FINM 34500/STAT 39000
Winter 2025 Problem Set 7 (due Feb 24)
Reading: 5.1 — 5.3

Exercise 1 *Book, Exercise 5.1*

Exercise 2 *Book, Exercise 5.2*

Exercise 3 *Book, Exercise 5.3*

Exercise 4 *Let B_t be a standard Brownian motion with $B_0 = 0$. Let $m > 0$ and let $X_t = e^{-mB_t^2}$.*

1. *Find a function g such that if*

$$M_t = X_t \exp\left\{\int_0^t g(B_s)\, ds\right\},$$

then M_t is a local martingale.

2. *What SDE does M_t satisfy?*

3. *Let Q be the probability measure obtained by tilting by M_t, that is, if V is $\mathcal{F}_t$-measurable,*

$$Q(V) = \mathbb{E}\left[M_t\, 1_A\right].$$

Find the SDE satisfied by B_t with respect to a Q-Brownian motion.

4. *Explain why M_t is actually a martingale and not just a local martingale.*

Exercise 5 *Let B_t be a standard Brownian motion with $B_0 = 1$. Let $T = \min\{t : B_t = 0\}$. Let $r > 0$ and let $X_t = B_t^r$.*

1. *Find a function g such that if*

$$M_t = X_t \exp\left\{\int_0^t g(B_s)\, ds\right\},$$

then M_t is a local martingale for $t < T$.

2. *What SDE does M_t satisfy?*

3. *Let Q be the probability measure obtained by tilting by M_t. Find the SDE satisfied by B_t with respect to a Q-Brownian motion.*

4. *For which values of $r > 0$ is is true that*

$$Q\{T < \infty\} = 0?$$