

Assignment - 1

1. Given the following data of Temperature ( $^{\circ}\text{C}$ ) and Power consumption (kWh).

Temperature ( $^{\circ}\text{C}$ )	Power consumption (kWh)
(x)	(y)
10	300
12	310
14	320
16	330
18	345
20	360
22	370
24	390
26	420
28	450

- a) Derive the regression equation  $Y = a + bx$ , using least squares method and calculate  $a$  (intercept) and  $b$  (slope). Also compute the value of  $\sum X$ ,  $\sum Y$ ,  $\sum XY$ .

no. of Observations  $n = 10$

$$\sum X = 10 + 12 + 14 + 16 + 18 + 20 + 22 + 24 + 26 + 28 \\ = 190$$

$$\sum Y = 300 + 310 + 320 + 330 + 345 + 360 + 370 + \\ 390 + 420 + 450 \\ = 3595$$



$$\begin{aligned}\sum XY &= 10 \times 300 + 12 \times 310 + 14 \times 320 + 16 \times 330 + \\ &\quad 18 \times 345 + 20 \times 360 + 22 \times 370 + 24 \times 390 + \\ &\quad 26 \times 420 + 28 \times 450 \\ &= 70910\end{aligned}$$

$$\begin{aligned}\sum X^2 &= 10^2 + 12^2 + 14^2 + 16^2 + 18^2 + 20^2 + 22^2 + 24^2 \\ &\quad + 26^2 + 28^2 \\ &= 3940\end{aligned}$$

Least squares formula :-

slope :-

$$\begin{aligned}b &= \frac{n \sum XY - (\sum X)(\sum Y)}{n \sum X^2 - (\sum X)^2} \\ &= \frac{10(70910) - 190(3595)}{10(3940) - 190^2} \\ b &\approx 7.89394\end{aligned}$$

Intercept

$$\begin{aligned}a &= \frac{\sum Y - b \sum X}{n} \\ &= \frac{3595 - b(190)}{10}\end{aligned}$$

$$a \approx 209.51515$$

Regression equation :-

$$\hat{Y} = 209.51515 + 7.89394X$$



b) Using your predicted values ( $\hat{y}$ ), compute  $R^2$ .

x	y	Predicted ( $\hat{y}$ )
10	300	$\hat{y} = 209.51 + 7.89(10) = 288.45$
12	310	$\hat{y} = 209.51 + 7.89(12) = 304.24$
14	320	$\hat{y} = 209.51 + 7.89(14) = 320.03$
16	330	$\hat{y} = 209.51 + 7.89(16) = 335.81$
18	345	$\hat{y} = 209.51 + 7.89(18) = 351.60$
20	360	$\hat{y} = 209.51 + 7.89(20) = 367.39$
22	370	$\hat{y} = 209.51 + 7.89(22) = 383.18$
24	390	$\hat{y} = 209.51 + 7.89(24) = 398.96$
26	420	$\hat{y} = 209.51 + 7.89(26) = 414.75$
28	450	$\hat{y} = 209.51 + 7.89(28) = 430.54$

To compute  $R^2$ ,

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}$$

$$\sum y = 3595$$

$$\bar{y} = \frac{3595}{10} = 359.5$$

$$\text{mean}(\bar{y}) = 359.5$$



$$SSE \text{ (Sum of squared Errors (residuals))} = \sum (y_i - \hat{y}_i)^2$$

↓  
Errors between actual & predicted values.

$$SST - \text{Total sum of squares} = \sum (y_i - \bar{y})^2$$

↓  
Total variation in  $y$  from its mean

$y - \hat{y}$	$(y - \hat{y})^2$	$y - \bar{y}$	$(y - \bar{y})^2$
11.55	133.26	-59.5	3540.25
5.76	33.15	-49.5	2450.25
-0.03	0.00	-39.5	1560.25
-5.82	33.88	-29.5	870.25
-6.61	43.70	-14.5	210.25
-7.39	54.60	0.5	0.25
-13.18	<del>54.61</del> 173.68	10.5	110.25
-8.97	<del>173.68</del> 80.53	30.5	930.25
5.24	27.46	60.5	3660.25
19.45	378.44	90.5	8190.25

$$SSE = 958.79$$

$$SST = 21,522.5$$

$$R^2 = 1 - \frac{SSE}{SST} = \frac{958.79}{21,522.5}$$

$$R^2 = 1 - 0.04453 = 0.95545$$

$$R^2 = 0.95545 \text{ or } 95.5\%$$