FINM 34500/STAT 39000
Problem Set 4 (due February 3)
Reading: Notes through Chapter 3 (remember: indented parts are optional reading)

Exercise 1 *Use Itô's formula to find the stochastic differential* $df(t, B_t)$ *where* B_t *is a standard Brownian motion and*

1. $f(t, x) = x\, e^{-x}$
2. $f(t, x) = x\, t\, e^{-tx}$
3. $f(t, x) = t + [\sin x]^3$
4. *Repeat these exercises for* $f(t, X_t)$ *where*
$$dX_t = X_t\,[2dt + 2\,dB_t].$$

Exercise 2 *Suppose*
$$Z_t = \int_0^t A_s\, dB_s$$
where A_s *is a bounded, adapted process with continuous paths. Let*
$$M_t = Z_t^2 - \langle Z \rangle_t.$$
Show that M_t *is a continuous martingale (with respect to the filtration where* $\mathcal{F}_t$ *is the information in* $\{B_s : s \leq t\}$*).*

Exercise 3 *Suppose an asset follows the following geometric SDE,*
$$dX_t = X_t\, \left[2\, dt + dB_t\right].$$

1. *Write the exact solution of this equation. In other words, find* X_t *as a function of* B_t*.*
2. *Suppose* $X_0 = 1$*. What is the probability that* $X_1 > 3$*?*
3. *Suppose* $X_0 = 1/2$*. What is the probability that* $X_2 < 3$*?*
4. *Let* $Y_t = \log X_t$ *where* $\log$ *denotes the natural logarithm. Find the equation that* Y_t *satisfies (your answer should be in terms of* Y_t *and* B_t *and should not include* X_t*).*

Exercise 4 *Suppose that two assets* X_t, Y_t *follow the SDEs*
$$dX_t = X_t\,[2, dt + 2\, dB_t],$$
$$dY_t = Y_t\,[3\, dt - dB_t],$$
where B_t *is a standard Brownian motion. Suppose also that* $X_0 = Y_0 = 1$*.*

1. *Let* $Z_t = X_t Y_t$*. Give the SDE satisfied by* Z_t*.*
2. *Let* $Z_t = X_t / Y_t$*. Give the SDE satisfied by* Z_t*.*

Exercise 5 *Book, pp. 123–124, Exercise 3.7*