

FINM 34500/STAT 39000

Winter 2025

Problem Set 8

Reading: Remainder of Chapter 5. Some of Chapter 6 will be discussed on Wednesday, but this will not be on the final.

- The final exam on Monday, March 10, 3:00 – 4:30 pm. Classrooms will be announced later. The rules for the exam are the same as for the midterm. It will cover material through and including the lecture of Monday, March 3. The material in the final lecture (March 5) will not be on the final exam. It will be cumulative but with some emphasis on the later material.

Exercise 1 Consider the following measures on the real line. For each pair of these state which, if any, of the following is true: $\mu_j \ll \mu_k, \mu_k \ll \mu_j, \mu_j \perp \mu_k$. Give reasons.

- μ_1 is the distribution of a Poisson random variable with $\lambda = 1$.
- μ_2 is the uniform probability measure on the interval $[0, 1]$.
- $\mu_3 = \mu_1 + \mu_2$, that is, for every $V \subset \mathbb{R}$,

$$\mu_3(V) = \mu_1(V) + \mu_2(V).$$

- μ_4 is the counting measure on the integers, that is, for every V ,

$$\mu_4(V) = \text{the number of integers (positive, negative, or zero) in } V.$$

(Note that this is an infinite measure, $\mu_4(\mathbb{R}) = \infty$.)

- μ_5 is length (Lebesgue measure),

$$\mu_5(V) = \text{the length of } V.$$

(Note that this is an infinite measure, $\mu_5(\mathbb{R}) = \infty$.)

Exercise 2 Book, Exercise 5.7

Exercise 3 Book, Exercise 5.8

Exercise 4 Book, Exercise 5.9

Exercise 5 Book, Exercise 5.11