a setting where labels or feedback is expensive to obtain. AL consists of three key components: a query function, \mathbf{Q} , which determines which samples from the unlabeled data pool should be selected for obtaining feedback, an oracle, which provides feedback to create a labeled dataset, $\mathcal{D}_{\text{feedback}}$, and a training protocol, which defines a loss, $\mathcal{L}_{\text{feedback}}$, and update procedure for how the model will incorporate said feedback.

Query functions (Ash et al., 2020; Wang and Shang, 2014; Ducoffe and Precioso, 2018) are broadly designed to identify the samples where labeling will have the most impact. Effective functions often use sample uncertainty, difficulty or coverage to select points. The oracle serves as a proxy for an expensive but reliable labeling procedure, for example human annotators or wet-lab experiments. The training protocol is designed to ensure stability, and avoid over-fitting when operating over small batches of data. While some graph AL strategies have been recently proposed, these methods focus on semi-supervised node classification and are not directly applicable to our problem setting.

Moreover, we emphasize that while AL traditionally assumes that (i) the oracle is trust-worthy, we do not know apriori the reliability of the LLM's feedback and (ii) our problem is unsupervised, so existing AL query functions, and training protocols may not be well-suited (Li et al., 2021; Liu et al., 2020; Ostapuk et al., 2019). Lastly, we note that while it is possible to receive dataset-level or task-level feedback, we focus on node-level feedback as it is more scalable for larger graphs (only a subset of nodes will receive feedback), and is more amenable with contrastive and pooling-based graph clustering algorithms, as they already pro-

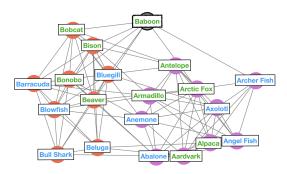


Figure 2: Unaligned Notions of Similarity. The following stochastic block model graph has clusters that correspond to whether a particular animal's name begins with "A" or "B." However, an alternative clustering according to "land" vs. "aquatic" animals is also valid and more semantically interesting. Indeed, when GPT-3.5 is asked whether a "Baboon" is more similar to a "Bluegill" or "Antelope," it replies with "Antelope" as it is also a land mammal. This emphasizes that (i) simple pairwise comparisons may not be sufficient for providing feedback and (ii) LLMs and GNN clustering algorithms may utilize disparate notions of similarity.

vide node-level embeddings and assignments. In subsequent sections, the design of \mathbf{Q} and $\mathcal{L}_{feedback}$ for clustering on TAGs is discussed in detail.

3 GCLR: Graph Clustering with LLM Refinement

In this section, we formally introduce GCLR, our framework for graph clustering with LLM refinement (Fig. 1). We begin by discussing how to obtain useful feedback for graph clustering from LLMs and then present how to identify and refine the initial solution accordingly.

3.1 Eliciting Feedback from LLM for Graph Clustering

While feedback in traditional AL typically corresponds to an oracle selecting a label from a predefined set of classes, it is less clear what form the feedback should take when performing clustering. Intuitively, feedback should help improve the similarity of the queried node with the cluster that it belongs to. However, the precise form of the feedback may vary, and it's unclear how to prompt the LLM to accurately ascertain this information.

To this end, we discuss the advantages and disadvantages of three different strategies for prompting the LLM to obtain clustering feedback. We begin by discussing a recently proposed strategy for LLM guided text clustering.

Triplet-Based Prompting. ClusterLLM (Zhang et al., 2023b) is a recently proposed state-of-the-







(a) Triplet Based Feedback

(b) In-Context Based Prompt

(c) Concept Based Prompt

Figure 3: **Example of LLM Feedback.** Using the graph in Fig. 2, we prompt chat-gpt-3.5-turbo with different strategies to demonstrate the importance of aligning the LLM's and GNN's implicit similarity functions. Indeed, we see that triplet-based prompting can be unreliable as it does not allow the LLM to infer the underlying similarity. For example, with the query, "Baboon" with triplets containing the land animals from from Cluster 1 (starts with B) and aquatic animals from Cluster 2, the LLM assigns Baboon to cluster 1, which is consistent with the graph solution. However, when we prompt chat-gpt-3.5 with a triplet containing *aquatic* animals from Cluster 1 and *land* animals from Cluster 2, the LLM assigns the query to Cluster 2 as it is also a land animal. In contrast, we find that both concept-based and incontext-based prompting are able to correctly infer the GNN's similarity function.

art LLM guided *text* clustering method that first selects uncertain samples (e.g., queries), Q_i , and two random samples from each query's two nearest clusters, and then prompts the LLM to predict which of the two samples is "more similar" to Q_i ; the more similar sample is considered a "positive" sample and the other is a "negative" sample. Here, $\mathcal{D}_{\text{feedback}}$ corresponds to the set of triplets (query, positive, negative) determined by the LLM and $\mathcal{L}_{\text{feedback}}$ is InfoNCE. While such an approach can conceptually be applied to graph clustering, there are some limitations.

Insofar as clustering requires learning a similarity function that can be used to partition samples into meaningful groups, it is important that the oracle is aware of this function so the resulting feedback is aligned to the existing partitioning. In text clustering, since both the encoder (BERT, E5, etc) and the larger, oracle LLM (Chat-GPT, Llama) are text based models, they share a similar prior for this similarity function. In contrast, when performing graph clustering, the GNN incorporates topological information unavailable to the LLM and may utilize a different function than the LLM. Indeed, in Fig. 2, we construct a simple synthetic example where the GNN and LLM utilize different similarity functions to identify concepts by design. We observe, in Fig. 3a, that the oracle (chat-gpt-3.5-turbo) provides unreliable

feedback when the triplet prompt contains random samples that do *not* overlap with the GNN's similarity function, but is reliable when the random samples are selected to align with the LLM's implicit similarity function.

Finally, we note that the performance of tripletbased feedback is closely tied to the quality of the initial clustering solution, artificially handicapping the LLM's performance. Given that the initial clustering solution is imperfect, randomly selecting samples from the two closest clusters can create triplets that do not actually represent the corresponding clusters, leading the LLM to perform a meaningless selection. Moreover, there is a loose upper-bound of the triplet formulation as the queries' "correct" cluster must be within the top-2 closest clusters. If this is not the case, the LLM will necessarily have to respond to an ill-formed triplet and will provide incorrect feedback. Due to the rapidly increasing capabilities of LLMs, it is possible that future LLMs will achieve perfect performance on valid triplets, however, the error incurred by ill-formed triplets is irreducible.

In-Context Similarity Learning. As discussed above, it is critical that the LLM can infer the similarity function implemented by the GNN. Given the impressive in-context learning capabilities of LLMs (Kaplan et al., 2020; Brown et al., 2020), we consider a prompt that allows the LLM to directly

infer it by providing several examples of the node's raw text and their corresponding cluster IDs, and the text of the unlabeled query (See Fig. 3b for an example.) Here, the LLM can be seen as performing a prediction task amongst pseudo-labels defined by the initial clustering, where $\mathcal{D}_{\text{feedback}} =$ $\{([0, \dots K]|i \in \mathcal{Q}\}.$ We note that the choice of $\mathcal{L}_{\text{feedback}}$ is flexible and discuss it in detail later. Notably, by ensuring that the prompt contains samples from all clusters, the LLM can (i) more holistically infer what concepts underlie clusters and (ii) predict an assignment for a query that does not belong to the top-2 clusters. This allows us to circumvent the previous issue where the upper-bound on refined performance was restricted by the number of samples where the preferred assignment was contained in the top-2 clusters.

However, directly inferring the similarity function from in-context examples becomes more difficult as the number of clusters grows as (i) the number of exemplars must correspondingly reduce to remain within the context length and (ii) if the number of clusters is sufficiently large, it is not possible to provide exemplars from all clusters. Furthermore, the selection and ordering of exemplars can have a significant impact of the LLM's ability to correctly predict a query's assignment, leading to potential loss of performance during fine-tuning.

Concept-based Prompting. the aforementioned issues with incontextprompting, we draw inspiration from topic modeling (Viswanathan et al., 2023; Pham et al., 2023) and design an additional "concept-based" prompting strategy where we first prompt the LLM to infer the concepts that were used to group samples and then create a prediction task where the LLM is prompted to select amongst the generated concepts. (See Fig. 3c for an example.) To generate concepts, we provide the LLM samples from each cluster and ask it to provide a "title" and "short description" that explains how these samples are grouped together. These generated titles and descriptions are then provided as options for the LLM to identify the most similar cluster for a particular query. Notably, by providing the titles/descriptions of all clusters, we can avoid the upper-bound encountered by triplets while simultaneously allowing the LLM to at least partially infer the GNN's similarity function.

Moreover, by using cluster titles/descriptions instead of multiple exemplars per cluster, conceptbased prompting uses much shorter prompts and better scale as the number of clusters grows in comparsion to in-context prompting. Indeed, as the number of clusters grows, In-Context prompting would require decreasing the number of exemplars per cluster to fit the context length. Moreover, this context must be passed every time feedback is obtained. In contrast, the titles/descriptions are generated once in a preprocessing step, and subsequently reused through a shorter, multiple choice-style prompt. Finally, we note that creating titles/descriptions may help denoise the exemplars as the LLM seeks to understand how they were grouped together.

Experimental Setup. We verify the effectiveness of the proposed feedback elicitation strategies on several public graph datasets, where the provided node labels serve as ground-truth cluster labels. mixtral-8x-7b is used as the oracle, and four different graph clustering backbones are used to obtain the initial clustering solutions. We sort the samples according to the entropy of the distance to the two nearest clusters (a proxy for sample difficulty) and prompt the LLM for each sample as per the discussed strategies. Please see App. E for example prompts, comprehensive experimental details and dataset statistics.

Results. The following observations are made from Table 1. We observe that across datasets and clustering methods, that the "concepts" strategy is the best or second best performing prompting strategy most often. While In-Context prompting achieves comparable performance on some datasets, we note that it is significantly more expensive. Indeed, every InContext prompt contains multiple exemplars per cluster, while "concepts" only processes these exemplars once to obtain the generated titles and descriptions, which are then directly used in the prompt. "Triplets" is the cheapest strategy in terms of token length, but lags behind on performance, failing to achieve the best performance on any dataset. Lastly, we note that the GNN outperforms the LLM on full dataset (100th percentile) accuracy on 9/12 settings, indicating that, in addition to being prohibitively expensive, prompting the LLM for every node would not be as effective as the initial GNN solution. Indeed, there are several situations where the LLM's feedback is less effective than the GNN's, highlighting that care must be taken when updating the GNN.

Table 1: **Reliability of LLM as an Annotator.** The accuracy of the GNN-based clustering solution and three prompting strategies are reported at the 10\50\100-th most difficult percentile of the dataset. The best performance overall is **bolded**, while any prompting-based method is colored if it exceeds the accuracy of the GNN, and the 2nd best prompting based method is underlined.

Dataset	Method	GNN	Concepts	Incontext	Triplets
		Graph Only		LLM Only	
citeseer	diffpool dinknet	32.1\36.2\49.7 40.6\54.7\70.3	36.2 \ 41.1 \ <u>49.1</u> 30.8\32.9\47	34.6 \36.2 \46.7 48.7\48.3\59.6	29.2\34.1\44.0 43.1\50.6\62.1
	dmon	36.5\38.2\44.1 35.8\52.2\66.5	40.9\39.9\43.9 38.4\46.1\58.5	36.2\37.7\42.9 42.1 \50.5\60.5	36.8\38\41.7 34.3\46.5\57.1
cora	diffpool	32.6\40\54.7	35.6\36.0\37.7	34.4\36.6\ <u>50.2</u>	33.7\36.9\48.8
	dinknet dmon	37.4\50.7\65.8 42.6\52.4\60.9	32.2\36.8\39 36.3\41.4\40.7	24.8\36.0\52.7 46.3 \51.3\56.9	35.2\47\58.2 40\47.9\54
	mincut	40\53.6\68.4	<u>42.2</u> \46.5\55.7	43.7 \ <u>50.5</u> \ <u>63.3</u>	37.8\49.8\60.9
wikics	diffpool dinknet	25.5\32.2\ <u>48.3</u> <u>37.7\51.2</u> \ 66.5	36.0\40.4\52.7 51.2\56.5\64.8	33.9\37.1\47.9 35.8\36.9\51.1	25.9\30.8\44.2 35.0\44.5\54.8
	dmon mincut	28.1\31.2\36.9 36.5\24.4\26.9	55.2\55.2\57.2 31.9\ 29.6 \ <u>29.8</u>	39.9\41.3\41.3 37.5\27.9\31.0	28.7\31.2\35.8 32.4\24.1\25.2

3.2 Refining GNN-Based Clustering with Feedback

While the proposed prompting strategies help improve the LLM's feedback, we must now incorporate this imperfect feedback into the GNN to scalably improve the overall clustering solution.

Finetuning Setup. While reconstructive (Zhang et al., 2023a; Bo et al., 2020) and adversarial frameworks (Gong et al., 2022) were initially popular for graph clustering, we focus on more recent contrastive (Liu et al., 2023; Xia et al., 2022; Thakoor et al., 2022; Devvrit et al., 2022) and pooling-based methods (Tsitsulin et al., 2023; Bianchi et al., 2020; Ying et al., 2018) as they are more scalable and performative. Furthermore, there is extensive literature on fine-tuning contrastively pre-trained models (typically for supervised tasks) that we can leverage when defining $\mathcal{L}_{feedback}$. Indeed, both in-context and concept-based prompting induce a dataset, $\mathcal{D}_{feedback} = \{([0, \dots K] | i \in \mathcal{Q})\}$, that consists of queried nodes and their predicted cluster assignments. Thus, we can consider refinement as a supervised task with LLM-provided pseudo-labels.

When working with pooling-based methods (DMon, MinCut, and DiffPool, etc), \mathbf{F} directly predicts the cluster assignment as the node features are pooled to the number of clusters. For contrastive methods like DinkNet, we can initialize a classifier using parameterized cluster centers or those obtained using KMeans. Then, given the classifier and $\mathcal{D}_{feedback}$, we can naturally define $\mathcal{L}_{feedback}(\mathcal{D}_{feedback}, \mathbf{F})$ using the cross-entropy loss. While other losses, such as triplet (Hinton and Roweis, 2003), InfoNCE (), SupCon (Khosla

et al., 2020), are certainly possible, we empirically find that cross-entropy is effective. However, since $\mathcal{D}_{feedback}$ is expected to contain incorrect labels, but the error-generating process is unknown, naively training on the labels may diminish performance. Thus, we consider the following simple but effective strategies for improving the finetuning performance.

Strategies for Handling Noisy Labels. Given that our prompting strategies induce a classification task, we use the model's predicted confidence in order to eliminate potentially noisy labels. Namely, we compute the LLM's confidence in its predictions by obtaining log-probability of the top-2 tokens corresponding to cluster predictions. Alternative prompting strategies and specialized losses have been proposed for better calibration (Tian et al., 2023; Yin et al., 2023; Zhou et al., 2023) but we do not consider them due to their additional expense. Empirically, we find that token-level log probability is sufficient.

To further stabilize and improve training, we augment $\mathcal{D}_{feedback}$ with samples well-clustered by the GNN, where probits of the predicted clusters are used to identify confident assignments. The loss is computed separately for the LLMlabeled and GNN-labeled samples, and aggregated as $\alpha \mathcal{L}_{finetune,LLM} + \beta \mathcal{L}_{finetune,GNN}$, where α and β are constrained to be a convex combination. By varying α and β , we can express different levels of certainty in the feedback. In practice, we find setting α and β to 0.5 leads to strong performance. Since the optimal weighting is not known apriori, creating a simple deep ensemble (Lakshminarayanan et al., 2017) by varying α , β to train multiple independent models can further improve performance. Though this incurs additional training expenditure, it is not substantial with respect to training the initial model as clustering losses often approximate quadratic operations, or obtaining feedback. We assess the effectiveness of each of these components and GCLR as a whole in the following section.

Scaling to Larger Graph. GCLR's scalability is determined by the budget and underlying GNN clustering solution. Indeed, larger graphs may require a larger budget in order to obtain feedback on similar portions of the dataset. However, we note that fine-tuning remains scalable due to use of the cross-entropy loss, which can be easily batched, and will not be as expensive training the initial clustering solution. Moreover, GNNs are fairly small,

Table 2: **LLM Labels Provide Complementary Information For Active Learning.** Here, we compare the performance of different feedback mechanisms and finetuning losses. We observe that (i) while both LLM (9/12 Acc.) and GNN (10/12 Acc) feedback generally improves performance over the initial starting solution, that LLM feedback with the cross entropy loss achieves the best accuracy overall (8/12), though performance on intrinsic metrics is more mixed; (ii) on Cora, where GNN feedback was more reliable than LLM feedback, we see that using the GNN pseudo labels is more effective; (iii) on WikiCS, where LLM feedback is much more reliable, we see dominant performance by LLM feedback with cross entropy loss; and (iv) we see that the cross entropy loss (9/12 Acc., 7/12 Modularity, 7/12 NMI) is more effective than the triplet for finetuning.

		(startir	ng performance) \ GNN Feedba	ck + Cross Ent. Loss \ LLM Fe	edback + Triplet Loss \ LLM 1	Feedback + Cross Ent. Lo	SS
Dataset	Method	Acc. (†)	NMI (↑)	F1 (↑)	ARI (↑)	COND (\downarrow)	MOD (†)
	diffpool	(47.09) \54.69 \48.05 \58.96	(25.59) \25.94 \21.50 \26.84	(23.08) \23.57 \14.65 \19.70	(43.09) \43.33 \33.22 \41.41	(0.23) \0.23 \0.25 \0.24	(0.56) \0.56 \0.45 \0.50
citeseer	dinknet	(66.46) \66.43 \67.36 \67.40	(43.08) \43.30 \19.16 \36.97	(42.43) \41.30 \16.37 \27.16	(60.39) \60.58 \42.49 \47.91	(0.07) \0.07 \0.29 \0.09	(0.70) \0.70 \0.51 \0.62
cheseer	dmon	(47.89) \49.85 \48.75 \49.87	(28.49) \28.77 \27.11 \27.12	(24.29) \24.61 \18.86 \14.46	(43.65) \43.71 \34.14 \29.87	(0.19) \0.19 \0.25 \0.15	(0.60) \0.60 \0.45 \0.47
	mincut	(64.18) \66.70 \ 69.82 \ 67.51	(44.41) \ 46.21 \40.48 \39.60	(41.95) \43.25 \38.54 \35.81	(61.72) \62.11 \59.54 \59.81	(0.08) \0.09 \0.13 \0.17	(0.73) \0.73 \0.67 \0.64
	diffpool	(59.97) \63.6 \43.38 \51.35	(43.46) \42.70 \20.97 \22.21	(36.58) \35.65 \7.83 \6.49	(56.76) \55.64 \29.3 \29.05	(0.24) \0.25 \0.38 \0.32	(0.60) \0.60 \0.33 \0.34
	dinknet	(68.26) \66.84 \67.32 \65.16	(51.98) \50.87 \25.01 \23.42	(44.21) \40.50 \15.16 \9.25	(62.09) \59.20 \41.86 \27.40	(0.12) \0.11 \0.30 \0.08	(0.70) \0.67 \0.49 \0.29
cora	dmon	(57.56) \60.27 \59.06 \56.70	(41.60) \42.24 \30.18 \30.06	(33.76) \34.64 \20.66 \13.67	(50.94) \51.40 \39.44 \29.40	(0.27) \0.26 \0.38 \0.12	(0.56) \0.58 \0.42 \0.33
	mincut	(64.17) \66.63 \59.91 \61.62	(48.92) \48.92 \39.74 \41.61	(40.35) \40.35 \29.43 \30.54	(58.33) \58.33 \47.28 \54.01	(0.14) \0.14 \0.21 \0.28	(0.70) \0.70 \0.56 \0.54
	diffpool	(43.15) \49.69 \55.44 \58.03	(26.27) \26.36 \37.20 \35.03	(18.87) \19.50 \31.10 \26.28	(39.88) \39.70 \41.12 \46.48	(0.34) \0.35 \0.30 \0.34	(0.48) \0.47 \0.36 \0.44
wikics	dinknet	(66.80) \73.65 \67.48 \74.00	(49.00) \51.84 \47.49 \51.25	(47.80) \53.04 \46.18 \51.57	(56.23) \63.06 \56.97 \63.06	(0.23) \0.21 \0.28 \0.23	(0.55) \0.55 \0.52 \0.54
WIKICS	dmon	(38.60) \39.68 \43.28 \51.87	(27.47) \27.49 \29.33 \32.51	(20.55) \20.65 \27.48 \31.04	(34.02) \34.18 \34.48 \36.49	(0.48) \0.47 \0.42 \0.26	(0.33) \0.33 \0.33 \0.31
	mincut	(24.70) \32.84 \38.52 \46.36	(6.14) \8.32 \17.99 \16.52	(-0.37) \-0.32 \18.02 \4.8	(7.91) \8.45 \24.71 \24.36	(0.04) \0.04 \0.45 \0.47	(0.03) \0.05 \0.30 \0.27

and neighborhood samplers, as well as other techniques for dealing with large graphs, can be used to further support finetuning. Thus, GCLR can be seen as scaling linearly with the size of the dataset, if we assume that the same portion of the initial solution is selected for obtaining feedback.

4 Experiments

In this section, we verify the effectiveness of GCLR in refining graph clustering solutions across several public datasets with different graph clustering algorithms.

Experimental Setup. Our set-up is as follows. Baselines. We consider the following graph clustering baselines: MinCutPool (Bianchi et al., 2020), DMoN (Tsitsulin et al., 2023), DiffPool (Ying et al., 2018), and DinkNet (Liu et al., 2023). Metrics. As we use public datasets with available groundtruth clustering, we report accuracy, Normalized Mutual Information, F1, and Adjusted Rand-Index between the predicted and labeled clusters. We intrinsically assess the clustering quality using conductance and modularity (see App. F for their precise definitions). We use embeddings obtained from SBERT as node features for all experiments. Datasets. We provide the dataset statistics in Table G. Training. Both the initial GNN and subsequently finetuned models are trained with earlystopping and the learning rate is tuned amongst 1e-4 and 1e-3.

GCLR. Unless otherwise noted, we use mixtral-8x-7b as the oracle LLM and seek feedback on 10% of the nodes in the dataset. (Please see D for additional results with ChatGPT.) The query function, Q, is defined to select nodes according to

prediction entropy (Wang and Shang, 2014). Here, high entropy nodes are less well-clustered, and labeling them would provide useful information. α and β are both set to 0.5, unless otherwise noted. Results are averaged over 10 seeds.

Results. The following observations are made using Tables 2, 3 and 4.

Observation 1. We begin by confirming that the LLM provides valuable information through its feedback by demonstrating, in Table 2, that subsequent finetuning not only improves performance over the starting clustering solution but also over finetuning on GNN pseudo labels, when reliable. Additionally, we find that using the cross entropy loss is more effective than the triplet loss when finetuning using the LLM feedback. This is in contrast to ClusterLLM, which focused on triplets. Lastly, we note that confidence filtering and ensembling are not applied in Table 2, so performance can further be improved.

Observation 2. Next, we seek to understand how filtering samples according to confidence can improve GCLR's performance. We do note that both the GNN and LLM feedback are not guaranteed to be calibrated, but nonetheless empirically find their confidences useful. In particular, in Table 3, we set $\alpha=0.5$ and $\beta=0.5$, and consider 2 different filterings: one where the GNN's confidence interval is high and the other where the LLM's confidence interval is high. We find that updating the model using only high confidence LLM feedback (80th percentile) and GNN feedback at lower percentile improves the accuracy 8/12 times. We posit that the relatively large set of low confidence GNN samples

Table 3: **Effect of Confidence Filtering.** Though feedback reliability is unknown apriori, prediction confidence can be used to select samples where the feedback is more likely to be reliable. Here, samples are filtered based on ascending confidence percentile (increasing difficulty). (See Table 7 for full results.) We observe that filtering improves performance without filtering (11/24 Acc.) and over the starting, GNN solution (17/24 Acc.). In particular, 80% LLM and 20% GNN filtering improves performance over no filtering (8/12 NMI, 10/12 Mod.) **Best performance.** (Starting GNN Acc.)

Dataset	Method	LLM	GNN	Acc.	NMI	F1	ARI	COND	MOD
	diffpool	20	80	53.04	22.67	15.06	34.93	0.31	0.45
	(47.09)	80	20	56.71	26.94	23.18	41.90	0.21	0.56
	(47.09)	0	0	58.96	26.84	19.70	41.41	0.24	0.50
	dinknet (66.35)	20	80	67.61	38.14	32.03	50.99	0.08	0.64
		80	20	67.43	40.23	37.88	56.47	0.10	0.67
citeseer		0	0	67.40	36.97	27.16	47.91	0.09	0.62
	dmon	20	80	51.21	26.85	18.27	31.64	0.15	0.50
	(47.89)	80	20	51.14	30.06	25.30	41.72	0.17	0.59
	(47.89)	0	0	49.87	27.12	14.46	29.87	0.15	0.47
	mincut	20	80	61.42	31.79	26.94	47.84	0.26	0.56
	(64.17)	80	20	65.40	41.32	38.01	59.37	0.13	0.69
	(04.17)	0	0	67.51	39.60	35.81	59.81	0.17	0.64

help stabilize training, while the high confidence LLM feedback helps enhance the overall clustering solution.

Observation 3. In settings where the LLM's feedback is less reliable than the GNN's, it is possible to harm the initial clustering solution when updating the intial clustering solution. For example, in Table 1, on Cora, the LLM's feedback is less reliable than the GNN's, and in Table 2, we see finetuning on GNN feedback leads to better performance than the LLM's. However, we note that even if the LLM's feedback is unreliable it may still contain valuable information. To this end, we create a simple deep ensemble that captures different levels of certainity in either source's feedback by varying α and β when aggregating the loss. In particular, we train 5 different models, where we sample $\alpha \in [0, 0.1, \dots 0.5]$ and $\beta \in [0.5, 0.6 \dots 1]$ at evenly spaced intervals. In Table 4, we show that using this ensemble can improve performance over a single model where $\alpha = \beta = 0.5$, and see that GCLR improves over the initial clustering solution as desired.

Observation 4. While the above experiments identify query samples according to their entropy, other query functions are viable (Wang and Shang, 2014; Ducoffe and Precioso, 2018). In Table 5 (Appendix), we consider the following alternative query functions: random sampling, sampling the least confidence queries, and sampling queries with the smallest margin between the top-2 predicted clusters. While random sampling incurs some loss in performance, we find that margin sampling per-

forms similarly to entropy sampling and sampling according to least confidence actually improves performance in some cases.

Observation 5. Traditional active learning generally benefits from increasing the labeling budget as the oracle provides additional reliable feedback. In contrast, we find in Table 6 that increasing the budget does not have a substantial impact on performance. We believe this is partially due to an imperfect oracle and the bootstrapping that occurs from stabilizing training with GNN provided pseudolabels.

Table 4: **Ensembling Improves Performance with Unreliable Feedback.** Here, we create a deep ensemble by sampling different α and β to simulate different levels of confidence in each ensemble source. On Cora, where the LLM's feedback is known to be unreliable, we find that ensembling improves the performance of over a single model where $\alpha=0.5$ and $\beta=0.5$, and surpasses the performance of the starting solution as desired. Overall, this indicates that GCLR can help improve the initial clustering solution (highlighted in gray) even with unreliable feedback.

Method	Ens?	Acc.	NMI	F1	ARI	COND	MOD
	starting	59.97	43.36	36.58	56.76	0.24	0.60
diffpool	Х	51.35	22.21	6.49	29.05	0.32	0.34
	✓	61.88	45.74	38.97	58.20	0.22	0.62
	starting	68.26	51.98	44.21	62.09	0.12	0.70
dinknet	Х	65.16	23.42	9.25	27.40	0.08	0.29
	1	69.36	52.66	45.28	63.12	0.12	0.70
	starting	57.56	41.60	33.76	50.94	0.27	0.56
dmon	Х	56.70	30.06	13.67	29.40	0.12	0.33
	✓	60.60	43.25	37.60	52.41	0.24	0.58
	starting	64.17	48.92	40.35	58.33	0.14	0.70
mincut	Х	61.62	41.61	30.54	54.01	0.28	0.54
	✓	64.63	48.96	40.77	58.79	0.14	0.70

5 Discussion

In this work, we proposed GCLR to improve graph clustering solutions on text attributed graphs by eliciting feedback from LLMs. In order to avoid large prompting expenditure, GCLR actively queries the LLM on only uncertain nodes and uses various prompting strategies to obtain clustering feedback. This feedback is then used to update the initial GNN based clustering solution. Since LLM and GNN feedback can be unreliable, confidence filtering and ensembling are used to further improve performance. Given that GCLR's efficacy is constrained by the quality of the LLM provided feedback, future directions of work include designing prompting/training strategies to improve the reliability of the LLM oracle.

6 Limitations

There are several limitations of GCLR that primarily arises from the reliability of the LLM and usefulness of textual attributes. Indeed, if the textual attributes are not amenable to the LLM or relevant for improving the gnn-based clustering solution, then GCLR is unlikely to lead to improvements by relying upon the LLMs. For such applications, it is more valuable to seek alternative sources of feedback or information that can improve the solution. Moreover, if the LLM is consistently unreliable, finetuning on this feedback is effectively adversarial, and will lead to decreased performance relative to the starting GNN solution. Similarly, there is a risk that biases in the LLM can be propagated to the GNN as well.

7 Potential Risks

We do not believe there are notable societal impacts or risks from this work. We use existing large language models so their is potential to inherit their biases and hallucinations. We used ChatGPT to help with editing the writing.

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A Appendix

- Related Work (Sec. B)
- Ablation Results (Sec. C)
- Additional Results with ChatGPT (Sec. D)
- Prompt Examples (Sec. E)
- Details about Metrics (Sec. F)
- Reproducibility (Sec. G)

B Related Work

In this section, we briefly introduce deep attributed graph clustering and relevant works for combining LLMs and GNNs when working with TAGs. Please see Liu et al. (2022) and Jin et al. (2023), respectively, for comprehensive surveys.

Deep Attributed Graph Clustering. While unattributed graph clustering has a rich history in network analysis through modularity maximization, spectral clustering, and cuts-based approaches, the success of GNNs in graph representation learning has lead to growing interest in deep clustering methods that efficiently leverage both node-level attributes and topology. Broadly, such methods either (i) learn node representations using a self-supervised or unsupervised objective, and then perform clustering given these representations or (ii) learn both the embeddings and clustering assignments end-to-end through specialized clustering-based losses. While reconstructive (Zhang et al., 2023a; Bo et al., 2020) and adversarial frameworks (Gong et al., 2022) were initially popular, in this work, we focus on contrastive (Liu et al., 2023; Xia et al., 2022; Thakoor et al., 2022; Devvrit et al., 2022) and pooling-based methods (Tsitsulin et al., 2023; Bianchi et al., 2020; Ying et al., 2018). Such methods, which, respectively, use contrastive losses to learn discriminative node representations or propose novel pooling layers that optimize for clustering-based losses (e.g., spectral relaxations of modularity or mincut), are more performative, efficient, and scalable than adversarial or reconstructive approaches. Moreover, as we will discuss in Sec. 3.2, these methods are more amenable to fine-tuning. Indeed, fine-tuning contrastively pre-trained representations is well-known to induce state-of-the-art performance on a variety of supervised tasks in both vision and graph representation learning.

LLMs + Graphs. Recent approaches that seek to combine graphs/GNNs and natural-language/LLMs can be categorized as being "predictors" (the LLM provides predictions), "encoders" (sentence transformers or other LLMs are used to provide input node features), or "aligners" (GNNs and LLMs jointly trained to perform the task) (Jin et al., 2023). Various mechanisms, including prompting (Jiang et al., 2023), fine-tuning (Liu et al., 2024), variational expectation maximization (Zhao et al., 2023b), joint optimization (Li et al., 2023), and distillation (West et al., 2021), have been proposed to fulfill these roles, typically on supervised tasks. Instead, GCLR uses the LLM as a refiner and enhancer, as the LLM is only prompted to provide feedback for updating the underlying GNN-based graph clustering solution and sentence transformers are used to provide input node embeddings. This allows us to avoid the expensive fine-tuning of either LLMs or pre-trained language models, as well as exploit the scalability of graph clustering algorithms.

Moreover, we note that existing work has primarily focused on supervised tasks (mostly node classification and to a lesser extent link prediction), and does not assume budget constraints, prompting over the entire graph or finetuning PLMs/LLMs. For example, TAPE (He et al., 2023), a recent prompting focused LLM-as-Encoder method, prompts the LLM at every node for a class prediction and explanation, before fine tuning a pretrained language model to obtain embeddings. Prompting for every node can be extremely expensive in the case of large graphs, and, in our setting, we do not have pre-determined class labels to simplify how feedback is obtained from the LLM, making it challenging to finetune the PLM. Similarly, SimTeG (Duan et al., 2023), a fine-tuning based LLM-as-Encoder method, uses LoRA to train the LLM directly on the downstream node classification task, before extracting embeddings for training a GNN. Such an approach requires both supervision (which is unavailable in graph clustering) and fine-tuning of language models, which can incur expensive hardware and skills requirements. LLM-GNN (Chen et al., 2024) is a concurrent LLM-as-Encoder method that selectively prompts the LLM for feedback, but only considers a node classification task. Here, provided class labels ensure that the GNN and LLM are using aligned similarity functions, making it easier to obtain useful feedback. In contrast, on graph clustering, the LLM must infer as well as align with the GNN's implicit similarity function to provide meaningful feedback. On the other hand, LLM-as-Predictor methods seek to pass structural and textual attribute information directly to the LLM to make predictions. However, in our setting, where we assume a limited budget, it may be infeasible to prompt every node to obtain a cluster assignment. Other LLM-as-Predictor methods seek to perform graph-aware finetuning of PLMs and LLMs (Zhu et al., 2023), which can also be expensive. Lastly, we note that to the best of our knowledge, graph clustering has not been explored by

LLM-as-Predictor methods, so it is unclear if LLMs are able to infer sufficient topological information to effectively assign clusters.

C Ablation Results

Table 5: **Query Function Ablation.**We report performance on the following query strategies: random sampling \ entropy sampling \ least confidence \ margin sampling. We observe that while there is a slight decrease in performance when using random sampling as the query function, overall margin sampling perform similarly to entropy sampling. Least confidence sampling, in fact, improves performance on a few cases.

Dataset	Method	Acc.	NMI	F1	ARI	COND	MOD
citeseer	diffpool	49.45\59.56\60.19\59.38	26.47\27.75\28.38\23.21	8.47\13.16\12.88\8.74	33.87\31.93\31.16\29.23	0.16\0.11\0.09\0.1	0.39\0.34\0.33\0.29
	dinknet	46.14\56.99\58.16\57.9	6.62\35.41\35.8\36.2	2.81\22.96\22.28\22.96	10.65\37.79\37.39\40.21	0.02\0.22\0.22\0.21	0.07\0.43\0.43\0.44
	dmon	37.94\51.74\52.18\51.31	7.73\27.88\28.34\27.54	2.53\18.42\18.2\17.98	11.86\33.67\33.88\32.75	0.06\0.15\0.15\0.15	0.16\0.51\0.5\0.5
	mincut	64.08\63.4\63.56\61.16	34.45\34.74\35.16\39.89	30.4\31.31\31.42\30.13	55.48\54.43\55.27\49.44	0.24\0.23\0.23\0.31	0.56\0.58\0.58\0.52
cora	diffpool	68.7\66.53\66.55\66.52	43.99\45.88\45.32\45.52	36.35\42.32\42.02\41.83	55.24\54.14\53.08\53.96	0.32\0.26\0.26\0.26	0.5\0.53\0.53\0.53
	dinknet	35.33\42.7\42.4\42.67	14.46\26.75\26.51\26.92	10.65\19.01\18.61\19	11.3\30.6\30.01\30.12	0.08\0.39\0.38\0.37	0.13\0.29\0.29\0.29
	dmon	46.14\56.99\58.16\57.9	6.62\35.41\35.8\36.2	2.81\22.96\22.28\22.96	10.65\37.79\37.39\40.21	0.02\0.22\0.22\0.21	0.07\0.43\0.43\0.44
	mincut	61.16\61.52\60.5\60.61	39.89\40.94\39.78\40.05	30.13\29.34\29\30.1	49.44\50.6\50.34\50.5	0.31\0.31\0.33\0.32	0.52\0.51\0.5\0.5
wikics	diffpool	37.94\51.74\52.18\51.31	7.73\27.88\28.34\27.54	2.53\18.42\18.2\17.98	11.86\33.67\33.88\32.75	0.06\0.15\0.15\0.15	0.16\0.51\0.5\0.5
	dinknet	64.08\63.4\63.56\61.78	34.45\34.74\35.16\33.38	30.4\31.31\31.42\29	55.48\54.43\55.27\54.02	0.24\0.23\0.23\0.25	0.56\0.58\0.58\0.57
	dmon	35.33\42.7\42.4\42.67	14.46\26.75\26.51\26.92	10.65\19.01\18.61\19	11.3\30.6\30.01\30.12	0.08\0.39\0.38\0.37	0.13\0.29\0.29\0.29
	mincut	46.4\46.92\44.46\45.76	22.06\18.61\20.11\18.19	11.57\5.6\6.79\9.08	28.09\22.16\22.67\22.37	0.27\0.24\0.28\0.21	0.22\0.16\0.2\0.16

Table 6: **Ablation on the Labeling Budget.** We report performance when the LLM labeling budget is $20\% \ 40\% \ 80\% \ 100\%$. We find that increasing the budget does not substantially increase performance, unlike traditional active learning. We hypothesize this is partially due to regularizing training using GNN pseudo-labels and the imperfect LLM oracle.

Dataset	Method	Acc.	NMI	F1	ARI	COND	MOD
	diffpool	54.43\53.82\52.77\53.05\54.66	23.52\22.7\22.59\22.76\22.21	15.33\15.3\15.2\15.54\15.23	35.44\36.46\36.72\36.88\36.52	0.3\0.3\0.31\0.32\0.32	0.45\0.45\0.45\0.44\0.44
citeseer	dinknet	69.81\69.84\69.81\69.81\69.81	34.15\36.98\36.61\36.38\36.19	26.36\29.83\29.23\28.97\28.82	45.51\48.06\47.35\47.13\46.93	0.07\0.07\0.07\0.07\0.07	0.6\0.62\0.61\0.61\0.61
Citeseei	dmon	51.81\51.63\51.62\51.3\51.25	28.17\29.15\28.99\29.19\29.1	18\18.82\18.37\18.65\18.56	31.84\32.72\32.66\32.53\32.45	0.13\0.14\0.15\0.15\0.15	0.5\0.5\0.5\0.5\0.5\0.5
	mincut	63.57\61.68\63.12\63.98\64.14	34.44\32.88\33.4\32.94\32.7	29.95\27.77\28.04\27.41\27.23	55.75\54.3\54.29\53.44\52.27	0.26\0.31\0.3\0.31\0.3	0.55\0.51\0.51\0.51\0.51
	diffpool	55.55\55.44\55.61\56.67\56.53	24.17\24.81\24.9\24.97\25.18	9.91\10.06\10.35\10.9\11.02	31.91\32.92\33.57\34.3\34.25	0.41\0.43\0.44\0.44\0.44	0.34\0.33\0.32\0.33\0.34
	dinknet	59.97\60.01\60.01\59.97\60.04	24.84\25.72\25.36\25.53\25.23	10.19\10.89\10.43\10.43\10	30.36\30.74\30.58\30.79\30.47	0.09\0.09\0.09\0.09\0.09	0.32\0.33\0.32\0.32\0.31
cora	dmon	58.14\58.89\59\58.91\58.91	35.71\36.31\36.95\37.39\37.5	22.31\21.97\21.82\21.85\22.25	38.78\37.71\37.93\39.24\39.44	0.21\0.22\0.21\0.2\0.19	0.43\0.43\0.44\0.44\0.45
	mincut	60.43\62.17\59.4\60.54\60.76	38.36\37.51\38.47\38.18\38.84	28.27\27.65\28.51\28.37\29.36	48.36\47.79\48.61\47.87\47.84	0.36\0.37\0.38\0.37\0.38	0.47\0.46\0.45\0.46\0.46
	diffpool	49.75\51.71\52.18\51.31\52.5	30.03\30.54\30.14\31.23\30.73	20.62\22.71\19.96\19.74\20.97	40.33\40.68\37.45\38.23\39.88	0.43\0.39\0.39\0.39\0.41	0.39\0.42\0.42\0.38\0.4
subtagwikics	dinknet	66.56\66.54\66.54\66.51\66.52	46.13\45.77\45.73\45.68\45.62	42.49\42.43\42.31\42.14\42.05	54.59\54.27\54.16\54.14\54.21	0.26\0.25\0.26\0.26\0.26	0.53\0.53\0.53\0.53\0.53
subtagwikics	dmon	42.99\42.83\42.9\43.05\43.13	27.31\27.7\27.83\28.09\28.32	19.24\19.35\19.57\19.73\19.97	30.42\30.68\30.76\31.06\31.12	0.37\0.37\0.37\0.36\0.36	0.29\0.29\0.3\0.3\0.3
	mincut	44.91\44.01\44.53\43.05\44.98	18.36\17.12\20.1\19.83\18.77	5.11\5.97\9.44\9.48\6.2	21.5\18.35\25.49\18.43\19.59	0.28\0.22\0.3\0.3\0.26	0.16\0.14\0.15\0.17\0.19

Table 7: **Effect of Confidence Filtering.** While we do not know the reliability of either the LLM or GNN's feedback apriori, we can use their confidence to select samples where the feedback is more likely to be reliable to avoid finetuning on misleading samples. Here, we filter samples based on the ascending confidence percentile, so the 80th percentile corresponds to samples whose confidence is greater than or equal to 80% of total samples. We observe that filtering improves performance without filtering (11/24 Acc.) and over the starting (no finetuning) solution (17/24 Acc.). In particular, 80% LLM and 20% GNN filtering improves performance over no filtering (8/12 NMI, 10/12 Mod.) On WikiCS, no filtering performs the best, suggestive of the LLM's better reliability. Best performance is **bolded** and accuracy of the starting solution is in parentheses.

Dataset	Method	LLM	GNN	Acc.	NMI	F1	ARI	COND	MOD
	diffpool	20	80	53.04	22.67	15.06	34.93	0.31	0.45
	(47.09)	80	20	56.71	26.94	23.18	41.90	0.21	0.56
	(47.09)	0	0	58.96	26.84	19.70	41.41	0.24	0.50
	dinknet	20	80	67.61	38.14	32.03	50.99	0.08	0.64
	(66.35)	80	20	67.43	40.23	37.88	56.47	0.10	0.67
citeseer	(00.55)	0	0	67.40	36.97	27.16	47.91	0.09	0.62
	dmon	20	80	51.21	26.85	18.27	31.64	0.15	0.50
	(47.89)	80	20	51.14	30.06	25.30	41.72	0.17	0.59
	(47.65)	0	0	49.87	27.12	14.46	29.87	0.15	0.47
	mincut	20	80	61.42	31.79	26.94	47.84	0.26	0.56
	(64.17)	80	20	65.40	41.32	38.01	59.37	0.13	0.69
	(04.17)	0	0	67.51	39.60	35.81	59.81	0.17	0.64
	diffpool	20	80	55.28	29.53	16.07	39.33	0.39	0.39
		80	20	61.94	41.64	36.77	55.67	0.27	0.57
	(59.97)	0	0	51.35	22.21	6.49	29.05	0.32	0.34
	dinknet	20	80	67.15	36.21	24.09	42.83	0.13	0.50
	(66.20)	80	20	67.87	48.03	36.82	52.04	0.12	0.66
cora		0	0	65.16	23.42	9.25	27.40	0.08	0.29
	dmon	20	80	58.07	36.72	24.99	40.19	0.23	0.47
	(57.55)	80	20	62.06	41.79	35.56	50.52	0.25	0.57
	(37.33)	0	0	56.70	30.06	13.67	29.40	0.12	0.33
	mincut	20	80	61.04	38.40	28.22	48.95	0.34	0.50
	(64.17)	80	20	64.55	47.15	38.89	57.82	0.19	0.65
	(04.17)	0	0	61.62	41.61	30.54	54.01	0.28	0.54
	diffpool	20	80	51.53	27.52	17.87	37.83	0.41	0.39
	(43.34)	80	20	50.60	24.03	16.68	34.87	0.40	0.42
	(43.34)	0	0	58.03	35.03	26.28	46.48	0.34	0.44
	dinknet	20	80	66.51	45.90	41.76	54.10	0.26	0.53
	(71.25)	80	20	66.79	48.39	41.85	55.66	0.23	0.54
wikics	(71.23)	0	0	74.00	51.25	51.57	63.06	0.23	0.54
	dmon	20	80	42.81	27.14	19.03	30.33	0.37	0.29
	(37.515)	80	20	40.92	28.11	20.24	32.39	0.46	0.33
	(37.313)	0	0	51.87	32.51	31.04	36.49	0.26	0.31
	mincut	20	80	42.79	19.16	7.50	17.07	0.27	0.23
	(24.70)	80	20	43.58	14.74	3.77	19.57	0.30	0.14
	(24.70)	0	0	46.36	16.52	4.80	24.36	0.47	0.27

D Additional Results with ChatGPT

In this section, we evaluate GCLR using feedback obtained from ChatGPT-3.5-Turbo, instead of Mixtral-8b, to demonstrate its robustness to choice of LLM. We note that obtaining feedback from ChatGPT is fairly expensive for us, so we only obtain feedback on 200 nodes. We select the 200 most difficult nodes for feedback, where difficult is defined according to the entropy of the distance to a sample's two nearest clusters. Here, a sample that is equidistant and relatively from the cluster centers would is more difficult and is selected over a sample that is close to a single center (well-clustered). Given the strong performance of GCLR with even this limited number of samples from a very powerful LLM, suggests that performance would be further improved with a larger budget. We note that due to the limited number of feedback samples, we perform a single round of fine-tuning to prevent over-fitting to feedback samples, instead of dividing the feedback over multiple rounds. Finally, please note that we had to retrain the base GNNs for these experiments, so the starting accuracy of the original GNNs may be slightly different that those reported in the main paper. All results are reported using the "concepts" feedback strategy unless otherwise noted. We strongly emphasize, however, that we are interested in observing the improvement of GCLR relative to the starting model, and we clearly observe its benefits in the following tables.

Table 8: **Feedback Elicitation.** We evaluate three different strategies for obtaining LLM guidance by measuring their accuracy in predicting the correct cluster assignment (wrt to known ground-truth label) on the 200 *hardest* samples as per the initial GNN clustering. The GNN's accuracy on **ALL** samples is reported in parenthesis. We observe that the Concepts strategy achieves the best performance on 10/12 datasets.

Dataset	Clustering Method	Concepts	InContext	Triplets
	diffpool (0.496)	0.295	0.24	0.26
citeseer	dinknet (0.703)	0.385	0.385	0.38
	dmon (0.441)	0.415	0.34	0.35
	mincut (0.665)	0.415	0.33	0.37
	diffpool (0.547)	0.14	0.29	0.29
ooro	dinknet (0.658)	0.355	0.235	0.295
cora	dmon (0.609)	0.355	0.15	0.31
	mincut (0.684)	0.54	0.25	0.235
	diffpool (0.483)	0.365	0.290	0.235
WikiCS	dinknet (0.665)	0.24	0.240	0.330
WIKICS	dmon (0.370)	0.335	0.235	0.27
	mincut (0.269)	0.08	0.01	0.015

Table 9: **ChatGPT Provides Complementary Information When Finetuning** In order to demonstrate ChatGPT provided labels capture complementary, *beneficial* information to the GNN, here, we compare performance of models that were *only* fine-tuned with GNN pseudo-labels and those that were fine-tuned with *GNN and LLM pseudo-labels*. Notably, we do *not* filter the LLM's nor the GNN labels for high confidence; allowing the mistakes from either source. The better result is underlined between (GNN Only / LLM+GNN). We observe that incorporating the raw LLM feedback improves the clustering solution noticeably on the extrinsic metrics (7/12 Acc), (10/12 NMI), (9/12 F1) but has mixed, but competitive performance on extrinsic metrics.

Method	Dataset	Acc	NMI	ARI	F1	Cond	Mod
DiffPool	Citeseer	54.110 / 55.740	33.710 / 36.240	27.430 / 30.920	45.290 / 49.100	0.146 / 0.164	0.633 / 0.630
DinkNet	Citeseer	$69.520 / \overline{69.718}$	$45.200 / \overline{45.733}$	$44.370 / \overline{45.343}$	$65.330 / \overline{65.570}$	$\overline{0.068}$ / 0.065	$\overline{0.701}$ / 0.706
Dmon	Citeseer	46.400 / 49.030	$29.585 / \overline{30.550}$	$24.295 / \overline{26.670}$	43.395 / 44.230	$0.210 / \overline{0.199}$	$0.582 / \overline{0.573}$
MinCut	Citeseer	67.360 / <u>67.950</u>	$46.520 / \overline{46.960}$	$44.820 / \overline{46.000}$	65.160 / <u>65.420</u>	$0.081 / \overline{0.078}$	<u>0.726</u> / 0.720
DiffPool	Cora	60.160 / 59.270	45.790 / 39.550	40.320 / 29.880	52.350 / 51.340	0.200 / 0.211	0.610 / 0.511
DinkNet	Cora	60.860 / 64.700	47.930 / 50.420	33.520 / 36.440	50.530 / 55.940	$\overline{0.124}$ / 0.110	$\overline{0.620}$ / 0.642
Dmon	Cora	$62.080 / \overline{61.410}$	$42.345 / \overline{42.615}$	$35.055 / \overline{33.995}$	$54.220 / \overline{53.885}$	$0.241 / \overline{0.253}$	$0.581 / \overline{0.574}$
MinCut	Cora	68.650 / <u>71.530</u>	$52.270 / \overline{53.830}$	47.050 / <u>49.950</u>	63.740 / <u>64.960</u>	$\overline{0.146} / 0.152$	<u>0.705</u> / 0.691
DinkNet	WikiCS	63.510 / 62.770	49.680 / 49.230	44.050 / 43.730	59.130 / 58.520	0.243 / 0.245	0.536 / 0.540
DiffPool	WikiCS	52.390 / 52.070	37.500 / 39.500	27.520 / 28.820	48.230 / 46.820	$\overline{0.304}$ / 0.294	$0.504 / \overline{0.513}$
Dmon	WikiCS	38.420 / 38.390	$30.420 / \overline{30.910}$	$23.140 / \overline{23.400}$	33.380 / 33.390	$0.444 / \overline{0.438}$	$0.358 / \overline{0.367}$
MinCut	WikiCS	30.430 / 34.370	$17.040 / \overline{21.750}$	$0.900 / \overline{4.490}$	$12.810 / \overline{16.160}$	0.054 / 0.073	$0.118 / \overline{0.134}$

Table 10: GCLR with ChatGPT Improves the Performance of Graph Clustering Solutions. Here, we consider GCLR's performance across different confidence filtering levels (for both the GNN and LLM), and compare its performance when using the triplet loss (instead of cross entropy). In particular, we consider two different confidence percentiles, 20% and 80%, denoted low and high below respectively. Aside from DinkNet, which uses a contrastive loss during training, we find that GCLR with cross-entropy and confidence filtering improves the performance over the starting GNN solution. The performance of starting GNN solution is denoted in parenthesis. Second Best, First. (Cross Entropy /Triplets).

Citeseer DiffPool (49.6) low low 53.360 / 49.560 34.460 / 38.730 30.940 / 26.320 46.430 / 44.770 0.157 / 1.09 0.619 / 10.606 Citeseer DiffPool liph low 51.570 / 49.690 36.200 / 28.920 27.670 / 23.830 45.110 / 44.160 0.160 / 0.232 0.618 / 0.650 0.639 / 0.606 Citeseer DiffPool liph low 69.400 / 71.030 45.400 / 46.370 44.600 / 49.500 65.130 / 65.640 0.072 / 0.066 0.693 / 0.608 Citeseer DiffNot low liph low 69.240 / 71.030 44.650 / 46.320 44.130 / 49.430 64.680 / 65.610 0.072 / 0.066 0.694 0.655 / 0.717 Citeseer DiffNot low low 48.780 / 71.030 44.650 / 46.320 44.130 / 49.430 64.680 / 65.610 0.072 / 0.067 0.696 / 0.720 0.	Dataset	Method	LLM Conf.	GNN Conf.	Acc	NMI	ARI	F1	Cond	Mod
Citeseer DinkNet (70.3) Low Low 69.400 / 71.030 45.440 / 46.370 44.600 / 49.500 65.370 / 44.830 0.153 / 0.200 0.639 / 0.660 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 / 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 / 0.720 0.066 / 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 / 0.720 0.066 / 0.072 / 0.066 0.072 / 0.066 0.072 / 0.066 / 0.720 0.066 / 0.06	Citeseer	DiffPool (49.6)	low	low	<u>53.360</u> / 49.560	<u>34.460</u> / 28.730	<u>30.040</u> / 26.320	<u>46.430</u> / 44.770	0.175 / 0.199	0.619 / 0.606
Citeseer DinkNet Din	Citeseer	DiffPool	low	high	<u>52.480</u> / 47.390	32.050 / 25.990	<u>27.670</u> / 23.830	<u>45.110</u> / 41.160	0.160 / 0.232	0.618 / 0.565
Citeseer DinkNet Low high 64,780/70.090 42,740/45.600 38,840/47.720 59,130/65.390 0.066/10.064 0.655/0.717	Citeseer	DiffPool	high	low	<u>51.570</u> / 49.690	<u>36.200</u> / 28.920	<u>29.870</u> / 26.460	43.270 / <u>44.830</u>	<u>0.153</u> / 0.200	<u>0.639</u> / 0.606
Citeseer DinkNet high low 69.240 / 71.030 44.650 / 46.320 44.130 / 49.430 64.680 / 66.610 0.072 / 0.067 0.696 / 0.720 Citeseer Dmon (44.1) low 48.780 / 45.135 30.090 / 29.680 26.050 / 27.645 45.570 / 36.660 0.219 / 0.191 0.574 / 0.547 Citeseer Dmon high low 48.590 / 45.200 30.620 / 29.690 26.110 / 27.675 45.650 / 36.735 0.208 / 0.191 0.583 / 0.546 Citeseer MinCut low high 68.880 / 71.903 47.370 / 48.660 46.950 / 48.60 65.50 / 66.170 0.075 / 0.061 0.719 / 0.729 Citeseer MinCut low high 68.080 / 71.003 47.370 / 48.660 47.260 / 48.300 65.50 / 66.130 0.075 / 0.061 0.719 / 0.729 Citeseer MinCut high low 68.080 / 71.003 46.260 / 48.600 65.50 / 66.180 0.075 / 0.061 0.719 / 0.729 Cireseer MinCut high how low 59.710 / 53.210 41.250 / 39.010 33.440 / 32.120 51.400 / 49.130		, ,				45.440 / <u>46.370</u>	44.600 / <u>49.500</u>	65.130 / <u>66.640</u>		0.696 / <u>0.721</u>
Citeseer Dmon (44.1) low low 48.780/45.135 30.090/29.680 26.050/27.645 45.570/36.660 0.219/0.191 0.574/0.547				_						
Citeseer Dmon Iow Inigh So.410/46.030 32.230/30.945 27.880/29.435 44.750/37.540 0.1937_0.167 0.577_0.571 Citeseer Dmon Inigh Iow 48.590/45.200 30.620/29.690 26.110/27.675 45.650/36.735 0.208/0.191 0.583/0.546 Citeseer MinCut Iow Inigh 68.490/70.330 47.370/47.650 46.950/48.360 65.620/66.170 0.075/0.061 0.719/0.740 Citeseer MinCut Iow Inigh 68.680/71.030 47.570/47.650 46.950/48.360 65.570/67.350 0.072/0.065 0.717/0.729 Citeseer MinCut Inigh Iow 68.080/70.030 46.950/47.690 46.260/3.660 65.570/67.350 0.072/0.065 0.717/0.729 Cora DiffPool Iow Inigh Iow 61.630/53.360 42.590/3.9110 40.430 30.640/3.3220 49.860/49.720 0.223/0.245 0.506/0.587 Cora DiffPool Inigh Iow 61.630/53.360 42.590/3.9110 40.430/3.23.40 33.510/49.220 0.223/0.245 0.506/0.587 Cora DiffNot Iow Inigh Iow 63.070/65.030 47.270/49.510 36.480/42.510 53.760/54.900 0.127/0.118 0.639/0.680 Cora DiffNot Inigh Iow 61.630/55.030 47.270/49.510 36.480/42.510 53.760/54.900 0.127/0.118 0.639/0.680 Cora DiffNot Inigh Iow 61.630/55.650 42.910/3.6625 33.880/2.6955 54.080/47.360 0.144/0.119 0.639/0.680 Cora Dimon Iow Inigh 44.500/56.280 32.140/3.8110 15.390/27.680 33.800/48.590 0.201/0.282 0.425/0.526 Cora Dmon Inigh Iow 62.520/55.559 42.380/36.620 33.880/2.6895 54.860/47.365 0.233/0.295 0.552/0.514 Cora Dmon Inigh Iow 54.330/52.100 40.280/31.260 32.90/2.4800 48.500 47.305 0.233/0.295 0.592/0.514 Cora MinCut Inigh Iow 54.240/51.930 53.750/54.510 50.500/4.8790 0.674/0.0230 0.552/0.526 0.552/0.526 0.580/4.8790 0.580/4.7365 0.233/0.295 0.592/0.704 WikiCS DiffPool Inigh Iow 54.240/51.930 37.510/3.100 32.900/2.7300 48.900 48.500 0.233/0.2230 0.546/0.530 0.552/0.5490 0.552/0.530 0.552/0.5490 0.552/0.530 0.552/0.5490 0.552/0.530 0.552	Citeseer	DinkNet	high	low	69.240 / <u>71.030</u>	44.650 / <u>46.320</u>	44.130 / 49.430	64.680 / <u>66.610</u>	0.072 / <u>0.067</u>	0.696 / <u>0.720</u>
Citeseer Dmon high low 48.590 / 45.200 30.620 / 29.690 26.110 / 27.675 45.650 / 36.735 0.208 / 0.191 0.583 / 0.546 Citeseer MinCut (66.50) low high 68.680 / 71.940 47.370 / 47.650 46.950 / 48.360 65.620 / 66.170 0.075 / 0.061 0.719 / 0.740 Citeseer MinCut high low 68.680 / 71.940 47.270 / 48.860 47.260 / 50.600 65.570 / 66.180 0.072 / 0.061 0.719 / 0.729 Cora DiffPool (54.7) low low 59.710 / 53.210 41.250 / 39.010 33.440 / 32.120 51.400 / 49.130 0.213 / 0.268 0.542 / 0.571 Cora DiffPool high low low 61.630 / 53.360 42.590 / 39.110 40.430 / 32.210 49.860 / 49.720 0.223 / 0.245 0.506 / 0.587 Cora DiffPool high low 61.630 / 53.360 42.590 / 39.110 40.430 / 32.210 49.280 / 49.220 0.226 / 0.270 0.571 / 0.568 Cora DinkNet (65.8) low high 61.630 / 53.60 47.270 / 49.510 <t< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>										
Citeseer MinCut (66.50) low low 68.490 / 70.030				high						
Citeseer MinCut low high 68.680 / 71.940 47.570 / 48.860 47.260 / 50.600 65.570 / 67.350 0.072 / 0.065 0.717 / 0.729 / 0.740 Citeseer MinCut high low 68.080 / 70.030 46.950 / 47.690 46.260 / 48.400 65.570 / 67.350 0.072 / 0.061 0.729 / 0.740 Cora DiffPool DiffPool Inigh low big 59.310 / 54.470 39.610 / 40.450 33.400 / 33.220 49.860 / 49.720 0.223 / 0.245 0.506 / 0.587 0.506 / 0.587 0.506 / 0.587 0	Citeseer	Dmon	high	low	<u>48.590</u> / 45.200	<u>30.620</u> / 29.690	26.110 / <u>27.675</u>	<u>45.650</u> / 36.735	0.208 / <u>0.191</u>	<u>0.583</u> / 0.546
Citeseer MinCut high low 68.080/70.030 46.950/47.690 46.260/48.400 65.540/6.180 0.072/0.061 0.729/0.740 Cora DiffPool (54.7) low low 59.710/53.210 41.250/39.010 33.440/32.120 51.400/49.130 0.213/0.268 0.542/0.571 Cora DiffPool high low 61.630/53.360 42.590/39.110 40.430/33.220 49.860/49.720 0.223/0.245 0.506/0.587 Cora DinkNet (65.8) low low 63.770/65.030 47.270/49.510 36.480/42.510 53.760/54.900 0.127/0.118 0.639/0.680 Cora DinkNet low high 63.070/64.550 45.450/50.640 35.810/42.200 53.600/54.490 0.151/0.110 0.639/0.687 Cora DinkNet high low 61.630/65.140 43.900/49.620 34.460/42.620 48.200/55.050 0.144/0.119 0.629/0.680 Cora Dmon low high 44.500/56.280 32.140/38.110 15.390/27.680 54.080/47.365 0.223/0.225 0.552/0.526	Citeseer	MinCut (66.50)	low	low	68.490 / <u>70.030</u>	47.370 / <u>47.650</u>	46.950 / <u>48.360</u>	65.620 / <u>66.170</u>	0.075 / <u>0.061</u>	0.719 / <u>0.740</u>
Cora DiffPool (54.7) low low 59.710 / 53.210 41.250 / 39.010 33.440 / 32.120 51.400 / 49.130 0.213 / 0.268 0.542 / 0.571 Cora DiffPool low high 59.310 / 54.470 39.610 / 40.450 30.640 / 33.220 49.860 / 49.720 0.223 / 0.245 0.506 / 0.587 Cora DiffPool high low 61.630 / 53.360 42.590 / 39.110 40.430 / 32.340 53.510 / 49.220 0.206 / 0.270 0.571 / 0.568 Cora DiffPool high low 61.630 / 53.360 42.590 / 39.110 40.430 / 32.340 53.510 / 49.220 0.206 / 0.270 0.571 / 0.568 Cora DiffPool high low high 63.070 / 64.550 45.450 / 50.640 35.810 / 42.200 53.600 / 54.490 0.127 / 0.118 0.639 / 0.687 Cora DiffNet high low 61.630 / 65.140 43.900 / 49.200 34.460 / 42.200 53.600 / 54.490 0.127 / 0.110 0.639 / 0.687 Cora Dimon (60.9) low low 61.340 / 55.650 42.910 / 36.625 33.880 / 26.955 54.080 / 47.400 0.244 / 0.296 0.585 / 0.514 Cora Dmon low high 44.500 / 56.280 32.140 / 38.110 15.390 / 27.680 33.090 / 48.590 0.201 / 0.282 0.425 / 0.526 Cora Dmon high low 62.520 / 55.595 42.380 / 36.620 35.390 / 26.895 54.860 / 47.365 0.233 / 0.295 0.592 / 0.514 Cora MinCut (68.4) low low 71.680 / 71.230 53.920 / 54.960 50.750 / 48.640 65.570 / 62.940 0.150 / 0.128 0.695 / 0.704 Cora MinCut low high 72.050 / 71.900 54.070 / 54.510 51.270 / 49.910 66.740 / 63.380 0.154 / 0.128 0.699 / 0.704 Cora MinCut high low 71.530 / 71.310 53.750 / 55.100 50.890 / 48.790 65.140 / 63.020 0.153 / 0.127 0.691 / 0.704 WikiCS DiffPool low high 54.240 / 51.960 40.350 / 29.060 32.290 / 26.800 47.040 / 45.330 0.229 / 0.301 0.520 / 0.490 WikiCS DiffPool high low 52.680 / 51.900 48.900 / 48.780 43.890 / 48.890 / 45.590 0.244 / 0.294 0.251 0.533 0.544 / 0.548 WikiCS DinkNet low high 62.760 / 65.800 48.900 / 48.780 43.890 / 48.800 / 45.590 0.244 / 0.243 0.244 0.545 / 0.552 WikiCS DinkNet high low 61.870 / 65.800 48.900 / 48.780 43.890 / 48.800 / 45.590 0.244 / 0.245 0.233 0.544 / 0.548 WikiCS DinkNet high low 61.870 / 65.800 48.900 / 48.780 43.890 / 48.780 43.890 / 48.590 0.245 / 0.250 0.243 / 0.243 0.244 0.545 / 0.552 WikiCS DinkNet high low 61.870 / 65.800 48.900 / 48.780	Citeseer	MinCut	low	high	68.680 / <u>71.940</u>	47.570 / <u>48.860</u>	47.260 / <u>50.600</u>	65.570 / <u>67.350</u>	0.072 / <u>0.065</u>	0.717 / <u>0.729</u>
Cora DiffPool low high 59.310 / 54.470 39.610 / 40.450 30.640 / 33.220 49.860 / 49.720 0.223 / 0.245 0.506 / 0.587 Cora DiffPool high low 61.630 / 53.360 42.590 / 39.110 40.430 / 32.340 53.510 / 49.220 0.206 / 0.270 0.571 / 0.568 Cora DinkNet (65.8) low high 63.770 / 65.030 47.270 / 49.510 36.480 / 42.510 53.600 / 54.490 0.127 / 0.118 0.639 / 0.680 Cora DinkNet high low 63.070 / 64.550 45.450 / 50.640 35.810 / 42.200 53.600 / 54.490 0.151 / 0.110 0.639 / 0.680 Cora Dimn (60.9) low low 61.340 / 55.650 42.910 / 36.625 33.880 / 26.955 54.080 / 47.400 0.244 / 0.296 0.585 / 0.514 Cora Dmon low high 44.500 / 56.280 32.140 / 38.110 15.390 / 27.680 33.090 / 48.590 0.201 / 0.282 0.425 / 0.526 Cora MinCut (68.4) low low 71.680 / 71.230 53.920 / 54.960 50.750 / 48.640 65.570 / 62.940 <td>Citeseer</td> <td>MinCut</td> <td>high</td> <td>low</td> <td>68.080 / <u>70.030</u></td> <td>46.950 / <u>47.690</u></td> <td>46.260 / <u>48.400</u></td> <td>65.540 / <u>66.180</u></td> <td>0.072 / <u>0.061</u></td> <td>0.729 / <u>0.740</u></td>	Citeseer	MinCut	high	low	68.080 / <u>70.030</u>	46.950 / <u>47.690</u>	46.260 / <u>48.400</u>	65.540 / <u>66.180</u>	0.072 / <u>0.061</u>	0.729 / <u>0.740</u>
Cora DiffPool high low 61.630 / 53.360 42.590 / 39.110 40.430 / 32.340 53.510 / 49.220 0.206 / 0.270 0.571 / 0.568 Cora DinkNet (65.8) low low 63.770 / 65.030 47.270 / 49.510 36.480 / 42.510 53.760 / 54.900 0.127 / 0.118 0.639 / 0.680 Cora DinkNet low high 63.070 / 64.550 45.450 / 50.640 35.810 / 42.200 53.600 / 54.490 0.151 / 0.110 0.639 / 0.687 Cora DinkNet high low 61.630 / 65.140 43.900 / 49.620 35.810 / 42.200 53.600 / 54.490 0.151 / 0.110 0.639 / 0.687 Cora Dmon (60.9) low low 61.630 / 55.650 42.910 / 36.625 33.880 / 26.955 54.080 / 47.400 0.244 / 0.296 0.585 / 0.514 Cora Dmon high low 62.520 / 55.595 42.380 / 36.620 35.390 / 26.895 54.860 / 47.365 0.231 / 0.296 0.255 / 0.526 Cora MinCut (68.4) low low high 71.680 / 71.230 53.920 / 54.960 50.750 / 48.640	Cora	DiffPool (54.7)	low	low	59.710 / 53.210	41.250 / 39.010	33.440 / 32.120	51.400 / 49.130	0.213 / 0.268	0.542 / 0.571
Cora DinkNet (65.8) low low 63.770 / 65.030 47.270 / 49.510 36.480 / 42.510 53.760 / 54.900 0.127 / 0.118 0.639 / 0.680 Cora DinkNet low high 63.070 / 64.550 45.450 / 50.640 35.810 / 42.200 53.600 / 54.490 0.151 / 0.110 0.639 / 0.687 Cora DinkNet high low 61.630 / 65.140 43.900 / 49.620 34.460 / 42.620 48.200 / 55.050 0.144 / 0.119 0.629 / 0.680 Cora Dmon (60.9) low low 61.340 / 55.650 42.910 / 36.625 33.880 / 26.955 54.080 / 47.400 0.244 / 0.296 0.585 / 0.514 Cora Dmon low high 44.500 / 56.280 32.140 / 38.110 15.390 / 27.680 33.090 / 48.590 0.201 / 0.282 0.425 / 0.526 Cora Dmon high low 62.520 / 55.595 42.380 / 36.620 35.390 / 24.860 55.70 / 62.895 54.860 / 47.365 0.233 / 0.295 0.592 / 0.514 Cora MinCut (68.4) low low 71.680 / 71.230 53.920 / 54.960 50.755 / 48.604	Cora	DiffPool	low	high	59.310 / 54.470	39.610 / 40.450	30.640 / 33.220	49.860 / 49.720	0.223 / 0.245	0.506 / 0.587
Cora DinkNet low high 63.070 / 64.550 45.450 / 50.640 35.810 / 42.200 53.600 / 54.490 0.151 / 0.110 0.639 / 0.687 Cora DinkNet high low 61.630 / 65.140 43.900 / 49.620 34.460 / 42.620 48.200 / 55.050 0.144 / 0.119 0.629 / 0.680 Cora Dmon (60.9) low high 44.500 / 56.280 32.140 / 38.110 15.390 / 27.680 33.090 / 48.590 0.201 / 0.282 0.585 / 0.514 Cora Dmon high 160 / 62.520 / 55.595 42.380 / 36.620 35.390 / 26.895 54.860 / 47.365 0.233 / 0.295 0.585 / 0.514 Cora Dmon high low 62.520 / 55.595 42.380 / 36.620 35.390 / 26.895 54.860 / 47.365 0.233 / 0.295 0.592 / 0.514 Cora MinCut low low 71.680 / 71.230 53.920 / 54.960 50.750 / 48.640 65.570 / 62.940 0.150 / 0.128 0.692 / 0.702 Cora MinCut high low 71.530 / 71.310 53.750 / 55.100 50.890 / 48.790 65.140 / 63.380 0.1	Cora	DiffPool	high	low	<u>61.630</u> / 53.360	<u>42.590</u> / 39.110	<u>40.430</u> / 32.340	<u>53.510</u> / 49.220	<u>0.206</u> / 0.270	<u>0.571</u> / 0.568
Cora DinkNet high low 61.630 / 65.140 43.900 / 49.620 34.460 / 42.620 48.200 / 55.050 0.144 / 0.119 0.629 / 0.680 Cora Dmon (60.9) low high 44.500 / 56.280 32.140 / 38.110 15.390 / 27.680 33.090 / 48.590 0.201 / 0.282 0.425 / 0.526 Cora Dmon high low 62.520 / 55.595 42.380 / 36.620 35.390 / 26.895 54.860 / 47.365 0.233 / 0.295 0.592 / 0.514 Cora MinCut (68.4) low low 71.680 / 71.230 53.920 / 54.960 50.750 / 48.640 65.570 / 62.940 0.150 / 0.128 0.695 / 0.704 Cora MinCut low high 72.050 / 71.900 54.070 / 54.510 51.270 / 49.910 66.740 / 63.380 0.154 / 0.128 0.692 / 0.702 Cora MinCut high low 71.530 / 71.310 53.750 / 55.100 50.890 / 48.790 65.140 / 63.380 0.154 / 0.128 0.692 / 0.702 WikiCS DiffPool (48.3) low low 54.330 / 52.100 40.280 / 31.260 31.920 / 27.480 48.	Cora	DinkNet (65.8)	low	low	63.770 / <u>65.030</u>	47.270 / 49.510	36.480 / 42.510	53.760 / 54.900	0.127 / 0.118	0.639 / 0.680
Cora Dmon (60.9) low low 61.340 / 55.650 42.910 / 36.625 33.880 / 26.955 54.080 / 47.400 0.244 / 0.296 0.585 / 0.514 Cora Dmon low high 44.500 / 56.280 32.140 / 38.110 15.390 / 27.680 33.090 / 48.590 0.201 / 0.282 0.425 / 0.526 Cora Dmon high low 62.520 / 55.595 42.380 / 36.620 35.390 / 26.895 54.860 / 47.365 0.233 / 0.295 0.592 / 0.514 Cora MinCut (68.4) low low 10w 71.680 / 71.230 53.920 / 54.960 50.750 / 48.640 65.570 / 62.940 0.150 / 0.128 0.695 / 0.704 Cora MinCut low high 72.050 / 71.900 54.070 / 54.510 51.270 / 49.910 66.740 / 63.380 0.154 / 0.128 0.695 / 0.702 Cora MinCut high low 71.530 / 71.310 53.750 / 55.100 50.890 / 48.790 65.140 / 63.300 0.153 / 0.127 0.691 / 0.702 WikiCS DiffPool low high 54.240 / 51.960 40.350 / 29.060 32.290 / 26.800 47.040 /	Cora	DinkNet	low	high	63.070 / <u>64.550</u>	45.450 / <u>50.640</u>	35.810 / 42.200	53.600 / 54.490	0.151 / 0.110	0.639 / 0.687
Cora Dmon low high 44.500 / 56.280 32.140 / 38.110 15.390 / 27.680 33.090 / 48.590 0.201 / 0.282 0.425 / 0.526 Cora Dmon high low 62.520 / 55.595 42.380 / 36.620 35.390 / 26.895 54.860 / 47.365 0.233 / 0.295 0.592 / 0.514 Cora MinCut (68.4) low high 72.050 / 71.230 53.920 / 54.960 50.750 / 48.640 65.570 / 62.940 0.150 / 0.128 0.695 / 0.704 Cora MinCut low high 72.050 / 71.900 54.070 / 54.510 51.270 / 49.910 66.740 / 63.380 0.154 / 0.128 0.692 / 0.702 Cora MinCut high low 71.530 / 71.310 53.750 / 55.100 50.890 / 48.790 65.140 / 63.020 0.153 / 0.127 0.691 / 0.704 WikiCS DiffPool low high 54.330 / 52.100 40.280 / 31.260 31.920 / 27.480 48.220 / 45.690 0.291 / 0.294 0.515 / 0.503 WikiCS DiffPool high low 52.680 / 51.920 37.510 / 31.090 29.500 / 27.230 47.040 / 45.	Cora	DinkNet	high	low	61.630 / <u>65.140</u>	43.900 / <u>49.620</u>	34.460 / <u>42.620</u>	48.200 / <u>55.050</u>	0.144 / <u>0.119</u>	0.629 / <u>0.680</u>
Cora Dmon high low 62.520 / 55.595 42.380 / 36.620 35.390 / 26.895 54.860 / 47.365 0.233 / 0.295 0.592 / 0.514 Cora MinCut (68.4) low low 71.680 / 71.230 53.920 / 54.960 50.750 / 48.640 65.570 / 62.940 0.150 / 0.128 0.695 / 0.704 Cora MinCut low high 72.050 / 71.900 54.070 / 54.510 51.270 / 49.910 66.740 / 63.380 0.154 / 0.128 0.692 / 0.702 Cora MinCut high low 71.530 / 71.310 53.750 / 55.100 50.890 / 48.790 65.140 / 63.020 0.153 / 0.127 0.691 / 0.704 WikiCS DiffPool (48.3) low low 54.330 / 52.100 40.280 / 31.260 31.920 / 27.480 48.220 / 45.690 0.291 / 0.294 0.515 / 0.503 WikiCS DiffPool low high 54.240 / 51.960 40.350 / 29.060 32.290 / 26.800 47.040 / 45.330 0.279 / 0.301 0.520 / 0.490 WikiCS DiffPool high low 52.680 / 51.920 37.510 / 31.090 29.500 / 27.230 4	Cora	Dmon (60.9)	low	low	61.340 / 55.650	<u>42.910</u> / 36.625	33.880 / 26.955	<u>54.080</u> / 47.400	0.244 / 0.296	0.585 / 0.514
Cora MinCut (68.4) low low 71.680 / 71.230 53.920 / 54.960 50.750 / 48.640 65.570 / 62.940 0.150 / 0.128 0.695 / 0.704 Cora MinCut low high 72.050 / 71.900 54.070 / 54.510 51.270 / 49.910 66.740 / 63.380 0.154 / 0.128 0.692 / 0.702 Cora MinCut high low 71.530 / 71.310 53.750 / 55.100 50.890 / 48.790 65.140 / 63.020 0.153 / 0.127 0.691 / 0.704 WikiCS DiffPool (48.3) low low 54.330 / 52.100 40.280 / 31.260 31.920 / 27.480 48.220 / 45.690 0.291 / 0.294 0.515 / 0.503 WikiCS DiffPool high low 52.680 / 51.920 37.510 / 31.090 29.500 / 27.230 47.030 / 45.590 0.294 / 0.293 0.506 / 0.503 WikiCS DinkNet (66.5) low low 62.470 / 65.800 48.900 / 48.780 43.380 / 44.380 58.020 / 59.610 0.243 / 0.233 0.546 / 0.548 WikiCS DinkNet high low 61.870 / 65.800 48.920 / 47.640 43.400 / 42.900 58.120 / 58.090 0.243 / 0.244 0.545 / 0.552 WikiCS DinkNet high low 61.870 / 65.800 48.750 / 48.780 43.380 / 44.380 57.220 / 59.610 0.245 / 0.233 0.544 / 0.548 WikiCS Dmon (37.0) low low 38.870 / 36.970 31.080 / 29.350 23.550 / 23.530 / 23.530 / 32.240 0.381 / 0.462 0.401 / 0.342 WikiCS Dmon high low 38.350 / 36.940 30.910 / 29.350 23.530 / 23.530 / 33.20 / 33.140 0.436 / 0.463 0.368 / 0.348 WikiCS MinCut (26.90) low low 37.230 / 26.860 22.270 / 10.920 13.870 / 3.400 19.520 / 11.750 0.083 / 0.337 0.182 / 0.139	Cora	Dmon	low	high	44.500 / <u>56.280</u>	32.140 / 38.110	15.390 / <u>27.680</u>	33.090 / <u>48.590</u>	<u>0.201</u> / 0.282	0.425 / 0.526
Cora MinCut low high 72.050 / 71.900 54.070 / 54.510 51.270 / 49.910 66.740 / 63.380 0.154 / 0.128 0.692 / 0.702 Cora MinCut high low 71.530 / 71.310 53.750 / 55.100 50.890 / 48.790 65.140 / 63.020 0.153 / 0.127 0.691 / 0.704 WikiCS DiffPool (48.3) low low 54.330 / 52.100 40.280 / 31.260 31.920 / 27.480 48.220 / 45.690 0.291 / 0.294 0.515 / 0.503 WikiCS DiffPool low high 54.240 / 51.960 40.350 / 29.060 32.290 / 26.800 47.040 / 45.330 0.279 / 0.301 0.520 / 0.490 WikiCS DiffPool high low 52.680 / 51.920 37.510 / 31.090 29.500 / 27.230 47.030 / 45.590 0.294 / 0.293 0.506 / 0.503 WikiCS DinkNet (66.5) low high 62.760 / 64.890 48.900 / 48.780 43.380 / 44.380 58.020 / 59.610 0.243 / 0.244 0.546 / 0.548 WikiCS DinkNet high low 61.870 / 65.800 48.750 / 48.780 43.170 / 44.380	Cora	Dmon	high	low	<u>62.520</u> / 55.595	<u>42.380</u> / 36.620	<u>35.390</u> / 26.895	<u>54.860</u> / 47.365	0.233 / 0.295	<u>0.592</u> / 0.514
Cora MinCut high low 71.530 / 71.310 53.750 / 55.100 50.890 / 48.790 65.140 / 63.020 0.153 / 0.127 0.691 / 0.704 WikiCS DiffPool (48.3) low low 54.330 / 52.100 / 40.280 / 31.260 31.920 / 27.480 48.220 / 45.690 0.291 / 0.294 0.515 / 0.503 WikiCS DiffPool low high 54.240 / 51.960 40.350 / 29.060 32.290 / 26.800 47.040 / 45.330 0.279 / 0.301 0.520 / 0.490 WikiCS DiffPool high low 52.680 / 51.920 37.510 / 31.090 29.500 / 27.230 47.030 / 45.590 0.294 / 0.293 0.506 / 0.503 WikiCS DinkNet (66.5) low high 62.760 / 64.890 48.900 / 48.780 43.380 / 44.380 58.020 / 59.610 0.243 / 0.233 0.546 / 0.548 WikiCS DinkNet low high 62.760 / 64.890 48.920 / 47.640 43.400 / 42.900 58.120 / 58.090 0.243 / 0.244 0.545 / 0.552 WikiCS DinkNet high low 61.870 / 65.800 48.750 / 48.780 43.170 / 44.380 57.2	Cora	MinCut (68.4)	low	low	<u>71.680</u> / 71.230	53.920 / <u>54.960</u>	<u>50.750</u> / 48.640	<u>65.570</u> / 62.940	0.150 / 0.128	0.695 / <u>0.704</u>
WikiCS DiffPool (48.3) low low 54.330 / 52.100 / 40.280 / 31.260 31.920 / 27.480 48.220 / 45.690 / 20.291 / 0.294 0.515 / 0.503 WikiCS DiffPool low high 54.240 / 51.960 / 20.290 / 20.600 32.290 / 26.800 / 47.040 / 45.330 / 0.279 / 0.301 0.520 / 0.490 WikiCS DiffPool high low 52.680 / 51.920 37.510 / 31.090 / 29.500 / 27.230 47.030 / 45.590 0.294 / 0.293 / 0.506 / 0.503 WikiCS DinkNet (66.5) low low 62.470 / 65.800 / 48.900 / 48.780 43.380 / 44.380 / 42.900 / 59.610 0.243 / 0.233 / 0.244 / 0.546 / 0.548 WikiCS DinkNet low high 62.760 / 64.890 / 48.920 / 47.640 / 43.400 / 42.900 / 58.120 / 58.090 / 0.243 / 0.244 / 0.545 / 0.552 WikiCS DinkNet high low 61.870 / 65.800 / 48.780 / 48.780 / 43.170 / 44.380 / 57.220 / 59.610 / 59.610 / 0.245 / 0.233 / 0.544 / 0.548 WikiCS Dmon (37.0) low low 38.870 / 36.970 / 31.080 / 29.350 / 23.550 / 23.550 / 33.700 / 32.560 / 33.10 / 32.240 / 0.381 / 0.462 / 0.401 / 0.342 WikiCS Dmon low high 44.690 / 37.260 / 33.850 / 27.830 / 20.350 / 23.530 / 23.530 / 23.530 / 33.140 / 0.462 / 0.463 / 0.368 / 0.348 <	Cora	MinCut	low	high	<u>72.050</u> / 71.900		<u>51.270</u> / 49.910	<u>66.740</u> / 63.380	0.154 / <u>0.128</u>	0.692 / <u>0.702</u>
WikiCS DiffPool low high 54.240 / 51.960 40.350 / 29.060 32.290 / 26.800 47.040 / 45.330 0.279 / 0.301 0.520 / 0.490 WikiCS DiffPool high low 52.680 / 51.920 37.510 / 31.090 29.500 / 27.230 47.030 / 45.590 0.294 / 0.293 0.506 / 0.503 WikiCS DinkNet (66.5) low low 62.470 / 65.800 48.900 / 48.780 43.380 / 44.380 58.020 / 59.610 0.243 / 0.233 0.546 / 0.548 WikiCS DinkNet low high 62.760 / 64.890 48.920 / 47.640 43.400 / 42.900 58.120 / 58.090 0.243 / 0.244 0.545 / 0.552 WikiCS DinkNet high low 61.870 / 65.800 48.750 / 48.780 43.170 / 44.380 57.220 / 59.610 0.245 / 0.233 0.544 / 0.548 WikiCS Dmon (37.0) low low 38.870 / 36.970 31.080 / 29.350 23.600 / 23.550 33.700 / 32.560 0.430 / 0.463 0.373 / 0.347 WikiCS Dmon low high 44.690 / 37.260 33.850 / 27.830 26.210 / 23.130	Cora	MinCut	high	low	<u>71.530</u> / 71.310	53.750 / <u>55.100</u>	<u>50.890</u> / 48.790	<u>65.140</u> / 63.020	0.153 / <u>0.127</u>	0.691 / <u>0.704</u>
WikiCS DiffPool high low \$\frac{52.680}{51.920}\$ \$\frac{37.510}{31.090}\$ \$\frac{29.500}{29.500}\$ \$27.230\$ \$\frac{47.030}{47.030}\$ \$45.590\$ \$0.294 \ \ 0.293\$ \$0.506 \ \ 0.503\$ WikiCS DinkNet (66.5) low low 62.470 \ \ 65.800\$ \$48.900 \ \ 48.780\$ \$43.380 \ \ \ 44.380\$ \$43.380 \ \ \ 44.380\$ \$58.020 \ \ \ 59.610\$ \$0.243 \ \ 0.233\$ \$0.546 \ \ 0.548\$ WikiCS DinkNet low high 62.760 \ \ \ 64.890\$ \$48.920 \ \ \ 47.640\$ \$43.400 \ \ \ \ 42.900\$ \$58.120 \ \ \ 58.090\$ \$0.243 \ \ 0.244\$ \$0.545 \ \ 0.552\$ WikiCS DinkNet high low 61.870 \ \ \ 65.800\$ \$48.780\$ \$43.170 \ \ \ \ 44.380\$ \$7.220 \ \ \ 59.610\$ \$0.245 \ \ \ 0.233\$ \$0.544 \ \ 0.548 \ \ 0.548\$ WikiCS Dmon (37.0) low low 38.870 \ \ 36.970\$ \$31.080 \ \ \ 29.350\$ \$23.600 \ \ \ 23.550\$ \$33.700 \ \ \ 32.560\$ \$0.430 \ \ \ 0.462\$ \$0.401 \ \ 0.342\$ WikiCS Dmon low high 44.690 \ \ 37.260\$ \$3.850 \ \ \ \ 27.830\$ \$26.210 \ \ \ \ 23.130\$ \$33.10 \ \ \ \ \ 32.20 \ \ \ \ 33.140\$ \$0.462\$ \$0.461 \ \ 0.368 \ \ 0.348\$ WikiCS MinCut (26.90) low low 37.230 \ \ \ 26.860\$ \$23.030 \ \ \ 12.770\$ \$13.870 \ \ \ 3.870 \ \ \ 3.400\$ \$1.750\$ \$0.083 \ \ 0.337\$ \$0.337\$ \$0.182 \ \ 0.182 \ \ 0.139\$	WikiCS	DiffPool (48.3)	low	low	54.330 / 52.100	40.280 / 31.260	31.920 / 27.480	48.220 / 45.690	0.291 / 0.294	0.515 / 0.503
WikiCS DinkNet (66.5) low low 62.470 / 65.800 / 48.900 / 48.780 / 43.380 / 44.380 58.020 / 59.610 / 59.610 / 59.610 0.243 / 0.233 / 0.244 / 0.548 / 0.233 / 0.244 / 0.548 / 0.552 WikiCS DinkNet low high 62.760 / 64.890 / 48.920 / 47.640 / 43.400 / 42.900 / 58.120 / 58.090 / 0.243 / 0.244 / 0.545 / 0.552 WikiCS DinkNet high low 61.870 / 65.800 / 48.780 / 48.780 / 48.780 / 48.780 / 43.170 / 44.380 / 72.20 / 59.610 / 0.245 / 0.233 / 0.544 / 0.548 WikiCS Dmon (37.0) low high 44.690 / 37.260 / 33.850 / 27.830 / 26.210 / 23.130 / 38.310 / 32.240 / 0.381 / 0.462 / 0.401 / 0.342 WikiCS Dmon high low 38.350 / 36.940 / 30.910 / 29.350 / 23.530 / 23.530 / 23.530 / 23.530 / 33.220 / 33.140 / 0.436 / 0.463 / 0.368 / 0.348 WikiCS MinCut (26.90) low high 40.750 / 28.660 / 22.270 / 10.920 / 13.870 / 3.400 / 19.520 / 11.750 / 0.083 / 0.337 / 0.182 / 0.139	WikiCS	DiffPool	low	high	54.240 / 51.960	40.350 / 29.060	32.290 / 26.800	47.040 / 45.330	0.279 / 0.301	0.520 / 0.490
WikiCS DinkNet low high 62.760 / 64.890 do. 48.920 / 47.640 43.400 / 42.900 do. 42.900 do. 58.120 / 58.090 62.23 / 0.244 do. 545 / 0.552 do. 552 do. 552 do. 59.610 WikiCS DinkNet high low 61.870 / 65.800 do. 48.750 / 48.780 do. 48.750 / 48.780 do. 43.170 / 44.380 do. 72.20 / 59.610 do. 245 / 0.233 do. 544 / 0.548 do. 57.220 / 59.610 do. 245 / 0.233 do. 544 / 0.548 do. 57.220 / 59.610 do. 245 / 0.233 do. 544 / 0.548 do. 57.220 / 59.610 do. 245 / 0.233 do. 544 / 0.548 do. 57.220 / 59.610 do. 245 / 0.233 do. 57.220 / 59.610 do. 245 / 0.245 / 0.233 do. 57.220 / 0.347 do. 57.220 / 59.610 do. 245 / 0.233 do. 57.220 / 0.245 do. 245 / 0.234 do. 245 / 0.245 do. 245 /	WikiCS	DiffPool	high	low	<u>52.680</u> / 51.920	<u>37.510</u> / 31.090	<u>29.500</u> / 27.230	<u>47.030</u> / 45.590	0.294 / <u>0.293</u>	<u>0.506</u> / 0.503
WikiCS DinkNet high low 61.870 / 65.800 48.750 / 48.780 43.170 / 44.380 57.220 / 59.610 0.245 / 0.233 0.544 / 0.548 WikiCS Dmon (37.0) low low 38.870 / 36.970 31.080 / 29.350 23.600 / 23.550 33.700 / 32.560 0.430 / 0.463 0.373 / 0.347 WikiCS Dmon low high 44.690 / 37.260 33.850 / 27.830 26.210 / 23.130 38.310 / 32.240 0.381 / 0.462 0.401 / 0.342 WikiCS Dmon high low 38.350 / 36.940 30.910 / 29.350 23.530 / 23.530 33.220 / 33.140 0.436 / 0.463 0.368 / 0.348 WikiCS MinCut (26.90) low low 37.230 / 26.860 23.030 / 12.770 8.150 / -0.770 17.410 / 10.480 0.072 / 0.055 0.143 / 0.064 WikiCS MinCut low high 40.750 / 28.660 22.270 / 10.920 13.870 / 3.400 19.520 / 11.750 0.083 / 0.337 0.182 / 0.139	WikiCS	DinkNet (66.5)	low	low	62.470 / <u>65.800</u>	<u>48.900</u> / 48.780	43.380 / 44.380	58.020 / <u>59.610</u>	0.243 / 0.233	0.546 / <u>0.548</u>
WikiCS Dmon (37.0) low Migh 10w Migh 38.870 / 36.970 / 31.080 / 29.350 / 31.080 / 29.350 23.600 / 23.550 / 32.560 / 32.560 / 32.560 / 32.560 0.430 / 0.463 / 0.373 / 0.347 WikiCS Dmon low Migh 44.690 / 37.260 / 33.850 / 27.830 / 26.210 / 23.130 38.310 / 32.240 / 32.240 / 0.381 / 0.462 / 0.401 / 0.342 WikiCS Dmon high low 38.350 / 36.940 / 30.910 / 29.350 / 23.530 / 23.530 / 23.530 / 23.530 / 23.530 33.220 / 33.140 / 0.436 / 0.463 / 0.368 / 0.348 WikiCS MinCut (26.90) low low 37.230 / 26.860 / 23.030 / 12.770 / 0.920 / 13.870 / 3.400 / 19.520 / 11.750 / 0.083 / 0.337 / 0.182 / 0.139	WikiCS	DinkNet	low	high	62.760 / <u>64.890</u>	48.920 / 47.640	43.400 / 42.900	58.120 / 58.090	0.243 / 0.244	0.545 / <u>0.552</u>
WikiCS Dmon low high 44.690/37.260 33.850/27.830 26.210/23.130 38.310/32.240 0.381/0.462 0.401/0.342 WikiCS Dmon high 10w 38.350/36.940 30.910/29.350 23.530/23.530 33.220/33.140 0.436/0.463 0.368/0.348 WikiCS MinCut (26.90) low low 37.230/26.860 23.030/12.770 8.150/-0.770 17.410/10.480 0.072/0.055 0.143/0.064 WikiCS MinCut low high 40.750/28.660 22.270/10.920 13.870/3.400 19.520/11.750 0.083/0.337 0.182/0.139	WikiCS	DinkNet	high	low	61.870 / <u>65.800</u>	48.750 / <u>48.780</u>	43.170 / <u>44.380</u>	57.220 / <u>59.610</u>	0.245 / <u>0.233</u>	0.544 / <u>0.548</u>
WikiCS Dmon high low 38.350 / 36.940 30.910 / 29.350 23.530 / 23.530 23.530 / 23.530 33.220 / 33.140 0.436 / 0.463 0.368 / 0.348 WikiCS MinCut (26.90) low low 37.230 / 26.860 23.030 / 12.770 8.150 / -0.770 17.410 / 10.480 0.072 / 0.055 0.143 / 0.064 WikiCS MinCut low high 40.750 / 28.660 22.270 / 10.920 13.870 / 3.400 19.520 / 11.750 0.083 / 0.337 0.182 / 0.139	WikiCS	Dmon (37.0)	low	low	<u>38.870</u> / 36.970	31.080 / 29.350	23.600 / 23.550	33.700 / 32.560	0.430 / 0.463	0.373 / 0.347
WikiCS MinCut (26.90) low low 37.230 / 26.860 23.030 / 12.770 8.150 / -0.770 17.410 / 10.480 0.072 / 0.055 0.143 / 0.064 WikiCS MinCut low high 40.750 / 28.660 22.270 / 10.920 13.870 / 3.400 19.520 / 11.750 0.083 / 0.337 0.182 / 0.139	WikiCS	Dmon	low	high	<u>44.690</u> / 37.260	33.850 / 27.830	<u>26.210</u> / 23.130	38.310 / 32.240	0.381 / 0.462	0.401 / 0.342
WikiCS MinCut low high 40.750/28.660 22.270/10.920 13.870/3.400 19.520/11.750 0.083/0.337 0.182/0.139	WikiCS	Dmon	high	low	<u>38.350</u> / 36.940	<u>30.910</u> / 29.350	<u>23.530</u> / 23.530	<u>33.220</u> / 33.140	<u>0.436</u> / 0.463	<u>0.368</u> / 0.348
	WikiCS	MinCut (26.90)	low	low	<u>37.230</u> / 26.860	<u>23.030</u> / 12.770	<u>8.150</u> / -0.770	<u>17.410</u> / 10.480	0.072 / <u>0.055</u>	0.143 / 0.064
WikiCS MinCut high low <u>36.430</u> /26.620 <u>21.990</u> /13.200 <u>6.820</u> /-0.770 <u>16.090</u> /10.320 <u>0.058</u> / <u>0.058</u> <u>0.139</u> /0.069	WikiCS	MinCut	low	high	<u>40.750</u> / <u>28.660</u>	<u>22.270</u> / 10.920	13.870 / 3.400	<u>19.520</u> / 11.750	0.083 / 0.337	<u>0.182</u> / 0.139
	WikiCS	MinCut	high	low	<u>36.430</u> / 26.620	<u>21.990</u> / 13.200	<u>6.820</u> / -0.770	<u>16.090</u> / 10.320	<u>0.058</u> / <u>0.058</u>	<u>0.139</u> / 0.069

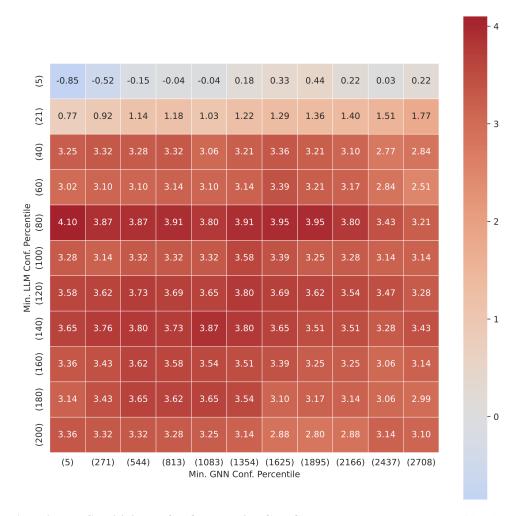


Figure 4: **Ablation on Sensitivity to Confidence with ChatGPT Feedback.** Here, we consider the sensitivity of GCLR to the confidence filtering percentiles. Namely, we take only the top [0,10,--,100]th percentile of the feedback data and report the change in accuracy to the starting solution using the CORA dataset. The number of samples at a particular percentile are indicated in parentheses, α and β are set to 0.5. We see that the best performance is obtained at a moderate confidence percentile for both the GNN and LLM.

E Prompt Examples

Table 11: Prompt Example: Triplets, CORA

PROMPT: Task: I'm clustering papers in a citation network according to research area and need help determining where a particular query sample belongs given its abstract and title. I will give you the abstracts/titles of two samples belonging to nearby clusters and you should select the abstract/title that is more similar to the query in terms of research topic. Please explain your reasoning and return your answer in a JSON format: {selection: [1,2,-1(neither or unsure)], reasoning: [your reasoning]}.

```
[SAMPLE 1]
<Sample from 1st (2nd) Closest Cluster>]
[SAMPLE 2]
<Sample from 2nd (1st) Closest Cluster>]
[QUERY]
<Sample of Query Sample>]
[ANSWER]
```

Table 12: Prompt Example: Incontext, CORA

PROMPT: [Example] <Sample> {Category: <GNN's Predicted Cluster>} ... [Example] <Sample> {Category: <GNN's Predicted Cluster>} [Task] Given the above examples, please identify the correct category for the following query sample. Please explain your reasoning and return your answer in a JSON format: category: [your prediction], reasoning: [your reasoning]. If you're unsure of an answer, select category -1. [QUERY] <QUERY> [ANSWER]

CONCEPTS GENERATION PROMPT: Task: I'm clustering papers in a citation network according to research area and need help coming up with cluster names. The following num-exemplars papers that have been clustered together and I'm going to give you their abstract/titles. Can you propose a < 7 word research topic and 2-3 sentence description for this cluster? Try not to make it too specific or too broad, and explain your reasoning. Return your answer in a JSON format: {topic: [your topic], description: [your description], reasoning: [your reasoning]}.

SAMPLES FROM CLUSTER:

Sample 1 Sample 2

. . .

Sample Num-Exemplars

Answer:

CONCEPT PREDICTION PROMPT:

[Task]

I'm currently working on clustering papers within a citation network based on their abstracts/titles. I'm seeking assistance in determining the cluster association for a specific uncertain sample. You'll be provided with the abstract/title of this sample, along with the titles and short descriptions of num-clusters potential clusters. Your task involves carefully reading each cluster title and description, taking a thoughtful approach, and selecting the cluster that best aligns with the confusing sample. Please provide your answer in JSON format, including the predicted cluster number, title of the predicted cluster, and your detailed reasoning. Your response should look like this: {cluster: [your predicted cluster number], cluster title: [title of predicted cluster], reasoning: [your reasoning for choosing this cluster]}. Take your time and ensure clarity in your explanation.

[CLUSTER TITLES]

1. <GENERATED TITLE>

Description: <GENERATED TITLE DESCRIPTION>

2. <GENERATED TITLE>

Description: <GENERATED TITLE DESCRIPTION>

•

NUM-CLUSTERS. < GENERATED TITLE>

Description: <GENERATED TITLE DESCRIPTION>

[UNCERTAIN SAMPLE]

QUERY

[ANSWER]

F Metrics

We consider the following extrinsic and graph topology-based metrics in our evaluation. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{T}, \mathcal{X}, [\mathcal{Y}])$ represent a graph with its respective node-set, edge-set, raw node based text information, embedded node attribute information (e.g., some embedding of a node's text), and optional ground-truth cluster assignment. Further, let N be the number of the nodes, M be the number of edges, C be the desired (or ground-truth) number of clusters, d the dimension of the hidden representation, $\mathbf{A} \in \mathbb{R}^{n \times n}$ be the corresponding adjacency matrix, $\mathbf{X} \in \mathbb{R}^{N \times d}$ be a matrix representation of \mathcal{X} , $\mathbf{Y} \in [0,1]^C$, \mathbf{d}_v be the degree vector of a particular node v, and c_v be the predicted cluster of a given node v.

• Modularity (Newman, 2006). Modularity measures the deviation with respect to nodes belonging to the same cluster against the expectation of the nodes being connected given a null model where nodes are connected randomly. Graphs with high modularity will have clusters where the majority of the edges are contained with some cluster and few edges that cross the clusters. Modularity falls within $[-\frac{1}{2}, 1]$, where a positive score indicates that the clustering structure that is above random, and is defined as follows:

$$Q = \frac{1}{2m} \sum_{ij} \left[\mathbf{A}_{[ij]} - \frac{d_i d_j}{2m} \right] \mathbb{1}[c_i = c_j].$$

• Conductance (Yang and Leskovec, 2012; Shi and Malik, 1997). Also known as the Cheeger coefficient, this metric measures how quickly a random walk on a graph will reach its stationary distribution. Given a particular cluster, \hat{c} , the number of edges belonging to that cluster (intra-cluster edges) can be computed as $r_{\hat{c}} = \sum_{u,v \in \mathbf{A}} \mathbb{1}[c_u = \hat{c}, c_v = \hat{c}]$, and the number of edges are not fully contained in \hat{c} (inter-cluster edges) can be computed as $s_{\hat{c}} = \sum_{u,v \in \mathbf{A}} \mathbb{1}[c_u = \hat{c}, c_v \neq \hat{c}]$. Then, conductance is defined as the average ratio of intra- and inter- cluster edges, where tight clusters are expected to have relatively fewer inter cluster edges.

$$\phi = \frac{1}{C} \sum_{\hat{c}}^{C} \frac{s_{\hat{c}}}{r_{\hat{c}} + s_{\hat{c}}}$$

Accuracy.

$$ACC = \sum_{i=1}^{n} \frac{\phi(y_i, map(\hat{y}_i))}{n} \tag{1}$$

 $\hat{y_i}$ represents the predicted cluster ID, while y_i indicates the ground truth cluster ID label. map(.) denotes the Kuhn-Munkres algorithm (Plummer and Lovász, 1986) which aligns the predicted cluster-ID with the class-ID, and indicator function $\phi(.)$ is formulated as:

$$\phi(y_i, map(\hat{y}_i)) = \begin{cases} 1 & \text{if } y_i = map(\hat{y}_i) \\ 0 & \text{else} \end{cases}$$
 (2)

Normalized Mutural Information.

$$NMI = -\frac{2\sum_{\hat{y}}\sum_{y}p(\hat{y},y)\log\frac{p(\hat{y},y)}{p(\hat{y})p(y)}}{\sum_{i}p(\hat{y}_{i})\log(p(\hat{y}_{i})) + \sum_{j}p(y_{j})\log(p(y_{j}))}$$
(3)

where p(y), $p(\hat{y})$, and $p(\hat{y}, y)$ represent the distribution of predicted results, distribution of the ground truth, and joint distribution of them, respectively.

Adjusted Random Index.

$$ARI = \frac{RI - expectedRI}{max(RI) - expectedRI} \tag{4}$$

where RI and expected RI signifies the Rand Index and expected Rand Index (Yeung and Ruzzo, 2001), respectively. An ARI of 0 suggests disagreement between real and modeled clustering in pairing, whereas an ARI of 1 indicates concordance between real and modeled clustering, representing identical clusters.

• F1-Score.

$$F1 = \frac{2.Precision.Recall}{Precision + Recall} \tag{5}$$

$$F1 = \frac{2.Precision.Recall}{Precision + Recall}$$

$$Precision = \frac{TP}{TP + FP}, \quad Recall = \frac{TP}{TP + FN}$$
(6)

where TP, FP, and FN indicate the number of true positive, false positive, and false negative samples, respectively.

G **Reproducibility Statement**

All code will be released upon acceptance. We dropped the computation linguistic and web-technology categories from WikiCS to create a more even and separate labeling for evaluation. We use the mixtral-8x-7b model, and a G.5 (8 gpu) instance on AWS. We repeat results over 3 seeds.

Table 14: Dataset Statistics.

Dataset	Num Nodes	Num Edges	Num Clusters
Cora (Bojchevski and Günnemann, 2017)	2,708	5,429	7
Citeseer (Yang et al., 2016)	3,327	4,732	6
WikiCS ¹ (Mernyei and Cangea, 2020)	10,601	204120	8

Example of Generated Titles

Table 15: Generated Concepts. Below, are examples of concepts generated by chatgpt-3.5-turbo on Cora with MinCut as the GNN clustering algorithm. While some concepts are imperfect, e.g., rule learning or theory, other topics are well captured. Applying self-refinement strategies could improve these generated concepts, at additional budget expenditure.

True	Generated
Reinforcement Learning	Reinforcement Learning and Dynamic Programming
Genetic Algorithms	Evolutionary Algorithms in Problem Solving
Rule Learning	Error Bounds in Learning Algorithms
Theory	Feature Selection in Machine Learning'
Probabilistic Methods	Bayesian Statistical Methods
Case Based	Improving Case-Based Reasoning Adaptation
Neural Networks	Neural Network Self-Organization