FINM 34500/STAT 39000
Winter 2024
Problem Set 3 (due January 27)

Reading: Sections 3.1 – 3.2.

Exercise 1 *For each of these problems $\{A_t\}$ will be a simple process that only changes at time $1/2$, that is*

$$A_t = 1, \qquad 0 \leq t < 1/2.$$

$$A_t = Y, \qquad 1/2 \leq t \leq 1,$$

where Y is a random variable measurable with respect to $\mathcal{F}_{1/2}$. Let

$$Z_t = \int_0^t A_s \, dB_s.$$

For each of these examples of Y, find $\mathbb{P}\{Z_1 \geq 0\}$.

1.
$$Y = \begin{cases} 0 & B_{1/2} \geq 0 \\ 1 & B_{1/2} < 0 \end{cases}$$

2.
$$Y = \begin{cases} 0 & B_{1/2} \geq 0 \\ -5 & B_{1/2} < 0 \end{cases}$$

3.
$$Y = \begin{cases} 1 & B_{1/2} \geq 0 \\ -1 & B_{1/2} < 0 \end{cases}$$

Exercise 2 *For the three cases in the last exercise, give*

$$\langle Z \rangle_t = \int_0^t A_s^2 \, ds.$$

In each case, the answer should look like two functions of t — one for the event $B_{1/2} \geq 0$ and one on the event $B_{1/2} < 0$.

Exercise 3 *For Case #2 in the last two exercises, verify directly the statement*

$$\mathrm{Var}[Z_1] = \int_0^1 \mathbb{E}[A_t^2] \, dt.$$