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Student ID

Educational Program:

## Exercise 1: Basics of convolution and Fourier transform

### Matlab Preamble

- During development of your m-code, you might want to inspect images and image-like data (such as Fourier magnitude spectra) on your screen for inspection. This is done as follows:
 

```
figure;                % create a figure window
imshow(im);           % write the image to the current graphical object
```
- If the data is out of range (for type doubles: not between 0 and 1), and it is an intensity (grey) image, you can adapt the range of screen by (this is also useful if the range is so small that contrasts are hardly visible):
 

```
imshow(im, []);       % adapt the range
```
- For color images, this does not work. If you want to adapt the scale, you use:
 

```
imshow(mat2gray(im)); % adapt the range
```
- For the report, you have to write the images to file first. This is done as follows:
 

```
imwrite(im, 'fname.jpg');           % without adapting the range
imwrite(mat2gray(im), 'fname.jpg'); % with adaption of the range
```
- For non-image like data, i.e. graphical data such as plots, you have to save the whole figure window to file. This is done as follows:
 

```
print -r300 -dpng fname.png % save the current figure window as a png
                             % file. The resolution ,300 dpi, is an
                             % example.
```

### The report and the m-code

You create a **single** m-file for all questions. The code must be such that **all** code can be executed by a single click on the “RUN” button (with the green arrow). Put the following line between each question:

```
%% Question 1 (etc)
```

You copy-and-paste all your code to the end of the report. Make sure that the line length does not exceed 95 characters since otherwise the code does not fit the width of a page.

### Questions

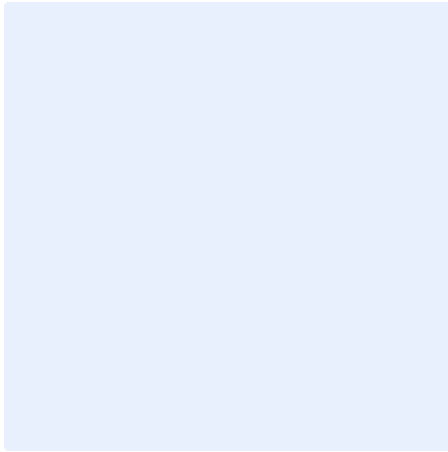
The image, stored in the file `car_dis.png`, is distorted. The objective is to apply a filter that removes the distortion without affecting the original image.

- Write and execute an m-file that:
  - clears the workspace and closes all possible figure windows (use `clear variables` and `close all`, respectively)
  - reads the image from file (`imread`), converts it to an intensity image of type double (`im2double`), and displays the image in a figure window on the screen (`imshow`).
  - calculates the Fourier transform and shows the log-amplitude spectrum as an image on the screen. Be sure to do so in a controlled manner: (i) the origin should be in the centre, and (ii) the logarithmic scale maps the magnitudes to intensities such that the interesting part of the range of magnitudes is displayed well. Include a title, and the correct x- and y labels to the axes. Write the resulting figure window as a png image<sup>1</sup> to file (use: `print -r150 -dpng Imlogmag.png`). **Do not use the copy-and-paste option, nor the ‘save as’ of the figure window as the result is not reproducible (it will depend too much on your screen resolution).**

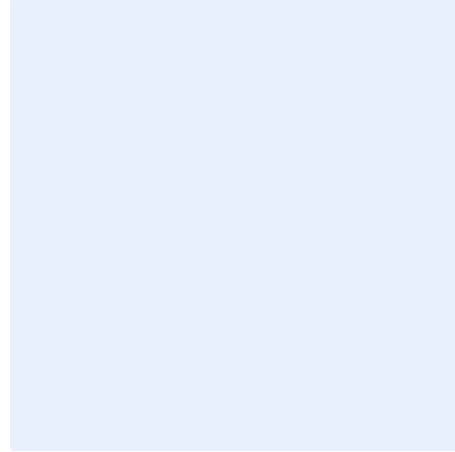
**Note:** Do not copy-and-paste text from this pdf into your Matlab editor: some fonts in the pdf might slightly differ from the ones that the editor expects

<sup>1</sup> The PNG (Portable Network Graphics) is a lossless image file format. For image analysis, a lossless coding is often preferred as a lossy encoding, such as JPEG, might induce loss of information. Another popular format is TIFF which supports both lossless and lossy encoding.

Insert the car\_dis.png



Insert the log-amplitude spectrum (Imlogmag.png)



2.a What kind of symmetry do you observe in the spectrum and how can you explain this?

2.b Suppose that the size of the input image `im` is  $N \times M$ . How large will be the fft `IM` of this image? The function `fftshift` shifts the origin of the  $u, v$  frequency domain to the centre. At which row and column index is the origin of this domain (that is,  $u=0, v=0$ ) located in the Matlab array `IM`, directly after application of `fft2`, and after application of `fftshift`. Give a mathematical expression express in Matlab notation using the given variables in this question AND the final numerical result for this case:

directly after <code>fft2</code> :	column =	
	row =	
after <code>fftshift</code> :	column =	
	row =	

2.c Suppose that the pixel size of the image `im` is  $\Delta \times \Delta$ . The distortion in the image looks like a harmonic function. By using the cursor of the figure window, created in question 1a, determine, i.e. measure, the wavelength of this harmonic function (expressed in pixel size  $\Delta$ ; that is the result could be, for instance,  $4 * \Delta$ ). Give the frequency  $\rho$  of this harmonic, also expressed<sup>2</sup> in pixel size  $\Delta$ :

lambda =	
rho =	

2.d Suppose that the distortion has a vertical orientation, so that in Cartesian coordinates we have  $u = \rho$  and  $v = 0$ . If the image size is  $N \times M$ , and its fft `IM` is centred with `fftshift`, at what row and column index one of the dominant frequency components of the distortion could be expected in the log-magnitude spectrum? Give Matlab expressions and the numerical result for this particular case:

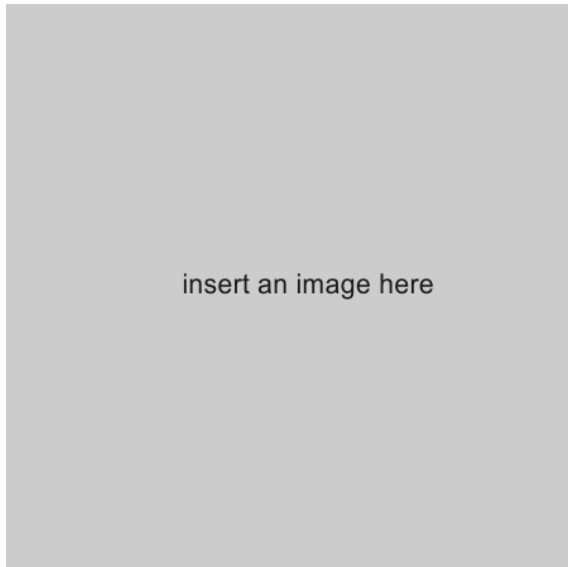
col =	
row =	

---

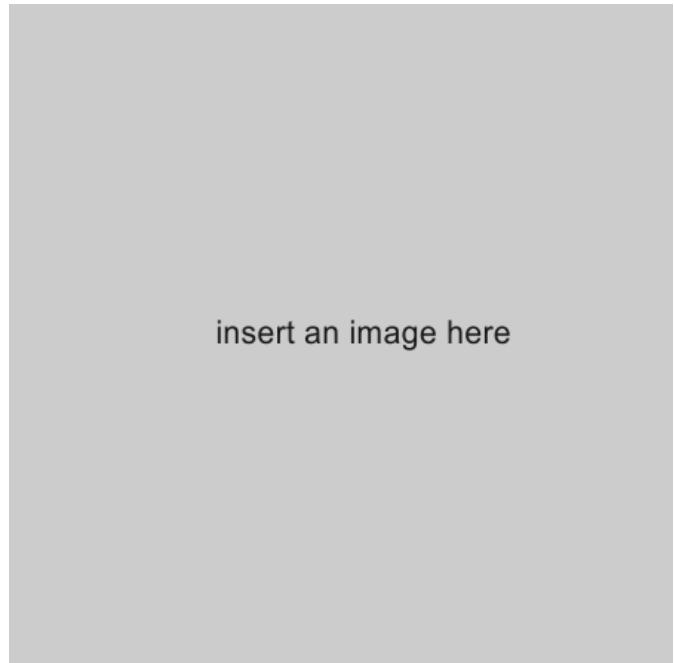
<sup>2</sup> Hint: useful information is on the slide entitled: “Discrete Fourier transform” of the lecture sheets.

3. Extend the m-file with code that filters the image with a rectangular PSF. In Matlab the corresponding filter is called 'average'. Select the horizontal and vertical sizes of the PSF such that the distortion is suppressed, but the image details are preserved as much as possible. (use `fspecial` and `imfilter`). Write the resulting image to file in JPEG format, e.g., `imwrite(imfil, 'IMfil.jpg')`; Calculate, display and store the log-magnitude spectrum of the filtered image, as was done in Q1.

Insert the filtered image here:



Insert the log-amplitude spectrum of the filtered image:



- 3.a What is the exact size of the PSF and motivate its specific width and length.

Nhorz =

Nvert =

- 3.b Explain what this filter should suppress in the spectrum. Does the filter also affect the informative part of the image? And is this inevitable, or could another choice of the PSF circumvent this?

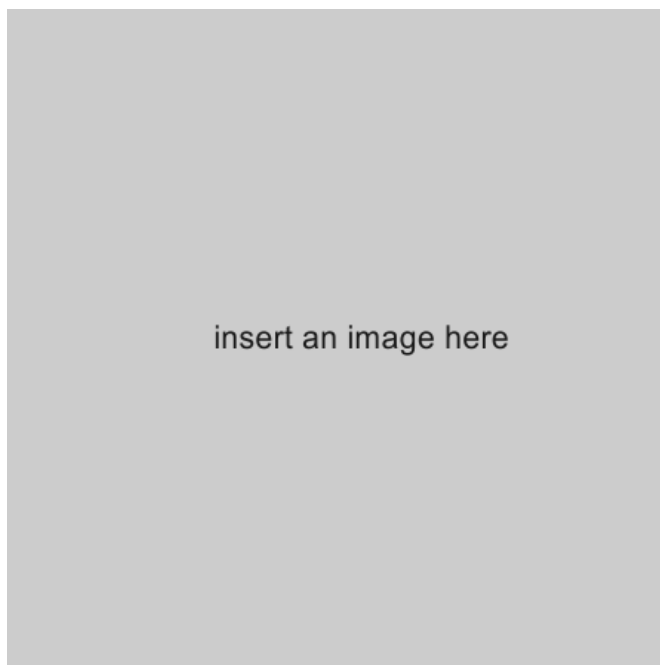
3.c What do you observe at the border of the image, and how do you explain this?

3.d Add one or option(s) to 'imfilter' (refer the documentation of this function) such that the result is as natural as possible (the natural result would be obtained if image data outside the image plane would have been available). Note: in future, use these option(s) whenever you want to use imfilter.

Option(s) used:

4. Extend the m-file with code that calculates the oftf associate to the filter found at Q3 and stores this in an array OTF. For that, use the function `psf2otf`. See the documentation. If you use this function with only one argument being the psf matrix, then the size of OTF will be equal to the one of the psf matrix (you can inspect that easily). With an additional option, you can calculate the oftf at a much higher resolution. Select the second option such that the resulting array has the same size as the image in Q3. Next, shift the origin to the centre of the graph, and show the magnitude of OTF as an intensity image in a figure window. Apply a linear scale to map the magnitudes to intensities. Don't forget to add the title and the labels at the axes.

Insert the magnitude of the OTF image:



4.b Inspect the imaginary part of the oftf, and explain the result.

`max(abs(imag(OTF(:)))) =`

Explanation:

4.b In Q2 you have identified the dominant frequency (and corresponding row- and column-indices) that can be associated with the distortion. Use this information, to extract from the OTF the attenuation factor of this particular frequency.

Attenuation factor is:

4.c Which type of this OTF represent (low pass, high pass, band pass, high emphasis, etc)?

Type of filter:

5. Instead of spatial convolution we can filter in the frequency domain. The strategy will be to define a filter transfer that blocks a range of frequencies that can be associated with the distortion. All other frequencies will be passed without attenuation.

5.a Suppose that `(row,col)` are the row and column indices of one of the dominant frequency as found in Q2d. We define a rectangular area around this dominant frequency. Suppose the size of the rectangle is set to  $s \times s$ , where  $s$  is an odd integer. In Matlab, a rectangular range of indices can be represented by `rstart:rend`, and `cstart:cend`. These four parameters must be chosen such that `(row,col)` is in the centre of this rectangle. Give matlab expressions for `rstart`, `rend`, `cstart`, and `cend`:

`rstart =`

`rend =`

`cstart =`

`cend =`

5.b An OTF matrix `H` with the same size of the image, and filled with 1's everywhere except for the frequencies of the rectangle can be created with<sup>3</sup>:

```
H = ones(size(im));
```

```
H(rstart:rend,cstart:cend) = 0;
```

However, this code will not yield an OTF with the needed symmetry. Extend the code such that the symmetry requirements are met.

Code:

5.c Apply the filter. That is, perform a pixel-by-pixel-multiplication between `H` and the FT of the original image. Next, calculate the inverse Fourier transform of the resulting image and file this in your report. Very small imaginary parts (that is smaller than  $10^{-14}$ ) of the resulting image are due to numerical round-off errors. These parts may be removed. If the imaginary parts are substantial, then the matrix `H` did not comply with the symmetry requirements. Suppose that `imresult` is the final resulting image, then execute:

```
max(abs(imag(imresult(:)))) =
```

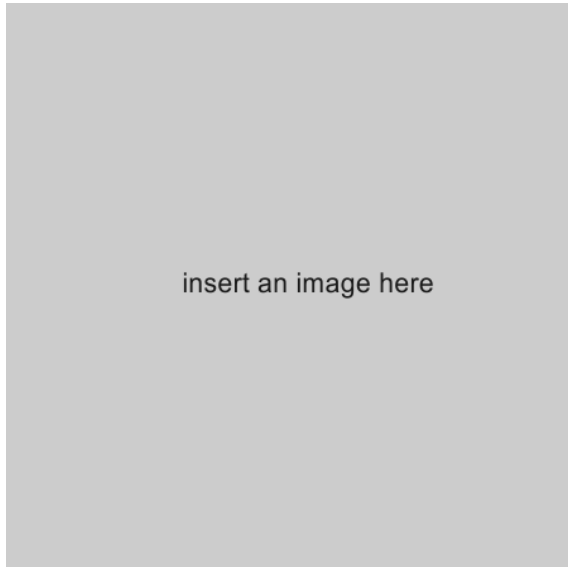


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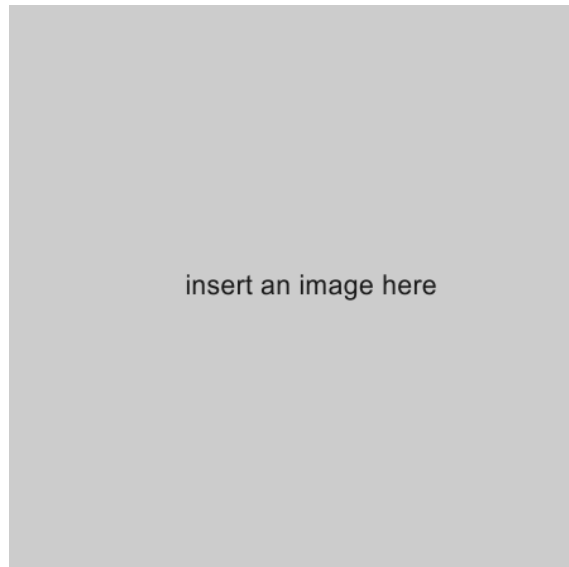
<sup>3</sup> Experienced Matlab programmers always try to avoid for-loops. Here, the colon operator `(:)` very useful. It produces concise and comprehensible code. Moreover, it may speed up Matlab considerably..

5.d Make graphical representations of the new OTF and the log-magnitude spectrum of the newly filtered image (as you did before).

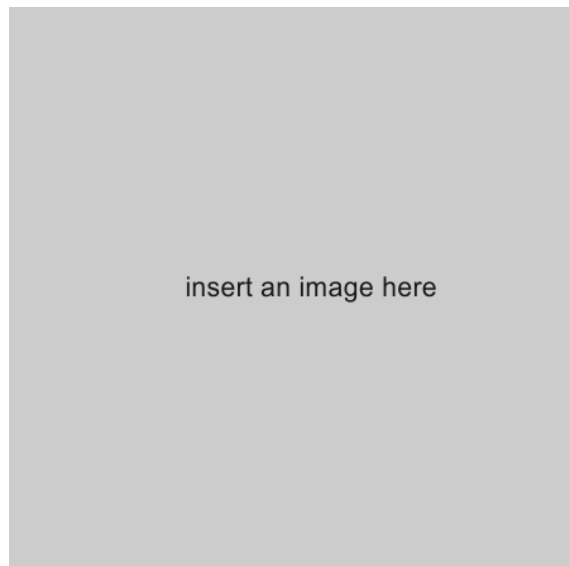
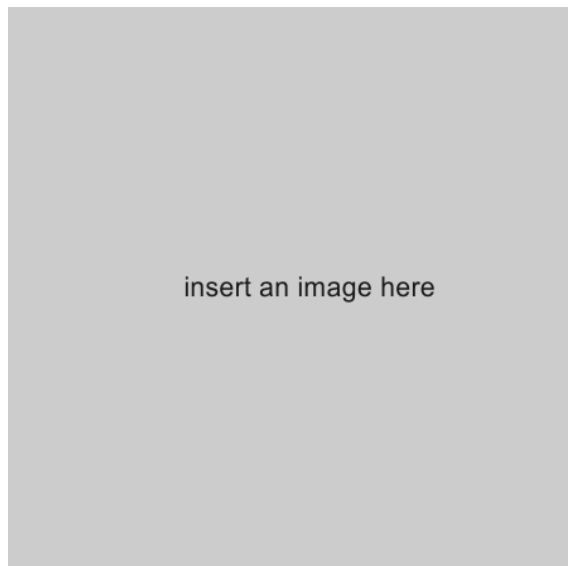
The magnitude of the found OTF image:



The log-magnitude of the newly filtered image



Show (for comparison) the distorted image and the newly filtered image:



6 In the resulting image, there will be some artefacts at the border of the image. These artefacts depend on the size of the chosen rectangle. Describe how, and why:

Make sure the m-code fits within the PDF-margin (also the added comments)!

**m-code:**

