THE OIL CONUNDRUM

vol 1: Decline | vol 2: Renewal

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PREFACE

About thirty years ago there was much talk that geologists ought only to observe and not theorise; and I well remember some one saying that at this rate a man might as well go into a gravel pit and count the pebbles and describe the colours. How odd it is that anyone should not see that all observation must be for or against some view if it is to be of any service!

— Charles Darwin

I began writing this book thinking that it would stay on topic. But then I started finding all sorts of supporting evidence for my original topic in other fields of natural science. Eventually I ran across this quote from Richard Feynman

> *"When you have put a lot of ideas together to make an elaborate theory, you want to make sure, when explaining what it fits, that those things it fits are not just the things that gave you the idea for the theory; but that the finished theory makes something else come out right, in addition."*

That is my excuse for why it this tract is so long, and why I find it endlessly fascinating to write about.

Volume 1 — Decline

"In any field, the Establishment is seldom in pursuit of the truth, because it is composed of those who sincerely believe that they are already in possession of it."

— E.T. Jaynes

Volume 1 discusses aspects of oil depletion and the potential downward trajectory with respect to fossil fuel energy availability we have found ourselves on.

Volume 2 discusses positive aspects of this crisis. What we can learn from studying oil depletion we can apply to potential new opportunities in renewable energy and in interacting with our environment.

I consider myself put in the same position as a huge fraction of the world's population. We don't understand the nature of the fossil fuel energy crunch that has hit us, and, even if we did understand it, we may remain unconvinced of the seriousness of the situation as we will likely never see the supporting data. Having seriously searched for the bottom-line numbers, I along with many other similarly curious people realize that widely-accepted data concerning world reserves of oil simply doesn't exist.

So why, and how, with the billions of dollars invested in the fossil fuel industry, can we not get a projection of our reality-based future? I bet that the oil industry actually doesn't want to know, and even if they did know they never seriously wanted to teach it to the engineers and scientists studying the discipline.

Volume 1 — Decline. I had several objectives for my approach to modeling fossil fuel depletion:

- Introducing something fundamentally new to the discussion.
- Going beyond the heuristics and empirical relationships advocated, to an approach mathematically-inclined people can understand.
- Resurrecting some old but perhaps forgotten techniques buried in the literature.
- Using the derived formulation to historically analyze or make predictions based on current data.
- Demonstrate an alternative to and weaknesses of the conventional approaches such as the logistic curve and gaussian (questioning empiricism).
- Come up with an open-source modeling environment, where I can make all source code and data available to the public.

Volume 2 — Renewal. The second volume plays off the first.

- Showing analogies to other physical processes, like an RC circuit in electronics or a 1^{st} order damped system in mechanical dynamics.
- Expand on the math to model domains and application areas that will become part of our renewable energy future.
- And above all, try to make sure the constituent parts fit together and don't contradict each other.

Although the arguments contains a bit of technical math, the modeling of oil depletion derive from some very simple and fundamental concepts. I tried to attack the premise from many different angles and eventually arrived at the same conclusion each time. This adds to the level of confidence I have for the models that I lay out.

CHAPTER 1

Introduction

Why we need to understand oil depletion

"Why don't they talk about the issues?! We're fighting for our LIVES here!"

— Marc Maron

"*It is difficult to convince a man of something when his salary depends on him not understanding it.*"

— Sinclair Lewis

As market consumers, we can imagine ourselves running out of some necessity or convenience every once in awhile. Though annoying, we usually don't worry, as we can remedy our local shortage by scooting off to the store and replenishing our supply. Occasionally, we also hear about the dire story of someone getting stuck in a precarious situation and facing a real shortage that they cannot do anything about. However, considering that we live in a technologically advanced society, we rarely find ourselves in a situation where we confront a complete and potentially long-lasting shortage. In the past, we have routinely replaced a shortage of one item with another one that served the same need. But what happens when the consumable has no replacement, and technology cannot provide an immediate solution?

That has never happened on a large scale before, yet we can try to understand possible scenarios. For example, how do we deal with what we would consider as necessities if they become in short supply? Invariably, the most critical of our supplies fall under the class of materials called **non-renewable resources**. Renewable resources differ from their non-renewable counterparts in that once a non-renewable gets consumed, it will never get recycled. If you think about it, this comprises a small portion of all of our categorized resources yet a significant part that we rely on daily to maintain our lifestyles. Fortunately, most materials do not simply vanish, as enterprising individuals can always recover their constituent parts, what we commonly refer to as recycling. We can partly thank salvage yards, pawn shops, and E-bay for this mechanism. Or else the materials gradually get recycled by natural processes. Either way they get recycled, these constitute our renewable resources.

So, in terms of recycling, we find materials such as iron and gold which fit the scrap renewable category — and water and carbon which fit the naturally renewable class of materials. And then we have the broad categories of plant and animal life that biologically get recycled, with extinction events proving the sure path to non-renewal to the inhabitants of the animal and plant kingdom.

[Ref 147]

The actual list of resources that fall under the completely non-renewable category remains fairly limited in scope. One that we normally don't think about, helium gas, essentially disappears into the ozone when we use it for just about any application, whether it frivolous (balloon toys) or critical (cryogenics). The big concern - the one that we have always worried about — hydrocarbon-based *fossil fuels* takes up the bulk of the remaining non-renewable resource category.

Although fossil fuels can technically become renewable, through plant and animal life interacting with geological forces, the fact that the rate of renewal takes place over the course of millions of years, makes this an inconsequential and therefore irrelevant detail. Facing the inevitable, once fossil fuels disappear, we won't see a plug-and-play replacement reappear anytime soon. *When* that depletion occurs, not what happens after, has become a challenge to predict, and a concern that many people share. Unfortunately, not *everyone* shares this concern and, in fact, much mis-information purposely gets broadcast to obfuscate the reality of the situation.

To understand the situation one needs to unravel the obfuscation surrounding the reporting of non-renewable energy reserves by energy companies, the corporate media, and the governments in cahoots with them. Starting with a few initial premises, lots of rigorous intuition, and incomplete data, I approach the investigation as an amateur detective would, piecing together the scrambled components of the puzzle. For the most part, we already know the bottom-line answer — that we will eventually reach a permanent stage of scarce, expensive fossil-fuel-based energy.[1] However, the glide path that we will follow to reach the inevitable conclusion, particularly in elementary mathematical and statistical terms, remains largely a riddle. So we need to do what most any technology savvy person would do; we practice a bit of reverse engineering. In the end we will transform the glide path into modeling of extraction that we can refer to as a *stochastic arc*; in other words a trajectory of possible pathways that we can structure policy around.

1. The concept of peak oil is not about when we run out, but when we reach a peak in production signifying that we have gone through about half of the easily accessible and cheap oil. We currently exist in the middle period.

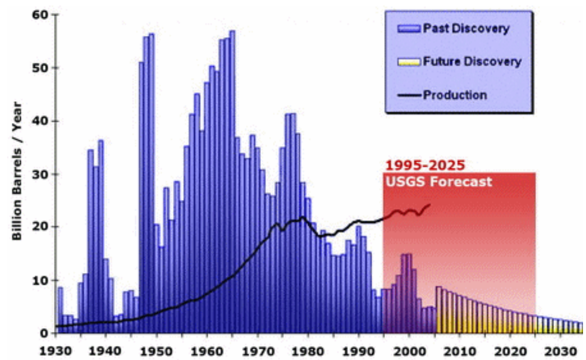
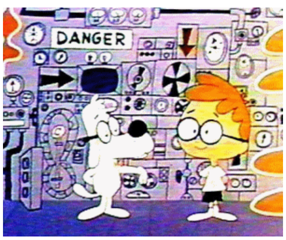


FIGURE 1-1. Global oil discovery and production curves [Ref 111] adapted from [Ref 112]



Start the Wayback Machine. Fossil fuels have sat in the ground, virtually untouched, for millions of years prior to the 19th century. Certainly, both liquid and vapor forms of the fuel, such as natural gas, have continuously seeped out of the ground over the ages, and people have longed used brown and black coal, but our knowledge of substantial reserves have only emerged in the last two centuries. With the help of humans that have mined or pumped it out of the ground, fossil fuels have provided a convenient combustion source for a relatively short time. In fact, humans have developed such convenient and efficient ways of combusting fossil fuel that we can quickly use up whatever we can find. Quite naturally, that free lunch cannot last forever as eventually we use up more than we can freshly discover. That and some time lag factor give us the concept of “peak oil”.

So we have the three stages of fossil fuel: the pre-history that lead up to the near simultaneous industrial discovery of oil in both hemispheres in the 1800’s, the production phase that has spanned the *oil age*, and the post oil age that we will eventually have to face. The first stage has become water under the bridge[2] — although it helps a bit to understand how fossil fuels formed and how we first discovered the oil-rich regions of the world, we will never again reproduce those conditions again. So can’t expect our oil resources to recover as a threatened animal species might. However, we can indeed learn much from the historical data and the modeling of trends describing the evolution of commercial oil production. As no better information exists, I have concentrated on the available industrial data and specifically tabulations of reserve and production estimates for this investigation.

2. Or water over the dam. I don’t know which fits the situation better.

Into the Future. We have no generally accepted model of oil depletion or the effect of constrained resources on the economy. The intensity of discussion surrounding economic theory during recessionary crises (such as the one which started in 2008) seems particularly perplexing considering this huge gap in our knowledge. So we don't really know how much of an impact our oil predicament has on an economic downturn, and may instead try to scape-goat the role of speculators and our financial institutions themselves. This becomes really a failure of the mainstream economists in the world, and extra fault lies with the oil companies[3] for not releasing any data on depletion. In the greater scheme of things, obtaining detailed historical oil production data should not turn into an unknowable problem[4].

Furthermore I find it fascinating that we focus on oil pricing as much as we do. From a resource perspective, everything revolves around supply and availability. In historical terms, we almost always see huge fluctuations in price as we start to reach a constrained limit on supply. The price becomes an indirect side-effect of the interaction between supply and demand, which makes it impossible to fundamentally predict the price of a commodity under these circumstances. Leave it to the economists to live in an overly abstracted, perfect world free of any constraints, where they can use abstract theories to predict growth, profit, and price.

"When the topic is oil there are far too many economists that express themselves far too much about something that they do not understand."
— Kjell Aleklett

> Interview with Milton Friedman, from *Economists and the Environment*
>
> *Ravaioli*: But there are many other environmental problems...
>
> *Nobel Laureate Friedman*: Of course. Take oil, for example. Everyone says it's a limited resource: physically it may be, but economically we don't know. Economically there is more oil today than there was a hundred years ago. When it was still under the ground and no one knew it was there, it wasn't economically available. When resources are really limited prices go up, but the price of oil has gone down and down. Suppose oil became scarce: the price would go up, and people would start using other energy sources. In a proper price system the market can take care of the problem.
>
> *Ravaioli*: But we know that it takes millions of years to create an oil well, and we can't reproduce it. Relying on oil means living on our capital and not on the interest, which would be the sensible course. Don't you agree?
>
> *Nobel Laureate Friedman*: If we were living on the capital, the market price would go up. The price of truly limited resources will rise over time. The price of oil has not been rising, so we're not living on the capital. When that is no longer true, the

3. Excepting those in the UK and Norway which have well-organized reporting mechanisms
4. Unlike the unknown effect of financial instruments on the global economy, I see an analogy between predicting peak accurately with the valuation of derivatives. We only have a vague understanding of how much oil is under the ground, and we only have a vague understanding on how much money is in derivatives. But then again, derivative valuation sounds like a virtual thing and it can get removed from the books at any time, whereas the oil is quite tangible, yet hidden from view.

> price system will give a signal and the price of oil will go up. As always happens with a truly limited resource.Why an illusion?
>
> *Ravaioli*: Because we know it's a limited resource.
>
> *Nobel Laureate Friedman*: Excuse me, it's not limited from an economic point of view. You have to separate the economic from the physical point of view. Many of the mistakes people make come from this. Like the stupid projections of the Club of Rome: they used a purely physical approach, without taking prices into account. There are many different sources of energy, some of which are too expensive to be exploited now. But if oil becomes scarce they will be exploited. But the market, which is fortunately capable of registering and using widely scattered knowledge and information from people all over the world, will take account of those changes. [Ref 146]

In this excerpt, Friedman treats oil as a completely abstract entity, and one that possesses an equally abstract replaceable form. Thankfully, none of our understanding needs to derive from premises developed in an abstract world — we don't have to comprehend quantum physics or general relativity, for example. In terms of understanding the dynamics of oil production and use, we deal with very concrete terms: volume of oil and the change of the volume of oil over time. Step aside from the myriad uses of the oil, and we really have to only consider the most tangible of quantities: the barrel of oil itself and whether it sits above ground — or — remains buried underground in reservoirs yet untapped. With the single assertion that we have knowledge of national and global production levels, we can prognosticate on the future evolution of our fossil fuel reserve. That and some detective work will get us to a better understanding; we may not know *how* an alternative strategy will come about but we will know *when* we need it.

So I see this effort as part science and part investigative journalism. The journalistic aspects come about simply because that remains the only avenue to dig out the data and the only means to get the word out to a broader audience. It looks like science because no one has looked at the analysis before from this particular perspective. Serious geologists may in fact know everything contained herein but (1) they haven't made this clear and (2) corporate forces or decisions may prevent this from seeing the light of day. Scarier yet, they may not even understand the relative ease in which we can cast our understanding.

Ultimately, if no one reports what they know in some practical capacity, then no one will hear about it. Simply describing the situation in dry, mathematical terms for a specialized scientific journal will not do us as a society any good. Therefore I will try to keep the math as grounded and practical as possible.

The Open Source Approach. Since much of this work sprang from real-time blogging efforts on internet sites frequented by people interested in peak oil topics, much of the vetting has come about from pragmatic or critical evaluations of others willing to dive into the details. I have referred to this as an open source approach, whereby through network forums participants can interactively create “blogo-rithms” and deal with “pragmathematics” . Whatever we call it, allowing an alter-native group of peers to contribute ideas and peer review each other’s work provides an excellent complementary avenue to the academic journals.

Typical of the exchanges I have had include this response from a petroleum engi-neer named “rockdoc123” to an analysis that I summarized online.

> *rockdoc123 wrote:...what you just created as I remember was being used some twenty years ago in the research labs (back when oil companies like Gulf and Shell had large research facilities and had no problem hiring people with a maths jones to sit in an office and fiddle with data). ...*
> *Today most oil companies are just not that interested in how much might be left overall (exception would be BP who still has a pretty big research compo-nent)...they spend their time modeling existing production and coming up with pre-dictions on whats left to be discovered in various parts of the world where they are working*

At the time, I wondered why such fundamental analyses never got transferred to textbooks. In other disciplines, lacking such basics becomes much like teaching first-year electrical engineering without introducing the fundamental theory of Kir-choff's Laws[5]. On a learning curve, we don’t learn anything complicated initially, and the math remains just rate equations and conservation of matter. In other words, you have to start from somewhere.

If anyone has taken classes in subjects such as ecology, early on you learn that all freshwater lakes go through a life-cycle, from birth through death[6]. No one seems afraid of admitting to this natural process.Yet, geology and petroleum engineering departments may not want to even broach the subject of the life-cycle of oil. By avoiding the teaching of such fundamentals, they essentially avoid discussing the possibility that an oil-based economy might become just a house of cards or a Ponzi scheme (arguably analogous to aspects of our present day economy), certain to eventually decline, and perhaps to collapse. This gap in understanding essentially defers accountability and a transition to other forms of energy from our current crop of engineers to future ones.

I figure that since we all have some interest in the production of oil, however tran-sient its existence may eventually pan out, that we might as well get to know its

5. Kirchoff’s Laws explain mathematically how electrical circuits behave.
6. The study of limnology introduces the concept of eutrophication. The term “entrophy” is an interest-ing combination of entropy and eutrophic describing the inexorable trend toward disorder.

life-cycle[7]. In the end, the vast majority of us propel ourselves by internal combustion engines or heat our homes with fossil fuels. The explanation for how we so quickly squandered such a miraculous convenience of fuel oil will hopefully enlighten us the next time we get tempted to feast at some new form of non-renewable energy.

So to develop our understanding, we reformulate the problem domain. We know all the hand-wavy arguments about why oil depletion will effect us, but I contend we should recast the "why" arguments into something more formal, yet avoiding undue complexity.

1. So as a general rule, we should strive to teach the ***first-order effects first***. Just about everything in engineering relates to first -order effects. If you don't do the first-order stuff first, you need to find a new line of work. They call it first-order for a reason.
2. We do not even necessarily need to specifically invoke geology. In other words, we use ***practical mathematical and probability models*** to represent geology. To open up the entry criteria for understanding oil depletion we need to use representative analogies and abstractions from other disciplines[8]. I fear that the minute we start to appeal to higher authorities such as experts in petroleum engineering, geology, or commodity economics on understanding this stuff, we start to lose the battle.

By avoiding an appeal to authority, this approach employs the Socratic dialectic as opposed to engaging in rhetorical arguments. To really understand this subject, one must transition from a *qualitative* understanding to a *quantitative* model. Such a model works to exercise and unify a common understanding while refuting alternative incorrect or contradictory arguments.

The Sunk Cost Effect. Collectively we have invested much time in trying to understand the dynamics of oil depletion — without ever achieving a non-heuristic quantitative model. By and large, society still don't understand the dynamics of oil depletion at a universally accepted level. Yet, we will likely retain most of the conventional wisdom and tribal knowledge due to inertia and group-think and without having any better information available.

As an appeal to the economists, who should know best to avoid this quandary, I refer to the *sunk-cost effect*. Sunk-cost relates to the dogged persistence in people

7. Or you can subscribe to insider oil periodicals for >$1000/year subscription fees.
8. We could just as easily discus the abundance of plant and animal species and the mathematical underpinnings would remain identical, see Volume 2 for this and other practical applications of the math.

that have invested their own money and ordinarily would show rational behavior if they happened across found money. The field of economics teaches us the fallacious reasoning involved in falling prey to sunk-cost arguments. This becomes problematic because it often leads to emotional rather than rational decision-making.

> *Research tended to be motivated by the internal logic, intellectual sunk capital and esthetic puzzles of established research programmes rather than by a powerful desire to understand how the economy works - let alone how the economy works during times of stress and financial instability. So the economics profession was caught unprepared when the crisis struck.* [Ref 1]

All the past mistakes in theorizing have become "sunk" and more practical and rational arguments provide an alternate and ultimately more realistic view for the path forward. If the emotion gets down to a personal level and each of us has to deal with our own experiences with resource constraints, the rationality of our plight might just hit home, with the sunk-cost and group-think fears cast aside.

> *... And why didn't a consensus of economists at universities and other institutions warn that a crisis was on the way?*
>
> *The field of social psychology provides a possible answer. In his classic 1972 book, "Groupthink," Irving L. Janis, the Yale psychologist, explained how panels of experts could make colossal mistakes. People on these panels, he said, are forever worrying about their personal relevance and effectiveness, and feel that if they deviate too far from the consensus, they will not be given a serious role. They self-censor personal doubts about the emerging group consensus if they cannot express these doubts in a formal way that conforms with apparent assumptions held by the group.* [Ref 3]

It appears that no one wants to confront the problem, no matter how much of a challenge oil depletion presents (or doesn't in some people's minds). If you look hard enough, you will find many qualitative arguments to explain peak oil, some from long ago. Read the section "Epilogue: A Historical Perspective of Peak Oil" for a flashback to 35 years ago and then you will realize how much things haven't changed. Further, when you attempt to understand what petroleum engineering academics say about our situation, you immediately sense a disconnect between what I think we should worry about and what the industry thinks.

> *Sooner or later it must be appreciated that we have discovered quite enough oil, the world is awash with it, and concentrate more on attaining a higher recovery of what has already been found than ever aimed at or attained in the past*. [Ref 172]

It essentially boils down to the issue of deciding between *"Sooner or later it must be appreciated that we have discovered quite enough"* as *"the world is awash with it"* versus *"we will never discover enough".* I contend that recovering what we have already discovered won't cut it, and adopting the "drill, baby, drill" mantra once we have exhausted this option, will also result in failure. Like proof of water eludes the fish, the oil industry has swum in oil for so long they no longer question and take the eternal existence of oil for granted. And the typical economist apparently can not even imagine a society without oil, as no other cheap energy substitute will provide the stimulus to propel their economic growth models.

The Questions. So far I have covered *why* we need to understand oil depletion. Since the name of the book includes the word *conundrum*, you can imagine that we have many remaining questions that need addressing. In the rest of the text, I will answer other *w*'s that seem pertinent[9].

1. Why we need to understand oil depletion? (this chapter)
2. Who has tried to qualitatively model oil depletion?
3. What fundamental ideas do we apply to the model?
4. Where do we find oil reservoirs?
5. When does the extraction kick in?
6. How do we model the depletion of oil?
7. How do we model the discovery of oil?
8. How does discovery affect production?
9. How do estimates of oil evolve?
10. How do we simplify the search model?
11. How do we simplify the extraction model?
12. Which sets of available data support the model?
13. Which conditions can negatively (or positively) impact the model?
14. How do we reconcile against overly-optimistic oil supply analyses?
15. How do other more pessimistic projections fit into the analysis?
16. What current situation do we find ourselves trapped within?
17. What can we extrapolate for the future?
18. How can we apply this to other resources besides oil (natural gas, phosphates)?
19. While we have gotten this far, what can we conclude?
20. Why should you believe any of this?

9. The famous 5 *w*'s of journalism are *who, what, why, when*, and *where.* We typically include *how* for good measure.

The answers that we have historically received in response to these questions have lacked a quantitative foundation. By trying to answer these to my own satisfaction, I figured that I could also help other mathematically-inclined people understand or at least gain insight into our current Peak Oil predicament. In short, this investigation essentially provides a logical and rigorous outlook from the perspective of an oil industry outsider, no different from what a journalist would do.

CHAPTER 2

The Problem

Who has tried to qualitatively model oil depletion?

> *"The people of ____ have been led in ____ into a trap from which it will be hard to escape with dignity and honour. They have been tricked into it by a steady withholding of information. The ____ communiques are belated, insincere, incomplete. Things have been far worse than we have been told, our administration more bloody and inefficient that the public knows... We are today not far from a disaster. "*
>
> — TE Lawrence

heuristic: involving or serving as an aid to learning, discovery, or problem-solving by experimental and especially trial-and-error methods

empirical: (1) originating in or based on observation or experience; (2) relying on experience or observation alone often without due regard for system and theory

Few people feel confident in the world-wide fossil fuel outlook as it stands. Rapid fuel price increases come out of nowhere and hit just about everyone. Analysts attribute much of the price hikes to speculation on future supply availability. Beyond that, things get muddled, and the constant flip-flopping of conventional wisdom has split us into factions. We have essentially two opposing camps of opinion: the *pessimists* and the *optimists*. Pessimists (aka *doomers*) worry about our future oil supply and of escalating costs as our supplies dwindle. Optimists (aka *cornucopians* [Ref 145]) remain upbeat and encourage people not to worry. The vast majority of the public fall somewhere in between the two camps, and as always, rely on their wallet to tell them the direction of the wind.

Unfortunately, given the relatively poor understanding we have for oil production trends, with so much of the analysis based on heuristics and empirical data, the battle between the pessimists and cornucopians will continue. One conundrum lies in the fact that we don't know the future until we get good data, but we will only get good data when the future arrives at our doorstep. In other words, nothing available gives us a model that can explain the trends in the data — except in hindsight a few years down the road.

The root of the problem I believe resides in a simple observation: most forecasts for oil production derive from predictions for demand. Plainly this approach does not take much in the way of thought and certainly does not require much in the way of courage. Since the data arises from measurements of customer demand, the industry analysts simply use that as a projection yardstick. Where we encounter elastic supplies, an increase in demand usually gets met with an increase in supply, And so we see the typical expectation level of an increasing demand met by an increasing supply of oil.[1]

But when the unrelenting force meets the immovable object, something has to give.

The give comes down to a human decision, as nature has never shown a propensity to yield. We basically want to explain why the fossil fuel supply won't budge from its inevitable trajectory and landing[2], and convince people that they will have to perform evasive and imaginative maneuvering to maintain current energy consumption levels. The oil industry takes an almost naive viewpoint to where the new supply of oil will come from. The following response to a question on how much effort it would take to model global oil depletion came from Les Magoon who retired as a geologist after working at the USGS for 30 years:

> "*It's never couched in terms of what is left. It's always couched in terms of what is there, how much I can sell it for, and is it going to be economic. The only people who are really concerned with what's left are the people who are trying to figure where peak oil is or what the ultimate base of the resource is.* ***Those are the only people who care****. The industry is looking for just a way to drill a well and find hydrocarbons in a profitable manner.*" [Ref 189]

This essentially states that the corporate engine will continue to try to serve up fossil fuels as it lacks a conscience and a throttle against over-exploitation. Unrelenting human greed and voraciousness got us into this predicament, and something else approaching a paradigm shift will provide a way to get us out of this mess.

> *"All current growth-based economic models imply massive use of non-renewable resources and environmental degradation. These models are not sustainable, even in the short term." [Ref 4]*

The Pioneers. To figure out a glide path, we can first look to the people that Les Magoon said seemed to care about our oil predicament. Engineers such as the geologist M.King Hubbert have sought to explain the finite nature of the resource. However his explanations universally lacked a real *quantitative* flavor and he ended up guiding much of the work via intuition and the use of heuristics. Laherrere and Deffeyes have also done much work, essentially picking up where Hubbert left off. This small group provided enough of a foundation to encourage others to kickstart

1. Just like "if it rains today, then it will rain tomorrow" becomes a first-order forecasting tool.
2. You might hear the terms *soft* or *hard landing*, which qualifies how smoothly society will adapt when confronted with a severe resource constraint.

an ongoing discussion. The widespread use of computers clearly has helped motivate many to analyze the situation for themselves.[3]

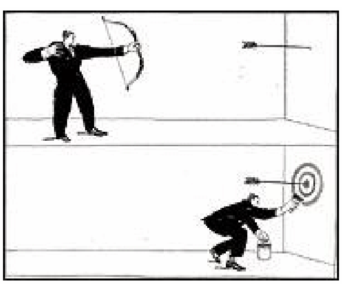
So let us understand the problem by explaining quantitatively how we got here.

1. Accelerating growth in technology and human consumption. The growth in consumption eventually leads to diminishing returns of supplies. We can explain this in terms of both micro and macro effects.
2. We sweep the volume of the earth's crust to explain past and future oil discoveries and the possibility of reserve growth. We can understand the problem by incorporating the concept of dispersion, which amounts to varying the rates of search with time and geographical region.
3. Production dynamics and the effects of perturbations. We present the shock model as an intuitive way of analyzing the situation.

If we want to understand the problem, we need to bridge the separation between the mathematical abstraction of economic/human flows and that of geology. Ideally, we could discuss the flow of jellybeans and the math concepts would remain nearly the same as modeling oil extraction.

By the same token, the ability to determine what constitutes a first-order effect and what make it second-order makes a big deal in whether you get mired in the details. This can lead to an impasse if you don't believe that such an approach would ever work. I had a fundamental disagreement with an on-line commentator with some expertise in the oil industry who stated this:

> *"my experience has been that analytical methods are great for developing an understanding of theoretical concepts, but empirical methods are better in application."*[4]



Without a model, what you hit becomes the target.

Of course we can describe or *emulate* anything we want to using empirical methods not backed by theory (what we call heuristics), but from what I can see, we still have no universally accepted theory. For instance, all the economists that try to predict when or whether we come out of any particular recession, or estimate GDP growth, work essentially with one hand tied behind their back without the benefit of a good oil depletion theory. The lack of that theory constitutes the problem and the steps outlined above provides a solution from which we can derive analytic and quantitative predictions and trends. This perspective comes from drawing on basic first principles of math, probability, and statistics.

3. Unfortunately, the same does not always hold for the engineers who blindly use a piece of simulation software without understanding the fundamentals of the algorithm inside.
4. Captured from http://www.theoildrum.com/node/4831#comment-440674

To the best of my knowledge, no one has tried to cast the problem in mathematically modeled terms. Not all highly regarded depletion analysts put much faith in such an approach. For example, Matt Simmons has shown similar reservations:

> *"Data always beats theories. 'Look at data three times and then come to a conclusion,' versus 'coming to a conclusion and searching for some data.'" [The former will win every time.]* [Ref 5]

"all observation must be for or against some view if it is to be of any service."

— C. Darwin

I would stress that data only tells you what happened and thus the only conclusions one can drawn on relates to precisely that — what happened. Consider that the scientist Charles Darwin advocated that theories should always supplant the observations. Take the analogy of observing the trajectory of a mile wide asteroid. The ultimate utility of the information depends on whether we have a theory that would allow us to project whether it will hit the Earth (and then to do some evasive action). Kind of absurd, yes, but without a theory we would fear any asteroid that looks like it headed toward us, not realizing the low probability of this event. So, in a way, an analytical theory supplements all the empirical derivations that have preceded us, stretching back to King Hubbert[5] and up to Matt Simmons. This essentially becomes an approach that can truly substantiate the trend-seeker's intuition, and we can perhaps effectively plan our fossil-fuel-based trajectory.

"Of all races in an advanced stage of civilization, the American is the least accessible to long views… Always and everywhere in a hurry to get rich, he does not give a thought to remote consequences; he sees only present advantages… He does not remember, he does not feel, he lives in a materialist dream."

— Moiseide Ostrogorski

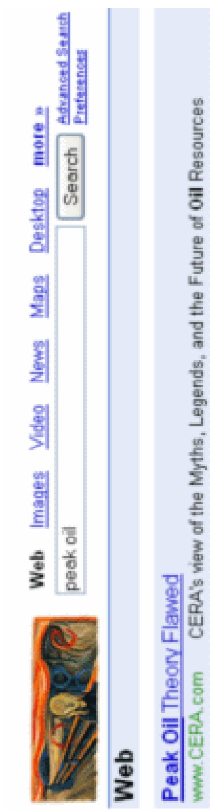
Moreover, generating a good quantitative approach provides needed ammunition to defend against cornucopian arguments. Optimists such as Michael Lynch and Peter Huber, who try to refute possibilities of energy shortages, regularly dispute the claims of oil analyst pessimists. Several of Lynch's papers try to debunk pessimistic scenarios and in pure rhetoric he nearly succeeds. By one measure, he makes a good case by poking holes in the incomplete nature of the heuristic theories of analysts such as Laherrere and Campbell. But Lynch's ideas have no better than a heuristic nature themselves. So it becomes a battle of heurism versus heurism or empirical data versus other empirical data. With a good formal counter-argument in place, Lynch and Huber would have to contend with justifying formal arguments of their own, and from what we know in regards to global warming arguments the critics have few theories of their own[6].

---

5. The math of stochastic processes only got popularized in the 1950's and early 1960's via some classic texts. This opened various disciplines to new analysis approaches. I have debated in my own mind whether King Hubbert knew of these ideas. And since he lead the way, the inertia of his heuristics have gained some weight which have unfortunately become hard to dislodge. Remember as well that the theory of plate tectonics only came about in the early 1960's so basic understanding in geology has always plodded along slowly. Deffeyes has said that Hubbert worked backward to find mathematical curves which would fit his intuition [Ref 96].
6. See "Cornucopian Conundrums. How do we reconcile against optimistic analyses?" . Humans do feel some intuition toward a good heuristic. After all, it does reinforce positive feedback, like a temporary warming trend would reinforce a positive feedback to dismiss global warming.

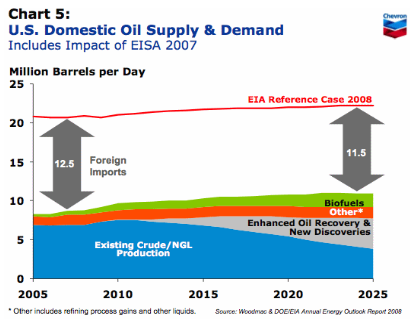
In addition to individual cornucopians such as Lynch and Huber, you have oil industry stalwarts such as CERA (Cambridge Energy Research Associates) who sell "insider" information and statistics to those who can afford to pay the steep price. But these people don't do too much better:

> **Impressive track record?**
>
> It is surprising the kind of standing CERA has among politicians and the media, given their track record. If you look at the report they published in May of 2005 their forecast for 2005 was a capacity of 87.85 million b/d. Yet by the release of CERA's October report in 2008 the 2005 figure had been revised down by 1.4 million b/d to 86,45. Not very impressive real time data.
>
> CERA makes a big issue out of their fantastic database, that it is why their forecasts are so superior to everyone else's. If they are far away from other forecasts the reason is simply that they claim they have much better data to back up their views.
>
> It is therefore an interesting experiment to size them up in a country where the data is simple, transparent and public, and where "a better database" therefore is not really a competitive edge. Let us take Norway. In their 2005 report they were expecting Norway to produce 3 million b/d of liquids in 2010. In their 2006 update this figure had been revised up to 3.15 million b/d.
>
> According to the latest IEA figures and the Norwegian Petroleum Directorate it is expected that Norway to produce about 2.1 million b/d in 2010. **This actually means that in 2006 CERA was more than 50% too high in their estimate for capacity 4 years down the road for a country with relatively few fields and the most transparent and public statistics in the world.**
>
> The UK is also a fairly transparent country. In CERA's 2005 report the UK should have a capacity of 2.1 million b/d by 2010. According to the latest forecast from the IEA the production in 2010 will be 1.2 million b/d. **This time they are 75 percent too high.**
>
> When they are that much off in such a near future in transparent countries like Norway and the UK, how likely is it that their forecast for countries like Azerbaijan or Iran 10 years into the future are meaningful at all? [Ref 6]



CERA has paid Google ad placements to refute peak oil theory. They have a point as no real theory apart from heuristics exists.

Wishful thinking, or Production Estimates = Demand Estimates? So we know that industry estimates trend too optimistic. Why again does that exactly happen? The following conference call in 2008 organized by the American Petroleum Institute with Chevron executives gives an idea of the corporate viewpoint:



(note the top line provides the EIA's estimate for domestic oil production) [Ref 157]

"John Felmy, API's Chief Economist, then pointed out what is easy to miss. EIA's top line is really an estimate of demand. Demand is estimated based on an economic model that includes the desired level of economic growth together with a growth in efficiency equal to what it has been in the past -- about 1.6% per year, plus the expected impact of the new fuel economy requirements from the 2007 legislation. Thus, Chart 5 does have some efficiency growth built into it, but even including the efficiency gains, it is indicating an increase in expected oil consumption." [Ref 157]

This provides more evidence that big oil and EIA perhaps never intended to predict anything realistic, instead opting to project their wishes for the future based on current demand and economic growth. Unfortunately, that doesn't fit into any branch of scientific forecasting methods that one would normally put any faith into. At best it presents as a case of "dead reckoning" for predicting a ship's course.[7]

At present we may have to come to the realization that we may have gone up and past peak without any real analysis, either economic or scientific, that has gone beyond heuristics (on the pessimistic side), or worse, plain wishful thinking (on the optimistic side).

Thinking even further, we may hit the *next* resource problem with the same lack of insight. We still need to think, which inevitably leads to arguments, and thus the problem with coming up with a good mathematical foundation.

7. The model for "dead-reckoning" is described later. Some would say this also follows the model of financial growth, as in the expectation that economic growth alone would increase the value of equities (such as homes), leading to an ever upward trend. Suffice to say, dead reckoning doesn't work real well, even for its originally intended navigational uses.

CHAPTER 3

The Premise

What fundamental ideas do we apply?

"Imperialistic house of prayer,
Conquistadors who took their share."

— Ian Curtis

Concerning oil depletion, we know that three things will happen in sequence.

1. Conventional oil reserves will peak.
2. Conventional oil reserves will decline.
3. Extraction and use of oil will become counter-productive.

The dates of these events remain unknown.[1]

Given those rather safe assertions and the underlying timing uncertainty, I would state as a premise that we need to believe that we can actually mathematically *understand* oil depletion. If we can't grasp the fundamentals we will end up relying on what the corporations and governments tell us. Fortunately, nothing in the oil discovery, extraction, or production process relies on any magical incantations; thus basic intuition paves the way for a deeper understanding. Further, I would say that the accompanying mathematics does not devolve into formal posturing, but instead comprises a set of pragmatic probability considerations which approaches the practical equivalent of a statistical bean-counting approach. Such relative simplicity gives us confidence that we can actually make some headway in modeling the global oil life-cycle.

1. Paraphrased from the first blog post http://mobjectivist.blogspot.com/2004/05/rationale.html

To get a handle on the entire life-cycle of oil, I prefer to break it down into three components that we can attempt to handle individually: *growth*, *discovery*, and *extraction*.

The first component drives the whole process. Intuitively, we need a rate *function* that describes how fast technology and consumption pressure stimulates the search and extraction process. This rate can either accelerate in cases of restraint-free growth or perhaps decelerate if we hit the law of diminishing returns.

The second component describes the search for undiscovered reserves. The basic premise actually describes a formidable constraint — we have a *finite* search space to deal with. The law of unintended consequences takes root if we do not have an understanding of constraints and limits. Clearly, we can no longer plan for the eventuality that will come about due to the finite nature of the world's resources.

The last of the three components describes the extraction process. Extraction can only happen after a discovery occurs. If you believe in this premise, which makes perfect pedantic sense, then you should convince yourself that we separate it from the discovery process and incorporate the extraction as an independent process. Which implies that we can *solve* them independently.

Although the components of growth, discovery, and extraction remain largely independent (mathematicians refers to these as *orthogonal* components), I assert that the invariant nature of human greed plays a role in supplying a consistent stimulus, thereby propelling the solution forward from the basic premises. If greed sounds like too disparaging a term, replace it with the combination of individuals wanting to pursue wealth and the effect of unrelenting consumer demand. In turn, this abstracted "greed" supplied the impetus of growth as well as the free-market result of consumers using the outputs of extraction (i.e. cheap energy) in whatever way they want.[2]

The Greed Invariant. The shorthand of greed serves as a constant reminder that the demand for oil existed since day one. In the early days, greed revealed itself as wasteful production practices and inefficient use due to the abundance of a cheaply available resource. One way or another "*Oil is always used as fast as it's pumped out of the ground*" [Ref 183]. This illustrates one variant of "the tragedy of the commons":

> *"Apart from considerations of economic waste, great physical wastage was also taking place due to the drilling of hundreds of unnecessary wells, permitting wells*

2. Some may suggest that energy availability drives consumer behavior, not vice versa. This essentially implies that oil fuels the human engine of clinical greed [Ref 182].

to flow unchecked (with a resulting high gas-oil ratio), wasting reservoir energy, and decreasing amounts of ultimate production." The most deplorable practice was the flaring of huge quantities of natural gas, which instead of being returned to the oil reservoir for the maintenance of pressure was burned in such volume that the glow from the flames in some fields was visible for fifty miles. Experts estimated in 1939 that 95 percent of the gas produced—some 134 billion cubic feet—was flared. [Ref 7]

Think about this. During the 1930's, albeit locked in a depression, crude oil prices dropped to a low of 10 cents *per barrel*. Unprocessed, this meant that you could get 4 gallons per penny. On the other hand in 2008, prices rose enough to make the refined product rise to at least $10 for that same four gallons. Much of that increase in price came about due to inflation, a fact that becomes quite apparent when you consider the inflation-adjusted price of oil has largely remained constant since the 1970's.

Texas Railroad Commission Milestones [Ref 8]

October 9, 1930 — East Texas Field discovered. Production of one million barrels per day drops price from $1.10 to 10 cents per barrel.

November 12, 1932 — Legislature passes law to limit production to market demand. RRC given jurisdiction.

April 13, 1935 — Comprehensive oil and gas law passed to prevent wasteful production.

March 17, 1947 — Fields ordered to shut down to prevent wasteful flaring of natural gas.

So even though conservation and efficient uses of oil has improved since the oil shock of the 70's and 80's, the fact that inflation-adjusted prices have remained constant implies that greed of the same order-of-magnitude exists. For the USA in particular, oil still doesn't cost us much to use in practical terms. In essence, we still see the same price-based consumer buying decisions that we always have. Which means that we can draw a straight-line from the early days until now, and use basically the same math. Greed becomes an invariant in mathematical parlance[3]. So the lack of a universal paradigm shift in fossil fuel consumption practices allow us to apply perturbations to the analysis to discover how scenarios such as current and future oil price shocks play out.

3. The *greedy algorithm* used in many mathematical algorithms makes no judgement calls.

I don't know if this set of premises exists as tacit knowledge[4] or not. Extracting any kind of knowledge from industry insiders remains one of the most frustrating problems we face. By pulling together a model describing the entire life-cycle, we hope to codify and make explicit this perhaps innate information hidden behind boardrooms and chalkboards in the corporate culture. As an oil industry optimist had to say: "*Here it is pertinent to note that peak oil forecasters do not enjoy an undiluted view of the state or corporate portfolios that contain these internal and hidden assessments which their models logically require*." [Ref 165] If I could cast one conspiratorial idea to substantiate such thinking: I would say that it essentially goes against the corporate oil culture's best interests for this knowledge to filter out and fall under a wider sphere of acceptance.

"*The only permitted answer to the effects of greed is more greed.*"

— George Monbiot

"*The world's real superpower is oil.*"

— Deepak Chopra

Consider this perhaps throwaway comment from an oil company executive:

> *"We're more likely to see other companies as collaborators rather than adversaries... We aren't so much competing with other as we are competing with the Earth. And maybe that's a healthy way to look at it."* [Ref 10]

The executive basically states that the only restraint to oil production stands in the way of a captive resource which we can freely capitalize on. This turns into the first-order rule when staging the model for the dynamics of oil depletion.[5] Unrelenting greed becomes the overriding factor in the stimulus. Model it as a process of grab, exploit, grab, exploit, and then exploit some more, and you have captured the oil production mentality. I really don't believe economics plays much of a role in the driving stimulus, as technology and human consumption turn it into a monotonically increasing function. Which makes it a good first-order rule. Like pigs at a trough, we constrain extraction only by how much room we have available to stick our collective snouts into.

These bits of tacit knowledge, and confirmation perhaps inadvertently provided by a oil honcho, basically outlines the entire premise of dispersive discovery and the oil shock model that we will explore. It basically says find as much as you can while you can, and turn on the taps as much as they can handle.

> *"We want to drill like we have never drilled before." "New technologies enables for — to be able to drill like we've never been able to do so before"*
>
> — G.W. Bush at a Rose Garden press conference April 29, 2008

4. tacit: not explicitly or often written down in the technical literature[Ref 9]
5. Conventional wisdom holds that attributing greed as cause-and-effect is akin to blaming gravity for airplane crashes. Yet, greed, like gravity, stays constant and always factors into the analysis.

Symmetry in the eye of the beholder: Granted that we may hit a peak in oil consumption (or have already), the conventional wisdom holds that this peak to look symmetric on the way up as well as on the way down. This leads to the familiar "bell-shaped" curve of peak oil. Shell Oil geologist King Hubbert in the late 1950's sketched a largely symmetric bell curve to reinforce the idea that maximum oil production would naturally and eventually occur.

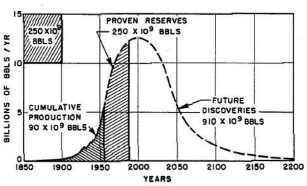


FIGURE 3-1. The classically symmetric Hubbert Bell curve.

The outcome of a symmetric curve has consequences for the pessimists and the cornucopians alike. By inspection one can see that the 50% consumption point obviously occurs at the peak for the Logistic curve, Gaussian normal, and other symmetric profiles. This rule for depletion has in fact become the conventional wisdom among certain analysts, dating back to Hubbert. On the pessimistic side, it means that we have used half our oil at peak. On the cornucopian side, it means we still have half of the oil left. In reality, only the most rudimentary and restricted models of oil depletion assume a symmetric production profile. But once again, asymmetry does not necessarily sway the outcome to either a pessimistic or cornucopian outlook. This geologist echoes Milton Friedman's words from the introduction:

> William L. Fisher, director of the school of geosciences at the University of Texas at Austin, contends that application of the Hubbert curve for resource assessment or for projection of production peaks is seriously flawed.
>
> *It assumes that the amount of oil or gas is known, which it is not. It assumes that the peak will come midway through the production of the resource, thus the symmetry of the curve, which is not necessarily the case. It also assumes that resources are inelastic, not responding measurably to economics and technology.[Ref 11]*

This argument implies that economic forces can extend the curve into asymmetric territory.

So it may turn out that in practical terms and only under certain circumstances, can we get the idealized symmetric curve. In the subsequent chapters, as we introduce the basic concepts for analyzing oil depletion dynamics, the presence or absence of symmetry will become apparent and thus better understood.

Understanding the plan. The comprehensive framework that I advocate has aspects of probability-based forecasting[6]. The salient reason for using probabilistic-based models has to do with reasoning in the face of uncertainty. We never have had and probably never will have perfect and complete data to accurately analyze, much less predict, our current situation. Lacking this, imperfect probabilistic

6. Limited by avoiding the prediction or psychology of collective human actions

approaches serve us very well in our understanding of the fundamentals of oil depletion.

The following figure lays out a flowchart of the current understanding of the different phases of the oil production life-cycle. The heading row provides short names for the life-cycle phases. The first row lists some of the conventional practices used to describe the phases, primarily as a set of heuristics. Below this row appear the model interconnections, which establish the architectural foundations of the comprehensive model. Distinct stages which traverse the conventional phases of the life-cycle draw from elements of probability theory which I use to model the behavior of the phases. Several surprising outcomes derive from the application of the model. For example, we can derive the legendary Logistic heuristic and explain the field size distributions observed. Further, we can use the dynamic elements to track shocks in the production process and extrapolate into the future.

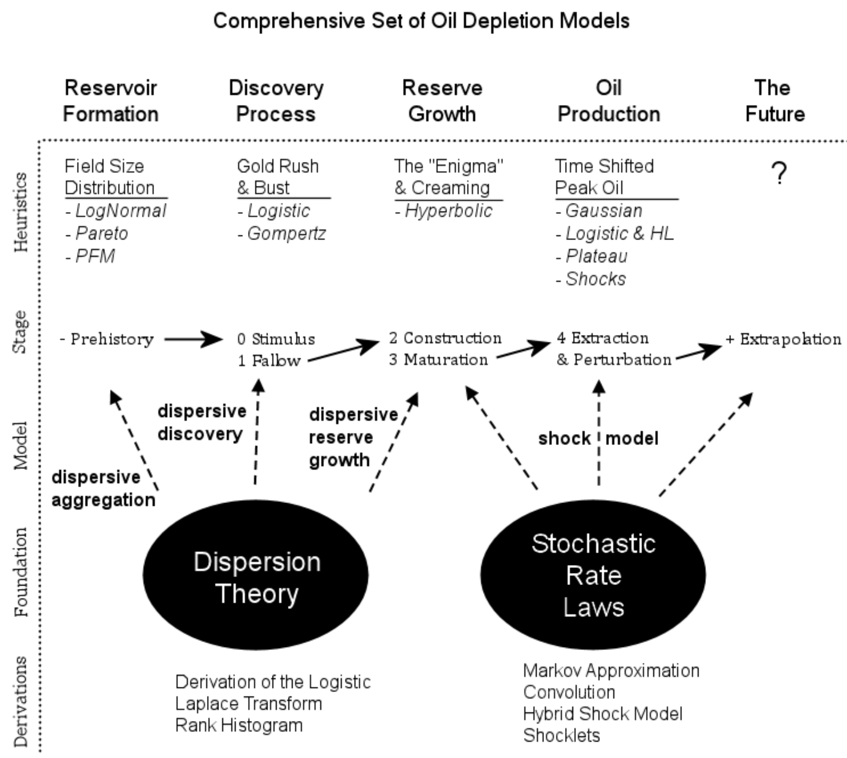


FIGURE 3-2. The roadmap for the analysis includes reuse of simple laws

Again, I don't consider the math behind the models that difficult to understand. The two darkened bubbles above contain the essential probability ideas that I use for parts of the discovery process along with the oil production model. In terms of a strong model, I find parsimony an important characteristic and the interpretations of the model replace a longstanding set of heuristics that in the past have served to cobble together a rather poorly-formed understanding of the aggregated oil production life-cycle[7].

7. In chronological terms, I formulated the front end of the comprehensive life-cycle last as ut took me the longest to generate a model for field-size distributions. So as a practical matter, and fitting into a reverse engineering theme, I spent most of my time working my way backward in the oil life-cycle timeline (right to left in the above figure). Although that gives a bit of the history into how I worked out the model, it makes sense to elaborate it fully from right to left, which follows the actual oil production life-cycle.

CHAPTER 4

The Facts in the Ground.

Where do we find oil reservoirs?

"If Mother Nature can, She will tell you a direct lie."

— Charles Darwin

Sound-proofed drilling rig in urban LA. [Ref 175]

As oil attorney Bruce Webster once complained to Los Angeles magazine: "They ruined a perfectly good oil field by building a city on top of it." [Ref 166]

Let us consider everything that occurred before the first real commercial extraction of oil began in the mid 1850's as *pre-history*. Suffice it to say that a small fraction of the population probably had some idea of petroleum's potential prior to this time via the commercial utility of coal and whale oil. But without a practically available petroleum source it reduces to a historical "what if" exercise, as understanding how prospectors came across fossil fuels doesn't really help explain what came afterwards. However, a few aspects of geology and fossil fuel formation become critically important as we look at the depletion process.

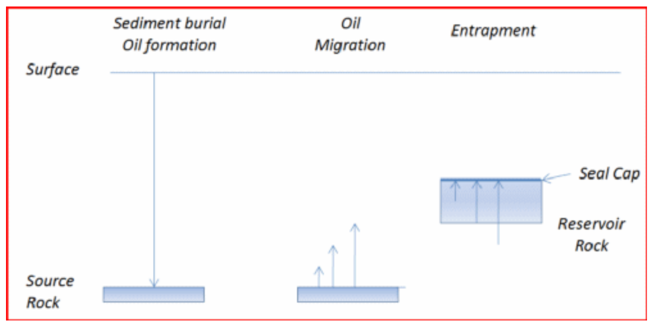
Let us consider first the geological process that leads to oil formations. A good model of this will effectively describe the randomness of oil sources, both in size and location. A fraction of oil that gets created eventually becomes trapped in natural occurring reservoirs that have laid dormant for many years. The locations and sizes of individual reservoirs probably has some form of pattern but for the most part has huge elements of randomness to it. The places that oil can congregate and readily accessible occur sporadically. Structurally the earth has to provide trapping layers, otherwise the oil becomes too dispersed within the earth's crust — in that case, we end up with oil shale or oil sands which contain a suspension of oil that becomes much more difficult to extract. Ideally the "best" traps occur in structural layers that may lie along fault lines (similar to those that can cause earthquakes).

In terms of a timeline, a portion of the oil that initially gets formed in huge beds of dead biological material subsequently migrates from a dispersed state through porous rock until it ultimately reaches these semi-permanent traps lined by impermeable rock. In John McPhee's book *Basin and Range*, he described the effect of geological forces that can move continents over the course of millions of years:

> *Oil also moves after it forms. You never find it where God put it. It moves great distances through permeable rock. Unless something traps it, it will move on upward until it reaches daylight and turns into tar. You don't run a limousine service on tar, let alone a military-industrial complex. If, however, the oil moves upward through inclined sandstone and then hits a wall of salt, it stops, and stays — trapped.* [Ref 180]

As fault lines and similar structural and strata-related defects don't occur uniformly, we end up not finding oil wherever we look. Add to that, the fact that oil itself did not form everywhere, we get left with a sporadic set of proverbial needles in a haystack.

FIGURE 4-1. The effect of structural formations with natural caps tends to trap migrating oil into reservoirs. (from Sam Foucher)



One can analogize the distribution of these structural traps with the amount of defects in so-called perfect crystals, such as a gem-quality diamond. Although occurring on a microscopic scale, the crystalline faults share much in common with their macroscopic cousins. They also play a big part in the electronics industry where defects such as these lead to poor performance and even failures in crystalline solid-state devices. Although difficult to find the defects through a microscopic scan, we have indirect means to quantitatively characterize their density. In essence, this becomes the problem that oil prospectors face: that of adequately characterizing the number of oil-bearing faults that the earth's crust contains. However, unlike the semiconductor industry, we want to seek out and maximize the number of these defects. Only by finding more structural anomalies do we have hope in finding more oil.

The other aspect has to deal with the sizing of individual reservoirs. Through years of discovering and then estimating reservoir sizes, analysts have empirically guessed at various *probability distributions* for reservoir sizes. In general, we unsurprisingly reach the conclusion that smaller reservoirs occur much more frequently than larger reservoirs. And the largest, the so-called super giants, occur very rarely.

With the empirical evidence for reservoir size distributions at hand, we can justify statistically to some degree how it came about. A few parameters have important consideration to how the sizes of reservoirs evolved**:** migration rate, available supply, and time. A concept that we will revisit several times over involves the randomizing factor of *dispersion*.

Filling the Reservoirs. If not for these structural traps, we may never have had the chance to even encounter reservoirs of oil. The natural driving force of entropy tends to mix materials to a uniform consistency over time and only the addition of energy or the formation of enclosures with a sufficient energy barrier allows some sort of homogeneity of matter such as we find with oil reservoirs. How the oil gets there we can reason with some clarity.

Assuming that the formation of oil over millions of years involves the following basic steps:

1. Formation of a layer of organic material (mainly prehistoric zooplankton, algae) at the bottom of a lake or ocean under anoxic conditions (no oxygen).
2. Sediment burial and diagenesis: the rise in pressure and temperature is transforming the organic materials into kerogen.
3. Catagenesis (or cracking): organic kerogen transforms into lighter hydrocarbons.
4. Migration: because most hydrocarbons are lighter they migrate in adjacent rock layers.
5. Entrapment: eventually the oil is collected within a reservoir rock below a seal or cap rock, with low permeability that impedes the escape of hydrocarbons from the reservoir rock.

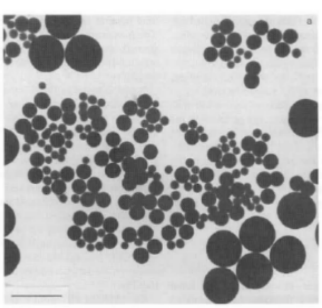


FIGURE 4-2.
Typical distribution of particle sizes in a suspension

From considerations of steps 4 and 5 and drawing parallels to material nucleation and growth processes[1], one can grasp the fundamentals that go into oil reservoir size distributions. Deep physical processes go into the distribution of field sizes, yet I contend that some basic statistical ideas surrounding kinetic growth laws may prove more useful than understanding the fundamental physics of the process. To make the case even stronger, I use the same ideas from the model of Dispersive Discovery which I will use in a later chapter to demonstrate how as humans sweep

1. In fact, I see many practical similarities between the two processes. For example, instead of individual atoms and molecules, we deal with quantities on the order of million-barrels-of-oil; yet the fundamental processes remain the same: diffusion, drift, conservation of matter, rate equations, etc. See Volume 2.

through a volume searching for oil leading to oil discoveries, so too can oil diffuse or migrate to "discover" pockets that lead to larger reservoirs.

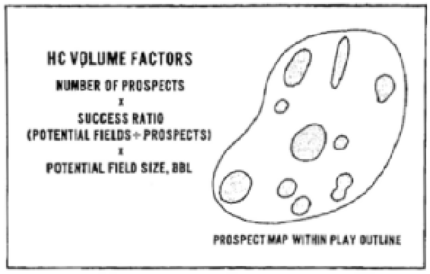


FIGURE 4-3. Range of oil field sizes in a "play", from Baker-Gehman

The premise that varying rates of advance can disperse the ultimate observable measure leads to the distribution we see. For oil discovery, the amount gets dispersed with time, while with field sizes, the dispersion occurs with time as well, but we see the current density as a snapshot in a much slower glacially-paced geological time. For the latter, we will never see any changes in our lifetime, but much like tree rings and glacial cores can tell us about past earth climates, the statistics of the size distribution can tell us about the past field size growth dynamics.

Laherrere estimates that worldwide we have had on the order of 10,000 crude oil discoveries; other estimates range up to 50,000. Pepper this over a range of 100 years and you get a relatively small sample size to deal with per year. This small sample number over a reservoir size distribution has traditionally followed a log-normal function[2], which has the nice property of preventing negative sizes by transforming the independent variable by its logarithm (i.e. logs of the values follow a normal distribution). This pattern also seems to work for natural gas reservoirs:

> *Lognormal distributions — a method based on the observation that, in large well-explored basins, field-size distributions approximate to a lognormal distribution. The method is most reliable with large datasets, i.e., in mature basins, and may be unreliable with small datasets.[Ref 12]*

2. Another distribution often cited to describe reservoir sizes is called the Pareto distribution, aka Zipf's Law [Ref 96]. This uses hyperbolic curves so it has convergence problems, so a truncation is usually applied.

As the variance tightens around the mean, the shape of the curve peaks away from zero. But importantly, a large variance allows the larger-than-average sizes (the "super-giants") to appear.

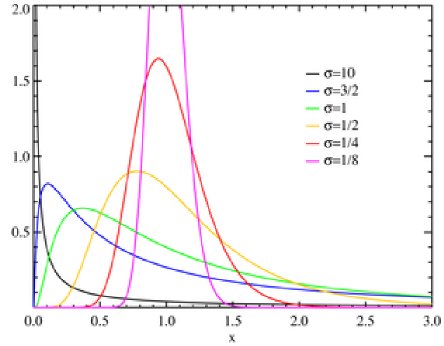


FIGURE 4-4. Various orders of a log-normal distribution purport to describe the relative probability of reservoir sizes. Low orders weight heavily to smaller sizes.

From the best understanding of the fundamental process, we can thus assert that the physical basis for a peaked distribution (away from truly small sizes) likely has to do with coalescence and agglomeration of deposits. Much like cloud droplet and aerosol particulate distribution (which also show a definite peak in average size due to coalescence), oil deposits have a similarity in structure if not scale that we can likely trace to fundamental processes.

Dispersive Aggregation Model

The model I will derive next seems to work better than conventional heuristic models (such as the Pareto, log-normal, and fractal) and it derives in a similar manner to the discovery process itself. If oil can tend to seek out itself or cluster via settling in low energy states and by increasing entropy via diffusing from regions of high concentration, this we can consider as a discovery process. So as an analogy I assume that oil can essentially "find" itself and thus pool up to some degree. By the same token, the ancient biological matter had a tendency to accumulate in a similar way. In either case, this process has taken place over the span of millions of years. After this "discovery" or aggregation takes place, the oil doesn't get extracted like it would in a human-accelerated discovery process but it gets stored *in situ*, ready to be rediscovered by humans. And of course consumed in a much shorter time, by many orders of magnitude, than it took to generate!

We first assume that oil does indeed migrate from its original creation point through permeable rock to such traps. The buried organic material exists at great depth where it transforms into lighter hydrocarbons by heat and pressure. Then the hydrocarbons eventually start migrating from the source rock to adjacent rock layers. The rate ***r*** at which it does this we treat as a stochastic variable with a probability density (oil migration acts as a random process whereby the diffusion rate follows an exponential law):

$$p(r|g) = \frac{1}{g} \cdot e^{-\frac{r}{g}} \qquad \text{(EQ 4-1)}$$

This introduces two concepts at once: the idea that we do not assume a single rate (i.e. assume instead *dispersion*) together with the idea that we can only assume at best a mean (as the growth rate ***g****) and* treat the standard deviation as equivalent to the mean. This type of assumption makes the least presuppositions as to what has happened — we know we have a mean value but beyond that, the rate can vary to the maximum entropy limit. To put a label on it I will refer to this mechanism as entropic dispersion[3].

If we next assume that a collection of these rates can act to sweep out a selected volume of somewhat uniformly deposited oil, then over time we can imagine that a structural trap can collect this migrating oil. Intuitively, we can imagine since these formed over many different timescales of the earth's history, that we will get a distribution of partially filled reservoir sizes according to how long they have collected migrating oil.

Let's say that the oil diffuses upwardly from the source rock, so for a given time period ***t***, oil will diffuse over a distance ***x***=***rt***, a simple variable change gives[4]

$$p(r|g) = \frac{1}{gt} \cdot e^{-\frac{x}{gt}} \qquad \text{(EQ 4-2)}$$

Over time, the probability that some oil will migrate at least a $\boldsymbol{x_0}$ distance is:

$$p(x > x_0|g, t) = \int_0^{\infty} p(x|g, t)dx = e^{-\frac{x_0}{gt}} \qquad \text{(EQ 4-3)}$$

Alternatively, the following relation tells us the cumulative probability of the distance covered by material after time ***t***. This again assumes a distance travelled ***x*** = ***rt***.

3. The idea of entropic dispersion will turn into a recurring theme throughout the text.
4. By nomenclature convention, we define two classes of probabilities, which differ by how the probability densities normalize
conditional probability: p(random variable | parameters)
joint probability: p(random var_1, random var_2)

$$P(x_0|g) = \int_{r = x_0/t}^{\infty} p(r)dr = e^{-\frac{x_0}{gt}} \quad \textbf{(EQ 4-4)}$$

This relation also crops up in terms of the *population balance equation*. It basically relates a conservation of particles law, in that we do not lose track of any material due to a flow.

If no oil trap (or seal cap) exists, all the migrating oil will ultimately dissipate and disappear.

So next we have to accumulate this over a volume or depth at which we think the oil exists within. Let us assume that a seal cap exists at some depth ***x***. The simplest approximation assumes that the oil gets distributed to a mean depth (***L***) with a similar exponential distribution

$$f(x|L) = \frac{1}{L} \cdot e^{-\frac{x}{L}} \quad \textbf{(EQ 4-5)}$$

Combining the two relations turns into an *a priori* probability for the expected cumulative transfer after time ***t*** through the volume. Integrating over the entire earth crust column (this vertical column has an horizontal cross-section of unity), gives the average oil trapped, ***U***, at a mean depth:

$$\overline{U}(t|L) = \int_0^{\infty} f(x|L) \cdot P(x|g)dx = \int_0^{\infty} f(x|L) \cdot e^{-\frac{x}{gt}}dx$$
$$\overline{U}(t|L) = \frac{1}{\left(1 + \frac{L}{gt}\right)} \quad \textbf{(EQ 4-6)}$$

For the last assumption, we note that if ***t*** gets evenly spread from the start of prehistory, some hundreds of millions of years ago, then the value ***gt*** becomes the effective collected thickness ***W*** of a distribution of reservoirs by $W = k \cdot g \cdot t$, where we add a factor ***k*** to indicate collection efficiency. The collection or trap efficiency factor works in conjunction with the migration drift factor ***g*** (understood as some product of reservoir rock porosity, oil saturation, formation factor and seal impermeability factor). Alternatively, we can interpret the stochastic variable ***W*** as the maximum net reservoir thickness that would develop over a diffusion time ***t*** if a perfect seal cap situated near the mean depth ***L*** (see Figure 4-1 on page 28). The term ***kL*** sets the potential maximum net thickness achievable if all the reservoir rock between the source rock and the seal cap saturates with oil, so it turns into a type of hyperbolic discounting probability distribution.

$$\bar{U}(T|L) = \frac{1}{\left(1 + \frac{k \cdot L}{W}\right)} \qquad \text{(EQ 4-7)}$$

This relation states that the cumulative probability of reservoirs of less than or equal to ***W*** starts at 0 for very small reservoirs and slowly approaches 1 (unity) for the largest possible reservoir. In practical terms, if ***L*** remains close to zero, nature has a greater chance to capture large amounts of migrating oil. On the contrary, if ***L*** takes on a large value, there will be no significant accumulation because of the large distance between the source rock and the reservoir rock[5].

From now on, we can work in terms of field size ***Size*** = ***W*A*** by integrating on a given geographical area ***A***

$$\bar{U}(Size|L) = \frac{1}{\left(1 + \frac{k \cdot L \cdot A}{Size}\right)} \qquad \text{(EQ 4-8)}$$

Note that if we set <***Size***> to the characteristic median field size (defined by the cumulative distribution equaling 0.5) then the equation reduces to:

$$\bar{U}(S < \text{Size}) = \frac{1}{1 + \frac{\langle\text{Size}\rangle}{Size}} \qquad \text{(EQ 4-9)}$$

This again describes the cumulative distribution of all reservoirs *below* a certain size. If we need to know the cumulative distribution *above* a certain size, we take the complement of this distribution, which results in the subtle difference of inverting the ratio in the denominator.

$$\bar{U}(S > \text{Size}) = \frac{1}{1 + \frac{Size}{\langle\text{Size}\rangle}} \qquad \text{(EQ 4-10)}$$

In a moment we will see how this gets compared to actual data in terms of a rank histogram, but we can glean some insight by looking at the probability density function corresponding to the derivative of the cumulative:

$$p(Size) = \frac{d\bar{U}}{dSize} = \frac{\langle\text{Size}\rangle}{(\langle\text{Size}\rangle + Size)^2} \qquad \text{(EQ 4-11)}$$

5. This significant result also serves as a building block for the discovery model

This clearly shows a declining probability of large reservoirs in comparison to small reservoirs. It also limits the frequency of small reservoirs to a finite probability. This occurs because of the relatively long passage of time and fast dispersers in the mix allow for initial nucleation of volumes away from size 0.

Comparison with real data. Michel provided a decent set of data for reservoir size distribution ranking of North Sea fields in 2008 [Ref 13]. In his paper, Michel tried to make the point that the shape follows a Pareto distribution, which shows an inverse power law with size.

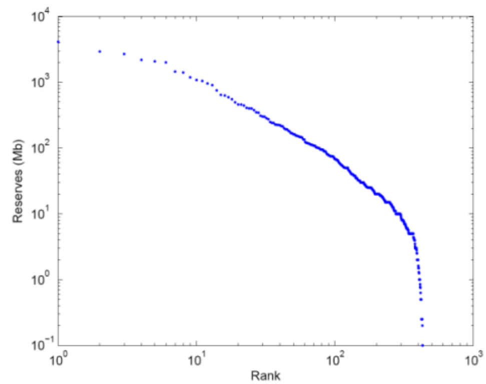


FIGURE 4-5. Rank histogram of reservoir sizes of North Sea fields. Relatively few large fields get discovered and smaller fields drop off in frequency as well.

This kind of rank plot is easy to generate and shows the qualitative inverse power law, close to 1/*Size* in this case. The curve also displays some anomalies, primarily at the small field sizes portion and a bit at the large field sizes.

This derivation competes against other heuristics. Background on the Pareto as well as the Parabolic Fractal Law is described in [Ref 14], with analysis of the log-normal as used by USGS in [Ref 15] and some case studies for Norway [Ref 16] and Saudi Arabia [Ref 17].

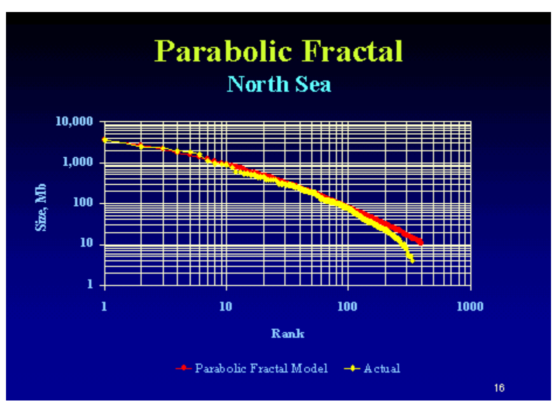


FIGURE 4-6. Parabolic Fractal Law fit to North Sea Data. Note that the PFL does not generate an asymptote at high rank values.

Neither the Pareto nor the Parabolic Fractal Law fit the extreme change of slope near the small field size region of the curve. The log-normal does better than this for small fields (as it does not blow up) but does not appear universally accepted[6].

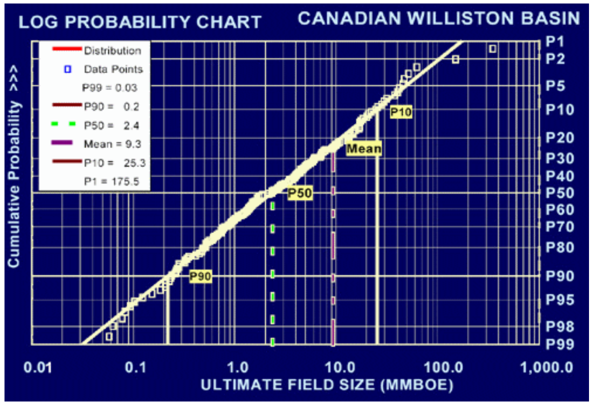


FIGURE 4-7.
Field size distribution plotted on a special log-normal graph to linearize the data

The following figure takes Michel's rank histogram and exchanges the axis to convert it into a regular (binned) histogram. The fitted curve assumes the Dispersive Aggregation model [7]. The curve essentially describes the cumulative $1/(1 + Size/\langle\text{Size}\rangle)$, where <Size>=20 Mb is the characteristic median dispersion size derived from the original exponential distribution used. The plotting algorithm for the original curve essentially traces the following expression where a normalized cumulative expressed as a fraction varies between 0 and 1. Note that this formulation makes it look a lot like a conventional odds function (see Volume 2)

$$Size = \langle\text{Size}\rangle \cdot \left(\frac{1}{U} - 1\right) \qquad \textbf{(EQ 4-12)}$$

For field sizes, we can equate this to a natural growth accumulation, where the average growth rate integrated over fixed a length of time would start to see the effects of aggregation above the median. In other words, the median value 20 Mb describes the equivalent size that a columnar aggregation process would need to sweep through before it reaches the 50% rank in population

6. The log-normal also gets used in derivations of small particle aerosol size distributions, which also show very few small particles because of energy considerations regarding surface tension in critical droplet formation (http://www.lassp.cornell.edu/sethna/Nucleation/). Also for molecular weight distributions.
7. This differs from the reserve growth of discovery discussed later only in the sense that the cumulative starts from 100% instead of zero; in other words, in a region near the origin, just about all reservoirs reach at least this size.

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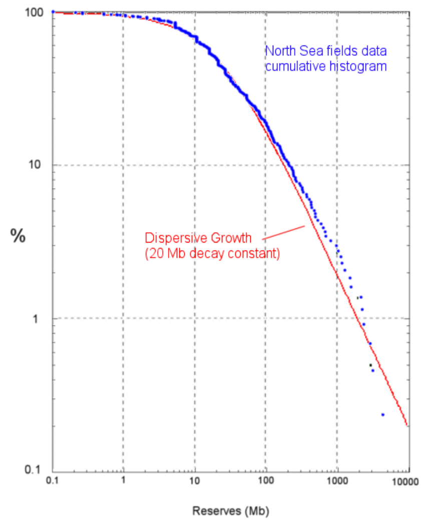


FIGURE 4-8. Dispersive Discovery model plotted alongside North Sea data, using a cumulative density format.

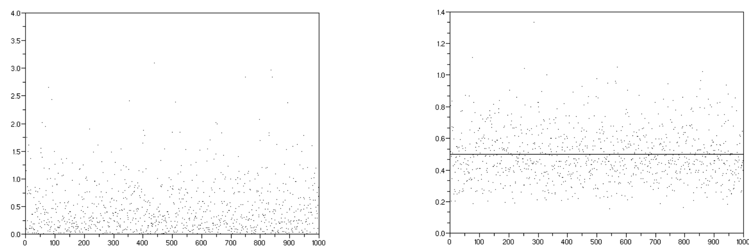


FIGURE 4-9. Damped exponential distribution (left) and clustered distribution after accumulation (right)

I can see another straight line through points which would give a slope of $1/Size^{0.96}$, but in general, for this region a single parameter controls the curvature via $1/(1+\text{Size}/(20MB))$. If put into the context of a time-averaged rate, where the inflection point $Size = k \times Time = 20$ Mb, and where k is in terms of average amount migrated per geological time in a region, you can get a sense of how slow this migration is. If *Time* is set to 300 million years, the constant k comes out to less than 1/10 barrel per year on average. The dispersion theory gives a range as a standard deviation of this same value, which means that the rates slow to an even more glacial crawl — as well as speed up enough to contribute to the super-giant fields over the course of time (millions of years in this case).

Even though the approach relies on kinetic (not equilibrium) arguments, this works out as much by a conservation of mass argument as anything else. If a volume gets completely swept out, via diffusion and seepage, and all the oil in a region congregates, it becomes the biggest possible reservoir with a rank equal to unity or 1. Yet, it has to equal the volume of the original distribution. The curve essentially shows cross-sections of the advancing mean at various stages of time, i.e. a moving finish line. So we assume that the last, *rank*=1, point is the finish line. Since the dispersion assumes a constant standard deviation relative to the mean, the stationary assumption implies that the rest of the distribution fractionally scales to match the extent of the fastest flow. So I have a feeling that this recasts the fractal argument, but only adding a starting exponential distribution which eliminates the pure 1/*Size* dependence of the fractal or Pareto distribution.

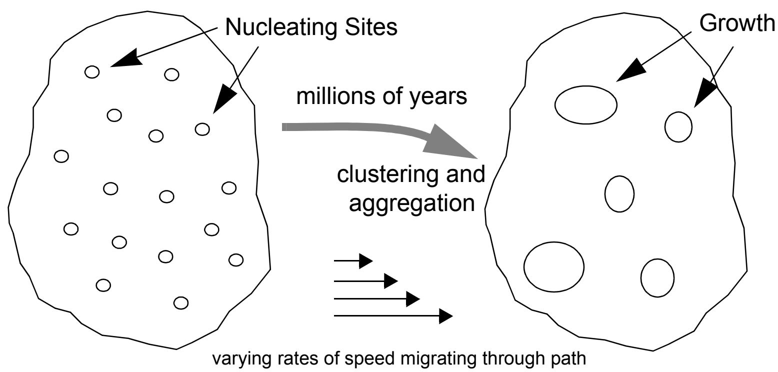


FIGURE 4-10. Schematic of reservoir growth demonstrating agglomeration of material under varying rates of drift or diffusion.

In a global context and given enough time, this simple kinetic flow model would eventually grow to such an extent that a single large reservoir would engulf the entire world's reserves. This does not happen however and since we deal with finite time, the curve drops off at the extreme of large reservoir sizes. We also can't wait for an infinite amount of time so we have never and likely will never see the biggest reservoir sizes (Black Swan events notwithstanding[8]).

So if we extended the following figure to show 1/*Size* dependence over an *infinite* range, this would of course only hold true in an *infinite* universe. I can't confirm because of the poor statistics we have to deal with, i.e. ***N***=small, but the super-giants might just sit at the edge of the finite time we have to deal with.

8. Also referred to as the "unknown/unknown", we will discuss this further in."Cornucopian Conundrums. How do we reconcile against optimistic analyses?"

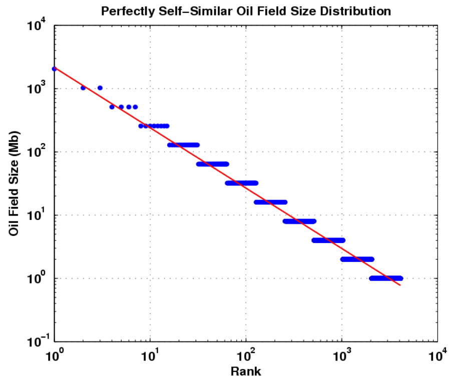


FIGURE 4-11. Rank histogram of a self-similar field size distribution with data "binned" into size ranges. This means it aligns well along an inverse power law curve.

Now we can return to the question of what other behaviors might influence the rate of accumulation and aggregation underground. Oil does move around through the processes of drift, diffusion, gravity drainage, buoyancy, and it does this at various rates depending on the porosity of the region. The reason that small particles, grains, and crystals show this same type of growth also has to do with a dispersion in growth rates. Initially, all bits of material start with a nucleating site, but due to varying environmental conditions, the speed of growth starts to disperse and we end up with a range of particle sizes after a given period of time. The size distribution of many small particles and few large ones will only occur if slow growers exponentially outnumber fast growers. The same thing must happen with oil reservoirs; only a few show a path that allows extremely "fast" accumulation (I say "fast" because this still occurs over millions of years). From studying the distributions of other naturally dispersive behaviors such as marathon races (see Volume 2), we essentially can confirm the same intuitive behavior. Only the fastest of the dispersers will maximize the amount of ground covered (or material accumulated) in a certain period of time.

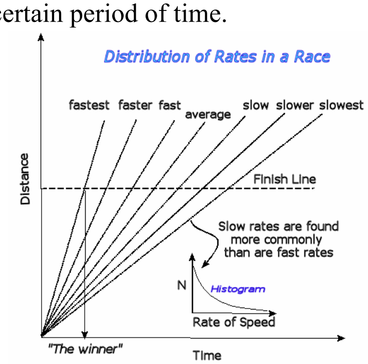


FIGURE 4-12. In Dispersive Aggregation, rates get distributed along a declining exponential probability density function. Slower rates occur more frequently and faster rates become exponentially more rare. The fastest rates will accumulate more in a given time than the slower rates. This race to the finish line in geological terms leads to the size distribution observed.

Before applying a more refined size distribution to the analysis, we have to incorporate some caveats and assumptions. Most importantly, one should not use field size distribution arguments alone to estimate Ultimately Recoverable Resources (URR) because no "top" exists for the cumulative size. This arises from an inability to represent the constrained size of the container that all the fields will eventually fit into.We can only do this proportionally according to the distribution we observe. So we know this would track to a value proportional to the volume of the earths crust, yet we can't pin down exactly this proportionality constant.

The dispersive discovery model considered later (which uses the same math) only considers the size of the container, while the dispersive aggregate figures out the distribution of sizes within the container. By explicitly including a URR-style limiting container, it becomes much more useful for extrapolating future reserves. I find it interesting though in how the two approaches complement each other. And as long as discoveries occur in a largely unordered fashion (I assert that we do not necessarily find large oil reserves first), applying the dispersive discovery curve independently makes the analysis more rigorous and straightforward[9].

Foucher and Laherrere make some good points concerning the Parabolic Fractal Law as some curves show significant "bending" as this size distribution from Mexico demonstrates:

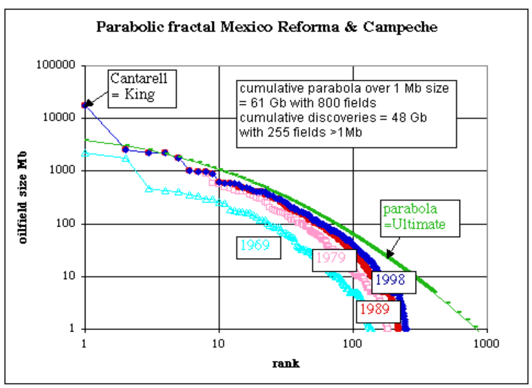


FIGURE 4-13. Field size distribution of Mexico oil fields. The various curves show maturation of the rank histogram estimates through the years. The PFL shows a continuous bending arc along the data points.

We need to run Monte Carlo simulations to determine what noise we can expect on the histograms and perhaps whether it accounts for deviations at extreme values. Let us go back to the North Sea data. The figure below shows a Monte Carlo run for the Dispersive Aggregation model where I sampled from the inverted distribution with *P* acting as a stochastic variate:

9. The USGS claims differently and assumes that reservoir sizing can predict ultimate reserves [Ref 15]. We will discuss this later in the section on reserve growth.

$$Size = c \cdot \left(\frac{1}{P} - 1\right) \qquad \textbf{(EQ 4-13)}$$

where P=[0:1]. For a run of 200 samples, the results look like:

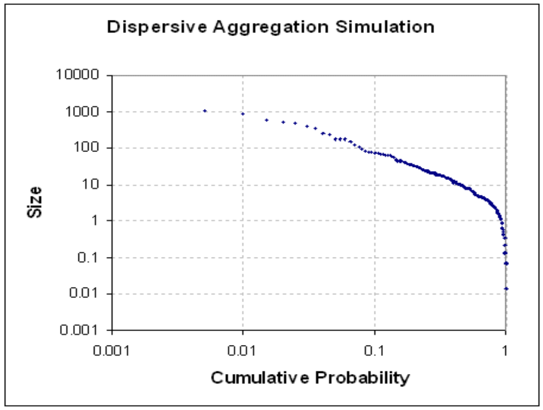


FIGURE 4-14. Monte Carlo simulation assuming Dispersive Aggregation. Because of the finite sample space, few large fields occur.

One can linearize this curve by taking the reciprocals of the variates and replotting. Note the sparseness of the endpoints which means that random fluctuations could change the local slope significantly (which has big implications for the Parabolic Fractal Model as well).

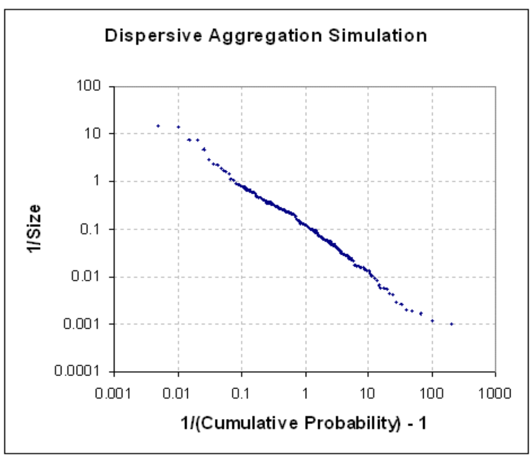


FIGURE 4-15. Linearizing of the Dispersive Aggregation simulation data allows us to provide better “eye-balling” in the curve fitting procedures. This relates to Hubbert Linearization discussed later on.

Plotting the Monte Carlo data simulated for 430 points on top of the actual North Sea data and we get this:

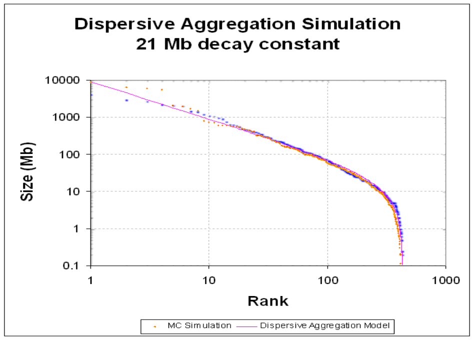


FIGURE 4-16. Dispersive Aggregation simulation plotted alongside North Sea data in a rank histogram.

The following figure gives a range for the single adjustable parameter in the model. For the North Sea oil, I replotted using the *MaxRank* and two values of *C* which bounded the maximum value.

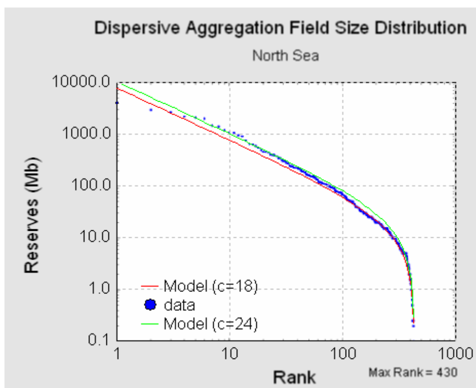


FIGURE 4-17. Replotting the previous figure demonstrates the sensitivity to the decay constant.

The parameter C acts like a multiplier so it essentially moves the curves up or down on a log-log plot.

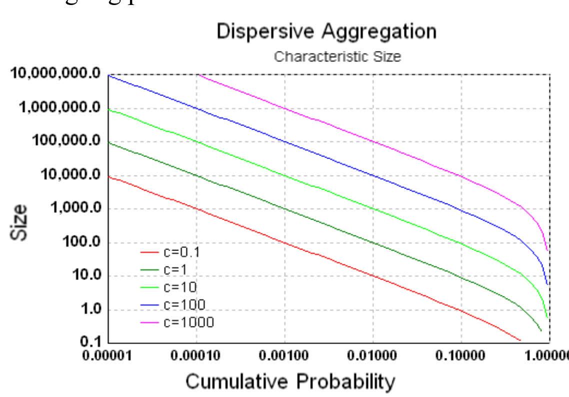


FIGURE 4-18. Varying the decay parameter characteristic size generates a family of curves.

One can try to estimate URR from the closed-form solution but as I said before, the lack of a “top” to the data makes it unreliable. The actual distribution goes like 1/*Size* so that this integrates to the logarithm of a potentially large number — in other words, it diverges. So unless one can put a cap on a maximum field size, *ala* the PFM’s curvature, the URR can look infinite. From the model’s perspective, one can emulate this behavior by eliminating/censoring a narrow window of probability for those large reservoir sizes.

Geologic Time and URR

In terms of geological time, we have one finish line, corresponding to the current time. Yet the growth lifetimes for the dispersion to occur correspond roughly to all the points between now and the early history of oil formation some 300 million years ago. So we have to integrate to average out these times.

$$P(Size) = \int_0^{Now} p(Size)dSize = \int_0^{Now} p(kt)dt \qquad \text{(EQ 4-14)}$$

where the value of ***Now*** you can consider as roughly 300 million years from the start of the oil age. Small values of t=*time* correspond to the start of dispersion at longer times ago and higher values result in values closer to the present (*Now*) time. The number t itself scales proportionately to the rank index on a field distribution plot if dispersion proceeds more or less linearly with time ($kt \sim Size$). Also, a rank value of unity corresponds to the largest value on a rank histogram plot from which can estimate the Maximum Field Size. Given a mature enough set of field data, this provides close to the ceiling for where fields cannot aggregate further.

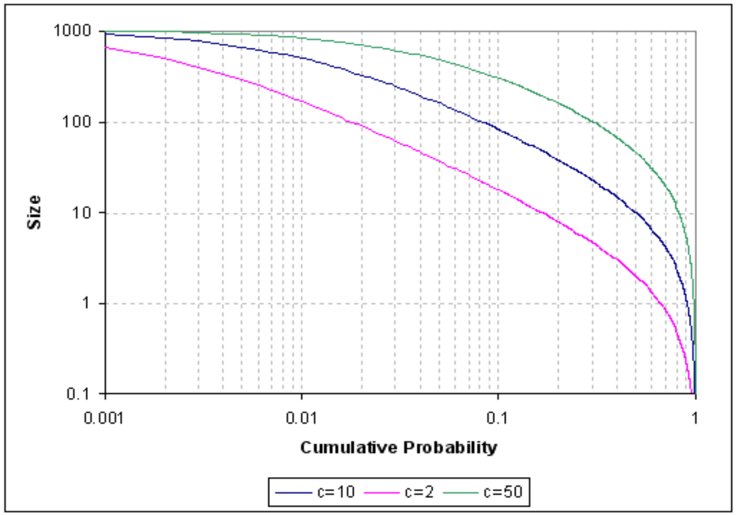
We essentially blank-out a probability window for field sizes above a certain value. This gives the following renormalization based on the Dispersion relation:

$$\begin{aligned} &\text{original:} \quad & P(Size) &= \frac{Size}{Size + \langle \text{Size} \rangle} \\ &\text{renormalized:} \quad & P(Size) &= \frac{Size(\langle \text{Size} \rangle + Max)}{(Size + \langle \text{Size} \rangle)Max} \end{aligned} \qquad \text{(EQ 4-15)}$$

and then inverting

$$Size = kL \cdot \frac{P}{\left(1 - P + \frac{\langle \text{Size} \rangle}{Max}\right)} \qquad \text{(EQ 4-16)}$$

The following set of curves shows the dispersive aggregate growth models under the conditions of a maximum field size constraint, set to L=1000.



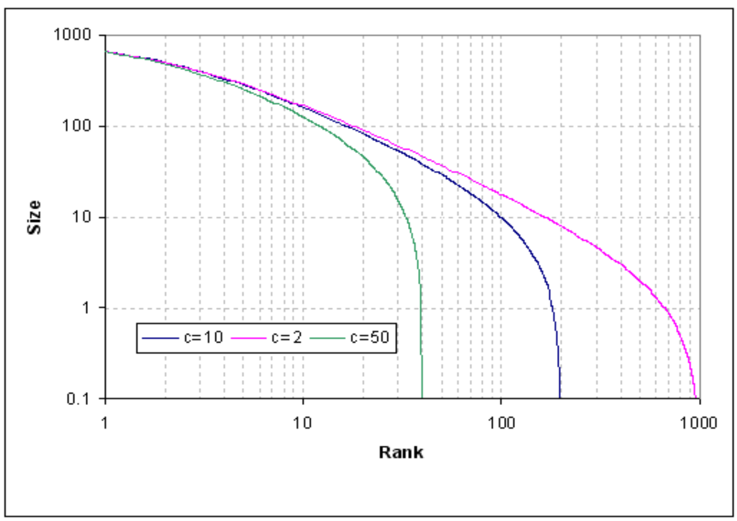


FIGURE 4-19. By applying a maximum field size constraint the probability field size distribution hits an asymptote (top). Plotting this same family of curves in terms of a rank histogram, the bending from linearity becomes more pronounced (bottom).

Converting this graph to a rank histogram and you will notice an interesting constrained transformation taking place. Since we now do have a constraint on field size, we can calculate an equivalent URR for the area under the curve.

We need to use the rank histogram to get the counting correct. Then the URR derives to:

$$URR = MaxRank \cdot C \cdot \left(\left(1 + \frac{C}{L}\right) \cdot \ln\left(1 + \frac{L}{C}\right) - 1\right) \qquad \textbf{(EQ 4-17)}$$

for most cases, this approximates to:

$$URR \cong MaxRank \cdot C \cdot \left(\ln\left(\frac{L}{C}\right) - 1\right) \qquad \textbf{(EQ 4-18)}$$

Note that the URR has a stronger dependence on the characteristic median parameter *C*=<*Size*> than the maximum field size *L*, which has a weak logarithmic behavior. I will discuss the case of the USA later in this chapter but keep in mind that Americans have drilled more oil fields by far than anyone else in the world, i.e. a huge *MaxRank*, yet our URR does not swamp out everyone else.

Case Studies

To test the model against reality, I retrieved field size data from Foucher's study on oil field sizes [Ref 14] and Laherrere's paper on "Estimates of Oil Reserves"[Ref 18].

- North Sea (see above)
- Mexico
- Norway
- World (minus USA/Canada)
- West Siberia
- Niger

Plus one estimate

- USA

This chart from Laherrere shows data from Mexico superimposed with the Dispersive Aggregation model (no field size constraint). Note that the super-giant field Canterell may fall in the predicted path and not form some sort of outlier[10].

10. As some have suggested due to its origination as a singular meteor impact event

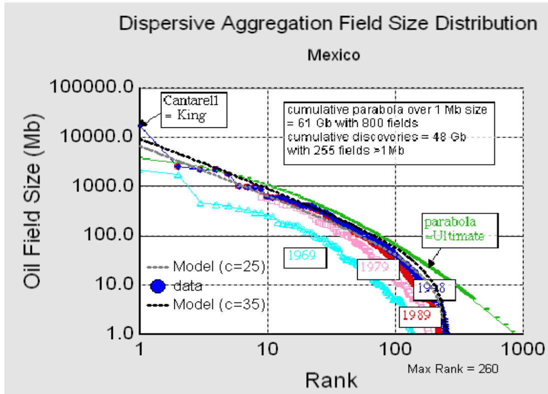


FIGURE 4-20. Dispersive Aggregate simulation data over-laid on Mexico field size data.

For Norway (courtesy of Foucher and Laherrere) we get the following two curves with data separated in time by several years. Note how the Maximum Rank shifts right as the value of *C* grows with time. Production decisions might play a large role in how this curve transforms over time. Producers may decide to activate smaller sized fields only later in a cycle and therefore rank data may get deferred for several years from the earlier histogram data. This points to real analysis ambiguity: we can't easily separate (1) the decision of not developing smaller fields from (2) an actual physical limit on the number of small fields that we count as production-level discoveries. Either of these choices, the first man-made and the second geologically dispersive lead to the bending of the curve at high rank.

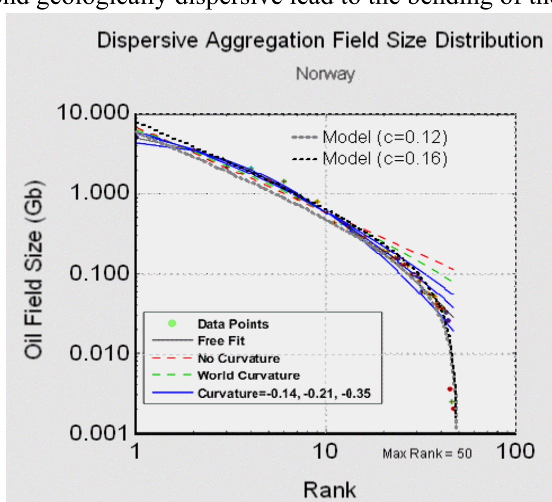


FIGURE 4-21. Dispersive Aggregation simulation overlaid on Norway field older size data. (Caution: The values for C are in Gb, so they have to be multiplied by 1000 to match the other C's in this set)

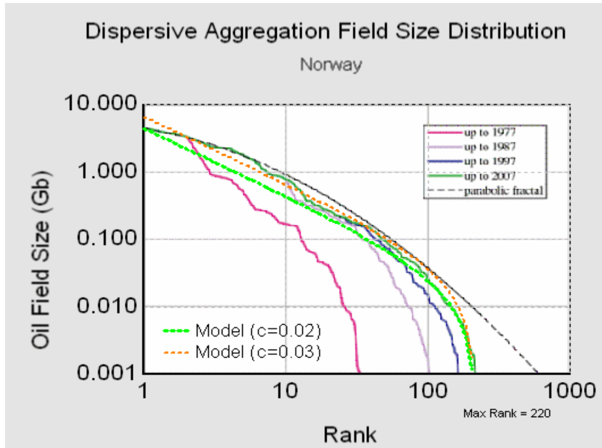


FIGURE 4-22. Rece nt data for Norway plotted along a PF law model.

The World data plot (excluding USA and Canada) from Laherrere does not collect rank info from the smaller oil fields, so the vertical asymptote shown here gives a prediction of a Maximum Rank, approximately 9000 fields worldwide.

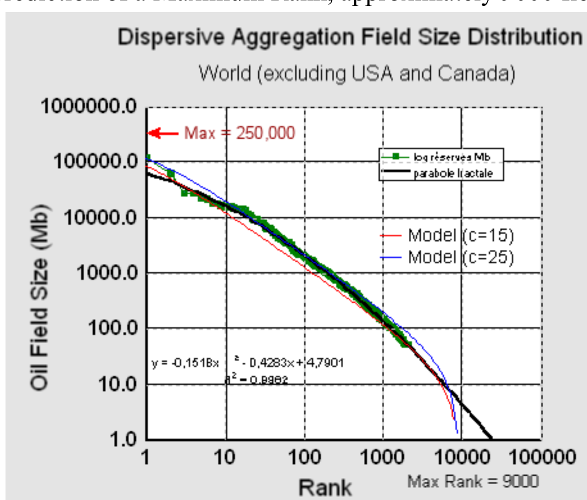


FIGURE 4-23. Dispersive Aggregation discovery model for the world, not including USA and Canada data. The data from the USA contains several times more fields than get included in the rank histogram shown here.

This gives a range in URR's from 1100 Gb to 1850 Gb, for values of ***C*** from 15 to 25 and a ***Maximum Field*** size of 250 Gb. I estimated the ***MaxRank*** for this model from Robelius' thesis [Ref 19]:

> An article by Ivanhoe and Leckie (1993) in Oil & Gas Journal reported the total amount of oil fields in the world to almost 42000, of which 31385 are in the USA. According to the latest Oil & Gas Journal worldwide production survey, the total

> number of oil fields in the USA is 34969 (Radler, 2006). The number of fields outside the USA is estimated to 12500, which is in good accordance with the number 12465 given by IHS Energy (Chew, 2005). Thus, the total number of oil fields in the world is estimated to 47 500.

From Foucher, the parabolic fractal model gives a low-end estimate ignoring the missing parts of the rank histogram:

> Using his (Laherrere's) parameters, we can compute a world URR (excluding the US and Canada, conventional oil) equals to 1.250 Trillions of Barrels (Tb) without considering oil fields with sizes below 50 Mb.

This chart from West Siberia bins histogram data on a linear plot.

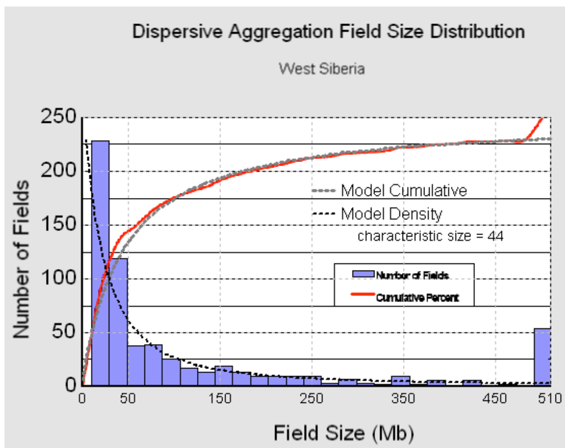


FIGURE 4-24. Dispersive Aggregation model plotted against West Siberia data, in a linear CDF format.

Niger Delta data does not work very well at all. This could potentially work well as a candidate for constrained reservoir sizes, yet we can not rule out the possibility that some large fields have avoided discovery thus far.

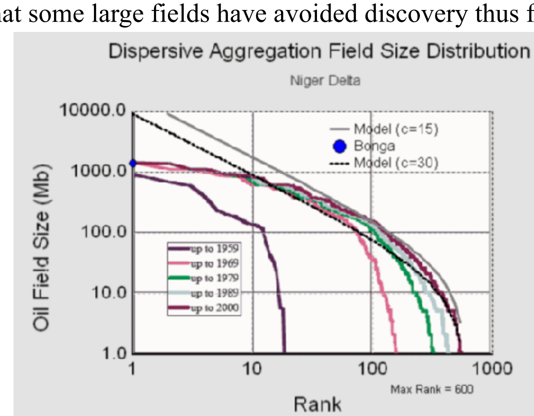


FIGURE 4-25. Dispersive Aggregation model plotted against Niger Delta data.

I have yet to find a field size distribution specifically for the USA, but I generated the following figure as a prediction. I used a maximum rank of 34500 from Robelius' thesis and came up with two curves, with one assuming a maximum field size of 10 Gb (lower curve). This latter curve corresponds to a URR of 185 Gb. If I used ***C***=0.7 and ***Max Field*** of 15 Gb, then I estimate a URR of 217 Gb. Later in this chapter I will add data from the USA to see how closely the Dispersive theory will agree with such a large (34,500) statistical sample[11].

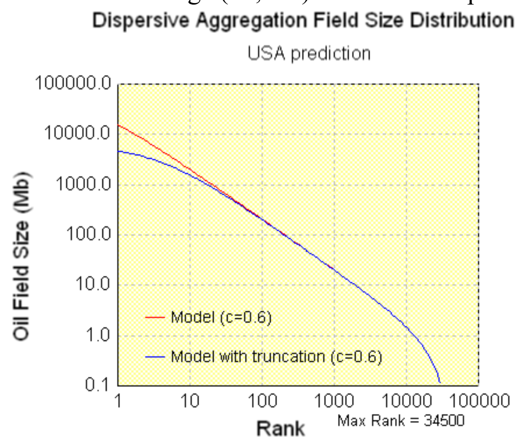


FIGURE 4-26. Dispersive Aggregation model for USA assuming the estimated max number of fields cumulatively put into production. This becomes the Max rank value. The Max Field size matches the largest field found so far.

Overall, most of the characteristic size (*C*) parameters for all the field size distribution curves fall in the range of 15 to 30 Mb (Siberia at 44), except for the USA which looks definitely less than 1 Mb. What exactly does this mean? For one, it might indicate that the USA has a much higher fraction of smaller oil fields than the rest of the world. Is this actually due to more resources invested into prospecting for smaller oil fields than the rest of the world? Or is it because the USA has a physical preponderance of smaller oil fields? This could result from different accounting mechanisms and the difference between a field and a reservoir.[12]

You can also well ask why the curve nose-dives so steeply near the maximum rank. It really only looks that way on a log-log plot. Actually the distribution flattens out near zero and this creates a graphical illusion of sorts. The dispersion model says that up to a certain recent time in geological history, many of the oil fields have not started dispersing significantly — at this point the slow rates have not yet made

11. See [Ref 20]. Thanks to David Norwood for this data
12. Yet, the latter does make some sense considering how much more reserve growth that the USA shows than the rest of the world (and the number of stripper wells we have, averaging about 2 barrels/day). Slower reserve growth occurs for exactly the same reason — slower relative dispersion in comparison to distance involved — as it does for dispersive aggregation. Again, I constructed the underlying model in identical ways, substituting natural discovery in Dispersive Aggregation for man-made discovery in Dispersive Discovery and Dispersive Reserve Growth. This foreshadows the analysis in subsequent chapters

their impact and the fast rates haven't had any time to evolve. This manifests as an unknown distribution of sizes for oil fields before this point[13]. The typical USA field perhaps has a much slower dispersive evolution than the rest of the world, so we have a much higher fraction of small fields that have not yet aggregated. Or, alternatively, we may have exploited more of these smaller fields. However, in terms of a bottom line, this extra exploitation does not help significantly in extending a region's URR.

An interesting and supportive subset of USA data comes from the former Minerals Management Service (MMS) now the Bureau of Ocean Energy Management, Regulation and Enforcement (BOEMRE) and their data of reservoir sizes in the Gulf of Mexico [http://www.gomr.boemre.gov/PDFs/2009/2009-064.pdf].

> *On the basis of proved oil, for 8,014 proved undersaturated oil reservoirs, the median is **0.3 MMbbl**, the mean is 1.8 MMbbl.*

Plugging the median number into the dispersive aggregation model, the cumulative size distribution of reservoirs (ranked small to large) goes as P(Size)=1/(1+0.3/Size) if we assume a median of 0.3. On the Preston plot scale that the MMS preferred to use instead of a rank histogram, we show very good agreement.

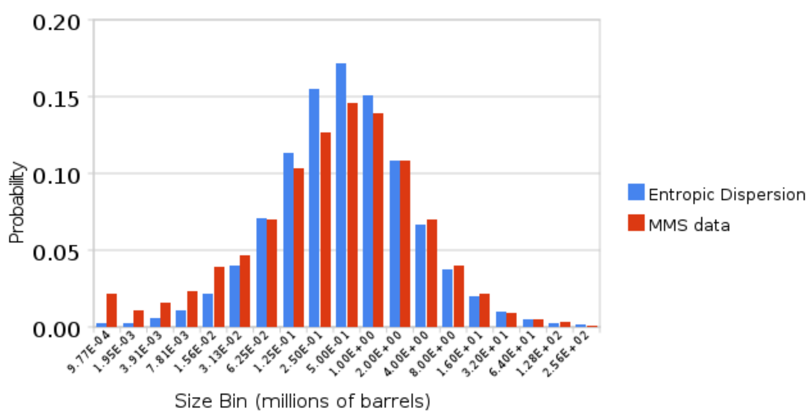
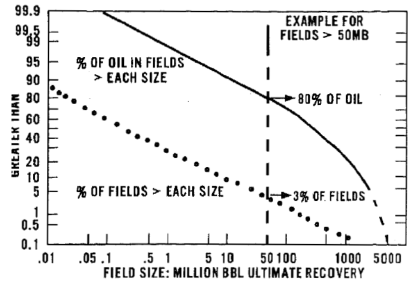


FIGURE 4-27. Preston plot of reservoir sizes in the Gulf of Mexico

13. If you plot a population's yearly income on a rank histogram you will see this same effect, in this case due possibly to a similar truncation due to a slow income growth early in a person's career, see Volume 2

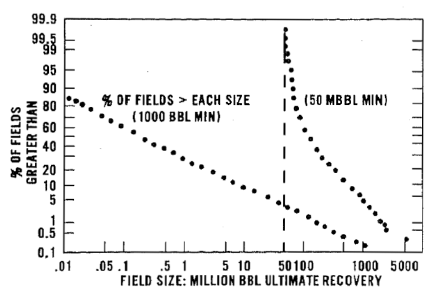
The USA as a Case Study. The somewhat dated paper (1986 [Ref 20]) on USA oil field size has a few interesting statistics. The authors roll up a few of the numbers from the curves.

The distribution of about 14,000 United States oil fields (Figure 2)—a partial sample of those in the lower 48 states—illustrates the importance of the larger fields. The sample includes almost all larger fields as known about 1970 and excludes many tiny fields as well as all of the more recent discoveries. The lower dotted line is made of 13,985 points representing the fields ordered according to increasing reserves size. Only 440 fields, or about 3%, are major ones larger than 50 million bbl. The upper curve tracks the percentage of the total oil volume occurring in fields greater than each size. From this curve, we read that the major fields, constituting only 3% of the total number, contain 80% of the total oil. Obviously, in this type of distribution one can account for the bulk of the oil by assessing the larger field possibilities only.



Selecting an effective minimum field-size cutoff is very important, because it affects every major factor in the assessment—the prospects to be counted and the success and risk levels, as well as the average field size. Normally, the minimum size is taken at or just below the assumed economic minimum for the area. This approach ensures that all prospects of real interest are included. It also avoids getting bogged down in hundreds or thousands of fields that are inconsequential to early exploration stages. Furthermore, the comparative data base for assessing sub-economic fields is very weak, as the true sizes of these fields have rarely been scaled. If desired, one can assess the small fields by statistical extrapolation or by estimating a lump-sum proportion from a volume curve like that of Figure 2.

Economic limits always truncate the lower ends of observed field-size distributions (Arps and Roberts, 1958; Kaufman et al, 1975; Grender et al, 1978; Drew et al, 1982; Vinkovetsky and Rokhlin, 1982). In nature's distribution, numbers of deposits probably increase progressively in successively smaller sizes down to droplets and molecules; such a distribution is not lognormal. But we deal exclusively with artificially truncated distributions whose plots almost invariably curve upward near the low-side truncation point (upper curve, Figure



3). Our United States distribution (lower curve, Figure 2) has no data below 1,000 bbl, and many of the data points below 10,000 bbl, where the graph ends, are questionable. If the plot were continued to the left, it would ultimately curve upward at the point of economic truncation beyond which there are no data.

We use the computational convenience of the lognormal distribution, appropriately truncated, but would not argue that this scheme is better or worse than other computational ones for strongly right-skewed distributions that have many more little fields than big fields. Some investigators (e.g., Ivanhoe, 1976; Folinsbee, 1977; Coustau, 1980) plot field size bilogarithmically against rank order. For our assessment approach, we must normalize field numbers at this stage by plotting "percent greater than" against log size. Depending on purpose and data, we may express field size as recoverable volumes of oil or of gas, or of oil plus gas on an energy-equivalent basis. [Ref 20]

The new numbers fill in the following points from Figure 4-26 on page 49.

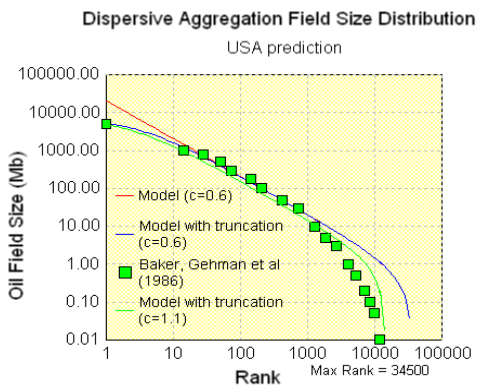


FIGURE 4-28. Historical data points from USA added to the previous Figure 4-26 on page 49

The authors state that 3% of the highest rank of fields contain 80% of the oil, which includes rank up to 440 in this chart. Or this means that 20% of the oil resides in the lowest 97% of the rank. The following relation provides a model dependent way of quantifying this effect:

$$\frac{U_p}{U} = \frac{\ln\left(1+\frac{Size}{C}\right) - \frac{1}{\left(1+\frac{Size}{C}\right)} + 1}{\ln\left(1+\frac{Max}{C}\right) - \frac{1}{\left(1+\frac{Max}{C}\right)} + 1} \qquad \textbf{(EQ 4-19)}$$

With the Dispersive Aggregation model for the extra 21,000 fields reported, the amount of oil contributed to those above 50Mb has dropped to 50%.

TABLE 1. Estimated cumulative number of production fields in the USA

Year	Size	Fraction of Total	#Fields
1986	>50 Mb	0.80	13,985
Today	>50 Mb	0.51	34,969

Another recent study by IOGCC showed 397,362 marginal oil wells produced 311 million bbl in 2004, or an average of 2.14 b/d/well [Ref 167]. Note that this makes sense with the total number of fields as multiple (>10) wells exist per field [Ref 169]. In the last 20 years, we probably have gained much extra mileage (though not necessarily huge boosts in URR) from the low volume fields, which we can also likely explain:

1. Smaller fields get deferred for production due to economic reasons
2. Smaller fields have a smaller cross-section for discovery so therefore show up later in the historical process. This 2nd order effect plays a smaller role in dispersive discovery than one would intuit.[14]

Placing this consideration of varying ***C*** into a global context, we can average the geological rates over a range of values. In this case, the overall fit works much better and we can account for a more gradual knee in the size distribution curve. I have found much similarity between this behavior with what ecologists observe with respect to relative species abundance. We will cover this further in Volume 2.

14. For field size distribution, Foucher plotted the top 170 fields discovered after 1920 in [Ref 21] The “big fields first” mantra keeps appearing, but it does not seem as strong a heuristic as most people claim. Physicists with an expertise in small-particle cross-section scattering would see reservoirs as the equivalent of small particles in a huge medium, and the size of the particle is completely overwhelmed by simple density and random collision considerations. This process may prove counterintuitive to the way people imagine that things would work. Size cross-section is therefore at best a 2nd order effect. We will further address this. See “The Effect of Field Size” on page 123..

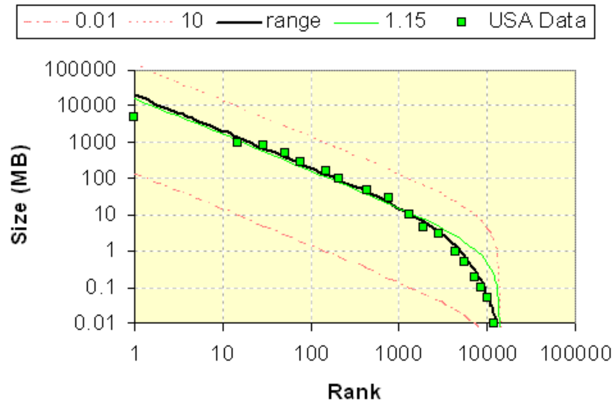


FIGURE 4-29. Dispersion of reservoir sizes for USA. Time aggregation of entropic dispersion does not affect the tails of the distribution, only the shape of the knee, making it less sharp.

Contrary to the Dispersive Aggregation model, which maps to a type of power-law, the results described in[Ref 20] assumed that the field size distribution followed a log-normal. The Dispersive Aggregation mimics the general shape of the log-normal under certain regimes, especially under a wide variance, while at the same time generating the heavy Pareto-like tail (i.e. $1/x$) that much of the data seems to indicate.

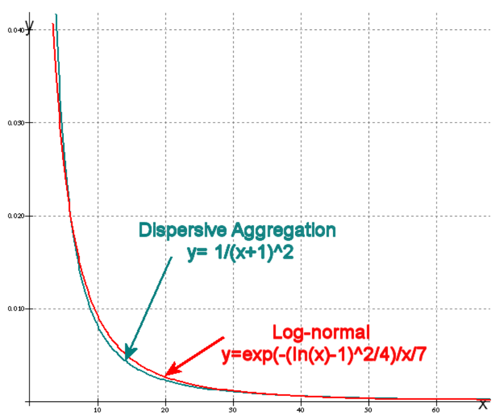


FIGURE 4-30. Comparison of Dispersive Aggregation model to a Log-Normal with most similar shape.

The following chart shows a cumulative field size distribution from the Canadian Williston Basin. The referenced article shows how they use a log-probability chart to map onto a log-normal curve; I pasted on a set of points corresponding to Dispersive Aggregation and as you can see, it also mimics the behavior of a log-normal distribution. Some of the other field size distribution models map similarly [15]

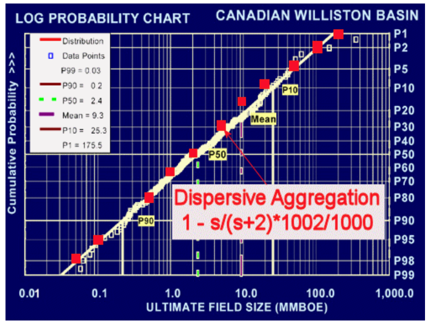


FIGURE 4-31.
Dispersive Aggregation model displayed on a log-normal plot showing deviations from log-linearity.

Setting the next stage. The Dispersive Field Size Aggregation falls into the dispersion theory category of models that appear to have a high degree of cohesion and universality. It looks like we can actually connect the dots from dispersive field sizes to the Logistic shape of the Hubbert Peak as the underlying statistical fundamentals have much commonality even across temporal and spatial behaviors.

We thus gain much by the way of fundamental understanding from working out the statistics of oil field size distribution. Estimates of potential reservoir sizes in terms of the next "super-giant" oil discovery gets mentioned enough by the media that it pays to quantitatively keep track of where it will fit into the global rank histogram. Unfortunately, without a real "top" to the probability of occurrences, we can't make definitive projections on URR. Thus we need statistics on the discovery process itself, which we can (and will) eventually explain.

15. The Parabolic Fractal Law looks to me like a heuristic as I can't find its derivation. Heuristics do not derive from physical models, only mathematical artifices. By definition, a heuristic does not have to explain anything, it just has to describe the results. And describing the results in a mathematical equation does not always work well. Most Wall Street quantitative analysts rely on models based on heuristics — and that has not arguably worked out as envisioned. We will later discuss the naive "derivation" of the Logistic, versus a well-formulated model leading to a curve, surprise, with the same character (See "The Derivation of "Logistic-shaped" Discovery" on page 183.).

CHAPTER 5

The Analysis Of Growth.

When does the extraction kick in?

"Tell people something they know already and they will thank you for it. Tell them something new and they will hate you for it."

— George Monbiot

If we knew the statistics and historical data for every oil production project ever put into play, we could simply sum the estimated reserves and then aggregate the actual production and arrive at a very accurate picture for our current global outlook (known as a "bottom up" analysis). The production data would tell us our current and past situation while the reserve data would allow us to extrapolate toward the future. Unfortunately, because of corporate (and national) secrecy and competitive advantage concerns, no one really knows the collective view of the historical and current production aggregates. Ideally, if we could obtain such data that would give us an optimal *deterministic* view of future estimates, what one might refer to as a dead-reckoning view of our path forward. All of the analysis work would essentially turn it into a "bean counting" exercise, similar to what we see for census data or when the government wants to collect our taxes.

Arguably, this deterministic approach gives us a level of confidence that we would normally not have; yet, all is not lost if we don't have the exact information. The key lies in the application of *stochastic* methods to fill in the missing pieces to the puzzle. The term stochastic indicates (in simplistic terms) that *randomness* plays an important part in the outcome. And the mathematical tools of probability and statistics allow us to predict likely outcomes given that we can place scopes and constraints on the randomness we observe or postulate. Just as we can estimate the census data using probability and statistics, via the stochastic elements of human population growth, so too can we do the same for a variety of other biological and physical processes — including the human-assisted process of oil production. If you believe in the idea that insurance companies can actually make money based on actuarial analysis, and that casinos make money (but of course!) based on detailed analysis of stochastic processes, then you have some reassurance that it will work just as well for oil depletion analysis.

Beyond the practical application of estimation, in certain ways the *understanding* of how oil production and discovery plays out becomes much easier if one applies stochastic principles. We don't actually need to know each and every one of the individual oil-producing reservoirs to understand how the trends have evolved over time. All the spikiness and noise observed in real-world data literally disappears, and we discover a smoothened outcome which proves eminently useful for guiding intuition, if not for making future predictions with greater confidence.

Basic Model

In lieu of the missing historical data to project our oil future, the key ingredient lies in the foundation of a good underlying model. Currently, many forecasts hinge on the interpretation of purely statistical trends in data, amounting to a generalization of the rule "whatever stays in motion, will continue in motion"[1]. As a starting point, everyone seems to at least eyeball and extrapolate from the slope of the last few known points. We can do much better than this though. Having at our disposal some simple and intuitive models allows us to feed the trend analysts the right dynamic inertia. So instead of predicting the equivalent of "rain for the next day, if it rains today", we can generate some long range forecasts that show some variation from a straight-line extrapolation.

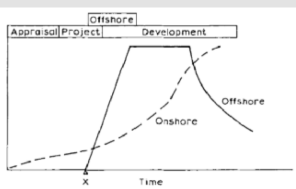


Fig. 1.1. Production profiles for onshore and offshore oilfields.

FIGURE 5-1. View of an oil project startup [Ref 172]

How it applies. Things don't schedule out deterministically for a collection of oil production projects. Certainly, for any particular development, the owners have planned a timeline that they will expect to meet. A deterministic schedule (by definition) would mean that all milestones get met according to pre-defined and pre-ordained milestones But realistically that schedule may go out the window, and we know that over a span of projects, variability in timelines will provide more exceptions to the rule than a single project could ever achieve. Often times this gets expressed as "the first casualty of war is the plan".

I will give examples of how variability in scheduling plays out. These all have some basis in common sense and the mechanics of how business and bureaucracies operate in the real world. We will go through the math in the next chapter, but first we will walk through the narrative described by the stages shown in Figure 5-2 on page 59.

1. Also known as the dead-reckoning model. Although, opinions on the origins of the term "dead reckoning" widely differ, one source puts it as a result of describing what happens if a ship keeps the same course oblivious to an iceberg in the path — they reckon to be dead.

1. Lying Fallow. In historical terms, the date of an oil discovery gets reported with very good accuracy. First of all we only have to go back to the middle-1800's for petroleum. As the discovery itself provides good marketing fodder for the investors it obviously pays off to advertise the date of an important find. So we basically know the discovery dates with some precision. The period after a discovery and before the start of the extraction process, I refer to as the *Fallow Stage*. We know less about the duration that a discovered reservoir stays dormant, waiting for extraction[2]. Clearly, the time a reservoir lays fallow can play out quickly under the right circumstances, or take much longer if, for example, the original prospectors don't own the land the reservoir sits on, or that it sits on a remote region, or restrictions get placed on land usage. The decision process can also include an appraisal period with exploratory testing. These all play a part in modeling a fallow period.

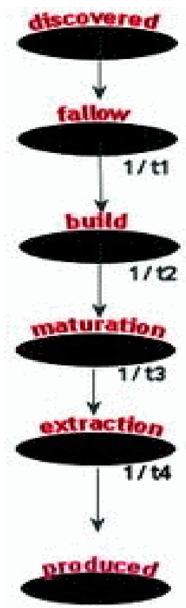


FIGURE 5-2. The planned stages of the oil production life-cycle shown as a serialized schedule. Each stage takes an average amount of time

Consider the following figure that shows several time-lines that start at the same point but have different lengths. If we make the connection between the lengths and time, then we have the analogy to a timeline, with the beginning dot indicating the start of an activity (or inactivity), and the ending dot indicating the finish. So we can generate a *distribution* in possible timeline lengths, corresponding to a range in fallow periods.

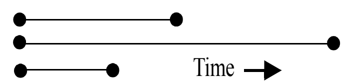


FIGURE 5-3. Generating a range of possible time lengths for a reservoir to remain fallow or dormant prior to development. We can apply this same treatment to ensuing phases as development progresses.

In practical terms, we might associate this distribution of values with an *average* value, and a *standard deviation* describing the range. The most conservative approach to model a two-term distribution leads to what we refer to as an exponential *probability density function* (PDF). The conservatism comes from not knowing the standard deviation accurately so that we set the mean to equal the standard deviation. This gives the widest range of values that also produces a finite mean, which agrees with the Maximum Entropy Principle (MaxEnt) popularized by E.T. Jaynes.

See the figure below for a PDF of a damped exponential function.

2. The text by Dake on reservoir engineering provides some good background [Ref 172].

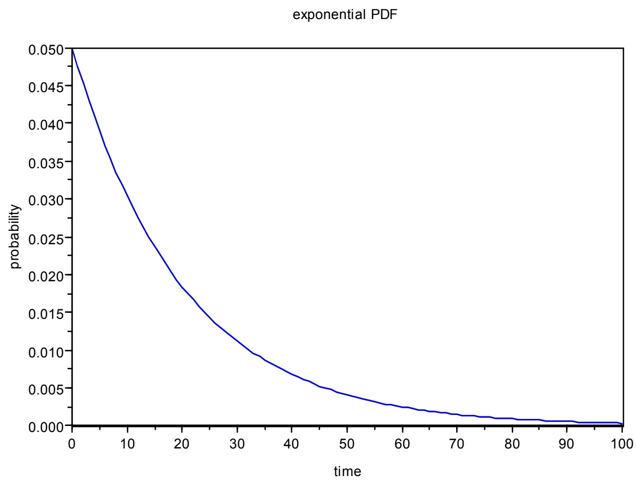


FIGURE 5-4. Decaying exponential probability density function of times.
`plot(time, 0.05*exp(-0.05*time))`

In practical terms, we can read the plot as indicating that we can have a fairly high occurrence of short fallow periods, but these get averaged out by the occasional long period of fallow time. Very long fallow periods rarely occur as they get suppressed by the strong exponential damping with time[3].

In statistical terms, the wide standard deviation becomes a MaxEnt estimate, made with the minimal knowledge available. It basically provides an automatic scaling on how much a value will change about its mean. Mathematically, about 85% of the values will land within plus or minus one standard deviation of the mean, or 63% of the values less than the mean (37% above the mean). Importantly, the premise disallows negative times, which essentially guarantees causality, simply put, the idea that things can't happen before the stimulus that provokes them occurs. For example, this prevents the start of work on extracting a discovery from occurring before the discovery actually happens! (as we do not wish to violate causality)

2. Building the Infrastructure. More than likely a significant fraction of people believe that a newly discovered oil reservoir immediately gushes oil and that by clamping a hose to the gusher we can start filling up oil barrels in a matter of minutes. Although this possibility does exist[4], in general, the actual production only

3. The Arctic National Wildlife Refuge oil field gives a prime example of an outlier that has sat fallow for a long period of time [Ref 22]. The damped exponential function says that these events rarely occur, and the Alaskan reserve provides the evidence via a calculated decision to not extract for some 30 years.

occurs after the extraction infrastructure gets put in place. The scenes of Jed Clampett of the "Beverly Hillbillies" accidentally shooting a hunting rifle in the direction of a reservoir (*"black gold, Texas tea")*, noticing bubbling crude, and then in the next moment living in luxury don't occur instantaneously[5].

Obviously, situations like that could only happen in Hollywood, yet such preconceived notions I imagine have had some impact on several generations growing up; picturing that petroleum existed as some sort of immaculate conception, with little effort required to reap the riches of a discovery. In fact, extracting the resources does require a finite period of time to build the infrastructure.

Once again, we do not know the exact distribution of the *Construction Stage*, but in probability terms it likely plays out in a similar manner to the *Fallow Stage*. Therefore we can use exactly the same mathematical setup. We elaborate a premise that an average time exists to construct the infrastructure, and that statistically a range of times describes the difficulty or complexity in completing the construction. On one end of the spectrum, Jed Clampett likely didn't have much trouble. Interpreting his fictional world, he likely sold the land to investors, who waited out the fallow period, and then constructed rigs quite quickly to tap a reservoir that appeared very close to the surface. On the other hand, prospectors who make a discovery in some deep off-shore location have lots of construction time to look forward to. In any event, an average finite time exists to construct a rig or platform, and we use the same conservative estimate to map a standard deviation on to the distribution.

In practical terms, this model premise means that within a sample set we will see quite a few construction times that finish well within the mean and we will find progressively fewer reservoirs that go on well beyond the mean. This has an economically limiting effect in that the longer the time it takes to construct an extraction facility (or the logistics surrounding the transportation, *etc*) the less likely that area will become financially viable.

3. Ramping up to Maturity. Once the extraction infrastructure gets put in place, you might expect that the peak extraction efficiency occurs and the flow reaches a maximum immediately. In reality, growing pains and business judgement usually suggest that the operators ramp up slowly.

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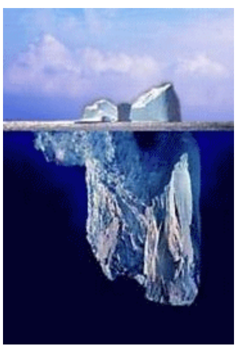
4. Some forms of high quality crude oil can in fact come out of the ground raw and get used directly by diesel engines [Ref 186]. This idea got tossed about occasionally during news reports of the Iraq occupation, where analysts devised scenarios whereby US military vehicles could get refilled in the oil rich areas of the country.
5. Perhaps a majority of oil finds need externally applied pressure or external stimulus to extract the oil, either via pumps or water injection.

Compared to the previous *Fallow Stage* and *Construction Stage*, the *Maturation Stage* serves more as virtual period whereby we can pinpoint when maximum flow occurs once they turn on the taps. If our ultimate goal remains to accurately model the average extraction of a collection of reservoirs from around the world, we also need to get a handle on what kind of *latencies* exist for the individual reservoirs or regions. Taken from discovery to actual production, each latency adds to deferring the aggregated peak from the point of discovery, and if we don't understand where these latencies come from, we will have difficulties interpreting the data. Therefore the maturation period likely plays a large role in the eventual peak value as either of the previous two stages.

The idea of *reserve growth* also plays into the concept of maturation. A cautious operation would only extract a steady proportional amount of the estimated reserve total so as to maximize his financial return. If, for example, the extraction infrastructure had way too high an extraction capacity compared to the size of the reservoir, the overhead costs would cut into profits. By the same token, not putting in a large enough extraction capability for a much larger reservoir would get impatient investors upset as they would not reap as much of a windfall.



By only looking at the tip of the iceberg, you don't realize that 90% of the volume may lie underneath the water

So, in terms of reserve growth, a specific production operation does not know the potential reserve until it reaches a typical maturation period. In perhaps more familiar terms, consider equating a hidden reservoir with an iceberg. You see the tip of the iceberg yet it may take a while to figure out how big a volume lies underneath it (perhaps 90%). If you knew the iceberg's size right away then you would never need to do a reserve growth analysis. So that over the passage of time, the early reserve estimates usually underestimate the available quantity of oil[6], and only as the operation gains some data, do they put in place their complete infrastructure. This, in fact, plays a significant role in the way the maturation stage plays out. As the operation starts out slowly, the investors feel happy that they can cut their losses if things turn sour, but then as reserve estimates improve, they can eventually maximize their profits with confidence. Hubris and greed play against each other, with greed eventually winning out.

The following set of interesting data from an on-line commentary [7]:

> *Average field distribution from 100 projects announced to be taken on-stream between the period 2005-2010 (taken from my own report, launch date now set at the end of October due to media-related reasons).*

6. Much of the underestimation comes from SEC regulations on reporting. This prevents potential for fraud in terms of dishonest speculators.
7. This reference from **Taskforce_Unity** at the PeakOil.Com site shows oil production ramp-up times (http://peakoil.com/post201356.html?sid=1efdc478f83a9275364a34e1c1fc6abf)

This analysis showed that:
• 23% of the projects were EOR projects (ed: enhanced oil recovery)
• 12% of the projects were discovered before 1980
• 7% of the projects were discovered between 1980 and 1989
• 29% of the projects were discovered between 1990 and 1999
• 18% of the projects were discovered between 2000 and 2004
• 11% of the projects were unconventional projects (tar sands and orinoco heavy oil)

This kind of data helps calibrate parameters as it includes estimates of the rolled up average value for how long a field stays in the *fallow + development* phases from the proposed analysis. Note how it shows the significant average lag and fairly wide stochastic distribution about the mean that we might expect. Foucher noted that these appeared to follow a Gamma distribution [Ref 23]. In fact, the gamma distribution follows from the time convolution of two or more exponentials, which in fact matches at least the first two of the three exponential phases, Fallow, Development, and Maturation, that the model will require as parametric inputs.

— The convolution of a delta function with a damped exponential results in a damped exponential.

— The convolution of a unit step function with a damped exponential gives an inverse damped exponential clamping to a value given by the amplitude of the step.

4. Extraction Starts the Decay. On an individual field or region, as a first-order approximation, we would extract at a rate proportional to how much oil in the reservoir we think we have available. This intuitively make sense in much the same way as someone who makes a million dollars a year will proportionately spend more money than someone who makes much less income, *on average*.

For illustrative purposes, let us look at a single hypothetical reservoir. At some point in the timeline, prospectors made a strike and then someone made an estimate of the discovered amount of oil. Based on that amount, the investors/owners applied a sufficient amount of development resources to extract the oil and make an appropriate amount of profit (i.e. the most possible).

In Figure 5-5 on page 64, I refer to this as a draw-down curve, as it represents the natural decline of an average reservoir, based on the original estimate of discovered reserves. However, without a replenishment curve, the reservoir will eventually deplete to the point at which the operators will decide to shut it down[8]. To maintain a production level, we require a replenishing supply of discoveries — what we can call a type of reserve growth. Depending on how things pan out, and under proper circumstances, a small yearly influx of new discoveries (or newly added reserve

8. The operator will on average apply proportionately fewer resources as the source supply dries up, which forms the basis of the exponentially-damped decrease, also referred to as an exponential decline.

estimates) allow the production level to continue at a constant pace. When the discoveries abate, the production levels decline.

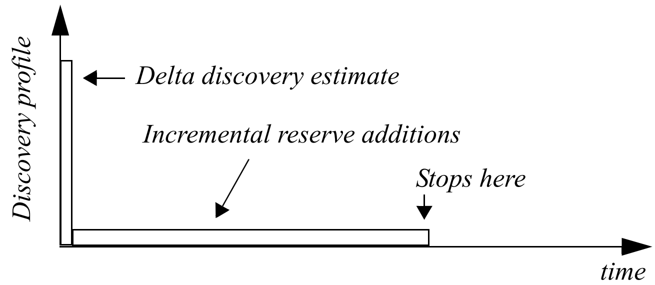


FIGURE 5-5. Idealized discovery profile incorporating incremental reserve growth over time.

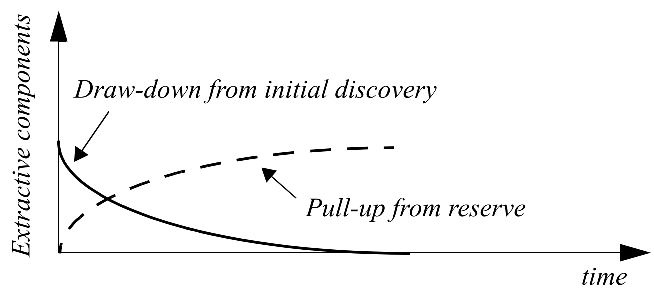


FIGURE 5-6. Applying reserve-proportional extractive pressure generates two curves that complement one another.

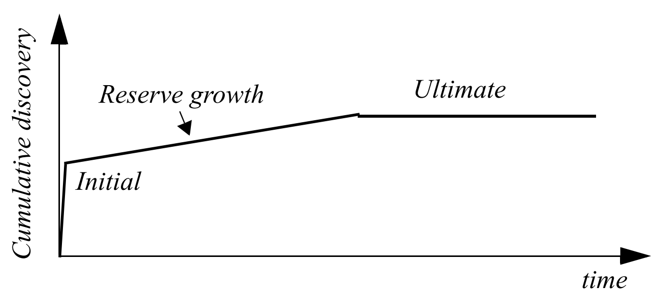


FIGURE 5-7. The cumulative reserve growth for previous figure.

The discovery term looks superficially similar to a hyperbola with a peak at time=0, and then a heavy tail providing a trail of reserve additions. This also appears suspiciously close to the reserve reporting in places such as Saudi Arabia and Kuwait.

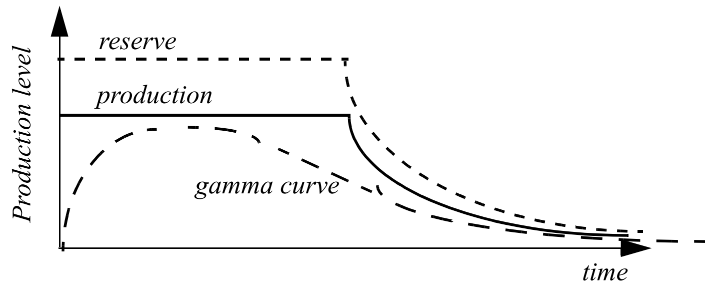


FIGURE 5-8. For this idealized case, a field can maintain a constant production level and non-cumulative (instantaneous) reserves.

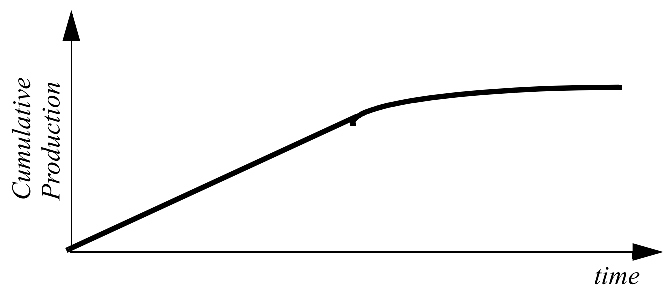


FIGURE 5-9. The cumulative production curve for the idealized case

The upshot: What happens if the operator does not proportionately extract in accordance with an exponentially damped regime but instead keeps the spigots running independent of the amount left? Then, sure enough, they can maintain production levels without the benefit of yearly incremental reserve additions, but when it does end, the output terminates abruptly and the tails disappear. For lack of a better description we can refer to this scenario as *constant* drawdown followed by *falling off a cliff*.

I consider proportionate drawdown and constant drawdown equally intuitive concepts but prefer the proportionate drawdown as it better matches the way statistics work out. For example, small reservoirs do not have the same constant drawdown rate as reservoirs 10 to 100 times the volume. In this case, proportionately serves as a useful scaling parameter, an ingredient missing from the constant drawdown model.

Above all, the concept of reserve growth plays an incredibly important role in how oil production evolves. The balancing act between extraction-induced depletion and reserve growth have kept us on an often-times precarious plateau that has the potential to turn south on us as soon as new discoveries or reserve additions abate. We will discuss this later.

Types of Growth

Historically, we can infer that growth in technology and economic growth play a large role in how fast we can discover and exploit resources. In particular, the implicit growth rate in discovery occurs as a volume of search space gets explored by scores of various prospectors. This can show a characteristic dispersive shape that we will address later on. [9]

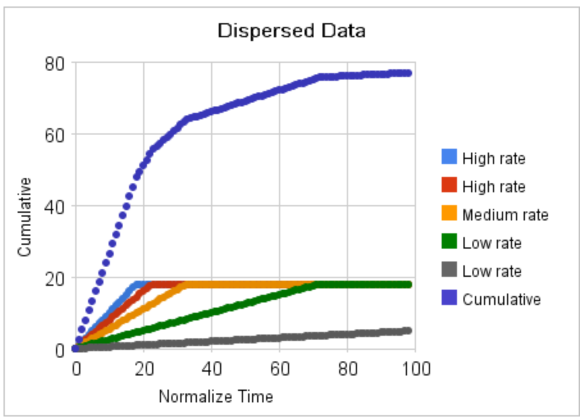


FIGURE 5-10. Aggregating dispersive growth by piecing together blocks of finite volumes of varying growth rates.

As an introduction, the basic types of growth needed to mathematically model what we empirically observe include:

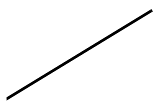
TABLE 1. Classification of types of growth and defining characteristic.

Types of Growth	Variant
1. Constant growth	
2. Accelerating growth	• Power growth • Exponential growth
3. Decelerating growth	• Parabolic (square root) growth

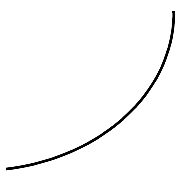
9. see http://spreadsheets.google.com/pub?key=prVeQf4uJHD26NosucXdIXw

TABLE 1. Classification of types of growth and defining characteristic.

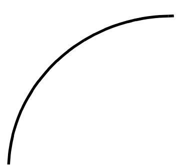
Types of Growth	Variant
4. Constrained growth	
5. Feedback-controlled growth	• First-order negative feedback
	• Logistic growth (looks like a negative feedback with an additional term preventing it from going negative)



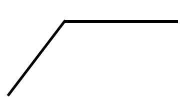
Constant growth. The most simple growth displays a constant cumulative increase in amount over time. This essentially models the deliberate pacing of resource usage or of searching a volume which doesn't change over time. Significantly, we can transform an accelerating growth into a constant growth by counting cumulative numbers of discoveries as the time scale, thus factoring out the temporal aspect. This becomes very useful for analyzing creaming curves where we might not have the original time scale to work with.



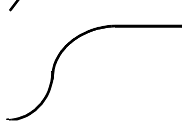
Accelerating growth. An accelerated growth occurs when either technological improvements, financial incentive, or increasing population resources serve to compound the growth rate. We will talk about this in depth in the next section, but quite clearly power-law and exponential growth remain the most fundamental types of growth.



Decelerating growth. This kind of growth usually occurs due to increasing difficulties in achieving some objective. As an example, a specific kind of diffusion-limited process leads mathematically to a fractional power-law growth rate, commonly referred to as parabolic growth. Importantly, this growth may never reach a finite limit but it obeys a kind of law of diminishing rate of returns, as the growth rate monotonically decreases to zero over time. Often, technological advancements can supplant and ultimately negate a decelerating growth so the net effect becomes a positive acceleration. As another interesting consideration, **all** exponentially damped growth rates will lead to a finite limit; this makes it hard to distinguish this from a constrained growth or a feedback-controlled growth.



Constrained growth. The growth hits some hard limit, either because we ran out of search space or run out of resources in a specific region. Statistical considerations prevent one from seeing the hard-stop and any slope discontinuity usually transforms into a smoothened levelling off.



Feedback-controlled growth. In the case of negative feedback, this behavior manifests itself as a slowing down in growth (and potentially the physically unrealizable negative growth). Due to feedback, what can look like as an accelerating growth can turn into a decelerating growth and then a constrained growth as it passes through an inflection point[10]. If we add a positive feedback this usually leads to an exponentially accelerating growth. Feedback control mechanisms can become incredibly complex — they can possess linear terms or non-linear terms and they can transform positive *real* physical variables turn into non-physical *negative* values if an analyst models the parameters incorrectly.[11] And in a stochastic context, they tend not to work well unless one becomes proficient with stochastic calculus.

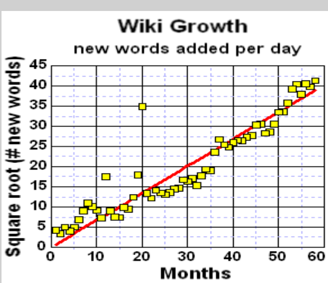
We will apply growth concepts to discovery first.

Growth in Discoveries

As a recurring theme, I assert that to explain discovery, one only needs to consider the rate at which we search a volume. This search rate gets calibrated only by the average size of discoveries per year — fluctuations in discovery sizes simply add to the noise. This subtle observation remains a fundamental concept; even though many analysts will try to judge future output by invoking reservoir size distributions, it really has little to do with the basic outcome. The finite search space alone places a cap on the ultimate number of discoveries.

Few models exist for the discovery dynamics of valuable yet *finite* resources. Consider the concept of quadratic growth. This kind of growth has an underlying mechanism of a constant acceleration term — in other words the rate of growth itself increases linearly with time. To first order, this explains scenarios that involve a rapidly increasing uptake of resources, and particularly those that spread by word of mouth. The growth of wiki-words in Wikipedia provides the best current-day example of quadratic growth (see panel on left). Unfortunately wiki-words grow out of an almost endless supply of alpha-numeric strings, which shows no signs of declining. However, for non-infinite resources we all know that growth ultimately

Consider a recent example of quadratic growth which comes from the world of the Wiki. At least one research team had noticed that the rate of increase of Wikipedia words follows a quadratic growth law. I extracted the following relationship from the Wikipedia

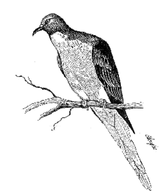


statistics table. Note that when we take the square root of the growth, it tracks a straight line. Also remember that quadratic growth does not equate to exponential growth. Exponential growth occurs when the rate of increase of a quantity proportionally scales to the amount of quantity at that specific time. This amounts to a much different swarming activity; in mathematical terms it amounts to a distribution of power-law growth factors.

10. Similar to the way a cruise control system works on a car. The feedback in this case arises due to an error signal coming from the difference between the car's current speed and the driver's intended speed. This works very well for this intended application but it may not in general work for arbitrary uncontrolled physical processes.
11. In the latter case, the popularity of one such feedback-controlled growth model, *Logistic Growth,* comes about primarily through an artifact of the feedback parameter clamping close to zero as soon as the decelerating growth term starts heading toward zero, and potentially negative values. Through a quirk of the math, this provides an extremely simplistic yet convenient heuristic to explain general trends. I have significant reservations about using this type of growth at all; in many cases it will violate causality as it may require some implicit knowledge of the future (i.e the intended limiting value), something that does not happen very often in physical systems. And where it does indeed happen, for example, in the speculative financial control, the psychology of human behavior can modify the calculated effect. More on the Logistic curve later., "The Context of Discovery. How do we simplify the search model?"

abates and (quite frequently) suddenly. I will first review two prime examples of this kind of dynamics: the old-fashioned gold rush and the extinction of species.

First, if we consider the gold rushes that occurred in places like California and Alaska during the 1800's, we invariably witnessed an accelerated search for the mineral as prospectors swarmed to a region. This accelerating growth in claims never lasted for long though — within a few years, the region became scoured clean and history usually records a decline typically more spectacular than the original rise. We uniformly have to agree that finite resources played an important role in this behavior, and numerous ghost towns remain the only concrete evidence that any type of culling activity even occurred.



The passenger pigeon extinction provides an even more dramatic example of accelerating growth followed by sudden decline. From historical accounts of the colonial days of the eastern USA, a few settlers started realizing that pigeon populations provided an easy or cheap source of food[12]. More importantly, other settlers joined in and discovered increasingly lethal ways of decimating the bird population. So this perhaps century-long accelerating increase in harvest numbers formed a framework for a precipitous decline in pigeon population within a few decades, ultimately followed by extinction of the species. The pigeon population essentially became a finite resource as reproduction dynamics could not overcome decimation by the sheer numbers and skills of the hunters[13]. Although I have found few reliable estimates of the actual numbers [Ref 24], no one argues that wild pigeons essentially became extinct within the span of a few years from the late 1800's into the first few years of the 20th century.

So I ask the question: can we create a model of this "gold-rush"-like discovery of resources which effectively matches those of gold, passenger pigeons, or perhaps ultimately oil? Or does oil discovery show a more gradual decline than the classical gold rush?

To answer this question, we can either consider growth in discoveries as a steady year-to-year increase or as an *accelerating* increase. The latter refers to a quadratic growth law commonly found in many situations where increasing numbers of resources get applied to a problem over time. Much like gold spawns a fevered rush of interest which seems to accelerate through a parabolic boom before finally busting, I offer that oil strikes might follow the same swarming pattern.

12. Near the peak, birds sold for 1 to 2 cents per bird.
13. For example, through creative uses of dynamite

“The California Gold Rush (1848–1855)... By 1850, most of the easily accessible gold had been collected, and attention turned to the task of extracting the gold from more difficult locations. Faced with gold that was increasingly difficult to retrieve, Americans began to drive out foreigners to get at the most accessible gold that remained” [Ref 25]

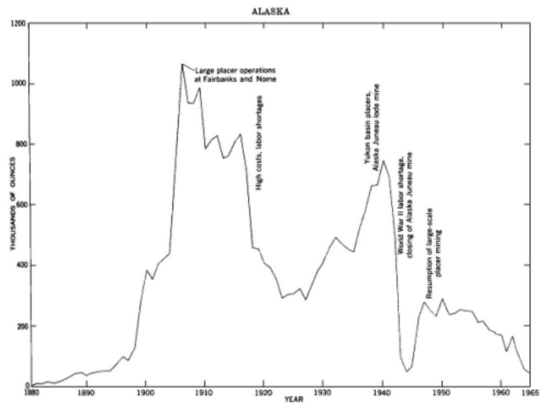


FIGURE 5-11. Fast buildup and decline characteristic of gold-rush dynamics noted in Alaska. http://www.akmining.com/mine/akgold.htm

The Effect of Noise

In the context of crude oil discovery, I curiously haven’t seen much written about the number of discoveries over time. The data often shows quite a bit of noise, which tends to hide or obscure much of the underlying trend. Yet, one should not fear noise. Noise can actually tell you a lot about the underlying physical character of a system under study. As an example of this, consider the historically noisy oil publicly released discovery curves. This chart of global discoveries appears unfiltered, but then production dynamics effectively filters this out (more on this later):

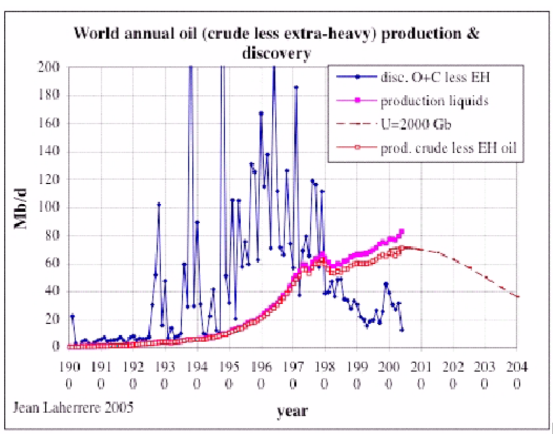


FIGURE 5-12. Yearly discovery data shows characteristic noise, with production smoothened and delayed by a substantial amount.

The following chart commonly cited has a 3-year moving average:

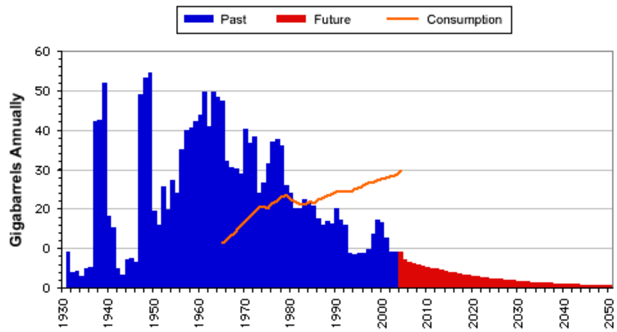


FIGURE 5-13. Global discovery data smoothened by a 3-year moving average.

Some say that the discovery curves approximate a bell curve hidden below the noise yet clearly the noise still exists even with a moving average applied:

> *The area under the complete discoveries curve must equal the area under the eventually completed global production curve, whatever it's math description - oil discovered must equal oil produced. The discovery process is controlled heavily by the math laws of probability with the bigger, easy-to-find pools of oil found first. Resource discoveries fall on bell curves too. Deffeyes makes the point that even with changing technology, this is the way discoveries play out. The global discoveries curve peaked in the mid 60s and, despite the immense tech revolution since then, the charted yearly discoveries have formed a pretty nice fit to a Gaussian bell curve, particularly if they are "smoothed" by grouping into 5 year bars in a bar graph [Ref 26].*

Although useful, this description contains many assumptions. First of all, you can only equate discoveries to production if one has good estimates of initial discovery or if the discovery estimates get continuously updated, i.e. via *backdating*. And according to Schopper:

> *"Pearson's r" test found no correlation between oil discoveries from one year to the next, i.e. discoveries appear to be random.* [Ref 27]

The fluctuations become very apparent essentially because of the limited number of discoveries we have had or can make in a finite amount of time. We can easily demonstrate this effect via simulation. By running a Monte Carlo analysis, one can see the natural statistical fluctuations which occur in yearly discoveries. The following data comes from several Monte Carlo trial runs of 10,000 samples with a log mean of 16 (corresponding to 9 million barrel discovery), and a log standard deviation (corresponding to 0.73 million barrel on the low side and 108 million on the high side). I then put a "gold rush" mentality on the frequency of discovery strikes; this essentially started with 8 strikes per year and rising to 280 strikes per year at the peak.

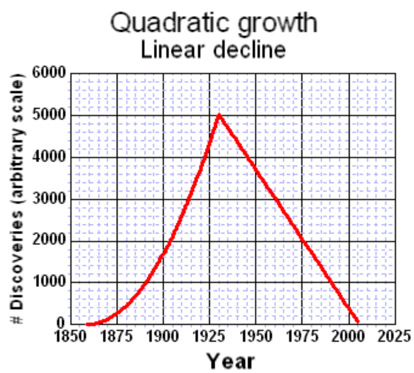


FIGURE 5-14. Idealized model showing quadratic growth in yearly discoveries followed by a linear decline after reaching peak. This artificial model provides an arbitrarily chosen stimulus to the production model.

The first chart below shows a typical sample generation, and the rest generate the discovery curves via the application of a steadily rising and then falling yearly accumulation factor; i.e. without the noise it would look like a triangular curve.

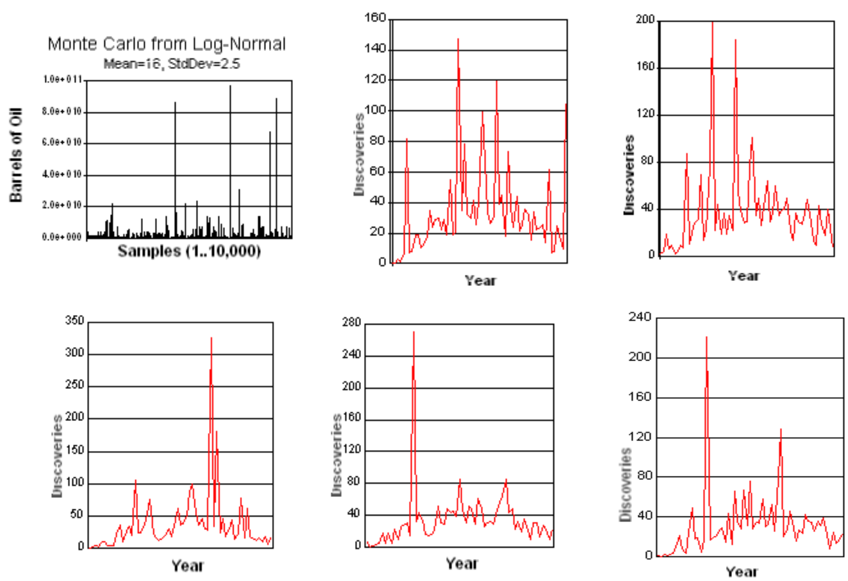


FIGURE 5-15.
A set of Monte Carlo simulations based on the discovery profile from the previous figure. Noise in the profiles arises from the finite set of samples and the field size distribution chosen. The rare large field discovered leads to a spike in the generated simulation.

The main thing to note relates to the essential noise characteristic in the system. The fluctuation excursions fairly well match that of the real data (*see* Figure 5-15 on page 72), with the occasional Ghawar super-giant showing up in the simulations, at about the rate expected for a field size distribution.

```
with Stoch;
with Text_IO;
with Ada.Numerics.Elementary_Functions;

procedure Stoch_Disc is
 use Stoch, Ada.Numerics.Elementary_Functions;

 Year: Integer:= 0;
 C: Float:= 2.0;
 L: Float:= 80_000.0;
 Y, P, Size: Float;
 Max: constant:= 1_000_000;
 Sizes: array (1..Max) of Float:= (others => 0.0);

begin

  for I in 1..120_000 loop
   P:= F_Random;
   Y:= (1.5*P/(1.0-P)) ** (1.0/8.0);
   Year:= Integer (100.0 * Y);
   P:= F_Random;
   Size:= C*P/(1.0+C/L-P);
   Sizes(Year):= Sizes(Year) + Size;
  end loop;

  for I in 1..300 loop
    Text_IO.Put_Line (I'Img & Sizes(I)'Img);
  end loop;

end Stoch_Disc;
```

I would never assert that the biggest fields get discovered first (Schoppers also sees no correlation and I use this conservative approximation elsewhere), only that they have a higher probability cross-section which overcomes their naturally lower frequency of occurrence, which we showed earlier[14]. The big ones may actually be found later because, over time, more resources get applied to exploration (increase in number of darts thrown at the dart board). And then eventually the resources get applied to more difficult exploration avenues as that dries up. That basically accounts for the noisy rise and noisy fall, until even the noise eventually disappears into the low overall level of discoveries.

So we can largely account for the origin of the noise. The real smoothing process comes about when we apply extraction to these discoveries, essentially dragging the data through several transfer functions that extracts at rates proportional to what is left. This does not result quite in a logistic curve, but something more closely resembling the *convolution* of the discovery data with a gamma distribution. Which leads us to the basic premise for the oil shock model I will use to describe extraction and production. The following figure foreshadows the result we will end up with:

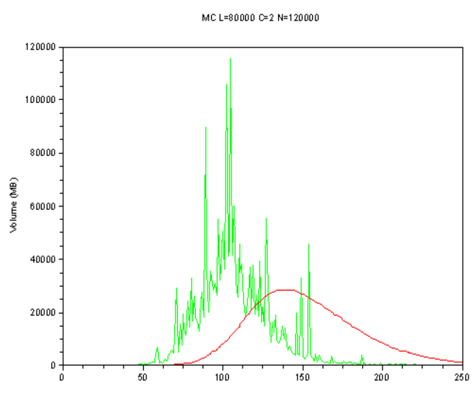


FIGURE 5-16. This algorithm generates a Monte Carlo simulation of a Dispersive Discovery growth curve (described later) along with an aggregated dispersion of reservoir sizes (see algorithm at left)

14. One professional geologist commented: "*Remember that the Megagiant field size sits on the 99 percentile of world field size distribution, meaning that the chance of finding another is pretty slim.... As to the likelihood of finding more of them diminishing every day... that is only true if exploration efforts in areas where they are most likely to be found has been aggressive.*"

The Strawman of Feedback

Consider again the quadratic growth figure which shows a quadratic growth followed by a linear decline. Clearly, this artificial break in quadratic growth does not follow from any physical process and I did this mainly to match empirical observations.

Consider a hypothetical assertion that the key to modeling a finite resource limited decline lies in combining a growth term (such as the quadratic) with a first-order feedback term describing the constraint. We start with the pure growth terms:

$$\text{Quadratic Growth :} \quad \frac{d^2}{dt^2}Q(t) = k$$
$$\text{Exponential Growth:} \quad \frac{d}{dt}Q(t) = a \cdot Q(t) \tag{EQ 5-1}$$

The second expression adds a variant of exponential growth to the quadratic form. The combined growth+feedback equation looks like the following, where $D(t)$ equals the discovery function:

$$\frac{d^2}{dt^2}D(t) = c - a^3 \int_{\tau=0}^{t} D(\tau)d\tau \tag{EQ 5-2}$$

Which basically says that the acceleration in discoveries follows proportionally to a constant suppressed by a drag factor that increases as discoveries accumulate. The drag term essentially describes the finite resource, which implies the rate of discoveries will decrease as we make more discoveries and thus approach the limit.

If, on the other hand, I switch the sign on the drag factor, it becomes an *additional* exponential growth term which eventually dominates the quadratic term, forming a type of positive feedback (which models population dynamics). However, for negative feedback, the acceleration eventually becomes negative and the discoveries get driven into the ground. You can see the behavior in the following figure, where I plot the square root of the quantity so you can see the divergence from pure quadratic growth depending on the feedback sign:

We can use calculus and Laplace transforms[15] to come up with the solution to the quadratic/feedback differential equation:

15. The Laplace transform of the quadratic/feedback differential equation: $P(s) = c/(s^3 + a^3)$

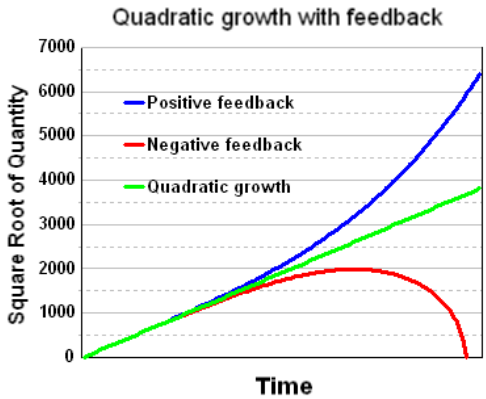


FIGURE 5-17. Possible deterministic trajectory of quadratic growth tempered by feedback terms. Note the scale of the y-axis. Positive feedback accelerates the growth while negative feedback decelerates the growth until it reaches a zero velocity.

$$D(t) = D_0 + \frac{c}{3a^2}\left(e^{-at} - e^{a\frac{t}{2}} \times \left(\cos\left(\frac{a\sqrt{3}t}{2}\right) - \sqrt{3}\cdot\sin\left(\frac{a\sqrt{3}t}{2}\right)\right)\right) \quad \textbf{(EQ 5-3)}$$

where D_0 = Initial Discovery. The term a acts like a characteristic value to the solution of a third order differential equation, while the value for c sets the amplitude.

Tempted, I decided to initially fit this kind of feedback model to historical estimates of oil discoveries, based on seeing the following kind of data reported:

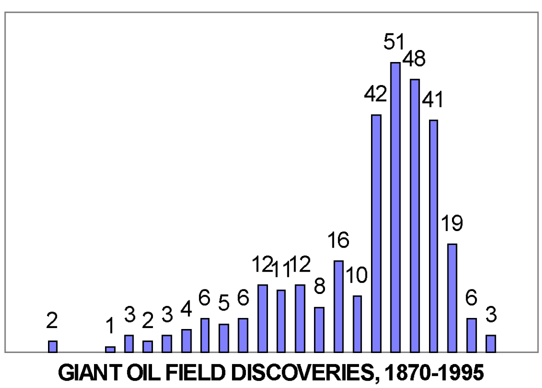


FIGURE 5-18. Histogram of yearly giant oil field discoveries (of unknown origin). Note that the discoveries appear skewed to later dates

Note that this figure shows a histogram of *numbers* of world discoveries which does not include the *individual size* of the discoveries; I consider this reasonable as the size forms a stochastically independent variable to the number of discoveries and random fluctuations would certainly modulate this profile — but only in a statistical sense.

Consequently, I decided to look more closely at USA data, as the discovery estimates provided by Laherrere [Ref 28]and the production numbers [Ref 29] generate a good dynamic range (see results section).

Affine transforms: Since the quadratic/feedback formulation shows self-scaling similarities similar to that of trig functions (i.e. period and amplitude), we can describe certain characteristics which depend on the a and c constants.

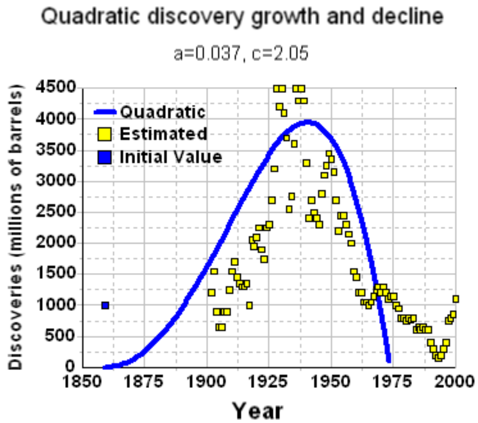
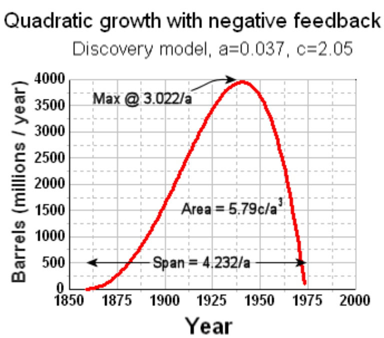


FIGURE 5-19. A deterministic negative feedback growth model alone will not account for longer tails in the discovery profile for USA data.



Via the properties of affine transforms, we can easily estimate the terms for quadratic/feedback growth by simply overlaying the scaled profile on potential discovery curves. Although the quadratic/feedback formulation looks promising, it does suffer from severe drawbacks. It has all the characteristics of a **"resource collapse" model** but clearly this should not happen in a realistic situation where there may exist many hidden pockets for oil to remain unexplored and undiscovered.

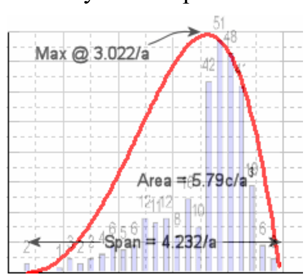


FIGURE 5-20. Unless we have data from the tails of the distribution, one can get fooled into predicting a "resource collapse".

If nothing else, the backside part of the curve probably needs modification — reflected by what I consider a different growth regime governed by a change in human dynamics:

$$\frac{d}{dt}\mathrm{Discovery}(t) = b_0 - c\int \mathrm{Discovery}(t)\,dt \qquad \textbf{(EQ 5-4)}$$

I would justify this by suggesting that once a permanent decline kicks in by the relentlessly diminishing resources available for discovery, the incentive to discover turns into a constant (i.e. a decline in "gold rush" participants), giving a damped exponential beyond the sharp decline (see the cubic example of this behavior below)[16].

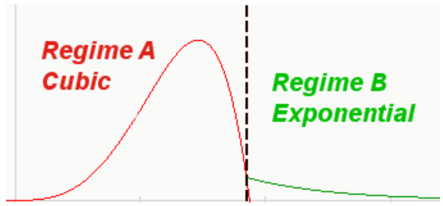


FIGURE 5-21.
A deterministic regime needs to account for the tails of a discovery profile. However, an artificial discontinuity between the regimes weakens the argument.

In general the shape of this curve tries to mimic the shape of the Logistic curve, an exponentially ramped up-slope and an exponentially damped down-slope. Of course, the solution does not match the simplicity of the Logistics curve, but we never intended to generate a concise solution; in my mind latching onto a concise thought process remains the ultimate goal.

Classical oil depletion modelers occupy a no-man's land situated between islands of poor data and a huge immovable object known as the Logistic curve. I always considered the Logistic formulation an empirical fit, widely used only because it generated a convenient, concise closed-form solution.

However, if we follow the path governed by feedback we reach a dead end in terms of understanding. Since this models a deterministic response, the parameters become nothing more than tunable factors into a mathematical artifice. With that in mind and assuming a more stochastic outlook, we will generate a more useful model for discoveries in the chapter "Finding Needles in a Haystack. How we discover oil" .

A Feedback Example. Just for completeness, we can add a slight variation to the feedback model to make it behave more like the classical Logistic model, particularly on the upslope. It essentially branches off from the premise of the quadratic and cubic discovery models. Keeping it simple, I switch the power-law dependence of discovery growth to an exponential law:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Discovery}(t) = b \cdot \mathrm{Discovery}(t) - c\int \mathrm{Discovery}(t)dt \qquad \textbf{(EQ 5-5)}$$

This has the property of the rate of discovery increase tracking the current instantaneous rate of discoveries. Although arguable in the validity of its premise, it has a basis in human nature that nothing attracts success like success, which translates into a "gold-rush" mentality for the growth in discoveries. The decline comes about as a finite supply of discoveries accumulate and provide the negative feedback in the integral term. This turns into a classic 2nd-order differential equation.

16. Some have observed this shape to resemble a "shark fin", and the shark fin as a harbinger of resource collapse. Although metaphorically intriguing, this sweeps a lot under the rug.

$$\frac{d^2}{dt}D(t) - b\frac{d}{dt}D(t) + cD(t) = 0 \qquad \textbf{(EQ 5-6)}$$

I used an online differential equation solver to seek out the regime which corresponds to the classic growth and decline in discoveries:

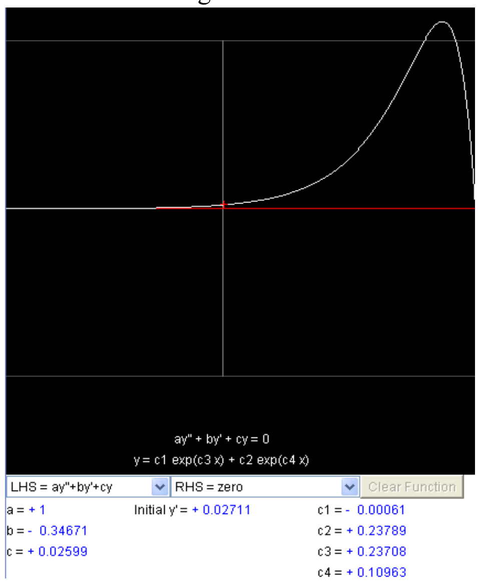


FIGURE 5-22. A variation of the polynomial growth model showing exponential growth modulated by negative feedback shows the same characteristic of collapse. This essentially superposes two exponentials, a slow one on the way up and a fast one on the way down.

This appears close in shape to the cubic growth model, but showing meatier tails and a sharper profile. It also needs an initial condition to kick in, as the solution degenerates to *Discovery*(t) = 0 without an initial discovery stimulus. *D*(0) and *D*'(0) provide the initial "gold rush" stimulus. Unfortunately it still lacks the tails characteristic of a gradual drop-off in discoveries.

We can always guarantee a crash by an accelerating discovery limited by a finite resource. The only way out lies in a discovery rate that slows as we find most of the oil. The feedback term only gives one side of the story, and we need to invoked dispersion to get long decline tails.

The Path from Discoveries to Production

I would like to continue to lay out the terms of a more complete and therefore valid discovery model at this point but we need to introduce the detailed model for extraction and production first. The interlocking mechanisms that tie discovery to production so tightly couple each other that it serves us best to foreshadow what happens with production before we go back and firm up the discovery model.

"It is difficult to exaggerate the importance of convolutions in many branches of mathematics."

— William Feller
An Introduction to Probability Theory and Its Applications.

The fundamental concept that links discovery to production, *convolution*, has a name that belies its elegance and simplicity. Most people probably think that the root of the term convolution has some relationship to that of a *convoluted argument.* In colloquial terms, this indeed has a bad connotation, and anyone that tries to press forward while admitting that their argument contains inordinate complexity or twisted logic immediately has two strikes against them. For better or worse, people will tune out when someone tries to explain things via what they think amounts to convoluted reasoning.

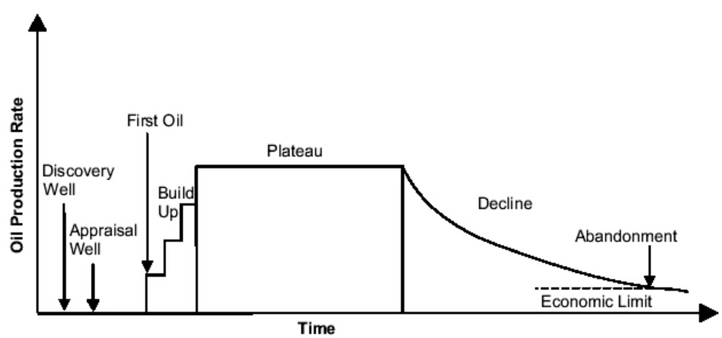
Yet, in fact, convolution in mathematical terms does not imply complexity, but instead forms the foundation of vast areas of engineering and scientific understanding. Via convolution, we can explain everything from how signal processing works to how to solve mechanical dynamics problems. In general, a signal or stimulus forms the front end of the convolution, and the response function forms the back end. So we need to understand the two ends (back and front) individually before we can really convey the result. And the result only works if we understand convolution[17]. So when you see this strange syntax:

$$Production = Discovery \otimes Extraction \quad \textbf{(EQ 5-7)}$$

don't become overly apprehensive. The operator ⊗ simply signifies that a mathematical convolution occurs to obtain a result.

In the next chapter we will try to understand how some of the intricacies of the production process come into play and how these get modelled. As an objective, we want to understand how the details of a chart like the following come about (from Robelius[Ref 19]):

FIGURE 5-23. Observed rates of production from an arbitrary field shows characteristic phases over its complete life-cycle (from Robelius).



17. Not all scientific and engineering disciplines use convolution. Softer sciences such as geology and economics may not include convolution in their curriculum. This would certainly explain the paucity of this line of reasoning in resource depletion topics.

CHAPTER 6

The Shock Model.

How we deplete oil

"Given a choice between a new set of matching tableware and the survival of humanity, I suspect that most people would choose the tableware."

— George Montbiot

"Truth hurts. Maybe not as much as jumping on a bicycle with a seat missing, but it hurts."

— Frank Drebin, Naked Gun 2½

Oil Extraction

Let us next develop the concepts of a peak oil model to help us understand the dynamics of extraction and production from a single reservoir. After that we can extend the concepts to an aggregation of reservoirs.

The Fundamental Link between Discovery and Production

We start by simplifying the relationship between reserves and depletion rates by relying on a first-order approximation: the rate of extraction (units per time) relates proportionately to the amount of oil left in a reservoir.

Reserves: Think of *reserves* as what you believe you have remaining in the *reservoirs* you have discovered.

$$\frac{d}{dt}U(t) = -r \times U(t) \qquad \text{(EQ 6-1)}$$

Lacking any additional information, this becomes the naive estimator for how something depletes; it also finds application in many other physical processes including thermal conduction and particle diffusion. In general, the relationship points to a reduced extraction rate as the availability or density of a resource depletes.

Of course, the first-order differential equation solves to a simple declining exponential (*aka* an exponential decline).

$$U(t) = K \times e^{-rt} \quad \textbf{(EQ 6-2)}$$

Obviously that doesn't complete the story as the exponential doesn't come close to approximating the roughly symmetrical Bell curve of the Hubbert peak.

A temporal driving force applied to the exponential allows us to mathematically intuit a better symmetry. To achieve this, we use the *a priori* assumption that **discoveries** provide the stimulus for extraction. Historically and intuitively, discoveries start at zero, reach some peak, and then start declining over time. We have long since reached peak in discovering oil wells, so this becomes valid empirical data that we can use to model depletion.

Given that we have (1) a depletion rate model and (2) an empirical discovery model, we need to combine the two by driving the transfer (depletion rate) function with a stimulus (discovery model) function. Mathematically, we solve the key third step by applying the convolution integral:

$$\text{continuous convolution} \qquad c(t) = \int_{\tau=0}^{t} a(\tau) \cdot b(t-\tau)d\tau$$

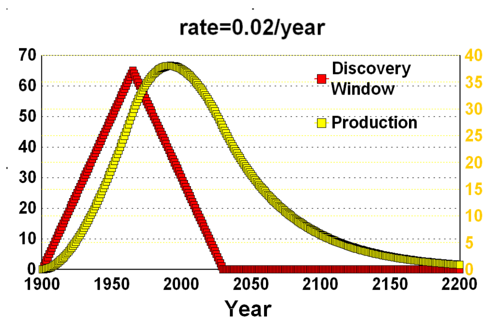
(EQ 6-3)

$$\text{discrete convolution} \qquad c[T] = \sum_{i=0}^{i=T} a[i] \cdot b[T-i]$$

The order in the functions doesn't matter; broadly speaking, it becomes nothing more than a moving average function applied over all points in time. I show the continuous variant of the convolution integral as well as the discretized version to illustrate how easily we can compute this integral simply by using a spreadsheet.

At this point I don't want to use the empirical discovery function. Instead I use a simple *triangular* function to serve as a heuristic and something that we can easily parameterize. The following figure shows the result of the convolution:

FIGURE 6-1. The first-order approximation of a rate proportional to reserve shifts the production in time from the discovery peak. This becomes the basis of the oil shock model.



Don't take the peak date in this figure too seriously. This profile essentially shows how easily a quasi-symmetrical Bell curve derives from such a simple model (one can actually mistake the slope in the right-shifted curve for a Gaussian if you don't look too closely).

When I first developed this analysis, I questioned whether anyone had ever done this kind of model before. From the literature I have yet to find much apart from references to *compartmental models*. Most analysts experiment with curves derived from Logistic equations, Ricatti equations, and Verhulst equations and use non-linear estimators to come up with best fits to coefficients. Those kind of exercise typically reduces to a set of heuristics that analysts refer to as *curve fitting*. As far as I can tell no one has developed intuitive mathematics and accompanying mathematical model which clearly shows the general trend of the Hubbert peak from the alternate perspective presented here.

‡ *see "Code Snippet #1"*

Assumptions. The assumption of first-order rate depletion (i.e. rate proportional to how much remains) has problems when considering extremely large reservoirs, where a constant depletion rate (or even increasing rate) can occur for a long period of time, extending until we start hitting hard limits.

So we need to use caution and understand that proportional drawdown does not always have to occur. For example some may view the situation of natural gas depletion as a non-proportional drawdown. I have seen many references that depletion in natural gas reservoirs does not follow the "rate-proportional-to-contents" empiricism. Like the nitrous oxide left in the whipping cream container, it maintains a steady flow of output while the can continues to hold pressure. After that, nothing. So we get no tails in the curve, as the production of natural gas drops off the table seemingly much more quickly than petroleum[1].

Yet even here, physics suggests that first-order rate depletion turns into a good approximation. Let us consider the ideal gas law, also known as Boyle's Law (or PVT relationship) to understand depletion from a reservoir of gaseous resources. Only material in the gas phase (such as natural gas) can compress with the following relationship between pressure (P) and volume (V):

$$PV = nRT \qquad \textbf{(EQ 6-4)}$$

1. Except for oil production drop-off aided by water injection recovery techniques. This not only maintains a high rate, it can spoil much of the contents as well.

This basically says that when pressure increases, volume decreases proportionately, all other factors remaining equal (oil geologists also refer to this as PVT data [Ref 172]). In other words, this essentially states mathematically what we all intuitively understand in terms of compression — we can compress gas but not a homogeneous liquid.

The other terms in the ideal gas law:

- n = the number of moles of gas
- R = the universal gas constant
- T = the absolute temperature

form a constant only if the individual terms remain constant. Yet in reality, through the process of extraction, we do remove material from a pressurized reservoir. This causes the number of moles (n) to decrease; a mole defining a unit of dimension corresponding to about $6 \cdot 10^{23}$ molecules of gas.

As pressure (P) defines the rate of release from a natural gas reservoir:

$$P = \frac{nRT}{V} \qquad \textbf{(EQ 6-5)}$$

and the volume (V) stays constant in the cavern, then the pressure must decrease as material gets removed from the reservoir. This assumes the premise that the exit of gas from the hole flows proportionally to the pressure of the gas within the volume.

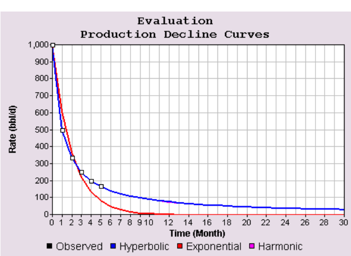


FIGURE 6-2. Distinction between exponential decline and hyperbolic decline. The latter has fatter tails.

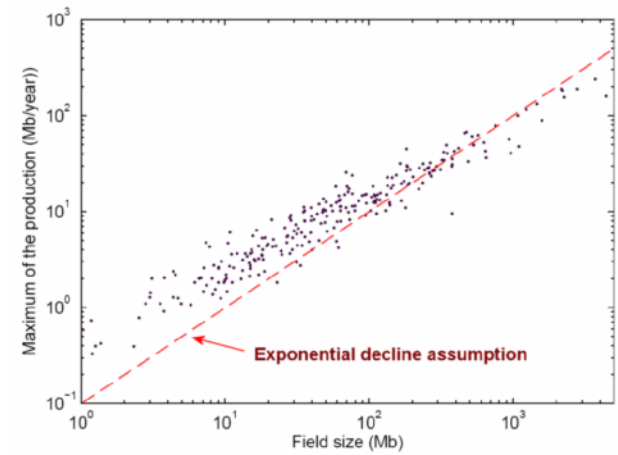
This gives us the proportionality, $P = kn$, whereby we draw down from any reservoir a linear fraction of the amount (n) left. This forms an alternative basis for the proportionate extraction model, this time applying it to natural gas or oil in a porous media. So this provides substantiating support that proportional drawdown remains a valid first-order approximation. This approximation also gets used heavily by petroleum engineers as they evaluate the production dynamics of a field, referring to it as *exponential decline,* or constant percentage decline[2]. An alternative approach uses an approximation called *hyperbolic decline*, which follows a sub-proportional drawdown rate, with the proportional rates declining as the actual production rate declines (we will look at this mechanism later in Chapter 9).

This brings up another interesting observation concerning the evolution of giant oil field production behavior. According to [Ref 30], a depletion rate approach works well for a large set of data. That study essentially observes a characteristic depletion

2. E.L. Dake discusses the lack of a physical basis for most decline behaviors in [Ref 172]

rate value at peak production for a range of 261 giant oil fields. The variance of this value remains relatively small.[3] Further, Michel shows maximum of production versus field sizes where the proportional draw-down linearity almost holds.

FIGURE 6-3. Data from Wood Mackenzie 2003 shows a large variance in max production rates but approximate linearity with size of reservoir. This extends to the assumption of proportional drawdown [Ref 13].



The characteristic rate becomes a more-or-less constant factor across a range of fields, providing more confirmation for a first-order model. So we can substantiate that both oil and natural gas follow this proportionate draw-down behavior, but not necessarily for the same reasons.[4]

Intuitively as well, a first-order extraction rate remains a valid assumption, as consumers historically have shown greediness in plundering any resource discovery. Market forces will tend to maximize the extraction in proportion to the amount available. Conservation of petroleum use during the late 70's caused a huge decrease in demand; before this time, people treated oil like an endless supply of water. In other words, in the old days, the spigot effectively had an opening proportional to the size of the reservoir.

Ignoring for a moment that the first-order exponential removal works for us at all, the following graph shows the zeroth-order approximation — extraction gets fixed

3. They refer to a "The Maximum Depletion Rate Model" as "resource constrained in the sense that the amount of oil in the ground ultimately puts a limit to the rate of production." [Ref 205]
4. That characteristic occurs commonly with a probabilistic model. Many classes of behavior get mixed in to the soup, and as long as the data follows a common distribution, the proportionate drawdown assumption remains valid.

to a constant rate for new discoveries. However, the total extracted remains the same as the first-order rate.

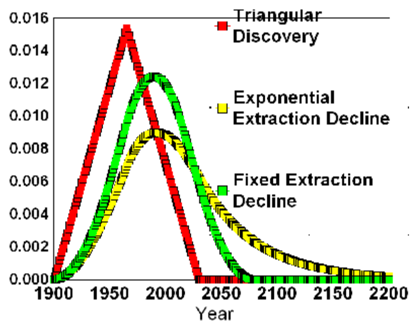


FIGURE 6-4. The assumption of a first-order extraction rate proportional to reserves leads to longer tails than a fixed yearly extra amount. The latter will abruptly hit bottom.

You can see the disappearance of the long tails as we remove the difficulty of oil extraction for depleted reservoirs, as it eventually hits bottom. The existence of stripper wells, capable of supplying proportionate drawdown effects, essentially cease to exist in this zero-order model. This remains the best counter-argument to the constant-rate argument — in that it doesn't make intuitive sense at the limits.

We can also change the discovery profile a bit to aid our intuition. Initially I set it as a symmetric profile which means that the peak discoveries occur at the midway point of the discovery life-cycle. However, we should equally consider the cases where we discover many of the reservoirs relatively early on (the "low hanging fruit" and "hunting elephants" phenomena). For completeness, I also added the late discovery profile to the chart.

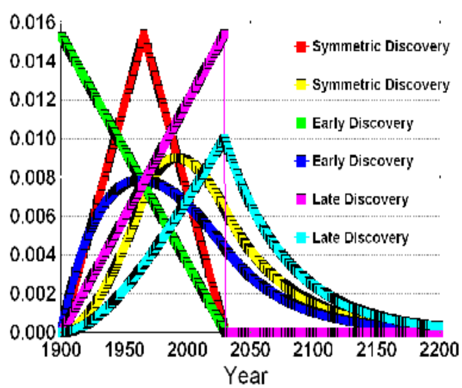


FIGURE 6-5. Changes in the symmetry of the discovery profile modify the production profile to that suggested by intuition.

Symmetric (Red=forcing, Yellow=response)

Early (Green=forcing and Blue=response)

Late (Magenta=forcing and Turquoise=response)

Even though I don't believe in a politically expedient "just-in-time" late discovery model, the shapes do imply some important considerations once the discoveries stop. For one, it means that once we discover the last reservoir, production starts its inexorable decline almost immediately. In general, the scales somewhat mirror the following (1) Early discovery - USA, (2) Symmetric discovery - The World, and (3) Late discovery - Parts unknown of Saudi Arabia and Iraq.[5]

Solving the Oil Shock Model

Stationary Processes — These models all have the stationary process property. Simply put, the transfer rate functions don't change with time. One can argue this point as we expect that technology and population growth to exert an ever upward growth to extraction rates. I don't initially include this variation as it defeats the purpose of providing a simple underpinning to the understanding of the curves. But quasi-stationarity holds as long as higher-order effects such as variations in extraction simply change the tails at a scale of 10's of years. This becomes clearly important for the actual peak oil date but not for understanding the general shape

The following derivation develops a relation suitable for an alternative grind-it-out numerical integration. This differs from the convolution approach in that it accumulates from the last value as a differential. The code solves the following differential equation:

$$\begin{aligned} R(t+dt) &= R(t) + (T(t) - R(t) \cdot E(t)) \times dt \\ P(t) &= E(t) \cdot R(t) \end{aligned} \quad \textbf{(EQ 6-6)}$$

where

$$\begin{aligned} R(t) &= \text{Current reserves} \\ T(t) &= \text{Triangular discovery curve} \\ E(t) &= \text{Extraction rate (yearly or daily)} \\ P(t) &= \text{Yearly (or daily) Production} \end{aligned} \quad \textbf{(EQ 6-7)}$$

‡ see "Code Snippet #2"

The equation basically states that the reserves accumulate by discovery but deplete by extraction *proportional* to the amount available for extraction, as asserted earlier. This latter proportionality allowed us do what amounts to convolution with an exponential, but the bookkeeping of a variable extraction rate caused by *temporal oil shocks* makes an analytical algorithm impossible. With a numerical integration, we can apply the proverbial mathematical sledge hammer to get at the results caused by perturbations in the extraction rate.

One can also now see how this differs from the logistic formulation favored by other depletion analysts:

$$Q(t+dt) = Q(t) + k\frac{Q(t)}{URR} \cdot (URR - Q(t)) \times dt \quad \textbf{(EQ 6-8)}$$

This solves for the classic Hubbert curve (where Q relates to the cumulative production as a proportion of the ultimately recovered resource, and k is a constant that sets the width of the peak). Unfortunately, this formulation has no meaning in terms of a physical model. I believe it important to mention the logistic approach at this stage because it provides the point of departure from a purely heuristic approach to

5. The shape of this reminds some commentators of a shark-fin.

what we will turn into a first-principles model of oil depletion, what I refer to as the Oil Shock model.

To give an idea of the kind of temporal oil shocks that we can include in the model, consider the real world oil shocks that occurred during our recent history. By using the time frame below showing historical oil price variations I make the intervals of shocks match the span of the political crisis in real terms.

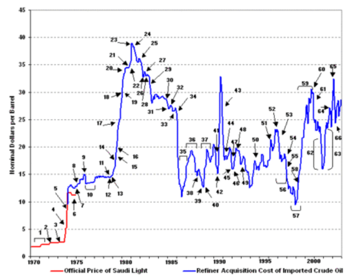


FIGURE 6-6. Disruptions in trade caused by geo-political, economic or environmental events can lead to shocks in price which percolate down to oil production decisions. These get modelled as shocks in extraction levels.

Obviously, it helps to know the dates at which the oil shocks occurred. The oil embargo started in late 1973 and lasted until the middle of 1974. The Iranian hostage crisis started in late 1979 and the early 80's recession officially ended by 1984. The gulf war started in 1990 and its associated recession ended in 1992. A simple algorithm interpolates the extraction rate over each of these intervals to make the suppressions in the curve a bit smoother (i.e. not as discontinuous a shock, reflecting reality).

With that in mind I developed the model to fit British Petroleum data along with the USGS cumulative value of 952 BBls. The following uses a symmetric triangular discovery curve starting in 1944 with a width of 87 years and normalized to an URR value of 2400 BBls.

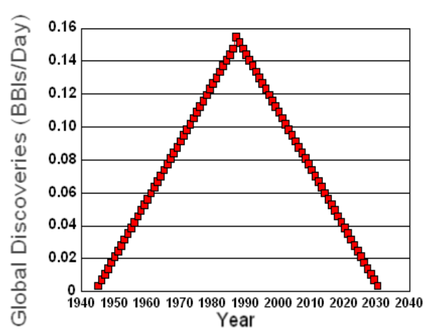


FIGURE 6-7. Another idealized discovery profile modelling the observed and smoothened historical global data.

The early years (pre 1970) of the model feature an oil extraction rate of 6% of total content per year. For an exponential drop-off, this gives a *1/e* point of approximately 16 years.

By adding three shocks in the years 1973-1974, 1979-1980, and 1991-1992, we can simulate the effects of the oil extraction rate changing dramatically. Because of the Markovian stationary properties of the model, I can simply adjust the rates of the normalized exponential term in the middle of the convolution without violating the stochastic nature of oil extraction over time. The shocks tend to momentarily suppress and flatten the production/consumption rate.[6]

TABLE 1. The three oil shocks correspond to the OPEC embargo, Iranian crises coupled with a deep recession, and the first Gulf war.

Year of Extractive shock	Rate	% change
pre-1973	6%	N/A - baseline
1973-1974	5.1%	-15%
1979-1980	3.4%	-33%
1991-1992	3.0%	-12%

‡ see "Code Snippet #2"

The output plotted against the British Petroleum data.

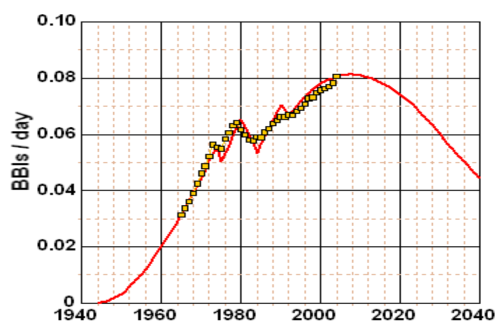


FIGURE 6-8. Fit to BP world-wide yearly production data using idealized discovery data with first-order shocks.

No doubt we will get more shocks in the future. The crucial finding in my mind: the shocks serve to delay *significantly* the onset of peak oil. Before the 70's, we used oil as if it came out of the tap; we have since made significant corrections in the extraction rate and our more conservative use of oil. When I originally analyzed this data in September of 2005, on first inspection I predicted that we would likely get a suppressive shock fairly soon. At the time I said that suppressive shocks would not disprove the peak oil hypothesis. It has in fact served to demonstrate how

6. As a technical aside, the suppressive dip in the output comes about mathematically from the exponential re-normalization, which essentially means a reduction in output.

unpredictable events can delay the peak from an assumed bell-curve shaped arc. In historic terms, I really believe (and the model shows) that we would have hit a peak several years ago without these 1970-1980 shocks in place.[7]

As a check, the red curve displays the output of the above code. The yellow curve shows what would have happened without the oil shocks. The production rates may in fact have increased significantly without the suppressive shock in place. And we may have experienced a more immediate decline.

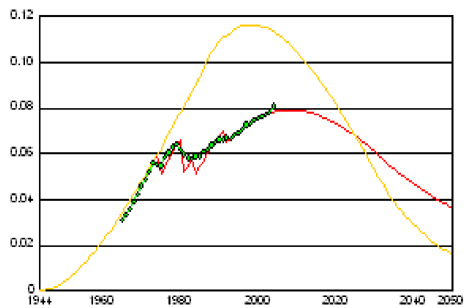


FIGURE 6-9. Super-imposed non-shocked model would show much earlier peak assuming the same rate of oil extraction. Suppressive shocks acted to defer the peak beyond the turn of the century. Note what happens after the peak without the shocks — a steady decline.

We will discuss this further in a later chapter, but by applying Hubbert Linearization (HL) to the model results, the asymptotic Ultimate Recoverable Resources (URR) fit looks like the following curve. Of course the asymptote hits 2400 BBls because of the triangular distribution I started with. Note that the curve only starts to show linearity in the out-years, indicating that the classic Logistic model used to derive Hubbert linearization probably does not reflect reality over the entire range. This points to an early indication that HL serves as an unreliable heuristic.

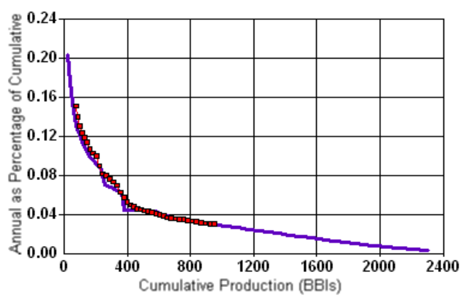


FIGURE 6-10. Hubbert Linearization on shocked model demonstrates quasi-linear behavior only the tails.

‡ see "Code Snippet #3"

7. In retrospect, and this will no doubt seem counter-productive, but recessions during the latter part of the 20th century actually probably did us some good — if you want to call delaying the inevitable beneficial.

Throughout this discussion, I have use the term "discovery" a bit loosely choosing some rather contrived discovery profiles. For the purposes of discussion, we also assumed that pumping activity started immediately upon discovery of the reserves. It should actually include some of the real latencies, described in the previous chapter, and which we will correct for in the model in the next step of the analysis.

Full Elaboration of the Shock Model

I start with an implicit assumption that any rate of extraction or flow always remains proportional to the amount available and nothing more; past and future history do not apply. This describes a first-order linear Markov approximation that allows one to either calculate analytically (in the simple cases) and computationally for more elaborate scenarios. This essentially describes a stochastic (random or seemingly random) trend of resource depletion over time.

The simple case reduces to the exponential model. Here, we assume two states: an **undepleted** state #1 (*P1*) that transforms into a **depleted** state #2 (*P2*) according to a Markovian rate term.

$$\begin{aligned} \frac{d}{dt}P_1(t) &= -\lambda P_1(t) & \quad P_1(0) &= 1 \\ \frac{d}{dt}P_2(t) &= \lambda P_1(t) & \quad P_2(0) &= 0 \end{aligned} \qquad \text{(EQ 6-9)}$$

The analytical solution to these two equations reduces to a couple of exponentials which match the initial conditions:

$$\begin{aligned} P_1(t) &= e^{-\lambda t} \\ P_2(t) &= 1 - e^{-\lambda t} \end{aligned} \qquad \text{(EQ 6-10)}$$

Graphically, we can express this set of two differential equations as a *state transition diagram*. For the visually inclined, we can graphically depict this as the following state transition diagram (STD):

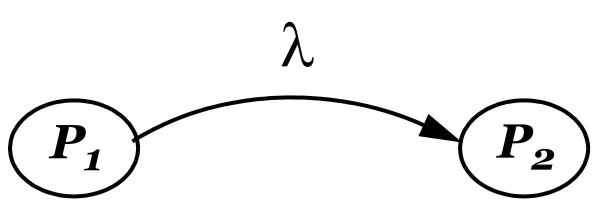


FIGURE 6-11. Simple 2-state model - transition diagram. Quantity flows from P_1 to P_2 with rate λ proportional to P_1

And given a value for the rate parameter assuming a particular time-scale, we can easily solve these differential equations through straightforwardly-derived numerical integration routines:

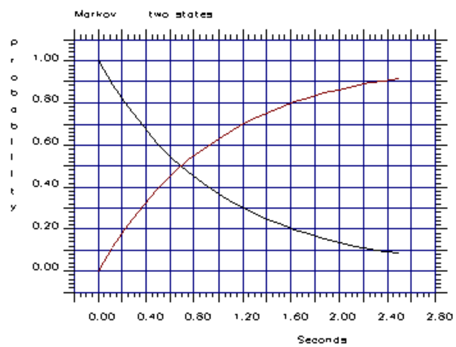


FIGURE 6-12. Temporal solution to state-transition diagram with initial conditions P_1=1 and P_2=0. The flow transfers from P_1 to P_2 with characteristic time constant of λ .

We provide detail to the model by adding rate terms that describe the other state transitions that occur during the oil production life-cycle.

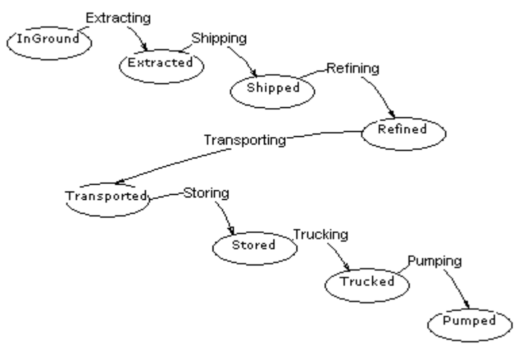


FIGURE 6-13. Adding more intermediate states to the state-transition pipeline cascades the flow and delays the ultimate transfer. In this case, we model transitions with sequential physical stages of production.

TABLE 2. Suggested rates of transfer for different phases of the oil production "pipeline". Rates greater than 100% means transfer rate greater than extraction rate.

Stage	Rate of Transition
Extracting	20% / year
Shipping	200% / year
Refining	100% / year
Transporting	100% / year
Storing	520% / year
Trucking	520% / year
Pumping	520% / year

Each transition follows a Markov rate, with the strength of the transition proportional to how quick we can "turnover" the amount in the previous state. In general, approximating the strength of extraction on the proportion left allows us to intuitively model such effects as the small amount taken from stripper wells and the infrequency of shipping small volumes of oil.

The initial conditions place all states at 0.0 except the **InGround** state which we normalize to 1.0 representing the full capacity of the reservoir.

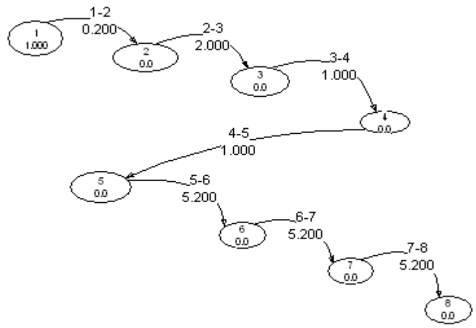


FIGURE 6-14. Transforming the symbolic transition rates into numbers. The initial state starts at unity and the others set to zero.

For the rate parameters chosen above, we can calculate the profile after 4 years of extraction (each state gets scaled by the rate going out of that state to capture the “in-the-pipeline” effect, something the consumer can most closely identify with):

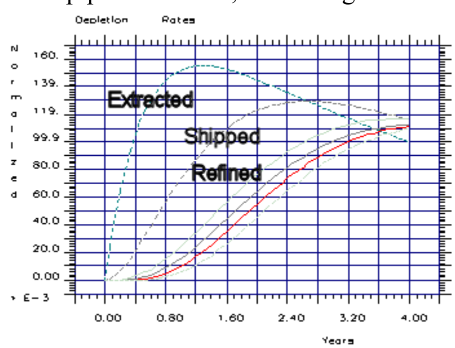


FIGURE 6-15. Solution of the cascaded state-diagram. Note that transitions of high rates show small shifts in time and therefore low latency of transfer. These can be ignored in many cases.

The snap-shot for the state diagram at 4 years shows the maximum available at the pump. Note that a maximum in the extracted state had already occurred.

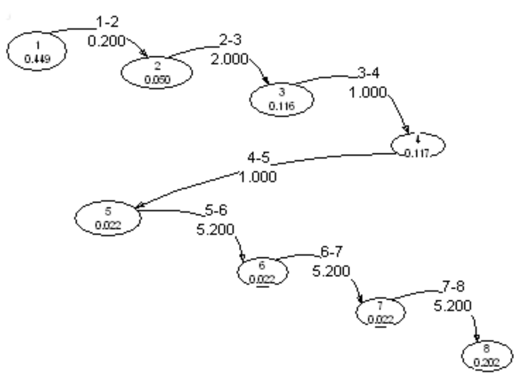


FIGURE 6-16. Snapshot of state-transition diagram at a point in time of the transfer process.

Latency at Sea

My rough estimate says that at any given time in recent years about 600 to 700 million barrels of crude oil are at sea, enroute from exporters to consuming countries.

As a shipowner, about the only cost I have much control over is my fuel cost. Financing and insurance costs are a function of time--so much per month or year. Maintenance is also mostly a function of time, but it might be deferred during economic downturns. Crew costs are also a function of time--so much per month, or per shift.

Fuel burned is a function of speed--the drag goes up as a function of the speed--roughly at the square of the speed--double the speed equals more than double the fuel, but half of all the other costs. So at any time, there is an optimum speed--the higher the cost of fuel, the slower to steam to optimize profits. The lower the fuel cost, the faster you show go, up to the limit posed by higher drag on the ship's hull.

We have just had the biggest drop in bunker fuel prices ever experienced, so every good charter captain just started steaming faster--anything else would be to leave profits on the table. How much faster? If 25%, then the amount of crude at sea would have dropped by 100 to 150 million barrels over the past 2-3 months, or about 1.5 to 2 million barrels per day. This destocking at sea would make it look like we have a worldwide oversupply of that amount. By the way, this same factor of ship-speed also explains the weakness in the BDI.

It also says that once oil prices(bunker fuel) starts back up, then that 2 million b/d will disappear from the markets as the steaming speed slows back down. I happen to think the destocking at sea is about to end, and that, together with the winter seasonal increase and OPEC cuts may remove some 6 million b/d from markets between now and February, and that is why Land-Lubber just might be right in his call for $150 crude by 2-29-2009. Just my 2 cents for discussion. I can sure tell you as a pilot I adjust airspeed to reflect fuel costs, and so do all the airlines. [Ref 323]

After 20 years, the depletion at the pump becomes clearly visible:

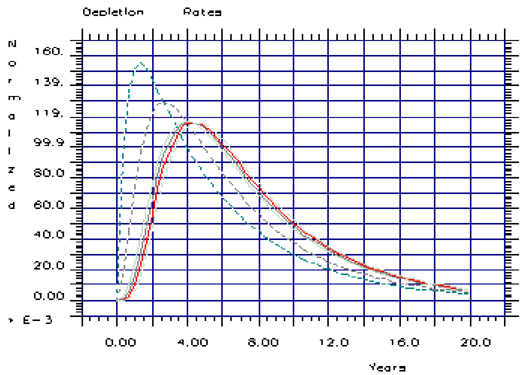


FIGURE 6-17.
Plotting the state-transition dynamics over a longer period of time, we see eventual depletion of all states, with the conserved quantity entering the "end"-state.

I can further interpret this analysis in terms of the larger global macroscopic view, but a few things to note from what we have modeled and simulated so far:

1. We observe an asymmetric peak from single reservoir depletion.
2. Depending on how we define the peak, it may depend on where we look in the state transition "pipeline"
3. Imagine sets of these curves laying on top of each other, representing independent reservoir depletion profiles.
4. Any reduction in the rates at any stage will push the peak to the right along the timeline
5. High relative rates in any of the transitions affect the peak location very little as these act to efficiently pass fluid flow quickly to the next state

A technically proficient analyst with access to symbolic math software can easily duplicate these curves by using the *convolution operator* on a set of exponentially distributed functions with appropriate coefficients. Alternatively, if the rates are all identical, we can simply plot each curve as an *n*-order Gamma curve (where *n* refers to the stage of the pipeline).

To intuit global peak oil (the *macro-economics*) at a larger scale we need to invoke a profile of oil discoveries along an appropriate timeline, but the shorter time frame described in this example (i.e. *micro-economics*) better describes effects due to local perturbations, including:

1. hurricanes
2. refinery explosions
3. transportation bottlenecks

A commenter pointed me to this extended abstract from the ASPO 2005 conference [Ref 27]. The researchers do a good job in empirically fitting various distributions to the global production profile (something that I didn't even attempt to try my hand at) but do raise the same point I made:

> *Contrary to popular belief, Gaussians are not good models for time series. The Central Limit Theorem applies to random walks through controllable dimensions. To apply to oil production, the theory would have to be that God dropped 2 trillion barrels directly above the year 2008, and the barrels scattered forwards and backwards through time from there. That's obviously silly.*

I wish the authors would have tried fitting to the **regional** oil production numbers (i.e. U.S., Europe, etc.). With the laws of self-similarity applying, they might have gotten some more insight into the asymmetry. However, as they wanted to concentrate on the **global** view, the tails have not yet emerged for them to match against any of the standard distributions to (either symmetric or asymmetric).

This gives us quite a bit of control in modeling our intuition. The characteristic asymmetry of the consumption curves (i.e. steeper rise than fall-off) arises due to the first-order assumption that humans extract petroleum at a rate proportional to the amount left in the ground. This essentially repeats the arguments in the last chapter[8]. For a good example of the proportionality principle, consider the rise and decay of U.S. wildcat operations. At one time, each significant find generated a large flow, but over time the reservoir contents became depleted enough that the contents reduces to the much smaller stripper well flow. The latencies involved in collecting and delivering the oil extracted from stripper wells at least partly contribute to the extended tails we see in the post-Hubbert-peak of U.S. oil production.

We still need to isolate the key latencies while incorporating the full discovery model to reflect the reality of global oil depletion.

The Canonical Curve

Most of the micro-latencies due to transportation and storage have little effect on the macro model. We actually want to concentrate on the "big hitters" that have a significant influence on the production dynamics.

The following narrative describes the virtual flow of oil through the system from discovery to production, providing a problem domain description to how an oil peak occurs. Then we will run through the accompanying stochastic differential equations.

- *Precondition*: Oil sits in the ground, undiscovered.
- *Invariant*: The amount of oil in each of its states cumulatively sums to a constant.
- The states of the oil in the ground consist of *fallow*, *construction*, *maturation*, and *extraction*.

The process starts by someone making a discovery and providing an estimate of the quantity extractable. Thereafter it follows a narrative with the connections labelled by "=>".

The oil sits in the *fallow* state until an oiler makes a decision or negotiates what to do with it. Claims must be made, etc. This time constant we call $\mathbf{t_1}$, and we assume this constitutes an average time with standard deviation equal to the average. This becomes a maximum entropy estimate, made with the minimal knowledge avail-

8. As a fan of Occam, I normally try to use the simplest and most parsimonious explanation for phenomenon that I can get away with

able.
=> A stochastic fraction of the discovery flows out of this fallow state with rate **1/ t_1**. Once out of this state, it becomes ready for the next stage (and state).

The oil next sits in the *build* state as the necessary rigs and platforms get constructed. This time constant we call **t_2**, and has once again an average time with standard deviation equal to the average.
=> A stochastic fraction of the fallow oil flows out of the build state with rate **1/t_2**. Once out of this state, it becomes ready for the next stage

The oil sits in the *maturation* state once the rigs get completed. We cannot achieve the maximum flow instantaneously as the necessary transport, pipelines, and other logistics are likely not 100% ready. This time constant we call **t_3**.
=> A stochastic fraction of the built rig's virtual oil flows out of the maturation state with rate **1/t_3**. Once out of this virtual state, it becomes ready for the next stage of sustained extraction.

The oil sits in the ready-to-extract state once the oil well becomes mature.
=> The oil starts getting pumped with stochastic extraction rate **1/t_4**. The amount extracted per unit time scales proportionally to the amount in the previous maturation state..

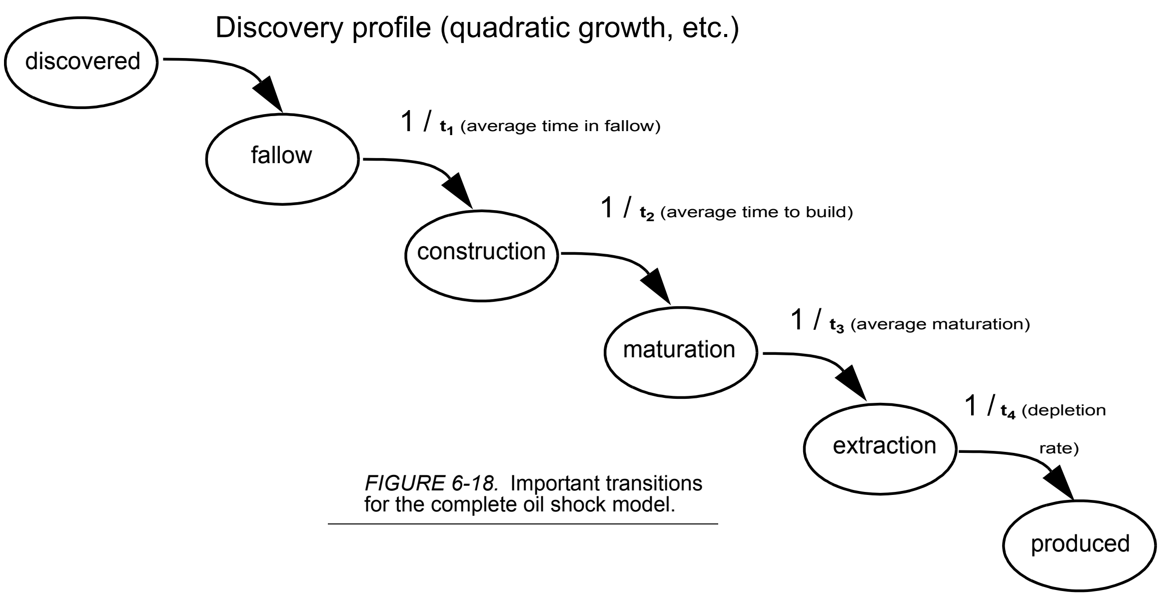


FIGURE 6-18. Important transitions for the complete oil shock model.

Post-condition: All oil eventually gets extracted at time=∞ . But because of the proportionality extraction rate assumed, this decline only asymptotically approaches zero at long time periods. Also, the cumulative amount extracted at time=∞ equals the total discovered. However, since we never achieve infinite time, cumulative extraction never matches cumulative discoveries, other than in an asymptotic sense.

We can consider each one of these states as a reservoir with a capacitive time lag associated with the time constant set for each stage[9]. In stochastic terminology the flow approximates a Markovian process, with a depletion rate not dependent on previous time-history values.

The extraction from the final stage gives the average production level. Since Markov processes have well-behaved linear properties and remain conditionally independent of past states, we can apply an entire set of discoveries as forcing functions to this process flow and the result will reduce to a convolution of the individually forced solutions.

The final production profile over time approximates the classic Hubbert curve. This narrative explains in very basic terms how and why the peak gets shifted well away from the discovery peak. Significantly, we observe no symmetry in the derived curve, as the nature of time causality rules long negative tails out.

Regarding the USA crude oil production curves, Staniford was able to make a very good fit over a few orders of magnitude using a Gaussian [Ref 31]. As for temporal properties of this curve over time, Staniford noted graphically that it has the property that:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = K \cdot (t_0 - t) \cdot P(t) \qquad \textbf{(EQ 6-11)}$$

where t_0=PeakTime. This relationship reads that the production increase slows down over time linearly, but also scaled by the amount in production at that time — matching the behavior of an linearly decreasing positive feedback turned into a linearly increasing negative feedback. At $t=t_0$, the production increase turns into a production decrease. Unfortunately, I can’t provide a problem-domain rationale for the gaussian formulation. Like the classical logistic curve formulation, I can’t shape it into intuitive terms. However, in terms of the Oil Shock Model, the Gaussian similarity makes some sense. I will discuss this further in “Special Case: The Central Limit Theorem” .

9. See Volume 2 for electrical analogies

This Must Be Laplace

Through some straightforward math, I can create a closed-form expression for the stationary solution to the oil shock model. Note that this assumes constant values for all associated rates over time and a delta value for the discovery term. As a sanity check, the solution matches that of the Gamma function in the special case of equivalent rates assumed during each phase.

$$\begin{aligned}
D(t) &= \text{Discovery stimulus}\\
R_1(t) &= \text{Reserve emerging from fallow state, Rate = a}\\
R_2(t) &= \text{Reserve emerging from construction state, Rate = b}\\
R_3(t) &= \text{Reserve emerging from maturation state, Rate = c}\\
R(t) &= \text{Reserve emerging from production state, Rate = d}\\
P(t) &= \text{Production curve}
\end{aligned} \tag{EQ 6-12}$$

The stochastic differential equations look like:

$$\begin{aligned}
\frac{dR_1}{dt} &= D(t) - a \cdot R_1(t)\\
\frac{dR_2}{dt} &= a \cdot R_1(t) - b \cdot R_2(t)\\
\frac{dR_3}{dt} &= b \cdot R_2(t) - c \cdot R_3(t)\\
\frac{dR}{dt} &= c \cdot R_3(t) - d \cdot R(t)\\
P(t) &= d \cdot R(t)
\end{aligned} \tag{EQ 6-13}$$

This forms a set of linear differential equations that we can alternatively cast in terms of convolution operators and the Laplace transform:

$$\begin{aligned}
P(t) &= D(t) \otimes F(t) \otimes C(t) \otimes M(t) \otimes E(t)\\
\text{where} \quad \otimes &= \text{Convolution operator}\\
D(t) &= \text{Discovery profile}\\
F(t) &= \text{Fallow lag}\\
C(t) &= \text{Construction lag}\\
M(t) &= \text{Maturation lag}\\
E(t) &= \text{Extraction rate}
\end{aligned} \tag{EQ 6-14}$$

$$A(t) \otimes B(t) = \int_{\tau=0}^{\tau=t} A(t-\tau) \times B(\tau) d\tau \qquad \textbf{(EQ 6-15)}$$

$$\mathcal{L}(A(t) \otimes B(t)) = A(s)B(s) \qquad \textbf{(EQ 6-16)}$$

If we take the Laplace transform of the set in EQ 6-13 and do the transitive substitution, we can get the production curve in *s*-space.

$$\begin{aligned}
r_1(s) &= d(s)/(s+a) \\
r_2(s) &= a \cdot r_1(s)/(s+b) \\
r_3(s) &= b \cdot r_2(s)/(s+c) \\
r(s) &= c \cdot r_3(s)/(s+d) \\
p(s) &= d(s) \cdot a \cdot b \cdot c \cdot d/(s+a)/(s+b)/(s+c)/(s+d)
\end{aligned} \qquad \textbf{(EQ 6-17)}$$

So we can either use the Laplace transform of differential equations or the Laplace transform of the convolution and arrive at the same result. This gives one some options depending on the application.

If we assume a single delta for discoveries, then $d(s)=1$. The inverse Laplace transform gives the following (unscaled) time-domain expression:

$$\begin{aligned}
&-\frac{e^{-at}}{(abc+abd+acd-bcd+a^3-a^2b-a^2c-a^2d)} \\
&-\frac{e^{-ct}}{(abc-abd+acd+bcd+c^3-ac^2-bc^2-c^2d)} \\
&+\frac{e^{-bt}}{(-abc-abd+acd-bcd-b^3+ab^2+b^2c+b^2d)} \\
&+\frac{e^{-dt}}{(abc-abd-acd-bcd-d^3+ad^2+bd^2+cd^2)}
\end{aligned} \qquad \textbf{(EQ 6-18)}$$

For values of rates very near 2.0, the production curve looks like this:

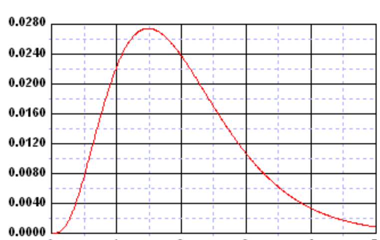


FIGURE 6-19.
Plot of analytical solution to the oil shock model with delta discovery . Note that the curve shows asymmetry from a perfect bell-curve.

In this specific formulation, we must make sure that no two rates identically equate or else the solution becomes degenerate as the multiple poles form singularities. I mention this because the formulation as described may prove useful in an optimization setting. By scanning through the ranges of the set of (*a,b,c,d*) one can quickly zero in on a first-order fit for a known discovery date and corresponding production data.

I prefer to use a numerical integration scheme to solve these equations, but the straightforward derivation provides a bit of insight into how the phased time constants arithmetically combine the exponentials into forming the asymmetric production profile. Ultimately, the simplistic assumption of a delta discovery and constant rates prevent me from recommending the closed-form solution for complex, highly-featured real-world production curves. This will have to wait for a real model of a discovery curve.

Limiting Distributions

Special Case: The Gamma Distribution

In typical use the oil shock model does not give a closed form solution. Because the input stimuli (normally provided by a set of discovery delta functions) needs to come from collected data and therefore displays a degree of randomness, we really should not expect anything approaching the symmetric simplicity of the Hubbert/logistic function for cumulative production:

$$\frac{\mathrm{d}U}{\mathrm{d}t} = k \cdot r \cdot e^{-rt} / (k + e^{-rt})^2 \qquad \textbf{(EQ 6-19)}$$

Yet, under a set of idealized conditions, a variant of the oil shock model does revert to a fairly simple representation, that of the gamma distribution, which involves the repeated convolution of an exponential curve with itself **N** times total. I mentioned this in the previous section “This Must Be Laplace” and it makes sense to repeat it again to close the loop.

$$U(t) = C(t) = M(t) = E(t) = \beta \exp(-\beta t) \qquad \textbf{(EQ 6-20)}$$

$$\mathcal{L}(\beta \exp(-\beta t)) = \frac{\beta}{s + \beta} \qquad \textbf{(EQ 6-21)}$$

$$\mathcal{L}(U(t)) = D(s)F(s)C(s)M(s)E(s) \qquad \textbf{(EQ 6-22)}$$

Then if we let $D(s)$ consist of two exponentials convolved with one another.

$$\mathcal{L}(U(t)) = \left(\frac{\beta}{s+\beta}\right)^6 \qquad \textbf{(EQ 6-23)}$$

Normalized, the gamma distribution derives from the inverse Laplace transform and looks like this depending on the order N:

$$\text{Gamma} = t^N \cdot e^{-t}/(N-1)! \qquad \textbf{(EQ 6-24)}$$

Plotted below with N=6, the gamma (in red) shows a distinct asymmetry with longer tails than the Hubbert curve (in yellow). Note that we needed one more convolution to get from U(t) to P(t).

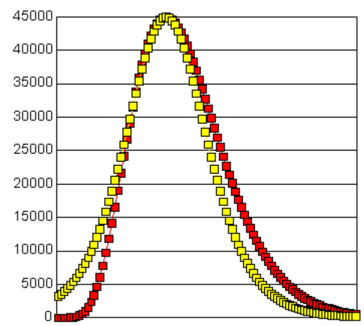


FIGURE 6-20.
Difference in symmetry between a normal (Gaussian) and a Gamma function. For equal areas, the Gamma shows a heavier tail.

I chose N=6 to mimic a set of discoveries (the first 2 exponentials convolved together) convolved with the remaining four exponentials representing the fallow, build, maturity, and extraction phases of the conventional oil shock model.

I would not typically use the gamma if I had discovery data available, but it does have the nice property of ease of use in data fitting applications and it has enough similarities to the Hubbert/logistic curve to serve as a replacement in traditional analyses. Plus its derivation rests on realistic first principles — something in which the logistic function falls short.

If we chose a different type of discovery function, $D(s)$, then the expression in EQ 6-23 would become:

$$\mathcal{L}(U(t)) = D(s) \times \left(\frac{\beta}{s+\beta}\right)^4 \qquad \textbf{(EQ 6-25)}$$

Clearly, this results in something more complicated than a Gamma.

Special Case: The Central Limit Theorem

The use of the gamma distribution provides some mathematical convenience that perhaps adds a bit clarity to the understanding of the Oil Shock Model. In mathematical terms it also plays a role in understanding the trending of certain distributions to the Normal distribution via the central limit theorem.

As mentioned earlier, one may often see a reference to the peak oil Hubbert curve describing it as a Gaussian or normal distribution. The reference to a Bell-shaped curve usually signifies some connection to a law of large numbers or central limit theorem argument — or less frequently to some type of rate law. I have never accepted the classical hand-waving justification for a Hubbert curve following a perfectly normal distribution, as it violates causality in my opinion. However, we can demonstrate a path to the central limit theorem just by using the oil shock model.

First consider this (wikipedia) statement describing the central limit theorem:

> *The density of the sum of two or more independent variables is the convolution of their densities (if these densities exist). Thus the central limit theorem can be interpreted as a statement about the properties of density functions under convolution: the convolution of a number of density functions tends to the normal density as the number of density functions increases without bound, under the conditions stated above.*

The first sentence basically reiterates the premise of the oil shock model. We made the assumption that the temporal dynamics of oil production rely solely on a set of random variables representing delays in extraction occurring after the discovery point. The densities (i.e. the probability density functions) of these delays follow a declining exponential in which the standard deviation equals the mean for each random variable. The second sentence indicates that the density of the sum (i.e. the sum of the variable delays) leading to a peak comes from the repeated convolution of the individual variable densities. The definition describes how even an arbitrarily shaped density function when convolved against itself several times leads to a curve that has a “Normal” shape:

As a refresher, let me give examples of the random variables that the central limit theorem refers to, in specific terms relative to the oil shock model.

$$\text{TimeDelay} = (T_1 + \ldots + T_n)\ \text{random variables} \qquad \textbf{(EQ 6-26)}$$

So for n=4, we may find for a specific well that

1. The discovery lays fallow for 3 years (T_1=3) while negotiations take place for ownership, rights, permits, etc.
2. Next, construction of the oil rigs and infrastructure takes 8 years (T_2=8)
3. After completion, it takes 5 years (T_3=5) for the reservoir to reach maturation (toss in reserve growth considerations)
4. Once pumping at full rate, the reservoir drains with a time constant of 10 years (T_4=10).

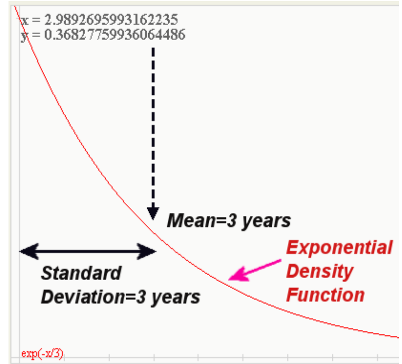


FIGURE 6-21. Properties of a damped exponential probability density function (PDF)

According to a deterministic setting, the sum of these values equals 3+8+5+10=26 years; or 26 years until the reservoir drops to its 1/e original value.

But in a stochastic world, the individual delays turn into density functions that we characterize completely by treating the delays as averages with a maximum entropy standard deviation, i.e. a decaying exponential.

So if we pair up the four stages as two sets of convolutions, we can generate intermediate density profiles.

$$e^{-at} \otimes e^{-bt} = \frac{(e^{-bt} - e^{-at})}{a-b} \quad \textbf{(EQ 6-27)}$$

The first convolution pairs the fallow stage with the construction stage. The peak of this curve occurs away from *Time*=zero even though the individual exponentials have peaks at zero. This demonstrates the initial impact of the central limit theorem.

The second convolution pair calculates the maturation+extraction shift. Note that the shift away from *Time*=0 illustrates the impact of a continuous maturation probability density function — extraction will not hit a peak until the area matures to an expected value.

Next, the two pairs get convolved together to show the total shift from the initial discovery delta, with a peak at around 18 years. Note that the profile continues to sharpen and become more symmetric.[10]

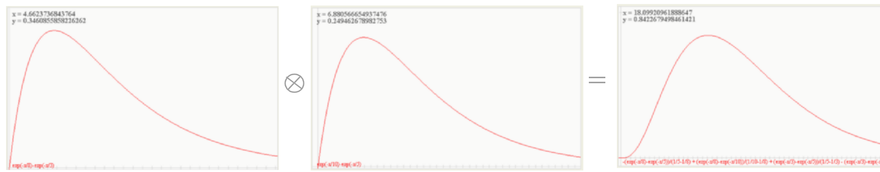


FIGURE 6-22. First, second and combined convolutions eventually leading to a normal distribution.

But remember that I generated this final production profile for a single discovery delta. Throw in a range of discoveries, perhaps following the quadratic or cubic discovery model I presented previously, and you will see the classical Bell-shaped curve emerge without requiring too many convolutions.

The outcome of this derivation suggests that we can use central limit theorem arguments to “prove” the existence of a roughly Bell-shaped curve without having to precisely match a Gaussian/normal profile. And again, the causal nature of the discovery/production process prevents us from achieving an exact match in the first place.

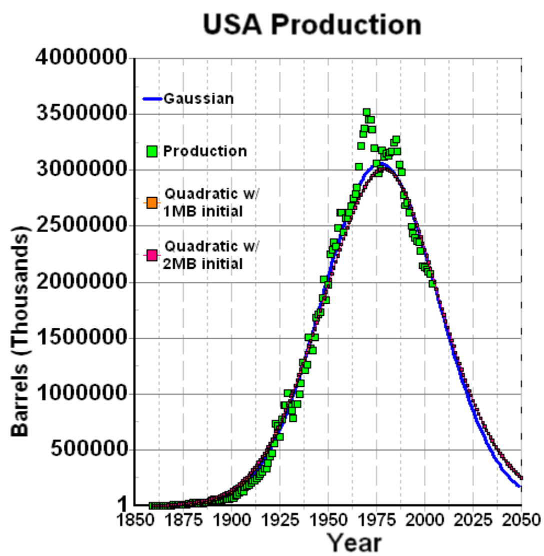


FIGURE 6-23. Fit to USA production using a smoothed discovery profile applied to the oil shock model.

As a variation of this special case, consider a hypothetical situation that a discovery profile fits a Gaussian density profile in time. Then consider that each shift in the

10. Note we can also generate the density profile via Laplace transforms, i.e. characteristic functions in central limit theorem parlance.

production history also follows a Gaussian mean with attached variance. It follows (in keeping with the formulation of the Shock Model) that a convolution of a Gaussian density function with a Gaussian results in another Gaussian[11]. The resultant width adds in quadrature and the Gaussian shifts by the relative offset of the two curves. This essentially means that after *N* repeated convolutions the peak shifts by *N* mean values and the width broadens by the square root of the sum of the squares of the *N* standard deviations. Both the Shock Model and this hypothetical analysis used the concept of the mathematical convolution to demonstrate how the oil production curve shifts in time from the initial discovery profile. In the global situation this shift often manifests itself by latency of dozens of years (the current shift runs at +40 years).

The repeated convolutions also cause the initial discovery profile to broaden due to probabilistic considerations, essentially explainable by the uncertainties in when production and other preceding phases actually start on a given discovered region. The only way that the profile can sharpen after discovery occurs by increases in the extraction rate as evidenced by demand, technological advancements, or other various production shocks. These non-stochastic properties can alone counteract the relentless advance of entropy, which ultimately leads to a broadening profile.

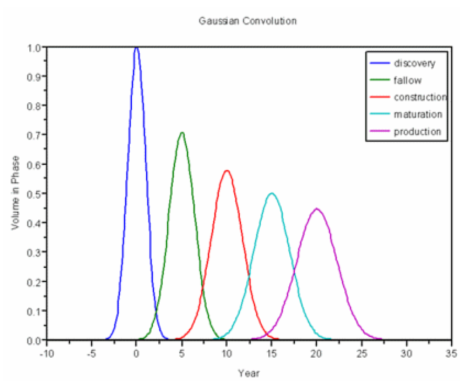
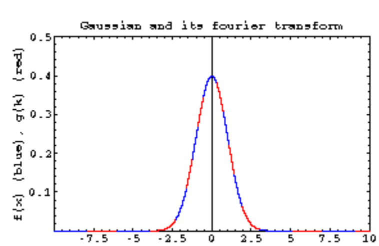


FIGURE 6-24. Shifting and broadening of the production peak after repeated Gaussian convolutions.

The curves above correspond to the equations below where standard deviation=w=1 and mean latency=dt=5 years:

11. Proof left to the reader. Hint: For an alternative derivation of the Gaussian convolution identity, look up the concept of Fourier transforms. The derivation works out simply if we use the identity that a convolution in the time domain corresponds to a multiplication in the frequency domain, and that the Fourier transform of a Gaussian results in a Gaussian.



$$P0 = \frac{e^{-\frac{t^2}{2w^2}}}{\sqrt{w^2}} \text{ Discovery phase centered at 0, width = w} \quad \text{(EQ 6-28)}$$

$$P1 = \frac{e^{-\frac{(t-dt)^2}{2(w^2+w^2)}}}{\sqrt{w^2+w^2}} \text{ Fallow phase, mean latency = dt, variance = w}^2 \quad \text{(EQ 6-29)}$$

$$P2 = \frac{e^{-\frac{(t-2dt)^2}{2(w^2+w^2+w^2)}}}{\sqrt{w^2+w^2+w^2}} \text{ Construction phase} \quad \text{(EQ 6-30)}$$

$$P3 = \frac{e^{-\frac{(t-3dt)^2}{2(w^2+w^2+w^2+w^2)}}}{\sqrt{w^2+w^2+w^2+w^2}} \text{ Maturation phase} \quad \text{(EQ 6-31)}$$

$$P3 = \frac{e^{-\frac{(t-4dt)^2}{2(w^2+w^2+w^2+w^2+w^2)}}}{\sqrt{w^2+w^2+w^2+w^2+w^2}} \text{ Production, draw-down time = dt} \quad \text{(EQ 6-32)}$$

The use of Gaussians to describe the convolutions allows one to obtain a closed-form solution for the result and at each interim stage. Note however, that the closed form solution only holds if the Gaussians have negative time histories, which unfortunately breaks the time causality of discovery and production. But as I introduced this special case, if we use this simply as a means to a better understanding in how the continuous model manifests itself, by truncating the negative tails and eyeballing the curves, we can essentially live with the approximation. It gives us a intuitive shorthand in understanding how the latencies add up and how the resultant peak can broaden given the underlying density functions.

The Role of Geology in Production

In this analysis, geology *per se* does not play a critical role. This suggests that the most difficult concept to convey remains that of the separation between the mathematical abstraction of economic/human decisions governing the flow of oil and that of geology. As I indicated earlier, we could be talking about the flow of jellybeans and would still use the same mathematical concepts.

By the same token, the ability to determine what leads to a first-order effect and what gets relegated to second-order makes a big deal in whether you get mired in the details. For example, I consider most of these flow latencies such as shipping at least second-order when placed on a global scale. The biggest effects lie in the main phases of the extraction process and that of discovery, which we will describe in detail next.

CHAPTER 7

Finding Needles in a Haystack.

How we discover oil

> *"The surplus has led us to believe in the possibility of universal peace and universal comfort, for a global population of 6 billion, or 9 or 10. If kindness and comfort are, as I suspect, the results of an energy surplus, then, as the supply contracts, we could be expected to start fighting once again like cats in a sack."*
>
> — George Monbiot

In grade school, we probably all remember routinely working out challenging mathematical word problems. I do recall occasionally getting one right, but more often ended up punting on the problem, and then waiting for the teacher to explain the solution in all its elegant simplicity. Of course, just about every real-world problem contains inherent ambiguities and incomplete information. So we rarely get to see the elegant solution in our day-to-day work life. Sometimes we get lucky and nail a problem, but in the majority of cases, we eventually resort to creating a limited model of the problem domain and deal with that. The problem at hand that we need to wrestle with has to do with *predicting future oil discoveries based on historical dynamics*.

Ideally, I want to reduce this to a solution that has the elegance of a word problem, and not have to deal with messy economic and geologic factors that would quickly turn it into a rat's nest of complexity. Call me an abstract idealist in this regard, but my intuition tells me that the solution remains as simple as the proverbial notion of *finding a needle in a haystack*. Perhaps not so in regard to the actual process, but simple as in the premise behind the problem. Let me explain why this provides a good primer to the oil discovery problem.

Scaled back to relative terms, the ratio of needles to hay compares intuitively to the ratio of oil to the earth's crust. So first and foremost, this rather naive analogy allows us to get our arms around a problem with just enough initial insight to get started — the description of which amounts to nothing more than imagining that the haystack acts like the earth's crust and the needles serve as the pockets of oil. Statistically speaking, happening across a random needle in a haystack has a lot in common with running across a pocket of oil. We can also add technology and human incentive to the mix to extend the simple analogy before we migrate to the real problem. So I present a starter word problem:

> Given a large number of needles dispersed in a random spatial manner throughout a good-sized haystack, at what point in time would we find the maximum number of needles? As a nod to technology we get to monotonically increase our search efficiency as we dig through the stack, and we can add human helpers as we progress.

Answer: Obvious, and we don't have to even lift a pen. On average, the maximum discovery of needles occurs as we sift through the last of the volume, and once finished, the discovery rate drops to nil. So the instantaneous "discovery" rate looks similar to the curve at the right. The acceleration upward in the curve occurs as we get more proficient over time and can attract some help. Note that if we mixed larger nails and smaller pins with the needles and instead measured total weight or volume instead of quantity, we would have the same curve (this has implications for the oil discovery problem).

Next, let's make the word problem a bit more sophisticated. Say that instead of dispersing the needles randomly through the *entire* haystack, we only do it to a certain depth, and to top it off, we do not reveal to the needle-and-pin searchers the value of this depth. They basically have to *oversample* the haystack to find all the needles. If you look at the following figure, we separate out the "easy" part of the search from the "difficult" part (i.e. difficult as in not finding much even though we expend the effort). The boxes represent monotonically increasing sampling volumes, which we use to sweep out the volume of the haystack.

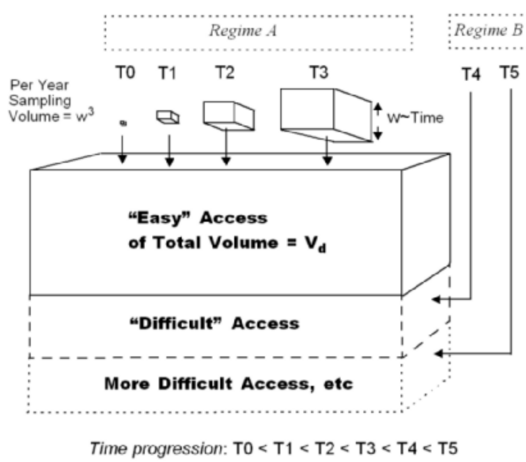
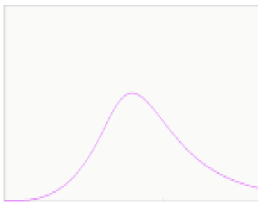


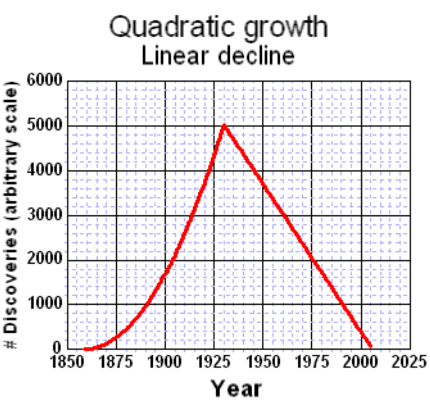
FIGURE 7-1.
Model for growth in discoveries showing an accelerating volume in search space with time. Concurrently occurring slower "dispersive" rates take longer to reach the more difficult-to-access regions.



Hand-Wavy Answer: Suffice to say, if we search top to bottom, we will similarly reach a peak, but the peak will also contain a gradual backside. Intuitively, we can sense that the sharpness of the peak reduces as the sampling volume overlaps the region that contains the needles with the region absent of needles. And then as the sampling volume drifts even deeper, the amount discovered drops closer and closer to zero.

For us to draw the peak as a smooth curve, we need to add stochastic behavior to the search process. This can occur, for example, if the individual searchers have varying skills.

> a *stochastic variable* is neither completely determined nor completely random; in other words, it contains an element of probability. A system containing one or more stochastic variables is probabilistically determined.



What really makes the haystack illustration different than the global oil discovery problem doesn't lie in the basic word problem but rather in the application of randomness or dispersion to the problem. We have much greater uncertainties in the stochastic variables in the oil discovery problem, ranging from the uncertainty in the spread of search volumes to the spread in the amount of people/corporations involved in the search itself. We don't just deal with a single haystack, but multiple haystacks all over the world. So the sharply defined geometric discovery profile shown to the left (and that we used in an earlier chapter) gets washed out as a result of the statistical mechanics of the oil industry ant-people hard at work, all individually looking for oil.

Final Exam Answer: So we next jump from haystacks to oil discovery. We solve the problem by making the generally useful assumption that the current swept volume search rate has an estimated mean, and a variance equal to the square of the mean. In other words, in the absence of having any knowledge in the distribution of instantaneous swept volumes, we assume a maximum entropy estimator and set the standard deviation to the mean. A damped exponential probability density function follows this constraint with the least amount of bias, maximum uncertainty, and a finite bound[1]. Figure 7-2 on page 112 demonstrates how the spread in values gets expressed in terms of error bars.

We essentially want to solve the discovery success rate of a swept volume realizing that part of the volume straddles empty space. In other words, to account for the effects of the dispersion of oversampled volume, we have to integrate the exponential probability density function (PDF) of volume over all of space, and determine the expected value of the cross-section. To solve the problem by baby-steps, we

1. The latter factor would rule out something like a log-normal or Pareto distribution, which can have an unbounded mean.

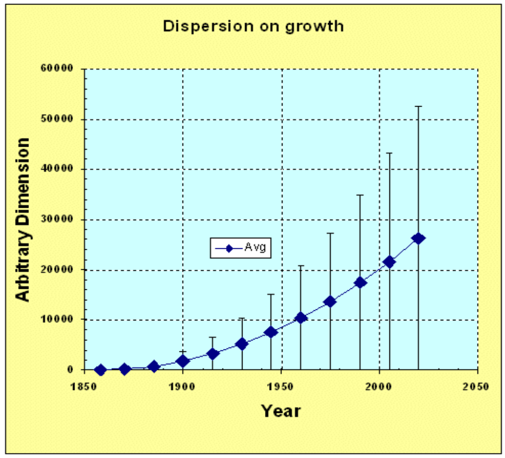


FIGURE 7-2. The dispersive aspects of growth serve to "spread" the range of the accelerating search rates. The fast search rates will cover lots of volume quickly, while the slower rates scour other volumes at a snail's pace. The latter lead to the tails in the discovery profile. The relative increase in variance with the mean indicates heteroscedasticity.

first take a look at the one-dimensional version of the problem, then extend it to three-dimensions, and finally add the time variation.

First, consider the following single-dimension formulation to solve the reserve growth enigma of a single reservoir [Ref 162].

We assume that any kind of prediction has to deduce from a probability model. To keep things simple, say that a volume has a finite depth L_0, which corresponds to a given extractable oil volume. Importantly, we do not know the exact value of L_0, but can make educated guesses based on a depth that we do have confidence in. We call this the "depth of confidence" and assign it to the random variable λ. This has the property that as we go beyond this depth, our confidence in our prediction becomes less and less concrete, or alternatively, more and more fuzzy. With this simple premise providing a foundation, we can use basic probability arguments to estimate a value for the unknown L_0 which we call $\bar{L}$.

Dispersive Discovery

This derivation has many similarities to modeling the reservoir size distribution described in an earlier chapter. With that in mind, consider the following line by line analysis of the derivation depicted in Figure 7-3 on page 113:

1. Probability density function for the depth of confidence.
2. Estimator for reservoir depth, a mean value.
3. Estimator shows a piecewise integration; the first part integrating to the actual depth, and the second part adding in a higher confidence factor as we probe below the actual depth.
4. Solution to the integration, giving the reserve growth dynamics as λ increases with time.

5. Parameterize the temporal dynamics of λ

a) If λ increases linearly with time

b) If λ increases with a parabolic dependence, matching a diffusional process.

c) If λ increases with a power-law dependence

d) If λ increases with a exponential dependence

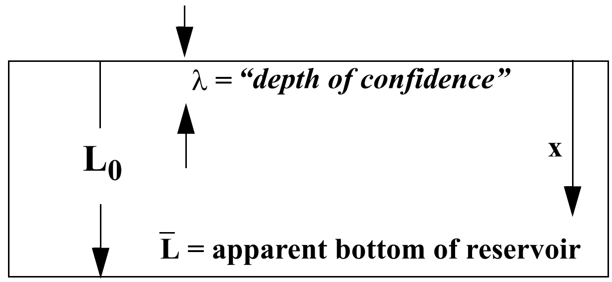


FIGURE 7-3. Derivation of dispersive discovery assuming a uniform finite bounded volume of search space L_0. Solving for a particular growth rate involves the parametric substitution for λ .

1. $$p(x) = \frac{1}{\lambda} \cdot e^{-x/\lambda}$$

2. $$\bar{L} = \int_0^{\infty} x \cdot p(x) dx$$

3. $$\bar{L} = \int_0^{L_0} x \cdot \frac{1}{\lambda} \cdot e^{-x/\lambda} dx + \int_{L_0}^{\infty} L_0 \cdot \frac{1}{\lambda} \cdot e^{-x/\lambda} dx$$

4. $$\bar{L} = \lambda \cdot (1 - e^{-L_0/\lambda})$$ **(EQ 7-1)**

5. **Parameterize**

$a)$ Linear growth : $\lambda = k \cdot t$

$b)$ Parabolic growth : $\lambda = \sqrt{(c \cdot t)}$

$c)$ Power-law growth : $\lambda = t^N$

$d)$ Exponential growth : $\lambda = a \cdot e^{bt}$

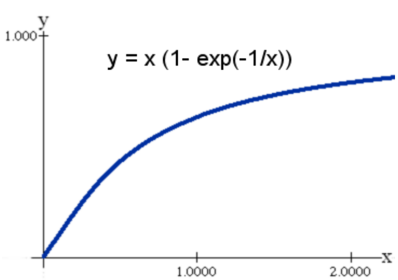


FIGURE 7-4. Reserve growth dynamics due to further discoveries in a uniform and randomly populated reservoir shows an asynptotically limited increase in the linear search regime.

The conservative nature of the estimation comes about because λ rises monotonically with time but *always* has uncertainty. We can either treat the spread in λ as a *fuzzy indicator of confidence* or as a *range in search rates* as shown in Figure 7-2 on page 112.

In the three-dimensional case, the stochastic variable λ represents the current mean swept volume, the term x integrates over all volumes, and L_0 represents the finite container volume V_d. The outcome $\bar{L}$ represents a kind of pro-rated proportion of discoveries made for the dispersed swept volume at a particular point in time.

By itself, the function corresponding to $\bar{L}$ doesn't look like anything special, and indeed looks a lot like the cumulative of the exponential PDF. However, the fact that λ monotonically increases with time, together with $\bar{L}$ appearing in the denominator, gives it interesting temporal dynamics as shown in Figure 7-4 on page 113, which I contend follows the empirical observations of cumulative oil discovery and that of reserve growth as well.[2]

From first principles, we would expect that swept volume growth approaches a power-law, and likely a higher-order law. For example, considering the "gold-rush" attraction of prospecting resources alone, we would expect that linear growths in (a) oil exploration companies, (b) employees per company, and (c) technological improvements would likely contribute at least a quadratic law[3]. In terms of the bottom-line, multiplying two linear growth rates generates a quadratic growth[4], and multiplying more linear rates leads to higher order growth laws. As an example, you can see this power-law increase play out as evidenced by the historical increase in average oil well depth over the years[5], see Figure 7-5 on page 115.

But of course, this only accounts for one dimension in the sampling volume. So if we make the assumption that the effective horizontal radius of the probe also increases with a quadratic law, we end up with a power-law order of $n = 2 \times 3 = 6$, where the 3 refers to number of dimensions in a volume. Because we actually use cumulative volume in the stochastic derivation, the order becomes 6 in the result shown below. When we make an assumption that the parameter k denotes a fraction of the swept volume that results in a cumulative discovery $D(t)$, we can replace V_d with D_d, where D_d is essentially equivalent to a URR for discoveries.

2. This function does reach a clear asymptote, given by L_0, however much it superficially looks as if it may grow indefinitely. This observation provides a breakthrough in understanding the "enigma" of reserve growth discussed later.
3. Note that parabolic growth is not the same as quadratic growth. Due to some historic conventions inherited from Silicon Valley, parabolic growth actually follows a fractional power-law growth, more precisely a square-root of time dependence.
4. See growth in wiki words for another real-world example of quadratic growth that occurs as we speak.
5. Maximum well depth chart culled from these sources: [Ref 135][Ref 136][Ref 137][Ref 138][Ref 139][Ref 140]

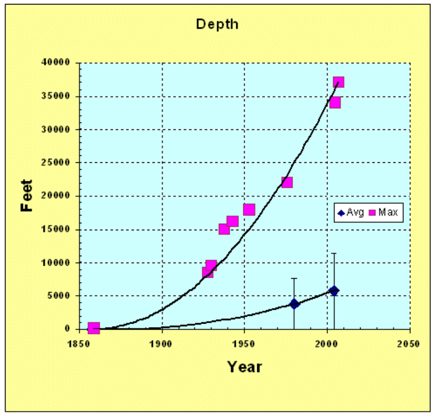


FIGURE 7-5. Empirical data supporting the accelerating rate of search rate with dispersion on the maximum excursions — cast into measures of maximum drilling depth per year. Note again the spreading variance with the mean indicating heteroscedasticity.

$$D(t) = kt^6 \times \left(1 - e^{-D_d/(kt^6)}\right) \quad \textbf{(EQ 7-2)}$$

and the derivative of this for instantaneous discoveries (e.g. yearly discoveries) results in:

$$\frac{\mathrm{d}}{\mathrm{d}t}D(t) = 6kt^5 \times \left(1 - e^{-D_d/(kt^6)} \cdot (1 + D_d/(kt^6))\right) \quad \textbf{(EQ 7-3)}$$

For a family of power-law growth functions, the trend looks like the following set of curves. The salient point to note relates to how we trend toward an asymptotic limit at the volume V_d as the power-law index gets larger.

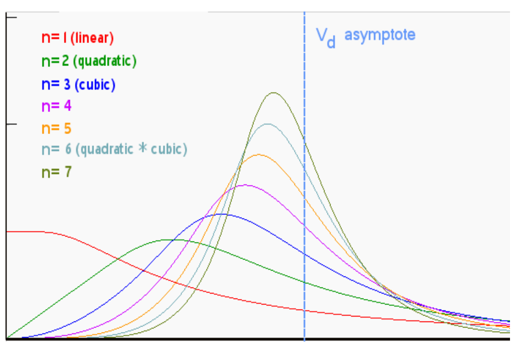


FIGURE 7-6. The effects of faster mean search rates on the dispersive profile.

To briefly summarize how dispersion of prospecting effort affects the discovery process, consider the curve below. Initially, as the sampling probe stays well within the V_d limit, the dispersed mean comes out as expected since we do not oversample the volume. However, as the standard deviation excursions of the cumulative volume starts to bleed past V_d, the two curves start to diverge and a rounded discovery peak results. We have a better hope of a "soft" landing rather than a hard gold-rush style crash due to this natural dispersion.

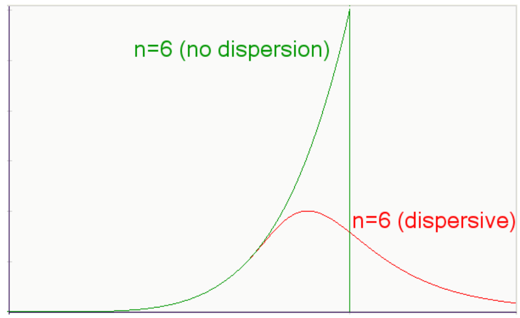


FIGURE 7-7.
The lack of dispersion on a fixed volume of search space leads to an abrupt fall-off as we hit the finite constraints. Collapse dynamics show up in non-dispersive growth models.

Examples of Non-Dispersive Peak. The following figure came from a fairly well-known study called "The introduction, increase, and crash of reindeer on St.Mathew Island"[Ref 32].

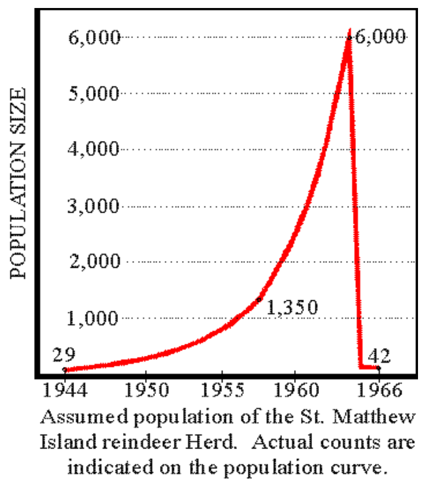


FIGURE 7-8.
Growth and collapse in reindeer population on Matthew Island.

This illustrates a classic example of non-dispersive growth hitting a hard physical limit. The absence of any variation caused by the extremely localized population on the isolated island essentially removed any rounding on the peak. Note the similarity between a hypothetical non-dispersive oil discovery model in the Figure 7-7 on page 116 and the reindeer growth. Like micro-organisms in a Petri dish, the reindeer basically scoured (i.e. searched) the island for food and then died off quickly.

Recall why we do not see this for discovery of oil — both the locations for oil and the effective search rates over a world-wide geographical area have such a large dispersion that we do not hit that sharp peak, and subsequent collapse. So instead we see the broadened peak in Figure 7-7 on page 116.

The harvesting of sturgeon from the Caspian area have empirically behaved similarly to the reindeer on St. Mathew Island — perhaps a little more dispersed, yet still geographically constrained with a focussed harvesting pressure. The rounding could also occur due to Gompertz dynamics. This results from acceleration in the proportional growth culling of a resource, which manifests as a law of exponentially diminishing returns instead of a hard constraint. The Gompertz has less sharp of a collapse, but a collapse nonetheless.[6]

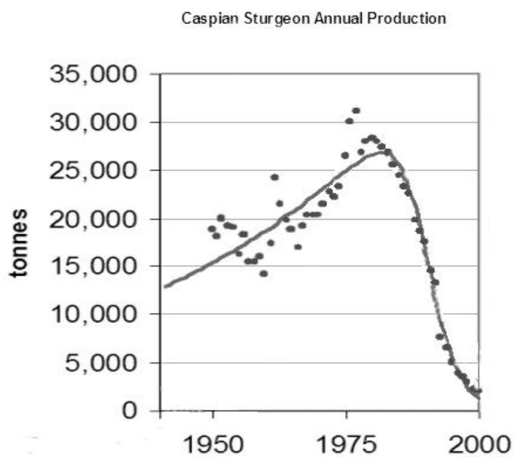


FIGURE 7-9. Growth and collapse in harvesting of Caspian Sturgeon. Like reindeer on an island, isolated population show little dispersion and culling becomes very efficient.[Ref 33]

The same behavior likely occurred for historical localized production of gold during the 1850's.[7] Essentially gold discovery became the production model, and any further smoothing of the curve due to shock model extraction phases essentially disappears. That phase smoothing due to construction, maturation, and extraction latencies basically explains why we rarely see the sharp asymmetric curve for oil production, even in the absence of discovery dispersion. In other words, we *may* see a collapse of discovery of oil in localized regions, but not for the final production, due to the accumulated latencies (or due to reserve growth which we will discuss later).

Dispersive Discovery as a Hubbert Curve. Scores of depletion analysts, including Laherrere, have pointed out the similarity of yearly discovery curves to the clas-

6. Gompertz dynamics will be discussed further in Chapter 10 and later.
7. I would really still like to find a curve or two for a localized Gold Rush discovery profile. It would really surprise me if, say, the take of gold from a place like the Klondike or Deadwood did not display similar peak behavior.

sic Hubbert curve itself. For the following discovery curve from Shell Oil one can see the same general trend, albeit buried in the noisy fluctuations of yearly discoveries.

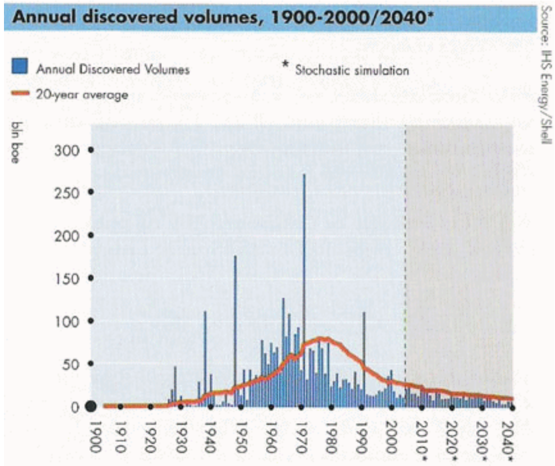


FIGURE 7-10. Discovery data from Shell Oil which includes "barrel-of-oil equivalent" discoveries. These likely include unconventional sources of oil. [Ref 34]

To remove the noise, we can generate a cumulative discovery curve. Apart from missing out on the cumulative data from the years post-1858 to the initial year of collected data, we can generate a reasonable fit to the curve with an *n=6* power-law dispersive growth function. (Note that the curve has a constraint to start in 1858, i.e. t=0, the "official" date which signalled the beginning of serious oil exploration)

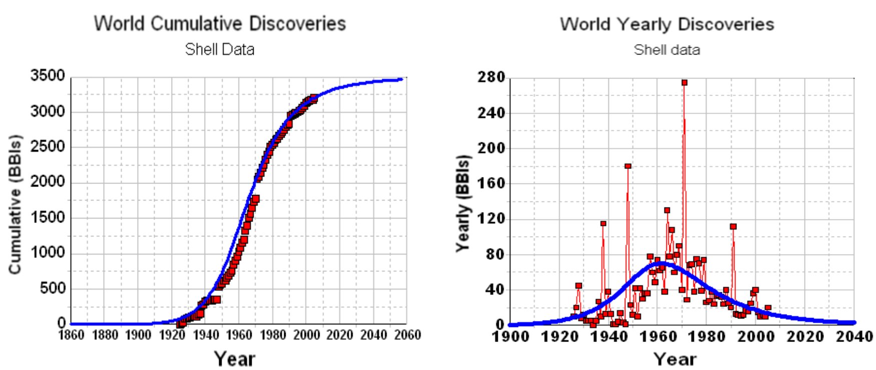


FIGURE 7-11. Fit to Shell Oil data using the dispersive discovery model. Cumulative (left) and yearly (right) — a nearly zero-parameter fit.

In general, the curve for oil production lags the discovery curve by several decades. Applying the modelled discovery curve to the Oil Shock production model, we can come up with the production extrapolation shown in Figure 7-12 on page 119.

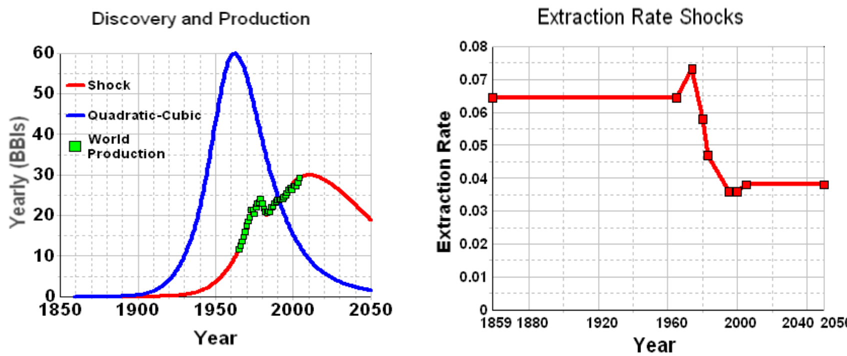


FIGURE 7-12. The oil shock parameters include a fallow latency of 6 years, a construction latency of 8 years, and a maturation latency of 10 years. It also includes the extraction rate shock profile to the right. Interesting that this gives a production peak around the year 2010, even though the effective URR from the Shell discovery data amounts to 3.5 trillion barrels -- much higher than the lowball 2+ trillion estimate commonly bandied about by pessimistic peak oil analysts (note that the Shell estimates uses the somewhat ambiguous "barrels of oil equivalent").

We can further substantiate the discovery fit by applying it to the USA data subset. For instance, let's consider what would happen if we used the same parameters from the global data to estimate U.S. discoveries. Note that the same constants (i.e. k and $n=6$) are used, but we change the D_d to reflect a fractional area of the US in comparison to the world.

World Land Area = 150,000,000.0 km^2
USA Land Area = 10,000,000.0 km^2

So to first-order, the D_d for USA is 1/15th that of the world's D_d[8]. The following figure lays the cubic-quadratic discovery curve on top of Laherrere's data.

Within an order-of-magnitude, the fit doesn't look out-of-place. In the context of swept volume, it means that the USA reached its limit of easily discovered oil quicker than the rest of the world, which makes sense as serious oil exploration started in the USA — and so likely would reach its limits first.

After the equations have been solved, the result can be translated back into the ordinary language.

Assumptions and Concerns. As far as word problems go, I don't consider the discovery model solution difficult in terms of the basic math. Perhaps we lack only an intuitive sense of how probabilities fit into the model. From one perspective, the uncertainty we have of the swept volume in relation to the finite volume of oil-bearing reservoirs reflects in our uncertainty with respect to reserve growth. In fact, I

8. A similar sanity check with reference to USA and world URR in [Ref 35].

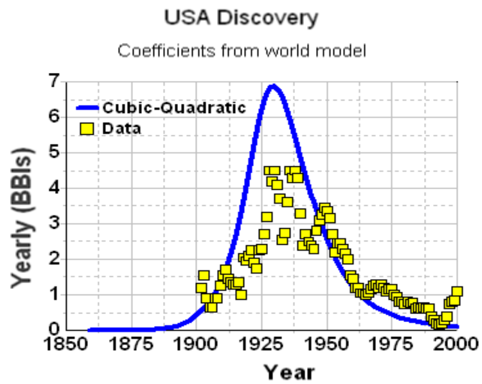


FIGURE 7-13. Scaled discovery curve applied to the USA using proportions of potential search space with respect to the entire world's search space.

originally derived the discovery model to understand the dynamics of reserve growth in a single reservoir (see chapter 9) and found that it has applicability to the larger global dynamics. Remember that the estimated discoveries themselves have uncertainties built into them and only become solidified with the passage of time. As shown in Figure 7-3 on page 113, the "depth of confidence" λ term represents a real uncertainty of how much volume we have actually swept out. Only after over-sampling the volumes do we sufficiently increase our confidence of our original estimate. Analysts typically use backdating to update earlier conservative estimates; in a way, we build backdating into the model by smearing out the estimate. Note that the roles of backdating discoveries and the maturation phase in the production model have a symbiotic relationship; if we have to deal with backdated data then the maturation phase takes longer and if we don't get backdated data, then the maturation gets reflected by delta discoveries that extend over time[9].

To account for potential criticisms, one could question the actual relevance of a power-law growth as a driving function. In fact the formulation described here supports other growth laws, including monotonically increasing exponential growth[10]. Furthermore, one could question whether we can sustain a power-law growth in the future, which together with extraction rate extrapolations, will have a significant impact on how future production will conceivably pan out. And to account for any

9. To address this detail, see the Hybrid Shock Model [Ref 23].
10. This variation of the derivation has an artificial constraint to it. It assumed that pockets of oil were distributed uniformly over a fixed/finite depth. The piece wise integration comes about because the first part integrates over the "filled" region and the second part over the "empty" region, below which you would find no more oil (but no one knows what this value is until you are well past it). This may be more intuitive to people not familiar with the properties of a stochastic variable, which has to have a cumulative probability of unity over its range. A better variation described later has the distribution of depths as a true stochastic variable; by making this a continuous PDF, we can relax the piece-wise constraints. This eventually results in the Logistic curve — caused by exponential acceleration of discovery rates over time. (see "The Context of Discovery. How do we simplify the search model?")

further reserve growth, the fact that much of the fitted curve occurs before the peak happens means that past discovery estimates have had a chance to mature and we have more confidence in the discovery decline profile. In my opinion, this makes it a fairly conservative estimator; to substantiate this take a look at the huge effective URR for the Shell discovery data, which in all likelihood includes reserve growth, and note how it only impacts the peak date a few years from the current day. Or, one could question the impact of super-giant discoveries on the smoothened discovery plot. Statistically, super-giants get treated like anything else in this model and they populate the volume with the same randomness, which we can demonstrate via Monte Carlo simulations. Predictably, one could also question the absence of deep geologic or economic considerations in the model. The canned response to that line of questioning becomes second nature to a seasoned statistical mechanic; physicists and other scientists apply such stochastic approximations all the time without a lot of fundamental problems. Why should this stochastic model become an exception to the rule?

I also have not opened up the future possibility of a levelling out or even general decline in discovery search effort. One can give this some serious consideration, but then you would realize that this would give too pessimistic a prediction and perhaps too much of an artificial constraint. This argument appears at best a second-order competitive effect as technology if nothing else will continue to accelerate the search rates.

Finally, one could question why no one else in the oil industry thinks in terms of this kind of discovery model; in other words, why hasn't someone else found this proverbial needle in a haystack algorithm? For all we know, an analyst in some energy corporation's back room has come up with the same idea and it sits filed as intellectual property with no hope of seeing the light of day [11].

But if this approach indeed has some originality and correctness to it, I can rationalize the idea's obscurity with a more mundane explanation[12]. Occam says to rely on the simplest explanation to a problem — consider then what happens when two sufficiently separate but equally fundamental explanations contribute to a greater understanding? To explain this rather philosophical point, I consider an oil depletion model as a two-stage word problem. The first part of the word problem relates to production (illustrated by the Oil Shock model earlier) and the second part provides a model of the discovery input used to feed production (i.e. the basis of the Dispersive Discovery model described in this chapter). The relationship of two

11. What good would it do the corporation financially? Or perhaps, a similar idea remains buried in some academic journal, which we have yet to uncover.
12. That comes from, in part, my experiences in solving problems in the research and software world. In many cases, we have to overcome the inertia of conventional wisdom.

interacting models has some similarity to an aspect of software debugging instanced by the occasional defect that takes enormous resources to resolve. Or it resembles in some ways to the laboratory anomaly that no one can pin down precisely by experiment. Invariably, the most difficult bugs to resolve result from two or more interacting defects. In my opinion, these remain the most elusive problems to solve simply because you don't normally think that more than one fundamental issue contributes to the cause of a root problem. And there you have an example of a real-world word problem — no one has thought to separate discovery from production in just this manner.

A Two Stage Model

While everyone wants to analyze oil depletion in the context of a *single* logistic curve, as though that contains *the* key to the kingdom, we realize that oil depletion may have two underlying forces at work — namely, the discovery process followed by the extraction process. And so we rely on the wisdom of a divide-and-conquer strategy — figure out the extraction/production problem all the while knowing that the discovery problem lays in waiting, or vice-versa. Now think back to the original "needle in the haystack" problem; notice that in that case, discovery and extraction occur at the same time. Once you find the needle you can extract it. But not so with oil, as discovery only starts the process that culminates in extraction and production. In my opinion, when we can understand the two problems individually, we have solved the penultimate word problem of our times.

CHAPTER 8

Applying Dispersive Discovery.

How discovery affects production

"The sneakiest form of literary subtlety, in a corrupt society, is to speak the plain truth. The critics will not understand you; the public will not believe you; your fellow writers will shake their heads."

— Edward Abbey

Model Details

The Effect of Field Size

Large oil fields have a significant impact on the accounting of total reserves: *"almost half of the world production is coming from less than 3% of the total number of oilfields"*. [Ref 36] This fact has evolved to the conventional wisdom that we find all the big reserves early on and we therefore should express even more concern about future supplies. Yet I contend, and the data supports (see Figure 8-2 on page 124), that we do not always necessarily discover the biggest oil reserves first.

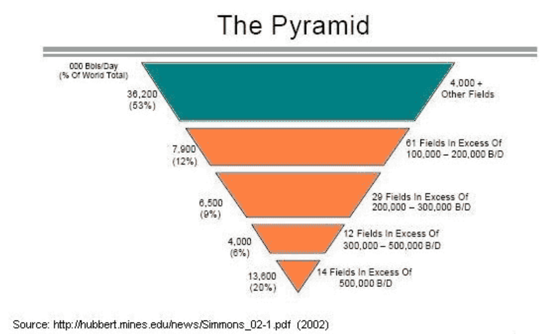


FIGURE 8-1. Reserve size distribution pyramid

So the intriguing notion remains, when do these big oil finds occur, and can our rather limited understanding of the discovery dynamics provide the Black Swan moment[1] that the peak oil skeptics hope for?

For a moment, putting on the hat of a skeptic, one could argue that we have no knowledge as to whether we have found all the super-giants, and that a number of these potentially remain, silently lurking in the shadows and just waiting to get discovered. Based on one report, the USGS has some statistical confidence that these exist and can make a substantial contribution to future reserves. Some research has duplicated the USGS results with the potential for large outliers — occurring primarily from the large variance in field sizes provided by the widely dispersed field size distribution empirically observed [Ref 15]:

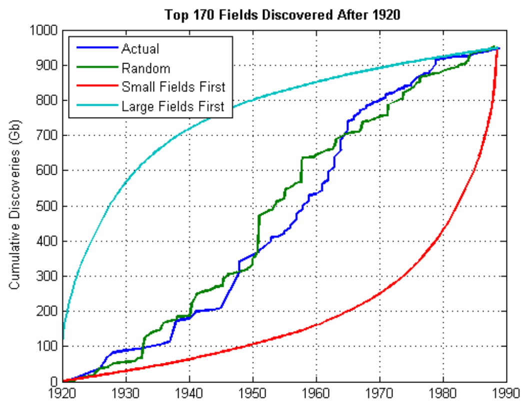


FIGURE 8-2. Plot of cumulative sizes of fields discovered. This shows little bias toward finding large (or small) fields first (by Sam Foucher).

The figure above does not show significant indication of finding big fields first. Yet this goes against some of the arguments I have come across which revolve around intuitive notions of always finding something big easily. Much like the impossibility of ignoring the elephant in the room, the logical person would infer that of course we would find big things first. This argument has perhaps tangential scientific principles behind it, mainly in mathematical strategies for dealing with what physicists call scattering cross-sections[2]. Scientifically based or not, I think people basically latch on to this idea without much thought. Deffeyes provides evidence that the cross-section approach has some validity but shows up only a weak bias in Kansas oil field discovery data [Ref 96].

But I have still have problems with the conventional contention, primarily in understanding what would cause us to uniformly find big oil fields first. On the one hand, and in historic terms, early oil prospectors had no way of seeing everything under the earth; after all, you can only discover what you can see (a bit of folk wisdom).

1. The discovery of a black swan occurred in Australia, which no one had really explored up to that point. The unlikely possibility of a huge new find hasn't as much to do with intuition, as to do with the fact that we have probed much of the potential volume. And the maximum number of finds occur at the peak of the dispersively swept volume. So the possibility of finding a Black Swan becomes more and more remote after we explore everything on earth See [Ref 37].
2. Where the probability of intersecting a particle rises with the size of the particle, however this does not guarantee that a large particle collides or gets detected first

So this would imply as we probe deeper and cast a wider net, we still have a significant chance of discovering large oil deposits. After all, the mantle of the earth remains a rather large volume.

Supporting this contention, the data does not convincingly back up the “big oil” early discovery model. Robelius dedicated his graduate thesis work to tabulating the portion of discoveries due to super-giants and it does in fact appear to skew to earlier years than the overall discovery data [Ref 19]. However, nothing about the numbers of giant oil fields found appears overly skewed about the peak as shown in Figure 8-3 on page 125:

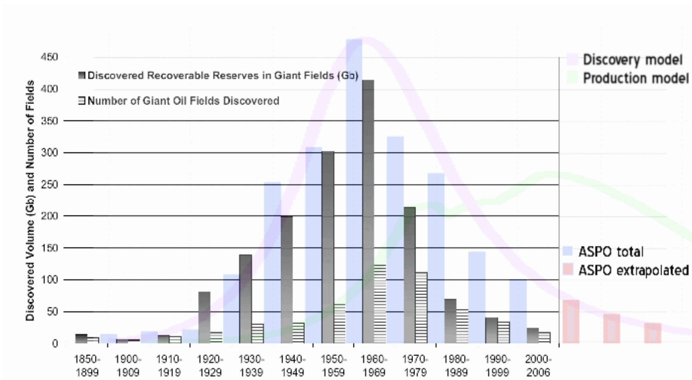


FIGURE 8-3. Discovery data from Robelius with ASPO total superimposed.

As typically shown in discovery data, I do spot some inconsistencies in the chart as well. I superimposed a chart of total discoveries due to ASPO ([Ref 38]) on top of the Robelius data and it appears we have an inversion or two (giants > total in the 1920’s and 1930’s). Another graph from unknown origins (Figure 8-4 on page 125) has the same 62% number that Robelius quotes for big oil contribution. Note that the number of giants before 1930 probably all gets lumped at 1930. It still looks inconclusive whether a substantial number of giants occurred earlier, and whether we can attach any statistical significance to the distribution.

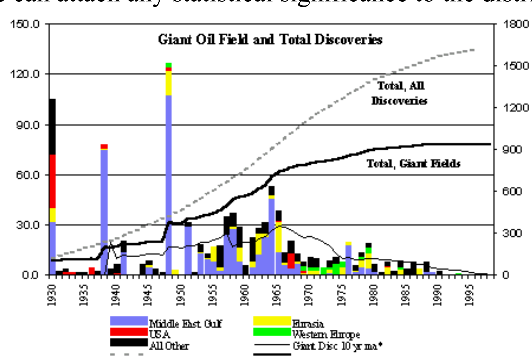


Figure 5. Giant Oil Fields = 62% of Total World Discoveries

FIGURE 8-4. Discovery data of unknown origins.

The somewhat suspect "BOE" discovery data provided by Shell offers up other supporting evidence for a more uniform distribution of big finds. As one can see in Figure 8-5 on page 126 due to some clearly marked big discoveries in the spikes at 1970 and 1990, the overall discovery ordering looks a bit more stationary. Unfortunately, I suspect that the big finds marked come about from unconventional sources. Thus, you begin to understand the more-or-less truthful BOE="barrel of oil equivalent" in small lettering on the y-axis[3].

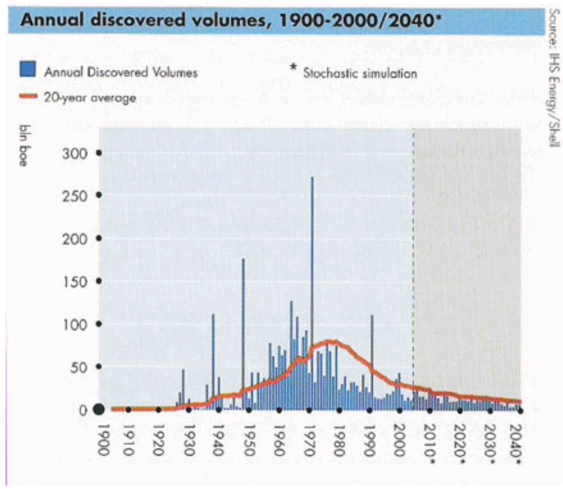


FIGURE 8-5. Shell Oil data (duplicated elsewhere)

Given the rather inconclusive evidence, I contend that I can make a good *conservative* assumption that the size of discoveries becomes a stationary property of any oil discovery model. This has some benefits in that the conservative nature will suppress the pessimistic range of predictions, leading to a best-case estimate for the future. Cornucopians say that we will still find big reservoirs of oil somewhere. Pessimists say that historically we have always found the big ones early. My assumption provides an unbiased and fair compromise between the two views.

In general, the premise assumes no bias in terms of when we find big oil, in other words we have equal probability of finding a big one at any one time.

Two Peas to the Pod

For my model of oil depletion I intentionally separate the Discovery Model from the Production Model. This differs from the "unitarians"[4] who claim that a single

3. I really don't understand what their "Stochastic simulation" amounts to — a simple moving average perhaps? — while Shell Oil doesn't have to disclose their methods
4. Unitarian: A believer in a single uniform heuristic to explain everything

equation, such as the heuristic Logistic, can effectively model the dynamics of oil depletion. From my point-of-view, the discovery process separates orthogonally from the subsequent extraction/production process, and that the discovery dynamics acts as a completely independent stimulus to drive the production model. I contend that the two convolved together give us a complete picture of the global oil depletion process.

I borrowed the term *dispersion* for the name of this discovery model to concisely describe the origin of its derivation. In the natural world, dispersion comes about from a range of rates or properties that affect the propagation of some signal or material. In terms of oil discovery dispersion, I model physical discovery as a maximum entropy range of rates that get applied to a set of exploratory processes. Some of these proceed slowly, others more quickly, while the aggregate shows dispersion. This dispersion becomes most evident on the far side of the discovery peak. That view essentially summarizes the contents of the last chapter.

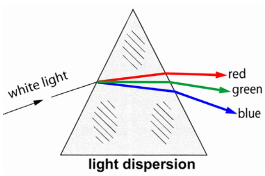


FIGURE 8-6. Physical example of dispersion as a light changes its frequency passing through a prism.

As for the production model, we use the Oil Shock Model (also described earlier) as a valid pairing to the Dispersive Discovery model. The Shock Model can take as a forcing function basically any discovery data, including real data or, more importantly, a model of discovery. The latter allows us to make the critical step in using the Shock Model for *predictive* purposes. Without the extrapolated discovery data that a model will provide, the Shock Model peters out with an abrupt end to forcing data, which usually stops at present time (with no reserve growth factor included).

As for the main premise behind the Shock Model, think in terms of rates acting on volumes of found material. To first-order, the depletion of a valuable commodity scales proportionately to the amount of that commodity on hand. Because of the stages that oil goes through as it starts from a fallow, just-discovered reservoir, one can apply the Markov-rate law through each of the stages. The Oil Shock Model essential acts as a 4^{th}-order low pass filter and removes much of the fluctuations introduced by a noisy discovery process. The "Shock" portion comes about from perturbations applied to the last stage of extraction, which we can use to model instantaneous socio-political events. [5]

5. I know the basic idea behind the Oil Shock Model has at least some ancestry. I refer to "compartmental models" for similar concepts, although no one has seriously applied it to fossil fuel production as described here

We just need to effectively show that we can reliably pair the two models up.

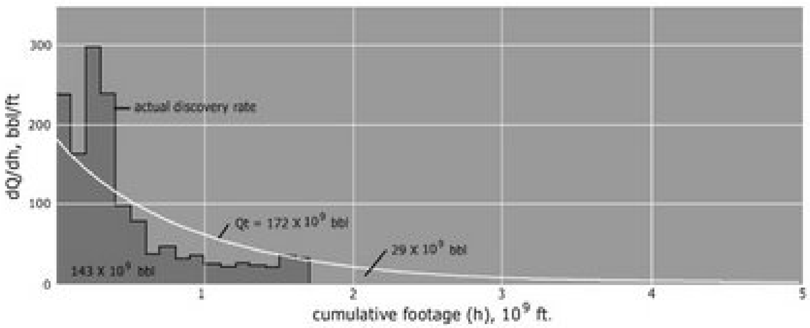


FIGURE 8-7. Hubbert's curve and fit for discovery/footage. Hubbert originally used a damped exponential to estimate cumulative. Note that this differs from the classic discovery profile as it uses footage instead of time.

Substantiating Dispersive Discovery

To substantiate the dispersive discovery model you can look for evidence of a time-invariant evolution of the cumulative growth component, and also in what oil analysts refer to as *creaming curves* or somewhat interchangeably as reserve growth curves. The derivation basically followed two stages: (Stage 1) a stochastic spatial sampling that generated a cumulative growth curve and (Stage 2) an empirical observation as to how sampling size evolves with time, with the best fit assuming a power-law with time. Hubbert himself generated some historical data demonstrating these trends[6]. Having supporting evidence reinforces the validity of the model.[7]

The shape of the curve as found by Hubbert has the characteristic of a cumulative dispersive swept region in which we remove the time dependent growth term,

6. Most of the growth charts are reproduced in the Epilogue as an article reprint collected from Senate hearings that Hubbert testified to.
7. I believe a set of intermediate results aids in validating the model. In effect, the stage-1 part of the derivation benefits from a "show your work" objective evaluation, which strengthens the confidence level of the final result. Lacking a better analogy, I would similarly feel queasy if I tried to explain why rain regularly occurs if I could not simultaneously demonstrate the role of evaporation in the weather cycle. And so it goes with the oil discovery life-cycle, and arguably any other complex behavior.

retaining the strictly linear mapping needed for the histogram, see the n=1 term in the figure below:

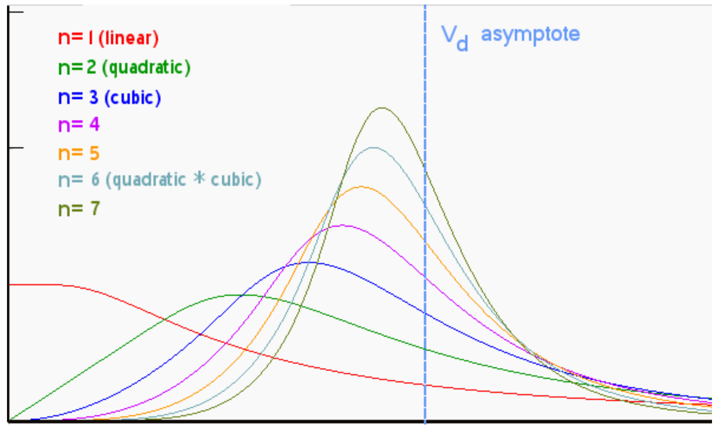


FIGURE 8-8. Order n=1 gives the cumulative swept volume mapped linearly to time.

For the n=1 solution, we get:

$$\frac{\mathrm{d}D}{\mathrm{d}h} = c \cdot \left(1 - e^{-\frac{k}{h}} \cdot \left(1 + \frac{k}{h}\right)\right) \quad \textbf{(EQ 8-1)}$$

where ***h*** denotes the cumulative depth. I placed an overlay with a scaled dispersive profile, which shows the same general shape in the figure below.

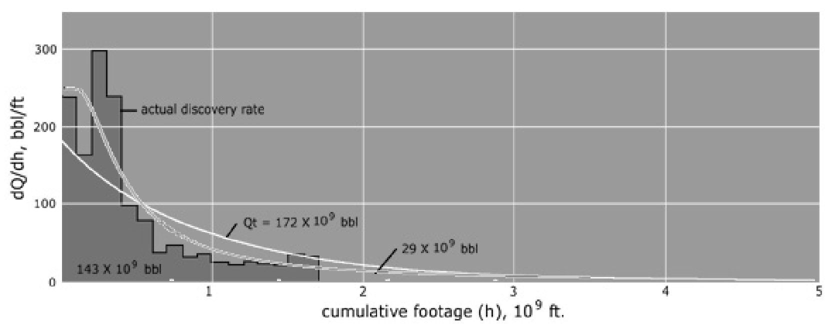


FIGURE 8-9. Hubbert data mapping delta discoveries to cumulative drilled footage. Superimposed with linear-growth dispersive model.

The ***k*** term has significance in terms of an effective URR. For this figure, I eye-balled the scaling as $k = 0.7 \times 10^9$ and $c = 250$, so I get 175 instead of the 172 that Hubbert deduced.

Since the results come out more naturally in terms of cumulative discovery[8], it helps to integrate Hubbert's yearly discovery curves. So Figure 8-10 on page 130 below shows the cumulative fit paired with the yearly (the former is an integral of the latter):

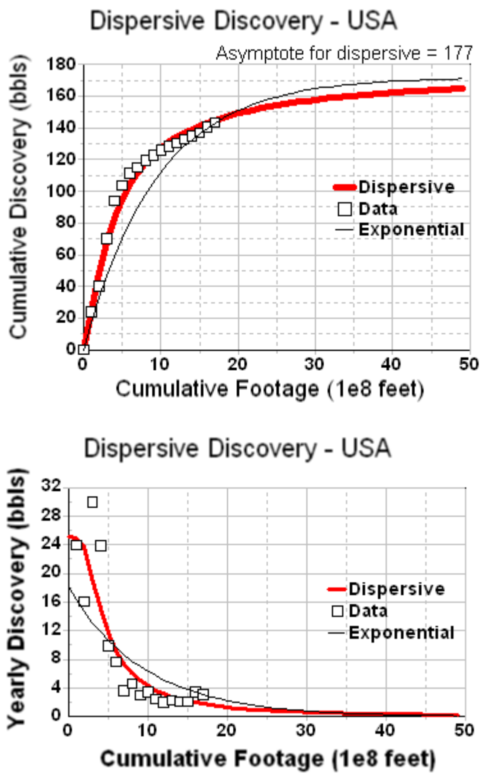


FIGURE 8-10. Dispersive Discovery fit for USA oil (top). Cumulative is the integral of yearly (bottom).

I did a least-squares fit to the curve that I eyeballed initially and the discovery asymptote increased from my estimated 175 to 177. I've found that generally accepted values for this USA discovery URR ranges up to 195 billion barrels in the

8. The details of the derivation essentially substantiate the derivation of the $\tilde{L}$ equations of the previous chapter. The key terms include λ, which indicates cumulative footage, and the $\bar{L}$, which denotes an average cross-section for discovery for that particular cumulative footage.

30 years since Hubbert published this data. This, in my opinion, indicates that the model has potential for good predictive power.

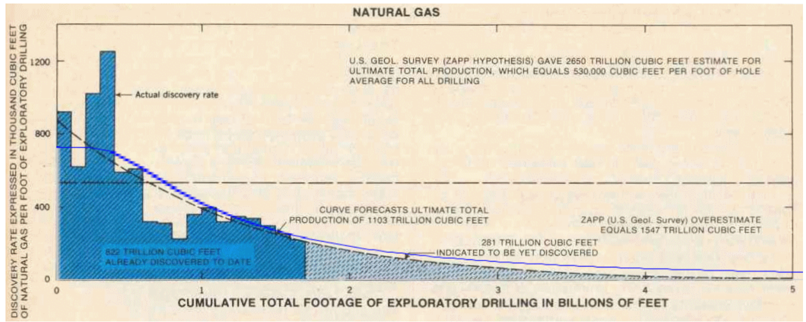


FIGURE 8-11. Hubbert's plot for USA Natural Gas, with the dispersive discovery profile overlaid.

Hubbert originally plotted yearly discoveries per cumulative footage drilled for both oil and natural gas [Ref 39]. Interesting that if we fit the cumulative discovery data to the naive exponential, the curve seems to match very well on the upslope (see Figure 8-12 on page 131 below) but that the asymptote arrives way too early, obviously missing all the dispersed discoveries covered by the alternative model. The dispersive discovery adds a good 20% extra reaching an asymptote of 1130, coming much closer to the value from NETL of 1190[9].

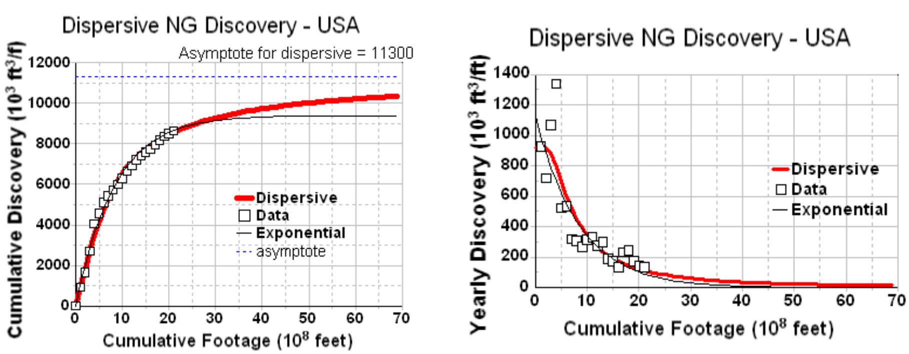


FIGURE 8-12. Dispersive Discovery fit for USA natural gas. Cumulative is the integral of yearly.

Although a bit unwieldy, one can linearize the dispersive discovery curves, similar to what oil analysts do with Hubbert Linearization (which we will address in a few pages). Referring to Figure 8-13, although it swings wildly initially, I can easily see the linear agreement, with a correlation coefficient very nearly one and a near zero extrapolated y-intercept. (note that the naive exponential that Hubbert used in Figure 8-11 on page 131for NG overshoots the fit to better match the asymptote but still falls short of the alternative model's asymptote, and which also fits the bulk of the data points much better*r*)

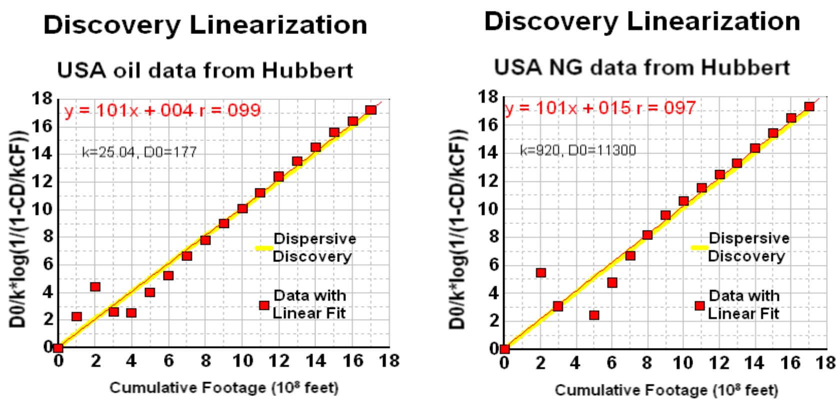
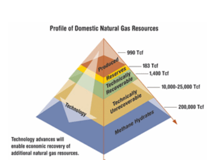


FIGURE 8-13. Linearization results for Dispersive Discovery Model of USA oil (left) and natural gas (right). Could use hyperbolic linearization here as well.

The aggregation of the data tends to corroborate that the dispersive discovery model works quite effectively in both providing an understanding on how we actually make discoveries in a reserve growth fashion and in mathematically describing the real data. So at a subjective level, you can see that the cumulative ultimately shows the model's strengths, both from the perspective of the generally good fit for a 2-parameter model (asymptotic value + search efficiency of discovery), but also in terms of the creeping reserve growth which does not flatten out as quickly as the exponential does. I suggest that this slow apparent reserve growth matches empirical-reality remarkably well. In contrast, the quality of Hubbert's exponential fit worsens when plotted in the cumulative discovery profile, only crossing at a few



9."The United States has proved gas reserves estimated (as of January 2005) at about 192 trillion cubic feet (tcf)" and from NETL [Ref 41]this: U.S. natural gas produced to date (990 Tcf) and proved reserves currently being targeted by producers (183 Tcf) are just the tip of resources in place. Vast technically recoverable resources exist — estimated at 1,400 trillion cubic feet —yet most are currently too expensive to produce. This category includes deep gas, tight gas in low permeability sandstone formations, coal bed natural gas, and gas shales. In addition, methane hydrates represent enormous future potential, estimated at 200,000 trillion cubic feet. This together with the following reference indicate the current estimate of NG reserves lies between 1173 and 1190 TCF (Terra Cubic Foot = 10^{12} ft^3).

US NG Technically Recoverable Resources US NG Resources (EIA, 1/1/2000, Trillion ft^3) (NPC, 1/1/1999, Trillion ft^3) Non associated undiscovered gas 247.71 Old fields 305. Inferred reserves 232.70. New fields 847. Unconventional gas recovery 369.59. Unconventional 428. Associated-dissolved gas 140.89. Alaskan gas 32.32. Alaskan gas (old fields) 32. Proved reserves 167.41. Proved reserves 167. Total Natural Gas 1190.62. Total Natural Gas 1779. [Ref 40] points and reaching an asymptote well before the dispersive model does (see Figure 8-9 on page 129).

Noise. The basis of the noise in the discovery data deserves some consideration, particularly in terms of how the effects of super fields would affect the model. You can see the noise in the cumulative plots from the Hubbert above (see Figure 8-5 on page 126) even though these also have a heavy histogram filter applied) and especially in the discovery charts from Laherrere in Figure 8-14 on page 133 below.

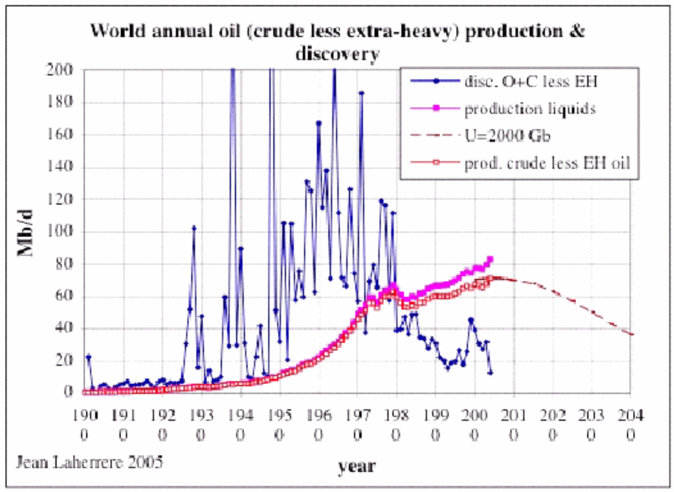


FIGURE 8-14. Unfiltered discovery data from Laherrere [Ref 18]

If you consider that the number of significant oil discoveries runs in the thousands according to "the pyramid" (Figure 8-1 on page 123), you would think that noise would abate substantially and the law of large numbers would start to take over. Alas, that does not happen and large fluctuations persist, primarily because of the large variance characteristic of field size distribution[10]. One can see this in terms of the derivation of field sizes in "The Facts in the Ground. Where do we find oil reservoirs?" , where estimating the URR from field sizes becomes difficult without a good knowledge of the extent of the data histogram and a field size maximum. By experimenting with Monte Carlo trials, I have found that the fluctuations do average out in the cumulative sense — but only if you have a dispersive model establishing a finite volume to constrain the analysis. The USGS unfortunately leave this rather important baseline out of their consideration. In other words, the Dispersive Discovery profile, more than anything else serves to place bounds on the URR.

10. To get some extra insight into how to apply the log-normal, and also for what I see as a serious flaw in the USGS interpretation, see [Ref 15]. The log-normal distribution alone necessarily leads to a huge uncertainty in cumulative discovery as it assumes an infinite sampling volume.

The following pseudo-code (Figure 8-16 on page 135) maps out the Monte Carlo algorithm I used to generate statistics[11]. This algorithm draws on the initial premise that fluctuations in discovering is basically a stationary process, and remains the same over the duration of discovery.

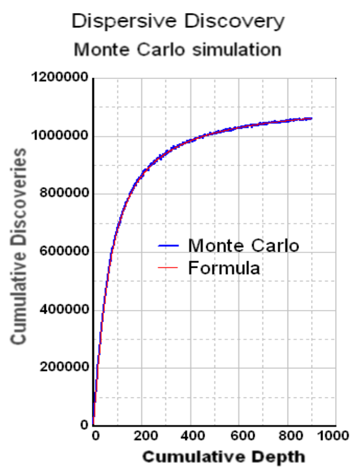


FIGURE 8-15. Result of MC simulation. Depending on the size of the sample, repeated Monte Carlo runs will eventually asymptotically approach the analytical result.

The basic idea says that if you draw a depth deeper than L0 (the maximum depth/volume for finding something), then cumulatively you can only scale to a L0 ceiling. This generates an asymptote similar to a URR. Otherwise, you will find discoveries within the mean depth multiplied by the random variable probe, *H*Lambda,* below. This gives you a general idea of how to do a stochastic integration. Remember, we only have an average idea of what probe depth we have, which gives us the dispersion on the amount discovered.

11. This uses the standard trick for inverting an exponential distribution and a more detailed one for inverting the Erf() which results from the cumulative Log-Normal distribution. I have since found it much simpler and more logical to use the inversion from "The Facts in the Ground. Where do we find oil reservoirs?" This fits the size distribution better.

```
1. for Count in 1..Num_Paths loop
     Lambda (Count):= -Log (Rand);
   end loop;
2. while H < Depth loop
     H:= H + 1.0;
     Discovered:= 0;
3.   for Count in 1.. Num_Paths loop
4.    if H * Lambda(Count) < L0 then
5.      LogN:= exp(Sigma*Inv(Rand))/exp(Sigma*Sigma/2.0);
6.      Discovered:= Discovered + Lambda(Count) * LogN;
      end if;
     end loop;
7.   -- Print H + Discovered/Depth or Cumulative Discoveries
   end loop;
```

FIGURE 8-16. Algorithm for generating MC cumulative discovery curve.

Basic Algorithmic Steps:

1. Generate a dispersed set of paths that consist of random lengths normalized to a unitary mean.
2. Start increasing the mean depth until we reach some artificial experimental limit (much larger than L0).
3. Sample each path within the set.
4. Check if the scaled dispersed depth is less than the estimated maximum depth or volume for reservoirs, L0.
5. Generate a field size value proportional to Lambda drawn from a distribution described in the chapter “The Facts in the Ground. Where do we find oil reservoirs?” [12].
6. Accumulate the discoveries per depth

If you accumulate over all depths, you will get something that looks like Figure 8-15 on page 134.

The series of Monte Carlo experiments in the next figures apply various size sampling distributions to the Dispersive Discovery Monte Carlo algorithm (we could also use a shortcut using the simple size distribution, see left). For both a uniform size distribution and exponential damped size distribution, the noise stays small for sample sets of 10,000 dispersive paths. However, by adding a log-normal size distribution with a large variance (log-sigma=3), the severe fluctuations become apparent for both the cumulative depth dynamics and particularly for the yearly discovery dynamics. This fact suggests why many oil-depletion analysts prefer to apply a running average on the discovery profiles. In terms of raw data, good analysis methodology would dictate to leave the noise in there, as I contend that it tells us much about the statistics of discovery.

```
for Depth in 1..Probing_Depth loop
 Amt:= 0.0;
 for I in 1..N loop -- reduces sampling noise
  Draw:= Random (0.0.. 1.0);
  Random_Depth:= -Depth * ln (Draw);
  if Value > L0 then
   Amt:= L0 + Amt;
  else
   Amt:= Random_Depth + Amt;
  end if;
 end loop;
 -- Print Depth + Amt / N
end loop;
```

FIGURE 8-16. Alternate algorithm.

12. In this case a log-normal size proportional to the dimensionless dispersive variable λ. The choice of distribution does not make that big a difference in the noise observed.

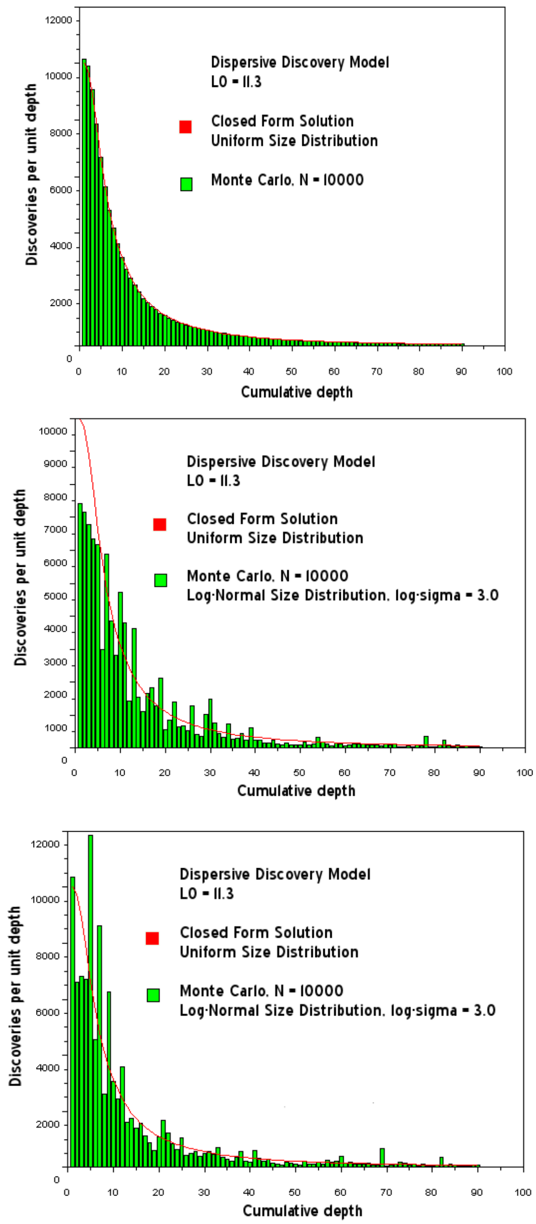


FIGURE 8-17.
(**top**) Dispersive Discovery Model mapped into Hubbert-style cumulative efficiency. The Monte Carlo simulation in this case is only used to verify the closed-form solution as a uniform size distribution adds the minimal amount of noise, which is sample size limited only.

(**middle**) Dispersive Discovery Model with Log-Normal size distribution. This shows increased noise for the same sample size of N=10000.

(**bottom**) Same as middle but using a different random number seed

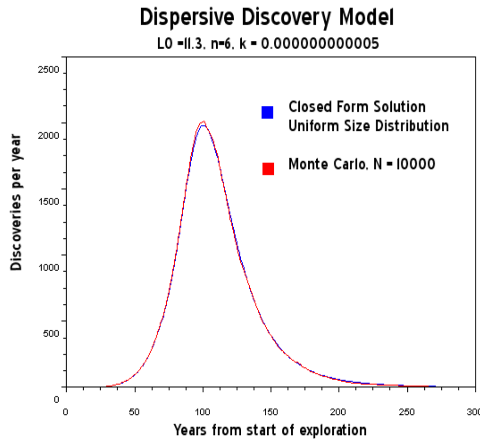


FIGURE 8-18. Dispersive Discovery Model assuming uniform size distribution

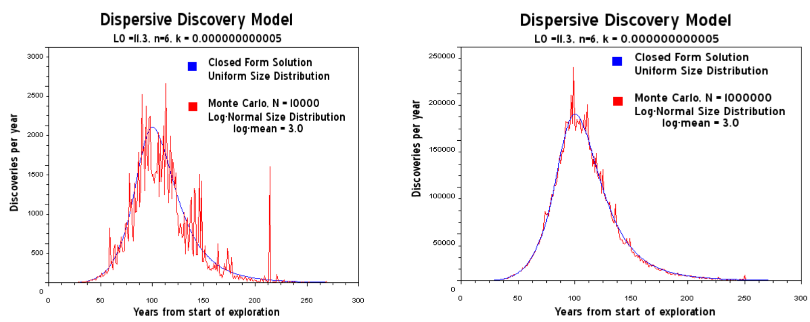


FIGURE 8-19. Dispersive Discovery Model assuming log-normal size distribution. Note that sample path size increased by a factor of 100 in the sample to the right from the left. This reduces the fluctuation noise considerably.

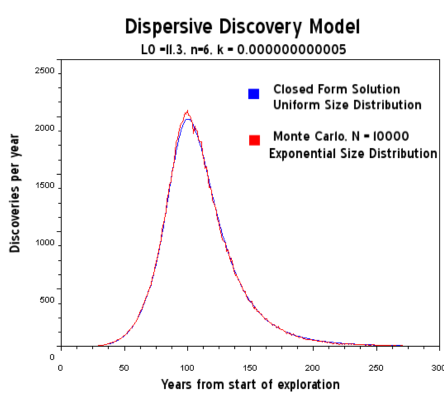


FIGURE 8-20. Dispersive Discovery Model assuming exponentially damped size distribution. The exponential has a much narrower variance than the log-normal.

The differences between the published discovery curves result primarily from different amounts of filtering. Figure 8-3 on page 125 uses a cumulative sums for each decade, which definitely reduces the overall fluctuations. However the discovery profile from Shell appears to have a fairly severe lagged moving average, resulting in the discovery peak shifting right quite a bit. Figure 8-21 shows little by way of filtering and includes superimposed backdating results. Figure 8-22 has a 3-year moving average, which I believe came from the unfiltered curve due to Laherrere shown in Figure 8-14.

I suggest instead of filtering the data via moving averages, it might make more sense to combine discovery data from different sources and use that as a noise reduction/averaging technique. Ideally I would also like to use a cumulative but that suffers a bit from not having any pre-1900 discovery data.

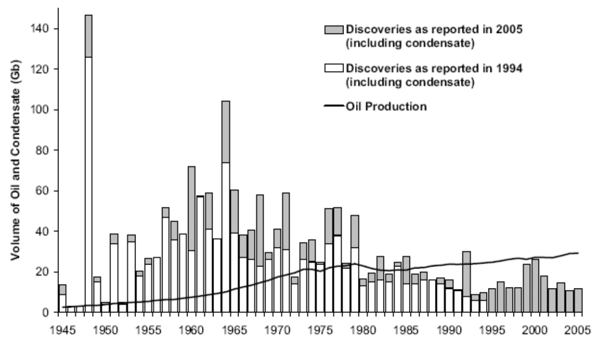


FIGURE 8-21. Discovery Data plotted with minimal filtering and reserve growth. The stacked bar segments represent reserve additions made by applying backdated data. This revises the previous estimate [Ref 23].

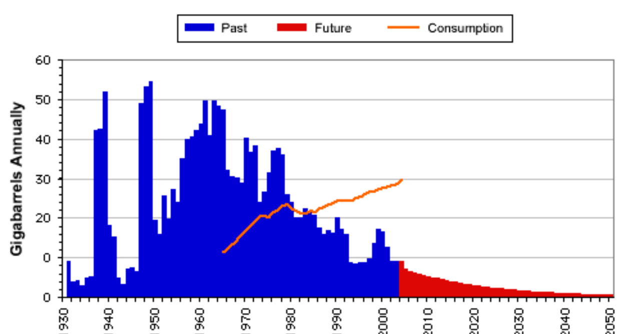


FIGURE 8-22. Dis covery data with a 3-year moving average (adapted from [Ref 112])

Applying the Combined Model to Global Crude Oil Production

In Figure 8-3 on page 125, I overlaid a Dispersive Discovery fit to the data. In this section, I explain the rational for the parameter selection.

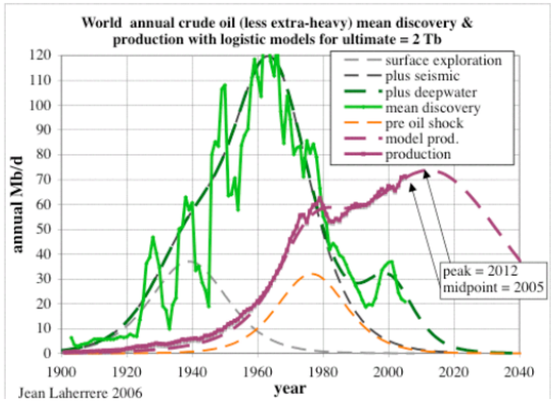


FIGURE 8-23. World crude discovery data overlaid with production [Ref 44]

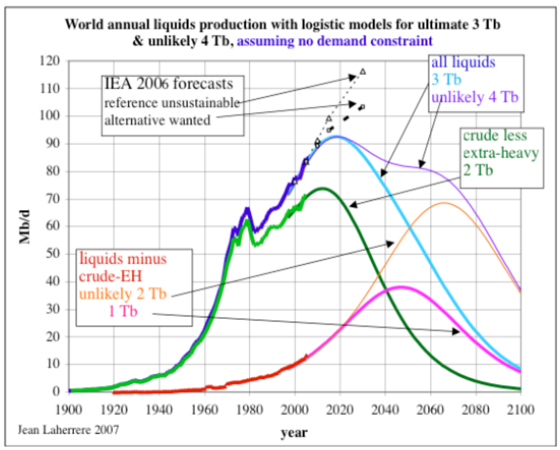


FIGURE 8-24. World crude production data including natural gas liquids [Ref 44]

Note that the NGL (natural gas liquids) portion of the "Crude Oil + NGL" data really has little to do with typical crude oil discoveries, so the combined discovery/shock model better represents the lower peak of *crude-only* production data[13]. This essentially scales back the peak by about 10% as shown in Figure 8-24. This also means that I could use a dispersive discovery model on discovery data, but needed to reduce the overcompensation on extraction rate to remove the "phantom" NGL production that crept into the oil shock production profile. This essentially will defer the peak because of the decreased extractive force on the discovered reserves. I fit the discovery plot by Laherrere to the dispersive discovery model with a cumu-

13. ;Finding oil only occasionally coincides with natural gas discoveries so we assume that the discoveries comprised only crude oil, and any NGL would come from separate natural gas discoveries

lative limit of 2800 GB and a cubic-quadratic rate of 0.01 (i.e n=6 for the power-law). This gives the blue line in Figure 8-25.

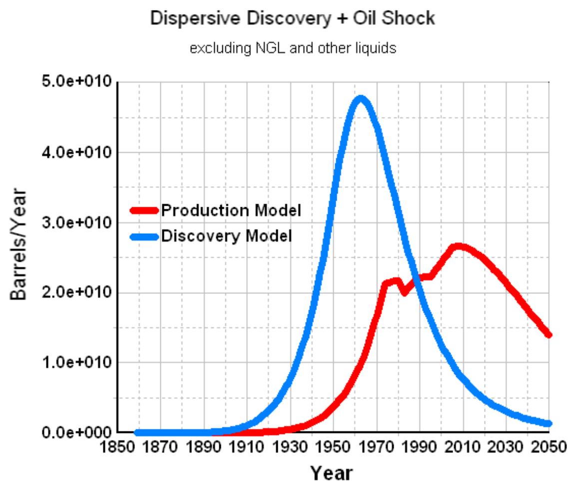


FIGURE 8-25.
Discovery Data + Shock Model for World Crude

For the oil shock production model, I used {fallow,construction,maturation} rates of {0.167,0.125,0.1} to establish the stochastic latency between discovery and production. I tuned to match the shocks via the following extraction rate profile:

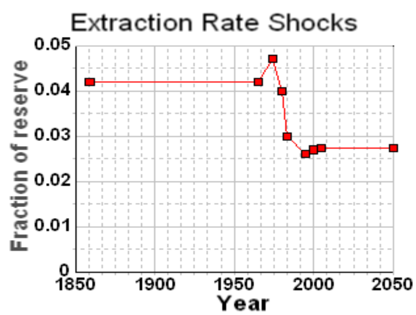


FIGURE 8-26.
Shock profile associated with Figure 8-25. Recent values are closer to 4% if you use actual reported reserves, but this under-estimates the reserve growth potential.

As a bottom-line, this estimate fits in between the original oil shock profile using a heuristic discovery profile that I produced initially and the more recent oil shock profile that used a model of the perhaps more optimistic Shell discovery data used earlier. Given we have confidence in the discovery data by Shell[14], this should probably better represent the total Crude Oil + NGL production profile; which makes the important point of understanding what grades of oil goes into discovery

14. Which had the cryptic small print scale "boe" (i.e. barrels of oil equivalent) indicating that the "oil" could possibly represent any kind of hydrocarbon

data. If we use discovery of conventional crude only, then you can't use it to predict an "all liquids" peak, which the original model had used.

Original Model(peak=2003) < No NGL(peak=2008) < Shell data of BOE(peak=2010)

I still find it significant how the peak position of the models do not show the huge sensitivity to changes that one would expect with the large differences in the underlying discovery URR. When it comes down to it, shifts of a few years don't mean much in the greater scheme of things. However, how we conserve and transition on the backside will make all the difference in the world.

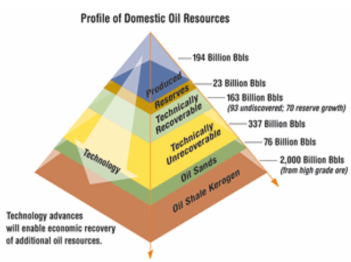


FIGURE 8-27. Oil Resources Pyramid [Ref 41]

Production as Discovery?

As an experiment, Sam Foucher applied the discovery equation to USA data [Ref 21]. Since the model should scale from global down to distinct regions, these kinds of analyses provide a good test to the validity of the model. In particular, Foucher concentrated on the data near the peak position to ostensibly try to figure out the potential effects of reserve growth on reported discoveries. For one, the Dispersive Discovery model should prove useful for understanding reserve growth on individual reservoirs, as the uncertainty in explored volume plays in much the same way as it does on a larger scale. In fact I originally proposed a dispersion analysis on a much smaller scale (calling it Apparent Reserve Growth [Ref 162]) before I applied it to USA and global discoveries.

As another example, I noticed that over the larger range of USA discoveries, i.e. inferring from production back to 1859, the general profile for yearly discoveries *would not* affect the production profile that much on a semi-log plot. The shock model extraction model to first order shifts the discovery curve and broadens/scales the peak shape a bit — something fairly well understood if you consider that the shock model acts like a phase-shifting IIR filter.[15] So I tried fitting the USA *production* data to the *dispersive discovery* model, bypassing the shock model response completely. I used the USA production data from EIA ([Ref 29]) which extends back to 1859 and to the first recorded production out of Titusville, PA of 2000 barrels ([Ref 45]). I plotted this in Figure 29 on a semi-log plot to cover the substantial dynamic range in the data.

15. An Infinite Impulse Response filter uses all previous time series values in averaging the filtered value.

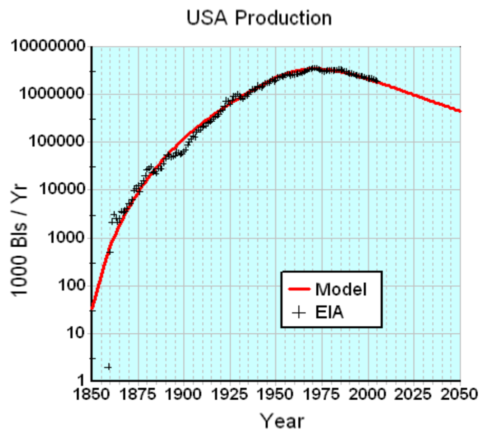


FIGURE 8-28. USA Production mapped as a pure Discovery Model

This curve used the n=6 equation, an initial t_0 of 1838, a value for k of 0.0000215 (in units of 1000 barrels to match EIA), and a D_d of 260 GB.

$$D(t) = kt^6 \cdot \left(1 - e^{-D_d/(k \cdot t^6)}\right) \quad \text{(EQ 8-2)}$$

$$\frac{d}{dt}D(t) = 6kt^5 \cdot \left(1 - e^{-D_d/(k \cdot t^6)} \cdot (1 + D_d/(kt^6))\right) \quad \text{(EQ 8-3)}$$

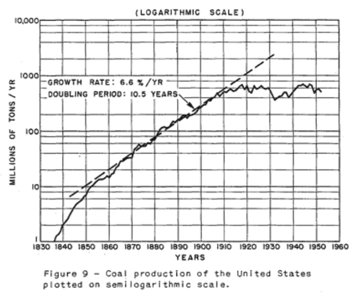
The peak appears right around 1971. I essentially set $P(t) = \frac{d}{dt}D(t)$ as the model curve.

I find this result very intriguing because, with just a few parameters, we can effectively fit the range of oil production over three orders of magnitude, hit the peak position, produce an arguable t_0[16], and actually generate a predictive down-slope for the out-years. Even the only point that doesn't fit on the curve, the initial year's data from Drake's Titusville well, figures somewhere in the ballpark considering this strike arose from a purely discrete and deterministic draw (see the Monte Carlo simulations above) from the larger context of a stochastic model.

Staniford originally tried to fit the USA curve on a semi-log plot[Ref 31], and had some arguable success with a Gaussian fit. Over the dynamic range, it fit much bet-

16. Thanks to S.Foucher for this insight.

ter than a logistic, but unfortunately did not nail the peak position and didn't appear to predict future production. The gaussian also did not make much sense apart from some hand-wavy central limit theorem considerations. Even before Staniford, Hubbert [Ref 47]gave the semi-log fit a try and perhaps mistakenly saw an exponential increase in production from a portion of the curve — something that I would consider a coincidental flat part in the power-law growth curve.



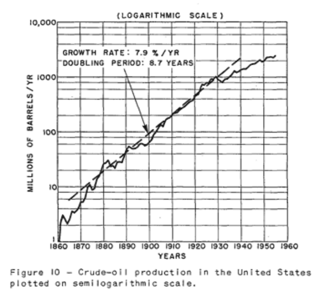


FIGURE 8-29. World Crude Discovery Data generated by Hubbert[Ref 47]

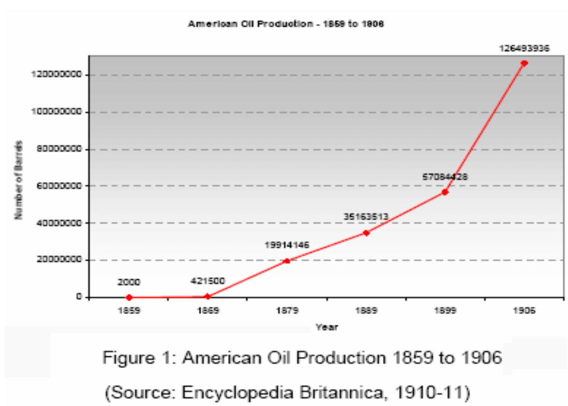


FIGURE 8-30. USA oil production early years from Encyclopedia Brittanica [Ref 46]

Discovered Conclusions

In this section we demonstrated that the Dispersive Discovery model shows promise at describing:

1. Oil and NG discoveries as a function of cumulative depth.
2. Oil discoveries as a function of time through an accelerating growth term.
3. Together with an appropriate field size distribution, the statistical fluctuations in discoveries. We can easily represent the closed-form solution in terms of a Monte Carlo algorithm.
4. Together with the Oil Shock Model, global crude oil production.
5. Over a wide dynamic range, the overall production shape. Look at USA production in historical terms for a good example.
6. Reserve growth of individual reservoirs.

All these factors improve our confidence in obtaining a good extrapolation on future production levels. Discovery, perhaps more than anything else, controls the ultimate peak oil profile. The other important aspect, that of reserve growth we will consider next.

CHAPTER 9

The Reserve Growth.

How estimates of oil evolve

"*We must accept finite disappointment, but we must never lose infinite hope.*"

— Martin Luther King Jr.

"*Reserving judgments is a matter of infinite hope.*"

— F. Scott Fitzgerald in The Great Gatsby

Definition

You might run across several meanings of the word *reserves* as it applies to oil. In one sense, it might indicate the amount of oil that we have, perhaps intentionally held back, as in "held in reserve". You will see this in reference to the emergency fuel store of oil maintained by the USA's DoE, referred to collectively as the Strategic Petroleum Reserve[1]. In a more general sense, the term reserve naturally extends from the term reservoir, and so it can also include the estimate of how much oil we have left in the reservoirs that we have collectively discovered.

So the value of estimated reserves ends up as an interim metric lying somewhere between the total oil discovered and the total oil produced. At any one time, the following invariant should always hold:

$$\text{Total Oil Discovered} = \text{Current Reserves} + \text{Total Oil Produced} \qquad \textbf{(EQ 9-1)}$$

Because of our inability to estimate the oil discovered in any area very accurately (at least initially), the continued estimates of actual reserve continue to play an important role as a reservoir matures. The producer will update the reserve value and an analyst can decide to *backdate* the original discovery estimate to reflect the

1. At any one time, the amount of oil held in the strategic reserve caverns approximates only one month worth of national consumption — a literal drop in the bucket compared to the reserves we will need to sustain us from new discoveries.

improved estimate. As we will see, the way we treat backdating effects the mathematical modeling of projected oil extraction.

We must also consider that estimates during an early discovery phase may not play out into recoverable oil. The optimistic discovery estimate we can call Original Oil in Place (OOIP) and the pessimistic value as Ultimately Recoverable Reserves (URR). The value of URR will invariably fall below the value of OOIP as we can usually not extract all the oil in place, much as you can't completely clean up an oil spill completely (some 70% of oil discovered will remain underground according to [Ref 172]). So the URR becomes the important number to watch for. And therefore the cumulative reserves becomes the recoverable portion of the original oil in place.

Possible. A few other terms that get discussed with regularity include the '3P's for estimating reserves — which essentially indicates qualitative confidence levels for *proven*, *probable*, and *possible* reserve estimates. These definitions of reserve have less relevance since our model works directly with probabilities so that in the end we get the best estimate available. If the modeled estimate turns out wrong, it will turn out wrong with an equal likelihood that it leans pessimistic or optimistic. To use terms like *possible* in a correct quantitative sense, you would need to give a pessimistic possibility and an optimistic possibility.

Probable. Bottom-line, the 3P classification scheme seems too subjective and could actually taint projections. For a given model, I believe the most important result remains the most probable outcome, and then perhaps a spread of results with different levels of confidence on either side of the most probable. In particular, the possible outcome seems quite arbitrary as it usually means possible on the optimistic side, yet we will also discover an equally possible outcome on the pessimistic side. This makes sense, especially if you tie in the notion of subjective probabilities.

Provable. As the most conservative, the provable amount seems to allow a case of a model-free accounting that offers little in the way of projecting potentially optimistic outcomes. Or is this just the "pessimistic" balance to the "possible" projection? However this turns into so much unnecessary qualification and hedging of bets, so that the point of modeling gets lost.

Note that these definitions become less ambiguous if we place the qualifier "at least in size" for each estimate. Then the possible, probable, and provable modifiers create a ladder of increasingly conservative estimates for reserves. As an example, if a provable reserve value has a 95% chance of occurring, you know that only a 5% chance exists for anything less than this. But again, you need to carry over all the Bayesian prior inferencing of your assumptions to create such an error margin on your original projection. Therefore, we will not pursue this particular approach, opting instead for only the most likely or probable outcome.

Recovery Factor

The recovery factor indicates how much oil that one can recover from the original estimate of OOIP. This has important implications for the ultimately recovery resources, and increases in recovery rate has implications for reserve growth.

First of all, we should acknowledge that we still have uncertainty as to the amount of original oil in place, so that the recovery factor has two factors of uncertainty.

The cumulative distribution of reservoir recovery factor typically looks like the following S-shaped curve. The fastest upslope indicates the region closest to the average recovery factor.

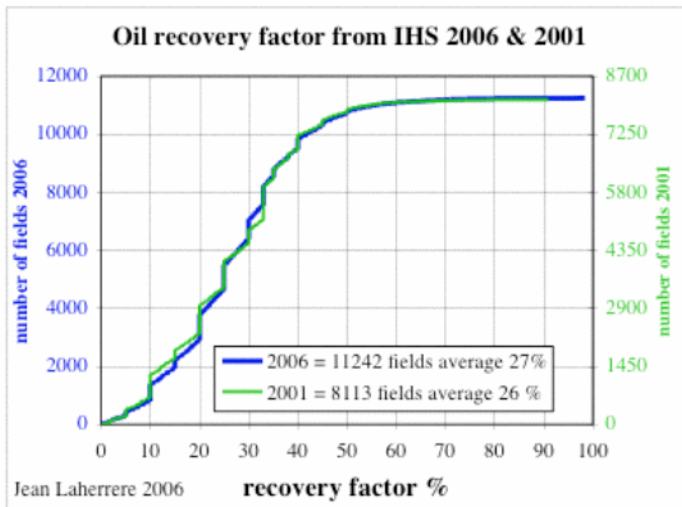


FIGURE 9-1. Recovery Factor cumulative distribution function (from Laherrere http://europe.theoildrum.com/node/7063)

To understand the spread in the recovery factors, one has to first realize that all reservoirs have different characteristics. Some are more difficult to extract from and others have easier recovery factors. One of the principle first-order effects has to do with the size of the reservoir: bigger reservoirs typically have better recovery factors and as one anonymous reservoir engineer has stated: *“Reserve growth tends to happen in bigger fields because thats where you get the most bang for your buck”*

So if we make the simple assumption that cumulative recovery factors, θ, have Maximum Entropy uncertainty or dispersion for a given ***Size***:

$$P(\theta) = 1 - e^{-k\theta/Size} \tag{EQ 9-2}$$

this makes sense as the recovery factor will extend for larger fields. Then add to the mix that reservoir sizes go approximately as:

$$Pr(Size) = \frac{1}{1 + Median/Size} \tag{EQ 9-3}$$

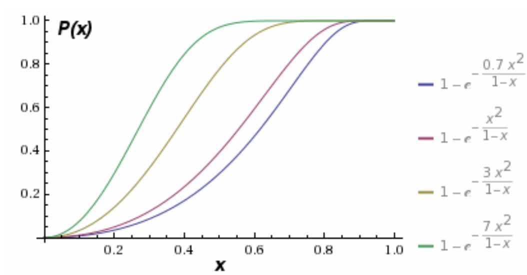
Then a simple reduction in these sets of equations (with the key insight that θ ranges between 0 and 1, i.e. between 0 and 100%) gives us

$$P(\theta) = 1 - e^{-\frac{k\theta^2}{Median \cdot (1-\theta)}} \tag{EQ 9-4}$$

the ratio ***Median/k*** indicates the fractional average recovery factor relative to the median field size.

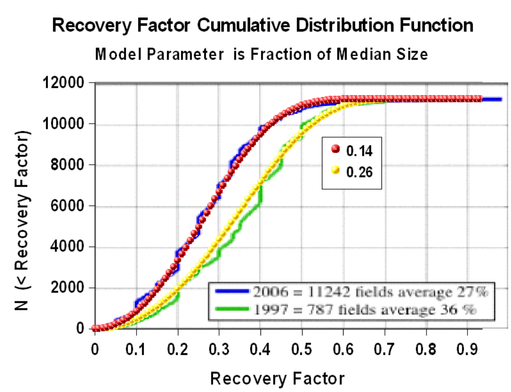
A set of curves for various ***k/Median*** values below:

FIGURE 9-2. Recovery Factor distribution functions assuming maximum entropy



From recovery factor curves originally supplied by Laherrere, I fit these to the ***Median/k*** fractions below.

FIGURE 9-3. Recovery factor curves from Rembrandt via Laherrere, alongside the recovery factor model described here.



Laherrere also provided curves for natural gas, where recovery factors turn out much higher.

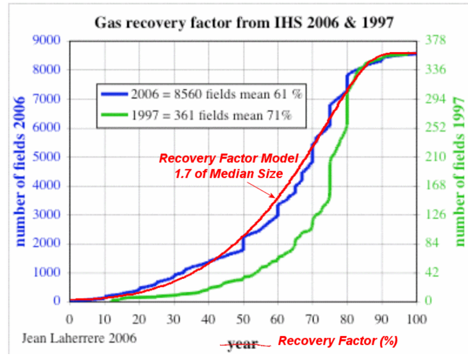


FIGURE 9-4. Recovery Factor distribution functions for natural gas. (Note: I had to fix the typo in the original Laherrere graph x-axis naming)

Note that the data shows a much higher recovery factor for natural gas than for oil.

It appears that this derivation may have universality as it parsimoniously models the recovery factor curves with only one sliding parameter. The parameter ***Median/k*** works in a scale-free fashion because both numerator and denominator have dimensions of size. This means that one can't fiddle with it that much — as recovery factors increase, the underlying uncertainty will remain and the curves in Figure 9-2 on page 148 will simply slide to the right over time while adjusting their shape. This gives us a good idea of where proportions of reserve growth will come from, with potentially more from smaller reservoirs. However, the relative efficiencies in the underlying recovery factors likely remain and thus we should see limitations in the ultimate smearing of the cumulative distributions. To reverse the entropic dispersion of nature and thus to overcome the recovery factor inefficiency, we will certainly have to expend extra energy.

We can model how the reserve growth behavior might evolve by looking at historical data over the years.

Solving the "Enigma" of Reserve Growth

As a good rule of thumb, when you have a promising model describing some physical process, you might as well put it through its paces. Not only do you shake out some stubborn corner cases, but you often find something new and revealing. We will do that later, where we derive the classic Logistic/Sigmoid-shaped Hubbert curve based on the generalized Dispersive Discovery model. In a similar fashion, I use the same discovery model to derive the upward climb of the cumulative reserve growth curve which we empirically observe on many oil reservoirs and oil-bearing regions.

Many analysts have found this reserve growth behavior both curious and, ultimately, very important. Two series of studies [Ref 23] and [Ref 48] have tracked this behavior as it plays an important role in how the peak will play out. Furthermore, I believe that the practice of "backdating" discoveries based on reserve growth updates has muddied the waters and stalled progress in the basic understanding of the fundamental growth process. Until we have a good model for the reserve growth dynamics we have resorted to using the heuristics supplied by USGS geologists, including the modified Arrington equation that others have successfully used in the past. I find nothing wrong with using a heuristic when appropriate and Foucher has explored the "un-backdating" approach with some excellent results. Still, a heuristic lacks some of the predictive power and room for insight that a fundamental model can provide.

The USGS crew have an interesting take on the reserve growth issue. Over the years, the geologists working for the government have labeled fossil fuel (both oil and NG) reserve growth an "enigma" and a "puzzle", also underscoring its importance [2].

> *For that reason the United States Geological Survey (USGS) considers [this] analysis "arguably the most significant research problem in the field of hydrocarbon resources assessment." [Ref 49]*

To try to solve the puzzle, this chapter runs though a stochastic analysis that essentially explains how reserve growth can happen, both from a bureaucratic point of view and then from the natural process of search.

Non-Speculative Estimates Only. In the USA, the federal government prohibits speculative estimates of the remaining oil in a field. The USGS defines the rule:

> *Operators in the United States are required by law to report only those reserves that are known with high certainty to be available for production, thus excluding resources that are at all speculative. It follows that one component of reserve growth is the difference between early estimates of recoverable resources, which in the presence of limited data are required to be conservative, and later estimates based on better knowledge of the field.* [Ref 50]

This means that oil producers can't use any heuristics based on previously measured reserve growth data. Those results, which have historically demonstrated a

2. Referring to something as an "enigma" obviously implies it has an unknown behavioral origin. Once we establish a cause, it no longer remains enigmatic.

growth of several times from an initial estimate, when applied to new fields then becomes classified as a speculative estimate. So, instead of coming up with an estimate based on any theory or even established heuristics, the field operators always undershoot the actual amount, to safely remain below the "speculative" point.

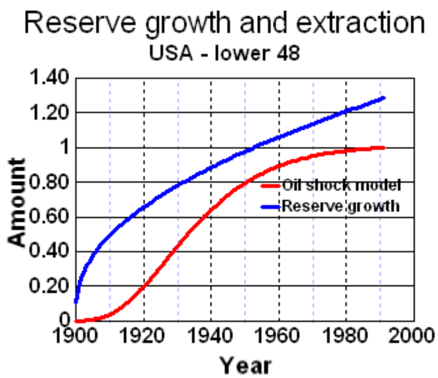


FIGURE 9-5. The oil shock model has a different shape showing a clear inflection point, with the cumulative production hanging below the parabolic growth at all points. As formulated, the long lag in extraction comes about from the serial application of the fallow, construction, maturation, and extraction phases. These combine to a *1/e* time constant of about 30 years, leading to the S-shaped curve shown.

Consider that reserve growth has a qualitatively different feel from conventional production curves. Note that the oil shock curve in Figure 9-5 on page 151 shows an inflection point due to the addition of fallow and construction times. Reserve growth rarely shows inflection but the cumulative values of discovery and production always do. You can understand this by considering that reserve growth bypasses the fallow and construction phases; since the reserve growth accounting only begins after production starts, and thus ramps up immediately. It also does not show an extended acceleration like a discovery or production curve does because it develops over a shorter time frame.

The end-result of the anti-speculative policies, particularly in the USA, means that all the original discovery curves need continual updating and therefore backdating of the data[3].

Censored Data. One of the technical issues that the USGS have ignored with respect to their reserve growth analysis involves the use of *censored data* [Ref 160]. This essentially says that you should take special care of extrapolating data backwards considering you have only a truncated time-series data set of recent vintage. The "sweet spot" for good data basically doesn't exist, with very few values for old data and also relatively few values for the latest data (mainly due to diminishing finds). Working with such a limited data set, the USGS massaged it the best they

3. Not only does the USA use somewhat arbitrary reserve estimation techniques, but other countries do as well. The Kuwaitis (http://www.washingtonmonthly.com/archives/individual/2006_01/008103.php) as well as other Middle Eastern countries appear to raise and lower their reserve numbers for no apparent reason

could and adequately normalized the fractional yearly growth, but they basically punted after this point and came up with only a heuristic to “explain” the trend. Pragmatically, not much that you can do about this [Ref 160]; in the long term, complaining about the possibility of statistical shortcomings still won’t explain most of the trend, which rises steeply enough to make the cornucopians hopeful for great prospects ahead[4].

Yet we still have left a riddle wrapped in a mystery inside an enigma. The actual problem with the flawed reserve growth analysis has become obscured by the trickiness with using censored data. The promise of optimistic growth really stems from a lack of a good value for the initial discovery estimate. Stating it bluntly, pick this number incorrectly and you can get numbers all over the map, with the possibility for some hugely absurd values.

To provide a path forward in unwrapping the riddle, I used the generalized Dispersive Discovery Model (described more fully in the next section). In terms of modeling reserve growth, the dispersion generates a tail for accumulating further discoveries after the initial estimate occurs. For constant average growth, the model looks like this:

$$DD(t) = \frac{1}{\left(\frac{1}{L} + \frac{1}{kt}\right)} \quad \textbf{(EQ 9-5)}$$

For the purposes of this analysis, I convert it to a reserve growth value $U(t)$ and set $T=L/k$ to make the math easier to handle later on:

$$U(t) = t \cdot T/(t+T) \quad \textbf{(EQ 9-6)}$$

Note that at time t=0, the discovered amount starts at zero and then the accumulation reaches some value proportional to L — what one should consider as the characteristic depth or volume of the reservoir. It takes time=T for most of the search to reach this median point. The basic premise of reserve growth and what USGS geologists such as Attanasi & Root [Ref 42] and Verma [Ref 43] frame their arguments on, has to do with the reserve growth considered as a multiplicative factor of the initial estimate. They see numbers that reach a value of nearly 10× after 100 years and claim (perhaps implicitly) that this has some real physical significance, almost offering up hope for still-to-come huge reserve benefits. Figure 9-6 on page 153 exaggerates the claim for effect, as I want to make you aware that a finite asymptote certainly exists, but it gets obscured by the data trends commonly reported.

4. Listen to the majority of talk radio programs or a financial cable program today for examples of this kind of unbridled optimism

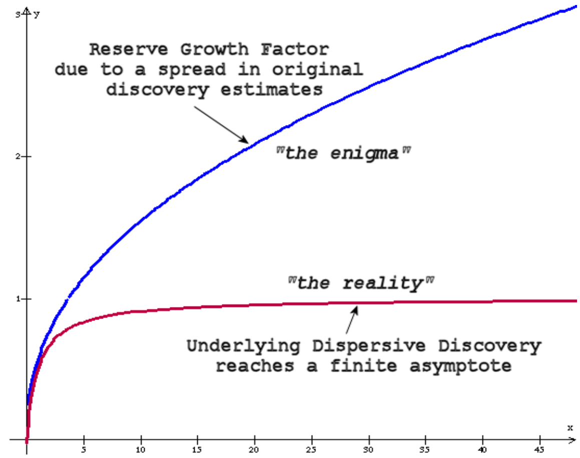


FIGURE 9-6. Potential for huge reserve growth in the "enigmatic" curve. The "reality" shows a finite asymptote.

The contrivance of the USGS explanation has to do with exactly when the original estimate becomes available. Conceivably you can make estimates that occur very early in the life-span of a reservoir, and you will get very low estimates for discovery size. To take it to one extreme, you might find the initial estimate to fill a sewing thimble. Now, if that estimate grows at all, you can get huge apparent reserve growth factors, some fraction approaching infinite in fact. In contrast, you can wait a couple of years and then report the data. The later years' growth factor will proportionately account for much less of an increase. Now if you consider that in other parts of the world, countries report reserves less conservatively then the USA, then the reserve growth factors can vary even more wildly. In my analysis, I used USA oil data from an Attanasi and Root paper [Ref 42]. Initially, I plotted the data as a fractional yearly growth curve, basically reproducing the trend that A&R report:

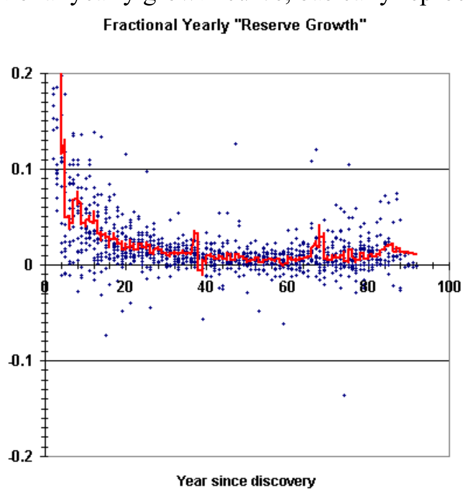


FIGURE 9-7. Trend duplicated from the A&R data. The blue '+' symbols represent the data and the red curve represents a small-window moving average. To generate the curve correctly, you must normalize the yearly growth per reservoir, as the reservoir sizes vary widely.

To understand the growth factor in terms of the Dispersive Discovery model, one has to first consider averaging the initial discovery point over a relatively small window of time starting from $t = 0$. This effectively scales the potentially infinite values of growth against some finite values, keeping the results finite. In other words, the use of the sampling/integration window brings down the potentially infinite (or at least very large) growth factor to something more realistic. The math on this derives easily into an analytic form, and we end up with this function, where A indicates the time integration window:

$$\bar{U}(t) = \frac{A - T\ln\left(\frac{t + A + T}{t + T}\right)}{A - T\ln\left(\frac{A + T}{T}\right)} \qquad \textbf{(EQ 9-7)}$$

The term on the right of the main divisor assures that the average $\bar{U}$ starts at 1 for time $t = 0$. Alternatively we can set A to some arbitrary value and skip the integration, which assumes that all initial discovery estimates start at $t = A/2$. This results in the conceptually simpler:

$$\bar{U}(t) = \frac{2(t + A/2)(A/2 + T)}{A(t + A/2 + T)} \qquad \textbf{(EQ 9-8)}$$

The latter equation obviously starts at unity and reaches an asymptotic value of $1+2T/A$. Both the values of A and T describe the ultimate asymptote, but the non-zero A serves to avoid generating a singularity at the origin. For T= 24.6 and A=6.6, Figure 9-8 on page 154 shows the negligible differences between using an integration window versus assuming a delta shift in the first estimate. The two curves essentially sit very close to one another.

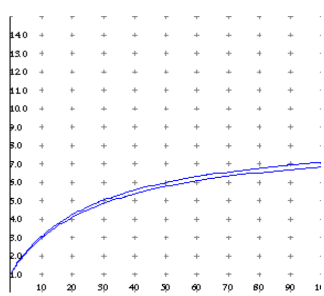


FIGURE 9-8. Very little difference occurs in using an initial value averager for reserve growth

So what do these numbers mean? Essentially, A=6.6 means that the first discovery estimate occurs on average 3.3 years after they first made the discovery. This makes intuitive sense because if we make the estimate too early, we end up with the equivalent of a thimble-full of oil. For the T=24.6 number, this means that it takes about 25 years for the majority (i.e. the fast part) of the dispersive search to take place. The rest of the long tail results from the slower dispersion. The curve does eventually reach the asymptote for a cumulative growth factor of 8.5.

In terms of a spreadsheet, you can turn the cumulative growth factor (CGF as A&R call it) formula into a discrete generating function, with the yearly estimates based on the growth factor of the preceding year. I plotted the Dispersive Discovery curve directly against the A&R data in Figure 9-9 and Figure 9-10. From the goodness of fit, I realized that this model has some nice understandable properties. It essentially generates growth factors based solely on the maximum entropy dispersion in the underlying model. In other words, the "enigmatic" reserve growth has turned from a puzzle into a mathematical result resulting solely from basic stochastic effects. Interestingly, for the curve shown the characteristic time in the figure (T=24 years) indicates the point at which the reserve growth factor reaches the median or 50% of its ultimate value.

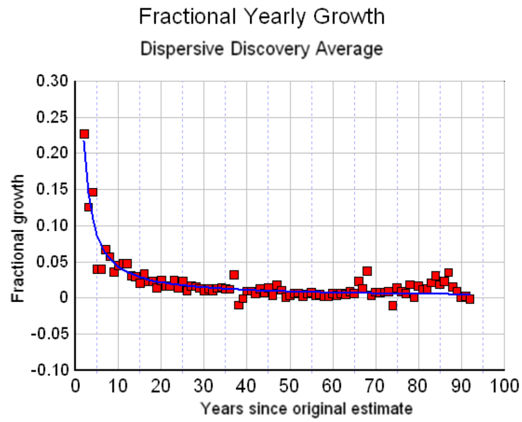


FIGURE 9-9. Fit to fractional yearly growth of the A&R data. Characteristic time T=24 years.

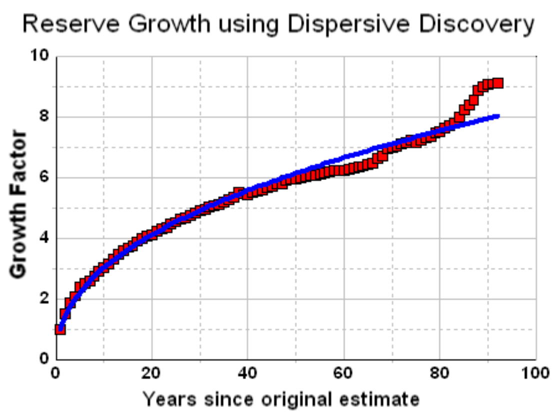


FIGURE 9-10. Fit to cumulative reserve growth of the A&R data. The data was calculated from the generating function in Figure 9-7. This correlates well with the direct use of the reserve growth equation, with value of T=24. The asymptotic value runs off the chart and reaches a value of 8.57 for the model.

Plotting the same data against an Attanasi & Root chart in Figure 9-11, it lays cleanly on top of it, showing discrepancies only on some very old outlier data[5].

A&R went through the rationale of discounting the outliers, but I consider all the data valuable, if it gets used in the context of a decent model[6].

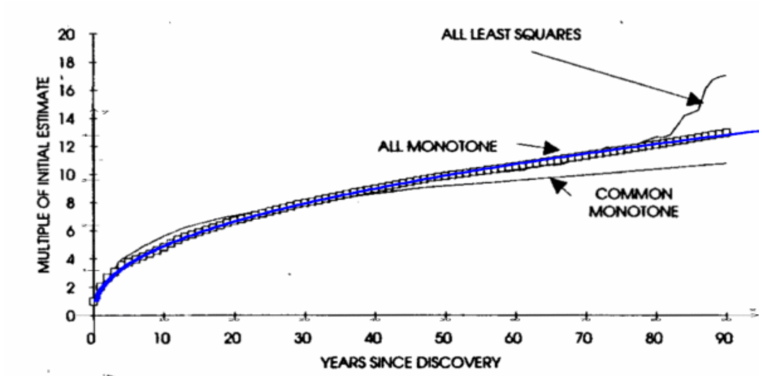


FIGURE 9-11. Fit on top of the original A&R chart. A&R forced a monotonic increase on their data to artificially generate a monotonically increasing curve. The blue line comes from the Dispersive Discovery reserve growth model.

Looking at the A&R data naively and with the limited data set available, the reserve growth looks like it will continue on and eventually reach infinite values — but this becomes a mathematical impossibility if we refer to the model and integrate out the “thimble”-sized initial estimates. As a bottom-line, if we continue to make poor initial estimates for discoveries, we will continue to pay the price for acting surprised at the “huge” reserve growth we come up with (by “we” I mean the oil industry and oil observers). In other words, the USGS, and condoned by the complicit oil industry, has pawned off a contrivance on us by making believe that their trend lines go beyond mere empiricism. If you look back at the previous model that the USGS’s Verma postulated [Ref 43] based on the modified Arrington approach, you will realize that the trend line comes about purely from heuristic considerations. Their equation shows a fractional order power-law growth, where YSD = years since discovery:

$$CGF = 1.738 \cdot YSD^{0.3152} \quad \textbf{(EQ 9-9)}$$

To top it off, this heuristic shows unlimited growth! (and an infinite upward slope at t=YSD=0!).

In historical terms, geologists and petroleum engineers who evaluate individual reservoirs, have apparently never truly understood the physical reasons for growth.

5. Outlier data mainly from heavy oil in the Kern River Basin extracted using enhanced recovery techniques, such as steam flooding. Most areas have not benefited from such an approach.
6. See [Ref 319] for a rationale explaining the importance of outliers.

Dake, who authored a definitive textbook on reservoir engineering wrote “*The reader may feel that the physical laws governing the subject of reservoir engineering are sparse.*” [Ref 172]

The cynical view may be that the USGS geologists decided that by reserving judgment on the possibility of a reserve growth asymptote, that they could pawn off a fast one on the public, and defer the reality for years to come?

Clearly the A&R paper can use a retroactive critical review since understanding the data really has such a simple statistical and mathematical underlying foundation that apparently the original peer review missed. Instead of invoking some “enigmatic” force, I truly find nothing complicated about the mathematics.[7]

Referring again to the literature, you find hints that support the dispersive effects of accumulated reservoir estimates. I am not sure that they use it exactly the way I do (I highlighted the word dispersion in the following passage).

> *A graphic illustration of the very broad URA data dispersion that occurs when grouping fields across geologic types and geographic areas was provided by the National Petroleum Council (NPC) and is reproduced with minor modification in Figure FE5.* [Ref 187]

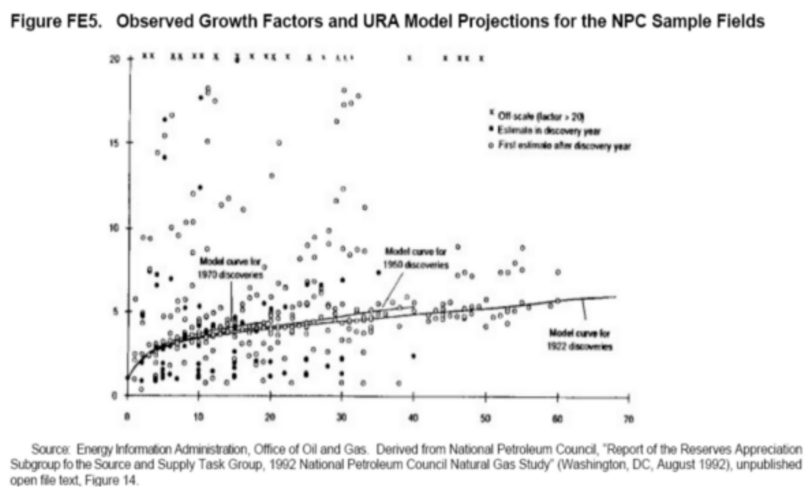


FIGURE 9-12. Reserve growth data (from [Ref 187]).

7. I call this combination of using simple models and using straightforward calculus and probabilities a form of pragmathematics — just something you pragmatically do to understand the physical foundation for the data we observe.

I claim that the dispersive discovery reserve growth model has the potential for filling in the back-dated discovery curves and providing better estimates of future production levels. As a path forward, I suggest starting with the USA data and using Foucher's approach for regenerating a profile of un-back-dated discoveries [Ref 23]and then applying the Oil Shock Model to estimate extrapolated production levels, using the maturation phase to model asymptotically-limited reserve growth. It may take some time to sort out the years of incomplete data, but we likely have all the pieces necessary to formulate a complete model-based projection[8].

Subjectively, one would almost infer that the actual enigma of reserve growth had more to do with the agenda of the USGS and the secrecy and inscrutability of the oil industry, not through any ignorance. You would think they would have figured out the reserve growth puzzle long ago[9].

Diffusional Growth

We can quite easily account for finitely constrained reserve growth using dispersive discovery arguments, but we can get some additional insight by considering an alternative reserve growth factor mechanism[10]. This has some basis in reality but the impact likely plays out as a second-order effect.

I preface the discussion by stating that the dynamics of fossil fuel "reserve growth" does not demonstrate compound growth by any stretch of the imagination. If it did in fact show such behavior, the growth would never reach a finite asymptote.

Compound growth in the traditional sense has a fixed proportional rate. Accumulated growth would thus show an accelerating slope. However, reserve growth has an apparently monotonically decreasing proportional rate over time which leads to a decelerating slope. Think of it this way — if the growth rate follows $1/x$, then any increase in x gets balanced by a smaller proportional amount or $\sim(1 + 1/x)^x$. I plotted a $0.5/x$ curve (in green) on top of the moving-average fit below.[11]

8. As another approach to time window averaging, I selected the incremental fractional curve to integrate for Figure 9-9 and Figure 9-10. The math gets a bit more complex. This results in a perhaps closer fit to the reserve growth data as the A&R approach uses the yearly fractional as a generating function as well. You can detect subtle differences only in the initial growth burst around t=0 as the integration tends to favor delta functions arising from the 1/t behavior close to the origin.
9. I guess they thought that deferring the reality would surely provide us with infinite hope.
10. Drift-diffusion models are covered in detail in Volume 2.
11. Dispersive discovery has a similar time dependence

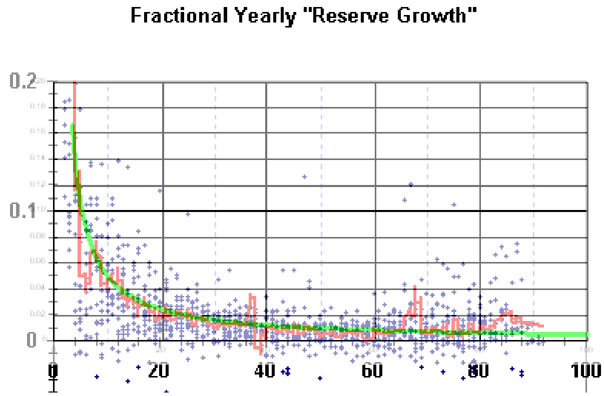


FIGURE 9-13. Smooth green curve shows how a diffusional factor *might* explain incremental reserve grwoth.

I see an analogy to silicon in the way that reserve growth could alternatively work. Take for example, the work of Andy Grove, one of the cofounders of Intel, who did his Ph.D. thesis in diffusion-limited oxide growth, a physical process critical to building integrated circuits. In a nutshell, silicon dioxide needs a source of silicon to form, but as the SiO_2 layer gets thicker, it becomes harder and takes longer for the Si atoms to diffuse to the surface and react with oxygen.

Grove used the physical analogy of diffusion to come up with a model for SiO_2 growth in this regime. In chemistry and materials science, engineers use the principle of diffusion frequently to estimate rates of flow. A simple formulation, known as Fick's first law, supposes that the rate of material flow remains proportional to the concentration gradient across an appropriate range of spatial distances. This works ideally for conditions of fixed concentration with respect to time at the boundary conditions[12]. The first law leads to a law of the following form, where ***F*(*t*)** gives accumulated thickness as a function of time:

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \frac{k}{F(t)} \qquad \textbf{(EQ 9-10)}$$

$$F = \sqrt{2kt}$$

Note that the fractional rate reduces to:

$$\frac{\mathrm{d}F}{\mathrm{d}t}/F = \frac{0.5}{t} \qquad \textbf{(EQ 9-11)}$$

12. Fick's second law makes the assumption that the concentrations can change over time, and a more complicated partial differential equation results. [Ref 174]

Note that this follows the "reserve growth" curve fit fairly well, where the fractional growth rate slows down inversely proportional to time. Microelectronics engineers refer to this as the *parabolic growth law* (a parabola sitting on its side, see the overlay on the green curve below). [13]

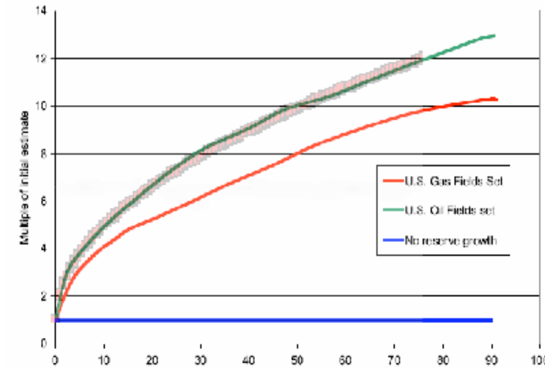


FIGURE 9-14. Results from Attanasi and Root Growth Functions for American Oil and Gas Fields

One could simply measure the volume as an approximation to how much oil producers have extracted, with the increase over time caused by diffusional diminishing returns. Much like a thick SiO_2 layer prevents fast oxidation, that drilling "deeper" into a field slows further depletion and you need to work harder and wait on average longer times to get at it. We want to demonstrate this effect via math.

This essentially describes the parabolic growth law. Yet, the parabolic growth law that individual oil reserve growth estimates seem to follow will likely hit some hard limit[14].

Under Fick's first law, the rate of transport (and therefore growth) tracks proportionally the concentration difference and inversely the distance between the opposing concentration layers. If the "growth" amounts to transferring material from one side of the layer to the other, the diffusivity *D* assists the flow from the high concentration area (*C(0)*) to the low concentration layer (*C(x)*), while the continued

13. Trying to fit such a curve seems pretty obvious, and of course had the semiconductor engineers of the last century thought that the oxide growth could only be "guessed" at, then we would never have advanced through the microelectronics revolution and process unpredictability would have killed us. We would still be working with crystal radio sets. No one would ever had established a process for creating million+ gate circuits!
The fact that material scientists and engineers like Andrew Grove quickly characterized the phenomena within a few years time (mid 1960's) and got their process down to a gnat's eyelash speaks volumes about the difference between engineering in the quest for improvement versus making predictions to support the business bottom-line, i.e. a future scarcity of oil. On the face of it, we lack the motivation to really want to explain enigmatic reserve growth — yet explaining an "enigmatic" oxide growth becomes necessary.

14. After all, no one really believes in an infinite supply of oil... do they?

growth on the other side starts to retard it (in a statistical sense due to random walk dispersion).

$$\frac{d}{dt}G(t) = D \cdot (C(0) - C(x)) / G(t) \quad \textbf{(EQ 9-12)}$$

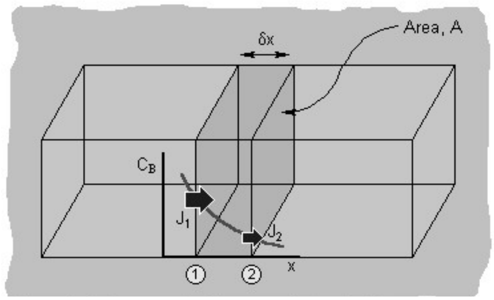


FIGURE 9-15. A gradient in the density of a mobile material results in the flow from regions of high density to low density. The greater the distance the lower the slope and therefore the slower the flow.

This first-order flow becomes self-limiting *only* in the sense that it starts to progressively slow down. However, given enough time, diffusion continues to happen and it will continue to grow indefinitely. In a material analogy, the oxides that form on silicon occur immediately and then start to slow down as the oxide gains thickness forming an increasingly impenetrable membrane. This generates the parabolic growth law — again perhaps better entitled the square root growth law — which states that G(t) ~ $\sqrt{t}$.

The connection to oil reserve estimates mostly comes from making an analogy — that the geologists can only predict what they can measure, and they can measure the oil at the low concentration layer, having to wait years or decades for the high concentration layer to "diffuse" across the barrier. I framed this ambiguously, as the "diffusion" I talk about may not symbolize real diffusion but rather a fixed difficulty in extracting material as we drill deeper, etc. We then will continually achieve diminishing returns, with an added real possibility of hitting hard limits.

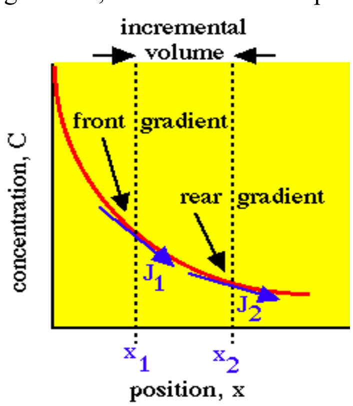


FIGURE 9-16. As the incremental volume increases, the rear gradient decreases and the diffusional flow slows down.

Drainage diffusion

The practical aspect of diffusion arises in the case of gravity drainage on lengthening the life-span on many fields. The fundamental concept of gravity drainage is contained in Darcy's Law [Ref 178] which states:

$$v = c(k/u)\frac{\mathrm{d}p}{\mathrm{d}s} \qquad \text{(EQ 9-13)}$$

For the case of gravity drainage

$$\frac{\mathrm{d}p}{\mathrm{d}s} = \frac{\mathrm{d}p}{\mathrm{d}h} \cdot \frac{\mathrm{d}h}{\mathrm{d}s} \qquad \text{(EQ 9-14)}$$

where dp/dh is the buoyancy of the oil or difference between the fluid gradient for oil and water, and dh/ds is the change in elevation over a distance, or simply sin(dipangle).

The v term above essentially provides the flow of oil into the region. Right away from the elements of the equation, one can tell Darcy's Law acts much the same as the ordinary Fick's Law in diffusion problems. So this casts Darcy's Law into Fick's Law of diffusion — which has a rather simple temporal behavior in the first-order case. The key involves the dh/ds term, the "dip angle", which provides a driving gradient at the heart of any diffusion process[15].

The ds term expresses the displacement in volume as the gravity drainage starts to move material from one volume to the other. So whatever goes from one side of "s" goes to the other side, the "v" side. This means that the length of the partially drained volume gets bigger and bigger with time.

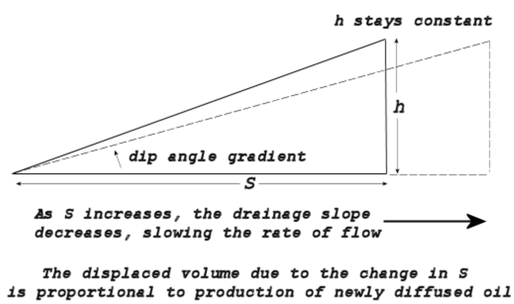


FIGURE 9-17. Darcy's law basically uses the concept of a diffusional gradient.

For a more three-dimensional view, look at the width in Figure 9-15 on page 161 where x corresponds to the s term with a diffusive flow in the opposite direction as

15. By not including diffusion Darcy's Law reverts to an equation that responds to pressure gradients, such as due to water injection, making it look more like Ohm's Law, but more complex as the phenomenon of ionized B in Si subject to an electric field [Ref 206].

above: So as x or s will get longer and longer over time, with a cumulative increase proportional to the integral of v. With the trigonometric small angle approximation for sin (dip angle) you get:

$$\frac{\mathrm{d}h}{\mathrm{d}s} = \frac{h}{s} \quad \text{(EQ 9-15)}$$

so rewriting this, replacing U with s to denote cumulative displaced volume

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \frac{k}{U} \quad \text{(EQ 9-16)}$$

this solves simply as

$$U(t) = k\sqrt{t} \quad \text{(EQ 9-17)}$$

which represents the time dependence of the Fick's First Law of Diffusion. As a bottom-line you get progressively diminishing return in oil production over time with this law. You can also see this by taking the first derivative.

$$\frac{\mathrm{d}}{\mathrm{d}t}U(t) = \frac{k}{(2\sqrt{t})} \quad \text{(EQ 9-18)}$$

Note that the rate of growth slows down inversely as the square root of time. As the long lifetime of a stripper well attests, the gravity drainage does exist but it also does have physical limits, mainly because of the diminishing rate of return coupled with the finite volume that it extracts from.

> *Gravity drainage fields can really confuse the villagers. I have one such fld that has produced for 50 years (20 mmbo so far) and will produce for another 100 years (maybe another 20 mmbo). When the angry villagers hear such tales they begin to think there really is help out there for them. The wells in this field make about 1 bb/ per day. It obvious has no bearing on PO. But the little ma and pa operators are slowly becoming millionaires. [Ref 106]*

So the production comes from the proportional drawdown of the initial reservoir and then from the reserve growth that occurs, due to whatever process causes such behavior. Another variant of hyperbolic behavior, the so-called *hyperbolic decline* becomes another piece of this puzzle as it can also describe the longer lifetime of wells. Incidentally, this has the same form as dispersive discovery growth examined earlier. Another form known as *harmonic decline*[16] often gets associated with grav-

16. The onset of harmonic decline occurs when the cumulative integral of production diverges under the assumptions of a hyperbolic decline. This happens when the inverse power is less than or equal to 1, as with the unbounded Fick's first law of diffusion.

ity drainage but always overestimates reserves, precisely for the same reason that Fick's first law shows infinite growth. As L.P. Dake discusses, some petroleum engineers have given up explaining the exponential, hyperbolic, harmonic, and any other form of decline in individual fields, as it doesn't really have any physical basis [Ref 172]. I would state this differently; you can't predict any one field, yet one can attempt to characterize an ensemble set through probabilities.

Adding Finite Constraints

Not wanting to work out Fick's second law[Ref 174], but sensing that the concentration changes with oil depletion, I worked out a modification to Fick's first law whereby I changed the *C(x)* term to track the growth term *G(t)*. This basically says that over time, the concentration differences start to level out.

$$\frac{\mathrm{d}}{\mathrm{d}t}G(t) = D \cdot \frac{(C(0) - (a \cdot G(t)))}{G(t)} \qquad \text{(EQ 9-19)}$$

Unfortunately, one can't find an analytical solution to this equation (except for the asymptotic behavior, which drops out straightforwardly). But, alas, we do have computers to do the grunt work of numerical integration. The following curve results for an ***a/C(0)*** ratio of **0.09** (the asymptote rather nicely goes to 1/0.09=11.1).

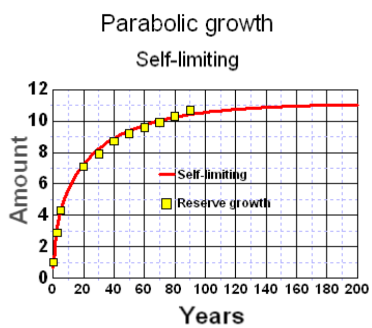


FIGURE 9-18.
Classic Fick's Law of diffusion assumes a limitless supply of material, but in reality, the source becomes depleted over time and so reaches an asymptote.

(As an alternative, we can arbitrarily cut off the parabolic growth law at a certain point in time, see the creaming curves in the next section. This will abruptly stop the diffusion process as if the source of the flowing material disappears. This seems a very workable model since, apart from the difficulty in predicting the constrained value, it has an analytical solution.}

I plotted the 90-year reserve growth from A&R (their "common monotone" data fit) on top of the curve so you can see one possible future extrapolation. Clearly, the enigma of miraculous parabolic growth starts to evaporate under this regime and extraction will continually eat away at the meager growth we will get in years to come.

So the basic premises sound similar and you would expect that the Fick's Law solution would have the same diminishing rate of return as the Dispersive Discovery case. But note the distinction and always realize that diffusion and dispersion stem from fundamentally different mechanisms, diffusion acts on its own while dispersion, like drift or convection, relies on a driving force.

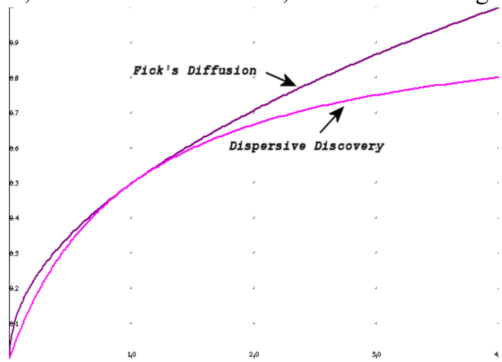


FIGURE 9-19. Simple diffusion in an unbounded volume shows no limiting behavior, but dispersive discovery reaches an asymptote.

This means that dispersive discovery, diminishing return diffusion, and other effects such as gravity drainage likely reinforce each other in terms of temporal behavior.

Dispersive vs. Diffusion Model of Reserve Growth

I assert that we can explain the enigma of reserve growth of oil reservoirs by simple physical considerations together with the dispersive formulation of growth rates. Most of the observed growth observed by the USGS seems to follow either a hyperbolic or “parabolic”[17] growth law that intuitively follows from the mathematics behind dispersion and diffusion. The characteristic solution to diffusive growth as described by Fick’s law shows an increase proportional to the square-root of time. Considering that one could model an arbitrary reservoir with a semi-permeable membrane that increases thickness with time, to first-order any concentration gradient across the thickness would show the characteristic square-root growth dependence.

However, we know that this growth law cannot sustain itself. We live in a finite universe, but the boundary conditions of Fick’s law assume an infinite supply to draw from, leading to a theoretically infinite growth. And contrary to the perpetualist notion of an infinite supply of oil, we know that we should apply some type of boundary condition to reserve growth. But how do we do that?

17. A bit of a misnomer, “parabolic” refers to the growth in time with thickness, not thickness with time, which would lead to proportional square-root growth.

A self-limiting parabolic growth law seemed to fit the data effectively (see Figure 9-18 on page 164), but it also veers away from Occam's territory a bit too much. A better, and more statistically and physically pleasing approach would include some considerations of the dimensionality of the reservoir volume and a maximum entropy spread in possible values for diffusive growth.

These actually fits the bill for a variation of dispersive growth with boundary conditions. By replacing a power-law growth rate in the original dispersive discovery model with a fractional (i.e. square-root) rate, we obtain the same "parabolic" growth curve initially — but it also hits an asymptote related to the fixed volume defined by the L0 parameter. Note the parabolic term in the growth law (see Figure 7-3 on page 113).

Figure 9-26 on page 171 shows the set of curves for various growth laws (both fractional power and integral power). I can't tell whether the data values match the fractional power-law of 0.5 (square-root) or 0.6 better, but the general trend demonstrates itself effectively in the following figure:

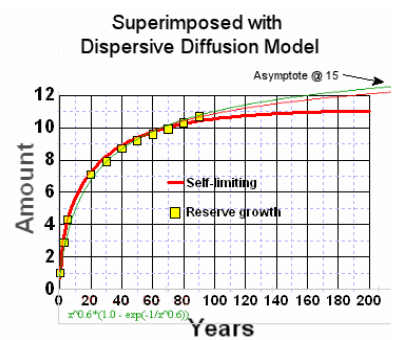


FIGURE 9-20. Dispersive discovery provides a more pragmatic explanation and generates well understood asymptotic behavior.

The green curve shows the 0.6 power-law, and both 0.5 and 0.6 curves reach an asymptote of 15. The asymptotic value obviously stays below infinity but it also approaches a value higher than the self-limiting numerical solution.[18]

Foucher has incorporated some of the Arrington and Verma reserve growth models to extrapolate the shock model and its hybrid variant to enable predictive forecasts ([Ref 23]). The dispersive discovery model may help in that particular analysis because of its finite asymptote and compact, closed form solution.

I can imagine that this model should also have some applicability to the analysis of creaming curves, where the important goal remains to identify the asymptote.[19] We can achieve a good macro understanding using nothing more than an expected

18. USGS's A&R have extrapolated to a value near this but give no asymptote

value for a hyperbolic function placed on a typical reserve growth model or creaming curve. I assert that the two curves basically amount to the same thing; in particular when we apply the numbers in a statistically valid way based on dispersive discovery and aggregated reservoir size we can get some notion of how the trend came about. This then fits nicely into the maturation phase of the oil shock model.[20]

The Hyperbolic Model

Let us look at the Dispersive Discovery model and the relationship describing cumulative reserve growth for a region. Omitting the diffusion-based parabolic growth law, and assuming linear growth and an exponential PDF for depth distribution, it looks like the following equation:

$$DD(t) = \frac{1}{\left(\frac{1}{L} + \frac{1}{kt}\right)} = U_{reserve}(t) \quad \textbf{(EQ 9-20)}$$

where t=*time* from the initial production. It gets a bit tricky to nail down the initial value for growth, as that has a big influence on the ultimate growth factor. The curve basically looks like Figure 9-21 on page 167:

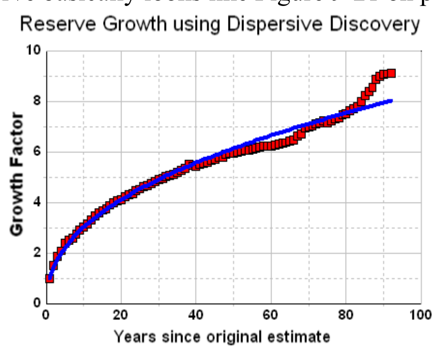


FIGURE 9-21. Fit to a reserve growth curve using the dispersive discovery model. For a linear growth of discoveries, this generates what Laherrere refers to a "hyperbolic" curve.

From noting in the last section that this same dependence can occur for field sizes, I realized that an interesting mapping into reciprocal space makes these curves a lot easier to visualize and to extrapolate from. So instead of plotting t against U, we plot $1/t$ against $1/U$

19. Peak oil contrarians such as Michael Lynch continue to have a field day in criticizing creeping increases in creaming curves — and without the benefit of a good model to confront his claims, I can imagine he has a point.

20. With a good comprehensive model, we have all the information needed to put to rest all the cornucopian concerns placed before us.

$$\frac{1}{U(t)} = \frac{1}{L} + \frac{1}{kt} \qquad \textbf{(EQ 9-21)}$$

On a linear-linear *x-y* plot where *x* maps into *1/t* and *y* into *1/U*, the linear curve looks like this:

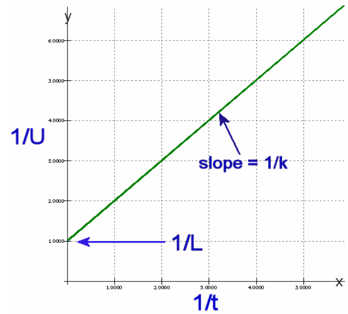


FIGURE 9-22.
We can linearize the dispersive discovery reserve growth model by plotting 1/*U* against 1/*t*. For collected data, this allows one to more easily perform linear regressions.

For field size distributions, which has the same hyperbolic form, it looks like the following on a log-log plot. This shows up clearly as a straight line over the entire range of the reciprocal values for the variants, if you pull the constant term into one of the two variants. This works out very well for size distributions if we scale the values to an asymptotic cumulative probability of 1.

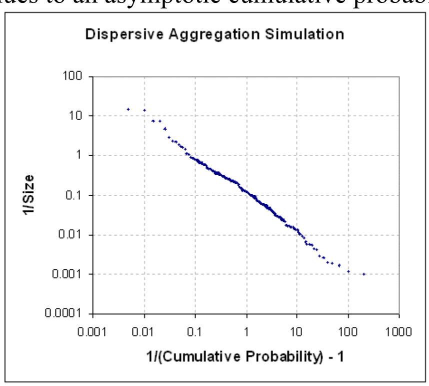


FIGURE 9-23.
Linearizing reserve growth data looks similar and uses the same concepts as linearizing reservoir/field size data.

Laherrere has long referred to “hyperbolic” plots in fitting to creaming and reserve growth curves. He describes how well he can match the growth temporal behavior to one or more hyperbolic curves [Ref 18]. However, I can find no mention of Laherrere’s description or derivation of the hyperbolic other than its use as a heuristic in describing the rate of production decline in an oil field[21]. Even the peak oil skeptic Michael Lynch could not find an explanation:

21. Hyperbolic decline in an oil field [Ref 172] differs from hyperbolic growth in reserves by definition. We will discuss further the unification of these ideas in a subsequent chapter. This overloading of terms results from a common entropy or disorder origin.

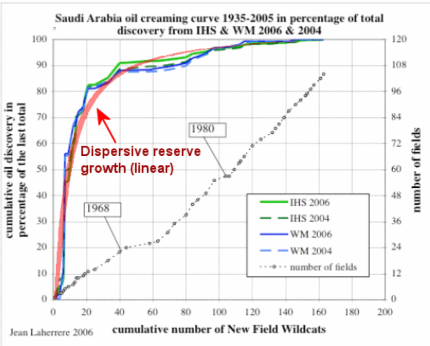


FIGURE 9-24. Adapted from Laherrere 2006, a dispersive discovery model with a linear progression corresponding to wildcats drilled shows asymptotic behavior.

No explanation is given for the "hyperbolic model" or why ordering by size is more appropriate than by date of discovery. [Ref 52]

One can naively assume that a hyperbolic function either means a slice through a conic section or the following type of dependence:

$$y = \frac{1}{x} + c \qquad \textbf{(EQ 9-22)}$$

However, none of these two behaviors really match what we see. But then I concluded that Laherrere could have meant the alternate version that I show in the prior equation [Ref 9-21]

$$\frac{1}{y} = \frac{1}{x} + c \qquad \textbf{(EQ 9-23)}$$

The following graph shows a typical Laherrere creaming curve analysis, where he fits to a couple of hyperbolic functions. Note that he refers to "hyperbola" in the legend. As the offset curve I plot the Dispersive Discovery cumulative. I had to slide it off Laherrere's hyperbola curve slightly since the two match exactly, which means that you couldn't tell the two apart and one curve would obscure the other.

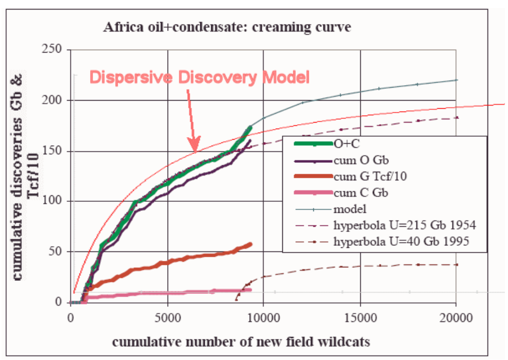


FIGURE 9-25. Dispersive discovery model laid on top of reserve growth data. I artificially shifted the model curve above the data points to keep it from disappearing into the thick line. The model actually goes through the origin.

I assert that Laherrere's hyperbolas map precisely into the linear Dispersive Discovery curves. So we have turned the hyperbolic fit heuristic into a model which comes about from a well understood physical process, that of Dispersive Discovery.

On another positive note, I think the reciprocal fitting (Dispersion Linearization) may supplement Hubbert Linearization. From what I have gathered, people seem to appreciate straight lines more than curves extrapolating trends.

Creaming Curves and Dispersive Discovery

Try looking up information on the oil industry term “creaming curve” via a Google search. In relative google terms, you don’t find much information. Of the top hits, this paper gives an Exxon perspective in regards to a practical definition for the distinctively shaped curve:

> *Conventional wisdom holds that for any given basin or play, a plot of cumulative discovered hydrocarbon volumes versus time or number of wells drilled usually show a steep curve (rapidly increasing volumes) early in the play history and a later plateau or terrace (slowly increasing volumes). Such a plot is called a creaming curve, as early success in a play is thought to inevitably give way to later failure as the play or basin is drilled-up. It is commonly thought that the “cream of the crop” of any play or basin is found early in the drilling history. [Ref 53]*

This seems a simple enough description and so you would expect a bit of basic theory to back up how the curve gets derived, perhaps via elementary physical and statistical processes. Alas, I don’t see much explanation on a cursory level beyond empirical hand-waving and statistically insignificant observations such as the Exxon paper describes. The paucity of fundamental theory combined with the fact that my own observations rank high on a naive internet search[22] tells me that we have a ripe and fertile field to explore with regards to creaming data.

The fresh idea I want to bring to the table regards how the Dispersive Discovery model fits into the dynamics of creaming curves. As the Exxon definition describes the x-axis in terms of time or number of wells drilled, one could make the connection that this corresponds to a probe metric that the linear Dispersive Discovery uses as the independent variable. The probe in general describes a swept volume of the search space. If the number of wells drilled corresponds linearly to a swept volume, then the dispersive curve maps the independent variable to the discovered volume, via two scaling parameters D_0 and k:

$$D(x) = D_0 \cdot x \cdot (1 - e^{-k/x}) \text{ — Uniform density} \qquad \textbf{(EQ 9-24)}$$

Or the alternative hyperbolic:

$$D(x) = \frac{D_0}{\left(1 + \frac{1}{kx}\right)} \text{ — Uncertain depth density} \qquad \textbf{(EQ 9-25)}$$

22. As of 2010, the #1 Google hit for “creaming curves” brings you back to my own site. This really indicates a paucity of understanding in the fundamental behavior behind creaming curves; hopefully this will change in years to come.

and then we map the variable x to the number of wells drilled. Changing the x parameter to time requires a mapping of time to a rate of increase in x:

$$x = f(t) \tag{EQ 9-26}$$

I assert that this has to map at least as a monotonically increasing function, which could accelerate if technology gets added to the mix (faster and faster search techniques over time), and it could possibly decelerate if physical processes such as diffusion play a role (Fick's law of parabolic growth):

$$\text{diffusion} \quad => \quad x = A\sqrt{t} \tag{EQ 9-27}$$

$$\text{accelerating growth} => \quad x = B \times t^N \tag{EQ 9-28}$$

$$\text{steady growth} \quad => \quad x = C \times t \tag{EQ 9-29}$$

The last relation essentially says that the number of wildcats or the number of wells drilled accumulates linearly with time. If we can justify this equivalence, then an elementary creaming curve has the same appearance as a reserve growth curve for a limited reservoir area. The concavity of the reserve growth curve or creaming curve has everything to do with how the dispersive swept volume increase with time:

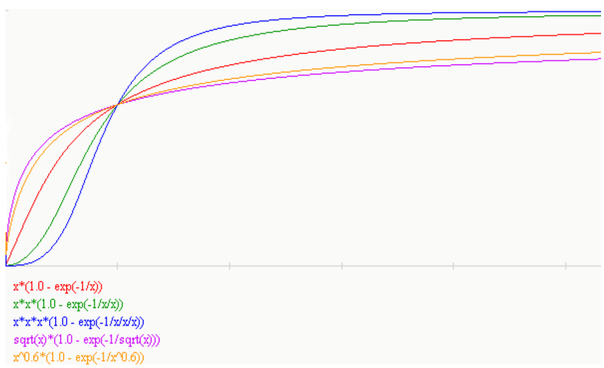


FIGURE 9-26. Dispersive discovery curves under various grwoth rates. As the acceleration of growth increases, a stronger inflection point arises.

Regarding the historical "theoretical" justifications for creaming curves, I found a few references to modeling the dynamics of the curve to a hyperbola, i.e. an $1/x^N$ shape. This has some disturbing characteristics, principle among them the lack of a finite asymptote. So we know that this wouldn't fit the bill for a realistic model. On the other hand, the Dispersive Discovery model has (1) a statistical basis for its derivation, (2) a quasi-hyperbolic climb, and (3) a definite asymptotic behavior which aligns with the reservoir limit.

For the curious, the *linear* Dispersive Discovery model also has a nice property that allows quick-and-dirty curve fitting. Because it basically follows affine transformations, one parameter governs the asymptotic axis and the other stretches the orthogonal axis. This means that we can draw a single curve and distort the shape along independent axis, thereby generating an eyeball fit fairly rapidly[23].

We can look at a few examples of creaming curves and their similarity to dispersive discovery.

The following figure shows a creaming curve for Norway oil.

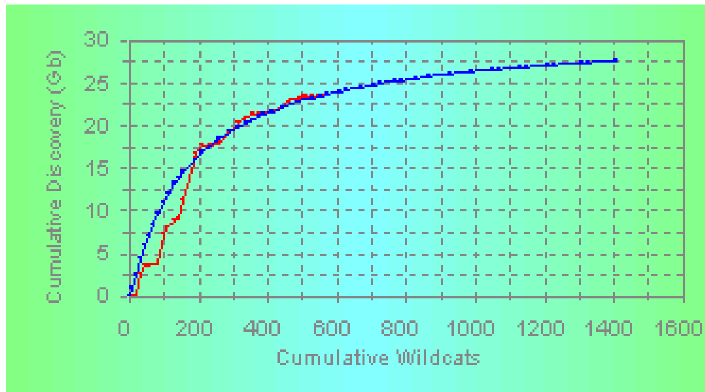


FIGURE 9-27. Creaming curve for Norway oil. Creaming curves differ from reserve growth curves in replacing the time axis with an ordinal that tracks time in some fashion (in this case, cumulative wildcats). [Ref 54]

At this point I overlaid a dispersive discovery curve over the "Theory" curve that one analyst alludes to[24]:

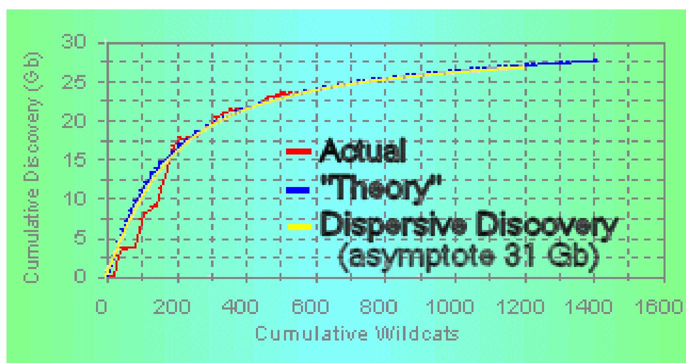


FIGURE 9-28. Undocumented creaming curve model labelled "theory" overlaid with dispersive discovery model (likely the same formula as Laherrere uses for his hyperbolic curve).

The figure does not come close to specifying the "theory" in any detail, but the simple dispersive discovery model lays closely on top of it with a definite asymptote.

23. Not true in general though. For other non-linear growth rates, such as the Logistic curve used in peak modeling do not have the affine transformation property, making curve fitting not eyeball-friendly.
24. "PolicyPete" http://policypete.com/background(8).htm

Another creaming curve analysis from the "Wolf at the Door" website results in this curve:

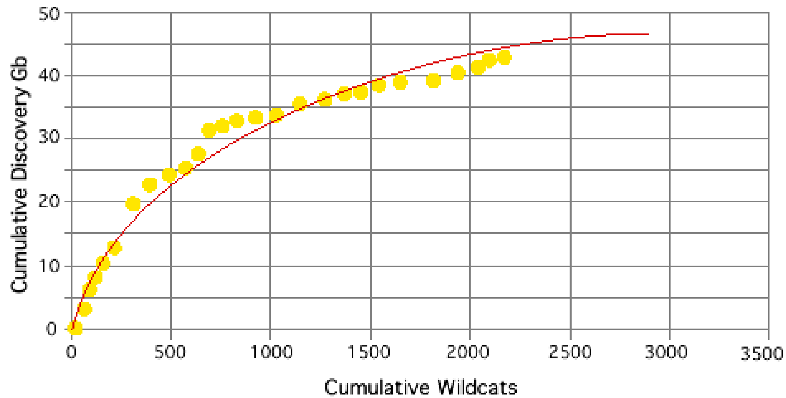


FIGURE 9-29. A Dispersive Discovery fit to another creaming curve data set. From http://wolf.readinglitho.co.uk/mainpages/reserves.html

The red curve above references a "hyperbolic" curve fit, while the figure below includes the Dispersive Discovery fit.

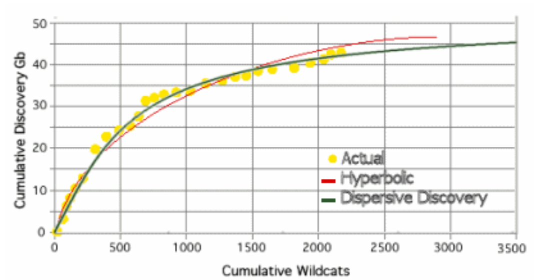


FIGURE 9-30. An example of a creaming curve where the actual values (yellow) compare to a hyperbolic curve (red). When the curve becomes asymptotically horizontal we can read the total contents of the field. The green curve of dispersive discovery shows an asymptote of 51 Gb.

The fit here proves arguably better than the hyperbolic and gives a definite asymptote that a hyperbolic would gradually and eventually overtake.

Natural Gas. One can apply the same model fitting to natural gas.The following chart shows the continuously updated creaming curve for USA NG..

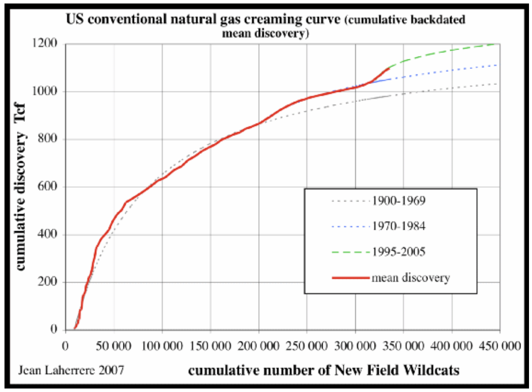


FIGURE 9-31.
Natural gas fields also follow creaming curve dynamics.

I originally did a non-creaming analysis (footage only from "Applying Dispersive Discovery. How discovery affects production") using Hubbert's 1970's data and arrived at an asymptote of 1130 Tcf using linear Dispersive Discovery. An updated curve using new field wildcats instead of Hubbert's cumulative depth drilled yields an asymptote of 1260 Tcf from a least-squares fit.

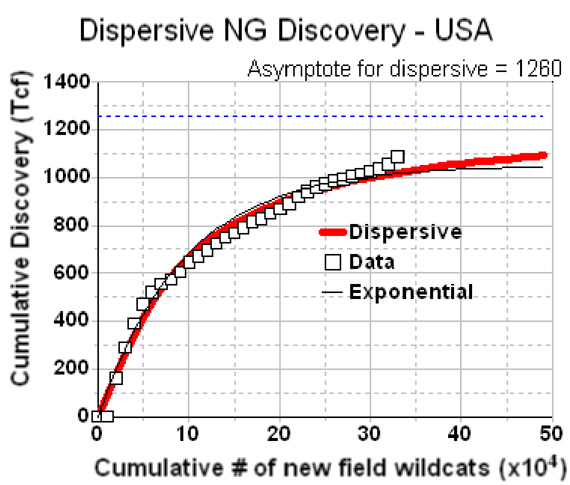


FIGURE 9-32.
Difference between dispersive reserve growth showing a gradual asymptotic behavior and Hubbert's limiting exponential showing rapid damping.

Hubbert's plot from the 1970's indicates the correspondence from cumulative footage to number of new wildcats; we assume that every new wildcat adds a fixed additional amount of cumulative footage. This allows a first-order approximation to dates well beyond what Hubbert had collected.

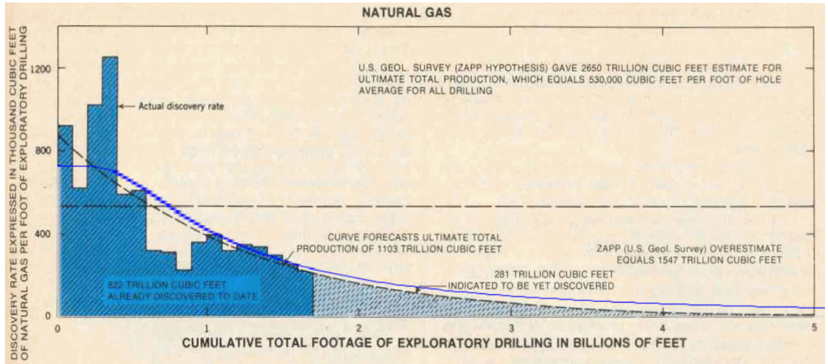


FIGURE 9-33. Dispersive growth model placed next to Hubbert's natural gas exploration efficiency. Hubbert used an exponential to estimate cumulative.

As a caveat to the analysis, I would caution that the number of wildcats drilled may not correspond to the equivalent swept volume search space. It may turn out that every new wildcat drilled results from a correspondingly deeper and wider search net. If this turns out as a more realistic depiction of the actual dynamics, we can easily apply a transform from cumulative number of wildcats to cumulative swept volume, in a manner analogous to like we did for mapping time to swept volume in the case of reserve growth.

Summary Response. Since Laherrere had pioneered much of the heuristic curve fitting of creaming curves and characterized the shape as hyperbolic, I posed a question based on a TOD post on Arctic creaming curves that he had written [Ref 55].

My question:

> *The one pressing question I always have is how the interpolated and extrapolated smooth lines get drawn on these figures. We all know that the oil production curves tend to use the Logistic as a fitting function, but we don't have a good handle on what most analysts use for discovery curves and creaming curves. In particular I have seen several references to creaming curves being modeled as "hyperbolic" curves yet find little in fundamental analysis to make any kind of connection.*
>
> *Based on statistical considerations I am convinced that the discovery and creaming curves result from a relatively simple model that I have outlined on TOD. I have a recent post where I make the connection from dispersive discovery to creaming curve.…*

In the following figure I apply the Dispersive Discovery function to one of the data sets on your graph. This function is simple to formulate and it produces a finite asymptote which you can use to estimate the "ultimate discoverable" (150 GBoe for NG in the following).

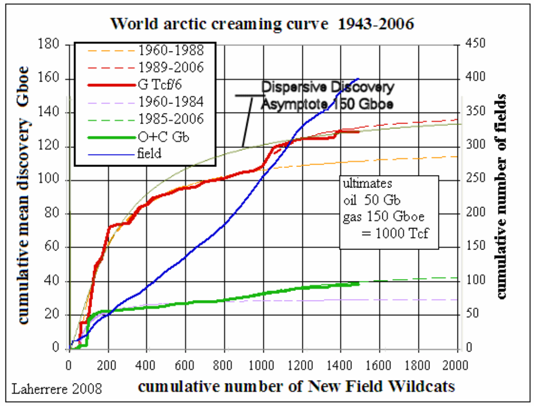


FIGURE 9-34. Creaming curve behavior for Arctic oil from Laherrere. With a dispersive reserve growth overlaid.

Response from Laherrere.

Every time that I plot a creaming curve, I am amazed to see how easy it is to model with several hyperbolas, but this doesn't explain why, except that on earth everything is curved. Linear is just a local effect (horizontal with the bubble, vertical with the mean) being the tangent of a curve. I found the same thing with fractals: it is a curve, so I took the simplest second degree curve: the parabola.

For creaming, hyperbola is the simplest with an asymptote. But the most important is to use several curves because exploration is cyclical. But another important point is to define the boundaries of the area. If the area is too big, it may combine apples and oranges making it difficult to find a natural trend. If the area is too small it will have too little data to find a trend. The best is to select a large Petroleum System which is a natural domain. The Arctic area is an artificial boundary and not a geological one.

I agree that the bigger the data set the better, as the statistics improve and local geological variations play less of a factor. But still, Laherrere doesn't claim any magic to the curve as he took the simplest function he could find.

I like how this all fits together like a jigsaw puzzle and we can get a workable unification of the concepts behind technology assisted discovery, creaming curves, and the "enigmatic" reserve growth. It also has the huge potential of giving quantitative estimates for the ultimate "cream level" thanks to the well-behaved asymptotic properties of the dispersive discovery model. And it basically resolves the issue of why no one has ever tried to predict the levels for the "hyperbolic" theory, as no clear asymptote results from any ***real*** hyperbolic curve without adding a great deal of complexity (both in understanding and computation).

Reserve Growth Composes Maturation

The likelihood of reserve growth adds a bit of complexity to the oil shock model but nothing it can't handle. The enigma of reserve growth, at least in the USA, shows a fairly fast rise in estimates in the first 10 years and then a slower growth spanning perhaps over the course of 90 years. If you look carefully at the growth, it appears to decompose into a fast exponential to which we superimpose a much more slowly damped exponential growth curve. If we assume that a fraction of the reserves get discovered early and the rest gets *discovered* as reserves later, then we can use the shock model to evaluate the effects on production by separating the two modes. (We can also use the dispersive discovery model to allow for deferred reserve growth just as well but this decomposition guides our intuition and might turn out more practically appealing)

For demonstration purposes, I used a single spike of discovery in 1899. The fast reserve growth comprises 45% of the discovery, while the slow reserve growth comprises 55%.

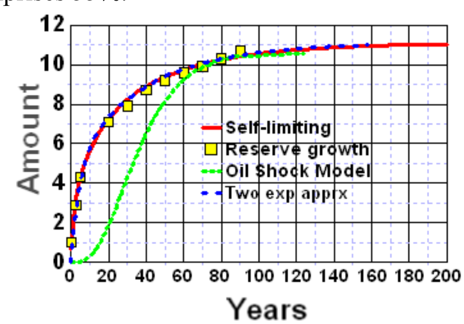


FIGURE 9-35. Reserve growth acts like the maturation phase of the shock model. No inflection is observed due to missing fallow and construction periods.

To model the production curve, I used 8 years for the fallow, build, and maturation phases for the quick growth. I replaced the maturation phase with a value of 40 years for the slow growth model. I chose an extraction rate of 5% of remaining reserves per year for both modes. The linear properties of this stochastic model allow us to decompose or compose the two modes freely.

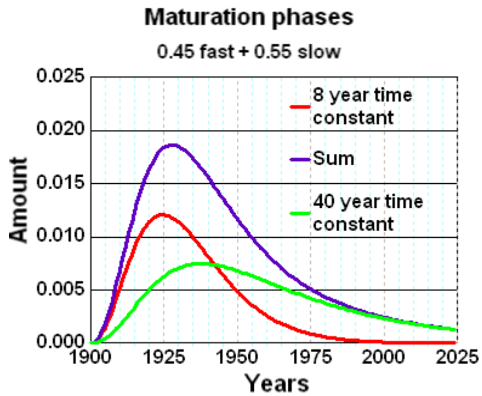


FIGURE 9-36.
Dispersive discovery reserve growth approximates to a composition of two exponentials, a fast portion showing the initial reserve increase and a slower phase describing long-term dispersion.

Note that the peak changes by at most 5 years with the addition of a significant long term reserve growth component. The model makes the assumption that the reserve growth actually comes about from continual discoveries or improvements in recovery technology over time. This tends to extend the tails out quite a bit and suppresses the height of production peak if we hold the proportional extraction rate constant.

Whether this assumption holds true world-wide, we probably can't yet say. We know that off-shore fields do not show extensive reserve growth because they shut down on the hint of production decrease. The same holds true for fields in other inhospitable or harsh regions or in fields that have had poor management practices over the years, in particular, via excessive water cuts. If the fraction of slow growth global reserves decreases with respect to the USA proportion, the broad peak will shrink.

The History of Wrong Predictions

The USGS has had a spotty track record in reserve estimates. Before Attanasi & Root, they gave us the Zapp Hypothesis [Ref 98][25].

> "Instead of remaining constant, discoveries per foot have fallen drastically during the last 35 years from a maximum value of 276 bbls/ft during the period 1928-1937 to a present figure of about 35 bbls/ft. From this, it is evident why the estimate derived deductively from the Zapp hypothesis is about 3.5 times the highest

25.Read "The Opposing View" to get a historical record of the USGS's analysis figure that can be justified by the discovery data — an over-estimate of about 425 billion barrels." [Ref 97]

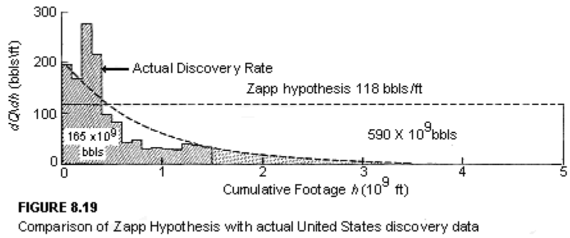


FIGURE 9-37. USGS geologist A.D. Zapp made a naive prediction for USA future reserves in 1962. [Ref 97]

This anecdote truly demonstrates how much room for improvement exists for reserve growth predictions.

CHAPTER 10

The Context of Discovery.

How do we simplify the search model?

"The theory of probabilities is at bottom nothing but common sense reduced to calculus."

— Pierre-Simon Laplace,
Théorie analytique des probabilités, 1820

Generalized Dispersive Discovery

In the following two sections, we try to further substantiate the oil discovery and depletion models by reducing them to more canonical and therefore simpler forms. This gives us greater insight and provides useful context against what we empirically observe.

Gold Rush Dynamics, a Dispersive Discovery Sanity Check

Dynamics such as those that lead to extinction events and of boom-bust periods first motivated me to generalize discovery dynamics in terms of dispersive effects.

If we look into an extinction event such as passenger pigeons in the 1800's, we find a steadily accelerating harvest per year until culling of the base population hit a critical point and then fell precipitously. The harvests went spectacularly to zero and so, unfortunately, did the pigeon population.

I can safely assume the same for boom-bust cycles, such as happened during the gold-rush days of the 1800's. In most cases, a boom occurred on the onset of an isolated discovery as many prospectors joined the search, enough time passed to enable the building of a huge infrastructure and then suddenly everything dried up with the infrastructure left standing in place.

But that hasn't necessarily happened with the discoveries of fossil fuel around the world. Although discoveries did increase at an accelerating pace until about the mid-part of the 20th century, reaching a peak a little after 1960, many discoveries continue to occur and the bottom did not fall out, unlike the cases of extinction and mini-boom-busts. We explain this by considering the role of dispersion in the discoveries. The following figure shows a non-dispersed discovery function, which reaches a sharp peak and then drops to zero as prospectors finish searching an isolated volume of potential finds.

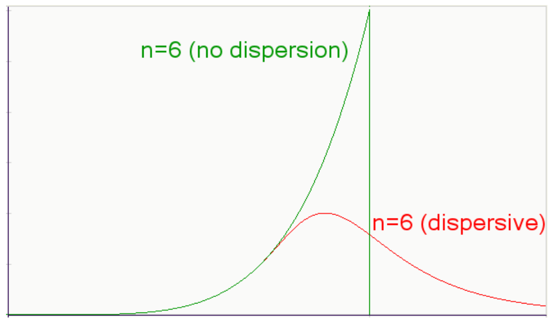


FIGURE 10-1.
Distinction between growth models in a finite container with and without dispersion.

This essentially happens when a highly localized search takes place, as with the case of the blanket coverage of passenger pigeon flyways by an efficient army of hunters[1]. It also happens with prospectors sifting everything with the equivalent of a fine-tooth comb in some localized gold strike area[2].

But the discovery of oil differs as dispersion in the rates of discovery in various parts of the world lead to a broad smearing of the bust peak. In fact, the effective peak determined by equal integrated volume only occurs on the backside of the dispersed profile (to the right of the dispersed peak). So, in fact, many busts have occurred already in various parts of the world, all due to dispersion. This makes consistent sense and provides a further argument against the use of the naive Logistic function[3] to model any of these kinds of search processes, whether they show dispersive characteristics or not.

As a fundamental concern, someone has to explain why a symmetric Logistic function (*ala* the classic Hubbert curve) makes no attempt to explain the steep drop-off displayed in many culling-forced extinction examples and of the bust drop-off in gold-rush cases.

1. The explosives used in forested areas were often supplemented with large nets to capture the falling birds.
2. Later, the brute force techniques took over which relied on aggressive industrial-strength processes to extract the gold.
3. The naive Logistic function relies on strict curve fitting. Otherwise it falls into a deterministic category of analyses, based on birth-death dynamics which does not model discovery.

Of course, that turns into a moot point as a heuristic by definition never can explain anything, it just has to mimic a behavior. This all gets the hand-wave treatment by the classically trained Hubbert modelers that use the Logistic function. Which I find really and truly odd as the Verhulst birth-death equations theoretically apply most effectively in localized Petri dish style experiments. Translation: analysis by Logistic approaches does not meet any formal sanity checks and only serves as a convenient stand-in... **unless we can derive it some other way.**

The Derivation of "Logistic-shaped" Discovery

This section addresses the origins and relevance (or lack thereof) of the Logistic sigmoid function as it is commonly used in projecting/modeling oil production forecasts. As far as I can determine, the analysis establishes the first instance of successfully deriving the Logistic model for oil from first principles.

Many people believe that the Logistic function adequately models the Hubbert peak. This comes with a few rationalizations:

1. We can (often/occasionally) get an adequate heuristic fit to the shape of the production data by matching it to a logistic sigmoid curve.
2. The logistic-growth formula $dU/dt = U(U_0-U)$ carries some sort of physical significance.
3. The logistic has hung around for a long time, in modern terms, therefore it must have some practical value.

I see nothing wrong with the first reason; scientists and analysts have used heuristic curves to fit to empirical data for years and a simple expression provides a convenient shorthand for describing the shape of a data set. In the case of the Hubbert peak, we get the familiar S-function for cumulative production, and a bell-shaped curve for yearly production — both characteristics that describe the Hubbert peak quite nicely as a heuristic.

As for point #2, we usually see hand-wavy arguments that point to an exponential growth that causes the peak oil curve to rapidly increase and then levels off as a negative feedback term in the equation takes over. What I consider circular reasoning with respect to Hubbert Linearization supports the idea that a physical process must drive this effect — perhaps something similar to the constrained growth arguments popularized by Verhulst[4]:

4. Robert Rosen has pointed out that even Verhulst had reservations concerning the use of the equations in stochastic environments [Ref 308].

> *Verhulst showed in 1846 that forces which tend to prevent a population growth grow in proportion to the ratio of the excess population to the total population. The non-linear differential equation describing the growth of a biological population which he deduced and studied is now named after him.* (http://www-history.mcs.st-andrews.ac.uk/Biographies/Verhulst.html)

Unfortunately, I have never seen a derivation of this idea to oil production, at least to my liking. Most proofs have simply asserted that the relationship fits our intuition and then the equation gets solved with the resulting sigmoid curve[5]:

$$U(t) = \frac{1}{\left(\frac{1}{U_0} + \frac{1}{Ae^{Bt}}\right)} \qquad \textbf{(EQ 10-1)}$$

I have problems with these kinds of assertions for a number of reasons. First of all, the general form of the resulting expression above can result from all sorts of fundamental principles besides the non-linear differential equation that Verhulst first theorized. For one, Fermi-Dirac statistics show the exact same S-curve relation as described by the $U(t)$ formula above, yet no respectable physicist would ever derive FD by using the $\mathrm{d}U/\mathrm{dt} = U(U_0\text{-}U)$ logistics-growth formula. Most physicists would simply look at the relationship and see a coincidental mathematical identity that doesn't help their understanding one iota.

Secondly, one can play the same kind of identity games with the Normal (gaussian) curve, which also gets used occasionally to describe the production peak. In the case of the gaussian, we can generate a similar non-linear differential equation $\frac{\mathrm{d}G}{\mathrm{d}t} \approx -(t \cdot G)$ which also "describes" the curve. But this similarly says nothing about how the gaussian comes about[6], instead it only shows how a mathematical identity arises from its parameterized curvature. This becomes a tautology, driven more by circular reasoning than anything else.

The last point of the logistic having implicit practical value has the historical force of momentum. This may seem blasphemous, but just because Hubbert first used this formulation years ago, doesn't make it *de facto* correct. He may have used the formula because of its convenience and mathematical properties more than anything else.[7]

---

5. A sigmoid is also known as an ***S***-shaped curve
6. See the previous chapter concerning the central limit theorem and the law of large numbers "Special Case: The Central Limit Theorem" .

The breakthrough uses the Dispersive Discovery model as motivation. This model doesn't predict production but I figure that since production arises from the original discovery profile according to the Shock Model, this should at least generate a first-principles understanding.

In its general form, keeping search growth constant, the dispersive part of the discovery model produces a cumulative function that looks like this:

$$D(x) = x \cdot (1 - e^{-k/x}) \quad \textbf{(EQ 10-2)}$$

The instantaneous curve generated by the derivative looks like

$$\frac{\mathrm{d}}{\mathrm{d}x}D(x) = c \cdot (1 - e^{-k/x} \cdot (1 + k/x)) \quad \textbf{(EQ 10-3)}$$

Adding a growth term for x and we can get a family of curves for the derivative: I generated this set of curves simply by applying growth terms of various powers, such as quadratic, cubic, etc, to replace x. No bones about it, I could have just as easily applied a positive exponential growth term here, and the characteristic peaked curve would result, with the strength of the peak directly related to the acceleration of the exponential growth. I noted that in an earlier blog post:

> *As for as other criticisms, I suppose one could question the actual relevance of a power-law growth as a driving function. In fact the formulation described here supports other growth laws, including monotonically increasing exponential growth.*

Overall, the curves have some similarity to the Logistic sigmoid curve and its derivative, traditionally used to model the Hubbert peak. Yet it doesn't match the sigmoid precisely because the equations obviously don't match — not surprising since my model differs in its details from the Logistic heuristics. However, and it starts to get interesting at this point, I can add another level of dispersion to my model and see what happens to the result.

I originally intended for the dispersion to only apply to the variable search rates occurring over different geographic areas of the world. But I hinted that we could extend it to other stochastic variables:

> *We have much greater uncertainties in the stochastic variables in the oil discovery problem, ranging from the uncertainty in the spread of search volumes to the spread in the amount of people/corporations involved in the search itself.*

7. Trying to contradict the use of the Logistic or searching for a fundamentally correct derivation is hard because of the sunk costs relating to its popularity.

So I originally started with a spread in search rates given as an uncertainty in the searched volume swept, and locked down the total volume as the constant $k{=}L_0$. Look at the following graph, which show several parts of the integration, and you can see that the uncertainties only reflect in the growth rates and not in the sub-volumes, which shows up as a clamped-asymptote below the cumulative asymptote: I rationalized that adding uncertainty to this term would make the result more messy than I would like to see at this expository level.

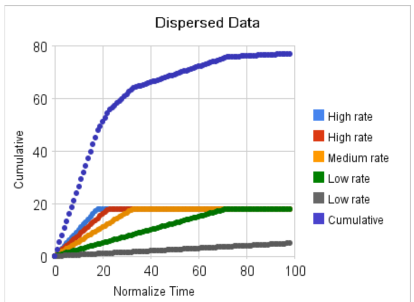


FIGURE 10-2. Piecewise approximation to dispersive growth in a linear regime. Fast growth rates lead to quick saturation while slower rates accumulate gradually leading to a long settling time before reaching an asymptote. A uniform bounding box is used as each curve reaches the same asymptote

But in retrospect, the extra step provides an intuitive result. That extra step involves a simple integration of the constant $k{=}L_0$ term as a stochastic variable over a damped exponential probability density function (PDF) given by $p(L) = e^{-L/L_0}/L_0$. This adds stochastic uncertainty to the total volume searched, or more precisely, uncertainty to the fixed sub-volumes searched, that when aggregated provides the total volume.

The following math derivation I extended from the original dispersive discovery equation explained in a previous chapter “Finding Needles in a Haystack. How we discover oil” (review this if you need motivation for the fixed size derivation). The first set of equations derives the original dispersive discovery which includes uncertainty in the search depth, while the second set of equations adds dispersion in the volume while building from the previous derivation.

Dispersive Discovery - fixed container L, dispersed depth λ

$$P(x) = \frac{1}{\lambda} \cdot e^{-x/\lambda}$$

$$\overline{D} = \int_0^{\infty} x \cdot P(x)dx = \int_0^{\infty} x \cdot \frac{1}{\lambda} \cdot e^{-x/\lambda} dx$$

$$D(\lambda, L) = \int_0^{L} x \cdot \frac{1}{\lambda} \cdot e^{-x/\lambda} dx + \int_L^{\infty} L \cdot \frac{1}{\lambda} \cdot e^{-x/\lambda} dx \qquad \text{(EQ 10-4)}$$

$$D(\lambda, L) = \lambda \cdot (1 - e^{-L/\lambda})$$

Double Dispersive Discovery - dispersed container size L_0

$$p(L) = \frac{1}{L_0} \cdot e^{-L/L_0}$$

$$DD(\lambda) = \int_0^{\infty} D(\lambda, L) \cdot p(L) dL$$

$$DD(\lambda) = \int_0^{\infty} \lambda \cdot (1 - e^{-L/\lambda}) \cdot \frac{1}{L_0} \cdot e^{-L/L_0} dL \qquad \text{(EQ 10-5)}$$

$$DD(\lambda) = \frac{1}{\frac{1}{L_0} + \frac{1}{\lambda}}$$

In the next to last relation, the addition of the second dispersion term turns into a trivial analytical integration from L=0 to $L=\infty$. The result becomes the simple relation in the last line. Depending on the type of search growth, we come up with various kinds of cumulative discovery curves.

$$\text{Power Law growth} \qquad \lambda = k \times t^{N}$$

$$\text{Exponential growth} \qquad \lambda = A \times e^{Bt} \qquad \text{(EQ 10-6)}$$

Note that the exponential term from the original dispersive discovery function disappears in DD(λ). This occurs because of dimensional analysis: the dispersed rate stochastic variable in the denominator has an exponential PDF and the dispersed volume in the numerator has an exponential PDF; these essentially cancel each other after each gets integrated over the stochastic range. In any case, the simple relationship that this gives, when inserted with an exponential growth term such as $A \cdot e^{B \cdot t}$, results in what looks exactly like the logistic sigmoid function, shown below with the label exponential discovery:

$$\text{Power Law discovery} \qquad DD(t) = \frac{1}{\frac{1}{L_0} + \frac{1}{k \times t^N}}$$

$$\text{Exponential discovery} \qquad DD(t) = \frac{1}{\frac{1}{L_0} + \frac{1}{A \times e^{Bt}}} \tag{EQ 10-7}$$

The last equation essentially describes the complete derivation of a discovery logistic curve in terms of exponential growth and dispersed parameters. By adding an additional stochastic element to the Dispersive Discovery model, the logistic has now transformed from a cheap heuristic into a model result. The fact that it builds on the first-principles of the Dispersive Discovery model gives us a deeper understanding of its origins. So whenever we see the logistic sigmoid used in a fit of the Hubbert curve we know that several preconditions must exist:

1. It models a discovery profile.
2. The search rates are dispersed via an exponential PDF
3. The searched volume is dispersed via an exponential PDF
4. The growth rate follows a positive exponential.

This finding now precludes other meaningless explanations for the Logistic curve's origin, including birth-death models, predator-prey models, and other *ad-hoc* carrying capacity derivations that other fields of scientific study have traditionally incorporated into their temporal dynamics theory. None of that matters, as the Logistic — in terms of oil discovery — simply models the stochastic effects of randomly searching an uncertain volume given an exponentially increasing average search rate. As an aside, you have to remember that Verhulst did not have the benefit of modern probability theory and the use of stochastic processes in the early 1800's, and came up with a very deterministic view of his subject matter.[8]

In the end, intuitive understanding plays an important role in setting up the initial premise, and the math has served as a formal verification of the understanding. You have to shoot holes in the probability theory to counter the argument, which any good debunking needs to do. As a very intriguing corollary to this finding, the fact that we can use a Logistic to model discovery means that we cannot use *only* a Logistic to model production. I have no qualms with this turn of events as production comes about as a result of applying the Oil Shock model to discoveries, and

8. As a matter of fact, the theory and application of stochastic processes only became popularized to Western audiences in the mid-20th century (with classical English books on the subject by Feller and Doob appearing in the 1950's) and for someone like Hubbert to make the connection would in retrospect have seemed very prescient on his part.

this essentially shifts the discovery curve to the right in the timeline while maintaining most of its basic shape[9]. In spite of such a surprising model reduction to the sigmoid, we can continue to use the Dispersive Discovery in its more general form to understand a variety of parametric growth models, which means that we should remember that the Logistic manifests itself from a specific instantiation of dispersive discovery. But this specific derivation might just close the book on why the Logistic works at all. It also supports the unification between the Shock Model and the Logistic Model that Foucher has demonstrated {Ref 23].

Linearization

A different question to ask: Does the exponential-growth double dispersive discovery curve (the "logistic") work better than the power-law variation? I find it interesting that the power law discovery curve does not linearize in the manner of Hubbert Linearization. Instead it generates the following quasi-linearization, where *n* becomes the power in the power-law curve:

$$\frac{\mathrm{d}U}{\mathrm{d}t} / U = \frac{n}{t} \cdot \left(1 - \frac{U}{URR}\right) \qquad \textbf{(EQ 10-8)}$$

Note that the hyperbolic factor (leading $1/t$ term) creates a spike near the U=0 origin, quite in keeping with many of the empirical HL observations of oil production {Ref 56]. Again, I don't think anyone has effectively explained the hyperbolic divergence typically observed. Although not intended as a perfect fit to the data, the following figure shows how power-law discovery modulates the linear curve to potentially provide a more realistic explanation of the dynamic behavior[10]. It also reinforces my conjecture that these mathematical identities (such as HL in particu-

9. This was demonstrated at the end of "Applying Dispersive Discovery. How discovery affects production"
10. The power law seems to explain production better over the entire timeline of USA production, see "Production as Discovery?"

lar) add very little intuitive value to the derivation of the models — they simply represent tautological equivalences to the fundamental equations.

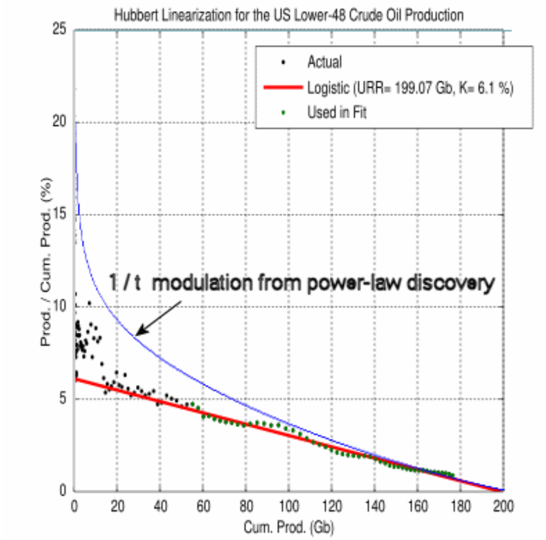


FIGURE 10-3. Deviations from linearized behavior in the HL regime can arise from power-law discovery

As another corollary, given the result:

$$D(x) = \frac{1}{\left(\frac{1}{L_0} + \frac{1}{x}\right)} \quad \textbf{(EQ 10-9)}$$

we can verify another type of Hubbert Linearization. Consider that the parameter $\boldsymbol{x}$ describes a constant growth situation. If we can plot cumulative discovered volume ($\boldsymbol{D}$) against cumulative discoveries or depth ($\boldsymbol{x}$), we should confirm the creaming curve heuristic. In other words, the factor $\boldsymbol{L_0}$ should remain invariant allowing us to linear regress a good estimate of ultimate volume:

$$L_0 = \frac{1}{\left(\frac{1}{D} - \frac{1}{x}\right)} \quad \textbf{(EQ 10-10)}$$

It looks like this will arguably fit curves better than with most other techniques. We used this relation in the chapter on Reserve Growth to “straighten out” hyperbolic reserve growth profiles as another form of linearization. In that case the linearization derived from an arbitrary power-law growth assumption, for exponential growth the linearization similarly becomes simple in algebraic terms. Importantly this establishes the equivalence of hyperbolic growth with the generalized form of dispersive discovery.

$$g(s) = \int_0^{\infty} f(t)e^{-st}dt$$

FIGURE 10-4. Pierre-Simon Laplace and his transform

General Dispersive Discovery and The Laplace Transform

Intriguingly, much of the mathematics of depletion modeling arises from considerations of basic time-series analysis coupled with useful transforms from signal processing. As a case in point, Foucher has postulated how the idea of loglet theory [Ref 57] fits into multi-peak production profiles, which have a close relationship to the practical wavelet theory of signal processing. Similarly, the Oil Shock Model uses the convolution of simple data flow transfer functions that we can also express as cascading infinite impulse response filters acting on a stimulated discovery profile. This enables one to use basic time series techniques to potentially extrapolate future oil production levels, in particular using reserve growth models ala Foucher's Hybrid Shock Model (HSM) or the maturation phase Dispersive Discovery.[11]

In keeping with this tradition, it turns out that the generalized Dispersive Discovery model fits into a classic canonical mathematical form that makes it very accessible to all sorts of additional time-series and spatial analysis. Actually the transform has existed for a very long while — it t turns into the Laplace transform of the underling container volume density; this becomes the aforementioned classic form familiar to many an engineer and scientist. The various densities include an exponential damping (e.g. more finds near the surface), a point value (corresponding to a seam at a finite depth), a uniform density abruptly ending at a fixed depth, and combinations of the above.

The following derivation goes through the steps in casting the dispersive discovery equations into a Laplace transform. The ***s*** variable in Laplace parlance takes the form of the reciprocal of the dispersed depth, $1/\lambda$.

Laplace Derivation. The basic idea behind dispersive discovery assumes that we search through the probability space of container densities, and accumulate discoveries proportional to the total size searched (see Figure 10-11 on page 192). The search depths themselves get dispersed so that values exceeding the cross-section of the container density random variable *x* with the largest of the search variables *h* getting weighted as a potential find. In terms of the math, this shows up as a conditional probability in the 3rd equation, and due to the simplification of the inner integral, it turns into a Laplace transform as shown in the 4th equation.

11. We can represent the Oil Shock Model as a statistical set of "shocklets" to aid in unifying with the loglet and HSM and DD approaches (see later)

General Dispersive Discovery - container L, dispersed depth λ

$$p(h,\lambda) = \frac{1}{\lambda} \cdot e^{-h/\lambda}$$

$$P(x|\lambda) = \int_{(h=x)}^{\infty} p(h,\lambda)dh = \int_{x}^{\infty} \frac{1}{\lambda} \cdot e^{-h/\lambda} dh$$

$$\bar{U}(\lambda, L) = \int_{0}^{\infty} f(x,L) \cdot P(x|\lambda)dx = \int_{0}^{\infty} f(x,L) \cdot \left(\int_{x}^{\infty} \frac{1}{\lambda} \cdot e^{-h/\lambda} dh \right) dx \qquad \textbf{(EQ 10-11)}$$

$$\bar{U}(\lambda, L) = \int_{0}^{\infty} f(x,L) \cdot e^{-x/\lambda} dx \qquad \text{—}LaplaceTransform$$

And the supplemental set provides a partial set of instantiations:

General Dispersive Discovery - container L, dispersed depth λ

$$\begin{bmatrix} \text{if exponential container} \quad f(x, L_0) = \dfrac{1}{L_0} \cdot e^{-x/L_0} \\ \bar{U}(\lambda, L_0) = \dfrac{1}{1 + \dfrac{L_0}{\lambda}} \end{bmatrix} \qquad \textbf{(EQ 10-12)}$$

$$\begin{bmatrix} \text{if point container} \quad f(x, L_0) = \delta(x - L_0) \\ \bar{U}(\lambda, L_0) = e^{-L_0/\lambda} \end{bmatrix}$$

$$\begin{bmatrix} \text{if uniform container} \quad f(x, L_0) = (u(x) - u(x - L_0))/L_0 \\ \bar{U}(\lambda, L_0) = \lambda \cdot (1 - e^{-L_0/\lambda})/L_0 \end{bmatrix}$$

The simplification starts when we realize that the container function *f(x)* becomes the target of the Laplace transform. Hence, for any *f(x)* that we can dream up, we can short-circuit much of the additional heavy-duty math derivation by checking first to see if we can find an entry in any of the commonly available Laplace transform tables.

In the square bracketed terms shown after the derivation in Equation 10-12, I provided a few selected transforms giving a range of shapes for the cumulative discovery function, $\bar{U}$. Remember that we still need to substitute the λ term with a realistic time dependent form. In the case of substituting an exponential growth term for an exponentially distributed container, $\lambda \approx e^{kt}$, the first example turns directly into the legendary Logistic sigmoid function that we derived and demonstrated previously.

The second example provides some needed intuition how this all works out. A point container describes something akin to a seam of oil found at a finite depth L_0 below the surface.[12] Note that it takes much longer for the dispersive search to probabilistically "reach" this quantity of oil as illustrated in the following figure. Only an infinitesimal fraction of the fast dispersive searches will reach this point initially as it takes a while for the bulk of the searches to approach the average depth of the seam. I find it intriguing how the math reveals the probability aspects so clearly while we need much hand-waving and subjective reasoning to convince a lay-person that this type of behavior could actually occur.

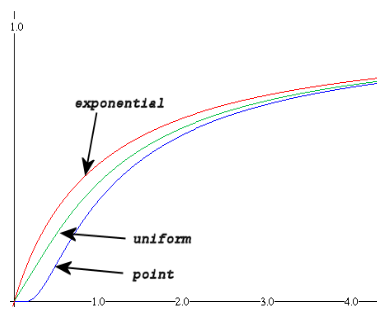


FIGURE 10-5. Cumulative discoveries for different container density distributions analytically calculated from their corresponding Laplace transforms. The curves as plotted assume a constant search rate. An accelerating search rate will make each of the curves more closely resemble the classic S-shaped cumulative growth curve. For an exponentially increasing average search rate, the curve in red (labeled exponential) will actually transform directly into the Logistic Sigmoid curve — in other words, the classic Hubbert curve.

The 3rd example describes the original motivator for the Dispersive Discovery model, that of a rectangular or uniform density. I used the classical engineering unit-step impulse function *u(x)* to describe th7e rectangular density. As a sanity check, the lookup in the Laplace transform table matches exactly what I derived previously in a non-generalized form, i.e. without the benefit of the transform.

Foucher also suggests that an oil window "sweet spot" likely exists in the real world, which would correspond to a container density function somewhere in between the "seam" container and the other two examples. I suggest two alterna-

12. I use depth and volume interchangeably for describing the spatial density. Instead of using depth with a one-dimensional search space, essentially the same result applies if we consider a container volume with the search space emanating in 3 dimensions (see Figure 7-1 on page 110). The extra 2 dimensions essentially reinforce the dispersion effects, so that the qualitative and quantitative results remain the same with the appropriate scaling effects. I fall back on the traditional group theory argument at this stage to avoid unnecessarily complicating the derivation.

tives that would work (and would conveniently provide straightforward analytical Laplace transforms). The first would involve a more narrow uniform distribution that would look similar to the third transform. The second would use a higher order exponential, such as a gamma density that would appear similar to the first transform example.

$$\frac{1}{(s+\alpha)^{n+1}} \qquad \textbf{(EQ 10-13)}$$

Interestingly, this function, under an exponentially increasing search rate will look like a Logistic sigmoid cumulative raised to the n^{th} power, where n gives the order of the gamma density.

The following figures represent some substantiation for the "sweet spot" theory as it plots Hubbert's original discovery versus cumulative footage chart against one possible distribution — essentially the Laplace Transform of a Gamma of order-2.

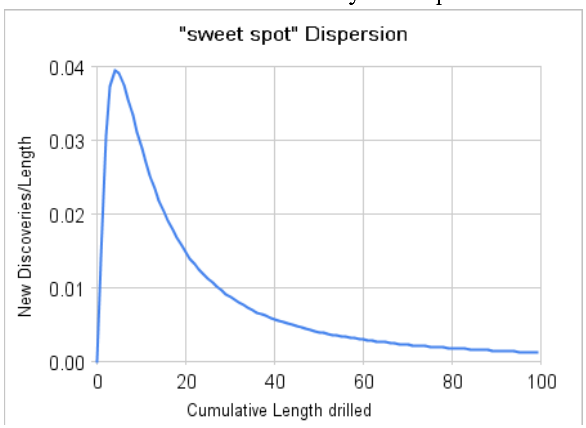


FIGURE 10-6. Derivative of the oil window "sweet spot" Laplace transform.

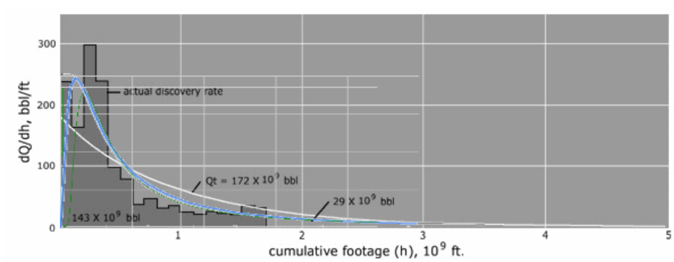


FIGURE 10-7. Eyeball fit to Hubbert's cumulative footage data.

The following scatter plots demonstrate how we can visualize the potential discovery densities. Each one of the densities gets represented by a Monte Carlo simulation of randomized discovery locations. Each dot represents a discovery.

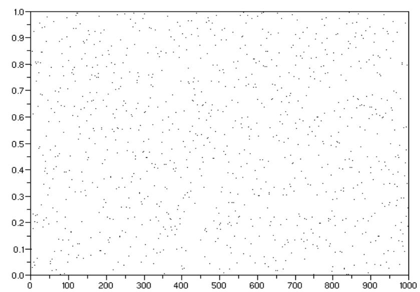


FIGURE 10-8. A uniform density of potential discoveries over a finite volume gives a normalized average value of 0.5. This distribution was the impetus for the original Dispersive Discovery model.

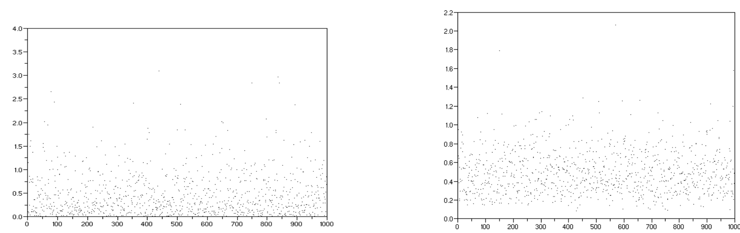


FIGURE 10-9. (left) A damped exponential density of potential discoveries over a finite volume gives a normalized average value of 0.5. When combined with an exponentially accelerating dispersive search rate, this will result in the Logistic Sigmoid curve. (right) A gamma order-5 density of potential discoveries over a finite volume narrows the spread around 0.5

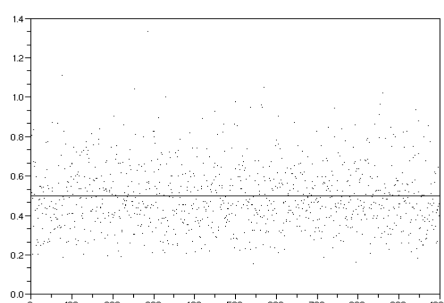


FIGURE 10-10. A gamma order-10 density of potential discoveries over a finite volume further narrows the spread around 0.5. At the limit of even high orders, the density approaches that of the “seam” shown as the solid line drawn at 0.5.

An interesting by-product results independently of the use of any of the distributions. It turns out that the tails of the instantaneous discovery rates (i.e. the first derivative of the cumulative discovery) essentially converge to the same asymptote as shown in Figure 10-11 on page 192. This has to do more with the much stronger dispersion effect than that of the particular container density function.

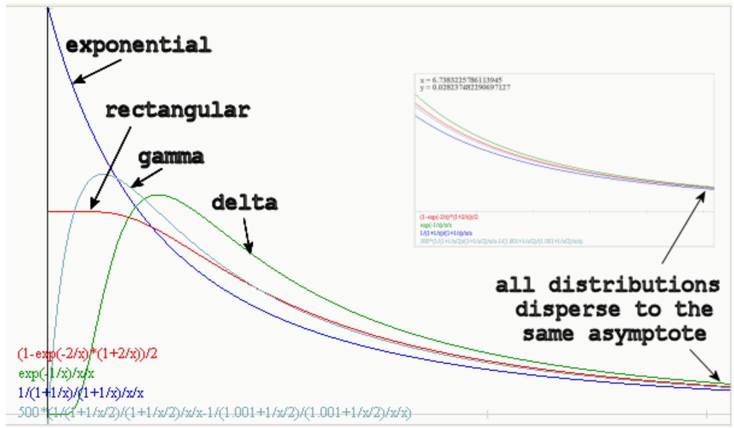


FIGURE 10-11. The set of first derivatives of the Laplace Transforms for various container density functions. Note that for larger dispersed depths (or volumes) that the tails tend to converge to a common asymptote. This implies that the backsides of the peak will generally look alike for a given accelerating search function.

In summary, using the Laplace Transform technique for analyzing the Dispersive Discovery model works in much the same way as it does in other engineering fields. It essentially provides a widely used toolbox that simplifies much of the heavy-lifting analytical work. It also provides some insight to those analysts that can think in terms of the mentally-challenging reciprocal space. Indeed, if one ponders why this particular model has take this long to emerge (recall that it does derive the Hubbert Logistic model from first principles and it also explains the enigma of reserve growth exceedingly well), you can almost infer that it probably has to do with the outside-the-box mathematical foundation it stems from.

Scaling and the Dispersive Discovery Growth Function

One search growth function I use for the Dispersive Discovery model follows a T^6 time dependence. The derivation comes from a quadratic growth term on top of a single dimension of volume. When the quadratic gets multiplied along the three dimensions of volume, the T^6 dependence results.

High-order growth terms such as T^6 have some similarity to exponential growth terms as a particular order in the Taylor's series polynomial expansion dominates over a certain interval. The following chart shows the cumulative dispersive discovery using T^6 plotted alongside an e^{kT} growth term inserted into the Dispersive Discovery equation. I normalized the two curves via an affine transformation so they intersect at T=1.

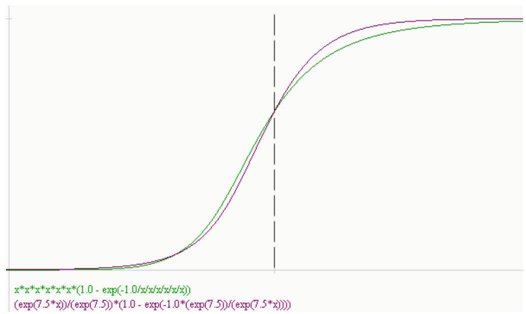


FIGURE 10-12. Comparison between a power law dispersive discovery growth and an exponential growth variant. The power law shows more gradual growth for large times.

Note that the doubling time for the exponential occurs at about 10% of T at T=1, which roughly coincides to the doubling time for the T^6 growth.

For world crude oil discoveries, the T=1 time point scales to approximately 100 years (the time period from 1858 to the early 1960's when we observed a global peak). This means that the discovery growth doubling time equated to roughly 10 years in historical terms — premised on that you believe the Dispersive Discovery model applies. If you look closely at the two curves beyond T=1, the exponential reaches the asymptote much more quickly than the T^6 growth curve. This makes perfect sense as the higher order polynomial terms in the Taylor's expansion of the exponential take over, and push to the asymptote more quickly, and thus minimizing the effects of dispersion.

Some might find the exponential growth model more understandable or intuitive, as this emulates technological advances such as those described by Moore's law (i.e. which shows doubling of microprocessor speed every two years), or approximates population growth and the demand and acceleration in prospecting effort that this implies.

Whether the exponential growth actually provides a more realistic picture of the dynamics, I can't say but know for certain that it requires a much stronger growth stimulus — thus implying that a doubling of search effort must occur every 10 years for the foreseeable future. On the other hand, a high-order function such as T^6, though it continues to accelerate, will show progressively longer doubling periods as T increases.

So what happens to the oil prospecting effort as we start hitting the walls remains unknown. In any event, it definitely will pay to start comparing and contrasting the exponential growth model in conjunction with the T^6 growth term. As the two complementary cumulative dispersive discovery curves don't show a significant amount of qualitative difference, while the underlying model shows a certain amount of predictability in terms of parametric variation. In particular, the exponential provides a good way of calculating differential margins should we want to assume a stronger post-peak discovery search pressure. While the T^6 variation will show weaker but longer-lasting tails.

The key to understanding how search occurs is to realize that much of it has become a "virtual" or "indirect" search through various seismic measurements as well as supercomputer simulations and advanced visualizations. Much of the recent accelerating search has progressed through advances in technologies[13] I would think that the seismic has covered a huge portion of the volume so far, with the slower areas bringing up the rear such as deep sea (i.e. the tails of the Dispersive Discovery model). Unfortunately, I expect that exact information won't be forthcoming as every oil exploration company would consider the proprietary data to serve them a competitive advantage.

From: stephen hubbard
Subject: Gestimate annual area searched for oil?

Hi, I was reading your comments on "Predicting Future Supply from Undiscovered Oil" on TOD and was wondering if there was a way to guesstimate the area searched for oil over the years. I work for a water company in Los Angeles so I really don't have access to such information, but perhaps some general relation can be developed to show the generally declining relationship between area searched and oil found. I know folks use rigs in service for drilling, is there industry data on search systems sold (land, airborne, ocean, etc.). If one assumed a set duty rate and scrapping rate, then with a guesstimate of the sweep/reach of such systems, one might be able to estimate the area searched.

Sincerely,
Pragmatic (Stephen Hubbard)

My Reply: I think your idea of indirectly monitoring usage of discovery equipment might tell us something. On the downside of this is that the DD model assumes ever increasing acceleration of search and if this were ever to slow down or stop, the model would become even more pessimistic.

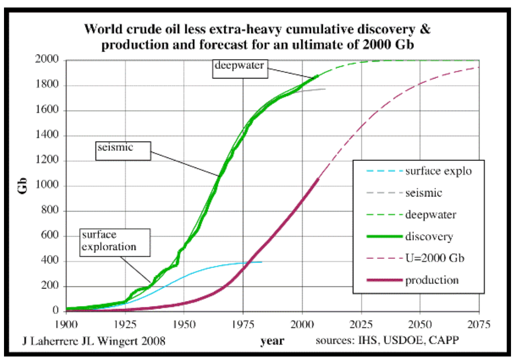


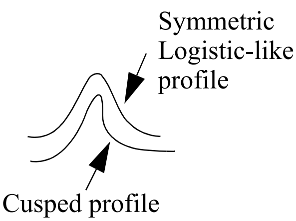
FIGURE 10-13. Laherrere cumulative discovery curve alongside shifted production. Seismic exploration provided a technology-assisted acceleration to the search space.

Ultimately, if this acceleration should eventually cease, we would see the effects soon. When it comes to oil exploration, I believe we have benefited from exponential (or power law) increases due to technology advancement. Initially, the search process methodically improved through lots of human labor. The first acceleration came about by huge influxes of prospectors who also brought in new exploration ideas. Eventually the industry went to seismic and then on to supercomputer simulations and visualization techniques. This supports the idea of a virtual search that was able to cut through huge swaths of the earth's crust. Each technology improve-

13. Look at this comment: http://www.theoildrum.com/node/4785#comment-438075 and this figure in particular http://www.theoildrum.com/files/lhcumdiscoveries200810.gif

ment and the trained people involved improved the search speed by perhaps an order of magnitude.

The entire derivation of Dispersive Discovery model and the special reduction to the Logistic function results from this exponential assumption. Interestingly, if the accelerating search suddenly went away, the discoveries would plummet much more quickly because of diminishing returns, resulting in a cusped profile[14]. We will still have a long tail in this case as the discoveries still occur but the rate reverts back to a reduced pace of past years.



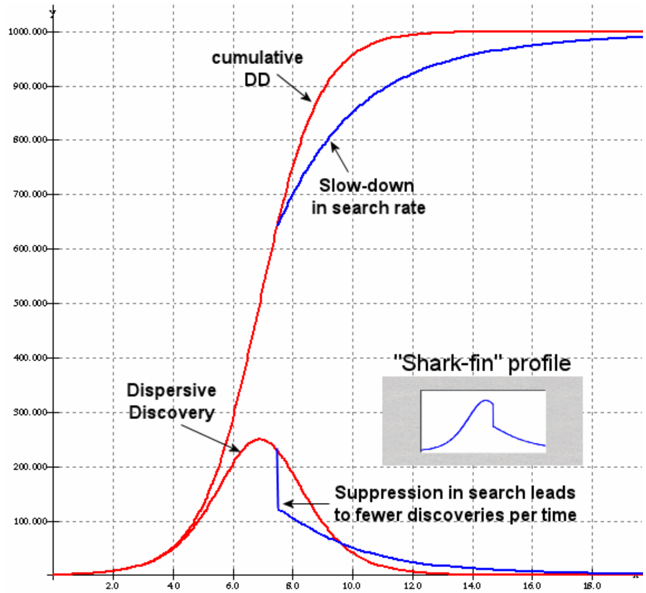


FIGURE 10-14. A sudden deceleration in discovery search speed will result in a finned or "shark fin" profile characterized by a drop in discoveries in a regime of diminishing returns. Long tails will remain if some discovery effort is maintained.

If we look at the trends of exponential discovery versus power-law discovery, we also see the effects of diminution of search effort. In the following figure note that power-law discovery will not hit as sharp a peak yet will show longer tails in the regime of diminishing resources. It all comes down to a fight between accelerating effort and rapidly diminishing resources.

14. Some also call this a "shark-fin" profile, but sharks display a variety of dorsal shapes.

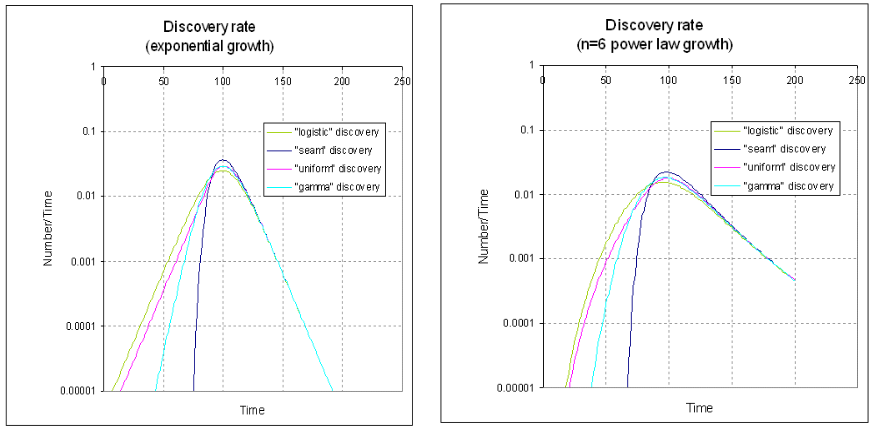


FIGURE 10-15. Dispersive Discovery curves for exponential growth search rates (left) and Dispersive Discovery curves for power-law growth search rates (right)

Discovery models have utility for situations where you lack much of the prior data. Through the process of back-extrapolation, one can fill in missing discovery data points and then apply a production model to gain insight into historical oil production. The following figure uses an older very contrived (i.e. non-dispersive) deterministic discovery model to estimate the historical trending of the observed data.

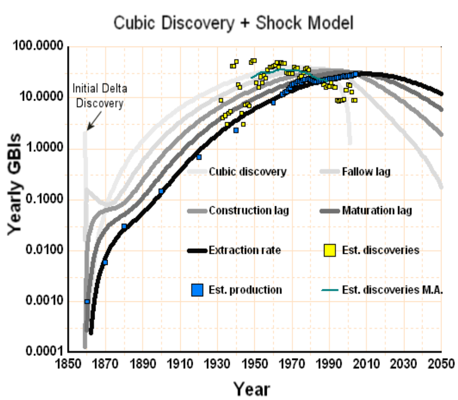


FIGURE 10-16. Global oil production appears time-shifted from discoveries when convolved with the oil shock model. To model the global oil production curve, values of 12.5 years for each of the shock model lags causes progressively deeply shaded curves from discovery to production. This uses the cubic growth/feedback (i.e. non-dispersive) discovery model to extrapolate back to early discovery

Placing the original discovery stimulus at year 1858 pinned the curve. If this deterministic stimulus did not exist the oil production profile would not match the model, which naively would suggest a starting value of 0 barrels/year. This adds an extra variable to the model, but it gives some insight as to how the discovery growth likely played out in the early years, not having any real good discovery data before 1930. Note that removing the determinism on the discoveries (i.e. using dispersion) will extend the tail beyond peak.

Removing the Dispersion

More fundamentally, what exactly happens if we remove the dispersion in search rates? In that case, we have a single accelerating search rate matched against a damped exponential volume density. Essentially the solution removes the dispersion from Equation 10-4 and simplifies to this:

$$\textit{Non}\text{Dispersive Discovery - dispersed container size L}_0$$

$$p(L) = \frac{1}{L_0} \cdot e^{-L/L_0}$$

$$ND(\lambda) = \int_0^{\infty} D(\lambda, L) \cdot p(L) dL \qquad \textbf{(EQ 10-14)}$$

$$ND(\lambda) = \int_0^{\infty} (u(\lambda) - u(\lambda - L)) \cdot \frac{1}{L_0} \cdot e^{-L/L_0} dL$$

$$ND(\lambda) = 1 - e^{-\lambda/L_0}$$

This states that as long as the depth random variable λ lies within the volume density of a given probability (denoted by the enclosed unit step function *u*), then that will contribute as a discovery. The non-accelerated cumulative solution appears as a damped exponential, with a characteristic length of L_0. As this solution does not derive from a dispersive search term, intuitively we can eliminate the possibility of fat tails, and sure enough, the damped exponential has much thinner tails than the slow decay described by either Equation 10-4 or Equation 10-5.

If we then apply an accelerating search rate to λ, such that:

$$\lambda = ae^{bt} \qquad \textbf{(EQ 10-15)}$$

we get a behavior consistent with the Gompertz equation that we will further describe in Chapter 16.

$$Gompertz(t) = 1 - e^{-(ae^{bt})/L_0} \qquad \textbf{(EQ 10-16)}$$

This acts like a double exponential in that as we race through an accelerating search, we start reach exponentially diminishing returns. The two exponentials reinforce each other and produce a fast diminution in discoveries once we near the tails of the search volume. Such a behavior contrasts to dispersive search in which the slower dispersed search rates in unexplored regions *compensate* for the faster search rates, thus smearing out the discovery peak.

The Gompertz curve shows the following behavior

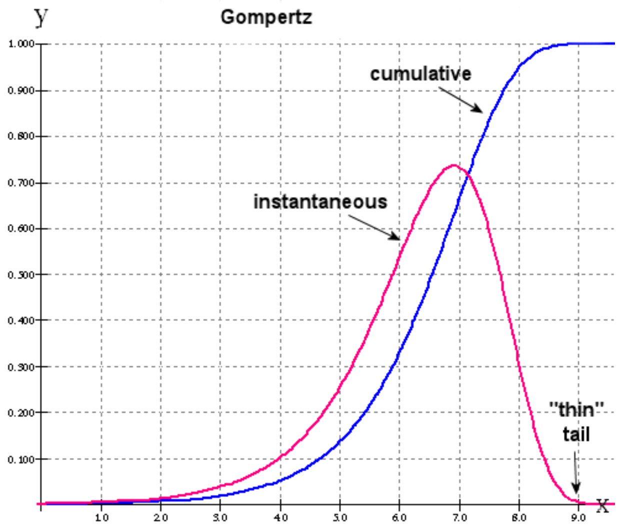
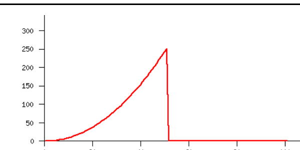
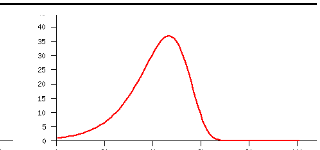
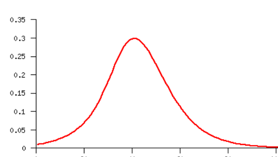
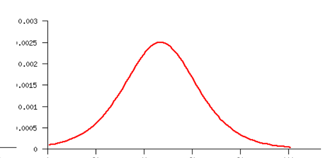


FIGURE 10-17. The Gompertz curve shows a rapid decline as it nears volume constraints, in contrast to dispersive discovery.

If we require fat tails, we need to invoke dispersion in search rates. On the other hand, if we can confine the resources enough so that we can control the dispersion to within a narrow window, then we can potentially apply the Gompertz. We will discuss if or when we should apply the Gompertz.This has huge implications on whether we will observe a crash or "cliff-like" behavior in discovery of resources. (See "Why We Can't Pump Faster" on page 338.)

TABLE 1. Depletion profiles for dispersive forms forming a 2x2 matrix.

	No dispersion in volume	Dispersion in volume
No dispersion in rates. Each curve has exponentially accelerating rate.	 Accelerating growth hits limits	 Gompertz-like behavior
Dispersion in rates. Each curve has exponentially accelerating *mean* rate. Standard deviation = mean.	 Slow rates create longer tails	 Symmetric Logistic curve

CHAPTER 11

The Oil Production Process.

How do we verify the extraction model?

"It is difficult to exaggerate the importance of convolutions in many branches of mathematics."

— William Feller,
"An Introduction to Probability Theory and Its Applications"

Shocklets

Decomposing Depletion to Individual Regions

The Oil Shock Model uses a simple rather unbiased multiplier to estimate the oil production response to a discovery input. To demonstrate this, let us take a particular sub-case of the model. If we assume that a discovery immediately becomes extractable, then the multiplying factor for instantaneous production becomes a percentage of what remains. This turns into a damped exponential for a delta discovery (delta meaning a discovery made at a single point in time). In practice, this means that for any particular discovery in the Oil Shock model, we immediately enter a regime of diminishing returns. You can see this in the following plot.

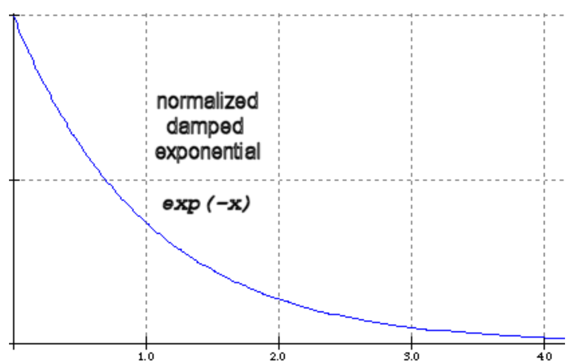


FIGURE 11-1. Plot of a normalized damped exponential. (normalized with a time constant of unity).

One could argue pragmatically that we rarely enter into an immediately diminishing return regime. In fact, we have all seen the classical regime of an oil bearing region — this often features an early constant plateau, followed by a drop-off after several years. We usually take this to mean that the oil producers deliberately decide to maintain a constant production rate until the wells no longer produce, in which case we then enter the damped downslope. Or else this could imply that the oil pressure maintains a certain level and the producers extract at the naturally established equilibrium. In fact, the reason that I chose the damped exponential for the Oil Shock model has nothing to do with the intricacies of production; instead it really has to do with the statistics of a spread or range of oil producing wells and regions. The unbiased multiplier really comes from the fact that bigger oil discoveries produce proportionately more oil than smaller oil discoveries, which naturally have less oil to offer. This model becomes nothing more or less than an unbiased statistical estimator for a collection of oil-bearing regions. In other words, the statistics of the collective reduces to a single instance of an average well if we want to think it through from a macro to a micro-perspective. So as a large discovered region starts to deplete, it tends to look statistically more and more like a small producing region, and therefore the fractional extraction estimator kicks in.

With all that said, I can come up with a simple discovery model that matches the behavior of the early plateau observations. We still assume the fractional extraction estimator, but we add in the important factor of reserve growth. Consider the following figure, which features an initial delta discovery at year 1, followed by a series of reserve growth additions of 10% of the initial value over the next 10 years. After that point the reserve growth additions essentially drop to zero.

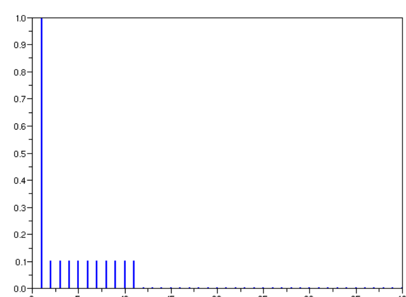


FIGURE 11-2.
Set of discrete discoveries over a period of time. The starting spike becomes the initial reserve estimate, with ensuing lower level spikes acting as reserve additions.

Next consider that the extraction essentially scales to the amount of reserve available, and so we set the extraction rate arbitrarily to 10% of the remaining reserve, calculated yearly. (The choice of 10% is critical if you do the math, explained below [1]) Therefore, for a discovery profile that looks like an initial delta function followed by a fixed duration reserve growth period, for the appropriate extraction rate we can come up with the classical micro-production profile.

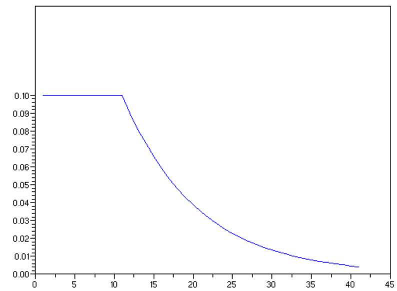


FIGURE 11-3.
Extraction proportional to the reserve amount leads to the production plateau shown. This perfectly flat plateau only occurs for a carefully selected reserve addition to extraction rate ratio.

We duly note that this micro-model approximates the classical observation of the early plateau followed by a damped exponential[2]. Not coincidentally, the plateau lasts for the same 10 years that the reserve growth takes place in. Rather obviously, we can intuit that this particular plateau maintains itself solely by reserve growth additions. So as the diminishing returns kick in from the initial delta, the reserve growth additions continuously compensate for this loss of production level, as long as the reserve growth maintains itself. After this, the diminishing returns factor eats

1. The solution to the delta plus finite constant reserve shock model is this set of equations:

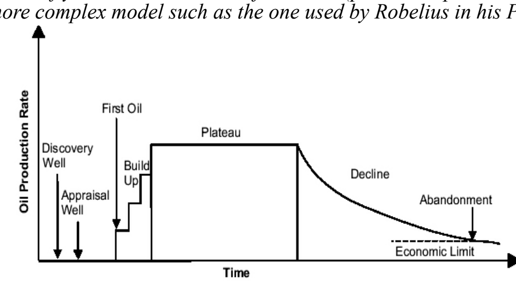
$$P_0(t) = ke^{-kt} \text{ — production from initial delta, for all } t \quad \textbf{(EQ 1)}$$

$$P_1(t) = C(1-e^{-kt}) \text{ — production from reserve, for } t < \mathrm{T} \quad \textbf{(EQ 2)}$$

$$P_2(t) = Ce^{-kt}(e^{kT}-1) \text{ — production from reserve, for t > T} \quad \textbf{(EQ 3)}$$

Where C is the magnitude of the yearly reserve growth, k is the extraction rate, and T is the duration of reserve growth. Clearly the P0 and P1 terms cancel for the choice of k=C. This will force P0+P1 to maintain a constant level for kT, the curve enters the damped exponential regime.

2. *"Now, if you have additional information (peak date, plateau duration, URR, etc.), you could use a more complex model such as the one used by Robelius in his Ph.D. thesis:*



Production figure from Robelius.

I'm currently working on a more complex model."
http://canada.theoildrum.com/node/3958#comment-373680

at whatever reserves we have left, with no additional reserve growth to compensate for it. Voila, and we have a practical model of the classical regime based on the Oil Shock model.

The troublesome feature of the classic plateau lies in its artifice. The underlying artificial discovery model consists of sharp breaks in the form of discontinuities. These manifest themselves in discontinuities in the production model. You see that in the kick-start to immediate stable production due to a delta function in discovery, and then a sharp change in slope due to the sudden end of reserve growth. In statistical terms, the discontinuities disappear when you look at an ensemble of data.

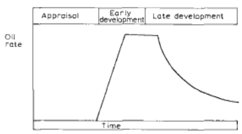


Fig. 5.68. Production profile for an offshore waterdrive field.

Production profile from Dake [Ref 172]

First of all, we know we must at least account for the build-up, or what I would call the Maturation phase in the Oil Shock model[3]. This next figure from Foucher aggregates several of these plateau-shaped production profiles from the UK and Norway (where the individual governments' requires close accounting of production levels from the field owners):

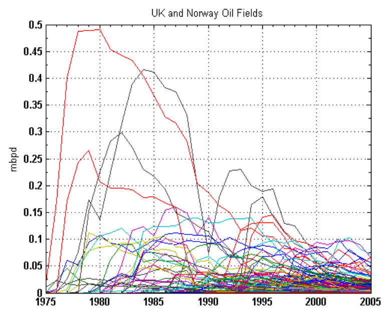


FIGURE 11-4.
Aggregated set of quasi-plateaued production profiles from the UK and Norway (from Foucher).

Even though the figure looks busy, note that almost all the profiles show the build-up phase fairly clearly. You can also observe that very few show a stable plateau, instead they mostly show a rounded peak followed by a decline. The asymmetry shows longer tails in the decline than the upward ramp during the build-up phase.

I contend that the Dispersive Discovery model of reserve growth composited with the Oil Shock model can handily generate these kinds of profiles.

The Idea of Shocklets

Foucher has investigated the idea of using loglets (similar to wavelets) in understanding and fitting to multiple-peak oil production profiles [Ref 57]. He also used characteristics of the loglet to construct the HSM (and I safely assume his more

3. The Oil Shock model also considers the Discovery phase and Construction phase but we can ignore these for the purposes of this discussion

complex model he hinted at above). As the basic premise behind these "*X*-let" transforms you find a set of sample signals that when scaled and shifted provides a match to the profile of, say, an oil production curve under examination or some other temporal wave-form. The Oil Shock Model does not differ much in this regard; this salient connection just gets buried in the details of the math[4].

So I suggest that we can visualize the characteristic oil shock model profile by constructing a set of "shocklets" based on the response from a discovery profile input stimulus. The shocklets themselves become micro-economic analogies to the macro view. The math essentially remains the same — we just use a different prism to view the underlying mechanism.

At the beginning of this discussion, we essentially verified the premise of shocklets by mimicing the plateau regime via a simple discovery/reserve/extraction shock model. That gave us the classical "flat-topped" or plateaued production profile. To modulate the discontinuities and flatness, we use the technique of convolution to combine the damped exponential extraction phase with a modeled maturation phase. The basic oil shock model proposed a simple maturation model that featured a damped exponential density of times; this described the situation of frequent fast maturities with a tail distribution of slower times over a range of production regions.

The exponential with exponential convolution gives the following "shocklet" (with both maturation and extraction characteristic times set to 10 years). Note the build-up phase generated by the maturation element of the model, with very little indication of an extended plateau[5].

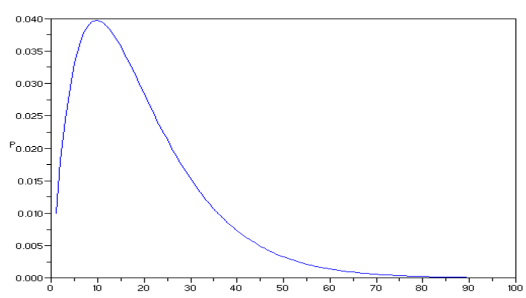


FIGURE 11-5. Plot of a "shocklet" with maturation and extraction characteristic time set to 10 years

4. Shifting, scaling, and then accumulating many of these sampled waveforms in certain cases emulates the concept of convolution against an input stimulus. For a discovery stimulus, this relates directly to the Oil Shock Model.
5. Bentley draws a heuristic with a similar shape without providing a basis for its formulation, see the figure to the side [Ref 58]

Now, with our previously established reserve growth model, we can replace the maturation curve with the empirically established reserve growth curve. I make the equivalence between a maturation process and reserve growth additions simply because I contend that extraction decisions primarily derive from how much reserve the oil producers think lies under the ground — other maturation effects we can estimate as second-order effects. This essentially makes the connection to and unifies with Foucher's deconvolution approach from backdated discoveries, where he applies Arrington's reserve growth heuristic to the oil shock model and its hybrid HSM [Ref 23]. We use the best reserve growth model that we have available, because this provides the most accurate extrapolation for future production.

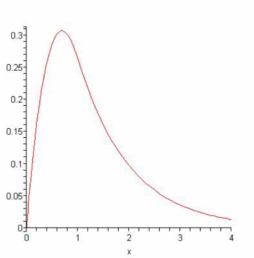


FIGURE 11-6. Bentley's heuristic [Ref 58]

We start with a general (non-cumulative) reserve growth curve $L^2/(L + Time)^2$ derived from the Dispersive Discovery model. The following figure looks much like an exponential (the characteristic time for this lasts 10 years) but the DD reserve growth has a sharper initial peak and a thicker longer tail. Compare this to the artificially finite reserve growth profile used to generate the idealized plateau production profile in Figure 11-1 on page 203.

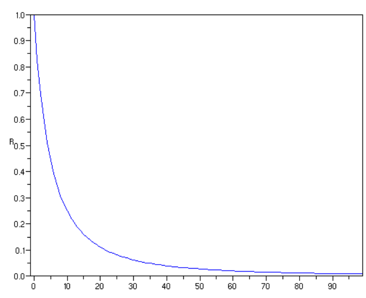


FIGURE 11-7. General reserve growth curve with characteristic time of 10 years

The shocklet for the DD reserve growth model looks like the following profile in the figure below. Note the build-up time roughly equates with the exponential maturation version, but the realistic reserve growth model gives a much thicker tail. This matches expectations for oil production in places such as the USA lower-48 where regions have longer lifetimes, at least partially explained by the "enigmatic" reserve growth empirically observed through the years. The lack of a long flat plateau essentially occurs due to the dynamics of reserve growth; nature rarely compensates a diminishing return with a precisely balanced and equivalent reserve growth addition. And this matches many of the empirically observed production profiles. The perfectly flat plateau does exist in the real world but the frequent observation of a reserve growth shocklet shape makes it much more useful for general modeling and simulation. Furthermore, the two parameters for characterizing the shape, i.e. an extraction rate and a reserve growth time constant, makes it ultimately very simple and compact as well.

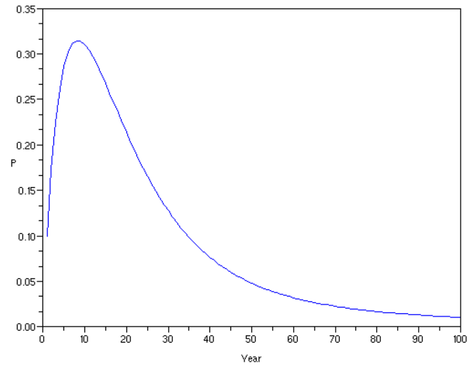


FIGURE 11-8. Extraction rate applied to the general reserve growth curve. Note that a flat plateau does not arise.

Shocklet Envelopes

Plotting a shocklet as a "cumulative *vs.* production" instead of against time allows one to infer asymptotic properties and generate an envelope. If we use a damped exponential maturation period we generate the family of curves in the following figure. Note the straight line envelope characteristic of exponential decline.

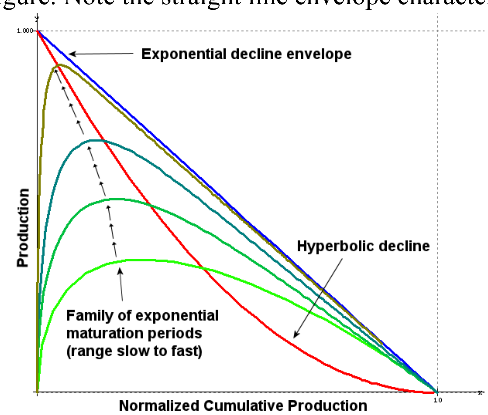


FIGURE 11-9. Using a damped exponential maturation, a family of shocklets follows an exponential decline envelope.

In the figure below we also show a hyperbolic decline envelope, a characteristic often observed in fields that show a decline rate that decreases as the production rate decreases [Ref 168], also known as Arp's hyperbolic decline. On the cumulative versus production plot, this shows up as a concave-up envelope. Interestingly, this same hyperbolic envelope occurs with the dispersive maturation shocklet. As you can see in the following figure, the creeping reserve growth makes the decline rate appear to slow down and the envelope shows the same concave-up character. This occurs because the rate of reserve growth essentially becomes the limiting factor as it continually shifts from one decline regime to another [Ref 170]. Should the reserve growth cease, then the envelope would switch back to an exponential decline.

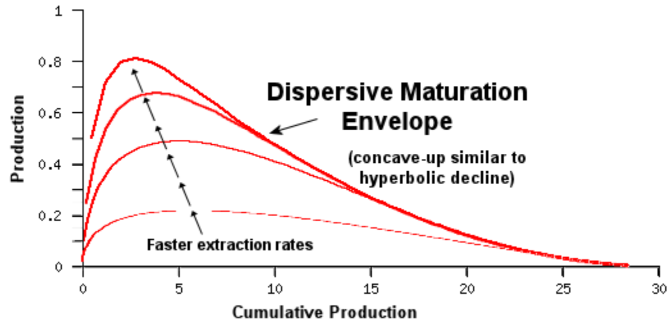


FIGURE 11-10. A family of dispersive maturation shocklets show a characteristic decline envelope.

This provides a nice intuitive model for how a hyperbolic decline can come about. Given this explanation, I don't think anyone should treat the classical heuristic derivation of the hyperbolic decline seriously, as it adds a "fudge" factor to the exponential decline extraction rate — i.e if R_0 = average extraction rate, then it slows it down with time according to $R = R_0/(1 + bR_0t)$. This factor allows one to analytically solve the modified differential equation, which petroleum reservoir engineers must find valuable as a heuristic[6]. Yet, it doesn't produce the rounded peak that normally occurs near the start of production — a pure hyperbolic with a maximum at zero unfortunately makes it less useful as a way to understand the common depletion dynamics (we will show this in the next section as we compare to data from the UK North Sea).

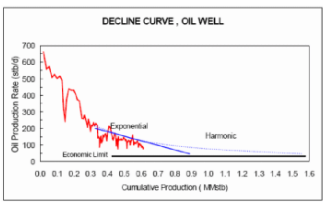


FIGURE 11-11. Hyperbolic decline lies between exponential decline and the infinite URR harmonic decline. At some point, you reach an economic limit of diminishing returns [Ref 179]

In general, the mantra of the Oil Shock Model continues to hold firm — every year we always extract a fraction of what we think lies underground. The role of reserve growth acts to provide a long-term incentive to keep pumping the oil out of certain regions. As the estimates for additional reserve keep creeping up over time, a fraction of the oil consistently remains available. And by introducing the concept of shocklets, we essentially provide a different perspective on the utility of the Oil Shock Model.

Shocklets in Action

I coined the term shocklet to describe the statistically averaged production response to an oil discovery. This building block kernel allows one to deconstruct the macroscopic aggregation of all production responses into a representative sample for a

6. In Volume 2, I explain this as the solution to a dispersive Fokker-Planck equation for solutes travelling through a porous media.

single field. In other words, it essentially works backwards from the macroscopic to an equivalent expected value picture of the isolated microscopic case. As an analogy, it serves the same purpose of quantifying the expected miles traveled for the average person per day from the cumulative miles traveled for everyone in a larger population.

In that respect the shocklet formulation adds nothing fundamentally new to the Dispersive Discovery (DD)/Oil Shock model foundation, but does provide extra insight and perspective, and perhaps some flexibility into how and where to apply the model.

Foucher noted a resemblance in post-processed oil production data to the basic shocklet curve [Ref 13]. Of the useful data reductions from the North Sea numbers, Figure 14 in Michel's paper contained a spline fit to the aggregate of all the individual production curves, normalized to fit the production peak rate and field size (i.e. final cumulative production) into an empirically established kernel function. The red curve below traces the spline fit generated by Michel.

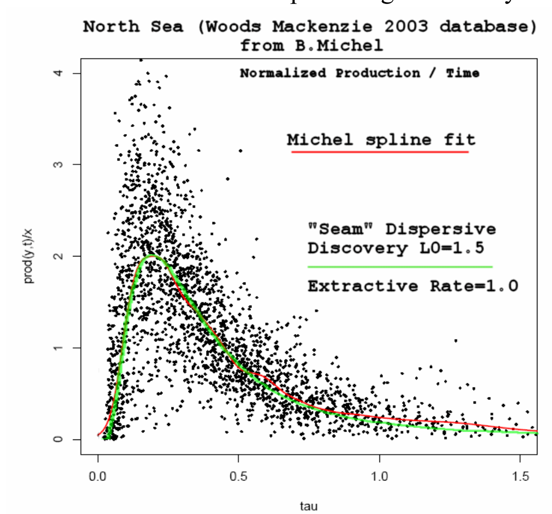


FIGURE 11-12.
Michel took production data from the UK North and normalized the lifetime of the fields to understand commonality. The dispersive discovery kernel applied to oil shock extraction (a shocklet) matches the data well.

The green curve represents the shocklet assuming a "seam" dispersive discovery profile convolved with a characteristic damped exponential extraction rate. As one of the key features of the shocklet curve, the initial convex upward cusp indicates an average transient delay in hitting the seam depth. You can also see this in the following figure below; digitized from data in Michel's chart and categorized vi a histogram averager to display the results. Unfortunately, the clump of data points near the origin did not get sufficient weighting, and so the upward inflecting cusp doesn't look as strong as it should (but more so than the moving average spline indicates, which is a drawback of the spline method). The histogram also clearly shows the noisy parts of the curve, which occur predominantly in the tail.

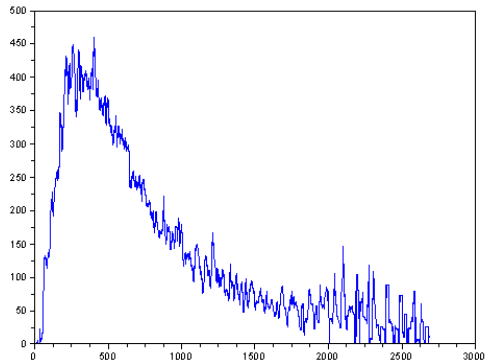


FIGURE 11-13. Time averaging of Michel's UK North Sea data.

I believe this provides an important substantiation of the DD shocklet kernel. The values for the two shocklet model parameters consist of the average time it takes to reach a seam depth and the average proportional extraction rate. Extrapolating from the normalized curve and using the scaling from Michel's figure 13, these give a value of 3.4 years for the seam DD characteristic time and 2 years for the extraction rate time constant (I assumed that Michel's Tau variable scales as approximately 20 years to one dimensionless unit according to Figure 11-12 on page 211). Note that the extraction rate of 50% per year looks much steeper than the depletion rate of between 10% and 20% quoted elsewhere because the convolution of dispersive discovery reserve growth does not compensate for the pure extraction rate; i.e. depletion rate does not equal extraction rate[7]. As an aside, until we collectively understand this distinction we run the risk of misinterpreting how fast resources get depleted, much like someone who thinks that they have a good interest-bearing account without taking into account the compensating inflationary pressure on their savings.

The tail of the shocklet curve shows some interesting characteristics. As I said earlier, the envelope of the family of DD curves tend to reach the same asymptote. For shocklets derived from DD, this tail goes as the reciprocal of first production time squared, $1/Time^2$.

Turning to Michel's paper, although I believe he did yeoman's work in his data reduction efforts, I don't agree with his mathematical premise[8]. His rigorously for-

7. Some analysts equate depletion rate with extraction rate. If both derive from instantaneous fractional rate lost from current known reserves, they should remain equivalent. The distinction in this case probably comes from the reserve growth model assumed.

mal proofs do not sway me either, since they stemmed from the same faulty initial assumptions.

1. **Field Sizes.** I contend that the distribution of field sizes generate only noise to the discovery profile (a second-order influence). Michel bases his entire analysis on this field size factor and so misses the first-order effects of the dispersive discovery factor. The North Sea in particular has a proclivity to of course favor the large and easy-to-get-to fields and therefore we should probably see those fields produced first and the smaller ones become shut-in earlier due to cost of operation. Yet we know that this does not happen in the general case; note the example of USA stripper wells that have very long lifetimes. So the assumption of an average proportional extraction rate across all discovery sizes remains a good approximation (as shown in the Figure below).

   Say we had one big reservoir of size V, or N smaller reservoirs of size v, where $V=Nv$. For exponential decline where the proportion extracted per unit time remains constant then $P=VR=NvR$. This means that at the tap you will not see much of a difference in production, where only the totality of reserves matters. Yet Michel places a large significance on the result that smaller fields contribute in a far different way than the exponential decline assumption would suggest.

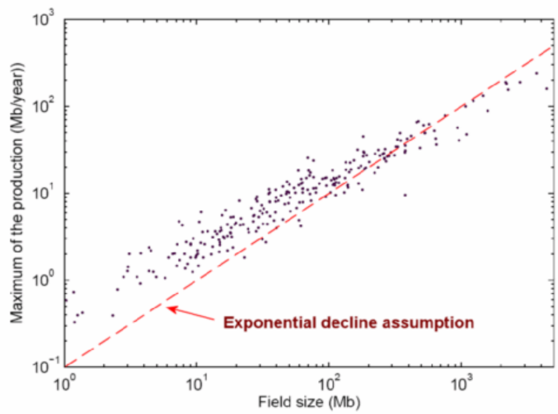


FIGURE 11-14. Max production follows closely the size of the field for North Sea data According to the scale invariant shock model it should follow according to exponential decline [Ref 13]. Note the absence of heteroscedasticity at larger field sizes.

   So the question is: Do we want to complicate the solution to account for second-order effects, and especially enforce the restriction that larger fields are always developed before smaller fields? The benefits/drawbacks on including this effect:
   • It won’t help a basic understanding that much, and adds an extra fudge factor.
   • It perhaps will improve the model fidelity and perhaps give us better predictions.

8. Although Michel had some insightful things to say, I thought he took an incorrect path by focusing on distribution of field sizes. In general, certain sections of the paper seemed to exist in an incorrectly premised mirror universe of what I view as the valid model.

• We would have to place probabilities on big fields before small fields, as exceptions would violate the strict rule. This could reduce the error in the model even more.

• If we account for the effect at all, we get a more conservative model, which becomes more conducive to fatter tails. In other words, the Michel adjustment would make a more pessimistic model for depletion as the smaller fields would deplete a bit faster. But since they don't make a huge contribution, in the long run it won't make that much of a difference to the depletion.

2. **Gamma Distribution.** This comes tantalizing close to independently reinforcing the Oil Shock model. Michel describes the launching of production as a queuing problem where individual fields get stacked up as a set of stochastic latencies. As I understand it, each field gets serviced sequentially by its desirability. Then he makes the connection to a Gamma distribution much like the Oil Shock model does in certain situations. However, he put forward an invalid premise, because the fields don't get stacked up to first-order but the production process stages do; so you have to go through the fallow, construction, maturation, and extraction processes to match reality. Remember, greed and *competition* rules over any orderly sequencing of fields put into play. The only place (besides an expensive region like the North Sea) that Michel's premise would work perhaps exists in some carefully rationed society — but not in a free-market environment where profit and competition motivates the producers.

So connecting this to a URR, we refer back to the field size argument. Michel can not place a cap on the ultimate recoveries from field size distribution alone, so ends up fitting more-or-less heuristically. If you base the cap on big fields alone, then a Black Swan event will completely subvert the scale of the curve. On the other hand, dispersive discovery accounts for this because it assumes a large searchable volume and follows the cumulative number of discoveries to establish an asymptote. Because of its conservative nature, a large Black Swan (i.e. improbable) discovery could occur in the future without statistically affecting the profile.

Besides, I would argue against Michel's maxRate/Size power-law as that convincing a tend. It looks like maxRate/Size shows about 0.1 for large fields and maybe 0.2 for fields 0.01× the size. So instead of a power of unity this varies perhaps as 0.8. The fact that the "big fields first" does not follow that strict a relationship would imply that the maxRate/Size works better by making it an invariant across the sizes. He really should have shown something like creaming curves with comparisons between "unsorted" and "sorted by size" to demonstrate the strength of that particular law. I suspect it will give second-order effects at best if the big/small variates are more randomized in time-order (especially in places other than the North Sea). The following figure shows a North Sea creaming curve. I have suggested earlier that cumulative number of wildcats tracks the dispersive discovery parameter of search depth or volume.

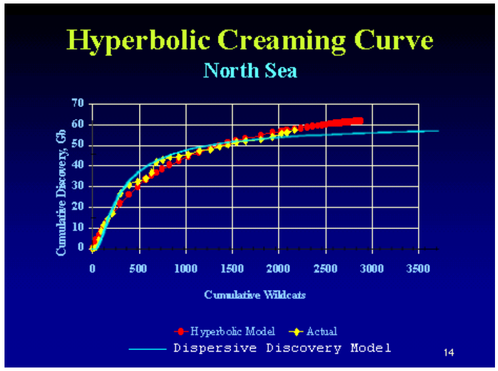


FIGURE 11-15.
North Sea creaming curve. I don't think the hyperbolic model is the same as Laherrere's [Ref 59]

The blue curve superimposed shows the dispersive discovery model with a cumulative of approximately 60 GB (the red curve shows a heuristic "hyperbolic" Laherrere curve with no clear asymptotic limit). So how would this change if we add a field size distribution to the dispersive discovery trend? I assert that it would only change the noise characteristics, whereas a true "large-field first" should make the initial slope much higher than shown. Now note that the authors of the North Sea creaming curve repeated the conventional wisdom:

> This so-called hyperbolic creaming curve is one of the more powerful tools. It plots cumulative oil discovery against cumulative wildcat exploration wells. This particular one is for the North Sea. **The larger fields are found first; hence, the steeper slope at the beginning of the curve.** [Ref 59]

I emphasized a portion of the text to note that we do not have to explain the steeper slope by the so-called law of finding the largest fields first. The dispersive discovery model proves the antithesis to this conjecture as the shocklet and creaming curve fits the data adequately. Most of the growth shown comes about simply from trying to find fields from a random distribution of needles in a haystack. The size of the needles really makes no difference on how fast you find them.

I submit that it all comes down to which effect will become the first-order one. Judge on how well shocklets model the data and then try to disentangle the heuristics of Michel's model. I just hope that the analysis does not go down the field-size distribution path, as I fear that it will just contribute to that much more confusion in understanding the fundamental process involved in oil discovery and production.

Convolve the Shock Model

Pipes and the Oil Shock Model

Few people outside of engineering seems to understand the usefulness of mathematical convolution. To me, it seems a natural operation, as obvious as other more well known statistical operators such as auto-correlation. Yet, you can't find a convolution listed in the @function list of a spreadsheet program such as Microsoft Excel. This has significance because convolution remains at the heart of the Oil Shock Model.

The shock model essentially expresses an oil production curve as a series of temporal convolution operators acting on an initial oil discovery stimulus profile. Each convolution corresponds to a phase of the lifetime of an average discovery, expressed probabilistically to account for the spread in potential outcomes. So that the original discovery profile gets convolved initially by a damped exponential representing the range in fallow times. This output gets followed by a convolution of another profile representing the range in construction times. In turn, the output of this gets convolved into another range of maturation times and then into the final extraction rate profile.

Importantly, the approach allows for the introduction of the shocklets described in the previous section in a very concise and canonical fashion.

Earlier I had referenced a full program that internally generated the entire set of convolutions, but one can just as well simplify the concept and use the idea of data flow and short scripts to perhaps make the concept more accessible (and potentially more flexible). Via UNIX or a Windows command shell, one can use the operating system ideas of pipes and the standard input/output streams to generate a very simple instantiation of the Oil Shock Model in terms of a short script and data files.

The pipes in the title of this section offer both an abstraction as to what occurs in the physical world as well as a useful mathematical abstraction to enable us to solve the shock model.

A typical invocation would look something like this:

```
cat discover.dat | conv fallow.dat | conv cons.dat | conv mature.dat | conv
extract.dat
```

Each one of the phases uses the standard output of the previous phase as a standard input via the operating system pipe command signified by the vertical bar "|". The term **`conv`** refers to a shell script or executable that takes the data piped into it and convolves it with data from a file given by the script's command line argument. The initial **cat** call reads from the discovery data file and pushes it into the first pipe.

In terms of a popular scripting language such as Ruby, the conv script looks like:

```
# conf.rb:

# convolution of stdio input array against array from file

def conv(a, b)
 length = a.length + b.length
 for i in 0..length do
  sum = 0.0
  i.downto(0) do |j|
   sum += a[i-j].to_f * b[j].to_f
  end
  puts sum
 end
end
conv(STDIN.readlines, IO.readlines(ARGV[0]))
```

The last line essentially calls the defined convolution function with two arrays generated automatically by using Ruby's readlines file parsing call. So that for each line in the file, representing a year's worth of data, an array is generated both for the standard input data stream, as well as the command line file's data. In other words, "a=the input data" and "b=the convolution data".

Operationally, to call this file using the Ruby interpreter, one has to invoke it with something akin to "`ruby conv.rb file.dat`". And the data in each of the profiles has to contain enough entries to cover the range of years that you desire to generate a production profile for. The convolution function takes care of the ranges automatically (i.e. the full convolution generates a range that covers the sum of the individual time ranges).

A typical data file for something with a 10 year damped normalized exponential profile would look like:

0.1
0.09048
0.08187
0.07408
…

The … would go on for *N* number of lines corresponding to *N* years. Of course, the data files themselves we can easily generate through other tiny scripts. The UNIX shell has a command called the back-tick "`" which when invoked within the command-line can generate a script call in-place. This means that we have many convenient ways, including providing a lambda function to Ruby itself, to generate the data profiles needed by the convolution operator.[9]

We can add the Shock function to the list of scripts. It essentially acts the same as a convolution, but since it relies on perturbations to a Markov (memoryless) process, we can only add it to the end of the sequence of convolutions. The file it works with contains a list of fractional extraction rates (acting proportionally to the current reserve) matched to the years since the first discovery occurred. For a stationary process, these rates stay relatively constant from year-to-year, but due to the possibility of global events, these rates can suddenly change, leading to sudden blips and dips in yearly production numbers.

The Shock script:

```
# shock.rb:
# Markov extraction of stdio data using perturbed rates from file

def shock(a, b)
 length = b.length
 temp = 0.0
 for i in 0..length do
  output = (a[i].to_f + temp) * b[i].to_f
  temp = (a[i].to_f + temp) * (1.0 - b[i].to_f)
  puts output
 end
end

shock(STDIN.readlines, IO.readlines(ARGV[0]))
```

The extraction rate file would look like this:

0.1
0.1
0.1
0.12
0.1
…

The fourth entry shows a 20% upward bump in the extraction rate. The complete shock model invocation would thus look like this:

cat discover.dat | conv fallow.dat | conv cons.dat | conv mature.dat | shock rate.dat

The following script takes the standard input and applies a constant extraction rate to the accumulated reserve data. Notice how the convolution simplifies given a Markov approximation.

The un-Shocked script:

```
# Markov of input array against arg value
```

9. In general, I find nothing really complicated about the convolution operation and find it really a shame that we don't have this functionality built into the set of standard spreadsheet operators. So even though this alternative pipe approach looks incredibly simple in principle, enough people stay away from the command line that it will never achieve the popularity of a spreadsheet formulation. Something like Matlab of course has the convolution operator built-in but it costs much more and caters to the engineering crowd (of course). Alas, for the moment we will have to satisfy ourselves with pipes, or resort to a complicated spreadsheet.

```
def exp(a, b)
 length = a.length
 temp = 0.0
 for i in 0..length do
  output = (a[i].to_f + temp) * b
  temp = (a[i].to_f + temp) * (1.0 - b)
  puts output
 end
end

exp(STDIN.readlines, ARGV[0].to_f)
```

This becomes a “shock”-less model and gets invoked as “`ruby exp.rb 0.1`", where the argument becomes a single floating-point value instead of a file. The extraction extends for as long as the input data sustains itself, which means that you need to extrapolate the input data if you want to better extrapolate into the future. I suggest this as a useful technique for every one of the scripts.

All of these scripts generate a granularity of only one year so don’t expect great results if the rates have time constants that get too close to one year. I would suggest that switching over to a smaller granularity than a year in this case; you just have to remember that the resultant output data will have this same granularity.

Digging Deeper

Figure 11-16 on page 219 demonstrates how Dispersive Discovery transforms into production via the application of the Shock Model. It works by clearly delineating the effects of discovery growth from the delayed action of extraction/production. Each colored band corresponds to a particular discovery year. The variation in the thickness of the bands over several years demonstrates the salient features of the shock model. The stacked bar chart essentially shows the effects of a multiple stage convolution on a discrete set of yearly discovery inputs.

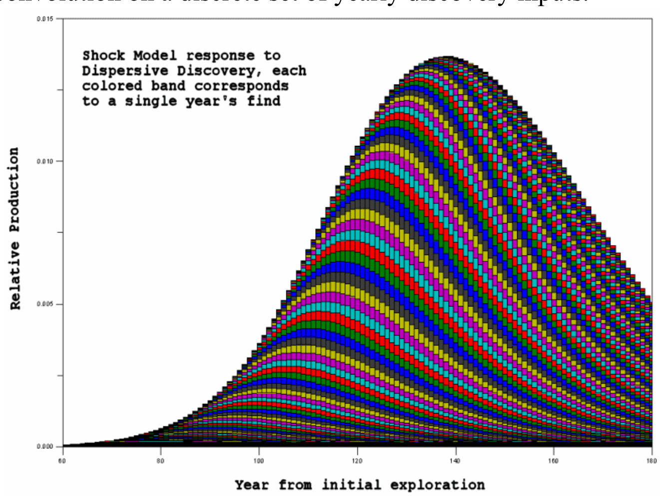


FIGURE 11-16. This curve features a damped exponential maturation phase. The color banding illustrates the effects of the convolution from each new year of discovery.

This type of chart rarely gets published for oil, unless it arises from very good accounting[10], but it has become very common for natural gas (see the next chapter for an example). I find that rather unfortunate, because if oil companies had provided detailed year-by-year paired discovery and production data, we would find ourselves in a much better position to analyze trends.

Summary of Analysis

This concludes the description of the analytical approach I take to model oil depletion. The amount of data that we can process and the number of views that we can interpret based on the models provides us with a lot of grist for the mill, especially if we have the data to play with and further analyze.

10. For example, you can find UK North Sea oil data drawn in this fashion. This comes about because the UK government mandates very accurate accounting of field production data.

CHAPTER 12

The Results.

Which data sets support the model?

*"Gas **was** great"*

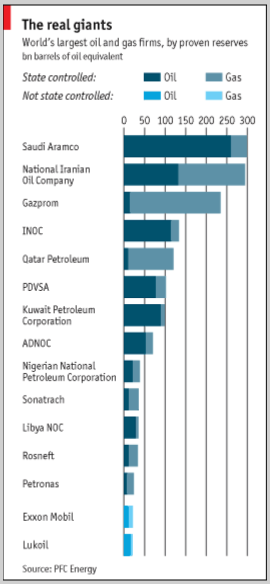
— Richard Smalley

We have much (at least partial) historical data available to attempt to substantiate various aspects of oil depletion models. This ranges from strictly accounted government records from places like the United Kingdom and Norway, to reconstructed data from incomplete and inferred data from other free-market or state-sponsored oil-bearing regions. In this section we look at data from the following regions:

1. USA
2. United Kingdom
3. Norway
4. Former Soviet Union (FSU)
5. Mexico
6. Alaska
7. Canada
8. Romania

Statistically speaking, when using a model as we have described, it makes sense to use as large a data set as possible. The biggest of the data sets — the aggregated world data — we have touched upon already. As we progressively shift to smaller data sets, the statistical effects give way to more deterministic and noisy outcomes, until we reach the smallest data set of Romania. Of the most inscrutable regions, such as that of Saudi Arabia, the lack of reliable and honest discovery and reserves reporting makes it less useful to model[1]. We essentially infer the results from the rolled up world discovery and production models. That remains one of the main benefits of using a model based on probability.

To refresh the discussion, I started by using the ASPO global discovery curve as an input forcing function to the oil shock model.



"Accounting from oil producers is more suspect from those governed by secretive states."

from [Ref 163]

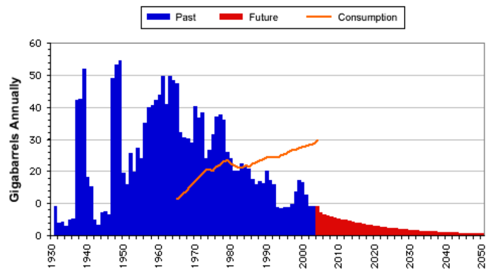


FIGURE 12-1. ASPO global discovery curve includes a 3-year moving average.

One first needs to consider all the latencies involved from the actual point of discovery, i.e. the initial "oil strike", to the point at which each discovery starts producing oil at a mature clip. So based on the elements of the oil shock model, we first need to justify linear composable latencies that served to shift the point of discoveries in time by the 36 years or so that others empirically observe.

> *The discovery curve mirrors approximately the production curve with a lag that varies from country to country. The US-48, for example, had a lag of 41 years whilst the UK North Sea production, with its urgency and technological basis had a lag of 25 years. The World's lag is estimated to be 36 years. [Ref 60]*

The composable stochastic latencies include:

- Mean time from discovery to decision to extract — The "fallow" period
- Mean time from decision to extract to completing rig construction — The "build" period
- Mean time from construction complete to maturity of production — The "maturation" period

If these times remain independent, we can use the convolution technique to generate a mature discovery window. I know that the *fallow* period can range well over 10 years; for example the industry knew about Alaskan oil well before they made the decision to start extracting. As for the *build* period, I have a few references that

1. The discovery curve of Saudi Arabia likely follows Hubbert Linearization better since it shows fewer effects of geopolitical events, and other shocks.

suggest that it takes a minimum of 3 years to construct an oil rig on land and 5 years for an offshore platform. And the *maturity* period includes all sorts of extraneous considerations, such as support features (building pipelines, etc.) and the possibility of dry wells caused by improper placement, which becomes part of the phase of maturation. I ended up choosing a Markovian 8-year average latency for each of these phases.

Finally, I added the previously described *extraction* phase to the model. The extraction rate basically relates the mean time to deplete a reservoir to *1/e* of its original value (or to 36.8% of its initial volume). As suggested before, I use as an implicit assumption that any rate of extraction or flow is proportional to the amount available and nothing more; past and future history do not apply.

And as the final necessary ingredient to match the spiky/notchy behavior of global production, I add the oil shocks which tend to suppress the production during critical geopolitical periods. The following fit to the BP data assumes the initial ASPO data as the forcing function[2], the 3 mean latencies occurring before mature production, an initial extraction rate, and the 3 shocked (or perturbed) extraction rates to match the dips. As the final perturbation, I then added a reverse shock starting after 2001 to match an uptick in production rate. In actuality, as I described in Figure 8-25, “Discovery Data + Shock Model for World Crude,” on page 140, the data I tried to fit included extra liquids not counted as solely crude oil discoveries, which meant that the I upped the extraction rates beyond realistic expectations.

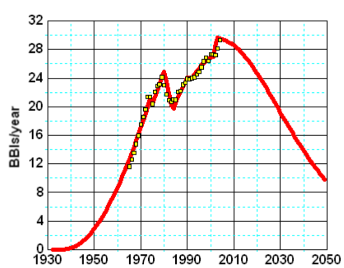


FIGURE 12-2. Early shock model fit that includes only crude discoveries which tries to match to an “all liquids” production curve. Extraction rates strain to meet the observed production.

This turned into one of the early estimates for extrapolating oil production and you can see that the lack of a continuous discovery stimulus beyond the last data point caused the production to immediately start declining henceforth. As a cautionary tale it really demonstrates the need for a more completely modeled discovery profile which allows for *future extrapolated* data points to predict the out-years.[3]

‡ *see “Code Snippet #4”*

2. This matched fairly well to the triangular discovery curve that I used in the past for the oil shock model. Of course the real discovery data does not show the piecewise shape of the triangular curve.

The limited expressiveness and poor predictivity prompted me to develop the more comprehensive model described in the chapter titled “The Shock Model. How we deplete oil” . This early model gives a good qualitative view but it doesn’t consider the impact of a good discovery model.

USA Discovery and Production

The federal government does not on its own keep track of all oil production records, nor does it completely force commercial interests to give up their detailed records. Instead agencies such as EIA (Energy Information Agency) and MMS (Minerals Management Service) keep track of the data. Other sources include consulting firms and the internationally-based IEA (International Energy Administration). One can find analyzed production and discovery data fairly easily on the internet, but you will have to bear with its quality.

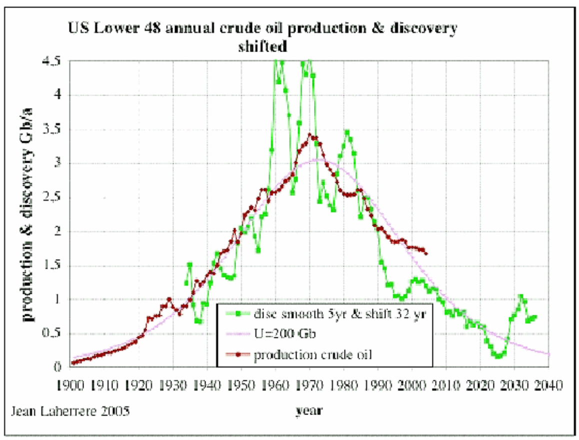


FIGURE 12-3. Production and shifted discovery for USA oil from Laherrere.[Ref 28]

Stuart Staniford reviewed the topic of USA production curves dating back to the first discovery made in 1859 [Ref 31]. He created some quite amazing fits to the entire USA oil production profile using a gaussian function, which looks like an inverted parabola on a semi-log plot:[4]

3. Foucher modified the shock model into something called the Hybrid Shock Model (HSM) to allow for an extrapolated reserve growth ala the logistic model. This prevents a sudden drop-off when the discovery data stimulus gets removed. [Ref 23]
4. Quadratic growth on a semi-log scale is concave downward. Exponential growth appears as a straight line.

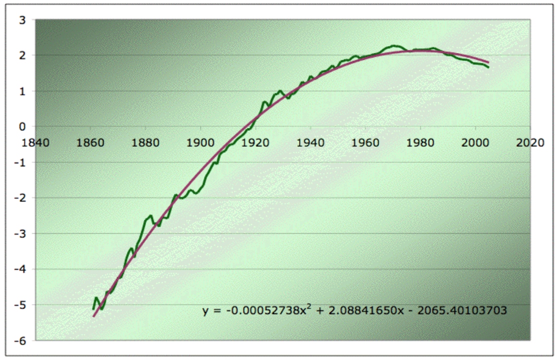


FIGURE 12-4.
USA production plotted on a logarithmic scale. Staniford noted a gaussian match over a substantial dynamic range. [Ref 31]

I used the USA lower-48 discovery table as a reference point and then eyeballed a quadratic growth factor to generate an accelerating rate of oil discoveries. The discovery peak hits about 1930 and then decreases after that point.

With the basic assumption that the size of strikes remains independent over time, I initially applied the oil shock model to the artificially constructed quadratic discovery data. I used about the same extraction rate that I previously used for the lower-48 model increasing it slightly to 0.08. I modified the *fallow, construction, and maturation* constants a bit from 8 years to 12 years. I did not apply any oil shocks, because they would not show up on this kind of scale in any case. This verifies that one can match the gaussian characteristic that Staniford observed.

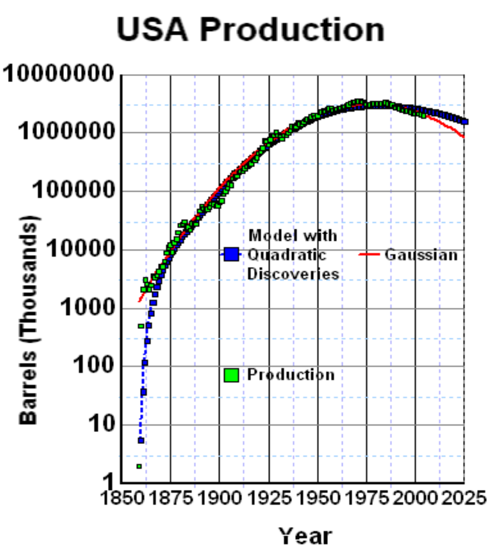


FIGURE 12-5.
An oil shock model stimulated with quadratic growth in discoveries also covers the dynamic range accurately. However, the gaussian suffers a sharper drop-off in the out-years.

Interestingly, the initial discovery estimate that the EIA provides, 5000 barrels, has a lengthy historical narrative.

On a brilliant Saturday afternoon, August 27, at 69 feet down, the drill suddenly dropped six inches into a crevice. Uncle Billy fished it out, wiped it off carefully, and knocked off for the Sabbath. But Monday seemed a long way off, and on Sunday Smith was back at the well, peering down the pipe, wondering if he really saw something glistening on the surface below. He grabbed a leftover end of pipe, plugged it up like a dipper, and thrust it down on a stick. It came back up filled to the brim with oil. A wild shout brought several mill hands running. Young Sammy raced off to town to notify Colonel Drake.
The whole village was buzzing; even townsmen who still couldn't imagine what might come of the find were eager to see it. A man from the nearby town of Franklin, on the Allegheny River, who visited Drake's well the following day, joining the eager crowds streaming in on every road in wagons, on horseback, and on foot, reported, "It comes out a flowing dark grease with a heavy white froth."
By then, the few pine barrels Drake had provided were already full. Drake took Margaret Smith's washtub from the engine-house shanty (she complained later she never could get it clean after that), then commandeered old whiskey barrels and sperm oil containers. And still Uncle Billy kept pumping and the oil kept coming; so did the crowds. (from [Ref 324])

The story also gives some older historical background. Arguably, we shouldn't even attach the discovery of oil in Titusville to any individual person. Many settlers had seen the residue in the oil over the years. So yes, in fact, it likely showed a *fallow* period, followed by Drake's *construction* period, and finally a *maturation* period.
Funny how things scale.

Note the EIA data points from 1859 and 1860 that Staniford omitted because he considered them as "outliers".

As an assumption I had to give the model an initial discrete stimulus to promote the discovery value above zero in 1859 (alternatively, I could have backed up the starting point a bit). This transient has little effect other than to keep the numbers on the graph. However, you can see that even the real data seems to plunge toward zero at the discovery — something that the gaussian curve cannot handle. The extrapolation of the gaussian would show 100 of barrels of oil in production many years before somebody officially discovered the oil! In other words, the idealized gaussian curve does not consider causality correctly.

Analysis for the quadratic/feedback model. I used the same oil shock model, and plotted the results for the USA below with a contrived quadratic/feedback without dispersion discovery input. Again it accounts for the causality in the initial discoveries by Drake in Titusville, PA in 1858 (i.e. something has to start out the "gold rush"). It also models the out-years quite effectively, as the decline will become much less steep than a Gaussian will predict, especially if we extended the profile much beyond the year 2050.

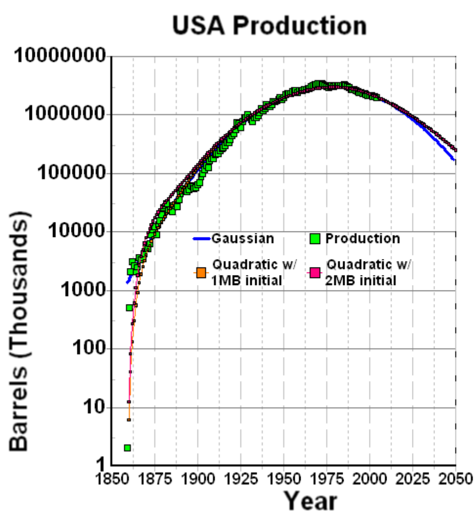


FIGURE 12-6.
Quadratic/feedback model for discovery stimulating oil shock model used to understand historical USA production.

Also notice when I plot the model on a linear scale (see below), it becomes nearly as symmetric as a Gaussian. One can easily explain this as the right-heavy asymmetry of the quadratic feedback model gets balanced by the left-heavy asymmetry of the gamma distributions that form the basis of the oil shock model. The convolution of the two models effectively cancels out the left/right asymmetries and a fairly symmetric model results.

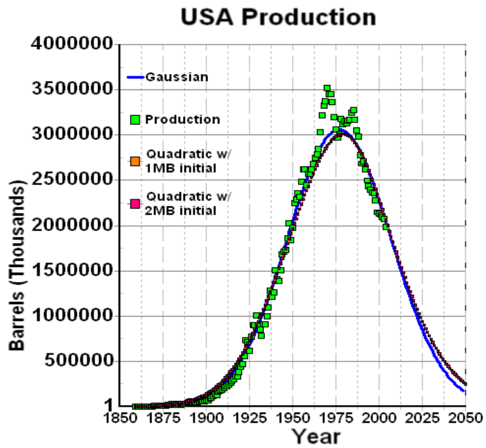


FIGURE 12-7.
The same model plotted on a linear scale.

The previous two sets of production plots used rather artificial discovery profiles to generate the final production data, since good discovery data does not extend much before the beginning of the 20th century. However, as we noted in Figure 8-28, "USA Production mapped as a pure Discovery Model," on page 142, the entire production curve eerily fits as a shifted dispersive discovery model where the model's parameters use the entire range of production data. As discussed earlier, the oil shock model's repeated convolution serve to simply shift the production model over a certain number of years, on which a log chart will blur the specific details.

Shock Model applied to USA lower-48

If we want to use the actual discovery data for the USA, we essentially need to ignore missing data before the early 1900's. I used the same base parameters as I used for the fit to the world data, namely each of the *fallow, build, maturation* phases set to a mean of 8 years, and the average Markov extraction rate set to 0.07 of volume/year (~14.2 year *1/e* time). The "unshocked" fit to the data turns out to match the observed production quite nicely.

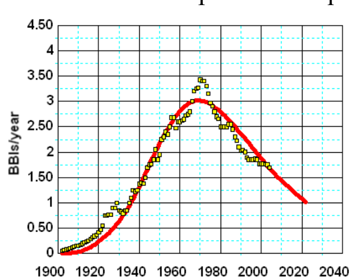


FIGURE 12-8.
USA production as described by the unperturbed oil shock model using discovery data post-1900 as stimulus.

But the fit can get better. First, ignore the poor fit during the 1930's (depression era). Next, as Laherrere has noted [Ref 63], we can likely account for the perturbations. The first one occurred during the late 1950's:

> *The initial "voluntary imports quota" of 1957 did not worked and President Eisenhower made them "mandatory" in 1959. The US were disconnecting their domestic oil market from that of the rest of the world, leaving to the large foreign producers the responsibility of managing the market, what they did by creating OPEC the following year, in September 1960.[Ref 61]*

So that clearly a quota-driven reduction in extraction occurred in the late 1950's, in other words, the first local USA shock. During the 60's, with the quota in place, the oil industry carefully modulated extraction (note that a big dependency on foreign oil did not exist), yet with a peak quickly approaching, extraction rates had to increase to make up for the continuing economic expansion at the time (and to feed the Vietnam War/Great Society war machine). Once the peak hit, the prorationing reached 100% and the oil industry moved to supplant domestic oil with that from foreign sources.

> *In March 1971, the balance of power shifted. That month the Texas Railroad Commission set proration at 100 percent for the first time. This meant that Texas producers were no longer limited in the amount of oil that they could produce. More importantly, it meant that the power to control crude oil prices shifted from the United States (Texas, Oklahoma and Louisiana) to OPEC. A little over two years later OPEC would through the unintended consequence of war get a glimpse at the extent of its ability to influence prices. [Ref 62]*

Actually, 1970 marked another turning point:

> *The transition. It started in 1969 when the Santa Barbara oil spill triggered some important US environmental laws (Clean Air Act, Clean Water Act). They slowed down the development of domestic energies (mostly coal) at the very time when the US indigenous oil production was going to peak (1970) before decline (see graph 1416). US net imports, which since 1959 were kept at about 20% of US consumption, were relaxed and grew by 25% each annum from 1970 (3.15 Mb/d) to 1973 (6.02 Mb/d), a growth perfectly matched by Saudi net exports. World oil market became tight, as revealed by the successive upwards price revisions in Tehran, Tripoli and Geneva before the October 1973 oil price explosion. The "Club of Rome" was right, oil was unable to fulfil all energy needs, but it was wrong because oil was only scarce in the US 48 lower states but abundant elsewhere.*
>
> *[...]*
>
> *Transport demand in the US was hardly affected by the first oil shock because the oil quota put in place by President Eisenhower in 1959 isolated the US oil pricing system from the international oil market However, demand fell strongly at the time*

of the second oil shock because its timing coincided with the liberalisation of the US oil prices.[Ref 61]

I used the following perturbation profile to try to fit to this evolving scenario.

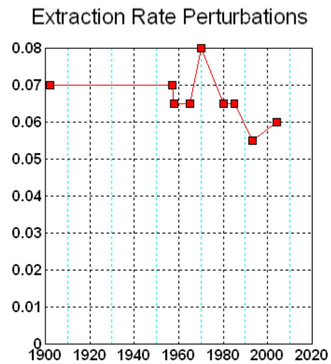


FIGURE 12-9. The selection of shock extraction rate perturbations needed to match the detail of USA oil production

Which gives this shock fit:

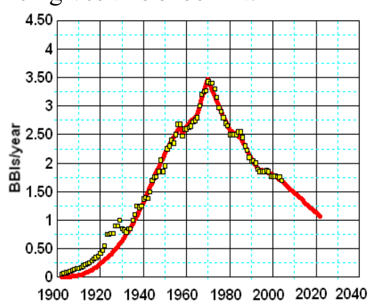


FIGURE 12-10. Extraction rate perturbations allow the oil shock model to match the details of historical USA oil production data.

This does show that the oil shock model demonstrates good order-of-magnitude scaling properties. If we can use the same shock model parameters on USA data as I did for the global data, it suggests that the rates have nearly a universal property. It makes some sense, if we consider that a few companies own the state-of-the-art in oil drilling technology and use the same expertise worldwide[5]. In general though, as Foucher has noted, decline rates do vary [Ref 64]. Yet, as it turns out, as long as the variability in decline rates have no huge systematic bias, one can always try to get by with a single *mean* extraction rate for each aggregated data set.

One can I ponder why, when Hubbert correctly predicted the date of peak oil in the U.S. around 1970, we still seem to extract a substantial fraction from domestic

5. This presentation [Ref 207] provides some good background on the interaction of the quota system with energy economics. And [Ref 208][Ref 2] gives some history on the politics behind the Texas oil men and the rise of Halliburton.

sources over 30 years later. In fact, several contingencies extended our run for years.

1. Alaskan oil — a form of discovery dispersion
2. Low throughput stripper wells — proportional extraction
3. Gulf oil and other offshore oil — another form of discovery dispersion
4. Easing of consumption rates — suppressive extraction rate shocks

I think of stripper wells like sucking a thick milk-shake through a straw; the rate limiting effect has nothing to do with your lung strength. The combination of throttling and working near the boundaries of the possible range of exploration effectively extended the back slope of the U.S. peak a bit beyond what we would have expected. Again, the caveat to this particular fit is that it uses actual discovery data instead of a model to extrapolate on future discoveries. This results in the selection of higher extraction rates than a proper discovery model would supply, as the built-in reserve provides a larger potential reservoir to deplete from.

United Kingdom North Sea

I next tried fitting the UK North Sea oil production data to the oil shock model. I transcribed the Laherrere data for discovery and production from the figure below:

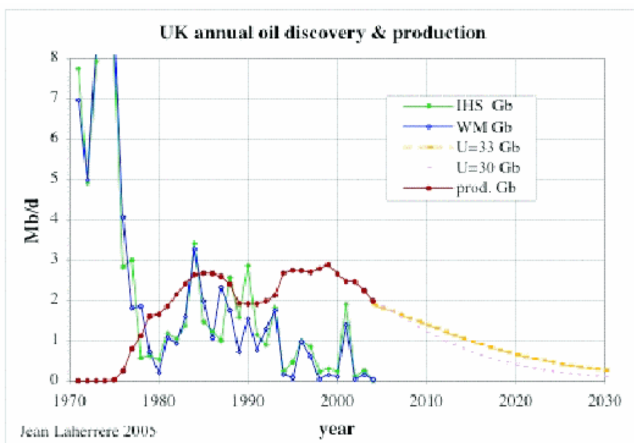


FIGURE 12-11. United Kingdom North Sea discovery profile and production data. [Ref 28]

The first cut I took assumed an immediate 4 year minimum time to construct an offshore oil platform with an exponential distribution of 1 year mean beyond that. The averages for the fallow phase and the maturation phase came out similarly low at 1 year. (Note that this differs for the worldwide fit where I assumed exponentials of 8 years for each of the fallow, construction, and maturation phases). I chose an extraction rate much closer to the world-wide average I used before of 0.07, adjusting it upward slightly to 0.1, i.e. 10% per year volume extracted. The unshocked results came out as the red curve below:

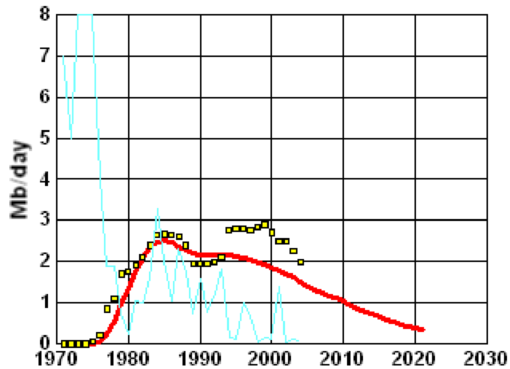


FIGURE 12-12.
Oil shock model fit with a constant extraction rate.

Note that a perceptible shoulder creeps up, largely due to the second set of discoveries. Importantly, the model can predict these secondary bumps given that the spacing between significant discoveries has a time gap beyond the reciprocal of the extraction rate (i.e. the characteristic time constant).

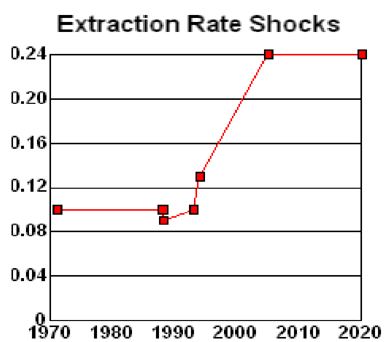


FIGURE 12-13.
Adding perturbations to the constant extraction rate of the UK North Sea oil shock model.

Assuming that the shocked model could account for the rest of the deviations through perturbations to the extraction rate, I eyeballed a shock profile in Figure 12-13 on page 231. I placed one regular shock causing a dip of 10% to the extraction rate in 1988; this corresponded to the accidental offshore fire on the Piper Alpha platform. After settling back to 10% extraction rate, I placed a fairly significant linearly increasing reverse shock starting in 1994. Whether this has any basis in reality, I can't really say for sure, but I needed this for the model to match the trending of the production curve.

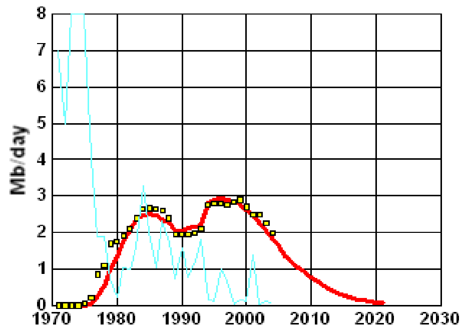


FIGURE 12-14. The additional shocks in extraction rate cause the second peak to elevate, matching the observed data for UK North Sea production levels.

So is this reverse shock indicative of a need to keep up with production demands?[6] Many people would prefer to introduce price into the oil depletion model. In my mind, I have no problem conceptually adding a profit factor to a model to trend it the right way. Maintaining profit for the North Sea oil companies has to exert a huge influence in their desire to keep the production levels at a constant rate in the face of declining reserves. This may in fact help explain how the North Sea shoulder turns into a bump *at the same level as the previous peak*. The reverse shock of increasing extraction rate economically acts as a short-term profit maintainer, at the expense of a long-term economic outlook. This indicates an *undulating plateau* on a fairly obvious time scale.

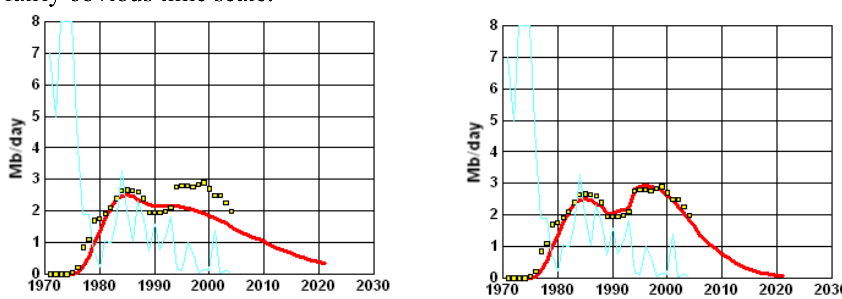


FIGURE 12-15. Side-by-side comparison between constant (left) and variable rate extraction rate (right) applied to oil shock model for UK oil production data.

Robert L. Hirsch has tried to understand North Sea production and the mystery behind sharp peaks and sharp declines.

SUMMARY

6. Or should we agree with Michael Lynch who suggests that the second North Sea bump as a repudiation of the general peak oil theory?

> To understand the possible character of the peaking of world conventional oil production, oil peaking in a number of relatively unencumbered regions and countries was considered. All had significant production, and all were certainly or almost certainly past their peak. The data shows that the onset of peaking can occur quite suddenly, peaks can be very sharp, and post-peak production declines can be comparatively steep (3 - 13%). Thus, if historical patterns are appropriate indicators, the task of planning for and managing world conventional oil peaking will indeed be very challenging. [Ref 65]

I could get the oil shock model to work for UK North Sea oil *if* I put a strong extraction increase near the peak of 1995. Up until that point the extraction rate was constant apart from some perturbations due to the Piper Alpha platform accident.

Since deepwater oil requires much more investment in upfront costs and maintenance, the oil company has an interest to pump out the oil as fast as possible. They can't afford to man an offshore platform over the years like they would a Huntington Beach stripper well sitting next to a McDonalds. So they go full blast once they detect peak coming — which produces a high, sharp peak, followed by a steep decline.

I have seen a particular style of production graph in only a few places, usually referencing natural gas.

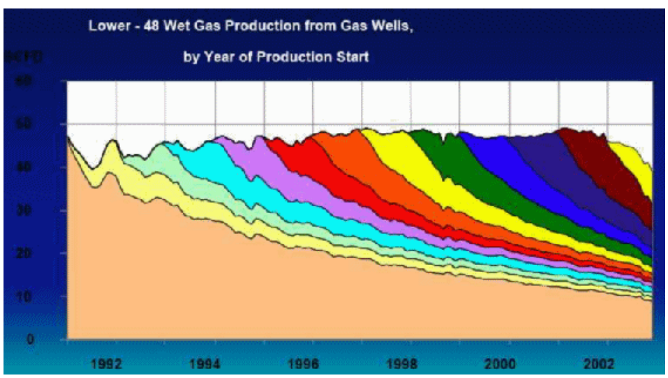


FIGURE 12-16. Layered natural gas production data showing depletion from fields introduced in a particular year. This provides a comprehensive view of both total and individual production declines.

This plot shows the decline of *individual fields* on top of the overall production curve, which essentially provides a compact and instructive look at trends. A UK study contained that kind of plot for North Sea oil. As the UK requires very detailed reporting of yearly production the plot comes out quite clearly.

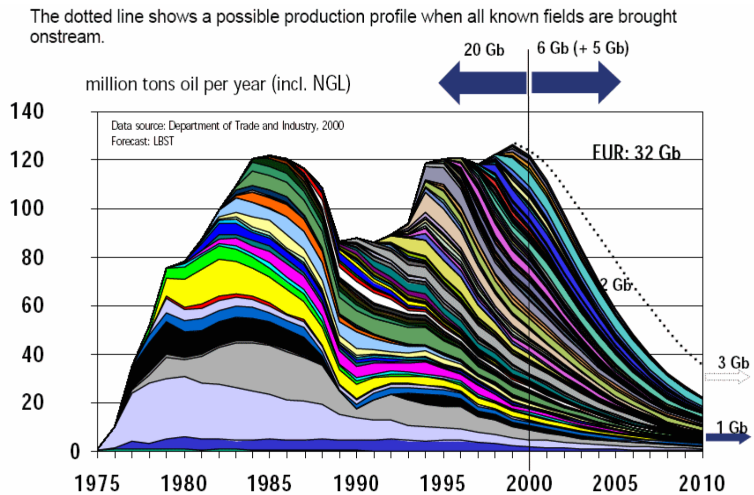


FIGURE 12-17. Layered field-by-field oil production for the UK. [Ref 66]

As this data only extended to the year 2000, with only an extrapolation given for the ensuing years. The shock model extends the profile to 2005.

After combining the two sets of data, the extrapolation follows the aggregated data set:

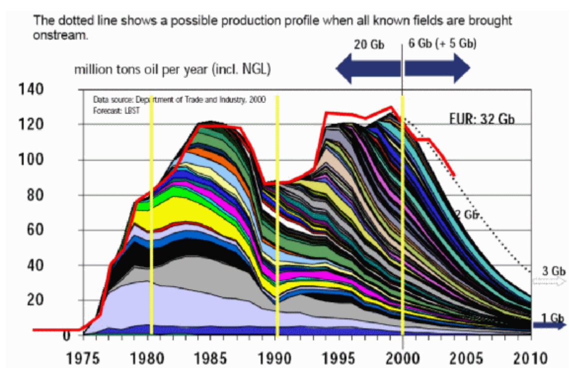


FIGURE 12-18. Extrapolation from the oil shock model (red) overlaid.

The red line indicates the updated data trend (corresponding to the yellow dots of Figure 12-15 on page 232).

In a sequence of updates over several months I checked the model with available data points

First North Sea Update (about one year later)

Oil production from the UK North Sea continues to plummet: Oil production growth dropped 4 per cent compared with May at 1,411,961 barrels per day (bpd) and down 13 per cent on the same month last year. From the previous year's oil shock model for UK North Sea, I added an additional data point (green star):

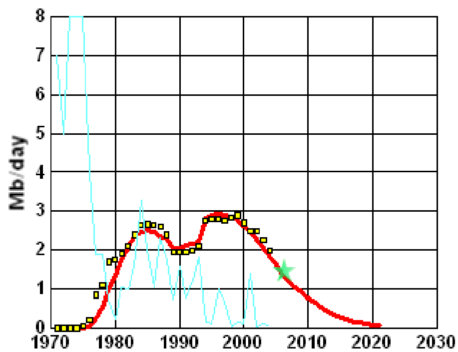


FIGURE 12-19. Recent data point added to the original UK oil shock model. No significant new discoveries during this time period.

Euan Mearns presented an extensive review of the UK North Sea oil production decline along with some of his own models **[Ref 67]**. I spliced the oil shock model from August 2005 on top of his chart below:

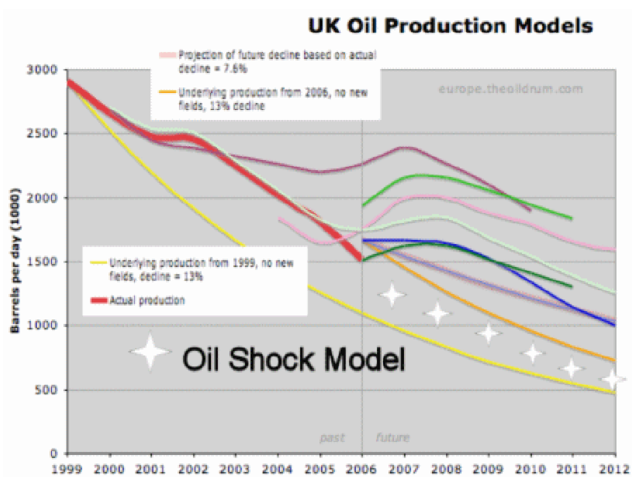


FIGURE 12-20. UK production data from Mearns [Ref 67]

The future bumps in some of Mearns model predictions come from new fields coming on line such as the Buzzard field discovered in 2001. This one barely got included in the discovery input for the oil shock model, but more recent discoveries such as Rosewood/Lochnagar in 2004 have not. The North Sea oil region more than anywhere else demonstrates how new discoveries remain the only way to halt the inexorable decline.

Second North Sea Update (three years later)

As of 2008, the latest information from the BERR [Ref 68][7] puts it at 1.2 million barrels per day, a bit higher than the oil shock model will predict without additional discoveries coming on line.

Norway

Next I applied the oil shock model to the Norway depletion curves starting from the Laherrere discovery data [Ref 28].

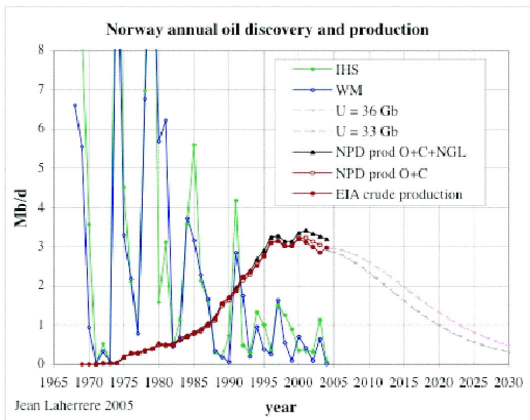


FIGURE 12-21. Oil discovery profile and production data for Norway. [Ref 28]

In the oil shock simulation, I used means of 5 years for the *fallow*, *construction*, and *maturation* periods, and a 10% depletion rate for years up to 1992. After 1992, like for the UK, I doubled the extraction rate over a 10+ year period. The fit is decent and it gives much more insight than the questionably derived logistic curve formulations.

7. multiply figures by 6.290 to convert cubic metres to barrels [Ref 68]

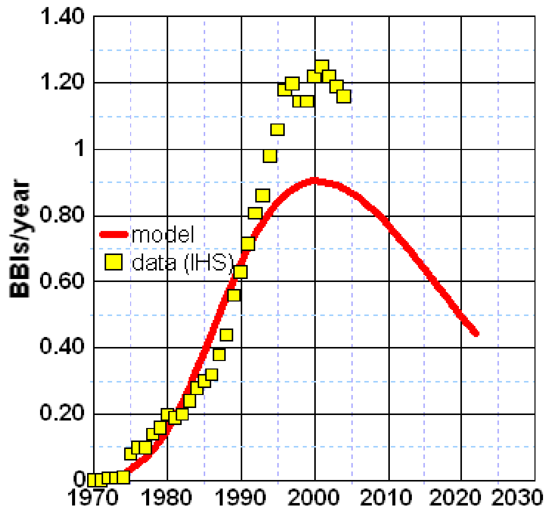


FIGURE 12-22.
Oil shock model for Norway oil production applying a constant extraction rate. The model peak does not reach the data peak. Unshocked -- 10% extraction rate

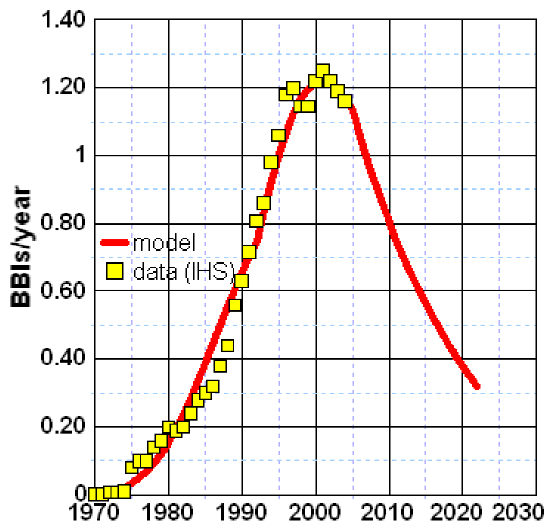


FIGURE 12-23.
Oil shock model for Norway oil production applying positive perturbation to raise the peak level. Shocked -- raised from 10% to 20%

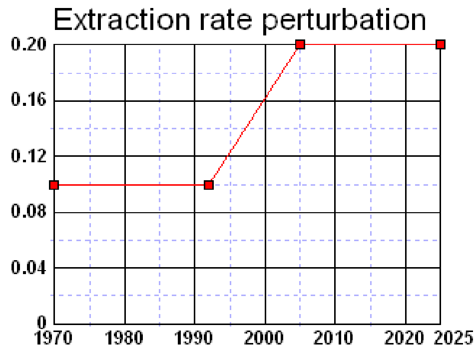


FIGURE 12-24.
Extraction rate perturbation used for the Norway oil shock model. Note the same general increase observed as for the UK North Sea model in the same time frame.

Note how close the profile of the shock perturbation approaches that of the UK North Sea model in Figure 12-13 on page 231. In both cases, the increases in extraction rate occurred right around 1992, and essentially targeted the same 3 MBls/day sustainable production rate.[8]

Like in the UK, the Norway model shows how the offshore areas can suffer rapid depletion. From the range of the parameters, the British developed and matured their rigs much more quickly than the Norwegians. The necessary increase in the extraction rates as the production curves started leveling off in the early 1990's became quite obvious; this basically forced the hands of each of the producers to effectively pump harder. Without new discoveries, and continuous hammering on the extractive technologies, they will certainly see a steep decline before they put the expensively maintained rigs into mothballs; this occurs as the North Sea producers finalize their output and avoid diminishing returns which would cut into ultimate profits[9].

First Norway Update

From the original oil shock model fit to the Norway production curve, more recent data coming in from the Petroleum Directorate of Norway reinforces the model's prediction of a rapid and steep decline in production.

> *OSLO - Norwegian crude production in February was down 18%, or 528,000 barrels a day, compared with February 2004, the Petroleum Directorate said Thursday. February crude production totaled at 2.46 million b/d, compared with 2.988 million b/d in the same month of 2004.* [Ref 69]

8. Market-based and competitive pressures perhaps?
9. Apparently, those in the oil industry call this falloff by the phrase *pump and dump* because of the high cost of extraction.

When overlay plotted on the previous model, it looks grim, even if we account for any calibration offset.

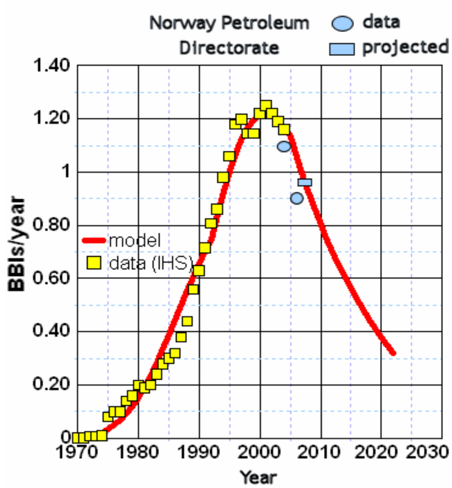


FIGURE 12-25.
Addition of recent production values to the Norway data set demonstrates how the extrapolated model tracks updates.

Projections of 2.64 million b/d in the 2007 and 2008 forecasts probably won't deter the slide much.

> *It forecasts average production in 2006 at 2.43 million b/d rising to 2.64 million b/d in 2007. Mathiesen said reserves grew by 975 million barrels of oil equivalent last year, with development of discoveries being approved and increased oil recovery projects. A production average of 2.64 million b/d is also forecast for 2008.* [Ref 69]

Second Norway Update

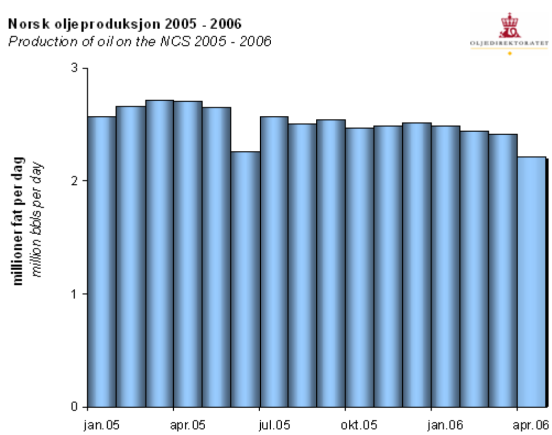


FIGURE 12-26.
Norway oil production for years 2005 to 2006. [Ref 70]

It still appears that oil production continues to drop like a rock with the March 2006 number.

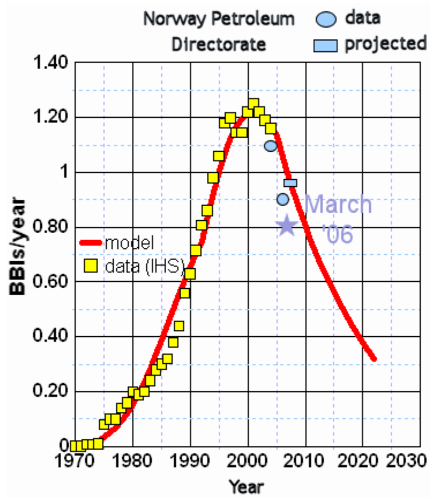


FIGURE 12-27. Even more recent data for Norway.

If anything, the extraction rate may have throttled down, below a value of 20% depletion of available reserves per year. This could mean that Norway's oil producers had decided to conserve oil in the anticipation of extending the time constant beyond that of 5 years expected for a 20% depletion rate.

Foucher used the data to come up with a multiple-oil-field production profile view. The more I look at it, the more it reinforces the notion that a definite bifurcation occurred in production rates. Around 1991, extractive effort has to have ramped up considerably, which has served to sharpen the production profiles in the individual regions. Look closely as the early individual production curves show rather broad profiles (with significant reserve growth in the oldest field) while the later production curves consist of half-widths of just a few years.

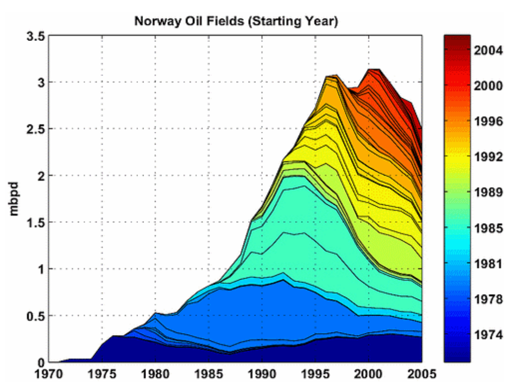


FIGURE 12-28. Layered oil production data for Norway. More recent fields show less volume and a more rapid decline. [Ref 71]

For 2008, Norway production stood at 122.7 million standard cubic metres or about 772 million barrels per year [Ref 72]. This remains below what the original oil shock model predicted of 800 million barrels out to 2010. Once again, Norway's oil producers may have intended to conserve reserves by throttling the production rate well below 20% per year.
As of 2010, Norway is on track to produce less than 800 billion barrels a year.

Former Soviet Union

Next, I fit the Former Soviet Union (FSU) oil production data using the oil shock model. This graph shows the discovery data superimposed on to the production data.

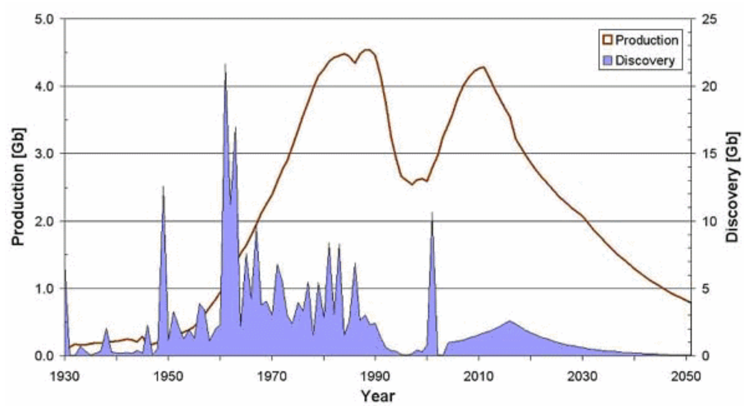


FIGURE 12-29. Former Soviet Union discovery profile and production data. [Ref 73]

I transcribed the data from the chart (ignoring the extrapolated data) and used the general oil shock model source code.

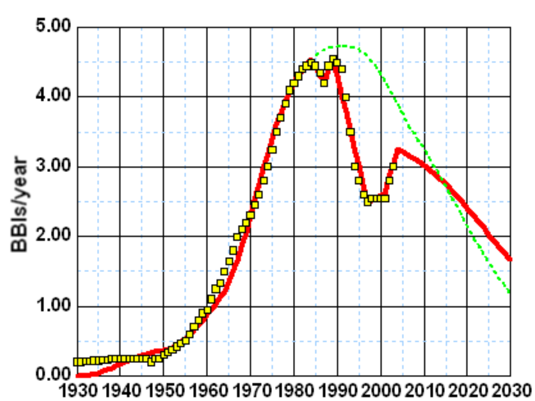


FIGURE 12-30. Fit to production data using a constant extraction rate (unshocked) and close fit assuming shocked perturbations.

I used a mean value of 5 years for each of the *fallow*, *build*, and *maturation* phases. This came out shorter than the 8 year latencies that I chose for the global and USA-48 model.

The green curve assumes a proportional extraction rate of 0.09/year (*1/e* time of about 11 years). This follows the production curve up until about the time of political instability in the Soviet Union. After that point, c.1984, I added perturbations according to the following chart, leading to the red curve which overlays the production data points.

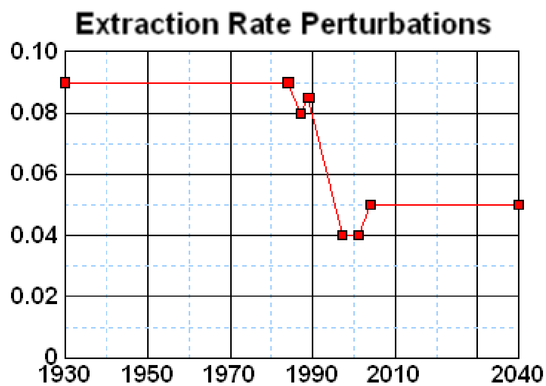


FIGURE 12-31. Extraction shocks for FSU production model fit. The initial dips occurred during and following the dissolution of the Soviet Union.

I see a more of a dip followed by a recovery rather than a strict oil production increase, which I imagine could result from a lagged reaction to market forces[10]. The Soviets had no choice but to shut down their pumps temporarily — they did not have the technology or reserves to increase production beyond what the Soviet machine could already provide. By the time 1988 arrived, it became too late for them to do anything about it, as the eastern bloc started to implode.

The post-Soviet transition showed immediate severe reductions in extraction rates. Clearly, the new FSU republics no longer had to produce oil to continue militariz-

10. The glitch occurring after 1984 looks benign but evidently carried some punch — according to one theory, Reagan had a hand in it:

> *It seems likely that the Reagan administration, which took office in 1981, bearing in mind the economic havoc produced when US production peaked in 1981, followed by the Arab oil embargo and the "oil crisis" of 1973-74 and the deep recession that followed, decided to use the "oil weapon" to destabilize the USSR. Reagan embarked on a major military buildup, putting the Soviet Union under pressure to keep up. Meanwhile, declining prices after 1981 forced the USSR to pump more oil to supply its clients in Eastern Europe and to sell in world markets for hard currency. Then in 1985 Reagan persuaded Saudi Arabia to flood the world markets with cheap oil. Again, the USSR had to increase output to earn hard currency. This led to the second peak in 1988.*[Ref 74]

ing their bloc countries and allies (including Cuba). True free-market forces replaced the fixed demand and the new capitalists also had their eye on the green curve (peak oil in 1990). Much like the USA after our peak in 1970, they likely wanted to modulate the supply.

So what explains the reverse shock (i.e. sudden extraction increase) in the last few years?[11] I don't know why, but OPEC production hasn't really gone up much over the past few years, and the FSU clearly has, or at least had the production capacity during the Soviet era. I would imagine that *somebody* has to make up the potentially widening supply and demand gap.

The model shows that at the current proportional extraction rate, the FSU has probably hit its secondary oil peak. They can conceivably try to keep on increasing the extraction rate[12]. But remember, the law of extraction: the steeper the hill, the faster the decline post-peak.

I repeat the numbers below, which show the FSU increasing at a perhaps unsustainable 10% per year, while OPEC showed a fairly significant increase of 7% in one year:

TABLE 1. Production rates

2000	**2001**	**2002**	**2003**	**2004**	
31354	30628	28855	30686	32927	OPEC
35583	35541	36056	35870	35916	Non-OPEC
08013	08659	09533	10499	11417	FSU
74950	74828	74443	77054	80260	Total

Recently, the production numbers have recovered from the post-Soviet transition.

11. Note that the BP data also shows this FSU oil production increase
12. The Soviet Union transformation into the FSU added lots of uncertainty into the amount of capability their infrastructure possessed. Lots of scrap metal likely traded hands.

Mexico

Foucher analyzed Mexico's oil production capacity [Ref 75]. He provided the following curve courtesy of ASPO.

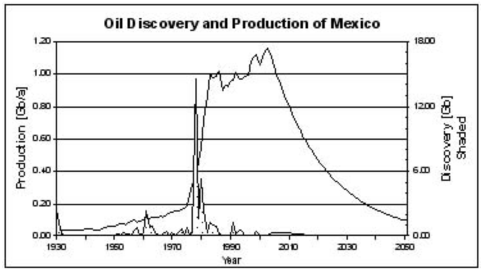


FIGURE 12-32. Oil discovery profile and production data for Mexico. [Ref 76]

Since the chart overlays a discovery curve alongside the production curve, I figured I could try out the oil shock model on the discovery data. I initially didn't have much confidence in getting good results because the production seemed to start to ramp up much too quickly after the big discoveries of the late 1970's (note that even though Mexico discovered Canterell in 1976, the overall discovery peak remains closer to 1979, the year that production on Canterell started).

I used the common approximation, last used on the Norway data, of equal values for the fallow, construction, and maturation phases of 5 years each. Initially, I also chose the same depletion rate as Norway of 10% of remaining amount per year. The fit (dashed green curve below) looked adequate, matching the general rise and estimated fall of the ASPO data[13].

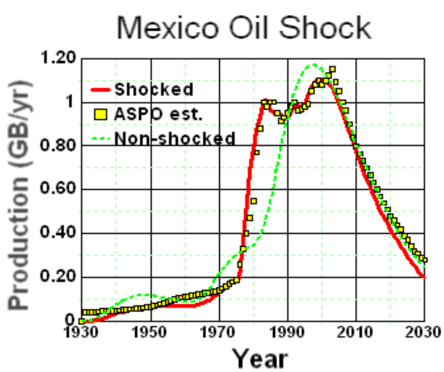


FIGURE 12-33. Oil shock model for Mexico using published discovery profile.

13. The ASPO newsletter does not reveal their Mexico production projections [Ref 76]. That is one of the problems of sharing oil depletion analysis in general, as individual teams don't always exhibit transparency, even though they may prove correct in the end.

As the estimated fall lined up with ASPO's extrapolations exceedingly well, I reasoned that the rest of the production profile could fit rather well with judicious modulation of the extraction rate in the form of a series of oil shocks. I achieved a much better fit with the following oil shock perturbation.

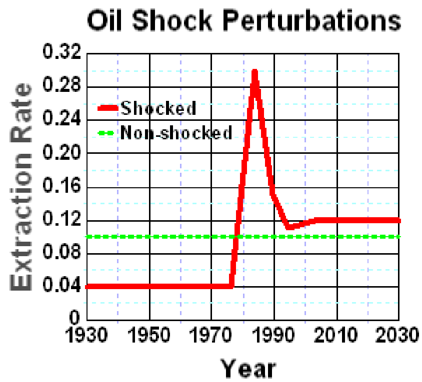


FIGURE 12-34.
Extraction perturbations necessary to match the production levels for Mexico oil. Rapid increase coincided with late 1970's to early 80's oil crisis.

Note that pre-1976, I selected an extraction rate of 0.04/year (an equivalent time constant of 25 years). Then, starting in 1976 and continuing until midway through 1983, I ramped up the extraction rate quite significantly, high enough that the effective extraction rate would drain about half of the reserves every 3 years. By 1990, the extraction rate flattened out to a steady state value 0.12.

Based on the dynamics of the curve, the late 1970's/early 80's perturbation impacted the reserves from pre-1976 as much or more than the Cantarell-era discoveries.

> *The (Canterell) field reached an early peak in production of 1.1 million barrels per day in April of 1981 from 40 oil wells.* [Ref 77]

This amounted to less than half of total production. Maturation also had a continuing impact as additional platforms came online in the mid 1990's. Moreover, I have a feeling that the USA had as much to do with this ratcheting up of production as anything else. Remember this occurred as the lower-48 started its depletion downturn, instability happening in OPEC and the Middle East, and Alaskan oil had not quite hit a peak. As the world adjusted to the new oil economy during the 1980's, oil production from Mexico similarly stabilized due to adjusted demand from the USA as it transferred to alternative sources in Alaska and the Gulf.

Mexico production does look quite strange in comparison to many other regions. It may have much do with the single super-giant (Canterell) oil field in the mix. This does have an effect on a stochastic analysis as it adds a highly non-random and deterministic component to the dynamics.

Even with that, a few nagging questions remain. At the time I remarked on a curiosity within the oil discovery profile from ASPO. As you can see below, the peak discovery appears around 1979, yet all references point to the discovery of the massive Cantarell oil field in 1976 (including production start that year).

As the oil shock model requires “fallow” and “construction” periods as parameters, and the discovery profile essentially squeezed this time frame to virtually nothing, I believe I erred in choosing a typical value of 5 years for these two parameters. As a by-product of selecting these numbers, I had to make the oil extraction shocks in the early 1980’s much too high to fit the historical oil production profile.

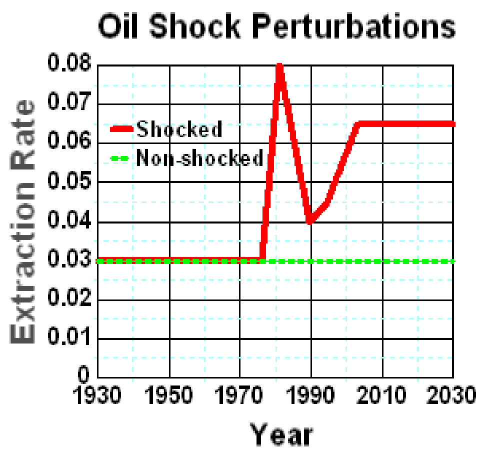


FIGURE 12-35. Adding an unperturbed oil extraction rate to the Mexico model.

So I re-visited the model with a “fallow” time constant of 1 year and a “construction” time constant of a bit higher at 1.25 (based on needs of the USA to find an alternate source after the 1970’s oil crisis). The resulting fit showed shocks much lower in extent, dropping from peak extraction rates of 0.3 to a more acceptable 0.08 per year value. As you can see, extraction peaked from the late 70’s into the 80’s and then dropped to a low right around the 1990 recession before rebounding during the 90’s.

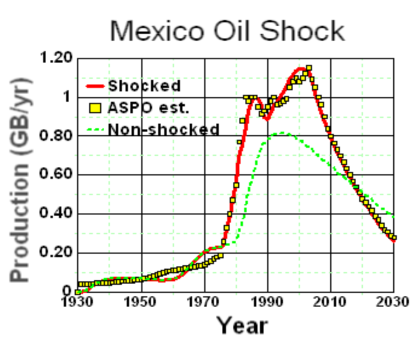


FIGURE 12-36. Superposition of a non-perturbed extraction rate on the Mexico oil shock model. The subsequent fit does not nearly reach the magnitude of peak production, which is why perturbations are effective for modeling.

What happens if we add a single large discovery to Mexico data?

Any Gulf of Mexico (GOM) finds can definitely effect Mexico oil production. Depending on it's size the so-called deepwater "Jack 2" discovery could remain insignificant for our future global oil production outlook.[Ref 78]

Industry analysts estimate that the GOM discovery could add 5 to 15 gigabarrels of oil to our reserve. In terms of the oil shock model, the discovery provides an additional stimulus to that model's world estimate. Putting the two together and using the optimistic value of 15 GB, the new out-year estimate appears below. Recall that the oil shock model uses a stochastic estimator, so the new curve provides a probability view of expected production and ultimate depletion.

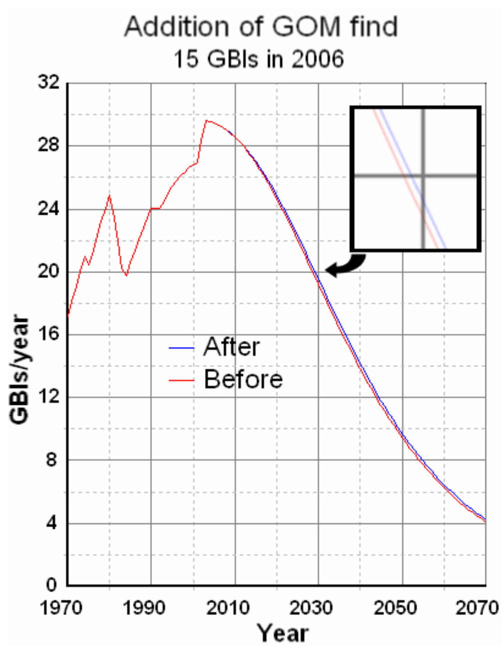


FIGURE 12-37. The addition of a relatively large new field to Mexico's reserve base does not alter the decline profile much at all. This occurs because the production gets stochastically spread through many out years, adding on average less than a GBl/year extra production.

Note the inset which provides a magnification of the two curves around the year 2030.

As of the end of 2008, Mexico's oil production dropped to 2.8 million barrels per day, or 1.022 billion barrels per day, very close to the 1 billion barrels per day shown by the oil shock model in Figure 12-36 on page 246. [14]

14. "Mexico's state-run oil company, Pemex, said its 2008 production of crude oil fell 9.2 percent from 2007. Pemex said Wednesday that daily production last year dropped to almost 2.8 million barrels compared to three million barrels the year before." [Ref 80]

Canada - The Weyburn oil field

Take a look at the following chart of production from the Weyburn oil field in Canada:

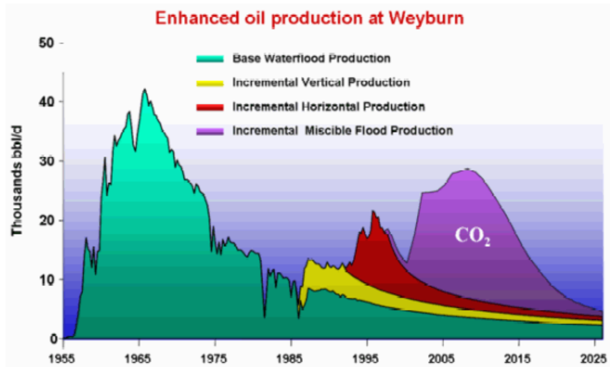


FIGURE 12-38. Oil production for the Canadian Weyburn field. Enhanced extraction occurred late in its life-cycle. [Ref 79]

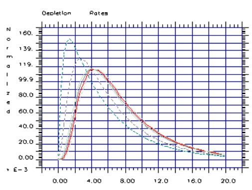


FIGURE 12-39. Family of Gamma curves

The classic shape of the green region looks similar to the shocklet variant of the shock model. Notice the asymmetric profile reminiscent of a gamma curve.

The Weyburn field made news as an experiment in CO_2 injection and sequestration (which gives the secondary peak in Purple). Using transcribed data [Ref 81] we can understand the shape in terms of the oil shock model which works for depletion profiles in the context of a much larger macro-depletion scope. I concentrated on the initial (non-enhanced) depletion behavior first.

TABLE 2. Oil Shock Model Parameters for Weyburn

Phase	**Parameters**
Discovery delta	51 million m^3 (320 million barrels) backdated to 1954
Fallow phase	1.25 years
Build phase	2.5 years
Maturation phase	2.5 years
Depletion rate	7% of remaining total per year

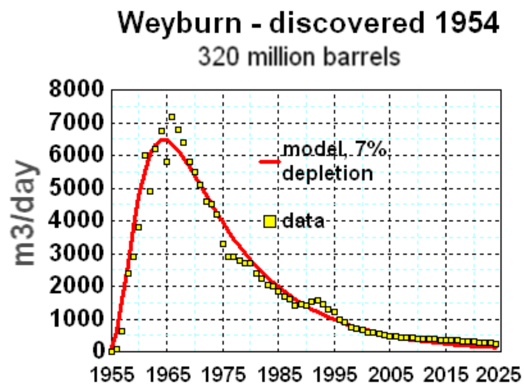


FIGURE 12-40. Oil shock model for Weyburn field.

The article (which shows the original curve in m^3 of crude) states that Weyburn has 1.4 billion barrels of original oil in place with about 34% of that available under water flood. That comes out to 476 million barrels available. The difference between the estimate and the model (320 million barrels) comes out to 154 million barrels extra from additional infill drilling.

> *An estimated 34% of the oil in place will be recoverable under waterflood. In excess of 80% of this oil has already been produced according to government statistics. Incremental production with injection of CO2 is estimated to be in the order of 15% of initial oil in place, in the area of the field to be flooded. This will produce an additional 130 million barrels (21 million m^3) of oil over the anticipated 25 year life of the tertiary recovery project.*[Ref 81]

If I put in the difference from the recent infill drilling, treating it as essentially a new discovery circa 1986, the new model looks like the following:

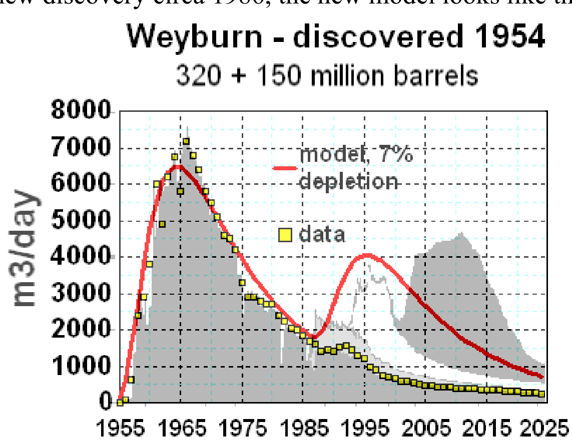


FIGURE 12-41. To model the secondary extraction at Weyburn, add a second delta discovery and plot the aggregate. This behaves similarly to reserve growtl as a slow supplementary discovery rate, or as a gradual technology improvement.

Interesting how well this model, which uses the same depletion rate as the USA lower-48 oil shock model, qualitatively fits the data from a much smaller exploration region. Even though the Weyburn field pales in comparison to the size of all the USA's (or the world's) fields combined, the essentially scalability of the oil shock model provides more evidence of its general applicability.

As for the remaining 15%, how much the substantial CO_2 injection will cost us in dollars and sweat remains a big question.

Alaska

The following Alaska data from Laherrere brings up some interesting issues to contemplate with regards to depletion modeling.

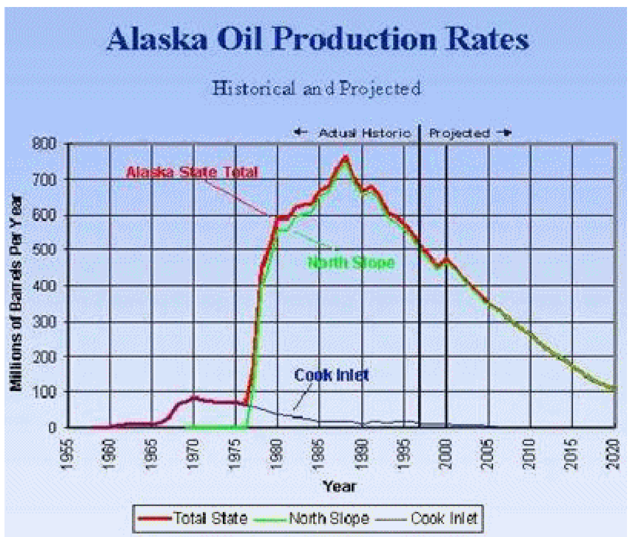


FIGURE 12-42. Alaska oil production showing several shoulders due to distinct discoveries. [Ref 82]

Oil companies discovered oil in Cook Inlet (Swanson River) in 1957 and on the North Slope (Prudhoe Bay) in 1968. I found a forecast for North Slope of 22.3 billion barrels [Ref 83] and a total extraction of 1.06 billion for Cook Inlet [Ref 84].

For North Slope I used the discovery date and the forecast as a stimulus to the oil shock model, and added the Cook Inlet model separately. For North Slope, I used values for fallow, construction, maturity, and extraction of 0.15 and for Cook Inlet, I used values of 0.2. The shock model production curve looked like:

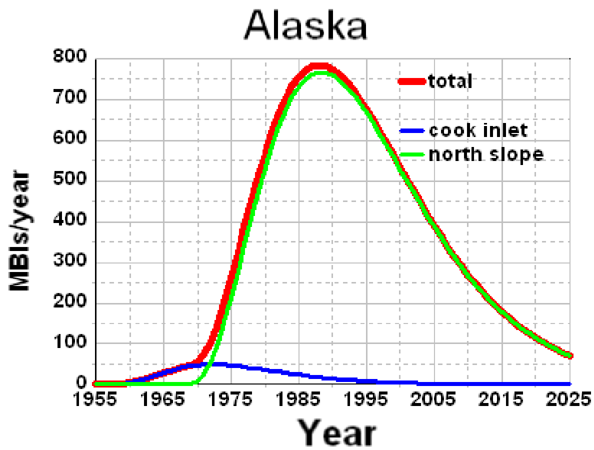


FIGURE 12-43. Oil shock model for Alaska modeled as two separate discovery stimuli.

I found it interesting that a strong discovery stimulus with typical rates adequately describes the curve:

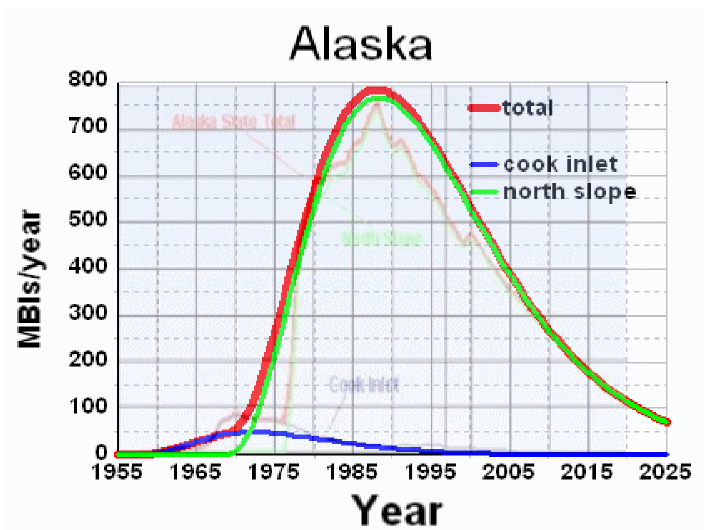


FIGURE 12-44. Overlay of oil shock model on Alaska production data.

In spite of the conflict between the stochastic premise of the model and the determinism implicit in a single field, the shape largely matches — except for one significant area. Production only commenced on Prudhoe in 1977, as soon as workers completed the Alaska Pipeline. So we see a sudden surge in production in the actual curve around 1977 which does not show up in the shock model. Since companies worked on construction of the rigs and pipeline in parallel, something has to give. I suppose that extracted oil prior to the completion of the pipeline might have got wasted or stored in reservoirs. Naturally this does not show up in production numbers but it has to pop out somewhere. Otherwise, one must suppress extraction until the pipeline opened up, which would have produced a large shock right around 1977 — something entirely doable within the context of the oil shock model.

So if I leave the extraction rate at some small number like 0.01 until 1976 and then jump up to 0.15 in 1977, the model looks like this:

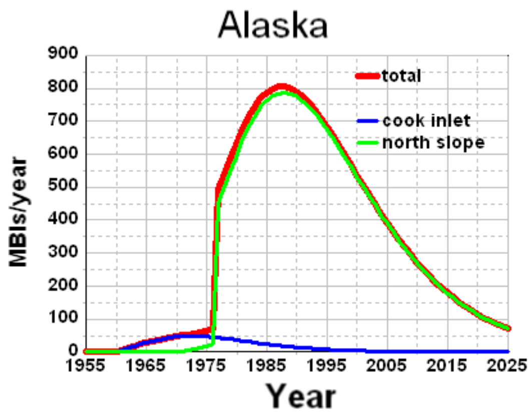


FIGURE 12-45. Perturbing the extraction rate for Alaska oil production. Production did not truly commence until the pipeline became available.

In general, a lot of this detail gets washed out as we take larger sets of reservoirs with varying discovery dates, yet the single set provides us with much insight — without invalidating the fundamental premise of the shock model. The use of the simplified shocklet model, which basically skips the fallow and construction stage may prove more useful in such cases.

Romania

Based on some intriguing results regarding historical Romanian oil production [Ref 85], this country provides an additional limited data set to compare against the oil shock model. Staniford plotted against the production data (shown Figure 12-46 on page 253), and effectively fit to the Hubbert Linearization method assuming a Gaussian shape.

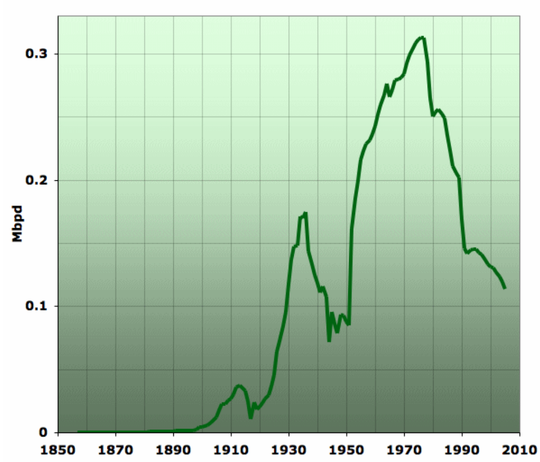


FIGURE 12-46. Romania oil production curve.

This curve looks similar to one that Laherrere published (which also includes estimates of discoveries)[15]:

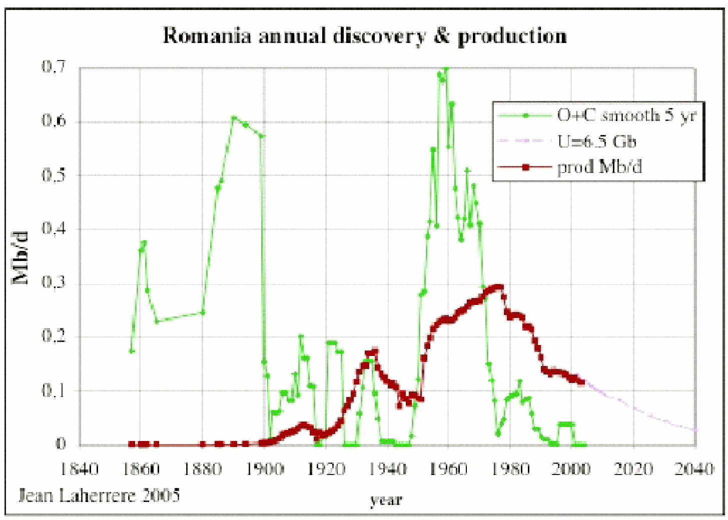


FIGURE 12-47. Romania oil discovery profile alongside oil production. [Ref 28]

15. Note that Laherrere/s discovery curve tries to "connect the dots" between years of discoveries, which has the unfortunate effect of indicating more earlier discoveries than warranted; check his cumulative curve to get the right data

The model used here derives completely from stochastic considerations and so one would generally apply more caution for someplace like Romania (or worse yet, an even smaller producer like France) to avoid the influence of deterministic effects. This comes about from a small sample size in these countries and the approximation of applying the maximum entropy estimator of equating the mean to the standard deviation on the rate random variables. In addition, since the Romania data crosses through a sweet spot that includes the years of WWII, one could imagine a huge perturbation occurring during that time.

In any case, I tried to keep the model as simple as possible and used means of 6 years for the *fallow*, *construction*, and *maturation* periods, and a 16.66% depletion rate for extraction (also corresponding to a 6 year *1/e* time).

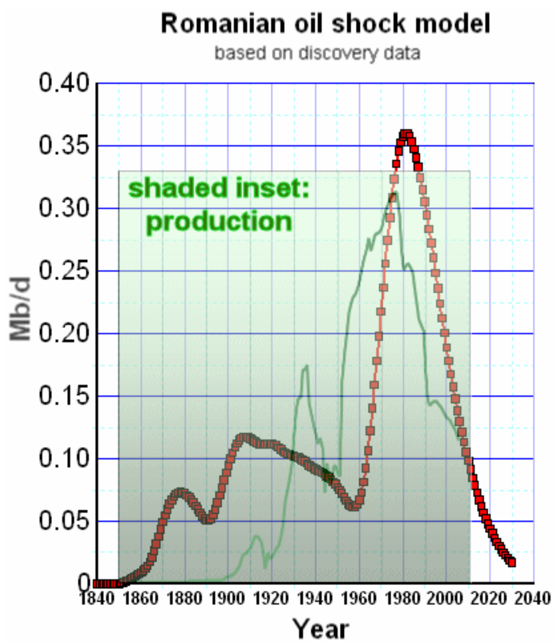


FIGURE 12-48. Romanian oil shock model. The stochastic approximation starts to break down on a region with a small amount of drilling prospects.

The oil shock model misses the actual reported peak by a lag of less than 5 years, but does catch up on the down slope. Earlier on in the production life cycle, the model does predict the two other significant peaks (corresponding to discovery deltas), but both the location and scaling appear way off. It almost looks as if much of the very early discovery estimates either did not deliver as much oil as predicted or, less likely, went to waste for whatever reason. Based on Laherrere's analysis that the cumulative oil production has not matched that from discoveries (see below), it would not surprise me if the “missing-in-action” pre-1900 discoveries account for the discrepancy.

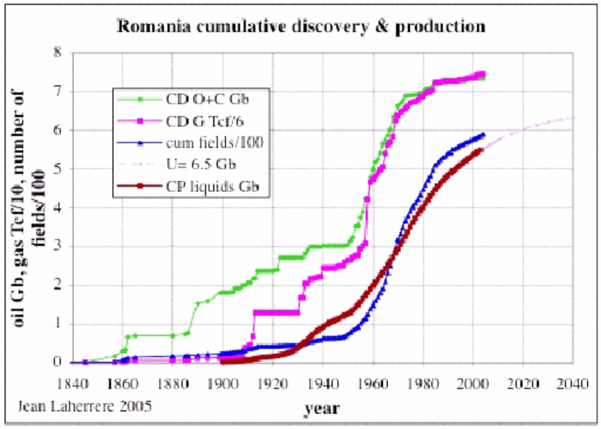


FIGURE 12-49. Cumulative discovery and cumulative production. Note that asymptotic oil looks like it will not match that predicted by discovery.

Natural Gas

USA

As a final comparison we consider the results obtained from modeling natural gas production. Laherrere provides data for discovery and production for USA natural gas in the following charts.

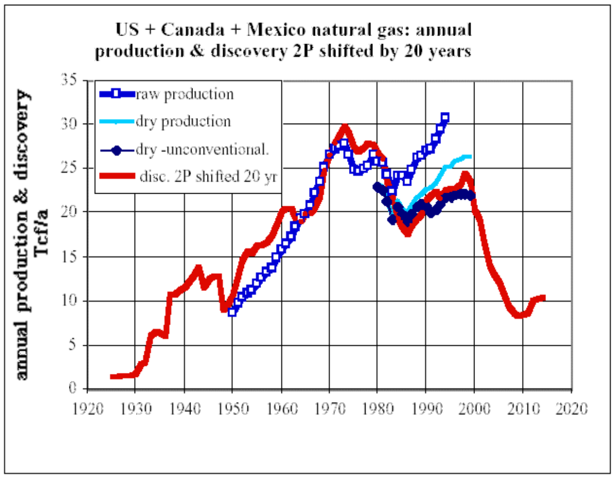
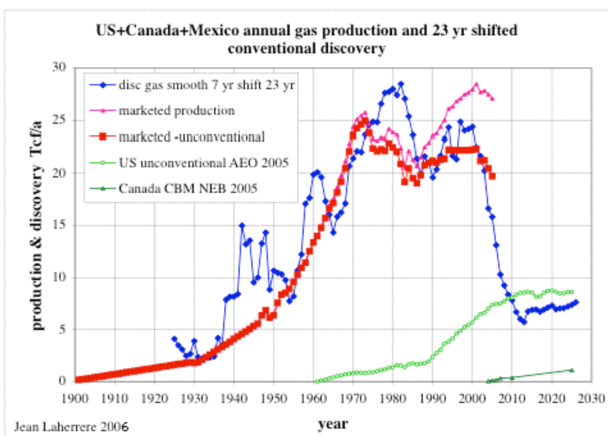


FIGURE 12-50. Natural Gas production and discovery from Laherrere. [Ref 86] (lower) additions due to unconventional sources.



Notice the obvious time-phased shadowing of the discovery curve by the production curve, and the implied reserve depletion that this portends[16].

16. Albeit, some of the gloom gets abetted by the rise in unconventional NG deposits.

I had not intended to use the Oil Shock Model to predict natural gas production, partly because of the lack of discovery data and partly because of the supposed abrupt dynamics of natural gas reservoirs[17]. The latter issue might imply that we can't quite as confidently assume a depletion rate proportional to the volume as a first-order estimate. On the other hand, big reservoirs, like big aquifers, do produce quantitatively more than small ones, so this approximation holds some merit.

Again, Laherrere typically time-filters his discovery profiles (see above figure, in this case, a 7-year average). The shock model naturally accommodates the raw data and can generate a meaningful production profile, noisy or not.[18]

In any case, given the fact that Laherrere provided *some kind* of discovery data, I decided to model the USA conventional NG data via the oil shock model. I chose fallow, construction, maturation, and depletion rates of 0.133/year (which is a fairly standard 7.5 year mean, in terms of *oil* depletion models) and I got the following dark line fit:

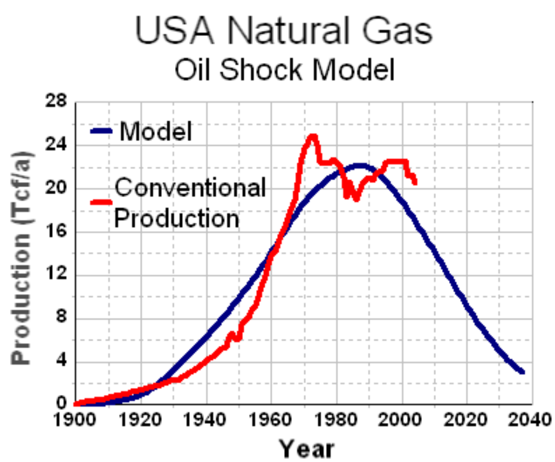


FIGURE 12-51. Oil shock model applied to natural gas, with no perturbations added to the fit.

What do the 0.133 numbers physically mean? In a stochastic world, all it means that it takes, on the average, about 7.5 years to start work on a discovered field, 7.5 years to get construction finished, 7.5 years for it to reach maturation, and simultaneously factor in a depletion rate of 0.133 volume per year. The aggregation of these terms causes the discovery profile to shift approximately 23 years (similar to Laherrere's annotation [Ref 164]) to match the production profile. Understand that

17. Do we accept the conventional wisdom that they shut down quickly at the end of their lifetime? Results from recent NG shale deposits support this notion, which indeed demonstrate very short production life-cycles.

18. See New Zealand natural gas in the next section.

this analysis does not necessarily work on individual fields, but to first-order (a Markovian process) it does explain everything you need to know on an aggregated set of fields, any one of which can vary according to its own specific parameters (i.e. how long it sat fallow, how long construction took, *etc*.).

The deterministic, or shock, aspects to the model come about when political or economic effects are taken into account. Even though I did not add it to this particular model, rapid changes, due to collusion or world events or technology, can cause the extraction rate to adjust at certain points in the curve. This allows one to get insight into deviations from the general trend.

From the residual errors, you can see max deviations around depression/WWII, 1971, 1984, and 2000 (I question data before 1920 because it looks extrapolated as a straight line). I would consider these candidate time points for introducing changes in the extraction rate.

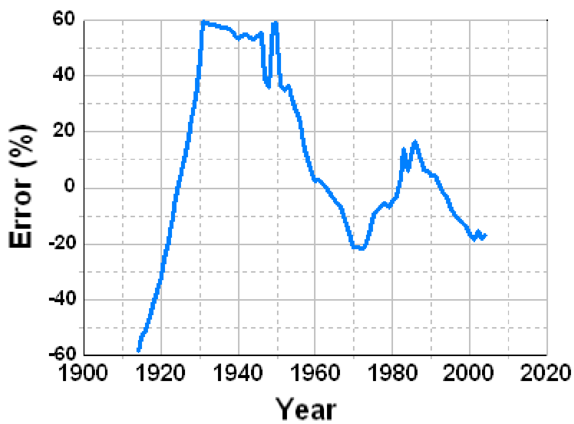


FIGURE 12-52. Residual errors from the unperturbed oil shock fit.

Alternate Fit. The previous model assumed a constant depletion rate over time and fit the real production curve in scale only. I prefaced use of the model with the caveat that natural gas reservoirs may not necessarily deplete at a rate proportional to the amount left, which forms the underpinnings of the oil shock model. One could imagine that a natural gas reservoir could deplete closer to an analogous water cooler, maintaining a steady flow until empty[19]. Yet gas reservoirs should follow Boyle’s law for a volume of gas, which implies a proportional extraction (see page 84).

19. Which conversely also means that one can throttle the rate presumably just as easily.

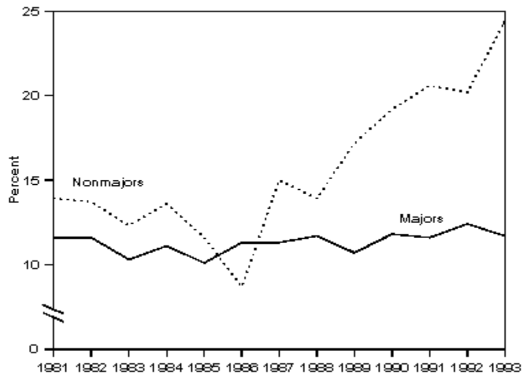


FIGURE 12-53. Extraction Rates for Offshore Natural Gas Reserves. Note: Extraction Rate = Production / (End-of-year reserves+ production). Nonmajors = Total United States (less 15 percent royalty interest) minus majors. Sources: Majors: Energy Information Administration, Form EIA-28. Total United States: Energy Information Administration, U.S. Crude Oil, Natural Gas, and natural Gas Liquids Reserves, 1981-1993 issues, DOE/EIA-0216 (Washington, DC). [Ref 87]

As the shock model does indeed allow a variation of extraction rate over time, I decided to fit the production curve again, but this time letting the extraction rate vary quite a bit.

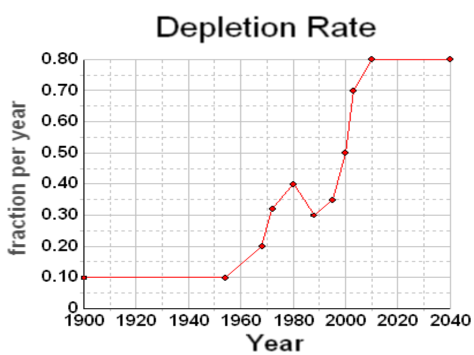


FIGURE 12-54.
The shock model applied to natural gas shows a strong increase in extraction rate.

As a first step I adjusted the (fallow,construction,maturation) rates down to 0.1 (10 year 1/*e* time) from the previous 0.133 to match the early evolution of the curve.

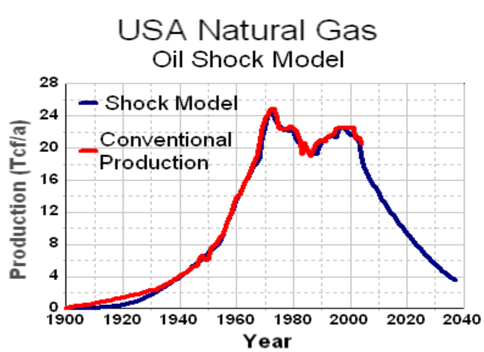


FIGURE 12-55.
Adding perturbations to the model in the form of extraction rate variations improves the fit to natural gas production date.

Combining the static rates with the variable extraction rates I generated the following fit and error curve.

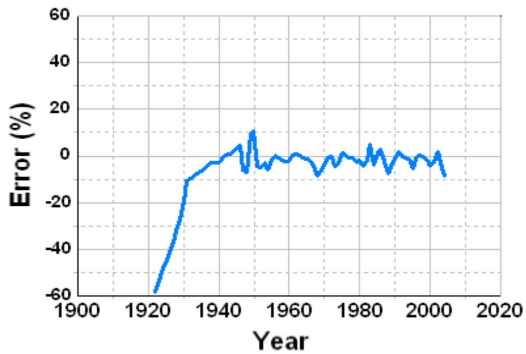


FIGURE 12-56. Decrease in residual errors of the natural gas shock model due to modifications of the extraction rate.

Remember that this pertains to conventional sources of natural gas; de Sousa covers the distinction between conventional and unconventional [Ref 86]. It appears that of recent sources of NG, many come from shale deposits which require fracturing and generate short production lifetimes. So this increase in extraction rate could come from improvements in technology allowing us to more than maintain the flow in the face of increases in demand. Withdrawing 10% per year of the volume worked fine for us for the better part of last century, but as we hit the 80% level, hard constraints have to follow. And that means we could still see the steep cliff heralded for conventional natural gas depletion. [20]

Yearly Delineated Natural Gas Analysis. Natural gas cumulative graphs typically use a unique presentation format. Note that you will rarely find graphs for oil whereby each year gets tallied separately [Ref 119].:

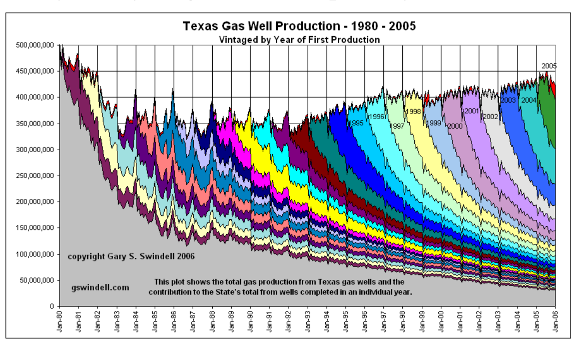


FIGURE 12-57. Layered natural gas production generates a long plateau.

20. On reviewing my only other natural gas model, New Zealand NG shows the same characteristic upswing in extraction rate on nearing a cliff. Note that USA extraction rate more than doubles in a similar time frame.

I presume the reason that oil cumulative graphs don't often get presented this way arises because the maturation of oil wells do not correspond to a given year very concisely. A variable maturation time essentially spreads the actual production over several years so that the chart wouldn't have the same columnar contrast. On the other hand, the draw from a natural gas reservoir occurs immediately and so the yearly data shows up very strikingly.

The Oil Shock Model handles the analysis passably well, as we want to handle only the short cycles. I basically set the maturation level to zero years and tried to emulate the chart's look.

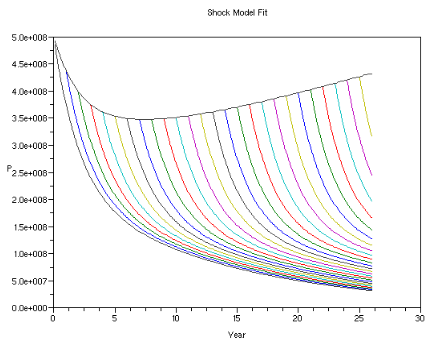


FIGURE 12-58. Fit to the yearly envelope of Texas Well production. Each year's production is modelled by a damped exponential with a small reserve growth component.

With the overlay below:

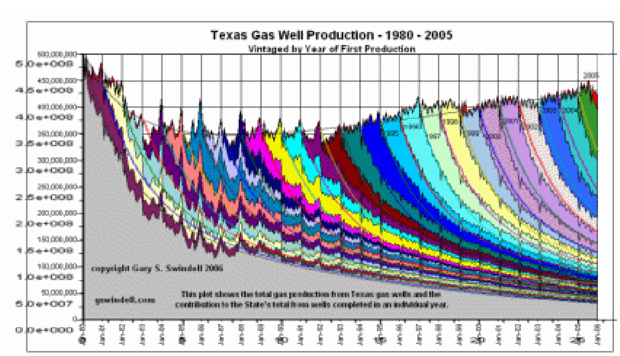


FIGURE 12-59. Model data overlayed on natural gas production.

Not having the actual discovery data available, I used empirical fits to generate the individual curves. The base curve for the years prior to 1980 results in a curve that follows the relationship:

$$Base = K \cdot (0.55e^{-T/2} + 0.45e^{-0.15(T/2)}) \quad \textbf{(EQ 12-1)}$$

This essentially gives a fast slope and a slower slope which approximates a reserve growth component that I have described earlier as "Shocklets in Action" . The individual yearly production curves also show a similar behavior (here small t corresponds from the date of the discovery):

$$Base = Gain \cdot (0.9e^{-t/2} + 0.1e^{-0.1(t/2)}) \quad \textbf{(EQ 12-2)}$$

The slow portion contributes only 10% of the bulk of the growth, so the reserve growth doesn't amount for much.

The other piece of the fit involved the contribution of the gain which basically generates the envelope of the curve, starting from the initial point in the data collection at 1980.

$$Gain = (1 - 0.7e^{-0.02T}) \quad \textbf{(EQ 12-3)}$$

This gain function contrives to demonstrate that continually greater amounts of Natural Gas get extracted per year but the trend does not show that a peak will arrive any time soon. It instead suggests that new wells get constructed to meet the demand for Texas. The yearly time constant for each year's output remains a short two years, so that when a drop-off in production occurs, it will happen fairly quickly.

The caveat to this analysis like the oil analysis for the USA, is that we have relied on actual reported discovery data instead of a model. Most analysis shows that the reserve situation remains more optimistic for natural gas (due to shale deposits, etc) than for oil in the USA. This too, will reveal as a slower decline after peak since we can extrapolate discoveries to the out years. Additionally, if we need more natural gas often it amounts to the selection of a new area to drill — as we have flow-rate-limited extraction in many regions the natural gas pipeline becomes the bottleneck and thus requires extra piping to carry the natural gas away. Oil does not suffer from this flow restriction as it travels globally wherever roads or ships exist.

New Zealand Natural Gas

New Zealand has a small set of natural gas fields that we can enumerate. The oil shock model has no real dependence on geology per se, as it simply models rates with first-order depletion (i.e. rate proportional to quantity left). As the list of fields reaches a handful, the statistics may not prove quite as useful.

Field	Year	Total reserves (Bcf)	Remaining reserves (Bcf)
Maui	1973	3438.83	319.16
Kapuni	1970	1365.8	339.55
McKee	1983	182.7	66.42
Tariki / Ahuroa	1987	115.48	34.04
Mangahewa	2001	72.5	42.85
Waihapa/ Ngaere	1987	28.84	0.38
Rimu	2002	21.99	19.58
Kauri	2005	42.28	38.81
Kaimiro / Ngatoro	1984	25.74	8.41
Total		5294.16	869.2

FIGURE 12-60. Production data from the limited set of New Zealand natural gas fields [Ref 88]

So, as a caveat, the statistical data for New Zealand NG does not contain any extra data. Just a few fields contribute to the sample space. This means that the stochastic approximations I make do not have as great a bearing on the results as a larger sample would, and deterministic effects consequently have a greater effect. However, the prospect of a *natural gas cliff* has some potentially huge ramifications; the New Zealand case study may illustrate this as well. So, if we model correctly, the cliff will show up — whether we have determinism or not.

I used mean time constants of approximately 6 years for the fallow, build, and maturation phases and also 6 years for the *1/e* "half-life" extraction time constant in computing the results. This generated the red curve in the following chart.

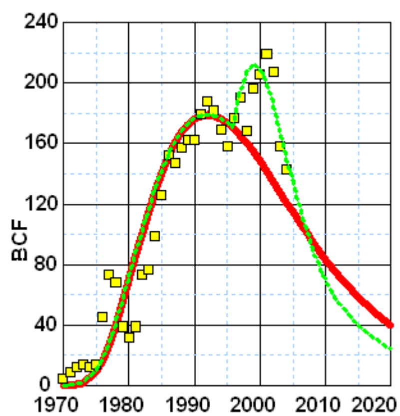


FIGURE 12-61. Shock model fit to the New Zealand natural gas production data assuming estimated reserves.

But clearly, right around the year 1996, you can see production (in billion of cubic feet) starting to ramp back up. To get this to work out, in the context of the oil shock model, I have to add a strong linear extraction rate increase (see the dotted green line).

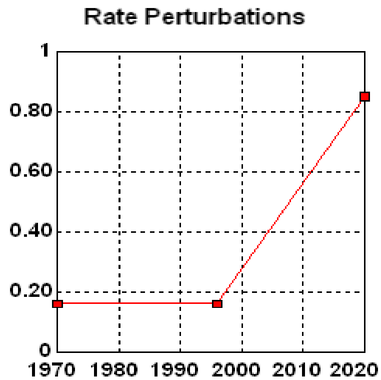


FIGURE 12-62.
Extraction rate perturbation needed to adequately fit the New Zealand production data.

Assuming no future discoveries, this shortens the half-life to just over a year by the time 2020 rolls around. Like the case of petroleum (i.e. UK North Sea oil), extraction rates have to increase to meet the needs of demand. Since New Zealanders can't get the gas from anywhere else, they basically have to follow the cliff down.

New Zealand Update. Since the previous model was created (November 2005 [Ref 91]), a few new natural gas fields came into production[21]. From recent data, an uptick of natural gas production came about from these recently discovered regions

> Shell's $650 million Pohokura gas field off Taranaki is on target to deliver its first contract volumes (2006). [Ref 90]

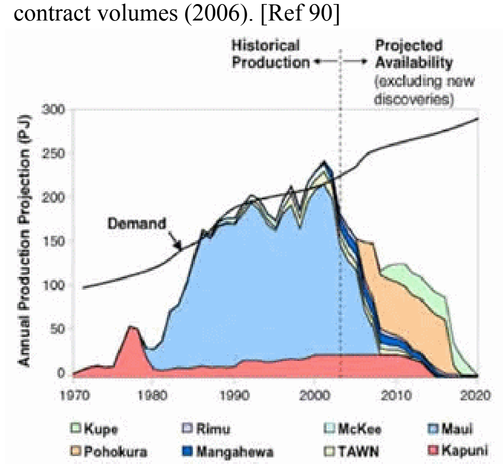


FIGURE 12-63.
New discoveries added to the New Zealand production data that initial discovery stimulus did not account for [Ref 89]. An extrapolated discovery model helps to account for future unknown discoveries.

This demonstrates that new discoveries not anticipated from a non-extrapolated data set, such as that in Figure 12-60 on page 263, will likely undershoot projec-

21. Thanks to Big Gav (proprietor of peakenergy.blogspot.com) for pointing this out.

tions (note that the Pohokura does not show up in the table). Again, one can remedy this shortcoming by using an model such as dispersive discovery to extend the discovery tails.

CHAPTER 13

The Discussion

Which conditions can impact the model?

"The Lord giveth, and the Lord taketh away, but he is no longer the only one to do so."

— Aldo Leopold

"The God/Creator doesn't do much these days. He must be emeritus."

— Robert L. Park

"*Earth Flat, Views Differ*"
— Newspaper headline suggested by Paul Krugman

As formulated, the comprehensive discovery+production model tracks the most probable trajectory of oil depletion based on historical information, with provisions for modifying its parameters to span future projections. How much the simulated trajectory differs from the actual outcome depends of course on the assumptions that we have made. Assuming we have derived the model correctly, we have to question whether we have chosen the right parameters and growth dynamics. By applying sensitivity analysis to the model, we can see how robustly it behaves in response to variations. As a worst case, which provides some practical insight as well, we can drive the model hard with excursions that span the range of outcomes.

Assumptions and Margins of Error

Some of the assumptions that crop up include:

- Reserve growth has *finite* limits — this has significant implications if it became *infinite*
- Finding big fields first occurs randomly — this tends the model to the conservative
- Discovery of oil includes only high grades of crude — see the effect of EROEI on lower grades
- Other "fat tail" or "black swan" phenomena — will another form of energy get discovered, mitigating a crisis?
- Recovery factor for oil — as estimated URR < OOIP and not all this gets recovered this usually means a more conservative outcome than anticipated

Of these assumptions, I feel that the possibility of higher reserve growth holds the most immediate interest.

Even Infinite Growth Won't Matter In The Long Run

As we found out earlier, the conventional wisdom of the USGS classifies reserve growth as an enigmatic phenomena. It appears likely to occur, yet the current research says that we don't understand it very well [Ref 42]. Moreover, when it comes right down to it, the potential size of the reserve growth may not even matter

Towards that end I want to present a simple idea to understand the concept of reserve growth. In general, we use the term *reserve* to indicate how much of something we have left. So on a micro-scale it can mean how much gas you have left in your car's tank. On a macro-scale, reserve indicates how much we have available in crude oil reservoirs as yet unextracted. The professional analysts further categorize reserves in terms of proven, potential, and possible. *Growth* in reserves indicates how estimates change, almost always upwards, over time.

As a frequently cited explanation for reserve growth, consider that since reserves get reported by the owners of the reservoirs, that they do not always want to run the risk of over-speculating. This means that the corporate analysts will consistently lowball their early estimates and then raise it over the course of time as the production starts to catch up to the initial reserve level. In general, they take a conservative approach in that they appear to not know how high the final estimate will run.

Yet, however high reserve growth may get, it may not deliver the promise of extending the time to peak as much as many would predict. As a premise, let me create a hypothetical situation. Say starting from right now, i.e. *Time*=0, we find a growth in reserves that goes like 1/(*Time*+*k*), where we assign "*k*" as some small number to keep the starting number finite. Let us say that this reserve growth falls in the provable category to indicate that we can extract it.

Three interesting results spring from this premise.

1. The amount of reserve left from now until eternity sums to an **infinite** volume. This derives from a property of integrating a hyperbola (1/*Time*) over all of time. In other words, we get an ultimate recoverable reserve (URR) of infinity from such a fat-tailed distribution.
2. If production follows a rate proportional to the current reserves (the classic "greed is good" assumption which explains man's and the free market's capitalistic instincts), the position of peak won't change too much. This has everything to do with rate considerations; as the rate of reserve growth cannot match consumption rates, and new discoveries clearly continue to dwindle.

And most importantly, the one aspect that explains a common puzzling question.

3. The draw-down from reserves can become vanishingly small in this scenario. Taking finite production from an infinite pool leads to the conundrum that we will continue to extract an infinitesimal fraction of that eventually available.

I consider the argument quite subtle, so that if interpreted incorrectly, it gives ammunition to the cornucopians, who can turn the argument around and assert that even pessimists anticipate that huge reserves lay in wait. However, in reality, since oil depletion occurs proportionally to current reserves, we end up seeing the classic effect of "diminishing returns". Of course this has real ramifications for a continuously growing energy-based global GDP economy, but the cornucopians will not spin it that way. They will instead point to a continually available reserve that doesn't get drawn down by as much as one's expectations can intuit. The value of the cumulative R/P (the so-called R/P statistic) essentially goes to infinity here [Ref 188].

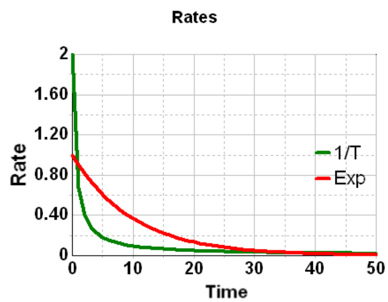


FIGURE 13-1.
Extraction rate and reserve growth factor differs in weighting. Reserve growth has a quick damping and longer tail.

The above figure shows an extraction term corresponding to an exponential and a reserve growth indicated by a $1/(T+k)$ function. The convolution of the two — shown below[1] — roughly gives an idea of the overall extraction. (This essentially becomes a variant of the shocklet described earlier).

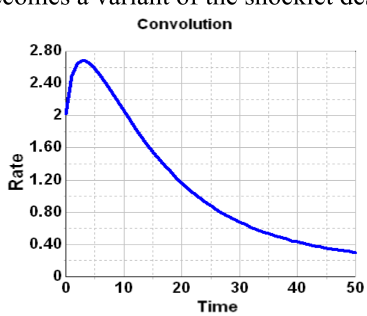


FIGURE 13-2.
The convolution of a fat-tailed reserve growth and fixed extraction rate generates an asymmetric production curve.

1. Spreadsheets don't have a convolution function as far as I have found, one can use the Fourier transform on each function, do a complex multiplication, and then do an inverse transform on the result. This does the convolution effectively, albeit in a roundabout way

Having to defend this argument, a cornucopian would have to propose a reserve growth rate that will keep pushing the peak into the future. Unfortunately, this would result in a growth even more aggressive that the 1/*Time* variant, which already has an infinite URR! This would rely heavily on strong technical growth to keep ahead of the curve.

Based on this kind of thought experiment, we have to continue to question potential reserve numbers. Obviously corporations and nations want to maintain their competitive advantage. Continuing to ask questions remain one of the few options that we have to counter the endless public rhetoric (such as whether to drill in the Arctic National Wildlife Refuge to maintain meager returns)[2].

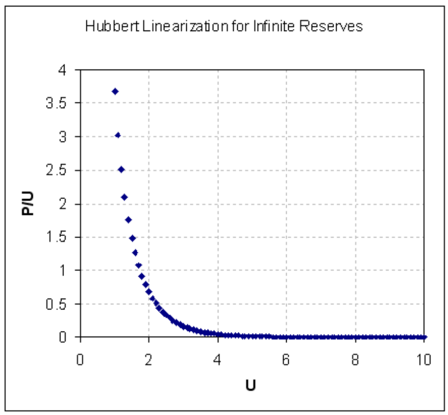


FIGURE 13-3. Hubbert Linearization of infinite reserve growth. Note that the tail will continue forever, asymptotically nearing zero but never reaching it.

Working with Reserves Only

Often we only have reserve information in the form of a discovery aggregate. Take the case of production data from the OPEC-member country of Libya. From ASPO Newsletter #34, we have a cumulative discovery and a cumulative production as our data set. If we approximate the cumulative discovery as a simple reserve growth, and apply a shock model to obtain the cumulative production, the trend becomes very easy to intuit, and then to plot.

2. As a recent and complementary example of where people have gotten hoodwinked in this fashion, google the "infinite horizon" argument to escalating Social Security costs. The Bush administration had actually suggested huge future costs of S.S. based solely on a hidden assumption of an "infinite horizon". The now senator Al Franken debunked this argument quite effectively by saying that, "yes we may have a huge SS deficit, but will have infinite time to pay it off, so it looks like our current funding is no problem".

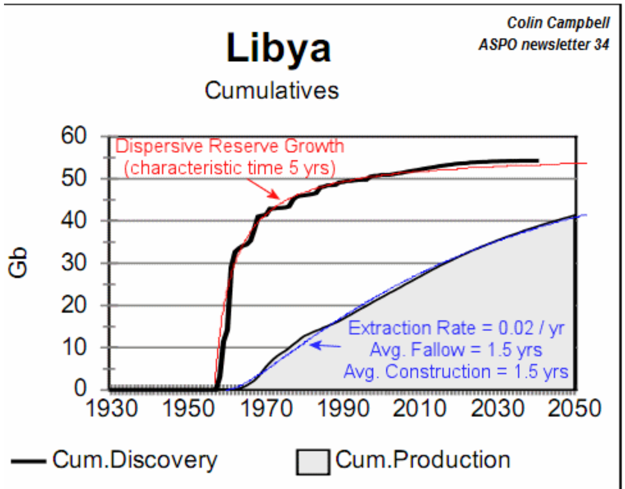


FIGURE 13-4. By looking at cumulatives only, we can extrapolate a model-based cumulative production profile for Libya. The main components include a reserve growth discovery component, and a constant extraction rate

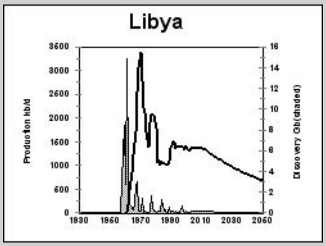


FIGURE 13-5. For Libya, the discovery peak occurred early so that the entire country behaved like a production shocklet, governed by extraction from a long reserve growth tail.

In other words, the fact that a discovery peak occurred followed by a decline in further discoveries, indicated that we can apply a reserve growth model for that profile. The country appears to look like a large basin that we can model with a Dispersive Reserve Growth model from the initial discovery stimulus. This in effect makes Libya an example of a huge oil production shocklet, perturbed only by minor fallow and construction effects.

Monitoring Reserve Peak

Staniford suggested looking at peaks in the estimated reserves of oil as another way of predicting future trends [Ref 92]. He plotted the reserve estimates on top of the production curves for several geographic regions and noticed that reserves typically lead the production curves, showing, in general, an earlier peak for reserve estimates than for production.

I can't argue with that logic and it does add a new dimension to looking at the data. Typically, we see the famous production curve (i.e. Hubbert curve) published and occasionally see a similarly peaked discovery curve that precedes the production curve by sometimes as much as 40 years. As this relationship usually holds up pretty well, the reserve peak ends up positioning itself somewhere in between the two peaks (the first peak in Figure 13-6 on page 272).

$$\text{Reserve} = \text{Cumulative(Discoveries)} - \text{Cumulative(Production)} \qquad \textbf{(EQ 13-1)}$$

Then, given the above relationship, we can pinpoint the reserve peak precisely in time, and at the very least just by eyeballing the two original curves. If the reserve peak occurs when the derivative of the above relationship goes to zero, and

$$\frac{d}{dt}\text{Cumulative}(x(t)) = x(t) \qquad \textbf{(EQ 13-2)}$$

then:

$$\frac{d}{dt}\text{Reserve}_{peak} = \text{Discoveries} - \text{Production} = 0 \qquad \textbf{(EQ 13-3)}$$

or the striking result that the reserve peak occurs when Discoveries/year = Production/year.

As an example, we can look at the Norway data (Figure 12-21 on page 236), which has recently hit a peak and has begun the decline down the backside. The following figure shows the reserve peak overlaid with production data:

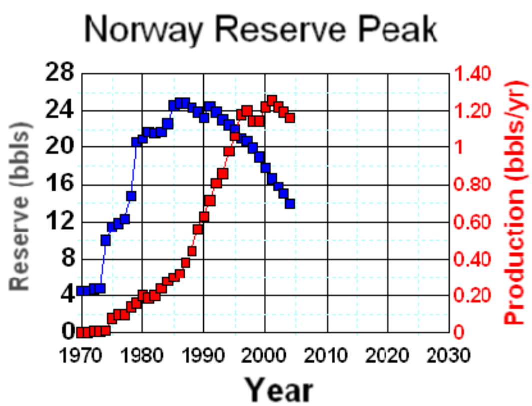


FIGURE 13-6.
Peak in oil estimated in reserve lies between the discovery peak (not shown here) and production peak. If an area lacks a discovery profile, a reserve profile works very well for analysis (including data from backdating).

Clearly, you can see the reserve peak occurring approximately where the noisy discovery peak on its downward slope meets the production curve on its upward slope (actually you see quite a few intersections corresponding to a number of local maximum). And, like I said, you can actually — with eyeball accuracy — pinpoint the comparable (mini) reserve peaks occurring at 1987 and 1992.

This does not prove that the reserve peak will stay put. Because of backdating of discoveries, it may move around a bit. And even if more reserve growth occurs, the reserve peak, like the discovery peak, will stay in our rear-view mirror.

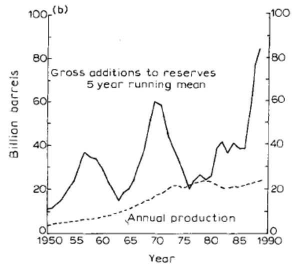


FIGURE 13-7.
Historical reserve growth from Dake referring to Odell [Ref 172]. "Much of the [price] increase can only be defined as a self inflicted wound on the fabric of western society arising from policies which reflected the unsubstantiated belief in an inevitable scarcity of oil." "the world is running into oil, not out of it"

Waiting for Further Growth

The slow growth of reserves in the dispersive mode means that future production will get evenly spread over the out-years, thus prompting *economic limit* considerations which eventually lead to *shutting-in* of the data. For a damped exponential increase of a reserve, the pay-off time it takes for the reserve to reach an asymptote averages out to a finite duration. This essentially gets formulated as an expected value of a duration t determined by weighting with the non-cumulative reserve growth normalized as a probability $P(t)$.

$$\bar{E}(t) = \int_0^{\infty} t \cdot P(t)dt \qquad \textbf{(EQ 13-4)}$$

For the exponential this becomes the reciprocal of the damping rate. The fast closing of the curve to the eventual asymptote ends generates little weight to longer times. However, for dispersive discovery, the decline in growth goes like $1/t^2$ so the expected value of the duration to complete reserve growth becomes infinite[3]. Clearly, no operator will wait forever to every last drop of oil so they will decide to shutdown depending on what percentile of reserves they have achieved — in other words, they will essentially observe the creaming curve converging arbitrarily close to an asymptote before it becomes uneconomical to continue waiting. That becomes the economic limit for halting further production. If this process looks subjective, that's because the lack of any moment computation, such as a mean or sigma, from hyperbolic growth disallows any quantitative measure.

3. The Cauchy distribution, another $1/t^2$ behavior, also has an undefined mean. This is used by Mandelbrot to invoke "fat-tail" behavior [Ref 184]. Also known as Lorentzian [Ref 209].

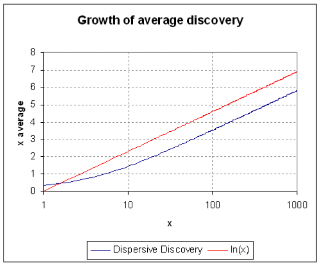


FIGURE 13-8.
Log increase in effective wait times, x=1/(1-P)

This negative form of dollar-cost-averaging, besides cutting into an oil producers potential profits against operating costs, has the effect of censoring some data from the model[4]. For dispersive discovery, the time/cost trade-off goes according to *log*(1/(1-*P))* where *P*=fraction of cumulative growth reached. So to go from 0.9 to 0.999 of asymptotic growth will take three times as long as to go from 0 to 0.9 You can refer to this as the curse of dispersion, as it describes another form of the law of diminishing returns. In this case the diminishing returns caused by slowly tracking down the last vestiges of oil becomes counter-productive from a financial point-of-view.

The Simplicity of the Model

Two issues that I consider very important, (a) using historical data and (b) using realistic yet simple models, will inevitably influence some people to take the results the wrong way. I think I have good arguments covering these concerns.

For understandable reasons, the use of historical data causes certain oil watchers to hesitate in their support of potentially applicable models. This thinking arises in the same sense as that famous bit of financial wisdom states: *"Past performance does not guarantee future success."* Yet they should realize that we can learn a lot, both in a Bayesian sense and in avoiding running across the same mistakes over and over again.

Otherwise, I can buy the remain skeptical argument only up to a point; for example, until we hit and go past the peak, we can't with 100% certainty predict the peak location. Having a real inflection point or an obvious concavity does wonders for this class of model. Yet, should new discovered reserves pop up, as in the New Zealand Natural Gas analysis., then relying on purely historical data illustrates real shortcomings with the approach. That essentially provides the rational for *extrapolating* a Dispersive Discovery model to fill in potential new finds in the "fat tail" of the distribution.

I also find the simplicity of the model important to defend. Unfortunately, simplifying to such an extent, by reducing it to a Logistic sigmoid curve, that one only has a single parameter to adjust, might make it look suspiciously fake. I find that much like having to make a prediction given only the *mean* of some data set, I would at least like to have a couple of parameters, say the mean and *variance*, to improve my chances. Yet, that in fact explains why Dispersive Discovery simplifies to the

4. This also describes the effect of marathon runners dropping out before completing a race (or never attempting one to begin with). Ideally, a distribution of marathon finishing times would show a wide dispersion due to differences in athletic ability, but in fact this self-censoring truncates the resulting data and one never sees the long tails. The pain and suffering does not make up for the reward. See Volume 2 for more insight into this behavior.

Logistic model in this situation. The standard deviation of the accelerating search effort always equals the mean and the sigmoid curve naturally falls out. As we usually find, a good physical model that provides good intuition and insight to the problem at hand has very practical uses.

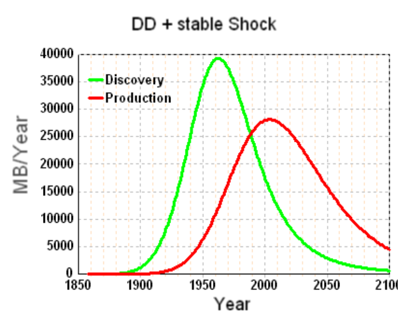
Statistical Precision

If we run Monte Carlo simulations of the Dispersive Discovery and Oil Shock model we can potentially get a handle on peak variability. I usually work the models out analytically because that gives the most probable outcome, but the Monte Carlo approach gives you some extra insight into how the statistics play out. In other words, each set of Monte Carlo runs gives a possible “alternate history” (AH) describing the passage of the oil age.

The fact that we can have significantly different alternate histories has to do with the fat-tails of the reservoir sizing curve. The rank histogram of the world’s large reservoirs suggests that we will likely find at most a couple super-giants nearing 100 GB in URR. Since these occur sporadically (as Taleb’s gray swans) yet have a significant impact on oil production, the Monte Carlo simulations should reflect the possibilities of super-giants occurring. So up to this point, we only have one alternate history to contend with, but the out years will likely show a variation from the expected analytical result due to the odd super-giant potentially still lurking.

The model used corresponds closely to the analytical Dispersive Discovery/Oil Shock model of Chapter 8. I did not retain the oil shock perturbations in the MC as the effects of the noise fluctuations in reservoir sizing can blur the distinction. So instead of the shocked curve of Chapter 8, we can analytically generate the unshocked curve below.[5]

FIGURE 13-9. Analytical Result of Dispersive Discovery plus Shock Model with a stable extraction rate (i.e. no shocks)



5. The simulation code is described here: http://mobjectivist.blogspot.com/2009/12/monte-carlo-of-dispersive-discoveryoil.html

The noisy curves in GREEN below represent discoveries per year, while the RED curves indicate production for that year. The noise in discoveries reflects the reality of the situation; compare the curves to the data points collected by Laherrere in the figure to the right (here 100 MB/day=36,500 MB/year).

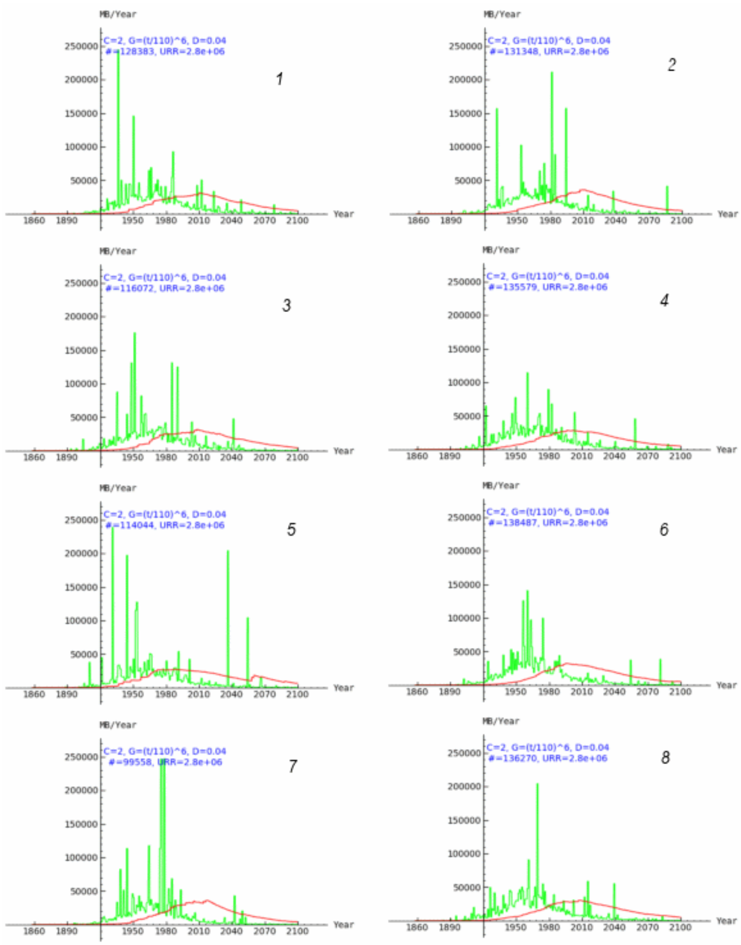


FIGURE 13-10. Monte Carlo runs of Dispersive Discovery + Shock Model

The most frequently occurring of the curves show a peak after the year 2000 but significant numbers occur in the adjacent decades. So even with average sample sizes of over 100,000 reservoirs, a few super-giants strategically placed earlier in the timeline can actually shift the peak quite a bit. Chart #5 place peak closer to 1980, largely due to a set of 3 super-giants (all bigger than Ghawar) occurring before 1950. By comparison, Chart #7 has the super-giants occurring right before 1980 which pushes the peak to the latter part of the next decade.

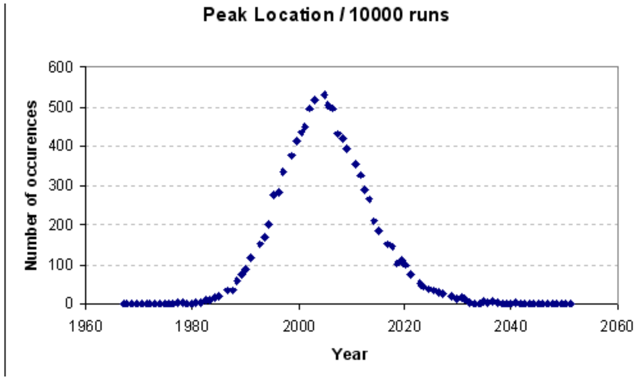
Yet, even with all this variability, if you look at the production level in the out-year of 2100, given a constant extraction rate, all the charts show approximately the same production level at that future point in time. In understanding what causes this common quasi-asymptotic behavior, just remember that all the fluctuations have settled out due to the cumulative number of reservoirs contributing to the signal. Many of the oil reservoirs now in play will continue to produce, just at reduced levels.

Overall the size of the fluctuations agree with that observed for sporadic nature of the super-giants. If the assumption of around 130,000 reservoirs (averaging 20 MB each) holds over time, then we can expect at most one or two more super-giants left. Although impossible to verify, the current cumulative count of reservoirs world-wide stands at likely over 50,000. As a sanity check, I use the formula I originally derived here to estimate the URR from the dispersive aggregation parameters. This should match approximately 2,800,000 MB.

$$\begin{aligned} \text{URR} &\sim \text{MaxRank} \cdot C \cdot (\ln(L/C) - 1) \\ &= 130,000 \cdot 2 \cdot (\ln(250,000/2) - 1) = 2,791,00 \end{aligned} \quad \textbf{(EQ 13-5)}$$

I would consider all of these simulations conservative in the sense that the URR remains on the high side of projections. Add to this the fact that size of discovery has no dependence on time (i.e. we remove the bias of anticipating finding the largest reservoirs first). So even with the large amount of potential outcomes, the number of reservoirs discovered and produced so far point to a relatively small spread in peak dates. The figure below shows the results of running the Monte Carlo simulation 10,000 times and plotting a histogram of the peak dates. The standard deviation in this profile is 8.5 years and the mean/mode/median peak date of 2004 agrees with that of the analytical result shown in Figure 13-9 on page 275, as that result gives 2004 as the predicted peak date as well.

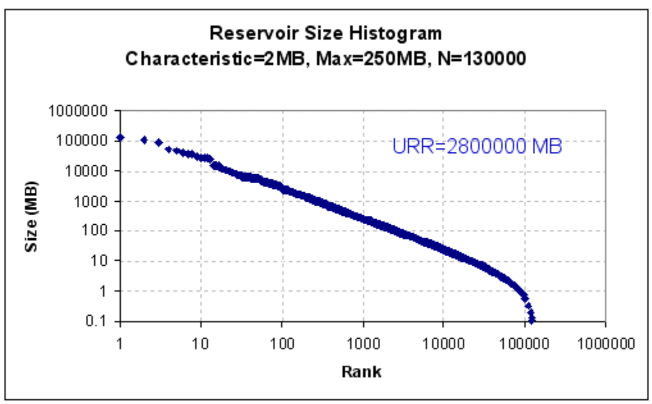
FIGURE 13-11. Peak location based on 10,000 Monte Carlo runs shows a spread in possible dates of several years.



Even though a substantial spread exists in potential outcomes, we have to consider that most of these have occurred in the past and we should discount negative results in any future projection. In other words, since nearly half of those that show large variance in peak date have occurred in the past, we can eliminate the possibility that an alternate history[6] will put the actual peak much beyond the next decade. One can justify this argument by simply considering adding Bayesian priors and running the Monte Carlo from the current date. I believe that this spread in outcomes has probably contributed (along with unanticipated reserve growth) to the usual problem of jumping the gun at predicting a peak date.

To gain an appreciation of the number of reservoirs that played in the simulation, I Monte Carlo generated a few rank histograms shown below. According to the dispersive aggregation algorithm, one can see only a handful of super-giants occur out of the tens of thousands of natural reservoirs.

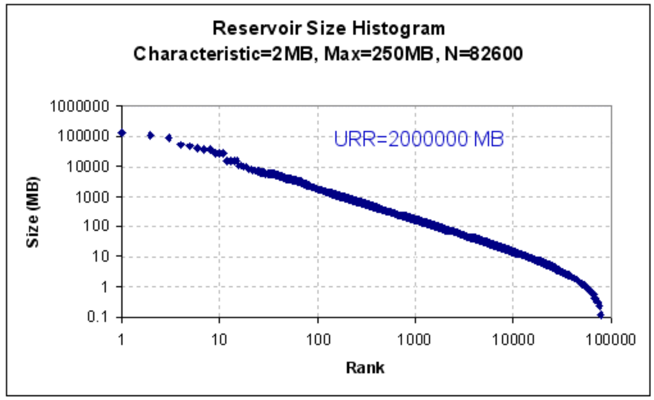
FIGURE 13-12. Reservoir size histogram generated from simulation



6. This term comes from a popular "parallel universe" genre of science fiction.

This next histogram resulted from approximately a 30% reduction in the number of reservoirs drawn. This better matches the current number of reservoirs at least 1000 MB in size (~130 according to Wikipedia's List of Oil Fields) and also the tally of known reservoirs (under 50,000 according to various sources). I expect the number of reservoirs to grow as the last geographic areas get exploited. So the rank histogram above gives an optimistic scenario for URR and the one below borders on the side of a pessimistic projection.

FIGURE 13-13. Reservoir size histogram generated from simulation, larger number of samples.

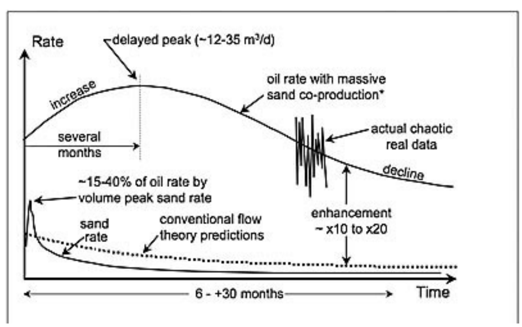


Hubbert Peak in Five Easy Pieces

Natural process can provide an alternate explanation for the classic Hubbert Logistic peak shape during a production cycle, independent of the discovery shape.

Geologists describe a process known as CHOPS (Cold Heavy Oil Production with Sand) which can enlarge a well's streaming throughput by promoting the formation of heavily eroded channels. The following picture of the possible outcome of the behavior [Ref 238].

FIGURE 13-14. CHOPS process. Sand particles acting as a strong abrasive driven along by the already high velocity stream of oil leads to increasing in the channeling and thus an even faster extraction rate.[Ref 238]



Note that the lower curve shows the typical output from a throttled flow. Above that curve, the modulated line shows the results of an accelerated extraction — note that a peak actually appears which pinpoints the maximum flow rate. In terms of the oil spill, we don't want this behavior because it gives us less time to fix or relieve the problem well. Yet, ordinarily we want this same behavior — that of fast extraction — in practical situations because we want and need the oil right now (so that oil companies can make money, of course).

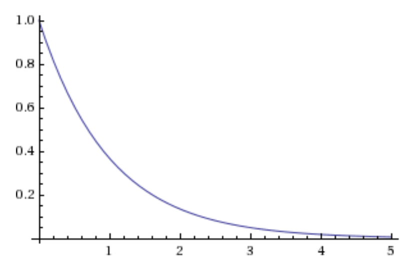
Which leads me to formulating the following very simple but physically correct model of Hubbert's Peak. You won't find this anywhere else, because this derivation does not jive with how geologists think about oil extraction. They get many of the pieces but they never put them all together.

I will offer up a derivation for this behavior leading to a Hubbert Peak in 5 easy pieces.

Piece 1. The standard assumption of draw-down from a reservoir results in an exponential decline over time. You can consider that the exponential shape results from a law of diminishing returns; in that a constant amount proportional to the remainder draws down per unit time. Or you can say that a maximum entropy range of extraction rates gets applied to the volume. A proportional extraction rate that we call $\boldsymbol{R}$ defines the mean and $\boldsymbol{U_0}$ is the reservoir size. $\boldsymbol{U(t)}$ gives us the cumulative reserve.

$$U(t) = U_0 \cdot e^{-R \cdot t} \quad \text{(EQ 13-6)}$$

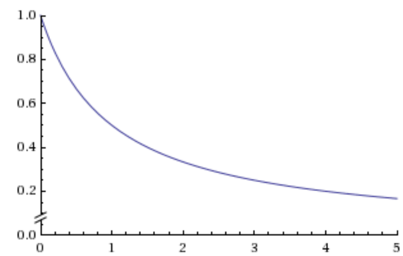
FIGURE 13-15.
Standard assumption of draw-down from a reservoir.



Piece 2. Next, we realize that we have uncertainty over the size of the reservoir; the U0 we have defined actually only serves as an estimate of the size. This means we have an uncertainty over the rate of proportional extraction as well. This turns into a form of hyperbolic decline and the cumulative draw-down actually looks like this.

$$U(t) = \frac{U_0}{1 + R \cdot t} \quad \text{(EQ 13-7)}$$

FIGURE 13-16.
If we assume uncertainty in the draw-down, as we do not know the exact size of the reservoir, we get a more gradual profile. Note that we get the characteristic fat-tail.

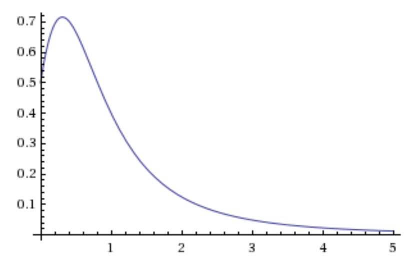


Piece 3. Next we assert that the constant but uncertain proportional extraction rate undergoes an acceleration starting from the original value, $\boldsymbol{R(t)} = \boldsymbol{R_0} + \boldsymbol{k*t}$. This acceleration equates to Newton's law, first-order with time. Then the instantaneous absolute rate of extraction from the remaining reservoir looks like:

$$RateOfExtraction(t) = -\frac{d}{dt}U(t) = \frac{U_0 \cdot (R_0 + k \cdot t)}{(1 + R_0 \cdot t + k \cdot t^2/2)^2} \quad \text{(EQ 13-8)}$$

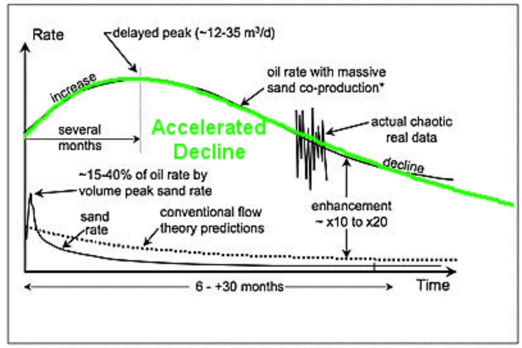
For $\boldsymbol{R_0}$=0.5 and $\boldsymbol{k}$=2, it results in this shape

FIGURE 13-17.
Accelerating draw-down generates a peak in production.



This curve we can scale and overlay on top of the CHOPS curve to validate our thought process.

FIGURE 13-18.
The accelerating decline model matches the CHPS curve.



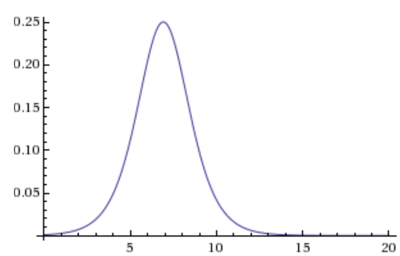
Piece 4. Over a larger set of reservoirs that experience a technical improvement over time, we can assume that the proportional extraction rate can accelerate even more strongly over time, R(t)=C*exp(k*t). This gives us a Moore's law form of acceleration, doubling every set number of years. Then

$$RateOfExtraction(t) = -\frac{d}{dt}U(t) = \frac{U_0 \cdot R(t)}{\left(1 + \int R(t)dt\right)^2}$$

$$= \frac{U_0 \cdot C \cdot e^{k \cdot t}}{(1 + C/k \cdot (e^{k \cdot t} - 1))^2} \quad \textbf{(EQ 13-9)}$$

For a small starting rate, the acceleration further accentuates the subtle peak that we observe in piece 3 and it turns into a full-fledged symmetric peak as shown in the next figure:

FIGURE 13-19.
The assumption of exponential acceleration leads to the classic Hubbert logistic curve.



Piece 5. No fifth piece actually. You haven't broken any rules and you have just derived the famed Hubbert Peak, also known as the Logistic Sigmoid function.

Consistency Check. Earlier we presented an alternate derivation for the corresponding discovery peak, which we called Dispersive Discovery. There, the uncertainty involves how much volume gets explored and at what rate, otherwise the math turns out exactly the same as a localized CHOPS analysis. Both derivations result from an assumed finite constraint but uncertainty in both rates and subvolumes. The only problem with relying completely on the 5 Easy Pieces peak derivation is that it builds on the premise that all extractions started at the same time (globally this would pin it to the year 1858). We know that this has not happened for global production, as extraction can only start after a discovery, and then some variable hold time. And we know that all discoveries did not occur simultaneously.

By using dispersive discovery, we get a larger spread in start years, and then The Oil Shock model generates the extraction curve. In general, if the discovery peak precedes the oil production peak by a number of years, I would use Dispersive Discovery, but if the two coincide, then extraction tracks discovery and it doesn't really matter how you interpret the rates. This explains why this particular derivation works well for more localized production areas that have seen significant technology changes. In contrast, the technology of discovery has undergone tremendous technology changes over the years, so that dispersive discovery works very well in terms of global modeling. This is actually not much of a caveat, as the more ways that you can find the same result, the more confidence you have that you have remained on the right track and you have a robust analytical technique[7].

The current derivation also points out the huge hole in the technique known as Hubbert Linearization (HL). As defined, HL derives from the observation that

$$\frac{\mathrm{d}}{\mathrm{d}t}U(t) = U(t) \cdot (U_0 - U(t)) \qquad \textbf{(EQ 13-10)}$$

Yet this only works for the one case where we can define R(t) as an exponential function, that of piece 4. The formula does not work for either piece 1, 2, or 3. Therefore, HL only serves as a curious mathematical identity for that one exponential case, which we know does not always occur.

The actual linearization takes the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}U(t) = -U_0 \cdot \frac{R(t)}{\left(1 + \int R(t)dt\right)^2} \qquad \textbf{(EQ 13-11)}$$

7. See Cosma Shahili on the utility of simple robust statistical models [Ref 239].

This may not prove as handy as HL perhaps, but it has the benefit of correctness, and it works well for certain cases.

CHAPTER 14

Alternate Approaches.

How do other pessimistic projections fit in?

"A theory has to be simpler than the data it explains, otherwise it does not explain anything"

— G. Chaitin

As you can tell, even though I try to remain objective, the model results cause the net oil outlook to tilt toward the pessimistic side. During the process of tracking oil depletion news and events on a routine basis, I run across many instances of equally pessimistic, yet ultimately unsupported claims. On the other side, you see unsupported overly-optimistic projections. I can't ascribe all these cases to ulterior motives, but many of these hint strongly at some pre-defined agenda, either corporate or political. I essentially see many of these flavors of views:

1. The wrong-headed analysis category
2. The catalog of discredited cornucopians
3. The tin-foil-hat theorizing category

Heuristic Arguments

I see no point in individually disputing every alternate point-of-view but enough uncertainties and confusion exist in some of the unproven heuristics used for serious oil depletion analysis, that it allows the questionable theories to maintain a life of their own. Only until we replace the heuristics (category 1 view) with alternate formal derivations can we put the perspectives from category 2 and 3 to rest. The weight of the counter-argument becomes that much stronger with a comprehensive model that we can start and end with.

In this section I will discuss symmetry, the classical Logistic model, Hubbert Linearization, and return to the Gaussian (to keep it symmetric).

The Misguided Pursuit of Symmetry

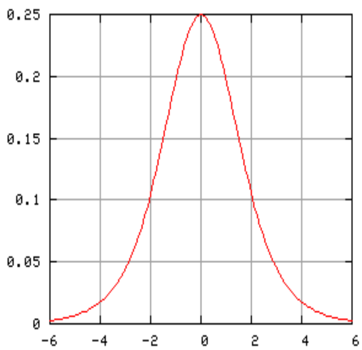


FIGURE 14-1. The Bell Curve has a long history of describing a set of measured numbers that feature a mean or average and variance around that mean.

Peak oil skeptics such as Michael Lynch often dispute depletion experts such as Colin Campbell over a seemingly trivial feature of the oil production profile — in particular that the curves show too much symmetry. I argue that much of the practical rationale for that argument arose (in the first place) from assorted media people who refer to the curves as describing a Bell shape or having a symmetric Normal or Gaussian distribution. This letter illustrates Lynch's attack angle:

> *Michael Lynch's July 14 article is a peevish exercise in intellectual dishonesty. To begin with, he makes an utterly misleading fuss because "oil production rarely follows a bell curve." Much ado about nothing! Hubbert's main point was that a fossil fuel's endowment is fixed; therefore its production curve "will rise, pass through one or several maxima, and then decline asymptotically to zero." "Energy from Fossil Fuels" also stated explicitly that such a curve may have "an infinity of different shapes." How can Lynch not know this? The bell-shaped curve is simply a stylized, idealized representation of the phenomenon of rise, peak, and decline of output, amenable to mathematical expression and analysis, useful as a pedagogical and forecasting device—in fact, the sort of thing economists do all the time. As Lynch should know, real-world data don't necessarily conform to idealized shapes generated by mathematics—and aren't expected to. The shape does handily illustrate the general phenomenon. So real-world data aren't a smooth bell curve. Big deal. What matters is the general pattern of rise, peak, and decline.* [Ref 93]

I agree that Lynch and other skeptical disciples latch on to this description and take the symmetry shorthand too literally. For certain, real curves can't show this exact degree of symmetry — for the simple fact that time-based processes have a non-negative starting point while symmetric curves contain tails that stretch to infinity in both positive and negative directions. A practiced statistician will never misuse the Normal distribution in this way; instead they use it in situations where the law of large numbers applies (see "Special Case: The Central Limit Theorem"). In that case, the long tails quickly dissipate to zero away from the mean; something that does not always occur for the Hubbert curves. Consider the most striking fact that the curves can't stretch to the left of the first discovery of oil in the mid-1800's![1] And of course, Lynch does not talk about that fact, demonstrating some intellectual dishonesty, but it doesn't help that the *believers* in oil depletion rarely if ever talk about this fundamental dichotomy[2]. So Lynch and other cornucopians can occupy a kind of safe "no-mans land", safely taking potshots at the naive interpretation of the classic oil production profile.

1. First oil strike occurred in the USA in the year 1858/1859 and around the same time across the Atlantic
2. A puzzling dichotomy occurs also in the understanding of the Logistic model, both for its similarities to the symmetric Normal curve but also via its questionable classical derivation.

The fact that many depletion experts (on the other side of the fence from Lynch) actually use the symmetric Bell-like curves for their analysis muddies up the waters quite a bit. In fact, the classic Hubbert curve gets expressed most simply as the derivative of a Logistic curve equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}P(t) = rP(t) \times (K - P(t)) \tag{EQ 14-1}$$

also known as the Verhulst equation, or as a symmetric profile in its most basic incarnation, the derivative of the *sigmoid*, where t is time and P is production:

$$\frac{\mathrm{d}P}{\mathrm{d}t}(t) = \frac{e^{-t}}{(1 + e^{-t})^2} \tag{EQ 14-2}$$

Along with a select group of people who study resource usage with a critical eye[3], I have problems with the derivation of the Logistic curve as it applies to depletion. In a hand-wavy fashion, I can understand how the differential equation can empirically match a physical process; unfortunately it contains the non-linear and deterministic factors that typically do not follow from any practical theory (see my attempts to apply differential equations in “The Strawman of Feedback”).[4] It essentially does not derive from any first-principles, and though it may make more sense to a population growth scenario, that doesn’t mean that it also has to apply to a depletion scenario. These two points make very little sense when looked at closely:

1. The rate of reproduction rises proportionally to the existing population, or more correctly (in the case of the Verhulst) cumulative population.
2. The rate of reproduction drops proportionally to the amount of available resources. Thus the second term models the competition for available resources, which tends to limit the population growth

It all boils down to whether you believe that oil molecules behave like living, reproducing species. If not, then the classic Logistics/Hubbert curve only applies as a heuristic, which empirically follows some of the measured production profiles. One feature of this heuristic involves a technique called *Hubbert Linearization* that has gained some popularity for its predictive power. However, until we get past the basics, we will defer talking about the empirical view and concentrate on the first principles. (And as we have demonstrated earlier, we do have one derivation of the

3. Robert Rapier has commented extensively on the use of the logistic and particularly to its use for Hubbert Linearization [Ref 94].
4. Shalizi points out the case of the Brusselator and other disproven nonlinear feedback models such as autopoesis in [Ref 239]. These serve as cautionary tales on the analysis of complex systems.

Logistic which has nothing to do with production, see "The Context of Discovery. How do we simplify the search model?" .)

Logistically Impossible

I really don't believe that the Logistic curve applies very well to the problem of estimating oil depletion rates. No matter what you hear and read, the classical formulation of the Logistic curve just doesn't have the correct physical basis to stand on its own feet. To many, this may sound like heresy, as oil analysts since Hubbert have used the differential equations (in particular, the Verhulst equation) describing the Logistic curve to estimate when peak oil would hit. More than anything, I think its utility came about more through coincidental properties that just happened to match those that **should** come about through a more mathematically rigorous and sound physical basis. In general, I think we violate Occam's razor by attributing something complicated and not physically possible in the oil depletion context (i.e. the Logistic Verhulst equation) to something that we can explain away much more simply and with a better qualitative understanding.

Of course, several issues crop up to make it difficult to stem the tide of use for the Logistic curve[5]. These include:

- Poor or closely-held data
- Data transformations that obscure content
- Inertia
- Convenience

I don't think we can do anything about the first issue. I try to use historical data wherever possible, preferring to fundamentally understand where we came from rather than try to predict the future. By and large, we can only predict the peak when we hit it. But the data remains so limited that any "good" fits to the Logistic curve gain extra weight.

As for the second issue, I consistently see strange data transformations applied to the data. This includes the common integral formulation, typically used to show cumulative productions as a function of time (or some other variable). Everyone should realize that integration acts like an excellent filter, which many times serves only to obscure the original data. And when the other variable does not show independence as in Hubbert Linearization, we compound the problem.[6]

5. The derivative of the Logistic curve gives the familiar "Bell-shaped" Hubbert curve familiar to most people.

Inertia in use of the Logistic curve results in sunk cost rationalization. The aforementioned data transformations seem to spontaneously appear whenever someone wants to "fit" to the Logistic curve. This tends to reinforce the application of the Logistic curve. Far too many analysts have seen the straight lines in their data, and immediately adopt the Logistic curve to do multi-parametric estimations and other fits to discern trends.

The convenience of the Logistic model sums up the current state-of-the-art with respect to its use. Consider again how many analysts plot $dQ/dt / Q$ against Q to try to see the famous "linear" behavior predicted by the Logistic curve. And that remains the crux of the problem. We should not contaminate one already dependent variable onto the axis of the other variable — unless you know for sure that this fits some realistic behavior, it may significantly bias the outcome. For example, to someone familiar with the easy-way-out of mathematical modeling it looks like the "drunk looking for his car-keys under the lamp-post" scenario. *Why does he look there?* Of course, that's where the light is! So, we all use this formulation because of its convenience, not realizing that it could lead us down the wrong path.

So how did the adoption of the Logistic curve come about? I don't know all the historical roots, but you invariably find many references to modeling of analogous "Predator-Prey" relationships [Ref 95], which invariably leads to the equation leading to the Logistic curve:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = k \cdot a \cdot (1 - a) \qquad \textbf{(EQ 14-3)}$$

In the logistic equation, you use the term "a" and "1-a" to refer to a *cumulative* quantity and its complement. Now I understand that most oil depletion analysts want to use "a" to refer to some economic scalar that grows exponentially, while "1-a" to refer to the oil reservoir itself. That makes absolutely no sense from a mathematical point of view, as in the familiar case of mixing apples with oranges. Unless someone establishes a physical relationship between "a" and "1-a", I wouldn't attempt solving this equation because it lacks a basis. And if a relationship did exist, it might not prove linear. In that case, the tidiness of the solution evaporates.

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6. For example, the more I look at it, the more I dislike plotting Production/CumulativeProduction against CumulativeProduction. My father once told me a story long ago about going to a talk by another engineer who got very excited about this great correlation he found in his data set. His data points aligned very well, all falling along a straight line … the engineer had plotted X against X! This sad tale demonstrates what happens when you start mixing dependent variables together; you can too easily get biased correlations.

In the normal predator-prey relationships, you can get away with this approach because you deal with discrete entities that have at least an intuitive empirical relationship. For example, it takes N rabbits to sustain a single fox. Or one virus to infect one unprotected computer. Or in chemistry, an anion and a cation to generate a molecule. So, understanding how analysts have used the logistic model in the past — as a variant of the "predator-prey" class of processes — I believe the modeling premise will have greater viability when applied to other pressing issue of the day, such as the potential spread of communicable diseases.[7] Important, yes indeed, but not a reason to use it for the study of peak oil.

Question: So one can ask, how can this $a \cdot (1 - a)$ relationship come up *hypothetically* in the context of oil depletion?

From the application of the oil shock model, the terms come about from the driving function to this differential equation:

$$\frac{\mathrm{d}R}{\mathrm{d}t} = U(t) - (E(t) \cdot R(t)) \qquad \textbf{(EQ 14-4)}$$

Which reads: *The rate of production equals the discovery rate minus the extraction rate applied to the current reserves*.[8] The tricky term *U(t)* essentially acts like a forcing function. In the past, I have used a triangular discovery function as a first cut. But in reality, discovery also acts as a self-limiting function, as in Dispersive Discovery, and I do not necessarily have to artificially constrain it with a triangular discovery window. The acceleration of discoveries over time naturally decreases. Ignoring for the moment the solution to Dispersive Discovery, it essentially does something like this:

$$a(t) = k \cdot (1 - t/T) \qquad \textbf{(EQ 14-5)}$$

This basically means that the acceleration in the number of discoveries decreases over time, much like the number of strikes during the Gold Rush days showed a maximum acceleration at first, but then declined over time until all discoveries stopped in practical terms.

7. In nature, when you use the logistic model, it starts with a small population of discrete entities, and you let it proceed to (consume/infect/kill/bond) one entity. Then you can sit back and watch as the reaction propagates. The predator prey relationships work best on homogeneous populations. Another, but not the most important, reason to stay away from it. It becomes touchy too with respect to initial conditions, which always gets conveniently swept under the rug in modeling discussions.
8. The expression also reminds me of an RC (resistive-capacitive) circuit or an economics "Supply/Demand" formulation.

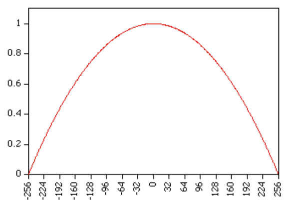


FIGURE 14-2. Welch window from DSP

But we do not actually see the acceleration in the real world; we instead see the velocity, or the number of discoveries made per unit time. And you get velocity by integrating acceleration over time:

$$v(t) = k \cdot t \cdot \left(1 - \frac{t}{2T}\right) \quad \textbf{(EQ 14-6)}$$

And this just happens to look like the curious $a \cdot (1 - a)$ Logistic term; an upside down parabolic function (aka the Welch window, see figure at the left) that provides the driving function to solve the oil shock model. Just a bit of pure calculus straight from Newton. (Furthermore, it doesn't give that much of a different solution to the Oil Shock Model than the triangular forcing function does)

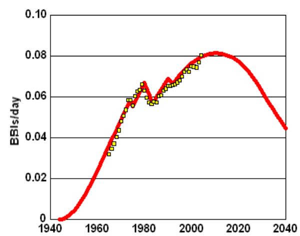


FIGURE 14-3. The curvature near the peak exhibits a characteristic parabolic shape.

Answer: So yes, you can get the $a \cdot (1 - a)$ relationship in the context of oil depletion, with a real physical basis. Unfortunately, the a term turns into a time, t. I have no idea where and how this whole formulation got all bent out of shape other than to attribute it to the natural evolution of a heuristic. A heuristic doesn't actually have to prove or signify any physical phenomena, it just has to duplicate the observations.

Contrary to what many analysts believe, nothing actually grows exponentially when applying this heuristic. With the correct model, we instead have a cumulatively growing set of tapped reserves, which takes work and time to find, ala Dispersive Discovery. This gets offset by a depletion activity which remains proportional to the amount of oil in each new reserve tapped. Unfortunately this logical probability-based description does not describe the Logistic's Verhulst equations, which I find more suited to the epidemiological and ecological sciences, and also to some established chemical reaction models[9]. In no way does this model work for oil depletion. It just happens to give an empirical fit. And people have started building heuristics around this model. Which remains a bad idea born out of fortuity and convenience.

9. See papers on birth-death models in chemistry and materials science [Ref 210]. These often reach carrying capacity based on purely stoichiometric considerations. The chemical constituents often reach a chemical equilibrium which can also undergo limit oscillations if perturbed. Oil does not do this because it never reverts back to equilibrium — it gets extracted and used!

This leads me into explaining the reasons for the good historical empirical match. The basic idea, first promulgated by Hubbert in 1982, and then applied by Deffeyes [Ref 96], states that the solution to the logistic equation leads to an empirical fit to real world data when plotted this way:

$$\frac{dQ}{dt}/Q = K \cdot (1-Q) \qquad \textbf{(EQ 14-7)}$$

I think it fairly straightforward to understand the asymptotic behavior without resorting to the logistic equation. Mathematically, consider that the differential equation governing extraction assumes a forcing function (i.e. discoveries) that have largely occurred sometime in the past when you enter the tail regime. To a good approximation, extraction stays proportional, first order, to how much is left (see stripper wells for the realization of this). So taken far enough to the future, the forcing function looks like a delta function, and the solution set matches the exponential function. Then when you plot $Y = \frac{dQ}{dt}/Q$ vs X=Q you get

$$\frac{e^{-kt}}{(1-e^{-kt})} \qquad vs \qquad (1-e^{-kt}) \qquad \textbf{(EQ 14-8)}$$

In the regime where the Hubbert linearization graph appears linear and it gets close to 90%, so does the exponential. And the match gets better if you put a bit of a spread in the delta function. Therefore you cannot tell the difference and the exponential model wins out because it matches a real physical process.

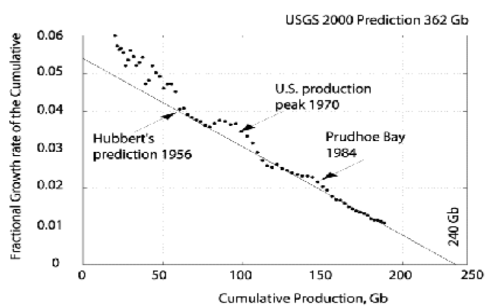


FIGURE 14-4.
USGS prediction applying Hubbert Linearization [Ref 240]

In other words, this has the asymptotic property of the extraction rate “appearing” to intercept the *x*-axis at the ultimately recoverable resources (URR) when *y* eventually reaches zero. However, it never gets there. It behaves correctly, but it has nothing to do with the Logistic curve. It acts more like a very thin man walking toward a wall, every second going halfway there, and then realizing mathematically that he will never hit the wall.

And then notice how the curves match best when we are deep into depletion (i.e. Texas). At that point, we do not need to figure out the URR; to use an electronics analogy, the process just discharges the capacitor in an RC circuit. The Logistic formulation just happens to work when you start fitting past peak because of the strong decline component. No one can prove that it works early on because of a weak premise and the fact that for some reason the plots get filled with “noisy” data in that regime. I believe it often looks much more hyperbolic than the data suggests. Only with the effects of Dispersive Discovery does the linearization work out precisely.

Why do I make a big point of this model? Partly to counteract the impacts of peak oil critics like Michael Lynch. Lynch and company have a field day in dissembling Logistic-based models. The common practice in skeptical circles leads to simply trashing another analyst’s model; in this case Lynch doesn’t even have to come up with his own[10]. If you don’t have a model fitting a specific agenda, it doesn’t matter that they don’t have one either — it suffices to make you look bad by applying weak standards. Lynch also objectively scoffs when he looks at the traditional Hubbert models. He doesn’t say it in exactly this way, but assuming Gaussians in particular breaks causality. He extends this to the Logistic curve sigmoids when he sees the long negative tail. While I don’t agree with this completely, as you typically start the sigmoid at some finite value, no one has ever articulated where it should start. But then again, we just demonstrated the original Logistic curve derivation requires bogus assumptions. Yet we do not need to fret as we can refer to the Dispersive Discovery model to derive the Logistic *correctly* from first principles.

The reason to have a formalized model exists so that peak oil skeptics doesn’t have a red herring to continuously beat up on. We need to go through the incorrect formulation carefully to truly understand all the misapplied assumptions.

The Misapplied Derivation of the Logistic

I do not agree with the fundamental use of the *logistic function* to model Hubbert’s peak oil curve. And I do not make this assertion out of some contrary empirical evidence, but rather from an understanding of how it got derived. So using the classical premise — *to answer your critics, you have to understand them* — the following explanation provides some valuable ammunition via a quick rundown of the logistic curves derivation.

10. Look at how well this strategy works in today’s political circles.

First off, the variations of the logistic curve go by a variety of names, such as the sigmoid function and solutions to the Verhulst equation, differing by the number of parameters and whether they pass through integration or differentiation phases. By most accounts, you generate the classical Hubbert curve by taking the derivative of the logistic function.

Of course, the logistic formulation comes about from studies of population dynamics, where the rate of birth and death follows strictly from the size of the population itself. This makes sense from the point of view of a multiplying population, but not necessarily from inanimate pools of oil. In any case, the derivation starts with two assumptions, the birth and death rates:

$$\begin{aligned} B &= B_0 - B_1 \cdot P \\ D &= D_0 + D_1 \cdot P \end{aligned} \qquad \textbf{(EQ 14-9)}$$

We base the entire premise on a the negative sign on the second term in the birth rate — in the event of limited resources such as food, the birth rate can only decrease with size of population (and the death rate correspondingly increases).

The next step involves writing the equation for population dynamics as a function of time.

$$\frac{\mathrm{d}P}{\mathrm{d}t} = (B - D) \cdot P \qquad \textbf{(EQ 14-10)}$$

This provides the underpinnings for exponential growth, however critically modulated by the individual birth and death functions. So if we expand the population growth rate, we get:

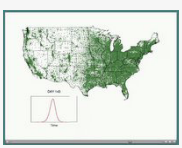
$$\frac{\mathrm{d}P}{\mathrm{d}t} = (B_0 - B_1 \cdot P - D_0 - D_1 \cdot P) \cdot P = (B_0 - D_0) \cdot P - (B_1 + D_1) \cdot P \qquad \textbf{(EQ 14-11)}$$

which matches the classic Logistic equation formulation:

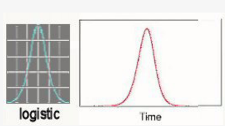
$$\frac{\mathrm{d}P}{\mathrm{d}t} = r \cdot P \cdot \left(1 - \frac{P}{P_\infty}\right) \qquad \textbf{(EQ 14-12)}$$

Where P_∞ becomes the *carrying capacity* of the environment. So the leap of faith needed to apply this to oil depletion comes about from analogizing population to a carefully chosen resource variable. The one that history has decided to select, cumulatively extracted oil, leads to the classical bell-shaped curve for instantaneous extraction rate, i.e. the derivative dP/dt. (Note that we can throw out the death term because it doesn't really add or mean anything to the discussion)

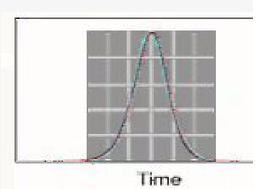
An interesting simulation (via DailyKos) of avian flu pandemic dynamics demonstrates the correct application of the logistic curve. Contrary to what old-school oil depletion analysts will tell you, the logistic curve (aka Hubbert curve) has no place in modeling a depleting resource, yet finds a much better fit in contagion dynamics. I took the following snapshot from a movie of the LANL simulation after the infected population reached a peak and had fully declined.



If you look at the lower left curve and compare it to a generic logistic curve, it fits (apart from a weak asymmetry) fairly well.



If you slide the one over the other, you can see the rather minor differences.



I have always had issues with both the upward part of the logistic curve derivative and the decline part. Trying to rationalize why instantaneous production would initially rise proportionally to the cumulative production only makes sense if oil itself drove the exponential growth. But we know that oil does not mate with itself as biological entities would, so the growth really has to do with human population increase (or oil corporation growth) causing the exponential rise[11]. That remains a big presumption to the model. The decline too has a significant interpretation hurdle as well. Why exactly the rate of growth after we start approaching and bypassing peak has that peculiar non-linear modifier doesn't make a lot of sense; the human population hasn't stabilized as of yet (even though oil company growth certainly has, technically declining significantly through mergers and acquisitions). We really have to face that a lot of apples and oranges assumptions flow into this interpretation.

The Naive Derivation of the Sigmoid. Next, to derive the actual sigmoid curve, we set up a relation representing population[12] growth (p) as a function of time (t):

$$\frac{\mathrm{d}p}{\mathrm{d}t} = r \cdot p \cdot (1-p) \tag{EQ 14-13}$$

If we can solve in closed-form for *p(t)* and then take the derivative, we ostensibly get the Hubbert curve. (The derivation can get a bit more complicated if we add additional constant factors, but that comes out in the wash in any case).

To solve, we convert the above equation into a differential that we can integrate from time t=0 to T:

$$dp/(p \cdot (1-p)) = r \cdot dt \tag{EQ 14-14}$$

This gets factored as:

$$\frac{dp}{p} - \frac{dp}{(1-p)} = r \cdot dt \tag{EQ 14-15}$$

The indefinite integral of this involves the natural logarithm:

11. Or as Deffeyes remarked: "Oil wells don't have babies". Yet, later he makes the odd association that "In a crude sense, oil wells do raise families. Drilling a discovery well brings on a bunch of new wells to develop the oil field. Understanding the geology of a new oil field leads to a search for similar fields." [Ref 96] This argument really taxes the imagination and stretches the analogy beyond usefulness. Besides, no one has tried to quantify this formulation.
12. Why and how oil extraction follows a population growth model frankly puzzles me. Oil molecules do not fit into any predator/prey or birth/death models that have crossed my path. I choose to ignore the flaw in this basic premise to concentrate on the mathematical derivation beyond this point.

$$\log(p) - \log(1-p) = r \cdot t$$
$$\log\left(\frac{p}{(1-p)}\right) = r \cdot t \qquad \textbf{(EQ 14-16)}$$

To make it definite, we put in the integration limits from t=0 to t=T, and p=P_0 to p=P.This results in:

$$\log\left(\frac{P}{(1-P)}\right) - \log\left(\frac{P_0}{(1-P_0)}\right) = r \cdot T$$
$$\log\left(\frac{P \cdot (1-P_0)}{(1-P) \cdot P_0}\right) = r \cdot T \qquad \textbf{(EQ 14-17)}$$

Converting this into the exponential, one gets:

$$\frac{P}{(1-P)} = \frac{P_0}{(1-P_0)} \times e^{rT} \qquad \textbf{(EQ 14-18)}$$

After a substitution of K=P_0 /(1-P_0), we get this function:

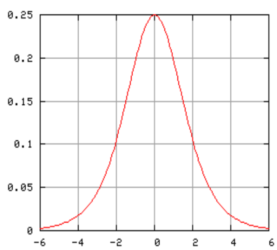


FIGURE 14-5. Derivative of a Logistic Sigmoid curve shows perfect symmetry

$$P(T) = \frac{k}{(k + e^{-rT})} \qquad \textbf{(EQ 14-19)}$$

The derivative of this gives:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = k \cdot r \cdot \frac{e^{-rt}}{(k + e^{-rt})^2} \qquad \textbf{(EQ 14-20)}$$

Note that the time progression or profile of the curve gets set by the value of $p(0)$, which gets buried in the k term. This fact has important implications. That an initial condition can determine the shape of the curve puts it in the class of *chaotic systems* based on a deterministic path. Linear systems such as the oil shock model do not suffer from this property.

Even more important than the chaotic property, the input stimulus of oil *discoveries* do not play into this specific logistic function formulation. If this stimulus in fact got added to the initial equation with something similar to the following delta function:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \delta(t) + r \cdot p \cdot (1-p) \qquad \textbf{(EQ 14-21)}$$

it would become impossible to solve in the same closed form as I have outlined above. In reality, only numerical techniques can solve this transcendental formulation. And even beyond that, we can drop one last monkey wrench into the works. As everyone who has studied chaotic systems understands, the most minor stimulus can have huge implications on how a system evolves.[13]

In the end, using the Logistic curve *in a context-free* situation only makes sense as a cheap heuristic, something that we can get a convenient analytical solution from. I have fundamental problems with the heuristic philosophy. Quite often you want something that *explains* the behavior and so we refer to the dispersive discovery model as a starting context for why we observe curves that look like the logistic, yet arises out of a completely different derivation.

Analogy to Growth. To add some more context to the dynamics of a Logistic curve, I will deconstruct an alternate calculus governing the Verhulst equation. Although deceptively simple in both form and final result, you will quickly see how a "fudge" factor gets added to a simple 2^{nd}-order differential equation (see the growth chapter) just so we end up with a convenient closed-form expression. This convenience of result essentially subverts a deeper understanding of the fundamental oil depletion dynamics[14].

If we consider the discovery rate dynamics in terms of a proportional growth model, we can easily derive a 2^{nd}-order differential equation whereby the damping term gets supplied by an accumulated discovery term. The latter term signifies a maximum discovery (or carrying) capacity that serves to eventually limit growth.

Now, if we refactor the Logistic/Verhulst equation to mimic the 2^{nd}-order differential equation in appearance, it appears very similar apart from a conspicuous non-linear damping term shown below.

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13. Look up what happens when a "butterfly flaps its wings in China" and you can get the picture. My brain hurts just thinking about the possible effects of a Hubbert Butterfly Effect and I would dread even attempting to go down this path.
14. I consider that a shame, as not only have we wasted many hours fitting to an empirical curve, we also never gave the alternatives a chance — something that in this age of computing we should never condone.

2nd-order Linear differential

$$\frac{d}{dt}D(t) = \underbrace{a \cdot D(t)}_{\text{growth}} - \underbrace{k\int_0^t D(t)dt}_{\text{damping}}$$

Logistic equation

$$\boxed{\begin{aligned} \frac{d}{dt}U(t) &= a \cdot U(t) - k \cdot U(t)^2 \\ D(t) &= \frac{d}{dt}U(t) \end{aligned}}$$

$$\frac{d}{dt}D(t) = \underbrace{a \cdot D(t)}_{\text{growth}} - \underbrace{2k \cdot D(t)\int_0^t D(t)dt}_{\text{non-linear damping}}$$

FIGURE 14-6. Comparison between a second-order linear differential equation, approximating an oil shock model with a damping term, and the logistic equation. Foucher used this similarity to create the hybrid shock model (See "Other Models" on page 716.)

That non-linear term in any realistic setting makes absolutely no sense. On the one hand, we assume an exponential growth rate based on the amount of instantaneous discoveries made. But on the other hand, believers in the context-free Logistic model immediately want to turn around and modulate the proportional growth characteristics with what amounts to a non-linear "fudge" factor. This happens to just slow the contrived exponential growth with another contrived feedback term. Given the potential chaotic nature of most non-linear phenomena, we should feel lucky that we have a concise result. And to top it off, the fudge factor leads to a shape that becomes symmetric on both sides of the peak since it modulates the proportional growth equally around $dD/dt = 0$, with an equal and opposite sign. Yet, we all know that the downside regime has to have a different characteristic than the upside (see the derivation of feedback growth for the explanation, and why the exponential growth law may not prove adequate in the first place).

Unfortunately, this "deep" analysis gets completely lost on the users of the Logistic curve. They simply like the fact that the easily solvable final result looks simple and gives them some convenience.

The Sigmoid Fraud: Why does this thinking persist? A majority of depletion analysts continue to view the Logistic Function as something that contains some deep and significant meaning.

To the contrary, the Sigmoid curve — as the simplest manifestation of the Logistic — remains a cheap empirical relationship that describes a value that increases and then saturates below some constrained limit. It indeed does follow from the solution of a non-linear differential equation, but this equation describes the temporal dynamics of a simplistic *birth-death* model used to describe interacting entities. One can choose populations of biological creatures or concentrations of chemical reagents to plug in to the equation. But you don't insert oil molecules into the equation and expect it to make any sense.

For example, the harvesting of whales to obtain oil produced a peak in the 1800's. Fair enough, whales do fall into a biological classification, and they do give birth and die. But whale oil harvesting never tracked a population rise in whales themselves. It actually tracked the reverse. So, instead of calling it a "birth-death" model we should refer to it as a "death-birth" model. The parameter "death" represents the culling of the whale population for oil and any residual "birth" comes about because the whales can reproduce themselves based on the size of their population. Then as an exercise for the reader, one can plug some values into the birth-death equations as described earlier.

But then we get to the real twist. Since whales do reproduce, if we play our cards right, then the amount of whale oil that we can harvest has no limit! The URR of whale oil essentially becomes infinite since the cumulative never abates. And unless we harvest the whales to extinction, the Logistic function will fail miserably in describing whale oil production. (In actuality, cumulative whale oil production likely saturated because crude oil replaced whale oil as a harvestable resource.) See passenger pigeons if you want to get closer to a saturated harvest driven to extinction.

On the one hand, oil does not reproduce like a biological entity nor does it act like a chemical reagent. So the equations themselves make no sense. But since oil only gets consumed and obeys the rules of a finite resource, it will eventually saturate. So the sigmoid falls into our lap in spite of itself. The fraud in our understanding survives in effect only because it looks like an S-curve!

Compare this to the use of the Dispersive Discovery formulation for discoveries and the Oil Shock model for extraction/production dynamics. Under one valid and intuitive assumption, the Sigmoid function falls out of the mathematics quite naturally, and you don't have to rely on empiricism that has the dynamic range of a dumb heuristic. The rise and fall of the oil culture deserves a better understanding than the classical derivation of the Logistic can ever offer. A fundamental first-principles functional form can offer some rationality.

Hubbert Linearization

You might get the impression that I don't believe that the Logistic curve provides much value, especially after hearing criticisms from several different angles. Of course, in terms of a deeper understanding of depletion dynamics I don't think it helps us much at all. Instead, we should understand it in terms of the dispersive discovery dynamics I outlined earlier.

However, the Logistic curve does possess some nice properties that make it useful for other applications. Granted, we can always do better, but the trick of Hubbert Linearization has become widely accepted and does indeed make some sense in limited application areas (i.e beyond peak).

Yet I discovered the dirty little secret behind Hubbert Linearization, hinted at earlier. The conventional wisdom basically states that plotting dU/dt / U versus U, where U refers to cumulative oil extracted, you can extrapolate a negatively sloped line that intercepts the axis at ultimately recoverable resources. Most analysts use the logistic curve or Verhulst equation to *prove* this limiting behavior. Whereas, in practice, any peak will do.

First take dU/dt, which gives the production rate. When plotted, this will give a Hubbert-like peak somewhere in its lifetime. Any somewhat symmetric peak when Taylor-series expanded about its center point looks like this — an upside-down parabola:

$$\frac{\mathrm{d}U}{\mathrm{d}t} = A \cdot (1 - k^2(t - t_0)^2) \qquad \textbf{(EQ 14-22)}$$

And then, any cumulative production increase looks like this near the peak — a linear trend upward:

$$U = a \cdot \left(1 + \frac{b}{a} \cdot (t - t_0)\right) \qquad \textbf{(EQ 14-23)}$$

Then make the substitution for time shifted around t_0, $T = t - t_0$, and you get this relationship:

$$\frac{\mathrm{d}U}{\mathrm{d}t} / U \sim (1 - kt) \cdot \frac{(1 + kt)}{\left(1 + \frac{b}{a} \cdot t\right)} \qquad \textbf{(EQ 14-24)}$$

The two positively increasing terms in the numerator and denominator more or less cancel, and you get

$$\frac{\mathrm{d}U}{\mathrm{d}t} / U \sim C \cdot (1 - kt) \qquad \textbf{(EQ 14-25)}$$

Which basically gives the famed Hubbert linearization term. Unfortunately, it doesn't give one any insight other than proof that you can linearize an upside-down parabola. In hindsight, someone might sarcastically say "big deal". We need way more insight than this to make headway in our understanding of depletion. (see "The Shock Model. How we deplete oil").

Foucher also had an interesting point concerning residual analysis from Hubbert Linearization. (substitute ***U*** for ***Q*** in the following derivation):

> *I'm skeptical about the use of this method to present production data because the relative error doesn't seem to be distributed uniformly. The relative error in the log domain of the vertical ordinates according to the logistic model is the following:*
> *D(ln(aP/Q)) = Dk/k - DQ/Q x Q / (1 - Q)*
>
> *where D stands for the greek symbol Delta, Dk/k and DQ/Q are the relative errors on k and Q which can be presumed constant. The error behaves has following:*
>
> *• Production start Q -> 0:*
> *D(ln(aP/Q)) = Dk/k*
>
> *• Production Q -> 1 (total URR has been extracted):*
> *D(ln(aP/Q)) = -infinity*
>
> *Because we are in the logarithm domain, D(ln(aP/Q)) = -infinity means that deviation around the asymptotic line will tend toward zero!*
>
> *That's why, we observe these wild deviations around the line when production is starting whereas it seems to converge nicely when Q tend toward 1. This behavior can be misleading for an observer because it seems to reinforce that there is some inexorable mechanism at work pushing the production data around the line. [Ref 95]*

I checked the math on this, and it really gets you thinking about what data visualization expert Tufte [Ref 99] says about graphing data in a biased fashion. That convergence on a continuously shrinking error acts like a laser beam and gives people the impression of an excellent fit that may have dubious value at best.

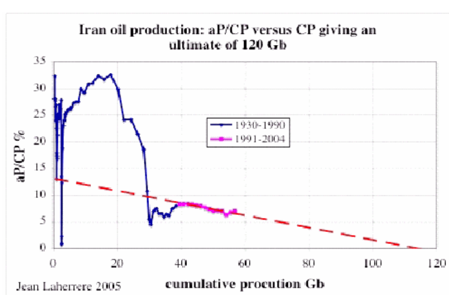

FIGURE 14-7.
Hubbert Linearization of Iran oil production

The technique of Hubbert linearization for estimating oil URR works perfectly for only one class of models: those which obey the logistic curve. Although a linearization technique may apply for other models, the fact that no one uses other models likely means that one does not exist — as of yet.

What else *seems* to work? Staniford fit a Logistic curve to oil production data from BP's report on World Energy 2005. He noted:

> *In the beginning, the data are crazy, but after about 1958, they settle down into pretty much a linear regime (with a little noise) that has held good ever since. The nice thing about this method is that you do not need to input an estimate for the URR. Instead, you extrapolate the straight line, and it tells you the URR. [Ref 100]*

I find the highlighted part revealing in that Staniford conveniently sweeps perfectly good data under the rug. I would never attribute to crazy what probably has a rational explanation.

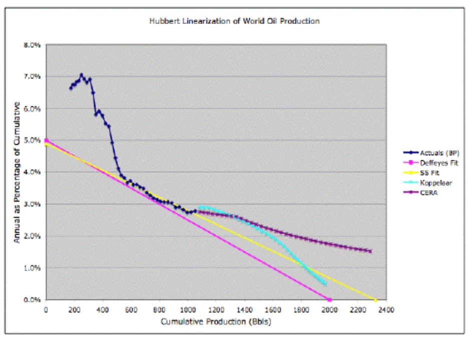

FIGURE 14-8.
Hubbert Linearization typically reveals departures from linearity for small cumulative values. This HL curve shows a cusp.

So what exactly causes that initial precipitous drop in the dark blue curve that nobody wants to fit to? And why do analysts want to work in the so-called "stable" regime where the Logistic curve seems to work so well? Well, I believe it has to do with the use of the Logistic curve itself, which has a narrow range in applicability to how discovery actually works.

Just by using a naive oil shock model, we can test this conjecture using Figure 6-1 on page 82: To match up against the Logistic curve plot, which Staniford and Deffeyes use to "linearize" the data,

$$\frac{\mathrm{d}Q}{\mathrm{d}t}/Q = k(1-Q) \qquad \textbf{(EQ 14-26)}$$

I simply had to integrate the simple production curves to obtain the cumulative (Q) number. The plot looks like the following, with a depletion rate of 0.03/year tacked on to the 130-year-span triangular discovery curve:

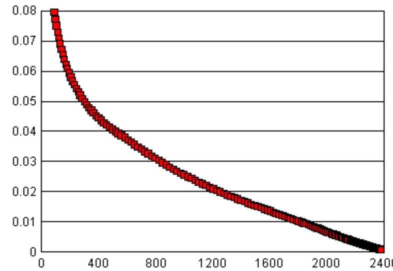

FIGURE 14-9.
Hubbert Linearization of a shock model shows a deviation from a straight line at small cumulative values.

Notice that we get a perfectly understood drop-off (perhaps quasi-hyperbolic) before it settles into a more-or-less linear regime as at heads to a zero production rate when we hit the cumulative production limit. The mistake that Staniford made in applying the Logistic model to BP's data came from the wrong initial (pre-1965) cumulative production. See Figure 14-8 on page 302 and you see a telling "cusp" at the top of the curve.

That cusp disappears when one applies the right initial cumulative data; Staniford added about 100 Billion Barrels (BBls) too much to this data. This becomes obvious when you notice that his cumulative as of 2004 sits at about 1050 BBls — 10% over the generally accepted value of 952 BBls from the **USGS**. It also appears a bit more optimistic than the Logistic fit, as it tends to flatten out in the out years.

The fact that a logistic curve forms a straight line with negative slope if plotted as *Rate/Q* vs. *Q* (where *Q* = cumulative), means that it gets a lot of head-nodding agreement when data seems to fit the linearization. Of course the oil shock model will not linearize the same way as a logistic curve will (as the curves themselves have distinct difference regarding symmetry, etc.). I find it also instructive to plot a typical *delta-input* (single discovery, all rates equal) oil shock depletion model with the same data transformation as the logistic curve.

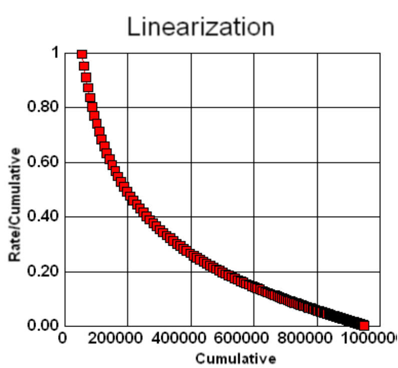

FIGURE 14-10.
Hubbert Linearization of a shock model with a delta discovery also shows a deviation from a straight line at small cumulative values.

If plotted on a ***semi-log*** chart, you can see the salient feature of the depletion model in a linearization context. At some point, further incremental additions of production will not affect the cumulative amount. This forms the oil extraction analog of “the law of diminishing returns”.

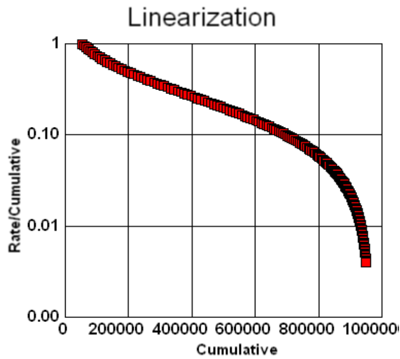

FIGURE 14-11.
For a Hubbert Linearization plotted on a semi-logarithmic graph, the effects of diminishing returns become more apparent.

The Export Land Model

The Export Land Model doesn’t qualify as a particularly wrong-headed analysis, yet it doesn’t bring much to the table either. It basically says that an oil-rich region’s ever increasing own energy needs will reflect as a reduction in its oil exports. This becomes simply a zero-sum analysis as global energy use increases as well, and one might as well just analyze the net global decline, instead of essentially cancelling out imports and exports from individual regions. So that even a theory like ELM, which I think has a sound fundamental basis does not get at the underling depletion behavior. It works mainly as a diagnostic tool to help people understand in terms of a zero-sum game the direction of a downturn.

This clearly shows how much information can get “squished” when looking at Hubbert Linearization. We have to consider the trade-offs between the convenience of obtaining a straight line (which may not exist) for a heuristic versus using a more comprehensive model like the oil shock model and comparing the entire, uncompressed production profile.

What about the Gaussian?

Staniford applied Hubbert Linearization over a range of data of US oil production using a Gaussian curve, showing promising results [Ref 102].

Recall that one can represent a Gaussian curve in terms of a rate equation. The familiar bell-shaped curve of a Gaussian follows from this partial differential equation:

$$\frac{dP(t)}{P(t)} = K(T_1 - t) \quad \textbf{(EQ 14-27)}$$

where P = production, t = time, T_1 = the peak date, and K parameterizes the width of the Gaussian. I suppose we can interpret the expression as an exponentially growing production with the throttle linearly pulled back through to where it switches sign at the peak. However, the T_1 number looks suspiciously preordained as opposed to coming out of some intuitive process.

Although Staniford showed good empirical fits using Hubbert Linearization, he still doesn’t know why they work out so well:

> *However, we don't, at the moment, have a very good theoretical understanding of exactly why, so there remains room for doubt about whether it will continue to work as well in the future.*

and

> *Where HL works well (e.g. the US), it seems to be because the production curve is close to Gaussian. Presumably there is some kind of central limit theorem "adding lots of random variables together" or "random walk through oil exploration space" kind of reason for this. If so, the asymmetry of individual field profiles may not necessarily give rise to an asymmetric overall shape. However, since we lack a clear and persuasive account of why the Gaussian shape arises, it's hard to say.[Ref 101]*

The empiricism leads straight to the fact that no one has ever suggested a forcing function to cause the temporal behavior in any of these models, including the logistic curve. They remain at best a set of heuristics that tend "shadow" the production data. Without a clearly identified forcing function, it remains a curve fitting exercise.

On the other hand, I could side with Staniford and look at the Gaussian from a law-of-large-numbers/central-limit-theorem-flavored approach, which only makes sense in the context of the oil shock model if many more phases of latency get chained together. As it turns out, in the limit of many independent phases convolved together, the ensuing curve will approach a Gaussian (see "Special Case: The Central Limit Theorem"). Unfortunately this will not help us to much in understanding and applying the actual parameters.

Going beyond convenient arguments

One can rationalize that providing a fundamental kind of analysis prevents the critics such as Michael Lynch, Peter Odell, Peter Huber and others to continue to drive a tar-sands-size dumptruck through the occasional gaps in the logic of conventional oil depletion analyses. You basically have to get rid of all of the logistic or gaussian curve fitting arguments or you will fall into the peak oil skeptics' favorite tautology traps — notably that of impossible symmetry and causality violations.

Many of the pessimistic oil analysts stubbornly hang on to their belief in the logistic formulation, and so Lynch will continue to eat their lunch, in a figurative sense. We believe that the skeptics have it wrong of course, but his arguments amount to a perfect framing that every political tactician would be proud of.[15] Whether the oil companies actually fund Lynch, Yergin and others, I can't say, but the following tactic would make an effective **1-2** punch.

1. Withhold information, both data and analysis
2. Hire consultants and political cronies to run interference and obfuscate.

I will lay this out very carefully because my concerns involve a combination of the practice of "framing" along with negation of arguments that the industry condones via paid consultants. First consider how Mike Lynch has become one of the most visible debunkers of peak oil theory out there. And what does he use as ammunition? Lynch uses the *mathematical formulation of Hubbert curves* to allow peak oil advocates to effectively shoot themselves in the foot.[16]

How does he do this? Well, he starts with the Logistic and Gaussian curves that the traditional analysts use and starts poking holes in how they get formulated. His favorite argumentative weapons include "physically impossible symmetry" and "causality violations". The argument about symmetry stands out because one needs only to look at real-world depletion curves and see that most do not display any symmetry. Lynch points out that the curves should lack symmetry based on his "own studies". Yet, depletion modelers continue to use them without a fundamental basis. Score: Lynch 1, Modelers 0

Lynch's causality argument says that the Logistic and Gaussian curves have tails at time "minus infinity", which remains a physically impossible condition considering that humans only discovered oil in the mid 1800's. Peak oil analysts have nothing to counter this other than some magical truncation that occurs in the curves, without any supporting explanation. Score: Lynch 2, Modelers 0

Now, I would consider these minor blemishes on the overall peak oil theory, but Lynch constructs a strong rhetorical argument building from this foundation of negating or falsifying half-baked heuristics masquerading as theories.[17] Much of the conventional wisdom regarding (Hubbert curves, etc.) gets turned upside down and used by Lynch to "disprove" peak oil or at least cast suspicions into its relevancy.

What can we do about this? Simple. Come up with better models than the Logistic curve or the Gaussian. These basically show empirical relationships that prove nothing and provide no foundation for understanding. Instead, teach something that works. An electrical engineering professor would certainly get shown out the classroom door if he claimed a Gaussian curve as the output to a electrical circuit. Yet this figuratively happens when the Hubbert modelers keep on showing Logistic or

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15. Also known as the technique of "psychological and/or language patterning".
16. Or "hoisted on their own petard"
17. See in particular this paper [Ref 52].where Lynch questions the fundamental modeling approach.

Gaussian curves. I can clearly understand how Lynch calls them on their weakly formulated assertions. Yet many analysts would never, ever give up their Logistic formulations due to sunk costs and group-think (see “The Sunk Cost Effect.”). Lynch counts on this as well; that they will keep on showing the Logistic curves and Lynch will keep on debunking. Lynch has basically *framed the Hubbert modelers as inept mathematicians*.

So let us turn it around and call the conventional models as cheap heuristics, and get some decent ones — models that don’t violate causality for one. Then we can effectively ignore Lynch, because he will have no legs to stand on, relying on the usual rhetorical arguments that industry lobbyists have historically counted on. By living by the sword they will figuratively die by the sword, as good science knows no match.

CHAPTER 15

Cornucopian Conundrums.

How do we reconcile against optimistic analyses?

"The economists all think that if you show up at the cashier's cage with enough currency, God will put more oil in the ground."

— Kenneth Deffeyes

"The juicy stories keep popping out... like clowns from a clown car."

— James Wolcott

Do you realize that we may have gone up and past peak oil without any real analysis, either economic or scientific, that has gone beyond heuristics?[1] As we then hit the next problem with the same lack of insight, any projections we make, either optimistic or pessimistic, will turn into equally likely yet poor outcomes. This means we still need to think hard and make predictions, which inevitably leads to arguments. For any problem that comes up, one side will inevitably show a strong pessimism, while the other side will remain optimistic.

1. That is a rhetorical question of course, with the last chapter spelling this out clearly.

Watch closely how the current global warming/climate change (GWCC) discussion gets swayed by "heuristics" versus "real models". The heuristics crowd consist mainly of the anti-GWCC people, who do not have real theories of their own, but they twist and turn trends to fit their agenda. The pro-GWCC people do have the real derivations on their side, yet the heuristics angle that the optimists latch on to tends to sway them the other way. (i.e. "gee, its been warm for a few days in a row, heuristics say it will stay warm, and therefore global warming is not real"). So it comes down to whether we believe the scientists or the contrarian empiricists.

Rank Empiricism

On the other hand, in the oil depletion debate, we have consistently watched both sides brandishing heuristic-based arguments. In other words, the pessimists **and** the optimists use heuristics to argue their side's correctness. Economists, such as the late Milton Friedman, have a strained attitude toward mathematics and science. In his research, he held to a single-minded goal to verify his "empirical theories", however he could. But modeling adaptive human behavior usually doesn't work well and he had no real way to justify or prove any of his theories. Excerpt from the book "The Shock Doctrine" [Ref 104]:

> The challenge for Friedman and his colleagues was how to prove that a real-world market could live u their rapturous imaginings. Friedman always prided himself on approaching economics as a science as hard and rigorous as physics or chemistry. But hard scientists could point to the behavior of the elements to prove their theories. Friedman could not point to any living economy that proved that if all "distortions" were stripped away, what would be left would be a society in perfect health and bounteous, since no country in the world met the criteria for perfect laissez-faire. Unable to test their theories in central banks and ministries of trade, Friedman and his colleagues had to settle for elaborate and ingenious mathematical equations and computer models mapped out in the basement workshops and the social sciences building.
>
> A love of numbers and systems is what had led Friedman to economics. In his autobiography, he says his moment of epiphany came when a high-school teacher wrote the Pythagorean theorem on the blackboard and then, awed by its elegance, quote from John Keat's "Ode on a Grecian Urn": "'Beauty is truth, truth beauty.' -- that is all / Ye know on earth, and all ye need to know." Friedman passed on that same ecstatic love of a beautiful all-encompassing system to generations of economics scholars -- along with a search for simplicity, elegance and rigor.
>
> Like all fundamental faiths, Chicago School economics is, for its true believers, a closed loop. The starting premise is that the free market is a perfect scientific system, one in which individuals, acting on their own self-interested desires, create the maximum benefits for all. It follows ineluctably that if something is wrong within a free-market economy -- high inflation or soaring unemployment -- it has to be because the market is not truly free. There must be some interference, some distortion in the system. The Chicago solution is always the same: a stricter and more complete application of the fundamentals.

CDS's: "..., but no one knows for certain how much is out there in CDS."

If you think about it, such cornucopian visions also preceded every oil crisis we have struggled through (and the recession that followed). In each crisis, some people *did* understand the underlying problem, i.e. Hubbert and a few others, but we never fully quantified the certainty because of the imprecise foundation of our knowledge and the reliance on heuristics and the intuition of experts who fancied an ideal market.

One can see how this plays out when faced with the debating tactics of the oil cornucopians. Some of the cornucopians have ideas so far out there that we don't even have to discuss them further (such as abiotic oil). But other cornucopians use the

> *"These factors have led to criticism of the modelling methods of peak oil theorists. Cambridge Energy Research Associates, a US-based energy consultancy, is damning in its assessment, saying that peak oil theory is garbage. Highly-respected (???) energy economist Michael Lynch has described peak oil theorists as practising pseudo-science and claims that: "The quantitative models used by peak oil theorists would earn a university student in elementary statistics a failing grade".*
>
> *Lynch also questions the quality of peak oil research, noting that nearly all of it has been published on the internet rather than in peer-reviewed journals.*" [Ref 325]

pessimistic oil analysts' *own dependence on heuristic arguments as ammunition against them*. The cornucopians may not actually understand their own beliefs, but since they realize the pessimists have no real basis either, they can use this uncertainty as a weapon to stir up fear, uncertainty, and doubt[2]. Lacking any kind of scientific tie-breaker, the neutral observer would at best call the argument between the pessimists and optimists a draw.

One cornucopian, Michael Lynch, has frequently called out the lack of a basis for the heuristic arguments presented by peak oil analysts. I covered a few of his criticisms in the previous chapter. In another case Lynch discusses reserve estimation[Ref 52].

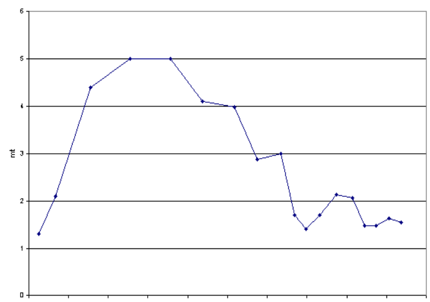


FIGURE 15-1. Michael Lynch's plot of production for the Cormorant field in the UK North Sea. [Ref 52]

Concerning the above chart (Figure 4 in his paper), Lynch suggests that asymptotic behaviors have no basis in reality:

> *Finally, Campbell and Laherrere use production data to estimate field size, "improving" on the IHS Energy data. By graphing production against cumulative production, as in Figure 3, they claim that a clear asymptote can be seen, allowing for a more accurate estimate of ultimate recovery from the field. The first problem with this is that there is no explanation for how often the method is employed.*
> *[...]*
> *Examining this data does confirm that some fields display a clear-cut asymptote. However, out of 21 fields whose peak production was above 2 mt/yr (or 40 tb/d), only 7 show such behavior. The rest do not show a clear asymptote (as in Figure 4), or worse, show a false one, as Figure 5 indicates. Clearly, this method is not reliable for estimating field size.*

So what happens if we plot Lynch's Figure 4, the UK North Sea Cormorant field, another way? It looks like this:

2. Also known as FUD

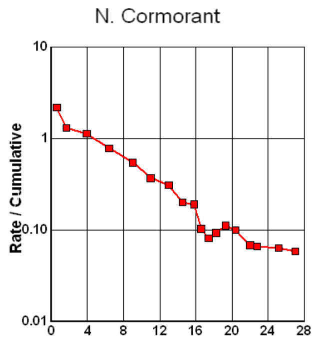


FIGURE 15-2. Plot of decline of Cormorant field. Note that Lynch did not plot the curve in the classical Hubbert sense, i.e. he forgets to divide production by cumulative along the vertical-axis, which describes Hubbert linearization

In the following figure I plot a simple oil shock model with the same axis as Lynch used. This curve may not follow Hubbert linearization but now Figure 15-1 on page 311 it looks similar to the oil shock model along much of its range.

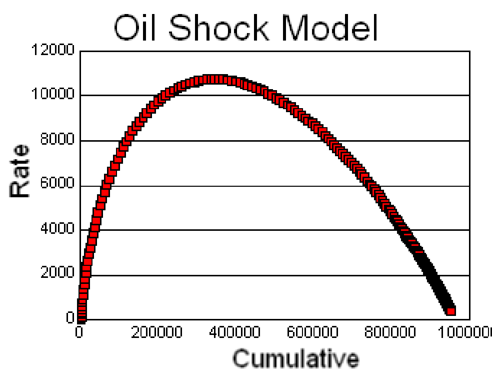


FIGURE 15-3. Plotting a delta-function driven shock model with the same visualization scaling as Lynch uses.

Now, take a look at some of the other curves in Lynch's paper:

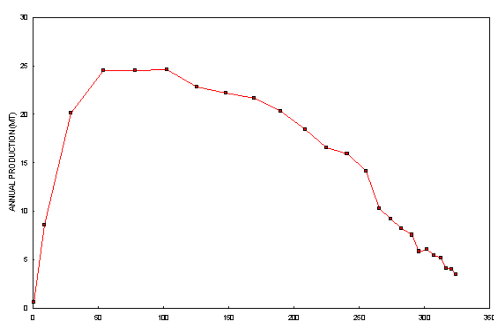


FIGURE 15-4. Another UK North Sea production profile plotted as a function of cumulative production. Note the asymmetry, as a logistic should show perfect symmetry.

Again the superposition of Lynch's curve with a model curve suggests that we can understand the behavior more than Lynch can admit.

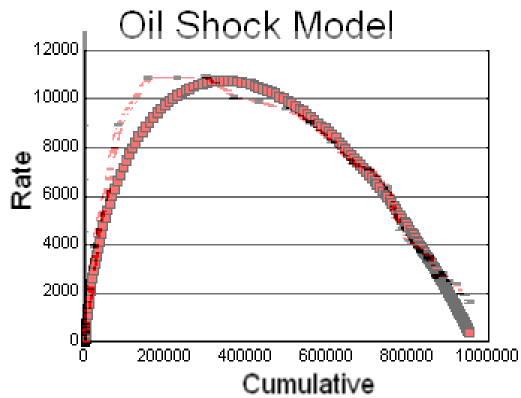


FIGURE 15-5. A shock model fit to the production profile with Fallow and Construction periods included.

Next, I plot another shock model with the first two time constants (Fallow period and Construction period) removed. This tends to make the curve more asymmetric and brings the peak in closer to the origin. (I gave all these curves the intuitive eyeball fit. And I normalized the curves by eye as well, since I had no discovery data.) Overall, I think this kind of "integration" linearization has some validity but it does transform data by compression, which tends to make the fit look better than if we kept the time domain in there and fit the original data as an "uncompressed" set of points. In essence, using the cumulative as an axis does a good job of filtering via integration.[3].

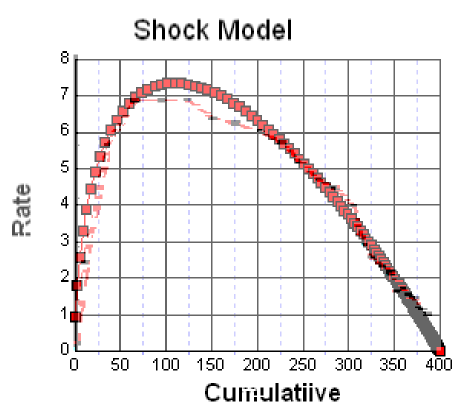


FIGURE 15-6. A shock model fit to the production profile with Fallow and Construction periods removed. This forces a greater asymmetry to the profile.

Does this discussion have a point? Yes, in that it basically shows how to respond to critics that attack improperly constructed heuristics. You simply need to counter with model-driven arguments that have intuitive basis, and only then can you hold

3. As noted in a previous chapter for the case of Hubbert Linearization.

sway and make a convincing pitch. The critics essentially have no model of their own, and you win by default. The empty rhetoric that they have brandished holds no match for a dialectic argument backed up by carefully modeled logic.

The Black Swan and Fat Tails

Many energy optimists like to think that we will eventually hit the jackpot, and discover an oil reservoir on this planet so large that it will service our needs for years to come. Nothing really supports this claim. The data we have collected suggests that this won't happen or, more conservatively, has a small likelihood of happening. This does not mean it can't. A controversial (i.e. interesting) analogy recently put forth relates to the case of *the black swan* [Ref 37]. The premise of the black swan asserts that straight probability and statistical analyses break down outside of a closed world assumption. In other words, when one considers that something odd can happen outside of the scope of your well understood reality, the infinite possibilities available can bring about some surprising new eventualities. The classic case that Taleb builds his argument around concerns the case of a previously undiscovered Australian black swan, of which European naturalists predicted would never occur, as the genetic probability of only white swans occurring amounted to 100%. But of course, the discovery of a new species of black swan turned the old paradigm on its ear and the genetics math code of the time proved worthless.

Some take this as an indication that anything occurring outside the normal (Gaussian or bell-curve) distribution has a more distinct possibility of occurring than previously imagined. Taleb refers to the fatter tails of the distribution outside of normal contributing to an unknowable factor that we need to take under consideration. However our analysis has made a great effort to avoid using a mindless bell-curve, in what would amount to using a heuristic. Many of our curves do not show the Gaussian drop-off and in fact have much thicker tails than the Gaussian (as does the Logistic). But one place it might come into effect: the mean=standard deviation Maximum Entropy approximation for probability densities precludes a significant amount of outliers.

$$\mu = \sigma \text{ — the unbiased estimator} \qquad \textbf{(EQ 15-1)}$$

Outside of this approximation we enter the "unknown unknown"[4] realm and anything becomes a guess. [5]

4. The phrase popularized by former Secretary of Defense Donald Rumsfeld to indicate unforeseen events. In pop culture, the phrase "No one expects the Spanish Inquisition!" from Monty Python also applies.
5. In contrast, "if you spend all your time thinking about Black Swans, you'll be so risk averse you'll never do a trade" [Ref 103] — the unboundedness of modeling Black Swans makes it nearly intractable.

Due to the popularity of The Black Swan book, some oil cornucopians think the allegory portends optimism for our fossil fuel future. Before I make a statistical case against any such eventuality, I will suggest that instead of a Black Swan representing new discoveries of oil, we have run into a counter-allegory called the Black Passenger Pigeon. Consider a simple scenario concerning swans and then an equally simple scenario concerning the now extinct passenger pigeons. First consider that because of a large population of swans, you will occasionally run across mutations leading to the evolution of a black species. I would consider this rare but mathematically possible. But it would seem vanishingly small to assume the possibility that a black passenger pigeon would suddenly appear as the overall pigeon population gets decimated and eventually becomes extinct. And this has nothing to do with a renewable resource like birds, but rather that we have pretty much scoped out every hiding area on the earth. Much like we can seek out every last passenger pigeon before it hits a critical level, so too can we find every last hidden oil reservoir. It takes many years of dispersive discovery to effect this, but it will eventually happen.

Theory: You ain't going to be finding any whales in a wading pool

Corollary: And when you start draining it, it won't make the event any more probable.

But that only disproves the allegory, as it doesn't say anything about the statistical foundation of the black swan from every happening. One just needs to consider the Dispersive Discovery model to back this up. In terms of the fat tails of dispersive discovery, we have to consider a strongly declining envelope compensated by small possibilities of huge spikes in chancing upon the occasional super-giant.

First let us acknowledge the conventional wisdom that large oil reservoirs get discovered first, or at least early on, during the prospecting cycle. This has some basis in intuition, as one might expect large objects, such as the proverbial "gorilla in the room" to get noticed first. Even if we couldn't prove that this routinely occurs, it would actually counter any black swan argument — so we actually err on the safe side and allow for even *more pessimistic* outcomes. In other words finding black swans early would essentially prove the exception to the rule, and so when you find all the "unexpected" black swans first, what do you have left to discover?

To make up for the loss of the black swan eventuality, oil company academics and USGS geologists have scrambled to come up with other mechanisms to provide a future surplus of oil reserves. One such scenario involves exaggerating the concept of reserve growth, and the other makes the case for generous probability distributions of discovery sizes. The optimistic analysts of the bunch have worked these mechanisms in tandem to create room for enough darkly-colored swans to populate a future projected oil supply.[6]

6. These projections likely hold little significance, and perhaps cynically amount to oil industry analysts' desperate attempts at meeting the expectations of their bosses.

The Conundrum of Reserve Growth

A significant conundrum that we face involves the large variation in reserve growth in various regions of the world. The USA has relatively large reserve growth rates, while production in other areas of the world show much lower rates.

I have gone back and forth a few times trying to understand the mechanisms behind the concept of "reserve growth". As much of this has to do with competing explanations presented by optimists and pessimists, I can volunteer the extenuating issues and points of contention:

1. Hard to distinguish between the ideas of reserve growth and plain new discoveries.
2. Some reserve growth rates, when extrapolated to the future, point to infinite URR.
3. Most reserve growth rates as extrapolated won't even begin to keep up with demand.
4. Areas that show reserve growth look good for awhile but when they get shut down, they quickly get removed from the data set. This makes the concept of asymptotic "reserve growth" mercurial because the contradicting evidence does not generally exist. In which case, cornucopians focus on the "still-growing" cases.
5. Certain areas show huge reserve growth (e.g. USA) but still hit peak oil. Other areas show little reserve growth, have honest reporting (e.g. UK and Norway) and of course hit peak oil.
6. Look up the concept of "creaming curves" and try to distinguish creaming from reserve growth. Hint: look at the x-axis variable.
7. Even though potentially promising regions in the world harbor dictators or fiefdoms or inhospitable natural environments, many people hold out for the hope of unknown amounts of reserve growth there. That involves counting on some daunting prospects or, at least, Black Swans to materialize.
8. Reserve growth predictions become heavily politicized and filled with legal technicalities because of the potential for fraud. The reserve growth effect itself could perhaps result more from an artifact of the reporting technique than anything else. For example, see the SEC regulations prohibiting "speculative" estimates.

In the following, I will give some pointers on how to argue this whole issue from a position of better understanding.[7] The fact that we have a workable model based on dispersive discovery to refer to helps to hone our intuition and to explain how people misjudged this effect so seriously.

The Enigma

A vintage and somewhat controversial oil depletion paper refers to the "enigma" of reserve growth as described in a paper by USGS geologists Attanasi and Root [Ref 42]. I assume they call this an enigmatic phenomena in that no one really understands reserve growth and why it occurs. Earlier, I contrasted the A&R paper to the alternate theory of dispersive discovery and dispersive reserve growth. So now that we have demonstrated how reserve growth can occur, let us note a couple of cornucopian insights from the USGS-supported research. The first attention-grabbing chart shows a growth that appears fairly significant, perhaps an order-of-magnitude effect on reserve growth for oil and gas. However the second and third chart shows much more moderate growth.

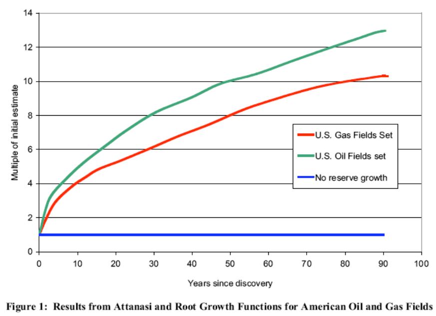


FIGURE 15-7.
Plot of reserve growth over as large set of USA oil fields since 1900.
[Ref 42]

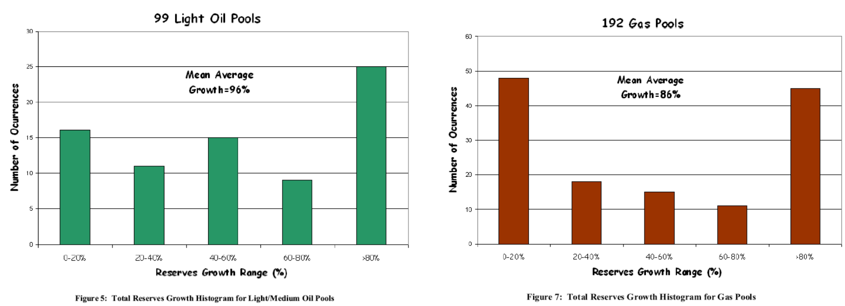


FIGURE 15-8. Histograms of reserve growth ratios for (left) oil and (right) natural gas. [Ref 42]

7. I struggled trying to resolve all these points simultaneously, achieving at best a progressive maturing in my own understanding. Much of this understanding occurred on the blogs. I started by looking at creaming curves. Next I saw some intriguing data on huge reserve growth in the US reported by the USGS, and tried to brush it aside a bit pedantically. Then I almost convinced myself that reserve growth would have little impact at all. I tried modeling reserve growth by applying conventional parabolic growth mathematics. I finally caught on to understanding SEC regulations on reporting and added some thoughts on asymptotic limits to growth. Eventually I found a few places where reserve growth showed little potential and developed a model that included uncertainty and dispersion. And then determined that you could achieve infinite URR without asymptotic limits but that it makes little difference in the end.

My take: Petroleum engineers and geologists in the employ of the oil industry no doubt can fall prey to job pressure and thus can easily slide into a catch-22 situation. They can either play with the truth and retain their employment, or they can risk losing it all by telling the truth. So, on the one hand, they can make these reserve predictions that err on the conservative side so that they can "grow" over time to maintain their job security. One can easily understand this, as nobody ever got fired for underestimating a certain cash cow.

As Lynch has demonstrated, so too do the geologists criticize pessimistic oil depletion analysts (such as Colin Campbell) for using the original and non-reserve growth numbers as proposed and continue to accuse them of "doomstering" because they keep missing the peak date. Of course you can perhaps fairly accuse Campbell of imperfection in his estimates, but some fundamental intellectual dishonesty prevents most everyone else in the oil industry from alerting us just why Campbell has made *his own mistake of worst case conservatism* — exactly the thing that the petroleum engineers practice and condone *within their own profession*[1].

So, by confessing to where these "bad" numbers come from, we can take geologists and petroleum engineers seriously. Right now I believe Campbell more than oil insiders because at least he does not fly the flag of hypocrisy.

I find it tempting to refer to **"reserve growth"** and rename it as **"bad estimate corrections"**.

1. EIA curves show worst case peak.

Notice that the first chart has a time scale that dates back 100 years. The authors do have the data that backs up the numbers on the chart, but you have to ask: how effectively did the wildcatters estimate reserves 100 years ago? Therein lies the enigma — what causes this mysterious reserve growth and will it continue as strong today as it did 100 years ago?[8]

Adding to the enigma, the authors haven't stated their results clearly, instead relying on qualitative extrapolations and inferences. Fortunately they do publish data so that we can muddle our way through it as well.

A&R basically developed an empirical approach to predicting reserve growth. If we plot the raw data as a scatter plot which shows only fractional increases per year since the year of discovery it looks like the chart below. Note that this plot does not visualize the "multiplier" approach which they used in Figure 15-7 on page 317; I would say they assume this rather unwisely, because it accentuates bad early estimates. The "fractional" approach provides an accepted statistical way to look at this kind of data which avoids amplifying regions with poor statistics.

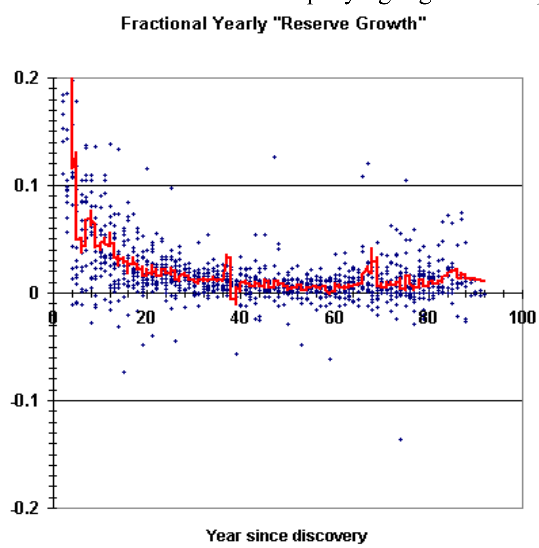


FIGURE 15-9. Reserve growth plotted as a fractional increase per year. This removes some of the censoring problems caused by extrapolating to full lifetime.

I placed a 20-point moving average filter to guide the eye (not weighted by size of field). Notice that in the sweet spot right in the middle of the chart, where we get the best statistics, the "reserve growth" fluctuates around 1% per year. You can see some growth for fields for fields older than ~80 years, but those have worse sampling statistics than the rest. The latest data also exhibits poor sampling statistics.

8. They also show another chart that has a large order-of-magnitude increase for heavy oil. What constitutes heavy oil in the USA and Canada? Does this include oil shale and tar sands?

Basically this approach demonstrates how to extrapolate assumed stationary data correctly.

Even so, with this moderate kind of growth, will 1% reserve growth of mature fields effectively compensate the 5% to 20% depletion rate per year routinely estimated for many fields? This reflects the promotion of the infinite URR as a cornucopian ideal that ultimately does not do us much good in the end. It merely diverts our attention away from the more serious problem of finding energy alternatives.

Diffusional Reserve Growth: No Panacea

If analyzed correctly, the concept of reserve growth should transform from enigmatic magic to a rigorous measure of extractability over time. The dispersive discovery model essentially accomplishes this and treats human diffusion (the speed of prospectors looking for oil) as a parameter.

I find this intriguing in light of the "enigma" article because diffusion and dispersion have a close association in terms of outcome. The phenomena of diffusion provides a spontaneous physical mechanism for material to move into a region of lower concentration. In this case, diffusional dispersion occurs because the material does indeed spread out over the volume, and the greater the volume or distance, the slower the relative dispersion proceeds. Humans can also work in terms of diffusion, as a barrier becomes harder to overcome the larger it gets. By the same token, I use dispersion in the Dispersive Discovery model to indicate that a search through a volume proceeds at many different effective rates, with the spread in rates always proportional to the cumulative volume searched. So, in the physical diffusion case, dispersion occurs as material fills up the new volume, and in the Dispersive Discovery case, humans do the work of randomly filling up the volume. In other words, diffusion implies the material comes to us and in dispersive discovery, we (as prospectors) go to the materials. And both of these processes occur randomly in terms of a quantifiable stochastic density function.

Yet the irresistible temptation to consider a diffusional model — that with an infinite supply at hand, diffusion of oil will grow a reserve without bounds. The most outrageous ideas would involve some complicated source of oil deep within the bowels of the earth (via mechanisms such as the abiotic oil "theory" [Ref 107]). How big can these reserves get? Remember that the reserve growth the USGS analysts predicted follows what looks like a parabolic growth law. The name of the law refers to the quadratic or parabolic shape you get when plotting *Amount* versus *Time* (*hint: tilt your head sideways to the right*). More typical, if we plot *Time* on the x-axis and *Amount* on the y-axis, it turns into a *square-root* relationship. If we stopped right now and relied only on reserve growth, *Q(t)*, then production can only proceed by at most the derivative of this number:

$$Q(t) = \int P(t)dt = k \cdot \sqrt{t}$$
$$\frac{d}{dt}Q(t) = P(t) = 0.5 \cdot k \cdot \sqrt{t} \qquad \text{(EQ 15-2)}$$

The production number shown drops off like $1/(\sqrt{t})$, which provides an even slower drop-off than the *1/t* dependence I conjured up earlier (in the Chapter 13). Like the hypothetical curve in Figure 13-1 on page 269, the USGS's curve also appears to generate an infinite supply of oil.

So, taken at face value, the geologists at USGS lead us to believe that reserve growth *will generate an infinite supply of oil*. I say they "lead us to believe" this, because no where do they say a limiting factor appears. This kind of sloppy analysis gives apologists like Lynch ample ammunition to put forward a cornucopian outlook to the population at large.

But even with such growth, this still leads to an incredible conundrum that peak oil deniers and cornucopians must face up to, based on their quotes of very large reserves. While reserve growth could hypothetically continue to grow as a square-root against time, the production will still drop off by the reciprocal of this number. If we indeed have reached a peak, then the reserve growth will still not make up for depletion losses and we will continue to face relentlessly diminishing supplies in the future. Only **many** new discoveries, sufficiently removed from known reservoirs will prevent the decline.[9]

Waiting For The Big One: No Panacea Either

Another of peak-oil denier Lynch's favorite arguments to counter the oil depletion pessimists (including Campbell and Laherrere, et al) has to do with questionable interpretation of the so-called "creaming curves"[10] from maturing fields.

9. The fact that drilling through harder and more impenetrable layers is also diffusional and thus eventually counter-productive.
10. The term creaming comes from the phrases "cream of the crop" or "cream rises to the surface", which essentially creates a good analogy for what oil producers desire.

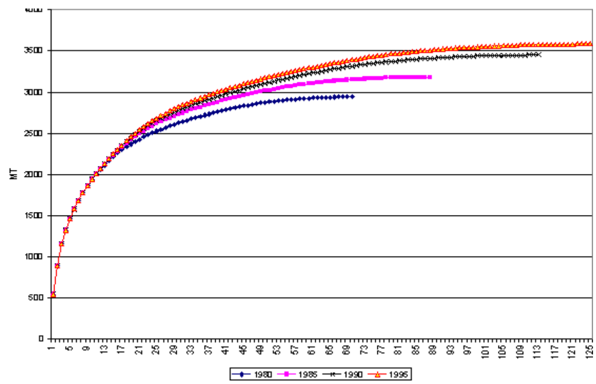


FIGURE 15-10. Set of curves which show a drift of asymptotic accumulated reserve upward with time. Note that the x-axis is not time!

Lynch essentially states that published creaming curves have a tendency to creep up over time, implying that more oil exists than anyone currently realizes. Unfortunately, Lynch has mistaken the asymptotic properties of finite regions with the pseudo-infinite scope that some creaming curves occur under.

As an example, consider this Lynch curve:

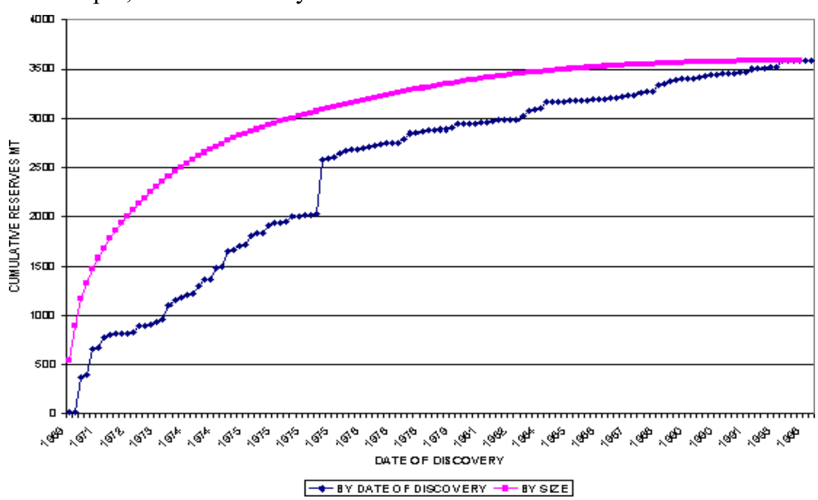


FIGURE 15-11. Creaming curve variations, as discovered and size ordered

Note that the latter curve gets plotted according to time progression. From the looks of it, it doesn't appear to have any asymptotic properties. On the other hand, when ordered by size (i.e. sorted), it shows a clear asymptote. Lynch likes to point out that people shouldn't look at the asymptotic curve because the other one keeps climbing. But Lynch has failed to point out an unwarranted assumption.

Within a finite field area, the asymptote eventually gets truncated artificially. The geologists or petroleum engineers declare the field “dry” when they stop finding strikes, go back and order the numbers, and figure out the creaming value. Ordering the values high to low and doing a cumulative sum gives the curve a filtered look that shows a horizontal asymptote. Graphically displaying the integration works out as a nice Power Point slide for management. Management then decides to go on to the next field, declaring the finished field at its *economic limit*. And the engineers and scientists don’t have to try to suck blood from a turnip.

Yet within a quasi-infinite or continuously expanding field, the asymptote continues to creep upward. You can’t make any assumptions on asymptotic behavior because the big discovery occasionally occurs, pushing the curves inexorably upward. In reality, only when you hit the limits of your quasi-infinite world can you make any serious interpretations on the creaming asymptote. In these situations, the big discoveries will likely still occur where you haven’t looked, thus invalidating any asymptotic trend that you may have counted on.

The big issue comes about because as fields become uneconomical and therefore “shut-in” today, they may turn profitable under tomorrow’s environment. So we still need to quantify the effect. If the reserve growth discoveries do follow an ordering according to size, big to small, you can still do some extrapolation. (The unsorted curve of Figure 15-1 on page 311 should show a noisy but quite straight linear upward trend if the sizes show independence with respect to discovery time)

So this illustrates an interesting property of the creaming curve. When ordered according to size, a histogram of the individual slope values gives the probability density function. Which means you can easily check against field-size distributions covered in “The Facts in the Ground. Where do we find oil reservoirs?” . If I could find a creaming curve for the entire world, we should get a good field size distribution to work with. This becomes very important as it unifies the concept of reserve growth with field size distribution. Cornucopians won’t like this because it formalizes the estimates and places them on equal footing.

Other Areas of the World: Low Reserve Growth?

Although the empirical A&R growth law does not show *accelerating* compound growth, neither does it show any signs of abatement — as it should continue to grow for years according to their heuristic formula in [EQ 9-9]. However, we still have the usual problem of varied inconsistent data from different sources.

Certain sets of data suggest that reserve growth as both an overrated effect and also significant, while we find some sets striking a middle ground [Ref 110].

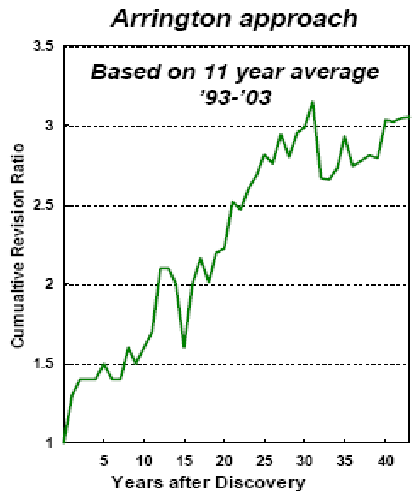


FIGURE 15-12. The Arrington approach consolidates A&R data into a heuristic to predict reserve growth [Ref 109]

While global reserve growth shows an apparently self-limiting growth of 3×, well below A&R's estimate for USA reserve growth of upwards of 10×, that still doesn't tell the whole story. For one, it all depends on when the reserve growth counter officially starts ticking. In the case of Russia, it appears the ticker only starts after 5-7 years have elapsed, potentially pushing the reserve growth to only 2× [Ref 108].

For another example, reserve growth in the UK North Sea and Norway appears even lower, hovering near the 1.3× level, or only about a 30% increase after a signal to go ahead with development (implying a long fallow stage).

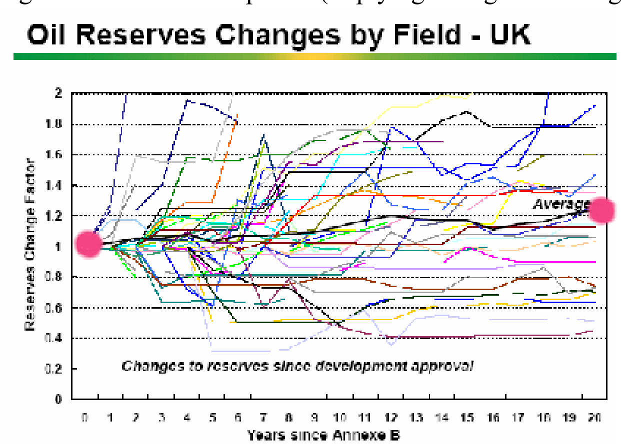


FIGURE 15-13. Reserve growth rates for UK North Sea fields. The average reserve growth (shown by large dots) is small despite large variance in data [Ref 111]

Further, note how in the Norway example that the delay prior to development approval serves to effectively stabilize the estimates. Estimates prior to year 0 actually show an average reserve decrease initially.

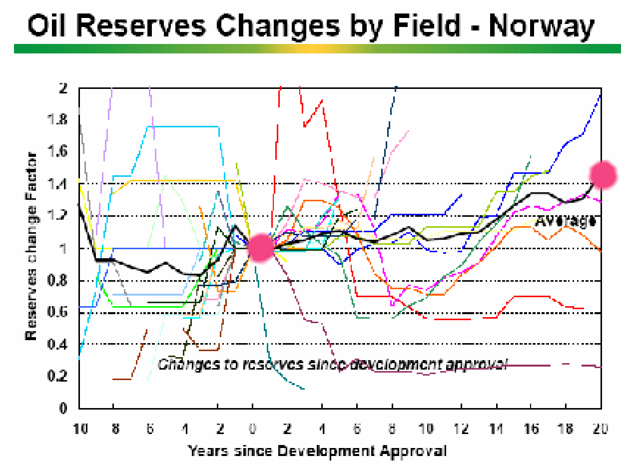


FIGURE 15-14. Reserve growth rates for Norway fields. The average reserve growth (shown by large dots) is small despite large variance in data [Ref 110]

But elsewhere, news comes out that Russia's reserve estimates have jumped.

> But there's one place — Russia — where reserve estimates just seem to go up and up. In its annual statistical survey of world energy, BP PLC (BP) has recently revised its estimates of Russia's total proven oil reserves to 69.1 billion barrels, 6% of the world's total, up from 45 billion bbl. in 2001.
>
> …
>
> According to a recent study by Dallas-based energy reserve auditors DeGolyer & MacNaughton, whose clients include leading Russian energy companies such as Gazprom and Yukos, Russia's true recoverable reserves are between 150 billion bbl. and 200 billion bbl.That's up from industry estimates of 100 billion bbl. a few years ago.
>
> Why such a big gap in the estimates? Because it's one thing to be sitting on oil reserves, another to be able to exploit them commercially.
>
> …
>
> "The biggest thing is the [new] technology being deployed in western Siberia. The results are beginning to show," says Martin Wiewiorowski, senior vice-president of DeGolyer & MacNaughton in Moscow. [Ref 133]

Yet USGS geologists had stated it rather differently as of a few years ago:

> As part of the assessment of reserve growth, Verma and others (2000) evaluated field growth in the Volga-Ural province of Russia. In their ongoing study, they also noted similar reserve growth in the West Siberian province of Russia. The Volga-Ural province has shown cumulative reserve (field) growth factors of 3-5 during the first 30 years since discovery of the oil fields. Because of the time required in

> Russia to develop a field, defining field growth is more complex than in the U.S. For the study on the Volga-Ural province, the field's effective discovery years is redefined as either the year of first significant reserve reporting or the year of first production.
>
> Volga-Ural and West Siberia provinces show most of their reserve growth in the first 5-7 years after discovery and little or no growth thereafter. It is difficult to compare the growth in Russian fields with those of the U.S. fields where growth continues even after 90 years, because in Russia oil fields are first evaluated over a 5-7 year period before being produced whereas in the U.S. both the evaluation and production of fields start shortly after their discovery. To further complicate the comparison, proved reserves in Russia generally include only primary and secondary (water flood) recoveries, although in the U.S. the reserves are revised regularly and include water flood and EOR recoveries. Other factors such as the Russian oil industry's lack of infrastructure, operational and economic problems in maintaining and developing fields, reporting requirements and documentation, and changes in the political system may have contributed to the difference in reserve growth. [Ref 108]

What appeared mature in 2000, now appears growing? According to the recent news, yes:

> The growth in Russia's proven reserves is mainly happening at existing fields in western Siberia, a supposedly *mature* region where production had been declining until recently. DeGolyer & MacNaughton predicts that western Siberia could boost its output to 10 million bbl. a day by 2012, up from less than 6 million at present, and keep production at that level for at least 10 years. The use of even newer technologies available by then means that western Siberian oil production may not decline for decades to come. Russia's reserve potential is vaster still when undeveloped regions, such as the Arctic, the Caspian, and in particular eastern Siberia, are factored in. [Ref 133]

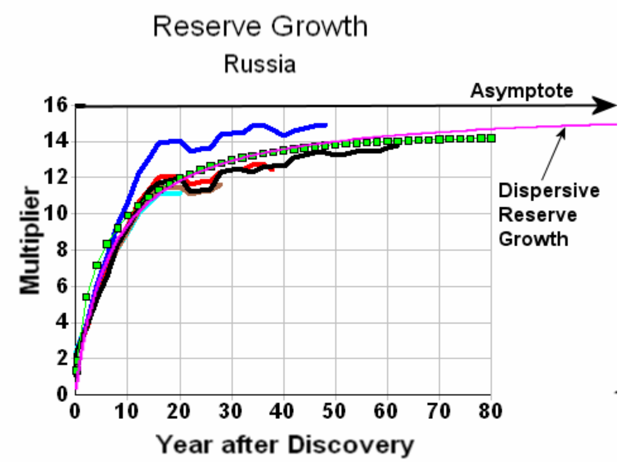


FIGURE 15-15. Over different time intervals, reserve growth from West Siberian Basin jumps around but appears to converge to the dispersive reserve growth asymptote of 16 since original discovery. Data from [Ref 48] [Ref 161].

In the case of Western Siberia, the Russians do emulate the USA's 10× reserve growth heuristic (see the figure above), even though they purportedly wait until they start pumping before they label it a discovery, and therefore have a much better estimate to begin with. The USGS really should do a follow-up to their original western Siberia findings as I see a number of inconsistencies cropping up. If we have to re-backdate *all* the FSU/Russian discovery curves, then a second peak will clearly reveal itself in the coming years. Laherrere has suggested that we should use the most probable reserve estimates and not use conservative numbers, as these tend to turn out more correct in the end [Ref 113]. This has to do with the effects of *apparent reserve growth* [Ref 162], as you can only account for the oil as you start to extract it in the most conservative approximation.

Daystrom: "Twenty years of groping to prove the things I'd done before were not accidents... seminars and lectures to rows of fools who couldn't begin to understand my systems... colleagues, laughing behind my back at the 'Boy Wonder' and becoming famous, building on my work -- building on my work!"

Bones: "Jim, he's on the edge of a nervous breakdown, if not insanity."

Suffice to say, dealing with inconsistent data gives cornucopians ammunition for their arguments, as it contributes to the FUD surrounding dwindling supplies. Not much we can do about this but to get better data.

More Creative Accounting

Reserve growth curves and creaming curves have a very similar shape even though they purportedly measure different things.

Several of these papers including Attanasi and Root (quote below) point to the difficulty in separating reserve growth from new discoveries that occur in close proximity to the original fields in some finite or slowly expanding area/volume. So in one case you have backdating and in the other case you just have additional discoveries — which get counted conventionally, i.e. not backdated, as a creaming curve.

> *"At the basis of all statistical extrapolations used to project field growth is the assumption that fields are well defined and that patterns of the past will continue into the future. The enigma of field growth is due, in part, to changing and ad hoc field definitions. In individual cases, fields are defined on the basis of convenience by regulatory agencies, by operators, or simply as artifacts of the historical discovery process. For example, the 1990 EIA OGIFF file had several hundred more fields than the 1991 file. Nearly all of the entries not in the 1991 file had been combined with other, larger fields. As new pools, reservoirs, and fields are developed, they are frequently included in older fields for geologic reasons and perhaps for the convenience of unit operators and regulators. When this happens, the discovery date of these more recently discovered hydrocarbons are backdated, and growth of the older fields is extended."* [Ref 42]

Clearly, the oil industry can use whatever means of bookkeeping they want to keep track of their reserves. This inconsistency in fact further perpetrates the enigmatic role of reserve growth. My comment prompted this reply from a peakoil.com thread [Ref 105]

> *strictly speaking this should not be a problem....increase in area would refer to a single field, if not a single reservoir. It is not a separate discovery which would mean it is not in hydrodynamic communication with the rest of the field....ie. a separate structural closure or a separate trap. What is being referred to is that often quite a bit of time after discovery additional work (3D seismic, outpost drilling etc.) identifies that initial assumptions regarding oil/water contact, depth conversion, location of bounding faults etc. were incorrect....as a consequence the area of closure in the field/pool can be increased. The unfortunate problem in all of this is that often the folks who play with the numbers have no idea about the reservoir dynamics. As an example people still like to speak of Ghawar as if it were a single field when in fact it is made up of a number of separate closures, and several reservoirs, some in communication some not. The geologists and reservoir engineers working on those particular projects know what is going on and report the information correctly....the powers that be decide they can group and lump that data. Hence folks simply grabbing that data from various sources will be led astray. Using Ghawar as a whole in any kind of statistical analysis skews the results beyond belief....for instance look at the backdated discovery curve with the huge peak in the sixties* [Ref 105].
>
> *You are right about the shape of the curves, but as I have pointed out on another thread it is pretty easy to get mislead by a particular creaming curve when you don't understand the underlying geology of the basin. Hence Hubbert missing new discoveries in the US, and creaming curves for Algeria created in 1990 missing out on about 6 billion barrels of new discoveries in the following decade. As to understanding the geology, Hubbert was a brilliant geologist (see my thread regarding his contributions) but the limit of our understanding of the subsurface at any point in time is often a product of the sampling density of that universe, which, when Hubbert was ruminating on the problem, was good but by no means stellar.*
>
> *I guess all I am trying to get across is that the idea of reserve growth makes perfect sense to us folks who deal with the discovery-development-production history of fields. The unfortunate thing is it just comes across as magic to those that haven't been directly involved in it. Not sure if there is a better way of explaining it.*

Claiming it as *magic* that the ordinary layman can't grasp sure seems a condescending attitude, especially since we know this doesn't involve quantum mechanics or relativity theory or some other non-intuitive concepts. I would rather attribute this lack of desire to impart wisdom to a combination of factors including laziness, arcane lingo, messy data, and double-talk politics that the business of oil production seems to thrive in.

I imagine that shrouding scant data in mysterious and vague terms will become the next step in cornucopian arguments. The optimistic reservoir engineer L.P. Dake had this to say in 2001 in his petroleum engineering text:

> The author has always believed that there should be a place in reservoir engineering for the very basic theory of physics which is (perhaps unfortunately) the Heisenberg "Uncertainty Principle" of quantum mechanics. This is not an original thought in the subject because as long ago as 1949 the ultimate reservoir engineer, Morris Muskat, had flirted with the same idea but concluded that: "*In its operational sense the principle of uncertainty, which is usually considered as limited to the realm of microscopic physics, constitutes the very essence of applied reservoir engineering as a science*."
>
> An excellent thought - but what can be done about it?
>
> Nevertheless, the subject is vulnerable to change, the latest approach being the adoption of "Chaos Theory". This would seem to be a convenient concept to hide behind in reservoir engineering but at the time of writing is still in its infancy - thank goodness

I don't understand what Dake was trying to imply, and unfortunately I don't think he knew either. After awhile, knocking down these cornucopian strawmen becomes a bit repetitious. The probability and statistics behind the reality-based concepts described in the preceding chapters is not that difficult to apply against the skeptics.

CHAPTER 16

An Oil Level Check

While we have gotten this far, what can we conclude?

"To state the facts frankly is not to despair the future nor indict the past. The prudent heir takes careful inventory of his legacies and gives a faithful accounting to those whom he owes an obligation of trust."

— John F. Kennedy

Bloomburg News reported that the Energy Department study found that conventional oil production reached "soft and sudden" peaks in Texas in 1972, North America in 1985, Great Britain in 1999 and Norway in 2001. These dates were predicted by formulas used by proponents of the peak oil theory to predict the crest of global oil production.

— September 2006
Houston Chronicle

As a bottom-line and in the greater scheme of things, the diagram to the right qualitatively describes our fossil fuel predicament — on a historic, geological time-scale, we have nearly passed through a mere window of usage which coincides with the entirety of the oil age.

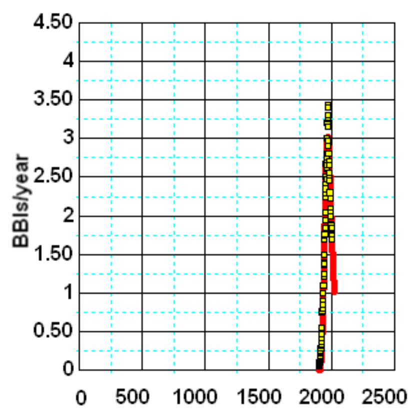


FIGURE 16-1. U.S. lower-48 oil production -- year 0 to 2050 A.D.

Exactly how have we concluded this from a modeling context?

(A) We know that oil exploration efficiency has progressively improved over time; every year we critically evaluate bigger and bigger swaths through the earth's crust, looking for new deposits of oil. Unfortunately, the peak discovery years likely occurred during the early 1960's and we have since started exploring the fringes of oil's geological range. This has contributed to a monotonically diminishing rate of discovery, albeit somewhat mitigated by reserve growth.

(B) We also know that once we discover a new reservoir of oil, it takes a while to start extracting and then to complete the life-cycle. We obviously have to consider that some period elapses as the region sits fallow, that some period elapses as extraction facilities get constructed, and that some period elapses as the new rigs come online and reach maximum production. These three factors have some average value that we can make an educated guess at. We can also guess at an average extraction rate that works out proportional to the amount of reserve that we have left. This extraction rate, more than anything else, responds fairly quickly and agilely to market and political considerations.

(C) We know that a good discovery model should help us quantitatively describe (A) and that a good extraction model should help us describe the dynamics of (B). With some work, we have arrived at a unified discovery/production model which captures this knowledge mathematically. The dispersive discovery model applies simple statistical ideas of a dispersive search space. The oil shock model uses a few intuitive stochastic parameters backed up by solid physical reasoning (at its core consisting of a time-stochastic phasing from discovery of oil regions to their maturation)

For an example of the steps A=>B=>C, the Shaybah field in Saudi Arabia, though discovered in 1968 only come on line in 1998 and hasn't matured yet. Located in the "Empty Quarter", a particularly desolate and imposing place, this field provides a typical example of how the latency of each of the stages adds up to explain the shift of the discovery curve to the production curve.

Although we can't say exactly how long a field remains fallow, or how long it takes to construct the rigs, or how long the maturation process takes, or even estimate the extraction rate, a global model would suggest a spread of these values representing the uncertainty/variability of these numbers from location to location and economy to economy. A good conservative estimator of the phases would lead one to guess at a mean with a standard deviation equal to the mean — this becomes a decaying exponential distribution of latencies. The convolution of this set of exponentials generates the shifted and spread production curve originating from the tighter discovery profile.

Qualitatively we have this set of parameters:

- Discoveries first started in 1858 (the year t=0)
- Swept volume increase per year = $f(t)$
- Total volume and fraction containing oil = URR
- Fallow period = X years
- Construction period = Y years

- Maturation period = Z years
- Extraction rate = R% /year

We fit estimated global discovery curves and global production profile to this limited set of unknowns given the premise of a discovery model and a rate driven extraction model.

Beyond expanding on that thumb-nail premise, I set as a context to (1) lay out new modeling ideas and (2) try to place these ideas in contrast to classical depletion analysis. The latter step became quite a task since the conventional wisdom of modeling relies on either a great deal of heuristics, or the grind-it-out work of methodical book-keeping. Since a fundamental and comprehensive view of depletion has never actually existed (in a formal way at least), most of the ideas that I presented got derived from first principles. Moreover, due to the *de facto* preliminary nature of this work, I would venture that much more interesting results will come out of the framework.

As for the fresh fundamental ideas to consider, I have a Top 10 list:

1. We derived a way to look at the distribution of reservoir sizes from first principles that matches empirical observation well — the ***Dispersive Aggregation*** model. Although it does not place limits on cumulative sizes, it characterizes our understanding beyond that of the heuristics of Pareto, log-normal, and Parabolic Fractal relationships.
2. We derived a way to describe growth, peaking, and decline in fossil fuel prospecting — the ***Dispersive Discovery*** model. This in fact allows us to put bounds on potential cumulative discoveries. The technique uses parametric substitution to allow one to input different growth profiles as the trajectory follows a *stochastic arc*.
3. We showed the independence of reservoir sizing effects from the behavior of dispersive discovery. This allows the DD model to serve as a conservative estimator as balance to the "find big fields first" heuristic.
4. We proved the relevance of the Logistic function as it reduces from a general form of the Dispersive Discovery model. Not surprisingly, this has little to do with the generally accepted mechanisms of "birth-death" behaviors that many have attributed to it.
5. We provided a canonical expression to generate dispersion envelopes via Laplace transforms.
6. We solved the "enigma" to reserve growth through the use of dispersion of rates — the ***Dispersive Reserve Growth*** model. This generates a constrained growth and explains the hyperbolic behavior of reserve growth and creaming curves.

7. We formulated and presented a comprehensive model for oil production based on stochastic waiting times and rates applied to the life-cycle stages — the ***Oil Shock Model***.
8. We applied the Dispersive Discovery model as input to the Oil Shock model to provide "what if" extrapolations and projections of future production levels. The shocks enable perturbations to the model for evaluations of overshoot and plateauing analyses. We also demonstrated the futility of depending on infinite reserves through several examples.
9. We derived a statistical kernel model of production on smaller fields — ***Shocklets***. This combines reserve growth and the shock model in a canonical simple relationship.
10. We provided a more flexible way to look at Hubbert Linearization including a way to linearize field size data and reserve growth curves.

If we refer again to our road map below, notice the "?"-mark under the last column marked "The Future". Although we can extrapolate from the fit to the historical data, we have to keep this last step open to interpretation due to the possibility of effects such as demand destruction. This becomes similar to the quandary of chasing a meteor in free-fall; desperately trying to keep up with it, you have nothing left to resort to in the end and you might just decide to go along for the ride.

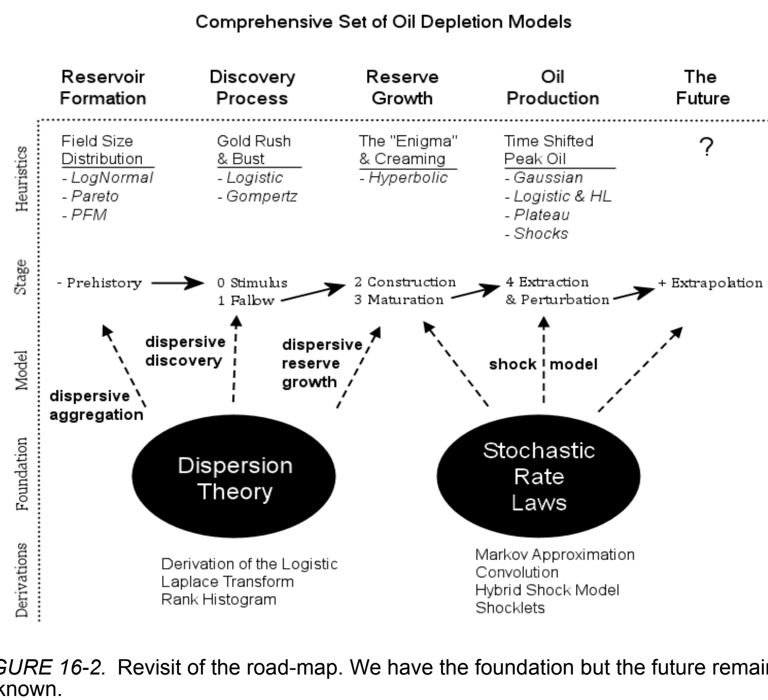


FIGURE 16-2. Revisit of the road-map. We have the foundation but the future remains unknown.

"*The picture's pretty bleak, gentlemen….the world's climates are changing, the mammals are taking over, and we all have a brain about the size of a walnut*."
— Dilbert's cartoon dinosaurs strategizing at the blackboard.

The fact that dispersion plays into several of these derivations comes as no coincidence. I believe that too many analysts have tried to understand oil depletion as a deterministic system, governed by something akin to a misguided birth/death formalism. This has thrown most everyone for a loop, leading us to neglect plain old randomness in the form of dispersion as the more significant effect in favor of the traditional approach of solving classical dynamical systems.

So, in current usage, the workman-like "theory" of peak oil exists only as a heuristic; since we have lacked a formally quantified theory explainable through some basic probability and statistics. The hand-wavy explanations for the Logistics sigmoid of the classic Hubbert peak rest on a premise that never made any sense from the start. In other words, we have no other choice but to reject the Verhulst and Volterra equations as invalid approaches.

Having showed that a simple yet perfectly valid math model does exist, working both on the discovery level as well as the production stage, we conceivably should start making some real headway.

Yet, it will take some effort to get this foundation to overcome inertia of the prevailing common knowledge and conventional wisdom (CW) presented just about everywhere we look. Consider that the following quoted excerpts of CW come from the same on-line article [Ref 142]:

> **CW #1.** *"Every non-renewable resource coming from a finite storage can be exhausted, this exhaustion process can be described* ***through a mathematical function*** *and represented by a depletion curve. This* ***theory of depletion*** *of non-renewable natural resources was first put forward in the 1950s by King Hubert, a US geologist, based on actual observation of oil wells' production".*

"*In light of this fact, it should be no surprise that the possibility that world oil production will soon reach a peak and then inexorably decline is a subject of great interest and intense debate. As noted by Dr. Greene, the 'pessimists', a somewhat pejorative label given to those who are convinced that the oil peak is imminent and that its consequences will be dire, assert that world oil supply is chiefly determined by the geology of oil resources.*" [Ref 141]

I agree generally with "Dr.Greene" in that geology determines the oil depletion arc almost completely. I would even take it a step further and also claim that probability and statistics plays an even greater role.

The mythologized version of peak oil theory largely rests on an intuitive gut-check (however brilliant) by Hubbert. Clearly, Hubbert never presented a first-principles mathematical model, and came well short of using any formality of theory[1].

> **CW #2.** *"Peak oil* ***theory is well grounded in physics and mathematics****, and there is little controversy that peak oil production for the world will eventually be reached at some point in the current century."*

The mathematical equations used for practical analyses such as Hubbert Linearization work essentially as heuristics based on previous empirical evidence. A recur-

1. "*Hubbert in 1956 was modelling bell-shaped oil production by hand without any equation (he adopted the logistic curve much later) and he was saying that the production curve mimics discovery curve with a certain lag. He was right and often I do not model production, I just show production and shifted discovery to guess the future production by looking at the discovery curve which is ahead of the production time*" Laherrere[Ref 164]

ring theme among traditionalists suggests that details in geology weigh more heavily than a stochastic probability model. I would not call this a balanced not well-grounded approach that considers both physics and mathematics. In spite of the informality of the arguments, we will certainly hit global peak oil production this century. This provides the annoying rub; we intuit these outcomes without providing a real basis for our arguments. As the data set gets larger this becomes less of an issue and I believe that a probability-based approach will describe the situation very well.[2]

2. CW #3. "This discussion is really a reprise, to a large extent, of the **Malthus theory on the evolution of population** and the evolution of resources to sustain it, particularly production of food resources."
To explain the profile of oil depletion, Malthus does not work out very well. Although we can acknowledge population dynamics as worthy of study, this point-of-view strays far afield from the fundamentals of the life-cycle of oil itself.

CHAPTER 17

The Diagnosis

What current situation do we find ourselves in?

"It has often been said that, if the human species fails to make a go of it here on Earth, some other species will take over the running. In the sense of developing high intelligence this is not correct. We have, or soon will have, exhausted the necessary physical prerequisites so far as this planet is concerned. With coal gone, oil gone, high-grade metallic ores gone, no species however competent can make the long climb from primitive conditions to high-level technology. "

Sir Fred Hoyle from "Of Men and Galaxies", 1964

Building from the Evidence

"How Did We Get Ourselves Into This Mess?". A significant fraction of the population thinks that oil production remains a simple matter of turning the spigot clockwise to get more oil, and that the big oil company, like the local water utility, takes care of the rest. They certainly don't understand the complexities of reserve growth. More than likely, they believe that raising the current reserve inventory becomes a measured, conservative decision made entirely under the control of humans. Kind of like deciding how much firewood you should place in your log-pile.

I am not an expert of the socio-economics of oil but have picked up a few nuggets from the tribal knowledge vault. Consider this bit of conventional wisdom: Overestimating the size of a reservoir has significant financial advantages. One can indeed potentially attract huge initial investment capital if you exaggerate a claim. But this practice can also attract charlatans eager to make a quick fortune. So I agree with the SEC's decision to put the stops on that practice, with big fines for fraudulent claims. And thus the oil companies and oil cartels have tried to make their reserve estimates as high as they can without overdoing it. But they still have lacked good ideas on how best to characterize their estimates (or at the very least make the data more publicly available). Some would suggest that the industry always makes "conservative" decisions based on efficient markets. Yet, I don't think that ambiguously-framed "conservative" decisions fares well in comparison with what you can accomplish with a good reserve growth model. I would describe the industry's approach not conservative but "non-predictive" and "safe". I can point to lots of other engineering areas where through the routine process of characterization one can make equally conservative estimates, but they back that up by doing an excellent job of predicting the asymptotic behavior with safety margins for any errors or noise that might creep in.

Bottom line, I believe the current industry estimates for reserve growth remain just as safe and non-predictive as a turkey thermometer, but that the underlying reality (having to do with the dispersion of searches through the volumes of potential reservoirs) can vastly improve our educated guesses. This makes it potentially predictive if the industry had the wherewithal to use better empirically-validated models. The simple model I initially used for reserve growth complements well to what Andy Grove did in the 60's.[1] Unfortunately, I also don't think that petroleum engineers have any control over marketing decisions. It's possible that insider oil technologists had knowledge similar to the characterization outlined here all along but management superseded their recommendations. Or else, perhaps they used it in their own internal book-keeping, but presented only a "safe" facade for the outside world to view. Or even worse, that keeping track of reserves has never progressed beyond the bewildering complexity and confusion as documented by Laherrere [Ref 113].

Unfortunately, we may have hit the plateau of peak oil, and have only the analysis of amateur sleuths and a few academics to explain the obvious that had confronted the establishment for years. If that slow a rate of progress applied to the rest of our situation, we should still use vacuum tubes in all of our electronics.

Linearity, Conservation, and Greed

Human beings appreciate linearity. We cope with an analog world by applying linearity. For example, most audio systems use principles of linearity to avoid distortion and provide a wide dynamic range. Linearity enables an audio amplifier to range from a whisper to a scream with nary a change in a waveform and allows the use of a single electronic analog filter independent of volume.

Here's the fundamental fallacy of linear thinking: if you think something is good, you just do more of it and pretend that this will make the future even better.

http://peakenergy.blogspot.com/2006/05/day-of-doombats.html

1. Grove pioneered characterizing silicon dioxide growth by evaluating simple diffusion-based models, and thus began Intel (in a nutshell). Getting oxide thickness correct could follow a process where someone checks the thickness every two minutes like they were watching a loaf of bread rise, but the technology and modeling has gone way beyond that now in Silicon Valley and China. Not so with the oil industry however; to the untrained eye it looks like they use the primitive trick of plunging a thermometer into the turkey roasting in the oven. This is safe but how primitive!

The oil shock model exhibits the same properties of linearity. Acting as a response filter on an initial discovery impulse, given the same *scaled* input stimuli, we should expect the same *scaled* production profile, assuming that we keep the same rate parameters.

But, in the real world, should we expect this scaling to naturally occur? In short: yes. And the simple reason why: greediness[2].

Say that the USA had a single delta discovery of **x** GBls of oil in 1930. And that this contributed to a production profile that peaked in 1970. Given the same extraction rates *per unit volume*, we should expect that the profile would look exactly the same except for the height of the peak, if instead of **x** GBls we had found an extra **10×** Gbls worth of oil in the original discovery.

But truly this only makes sense if we add the greed quotient to the equation. With human greed doing its part, the 10× worth of oil would have flowed like water. Civilization would have used the excess cheap oil for outdoor air-conditioners in Palm Springs, open forced-air doors in shopping centers, propelling the cruise ship Queen Elizabeth II at 50 feet per gallon, and a myriad of other activities that we would consider nowadays as wasteful.[3] For the same reason that we can suck every last drop out of the Colorado River or shoot every last passenger pigeon out of the sky, we would have used up the same fractional volume of oil by now, *no matter how much we originally had*.

Wherever you go, there you are

"*If you really want to reduce the amount of oil that you consume, you have to reduce the amount of gasoline you use*,"

— G. W. Bush, on a road trip to push energy initiatives

> By 1955, 5 million people lived in this basin (Los Angeles) and half of them had a car. Each day they burned 58,000 tons of natural gas, fuel oil, gasoline, and garbage, releasing more than 3,000 tons of air pollutants and blanketing the Southern California mountain ranges with up to 20 pounds of nitrogen compounds per acre. [Ref 116]

Conservation remains the only route to stem the tide.

To place this into a present-day context, consider the case of drilling in the Arctic National Wildlife Refuge. According to linear greed, extracting oil would just continue to feed the machine. But, alas, pointing out any opposition to drilling in the region, the pro-development side will implicitly suggest that environmentalists stand in the way of progress. So no matter its shortcomings economically or politically, conservation remains the only negative feedback in the loop to keep unconstrained growth in place. And the price of oil under a constrained supply might

2. Don't take this as disparaging, the "greedy algorithm" used in computer science works because it provides the most efficient path to solving problems.
3. Known as Jevon's Paradox, the theory goes that any conservation or efficiency gains made goes into use for leisurely or other consumptive activities.

provide the negative feedback to keep the use of oil down. Economists call this kind of feedback, *demand destruction*.[4]

Like an audio amplifier using negative feedback, one could perhaps sustain a lower consumption level, balancing greed with scarcity-driven price. Unfortunately this will only defer the inevitable.

> *"Economic systems provide sets of incentives. The incentives of all current systems are perverse, being designed to promote and reward maximum resource extraction and exploitation. This bias might have made sense in the 19th century, when the effects of technology were unknowable. The ignorance is now swept away, except for deniers and idiots; such incentives are economic and ecological suicide, now."*
>
> —Steve Ludlum
> http://theoildrum.com/node/4813

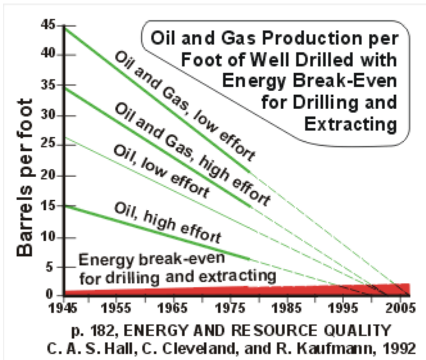


FIGURE 17-1. Under a diminishing return regime, you need to expend more effort to achieve the same results. [Ref 158]

Diminishing Returns

The dispersive discovery model states that various search rates combine to sweep out the volume of earth that we think oil may exist. We have a mix of high search rates that have effectively swept out a significant portion of the volume, and lower rates that take up the tail (so to speak). This combined with proportional extraction of the oil shock model, results in a 1-2 punch of diminishing returns. The chart in Figure 17-1 by Hall *et al* demonstrates this effect along orthogonal axes. Unless we consider the dispersed range of these *rates* along with a dispersed range of *volumes*, the end result can sneak up on ourselves and we get tempted to think a new discovery or two or three will turn the tide. But mathematically, that hasn't happened yet and won't in the future as we reside in the backside tail of the discovery peak.

Why We Can't Pump Faster

According to the Oil Shock model, to first order the rate of depletion occurs proportionately to the amount of reserve available. Overall, this number has remained high and fairly constant (I consider 5% per year high for any non-renewable resource). It takes effort to extract it substantially faster than this, but because of oil's value[5] we never have had a reason not to extract, and so it has maintained its rate at a steady level, the effects of OPEC notwithstanding. And because the extraction rate has never varied by orders of magnitude, we have limited historical insight into what we have in store for the future. In other words, even though we have no historical evidence for the possibility — can we actually pump faster?

4. Ask anyone who has built an audio amplifier to explain the utility of negative feedback, and you can better appreciate how it might apply.
5. They call it black gold for a reason.

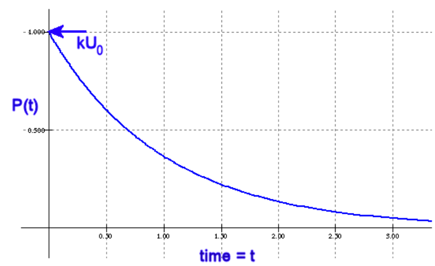


FIGURE 17-2. The law of proportional extraction states that the rate relates to current reserves by a constant *k*.

How. The following proportionality equation forms the lowest-level building block of the Shock model.

$$\frac{\mathrm{d}}{\mathrm{d}t}U(t) = -k \cdot U(t) \qquad \textbf{(EQ 17-1)}$$

Any shocks come about by perturbing the value of k in the equation. Painfully stating the obvious, the values of k can go up or down. Up to now the perturbations have usually spiked downward, often from OPEC decisions. In particular, during the oil crises of the 1970's, the model showed definite glitches downward corresponding to temporary reductions in the extraction rate imposed to member countries by the cartel.[6]

The characteristic solution of the first-order equation to delta function initial conditions derives to a damped exponential.

$$U(t) = U_0 \cdot e^{-kt} \qquad \textbf{(EQ 17-2)}$$

with extractive production following as the derivative of $U(t)$ (the negative sign indicates extraction):

$$P(t) = -\frac{\mathrm{d}}{\mathrm{d}t}U(t) = k \cdot U_0 \cdot e^{-kt} = k \cdot U(t) \qquad \textbf{(EQ 17-3)}$$

This gets back to the original premise: "rate of depletion occurs proportionately to the amount of reserve available".

As Foucher has pointed out via the Hybrid Shock Model (HSM)[Ref 23], this gives the behavior that production always decreases, unless additional reserves get added (the extrapolation of future reserves is the key to HSM). And if we reside near the

6. These perturbations formed one of the original motivations and basis for the shock model.

backside of a peak and without any newly-discovered additive reserves, it will go only one way … down.

Yet we know that plateauing of the peak will likely occur, at least at the start of any detectable decline. This will invariably come about from an increase in extraction from reserves. According to the shock model, this only increases if k increases. We can model this straightforwardly:

$$\frac{d}{dt}U(t) = -(k + ct) \cdot U(t) \qquad \textbf{(EQ 17-4)}$$

Regrouping terms to integrate:

$$(dU)/U = -(k + ct) \cdot dt \qquad \textbf{(EQ 17-5)}$$

$$\ln(U) - \ln(U_0) = -(kt + ct^2/2) \qquad \textbf{(EQ 17-6)}$$

this results in

$$U = U_0 \times e^{-(kt + ct^2/2)} \qquad \textbf{(EQ 17-7)}$$

$$P(t) = (k + ct) \times U_0 \times e^{-(kt + ct^2/2)} \qquad \textbf{(EQ 17-8)}$$

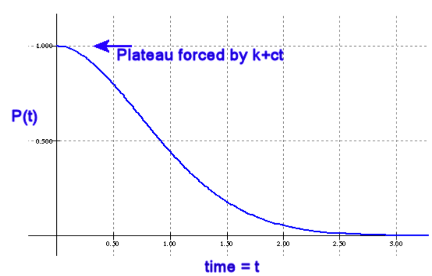


FIGURE 17-3. By increasing the extraction proportionality constant linearly with time, the onset of the downslope gets deferred or delayed slightly.

Notice how this gives a momentary plateau that soon gets subsumed by the relentless extractive force.

Instead of a 2nd order increase, we can try adding an exponential increase in extraction rate:

$$(dU)/U = -(k + a \cdot e^{bt}) \cdot dt \qquad \textbf{(EQ 17-9)}$$

$$\ln(U) - \ln(U_0) = -(kt + ae^{bt}/b) \qquad \textbf{(EQ 17-10)}$$

The solution to this results in a variation of the Gompertz equation:

$$U = U_0 \times e^{-(kt + ae^{bt}/b)} \qquad \text{(EQ 17-11)}$$

$$P(t) = (k + a \cdot e^{bt}) \times U_0 \times e^{-(kt + ae^{bt}/b)} \qquad \text{(EQ 17-12)}$$

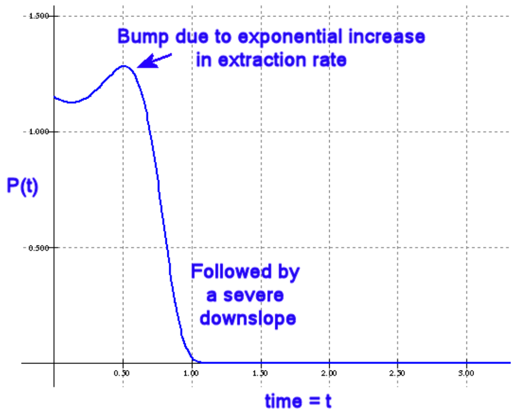


FIGURE 17-4. By increasing the extraction rate proportional factor exponentially, the downslope gets deferred even further. However, the slope becomes more severe once depletion sets in. This extends the life of a field artificially.

The uptick of the plateau has now become more pronounced. Overgeneralizing, “we” can now “dial-in” the extension of the plateau “supply” by “adjusting” the extraction rate at an accelerating rate. I place air quotes around these terms because I have a feeling that (1) no one knows the feasibility of improved extractive technology and (2) it will eventually hit a hard downslope. Under the best circumstances we may prolong the plateau somewhat.

Conversely, assuming the conditions of a non-rate-limited supply, oil producers can increase their output at the whim of political decisions. No one really understands the extent of this strategy; the producers who consider oil as a cash cow will not intentionally limit production, while those who emulate OPEC cartel practices will carefully meter output to meet some geopolitical aims. In this case, shareholders demanding the maximization of profit do not play a role, and greed plays a limited role.

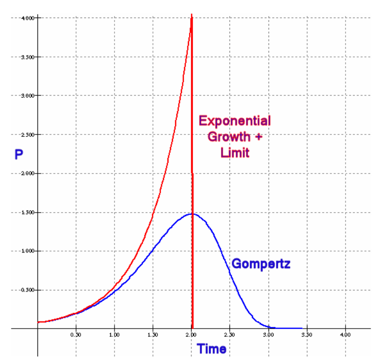


FIGURE 17-5. Gompertz curve shows similarity to non-dispersive growth hitting an abrupt resource constraint. The accelerating Gompertz extraction compresses the time frame to where a real constraint limit would occur. Qualitatively we see the same crash.

Can we delay peak by upping extraction rate?

Yes.

But at a downside. The downside becomes the downslope. Quite literally, upping the extraction rate under a limited supply makes the downslope that much more pronounced. The Gompertz curve shows a marked asymmetry that more closely aligns to exponential rise and abrupt collapse than the typical symmetric Hubbert curve.

I first created a simplified oil shock model[7] with a constant extraction rate (relative to reservoir size) and a clear primary peak. As an experiment I added a progression of linear increases in the extraction rate timed to start at the original peak position. I intended the increases to offset the decline in production and thus move the peak to the future.

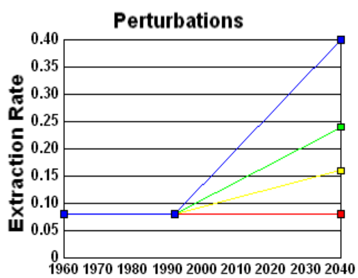


FIGURE 17-6.
A set of perturbations that show various rates of gradual upward shocks.

As the following curves show, this linear shock does shift the peak, but not as much as one might intuitively predict. The amount of shift seems to fairly quickly reach an asymptotic value of perhaps 8 years total "delayed gratification", which becomes *independent of rate increase*. And then we get punished on the backside.

The Cliff Notes summary

Try sucking the juice out of a Sno-Cone until you start seeing white ice. At that point, it doesn't matter how hard you suck. If you try real hard you might get one last slurp of flavor. But you might as well let it melt and get some Kool-Aid out of the rest.

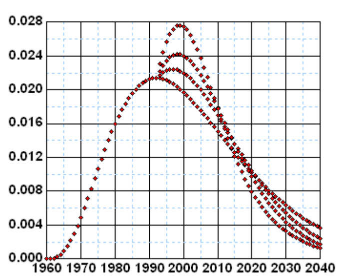


FIGURE 17-7.
The quicker the upward shock pressure, the more that an artificial delayed peak becomes apparent. A sharper decline follows a higher peak.

Why does this delay reach an asymptote? One could analytically demonstrate this effect, but it intuitively occurs because the real decline exhibits a sharp exponential

7. Single discovery spike in 1960, with rates of fallow=build=maturation=0.1/year.

damping which swamps any linear increase in extraction rate. The increases in discovery and extraction rate simply cannot keep up with this relentless decline in reserves. In the worst case, even adding a 40% extraction rate per year at 2040 won't cut it.

TOP: The Overshoot Point

The closer that global peak oil comes to fruition (if it hasn't occurred already), the more likely that sudden perturbations gain special significance. Unlike regional peak oil occurrences, where the oil companies can simply switch suppliers and thus ameliorate the effects of a local peak, we have nowhere else to go under a global peak oil regime. As I showed in the previous pages, upward pressures on extraction rates **can** prolong the peak's occurrence by several years, serving to obscure the true peak either via a plateau or a wiggly roller-coaster.

So, hypothetically, the world can delay the peak if extraction rates start to incrementally increase beyond an oil shock onset. Up to that point in time, extraction rates show a relatively constant value. Without further discoveries, a gradual increase remains the only way to maintain oil production at its current value (not to downplay the economically-driven increases required due to future demand pressures).

The following two hypothetical curves show production under two regimes. The more-or-less normal looking curve shows a clear peak under a constant extraction rate over time starting from day-one discovery. This makes the somewhat naive assumption that extraction rate remains at the mercy of technological limitations and that oil companies don't have a secret trick or two up their sleeve. The other, "flat-topped", curve comes about if the extraction rate starts to modulate upward right when we detect the peak.[8]

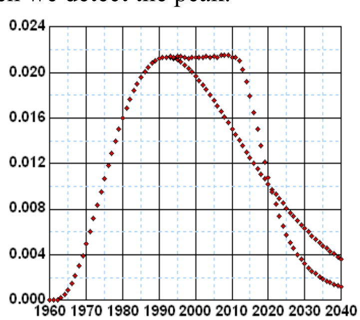


FIGURE 17-8.
The oil shock model can reproduce a plateau effect with an appropriate upward pressure of extraction as the natural decline sets in.

For the plateaued peak, the extraction rate profile looks like the following inflected curve. Taken from the parameters of the generic oil shock model solver, we can plot:

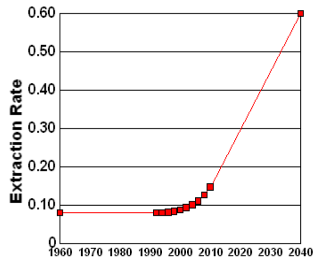


FIGURE 17-9. This upward extraction shock produces the plateau followed by a decline in the previous figure.

This clearly shows that we may have difficulty determining when peak oil officially hits if the oil companies soon start to modulate extraction upwards. Right near the peak, we only need subtle upward changes to counteract the relentless downward pressure of oil depletion. Mathematically, this comes neatly out of the wash any time we deal with a slope near zero, which occurs right at peak. However, in uncertain psychological terms, increasing the extraction rate through technical improvements or last gap measures will perhaps confuse people that believe in the sanctity of the symmetric Hubbert curve. No one can really predict how people will perceive a plateau, in the context of a long duration of continual growth.

To summarize, in a reality-based framework, peak does not have to occur when half the available oil gets used up, instead we really have to think in terms of a new metric to alleviate confusion.

I propose using the inflection point in extraction rate increase to more effectively describe when a pragmatic peak oil point hits us. I will refer to this as **The Overshoot Point**. Unfortunately, the overshoot only becomes apparent if you look at implicit values contained within a model. It will not show up in any explicit measures such as yearly oil production. A good model remains the only effective way to make sense out of this mess.[9]

We reach TOP as we keep trying to increase the extraction rate until the reserve completely dries up and the entire production collapses. Summarizing, we need an extractive acceleration high enough to maintain a plateau, but at some point the extraction rate needs to reach ridiculous levels to sustain a plateau, and if we continue with only a linear rate of increase it starts to give and the decline sets in.

8. The wolf at the door remains the vicious backside as we prolong the peak. *i.e.* No free lunch.
9. Good old hand-waving, the approach of the bullet-point crowd and of Michael Lynch, just won't cut it any longer.

Clearly, this approach won't sustain itself. And if we stop the increase completely after a lengthy period of artificial forcing instead of letting it fall naturally at the outset, the production falls that much faster (and possibly precipitously).

Demand Destruction

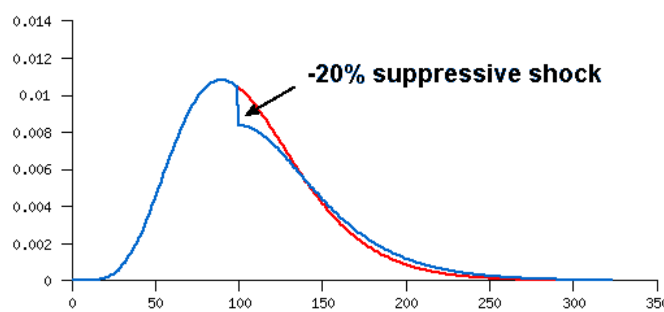
If we have reached a peak in oil production, we can only maintain a plateau if we accelerate the extraction rate, according to the oil shock model. If on the other hand we reach a recessionary period (which all evidence points to[10]), then we can expect concomitant decreases in oil production. A negative shock caused by a recession will do two things to a projected oil shock model production curve. First, it will cause an immediate dip in extractive output, and secondly, if the shock continues, then the tail will broaden out.

If this depression extends for any length of time, then it becomes that much harder to reach the previous peak, *even if the economy recovers.* The inexorable loss of cumulative reserves during this recessionary period reduces any kind of springboard effect that a larger reserve would have provided. Only a *huge* acceleration in extractive pressure during an economic recovery could get us even close.

But wait a moment, didn't that situation in fact happen in the late 1970's oil crisis with the peak deferred for a number of years? Yes, but consider that at that time, only the USA had hit a peak and the global reserves kept on increasing due to the latency of the production phases after the global discovery peak in the 1960's. The springboard effect as demonstrated by the depressive shock in the oil shock model could occur *then* but not *now.* The bulk of the discoveries lie much too far in the past to allow any real recovery.

I consider this a critical idea, as many people believe that the recession produces some sort of state of suspended animation that will revert back to a prior state when we wake up from it. Alas, it won't and the oil shock model verifies that intuition.

FIGURE 7-10. Demand destruction shock reduces peak, and sustains production only slightly.

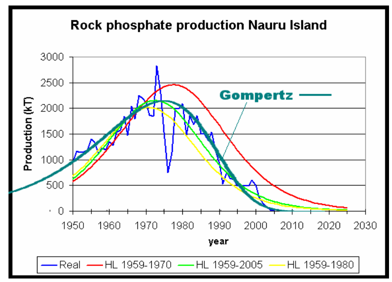


10. Some analysts have suggested arguments that peak oil in part may have caused the 2008+ recession.

The Gompertz in Practice

Do we ever see the Gompertz curve in practice? I venture to guess that yes we have.[11] Not a pleasant topic, but we should remind ourselves that fast developing extinction events may show Gompertz behavior. Oil depletion dynamics would play out similarly to sudden extinction dynamics, if and only if we assume that oil production immediately followed discovery in succession and the extraction rate then started to accelerate. Then when we compare to an event like the extinguishing passenger pigeon population (where very limited dispersion occurs), the culling production increases rapidly and then collapses as the population can't reproduce or adapt fast enough.

FIGURE 17-11. Rock phosphate production on Nauru Island in the Pacific shows Gompertz-like behavior.



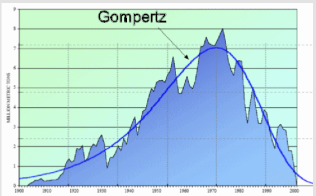
As a key to modeling this behavior, we strip out dispersion of discovery completely, and then provide a discovery stimulus as a delta function. For passenger pigeons the discovery occurred as a singular event along the Eastern USA during colonial times. The culling accelerated until the population essentially became extinct in the late 1800's.

To verify this in a more current context, we can look at the economically important worldwide phosphate production curve[Ref 114]. The search for phosphate started within a few decades of the discovery of oil in the middle 1800's. One may assume the shape of the phosphate discovery curve might also follow a logistic like curve. But I contend that this does not occur because of accelerating extraction rates of phosphate leading to more Gompertz-like dynamics.

The first plot that provides quite a gradual increase and sharp decline comes from phosphate production on the island of Nauru [Ref 115] to the left. Note that the heavy dark green line that I added to the set of curves follows a Gompertz function with an initial stimulus at around the year 1900, and an exponential increase after that point. The total reserve available defines the peak and subsequent decline. The long uptake and rapid decrease both show up much better with Gompertz growth

11. The long tail at minus infinity violate rules of causality; this means that the start of exponential growth has a finite value.

Sulfur Production



Mined USA Sulfur production between 1900 and 2000 also shows Gompertz behavior. Highly localized along the Gulf of Mexico and efficiently mined with the Frasch process, the recovery of sulfur dropped to zero by 2000. Now most sulfur produced comes from by-products of other processes [Ref 155].

than with the Logistic growth. This does not invalidate the role of the Logistic (of which the Dispersive Discovery model derives the mechanism), but it does show where it may not work as well.

Remember that all the phosphate on that island essentially became "discovered" as a singular event in 1900.[12] Since that time, worldwide fertilizer production/consumption has increased exponentially reaching values of 10% growth per year before leveling off to 5% or less per year.

> *As a result, fertilizer trade increased from about 2 million tons in 1950 to about 40 million tons in 1986.[Ref 117]*

Over this period, this rate compounds to 9% annually, clearly an exponential increase; this clearly implies that the fractional phosphate component of fertilizer increased at the same rate.

Then if you consider that most easily accessible phosphate discoveries occurred long ago, the role of Gompertz type growth becomes more believable. No producer had ever really over-extracted the phosphate reserves in the early years, as we would have had no place to store it, yet the growth continued as the demand for phosphate for fertilizer increased exponentially. So as the demand picked up, phosphate companies simply depleted from the reserves until they hit the diminishing return part of the curve. The producers can essentially pull phosphates out of the reserves as fast as they wanted, while oil producers became the equivalent of drinking a milkshake from a straw, as sucking harder does not help much.

For worldwide production of phosphates, applying the Gompertz growth from non-dispersive discoveries gives a more pessimistic outlook than what J.Ward calculated [Ref 115]. The following figure compares Ward's Hubbert Linearization against an eyeballed Gompertz fit. Note that both show a similar front part of the curve but the Gompertz declines much more rapidly on the backside. The wild production swings may come about from the effects of a constrained supply, something quite familiar to those following a commodities market for a scarce resource[13].

12. Not hard to imagine for the smallest independent republic in the world
13. Or the effects of constrained credit on speculative outlook in the bond and equity market

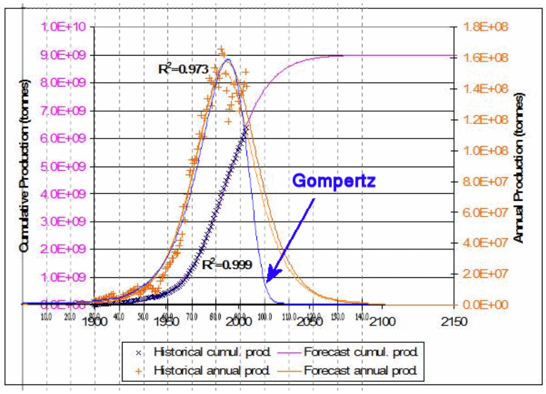


FIGURE 17-12.
Phosphate production as a Gompertz curve. Terrestrial phosphate deposits show little discovery dispersion and accelerating production controls the rise and decline.

So we have the good news and the bad news. First, the good news: oil production does not follow the Gompertz curve as of yet and we may not ever reach that potential given the relative difficulty of extracting oil at high rates. The fact that we have such a high dispersion in oil discoveries also means that the decline becomes mitigated by new discoveries. As for the bad news: easily extractable phosphate may have hit TOP. And we have no new conventional sources. And phosphate essentially feeds the world.

Read the rest of Ward's article for some hope:

> *Perhaps the best way to frame the debate from here is to suggest that, like oil, the world has been endowed with a given quantity of "easy" phosphorus (e.g. rich island guano deposits in places like Nauru) that can be – and have been – mined quite rapidly, as well as a larger endowment of lower-grade phosphate rock. While the easy phosphate has passed its peak, the low-grade phosphate should be considered separately. Figure 3 shows an example forecast where the total area under both curves (equal to RURR) is 24.3 billion tonnes, but the "easy" phosphorus (purple) is 9 billion tonnes as in Figure 2. Assuming the production history is mostly related to easy phosphorus, the fitting parameters (a and k) for the "hard" phosphorus cannot be established. Therefore, the height and timing of the secondary peak are unpredictable.*

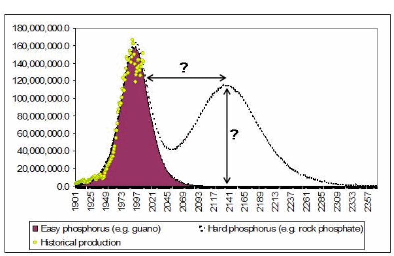


FIGURE 17-13.
Possibility for future phosphate production from other sources (from Ward).

Helium reserves

As natural gas supplies dwindle, and the accompanying helium reserves start to disappear, the military will likely have a hard time keeping the blimps aloft.

Other Circumstantial Evidence

Several pieces of circumstantial evidence pointing to a peak has already occurred [Ref 127].

1. There's a very good chance claimed OPEC reserves are exaggerated.
2. World production essentially stopped increasing in late 2004.
3. Decline rates of existing production are very high
4. Hubbert Linearization points to peak oil
5. At least one major oil company is warning us (of peak)
6. The price of oil keeps going up.
7. There is no evidence of Saudi spare capacity
8. There are geopolitical and climatic risks to the existing production level

The global oil shock model shows the same peak based on past discovery data. The cusp drops off like Figure 8-25 on page 140 assuming extraction rates don't continue to ramp up near term, while a continuing ramp-up will round the peak, either extending it a few years or creating a plateau, with an impending cliff highly likely. This also assumes that no oil company makes sudden large discoveries that come on-line quickly.

CHAPTER 18

The Implications

Why should you believe any of this?

"All models are wrong, but some are useful"

— George E.P. Box

"*Men argue; nature acts*."

—Voltaire

"*Peak Oil theory is garbage as far as we're concerned.*"

— Geologist Robert W. Esser, in 2006 [Ref 150]

Charles Darwin once stated: "*if Mother Nature can, She will tell you a direct lie*".[1] Since I use probabilities in this text and I believe in the concept of probabilities, you would have every right to call me a hypocrite if I didn't consider that there exists some probability that parts of this text are wrong and the extrapolations do not pan out. Yet, paraphrasing Harry Callahan, "*Do we feel lucky*?".

"In news the rule is that liberals will watch the news, and conservatives will watch conservative news. A liberal will watch to see what you think, the conservative will watch to see how much you agree with him. This is why the headline world is so far to the right even of the content."

— Stirling Newberry

Theory vs. Experiment

The models I espouse constitute a somewhat informal theoretical framework. In traditional science, theorists don't always have to defend their position. That's what experimentalists get paid to do and constitutes the classic approach to scientific verification. Theorists also don't necessarily mind getting attacked for their ideas as that has become part of the bargain of doing science. So until we get independent substantiation of the models, we have to entertain the skeptical views.

1. I recall the rallying cry my co-researchers and I would yell out when faced with challenging circumstances in the laboratory, man-made or not, *"Nature's fighting us. We must be on the right track"!*

This also echoes the sentiment of that what we do not understand, we tend not to believe. The struggle to understand takes time, and this text basically functions as a narrative in that understanding.

Earlier, I mentioned the fact that fields such as risk management and insurance used probability models to project future conditions. The fact that the practitioners in the insurance field have to at least make a living forces them to use a formal and robust approach to back up their projections. I have tried to use the same rigorous approach and rely on interactive review to verify the models. But of course that doesn't always work, and you end up with a disaster of cascading faulty projections. It often happens that one can't predict market psychology with any certainty. And with eager management at the helm egging the financial teams on, any Black Swan/Fat Tail events could easily drive the investments into losses. Personally, I consider the markets a very difficult modeling activity[2], and would rather tackle something straightforward such as oil depletion.

No one trusts the simulation except the engineer who ran the simulation. Everyone trusts the test except the engineer who ran the test.

M.Santori, Desktop Engineering Magazine, 2003

Given that optimism, then why do not economists discuss or analyze anything relating to very concrete matters of resource (e.g. oil, natural gas, etc) depletion? In comparison to the precarious nature of financial quantitative analysis, resource depletion remains simple and has nothing to do with belief systems, money-making opportunities, or gaming psychology; as it appears more like straightforward bean-counting. Yet, in the end no one has bothered to confront one of the most challenging problems of our times, and one that has huge economic implications.

The practice of economics as based on infinite resources assumes a forever expanding growth potential. Without that assumption, macroeconomic theory basically collapses on itself. And the economists (who ironically invented the sunk-cost model) do not want to see all their theoretical work go down the drain. This indeed has turned into pseudo-science cloaked by the religious belief in the free-market and subscribing to the comfort of group-think. It concerns me that if oil depletion analysis ever sustains the negative connotations of financial accounting, we will need to expend that much more effort framing the arguments.

> By their activities, quants admit that despite their misgivings they have at least given cover to some of the wilder schemes of their bosses, allowing traders to conduct business in a quasi-scientific language and take risks they did not understand. Dr. Goldenfeld of Illinois said that a decade ago when he posted scholarly articles, some of which were critical of financial models, on his company's Web site, salespeople told him to take them down. The argument, he explained, was that "it made our company look bad to be associating with Jeremiahs saying that the models were all wrong."

2. And only hope that it hasn't given mathematics and mathematicians a black-eye in the public's opinion.

"Real news, useful news that could predict the future is no longer in the MSM, precisely because hedge fund managers and people like that make money on the future. Knowing what is going to happen in the future is money in the bank. The more people who know the future, the less money the investor will make."

— cordgrass, 11/27/2008, dailykos.com

> Dr. Goldenfeld took them down. In business, he explained, unlike in science, **the customers are always right**. [Ref 159]

So, why should you believe any of this oil depletion stuff? *Because it describes the laws of nature in a self-consistent fashion, and not every law of nature exists for people to generate a financial windfall from.*In other words, no one will use an oil depletion model to make a fortune with. That said you have to approach the work as objectively as possible. Hopefully, the analysis can actually do some good and perhaps wean us off of oil sooner rather than later.

The environmental economist Nate Hagens summarized the current conventional wisdom in a response to a reported instance of an Exxon press release: [Ref 132]

"You can't fire me, I don't even work here"

— Kramer

> *I am beginning to believe it is an advantage to NOT work in the oil industry to understand oil. These people have been wrong, are wrong, and are about to be VERY wrong with their understanding of what peak oil means. Peak Oil has many definitions, but the most common is the all time high in world annual production of crude oil. [Human and technology-based] Resources have little to do with it. (There are probably 10 million earthworms on my property -but even with a team of people and the best equipment I might only get a fraction of them). Higher prices and higher technology have little to do with Peak Oil, which has to do with cheap, reliable flow rates. There is not the slightest evidence that market theories (or activities) has helped find any more oil and gas (in the United States) since price-induced drilling increases had essentially zero impact on the production (or finding) of oil and gas.*
>
> *Lets scrap the word 'peak oil' for the moment. To the economists and cornucopians at Exxon, the API, the EIA, etc. I ask these questions:*
>
> 1. *Do you expect oil production costs to get cheaper over time?*
> 2. *Have we past the point of cheap oil? (which is what matters - who cares if we can get an extra 20 mbpd if it takes more energy, more steel, more water and costs $500 per barrel)*
> 3. *Will the energy and other resources you use to procure oil and natural gas increase or decrease in the future?*
> 4. *Irrespective of resource or reserves, what will be the highest, reasonably low priced (say under $80 cost), FLOW rate that you can consistently provide that is not subject to geopolitical disruptions at the margin? (i.e. is there a perpetual cushion in case something goes wrong)*
> 5. *What is the error band and confidence interval you assign to your above answers? Are you willing to stake the future of industrial civilization on your answers? Even if there is a 5% chance that the resources you see translate to regular, cheap, flow rates of high quality oil, that is too big of a risk for society to take (and I think it is much higher than that).*
>
> *You people are asking the wrong questions, because you've been focused on what you believe is the most important aspect of the problem - where IS the oil. That is a small part of the many more important questions, yet the group think and myopia*

has created an enormous blindspot. A couple months ago, if I would have told you the Federal Reserve would DOUBLE its balance sheet since the end of September, would anyone have believed me? Well, they did. Rules and facts change. Correlations that worked in the past are now uncorrelated. What was uncorrelated in the past is now completely correlated. That's why economics isn't science. Its based on a moving target. Economists at the oil companies are trained to think in resources and price, not in energy costs and externalities. They will not see this Black Swan until it bites them in the ass. [Ref 132]

Hagens captures the "Whocoodanode?" sentiment well. For some people, an investment in knowledge will likely pay off more than investing in the stock market or speculating on oil futures[3]. We can almost rule out investing in new ways to find oil, especially in regards to the "*Drill, Baby, Drill*" mantra. We may have to brace ourselves as we will likely face a huge amount of resistance to any kind of conservation measures.

"*Have I told you yet today how much I hate these people?*"

— Mike Malloy

White House: Greed will help

"In the past few years, **we've seen too much greed and too little fear**; too much spending and not enough saving; too much borrowing and not enough worrying," Summers said Friday in a speech to the Brookings Institution. "Today, however, our problem is exactly the opposite."

In remarks to a private dinner at the U.S. Chamber of Commerce on Wednesday, Summers was even blunter, according to an attendee: "Before, we had too much greed and too little fear. **Now, we have too much fear and too little greed.**" [Ref 153]

Indeed, the relentless presence of greed will hamper our conserving of oil and getting off the oil spigot. Greed remained the only constant from the first moment we struck oil in the 1800's. It provided a virtually unconstrained tap of stimulus to extract every last bit of oil we could access. Not that I think greed will ever disappear, but I anticipate perhaps that we can modulate this effect somehow. The fear of increasing scarcity of oil may have that effect due to demand destruction, but due to the engine of the economy and its requirements for energy to propel increasing GDP, we will likely see greed once again drag production along, one way or another. Does this not sound familiar?

"*A crisis is a terrible thing to waste*"

— economist Paul Romer

So I contend that we need now, more than ever, to mathematically understand at a deep fundamental level the flow of physical resources on a global basis. Unfortunately with any down-turn in the economy, pessimists wielding mathematics often

3. If you look at the current global financial crisis, you realize that poor understanding of mathematical and statistical risk had a big hand in the economic downturn.

get the blame for practicing "defeatism" and thus reflecting a self-sustaining downward spiral.

The lack of information and fear, uncertainty, and doubt in the world's finances directly parallels the lack of knowledge we have in oil reserve accounting.[4]

What you cannot measure you cannot control. And that remains the primary reason why we have to model and simulate and analyze this stuff by our lonesome. The corporatocracy will never lift a finger to help.

4. *"The money Sanders is referring to is loans the Fed has made outside the TARP program. Bernanke says the loans are "over-collateralized," but opted not to disclose anything more about them, citing the "stigma" attached to receiving such loans. To that, I have only two comments.*
First, what stigma? The market is now assuming that every financial firm is in deep doo-doo. If anything, ***knowing which firms are receiving help and which are not removes one of the biggest uncertainties out there****. It might actually improve the markets"*
[Ref 154]

CHAPTER 19

The Prognosis

What can we extrapolate for the future?

"We fear change."

– Garth Algar

As suggested by the preceding diagnosis, we have the ability to model and thus understand what ails us — a quantifiable global supply deficit of easily refinable and accessible fossil fuel. From the trending of discoveries and the behavior of production we can safely predict things won't get any better. But we may yet find ways to alleviate the situation.

Amongst our options we can hope for, or at the very least prepare for include:

1. The saving Black Swan event
2. A high EROEI from alternative fuel sources
3. Intentional conservation and unintentional demand destruction

The idea of The Black Swan comes out of the writings of Taleb and refers to a surprising, often positive, outcome that no one could have predicted beforehand.[1] However, since no one can predict what ultimately exists outside of everyone's radar screen, this becomes a tautology, and almost a moot issue — serving more as a tantalizing hope or another puzzling conundrum, than anything we can quantify.

1. This compares to the New Orleans hurricane and flooding of 2005, which refers to a blatantly obvious negative outcome that astute observers predicted, but those in power chose to ignore. The entire Peak Oil issue is a Whocoodanode, unless it becomes overcome by a Black Swan event. We have yet to hear the phrase, "I don't think anybody anticipated the peak of the oil". And mirroring the Katrina infrastructure apologists, we may yet semantically argue the meaning of "overtopped" [Ref 120].

We can perhaps provide some bounds to the speculation, so earlier I presented the thought experiment of hypothesizing an infinite URR and interpreting how that would play out.

Prospects for a "Soft Landing"

Since I don't want to go beyond the scope of a modeling discussion we will concentrate more on what conservation, either intentional or unintentional, may accomplish in regards to what oil depletion analysts refer to as providing us a *soft landing*.

Price Volatility

As fuel becomes more scarce, prices will certainly rise on average, and this will lead to a forced form of conservation called demand destruction (see See "The Prognosis What can we extrapolate for the future?" on page 357.). We will also have to get used to higher volatility of gas prices from here on out. Large fluctuations in the price of a constrained resource fall out naturally from even the simplest analysis of supply/demand curves from *Econ 101*.

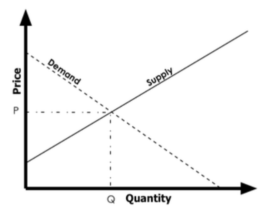


FIGURE 19-1.
Supply and Demand curves. Economists show the intersection graphically as the curves themselves are not always well understood or well behaved.

When the resource becomes constrained, the supply curve goes more and more vertical (i.e. it hits a hard constraint), implying that the price a supplier will offer for a good only depends on demand — describing the transition between the elastic and inelastic regimes.

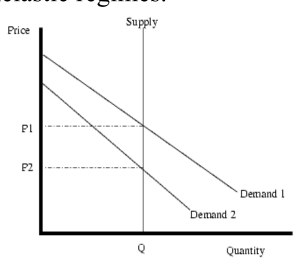


FIGURE 19-2.
When supply hits a hard stop then it abstracted as a vertical line, and demand affects price most sensitively.

Any sensitivity analysis performed against *Quantity* will show large excursions for a vertical supply curve. Investors have known this behavior from markets such as precious metals for some time now.

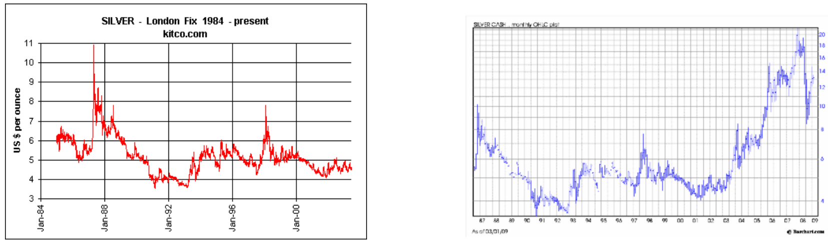


FIGURE 19-3. Silver prices show fluctuations of a constrained resource.

The other part of the equation, speculation *ala* the futures market, will further exaggerate the effect. Those familiar with signal processing know that a derivative-based projection will only amplify any noise already inherent in the system.

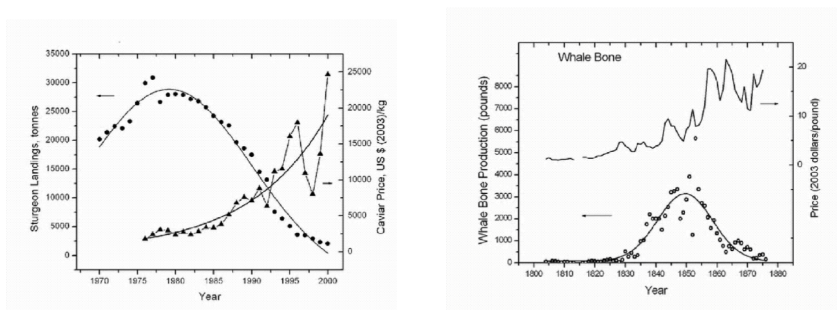


FIGURE 19-4. Price fluctuations as resource constrained sources for caviar (left) and whale bone (right) peaked.

In addition price functions as a proxy for other fundamental measures. Part of the price proxy applies to a concrete measure like supply, but a significant amount relates to it performing as a mechanism to game the system and others into making money. As long as this duality occurs —- a real metric versus a game theory element — it makes no sense to impose any kind of rationality to price. Unfortunately, the media and politicians who rejoice at the next drop in gas prices will pay later unless they plan for the eventuality of higher oil prices and work out some alternative energy strategies.[2]

2. But since most people have a patience horizon of a few days, they will treat the unpredictable roller coaster of price movements as a chicken would a reflection of itself in the mirror every morning. At least the chicken has an excuse for its short term memory loss.

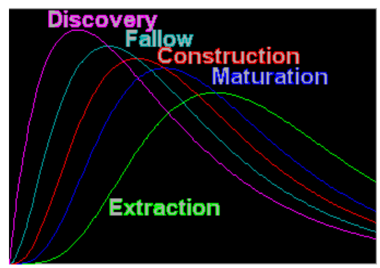
Geological Peak vs. Logistical Peak

As a step to understanding how a future accounting of our energy resources might work, consider the idea of a *geological* peak versus a *logistical* peak. A geological peak, which has certainly occurred in terms of discoveries, remains an eventuality we can't do anything about. However, we can defer the latter (or could have) by throttling the production in an optimal fashion — a very business-centric way of thinking. The chief of the oil company Total said:

> People are failing to deal with the reality of the price, which has nothing to do with speculators or even any lack of reserves, which are ample. *"It is a problem of capacities and of timing,"* de Margerie says. *"This is the real problem of peak oil."* [Ref 121]

I prefer to distinguish the two types of peak as residing on different phases of the oil shock model. Essentially, a geological peak occurs during the discovery process; we hit the peak when we think we have made the most volumetric discoveries per year. On the other hand, the logistical peak only occurs when we start extracting the oil, having to wade through the *fallow*, *construction*, and *maturation* phases prior to that point.

FIGURE 19-5.
Phases of the production curve according to the oil shock model.



Since each of these phases adds a cumulative lag term to the discovery peak and we can indeed modulate these terms via technology or business decisions, one really can't argue with de Margerie's pragmatism.

Unfortunately, it really doesn't help matters. An analogy to population dynamics might help here. Consider again the case of the decline of the North American passenger pigeon and how it compares to peak oil dynamics. First, we relate the geological oil peak to the historical observed peak of the pigeon population. That becomes the year that hunters decided that they had essentially discovered a maximum population that they considered worthwhile to harvest. Next, it turns out that we can simply equate the logistical peak with the maximum in the yearly passenger pigeon harvest. Note that the peak bird population preceded the maximum harvest by decades. In the long run it didn't matter that we deferred a semantically-defined peak — by that time, the pigeon population entered free-fall and could not recover.

But then the good news. Fortunately, a free-fall in oil production probably will not occur; extinction events have differing mitigating circumstances, such as whole-scale destruction of habitat that prevents sustaining of a population. Fossil fuel production only revolves around seeking every last pocket laying dormant somewhere, which will likely take a long while to play itself out.

Yet, as a sobering reminder, our own geological oil peak in discoveries occurred statistically in the early 1960’s.

An oil depletion trend line that should get more publicity plots the monotonically increasing ratio of production over estimated discoveries as a function of time. I show this value for each past decade in the following table:

TABLE 1. Ratio of P/D per decade

Decade	**Production/Discoveries**
1940's	0.14
1950's	0.2
1960's	0.29
1970's	0.62
1980's	1.07
1990's	2.0
2000’s	**?????**

I find it portentous that the numbers look like they increase at a roughly exponential rate, but we know that can’t keep going forever. At some point it needs to show an inflection point and start to level off and then creep back toward a value of unity — which means that we use as much as we find. Although I don’t show the data point for the 2000’s yet, the accumulated numbers have not yet budged from the previous decade. When we look back at the inflected curve, it will point out in hindsight the physical rationale behind what we see in purely economic terms — i.e. plotting the production peak alone only tells half the story.

Infinite Reserves

Every so often, someone will chime in on the potential futility of performing a URR analysis to estimate peak oil dates. I tend to side with these critics, but not for the reason they have in mind. As a case in point consider this post from a long-running discussion thread at the PealOil.com message board.

> I see. Well, with that in mind, let me volunteer an interesting little article I recently saw in some volume or another of Natural resources Research or some such technical rag. Maybe any of the other people who actually read the research on this topic can chime in and correct me if I paraphrase it incorrectly. This guy fit all sorts of the Hubbert curve stuff to oil production, the standard curve fitter type stuff which have been used to depict the end of world, over and over again, except he measured the goodness of fit of ANY of these curves when using different ultimates. He used Hubbert's ultimate, USGS ultimate, any other ultimate anyone had recently suggested, and he **discovered that the same curve which fit one estimate of ultimate could fit darn near any others, even with these huge, and some would claim, meaningful differences**. Apparently, these ultimate differences didn't matter much at all...literally. If you would enjoy a good read which quite reasonably dissembles the curve fitting approach, I'll go look it up as a reference for you. [Ref 122]

I highlighted a portion that I agree with — but for all the wrong reasons.

The commenter asserts that widely different parameterized curves, using a single parameter such as URR, can fit the same data equally well. Fair enough. But consider the case where we have a production curve that drops off as ***1/t*** where ***t***=Time. If we follow the progression of this curve it monotonically decreases each year — in other words, year after year *ad nauseum*. Unfortunately, the ultimate cumulative (the URR) for this curve happens to trend toward an infinite value! Try as you might, you cannot argue this result, as this property comes as a mathematical given. Take a look at the curve below — a simple hyperbola — and you would perhaps imagine that the area under the curve, the cumulative production, had a finite value. An intuitive guess perhaps, but wrong.

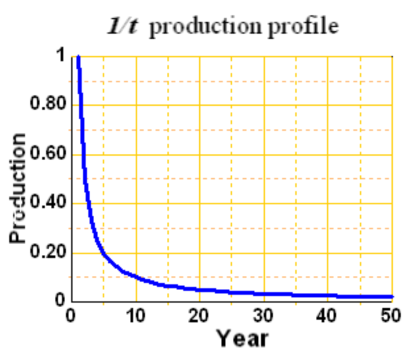


FIGURE 19-6.
Production profile that shows infinite URR. The long tails of the curve essentially hold an infinite supply of oil, yet yearly production monotonically decreases each year.

From this simple thought experiment, I find it not at all difficult to comprehend that a number of curves with vastly different URR's will fit the data equally well. After all, the difference between a finite URR and an infinite URR remains infinite, or at

least some large number depending on how fast production falls off. Which is exactly the depletion dynamics the commenter complains about.

However, this should not obscure the fact that production indeed decreases each year after we have hit the peak for the ***1/t*** curve. So whether or not we have an infinite URR, it does not matter when we have to confront a decreasing yearly supply in the face of a yearly increasing demand for oil.

Which brings us to the real issue. I think that many oil depletion analysts over-rely on the URR approach and risk missing the forest for the trees. The usual heuristic applied, that the peak occurs when cumulative production has hit URR/2, will not work in many cases, and will actually likely fail in every case of an asymmetric production curve. So for a constant URR, if we do indeed have longer tails then the peak occurs at < URR/2 (i.e. earlier), while if the tail shortens up then peak occurs > URR/2 (i.e. later). In the former case, it becomes a case of a terminal foreboding, while in the latter you have optimism right until you get hit by a truck traveling head-on at 80 miles-per-hour.

If that doesn’t strike you as pedantically convincing, I would refer back to the oil shock model, which does not use the questionable empiricism of the URR heuristic and work the model forwards. Because it instead uses historical discovery data combine with an extrapolated dispersive discovery model, the impact of declining production rates doesn’t get conflated with misguided assumptions related to the *ad hoc* URR/2 peak value. This will likely serve us best to prognosticate on future supplies, as we ween ourselves off of oil and activate conservation measures.

Peak oil and Climate change make strange bedfellows. I spotted this statement by a petroleum engineer in an online forum

> Depletion alone is only a part of the overall dynamics which go into worldwide production rate, without accounting for new discoveries, their sizes and future rates, old fields changing their depletion profile through better technology and reserve growth, new areas opening up to exploration as, say, the **Arctic seapack melts** and makes more areas available...

Upon which, I responded:

> Using the **melting Arctic seapack** as a rationalization for anything strikes me as a last gasp attempt at maintaining the status quo. If the seapack starts to melt at rates at which we need to replenish our oil supply we have a whole “boatload” of problems to start worrying about.

Some suggest large scale reserves. The USA’s Energy Information Administration (EIA) has historically forecast large reserves of oil for our foreseeable future. I see more than a few problems with the range of EIA oil production curves from 2006 shown in Figure 19-7 on page 363.

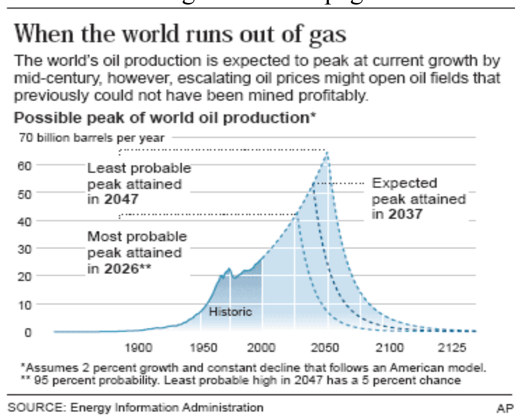


FIGURE 19-7.
EIA projection for the world’s oil. The band between least probable estimates has a 90% chance of occurring.

First of all, the sharply peaked nature of the production curves don't make much sense in a stochastic world view. We should really expect a continuously varying second derivative and not the discontinuity shown. In practice, the discontinuity could occur due to some severe oil shock, and the aggressive rise right before the shock likely due to an ever increasing extraction rate. We can only guess as to whether we could realistically sustain such an extraction rate increase, before hitting the overshoot point. And any large scale increase in reserves would likely modulate the peak, creating a mystery as to what the EIA intended to show with this chart.

Secondly, the EIA's energy projection [Ref 125] shows a linearly extrapolated expected growth in oil production. This assumes increases from a majority of the oil producing countries, including the USA which resulted in the chart Figure 19-8 on page 364.

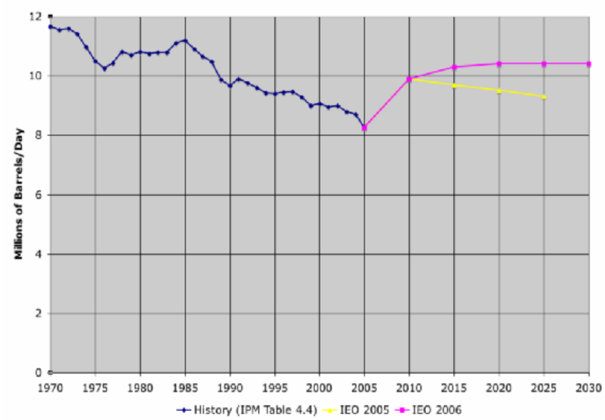


FIGURE 19-8. EIA prediction for USA oil production. From [Ref 124] extracted from EIA data.

Notice that the EIA thinks that we will reverse the downward trend of oil depletion, and start picking up production, significantly. Significant to the point that we will regroup to 1987 levels within the next 15 years!

Posing the strawman of large, even infinite, growth over a projected production envelope provides a strong talking point to use against the cornucopians who play with the numbers. In particular, Lynch provides a foil to test the argument against:

> **MICHAEL LYNCH:** Actually, I think the problem here is that Julian and a lot of the people making these arguments are not that familiar with the technical terms in the oil industry. The estimates that there's about two trillion barrels of oil resource are actually done by some very simplistic models, which have not always failed, but almost always failed on both the national and a global level. The oil conventional oil resource base, the oil in place, is about **eight to ten trillion barrels**. And right now, most estimates are that about 40% of that will be recovered, in other words, about **three, three-and-a-half trillion**. But the amount we'll recover will

> grow over time. So we're not -- we're really not even close to halfway through the conventional oil resource base. [Ref 123]

Here, Lynch claims that we have no worries as the global economy has over 3 trillion barrels left in reserves, i.e. a relatively healthy URR. Yet if we hypothetically counter with the claim that we can have an **infinite** supply left, we will still have to face a peak in production, in other words, an only slightly better prognosis.

To create such a scenario, we need only to create a production profile that — when integrated from now to eternity — tends to an infinite cumulative value. Simple enough, the following curve[3] does this quite nicely:

$$\text{Production}(T) = \frac{A}{(B + C \cdot \text{abs}(T - T_0))} \quad \textbf{(EQ 19-1)}$$

The curve, for a *T0*=2007, looks like the following:

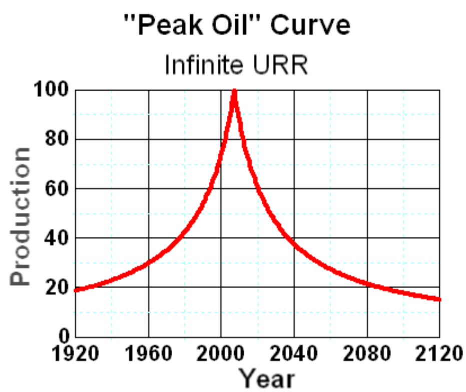


FIGURE 19-9. Another curve that shows an infinite URR.

It looks innocent enough on the way up, and then goes through what looks like a precipitous drop, a sure sign that we have entered an “end in sight” period. Right? Well, not quite. The long tail that this curve contains actually contains a infinite quantity of future returns.

So even rosy assessments, of which hope forms the initial premise, may not pan out as well as assumed. In reality, even excess reserves of 3 trillion, or an infinite amount, may play out as a decreased conservative usage over time.

3. If you think this curve a bit far-fetched, recall that some reserve growth predictions, by the USGS no less, generate parabolic growth laws. If real, these may even have longer tails than we show above.

EROEI Math

Some confusion exists around what Energy Return on Energy Invested (EROEI) means. In general it refers to a measure of efficiency for when a fossil fuel production process *reinvests* the recovered energy to obtain more of the same or to capture more-difficult-to-extract forms of fossil fuel. The question becomes how that will serve to aggressively deplete the supply as EROEI approaches unity. Consider when energy needed for extraction arises from a portion of the energy produced:

$$\begin{gathered} E = \text{Energy Returned on Energy Invested} = \text{EROEI} \\ P = \text{Fraction of Energy put to use elsewhere} \\ P = (E-1)/E \end{gathered} \qquad \textbf{(EQ 19-2)}$$

Notice that when E=2, we waste exactly half the energy in the regeneration process. When E=1, we waste all the energy.

Anything greater than unity (i.e. an E value of 1) means that the process can sustain itself. The problem occurs with the huge "burn" rate we get as EROEI approaches one. It becomes similar to the effect of "dying of thirst in the ocean" — perhaps lots of water available but it takes too much energy to extract the fresh water from the salt water to feed your thirst.

This has implications for global warming and the tremendous pressure on nonrenewable resources, which acts to hasten depletion much more than an energy source with a high EROEI would.

Alternatively, we can look at this with a more fundamental mathematical approach and cast the energy reinvestment as a geometric series (as you would by hand). This actually converges quite nicely if you can get the math right. The fraction "produced" over total energy reduces to:

$$\text{Energy returned on energy invested} = E$$

$$\text{Energy reinvested after N cycles} = \text{ER} = \sum_{i=0}^{N-1} E^i$$

$$\text{Energy produced after N cycles} = \text{EP} = E^N$$

$$\text{EP} = 1/E^N + 1/E^{N-1} + \ldots + 1/E = \sum_{i=1}^{N} (1/E)^i$$

o, $\text{ER}/\text{EP} = (1/E)/(1-(1/E)) = 1/(E-1)$

$$\text{Fraction produced} = P = \text{EP}/(\text{EP}+\text{ER})$$

$$P = 1/(1+\text{ER}/\text{EP}) = (E-1)/E$$

This becomes a form of *net oil* to refer to the loss due to reinvestment. One thing for certain — it will only get worse in the future as we use petroleum with lower and lower values of EROEI.

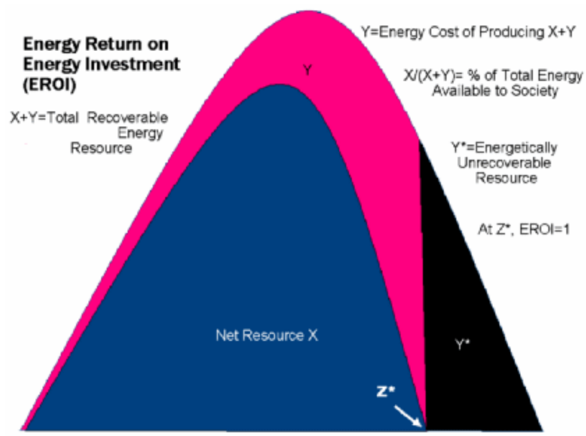


FIGURE 19-10.
An EROEI curve devised by Hagens from [Ref 124]

Someone may posit the question of why would we reinvest everything we extract? Apart from the fact that, yes, indeed no oil company would reinvest their entire stock, this analysis gives the upper bound on what the industry can potentially do. The more you reinvest, the more you can possibly get as payout. So the carrot in front of the horse remains human greed.

> *Let's say you happened across a broken slot machine in Las Vegas. You put in $1 and out pops $10. It happens three times in a row. Would you keep putting in the coin until someone stopped you? Or would you walk away with only $30?*

The answer to that question, albeit on a much larger scale, tells you the size of the dice that the energy industry plays with.

Other Forms of Energy

Optimists have pointed to other sources of energy such as nuclear, solar, wind and others to take the slack away from diminishing supplies of petroleum. The significant rub lies in the argument that none of these will easily fly an airplane or provide the convenience for our current way of life. Transforming other sources of energy into ethanol or hydrogen will work but we will have to face the facts of EROEI.

We will sit back and watch as the global “all fuels” oil production show increases labelled as “barrels of oil equivalent”, yet we have to remember that: [Ref 326]

1. This composite contains biofuels, which can have a disastrous consequences for several reasons and for which EROEI lurks probably not too far from unity.

2. Non crude oil liquids contain a large fraction of natural gas liquids (NGL's) which have only about 70% the volumetric energy content of crude. This component has been steadily rising as a fraction of total liquids.
3. EROEI of crude will likely continue to fall as newer and smaller fields and unconventional oil take up an increasing fraction of the crude total.

The Stochastic Arc

Still, I come from the school that the more models you run, the better job you do critiquing the assumptions of other models.

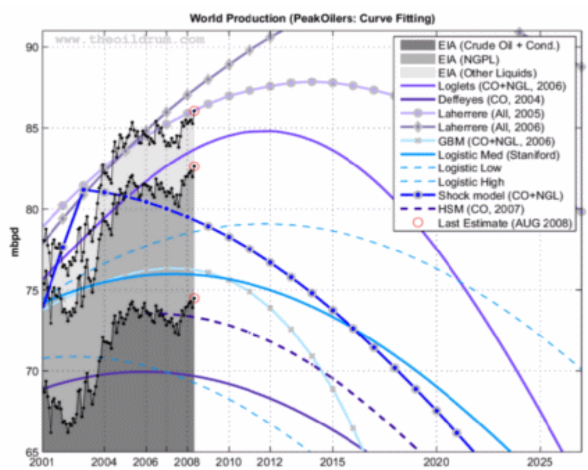
In the end, all we have is incomplete information, and we are forced to guess. I like having as many (prima facie valid) tests of those guesses at my disposal as humanly possible.

— ProfGoose @ TOD

By modelling the entire oil depletion process, we can project the most likely scenario for future availability and come up with a prognosis. This trajectory provides the best ammunition for planning purposes. Anything on either side of the curve — either more pessimistic or more optimistic scenarios — could occur but with lower probability. For planning, the stochastic arc will govern which the decisions we make. Depending on how smoothly and softly the trajectory lands, we can schedule a period of time to conserve or make investments in new energy technologies. Just like simulated global warming outcomes need to consider the most likely scenario, everything outside the envelope on the optimistic side will turn into a gamble if we decide to plan outside that arc.

Foucher [Ref 151]divides the prognostications into three classifications: (1) Business as Usual or BAU, (2) Bottom-Up Analysis, and (3) Curve Fitting models. The Shock Model falls into a separate class of model-based analysis. All the cornucopian models go into BAU, essentially because they go off the chart (so to speak). The painstaking and methodical bean-counting exercises comprise the Bottom-Up Analysis..

FIGURE 19-11.
The class curve-fitting models. The Oil Shock model here predicts a peak early based on truncated ASPO discovery data of crude oil. [Ref 152]



Despite having applied a set of mostly crude-oil discovery data to the original model and then applying it to crude-oil+ production data (See “The Results. Which data sets support the model?” on page 221.), it falls amidst some equally pessimistic projections. Trying to deconstruct a heterogeneous mish-mash of production data for a homogeneous input remains a challenging problem. If we want to reach the goal of pinning the peak date precisely, remember that a few years plus-or-minus with a beginning date of 150 years ago doesn’t sound so wildly off. In any case, the most recent shock model with a dispersive discovery input gives a value closer to 2008 (see “Applying the Combined Model to Global Crude Oil Production”)

“Original Model(peak=2003) < No NGL(peak=2008) < Shell data of BOE(peak=2010)”

On a wider time scale in the following figure, the oil shock model projection again sits right dab in the middle of the other pessimistic estimates, nearly matching Stanford’s median logistic model in the out years.

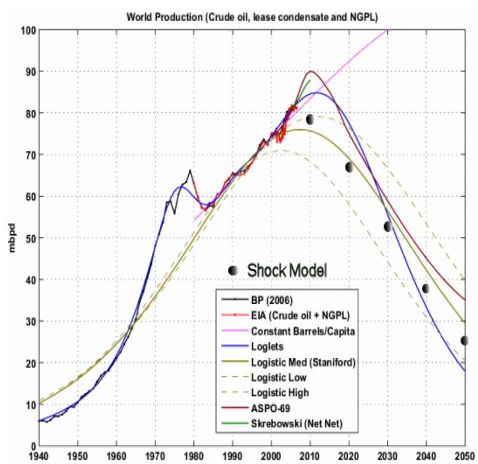


FIGURE 19-12. Production projections. Most recently I applied the oil shock model using the dispersive discovery model assuming only crude (no NGL) as a fit and came up with 2008 as the peak oil date.

The Default Prognosis

The oil shock model does not quite follow a fixed temporal shift in the discovery curve. It actually comprises a sequence of convolutions (or low-pass filtering stages) that has the appearance of shifting the curve. The convolutions also smooth out the discovery fluctuations, leaving only economic shocks to generate the fine detail. The filtering enables us to use a stochastic discovery model, in particular dispersive discovery, as a replacement for actual data. So as one looks at the discovery dates of the fields, one can basically imagine how the shock model will reliably play out by simply time shifting discovery forward by a certain amount to predict future oil flows.

You don't see any noise on this curve because it is pure probability, and it can extrapolate to out-years as it incorporates reserve growth as part of its derivation.

When I first worked the model out, I over-simplified by not separating the types of oil to use in matching to the production curve. The discovery curves essentially estimated only crude, but as non-crude became more important, the published production curves started including the other grades of oil. With the new mix of grades, one can appreciate that applying crude-discoveries to all oil production curves will consistently make the projections fall short by a few years. So by using the best estimate of crude discoveries (with reserve growth) and applying it to estimate basic crude production, the number comes out to 2008.

Yet looking at more optimistic discovery data from Shell Oil, which basically labels the discoveries as "Barrels of Oil Equivalent" and doing an extrapolated dispersive discovery fit, the peak only marginally shifts to 2010. This aligns against an "all oil" production profile, which essentially echoes the sentiment that Matt Simmons has also stated [Ref 309]: essentially the peak of crude coincides roughly with the peak of all oil. I think this happens because we often find high quality oil over the same time intervals as poor quality oil, and it becomes a matter of saving up the poorer quality of oil a couple of years before that also starts depleting. In reality, if we treat high quality crude like gold, and the poorer stuff as near-gold, nothing really rules out that the will not get developed just as fast, especially with developing and BAU industrialized nations involved. In other words, any echoes of "Eureka!" for lower grades of crude will fade away rapidly after we tap the prime oil.

Not putting in the effects of lower EROEI as we follow the trajectory down makes the model more optimistic than it might possible play out. But then it gets balanced by the fact that dispersive discovery does not include "big fields first", which makes the projection a bit more pessimistic, see "The Effect of Field Size" . So in general, we are shooting for minimizing the bias by not making any unwarranted assumptions. In other words, if we don't know, we leave the model alone. [4]

Future (Oil) Shock

As discussed earlier, dips in production due to effects like demand destruction do provide suppression or plateauing of the peak, but the continuing production maintains a flow great enough that a long-enough interval of this behavior will prevent

4. By the same token, using discovery of "BOE" numbers helps create an error margin for other kinds of oil. What exactly is the sensitivity to the effects of EROEI reduction? We know it is in one direction, but will it change things by a few years or how much faster exactly is the depletion rate then? Could the increase in BOE exactly offset the decrease in EROEI? Not likely, but it might turn it into a second order effect.

USA oil conservation push during WWII

by Weimer Pursell, 1943
Printed by the Government Printing Office for the Office of Price Administration NARA Still Picture Branch (NWDNS-188-PP-42)

the production from bouncing back up. Unless producers can increase the extraction rates enough to make up for depletion of reserves due to the sand-in-the-bottle of lost time, we will likely have lower production for the foreseeable future and likely longer. And that makes anything we do, including both demand destruction and limited conservation, clearly not sustainable activities. So any current or future recessions will act more like short-term modulators than a permanent kick-in-the-pants. Only during the 70's and 80's did we recover from a suppressive shock, and that had to do with a large reserve base.

Volume 2 — Renewal

"We scorn the abstract; we scorn it with passion."

— N.N. Taleb

In Volume 1, ***Decline***, we laid out an analytical framework to understand oil depletion. We had to justify our arguments by incorporating some unconventional mathematics and statistics that few other analysts have considered using before.

Although the approach showed much promise in its analytical utility, the relentless discussion of decline of our oil reserves in Volume 1 suggested some bleak prospects ahead. I would also like to look at the bright side; the investment in intellectual capital in this research can pay off in many other ways. For one, much of the mathematics that I had used for depletion analysis has vast practical utility in several other disciplines, many of them related to energy, ecology, and other factors in society and the environment. In some situations we can add to the body of knowledge such that it may perhaps lead to further breakthroughs.

The excursions that I took exploring these areas has a basis in intellectual curiosity, but initially I studied them to substantiate that the math and statistics I used would work in distinctly different areas. In certain cases I noticed only mild interest in exploring how some behavior manifested itself, such as in the statistics of popcorn popping. Yet the trajectory of a popcorn showed remarkable mathematical similarity to the lifecycle of oil extraction (a few pops, then maximum popping activity, and then decline). So what may have some understandable apathy, as in who cares how popcorn pops, may hold deeper significance.

"As a species, we worship growth. We absolutely hate sustainability. It runs counter to everything we were taught and believe in. It threatens our drive for expansion and multiplication, for the gratification of our personal wants, and our greed for ever increasing profit and wealth and power. Yet, sustainability is inevitable. We can only choose whether to live in sustainable misery or in sustainable comfort."

— Francois Cellier
http://europe.theoildrum.com/node/3871

The generally positive aspects of these models lead me to call this volume ***Renewal***. Enough of the analysis can get reused in research areas such as photovoltaics, wind energy, and other potential solutions lead me to deciding for that collective name.

Nothing really ties these pieces together, just as nothing concrete will establish a path to follow for the future. We will have to pick and choose from whatever the entropy of our environment has to offer.

CHAPTER 21 Applying Probabilities.

Consideration of disorder and uncertainty

"la matematica non è un'opinione"
Mathematics is not an opinion!

— Italian saying

We often see unrelated phenomenon that shows some rather similar characteristics — as in the popping of a popcorn serving and an oil depletion peak profile. In fact, the behaviors observed often have a common mathematical origin. In the situations that we cover in this volume, the behaviors by themselves don't necessarily diverge from basic intuition. Yet the effects of disorder and specifically that of entropy require us to use notions of probabilities to understand them. In this chapter we provide some of the intuitive background to help guide us through the case studies.

Professor Frink: *[drawing on a blackboard] Here is an ordinary square....*

Police Chief Wiggum: *Whoa, whoa - slow down, egghead!*

Odds and Uncertainty

In writings and correspondence, the scientist E.T. Jaynes took an opinionated approach to defending his ideas and challenging the status quo. Known best for relating entropy and probability to many areas of science and information technology, Jaynes in particular took on the proponents of the classical statistics school, known as the "frequentists". Although he did not necessarily disparage their work, he could never understand why the classical statisticians had such difficulty embracing alternate ideas, such as those coming from the Bayesian perspective. The "probabilistic" school (which included Jaynes) continued to make great practical strides in solving many thorny physics problems, yet the frequentists resisted the idea that Bayesian approaches could effectively subsume their doctrine. Jaynes showed in fact that ideas from probability could encompass some classical statistics ideas, going so far as to provocatively labeling probability theory as The Logic of Science. Similarly, the useful law known as Cox's Theorem justified a "logical" interpretation of probability.

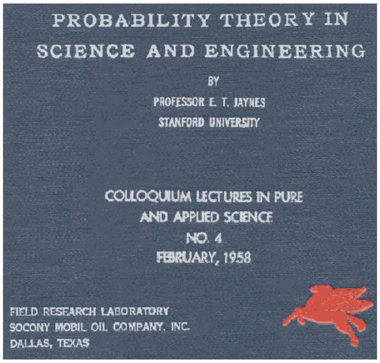
Jaynes has described how the mathematician Laplace had worked out many of the fundamental probability ideas a couple of hundred years ago (Jaynes lived in the 20^{th} century and Laplace in the 18^{th}), yet became marginalized by a few petty arguments. One of the infamous arguments Laplace offered, the *Sunrise problem*, has since supplied ammunition for opponents of Bayesian ideas over the years. In this example, Laplace essentially placed into quantitative terms the probability that the sun would rise tomorrow based on the count of how many times it had risen in the past. We can categorize this approach as Laplace's precursor of Bayes' rule, originally known as the *rule of succession.* In current terms we consider this a straightforward Bayesian (or Bayes-Laplace) update, a commonplace approach among empirical scientists and engineers who want to discern or predict trends. Yet, legions of mathematicians Laplace ridiculed for years since his rule did not promote much certainty in the fact that the sun would indeed rise tomorrow if we input numbers naively. Instead of resulting in a probability of unity (i.e. absolute certainty), Laplace's law could give numbers such as 0.99 or 0.999 depending on the number of preceding days included in the prior observations. Others scoffed at this notion because it certainly did not follow any scientific principle, yet Laplace had also placed a firm warning to use strong scientific evidence when appropriate. In many of his writings, Jaynes has defended Laplace by pointing out this *caveat*, and decried the fact that no one heeded Laplace's advice. For many years hence science had missed out on some very important ideas relating to representing uncertainty in data.

Jaynes along with the physicist R.D.Cox, have had a significant impact in demonstrating how to apply probability arguments. We inhabit a world rife with uncertainty and disorder. In some cases, such as in the world of statistical mechanics, one finds that predictable behavior can arise out of a largely disordered state space; Jaynes essentially reinterpreted statistical mechanics as an inferencing argument, basing it on incomplete information on the amount of order within a system.

In the oil world we know many of the cause-effect relationships (people need oil so oil gets depleted, etc), but we don't understand the evolution quantitatively and how much order versus randomness plays into the behavior. These missing pieces of data together with the lack of a good quantitative understanding motivate my attempts at arriving at some fundamental depletion models.

> *"The result of any transformation imposed on the experimental data shall incorporate and be consistent with all relevant data and be maximally noncommittal with*

regard to unavailable data"
— The First Principle of Data Reduction (due to Ables in 1974 after Burg).



Jaynes spent much time understanding how to apply the Maximum Entropy Principle (MaxEnt) to various problems. I happened to use the MaxEnt principle with regards to oil because I personally don't have access to all the oil production and discovery numbers apparently available. The approach works quite effectively in other application areas as well and perhaps in many future situations.

> "*Science is fully justified in finding some relation between these fields only after the equality of mathematical methods has been reduced to an equality of the real nature of the concepts.*" — A. Einstein.

These ideas have such a fundamental basis that you really need to stretch to discredit them, essentially making MaxEnt very much an Occam's razor argument. Only a non-obvious new physics explanation could ever displace the obvious and parsimonious choice.

> "*Any success that the theory has, makes it useful in an engineering sense, as an instrument for prediction. But any failures which we might find would be far more valuable to us, because they would disclose new laws of physics. You can't lose either way.*" — E.T. Jaynes

Volume 1 suggested that other oil depletion analysts haven't caught on to this approach yet. Mobil Oil actually published one of the early classic Jaynes texts based on a symposium they funded under the banner of their research laboratory.

During this era, academic geophysicists such as J.P. Burg had used Jaynes ideas to great effect. Burg essentially derived the approach known as Maximum Entropy Spectral Analysis. Not limited to geophysics, this technique for uncovering a signal buried in noise has become quite generally applied. The reliability researcher Myron Tribus pointed out this early success, demonstrating Burg's own personal victory whereby he ate his own dog food and used the algorithm at an abandoned oil field he christened "Rock Entropy #1". The profits he made from the oil he extracted helped to fund his own research [Ref 204].

This leaves us with the curious situation that the petroleum and geology fields contributed a huge early interest in the field of MaxEnt but never carried this forward. Jaynes has often pointed out that some of the applications work out so straightforward that a robot, given only the fundamental probability rules, could figure out the solution to many of these problems — presumably oil depletion included.

> "*We're not going to ask the theory to predict everything a system could do.* We're going to ask, is it possible that this theory might predict experimentally *reproducible phenomena*" — E.T. Jaynes

Jaynes has said that thinking about maximizing entropy parallels the idea that you place your bets on the situation that can happen in the greatest number of ways. Then because enough events and situations occur over the course of time, we end up with something that closely emulates what we observe.

> "*Entropy is the amount of uncertainty in a probability distribution*" — E.T. Jaynes

Yet, we don't get something for nothing, so we still have to guess at the underlying probability distribution. This sounds hard to do but, the basic rules for maximizing entropy only assume the constraints; so that includes things like assuming the mean or the data interval.

> *"No matter how profound your mathematics is, if you hope to come out with a probability distribution, then some place you have to put in a probability distribution"* — E.T. Jaynes

Given all that as motivation, we can look at oil reservoir field sizes and see what other ideas shake out. Based on a MaxEnt of the aggregation of reservoirs over time, we previously came up with the following cumulative probability distribution for field sizes:

$$P(\text{Size}) = \frac{1}{1 + C/\text{Size}} \qquad \textbf{(EQ 21-1)}$$

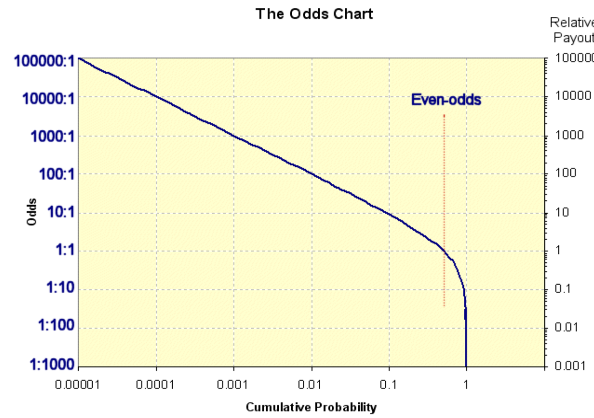
Everyone seems to understand how gambling works, particularly in the form of sports betting, where a man-off-the-street comprehends how the odds function works. Odds against for some competitor to win is essentially cast in terms of the probability ***P***:

$$\text{Odds} = \frac{(1-P)}{P} \qquad \textbf{(EQ 21-2)}$$

So in terms of odds, we can rearrange the first equation into the odds formulation by using either the definition of odds for, *Odds*=*P*/(1-*P*), or odds against, *Odds*=(1-*P*)/*P*.

When plotted the odds distribution looks like the following curve:

FIGURE 21-1. A plot of a cumulative rank histogram of the odds function



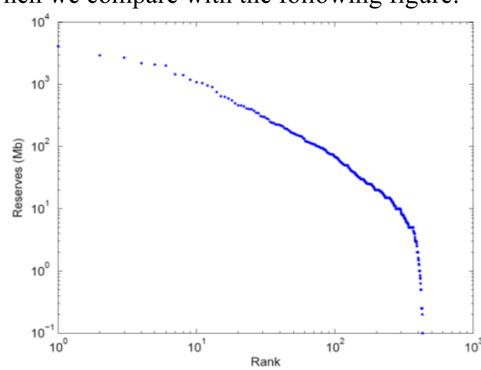
When rearranged, the odds of finding a reservoir larger than a certain size, assuming we randomly pick from the sample population come out to:

$$\text{Odds(Size)} = \frac{C}{Size} \qquad \textbf{(EQ 21-3)}$$

So we can give the odds of discovering a size of a certain reservoir in comparison to the median characteristic value just by taking the ratio between the two values. This equates well to the relative payout of somebody who beat the odds and thus beat the house median.

This becomes even more obvious when we compare with the following figure:

FIGURE 21-2. A plot of a cumulative rank histogram of a reservoir size distribution function. Note how closely it resembles the odds function of the previous figure. Higher odds against relate directly to the rare large size discovery.



This simple result gives us some great insight. It essentially tells us that the greater the size of the reservoir desired, the progressively smaller the odds that we would come across at least that size. For the USA, the value of C comes out less than 1 million barrels, so that finding a field of at least 10,000 MB is 1:10,000. This assumes that we randomly draw from a sample of newly discovered fields.

On the other hand, if we want the odds of drawing from the sample and expecting at least a 1 MB field, we put in the formula and get 1:1, or basically even odds. So if we want to somehow maintain our current rate of domestic production by placing safe bets, we have to find an awful lot of small reservoirs.

We could also place our bets on the long payoff but need to realize that the probability size distribution starts to asymptotically limit for large sizes and the odds factor blows up

$$P(\text{Size}) = \frac{\text{Size} \cdot (L + C)}{(\text{Size+C}) \cdot L} \quad \textbf{(EQ 21-4)}$$

as the odds does this

$$\text{Odds}(\text{Size}) = \frac{(\text{Size +C}) \cdot L}{\text{Size} \cdot (L + C)} - 1 \quad \textbf{(EQ 21-5)}$$

This gives similar odds for a small reservoir, still close to 1:1, but the odds for getting a large reservoir no longer scale. For example, if we use a max size ***L*** of 20,000 MB, then the odds of a size of 10,000 MB is one half the odds without the maximum size. And the odds for getting anything bigger than 20,000 MB become essentially 1 in infinity.

This all comes about from assuming a maximum entropy distribution on the accumulation of the reservoirs and then applying a constraint on the time that these reservoirs accumulate. As Jaynes said, we can do quite a bit with incomplete information.

The same arguments apply to the dispersive discovery model which places fixed limits on the cumulative production based on similar incomplete information. Why the researchers at Mobil Oil never extended these concepts, we will never know. They easily could have either known about this approach at some point and never wanted to disseminate the information to the masses, or never cared and focused strictly on the bottom line.

King Hubbert clearly never applied any of Jaynes' principles, except perhaps at some deep intuitive level. But as Jaynes himself might have concluded, that would have worked out just as well since one intent of probability theory has always tried to place quantitative terms on human insight, the so-called subjective probability approach. So Hubbert gave us some of the insight, and the rest of the probability-based models, such as dispersive discovery and the oil shock model in Volume 1 provides the mathematical foundation.

The Problem with Math

As a provocative statement, we should really understand this behavior all too well, because if we can understand gambling, then we should understand the math behind dispersion. What likely gets in the way is the math itself. People have a math phobia in that as long as they don't know that they need to invoke math, they feel confident. So the odds function becomes perfectly acceptable, as it has some learned intuition behind it. As Jaynes would suggest, this has become part of our Bayesian conditioned belief system. Enough processes obey the dispersive effect that it becomes second nature to us — ***only if*** we deal with it on a sub-conscious level.

The odds-makers don’t really have to think, they just make sure that the cumulative probability sums to one over the rank histogram. Then, since the pay-outs will balance out in some largely predictable fashion, they can remain confident that they won’t get left holding the bag. This becomes a rather intuitive way to think about probabilities if the math happens to get in the way.

Another obstacle that applied math faces concerns the abstraction of probabilities. The smooth curves of probability often look too artificial for their own good. In contrast, any raw Monte Carlo results when plotted has the feel of “real” data. The realistic look of the data has to do with the abundance of statistical fluctuations in the output. For some innate reason, I think that noisy profile gives people added confidence in the authenticity of the data. Yet, for most of the results of applied mathematics, you can also have a pure analytic result solved strictly by equations of probability. Of course these do not show noise because they provide the most likely outcome, essentially evaluated over an infinite number of samples. Yet, these do not seem to generate as much interest, perhaps because they appear to look “phony”: as in, no data in real-life can look that smooth.

Little do most people realize, but the Monte Carlo simulation results from an inversion of the analytical function, simply run through a random number variate generator. Many mathematicians do a Monte Carlo analysis to check their work and for generating statistical margins, but I also think having a bit of realistic noisy-looking output helps to reassure the reader that the results have some perceived greater “authenticity”.

So people like to see statistical noise and spiky conditions, yet these same fluctuations make the underlying trends harder to understand. By the same token, other people will dispose of “outlier” data as unimportant. Yet most outliers have great significance as they can reveal important fat-tail behaviors. Moreover, often these outliers do not show up in Monte Carlo runs unless the rest of the histogram gets sufficiently smoothed out by executing a large sample space. But then you run the risk that people will say that the output looks faked.

Like gambling, you never win. Go figure.

I will toss a diagram up here as an interesting experiment. Consider that the human mobility plot that I will refer to later on has an exceedingly simple rationalization. I have the following derived equation that gives the probabilities of how far a sample population has moved in a certain time, based on the dispersion principle.

$$P(x, t) = \frac{\beta}{\beta + \frac{x}{t}} \quad \textbf{(EQ 21-6)}$$

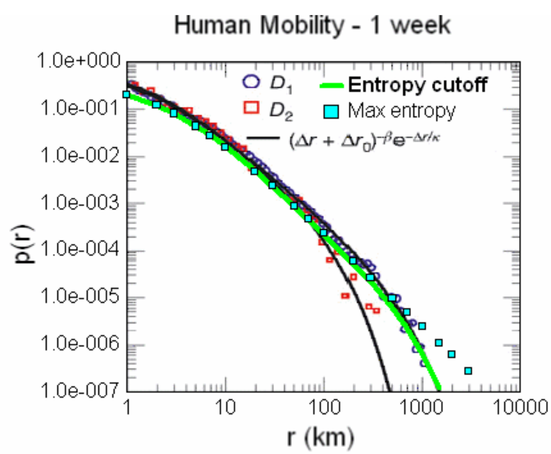
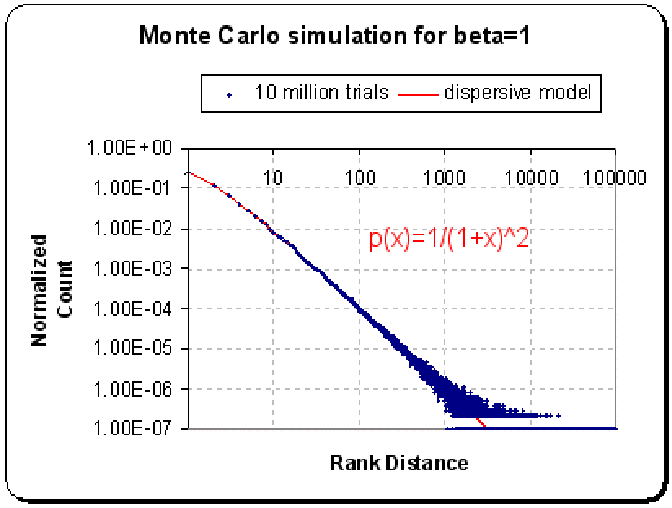


FIGURE 21-3. Human mobility specified as a cumulative rank histogram. The general trend follows a simple dispersion model shown in green and elaborated further in this volume.

To simulate this behavior, we need to take a few straightforward steps. First we simply have to draw from a uniform random distribution for distance (***x***) and secondly draw another number for a random time span (***t***). Or you can do it from two inverted maximum entropy exponential draws (doesn't really matter to achieve the fat-tail statistics). You then divide the two to arrive at a random velocity, i.e. ***v=x/t***.

We need nothing more simple than this formula or formulation. The ranked histogram for the Monte Carlo simulation of 10,000,000 trials of independent draws looks like the following points with the dispersion formula in red:

FIGURE 21-4. A Monte Carlo simulation of dispersion generates dispersion rates by e.g. deltaA/deltaB MaxEnt variates. The rarer events demonstrate noise from the counting statistics of a finite set of events. These exist at the low probability end of the scale.



The random draws definitely converge to the derived Maximum Entropy dispersion derivation. The program takes a few lines of source code.

Why can't the oil punditocracy make the same sense to our oil predictions, I only have a hunch. So we may fear change, but we should not fear uncertainty, at least when it comes to math. As you will see, we have many other disciplines where we can apply this technique.[1]

1. Speaking of odd, why do these rather simple arguments always seem to work so well, and, extending that, could dispersion work everywhere?

CHAPTER 22

Analysis and Behavioral Modeling

Considering analogies

"*Math can do anything ... it can fix the economy.*"

— IBM TV commercial 2009

Hollywood released the movie ***"A Serious Man"*** in 2010. In the film, the protagonists's brother takes the role of an almost savant numerologist, busy at work on a treatise he calls **The Mentaculus**. Filled with dense illustrations and symbology, it apparently functions as a "probability map" in what appears to spell out a *Theory of Everything*. It also apparently works to some extent:

> *We might guess that it makes no sense, but Arthur's "system" apparently "works" as intended, and he applies it to winning at back room card games. [Ref 307]*

Based on the events that eventually transpire, the theme of the movie revolves around the tug of war between rational determinism and random chance.

Outside of Hollywood and within the serious worlds of science and math, several authors have tried to rationalize the utility of probability and statistics in larger contexts. The first of these three you can find freely available on the internet.

1. ***Dawning of the Age of Stochasticity***, David Mumford [Ref 298]
   Mumford wrote a short position paper on the prospects of using probability to solve problems in the future. From the introduction: *"From its shady beginnings devising gambling strategies and counting corpses in medieval London, probability theory and statistical inference now emerge as better foundations for scientific models, especially those of the process of thinking and as essential ingredients of theoretical mathematics, even the foundations of mathematics itself"*.

2. ***Probability Theory: The Logic of Science***, Edwin T. Jaynes [Ref 299]
Jaynes almost finished his treatise on probability as a unifying field. From the body: "*Our theme is simply: probability theory as extended logic. The 'new' perception amounts to the recognition that the mathematical rules of probability theory are not merely rules for calculating frequencies of 'random variables'; they are also the unique consistent rules for conducting inference(i.e. plausible reasoning) of any kind. and we shall apply them in full generality to that end.*"
3. ***On Thinking Probabilistically***, M.E. McIntyre [Ref 300]
A white paper that provides a compatible view to Jaynes and R.T.Cox.
4. ***The Black Swan*** and ***Fooled by Randomness***, N.N. Taleb [Ref 37][Ref 301]
Popular books on probability in everyday life.
5. ***Critical Phenomena in Natural Sciences***, Didier Sornette[Ref 271].
The math behind what Taleb discusses.

Inspired somewhat by the mentaculus and admitting that this in no way explains everything, I tried to make a probability map of all the applications on what I refer to as *entropic dispersion* in Figure 22-1 on page 386.

	Stimulus			→	Accumulator		→	Response	
Physical Phenomena	Rate Description	Rate of change	What we know	MaxEnt Dispersion	Coverage	What we know	MaxEnt Uncertainty	Measure	Time Frame
Reservoir Size	Oil drifts into reservoirs	Constant	Mean	Exp	Swept Volume	Mean	Exp	Oil Volume	Evolving
Reserve Growth	Search a local area for more oil	Constant	Mean	Exp	Swept Volume	Mean	Exp	Oil Volume	Elapsed
Earthquake Size	Stress builds up on a fault	Constant	Mean	Exp	Critical Potential Energy	Mean	Exp	Magnitude	Steady State
City Size	Cities grow in size	Constant	Mean	Exp	Capacity	Mean	Exp	Population	Steady State
Species Diversity	Species adaptation rate	Constant	Mean	Exp	Adaptation Levels	Mean	Exp	Species Diversity	Evolving
Crystal Growth	Material attaches to nucleus	Constant	Mean	Exp	Local Volume	Mean	Exp	Particle Volume	Evolving
Human Transport	Human speed variations	Constant	Mean	Exp	Movement Time or Movement Distance	Mean	Exp	Distance Covered in Time or Time in Distance	Steady State
Project Completion	Workers meeting deadlines	Constant	Mean	Exp	Needed Effort	Fixed	Delta	Completion Time	Elapsed
Volatile Investments	Invest to receive rate of return	Constant	Mean	Exp	Cumulative Return	Fixed	Delta	Return on Investment	Elapsed
Hyperbolic Discounting	Perceived rate of payoff	Constant	Mean	Exp	Uncertainty in rate	Mean	Exp	Time advanced rate of payoff	Elapsed
TCP/IP Latency	Message travels between nodes	Constant	Mean with Max Speed	Reverse Exp	Network Distance	Fixed	Delta	Round-trip Time	Elapsed
Train Statistics	Train moves between stations	Constant	Mean with Max Speed	Reverse Exp	Station Distance	Fixed	Delta	Travel Time	Elapsed
Wind Energy	Speed of wind measured as a power	Constant	Mean Energy ~ Speed^2	Rayleigh	Accumulated Energy	Fixed	Delta	Time to reach energy	Elapsed
Signal Fading	Monitoring amplitude of signal	Constant	Mean Energy ~ Speed^2	Rayleigh	Accumulated Energy	Fixed	Delta	Time to reach energy	Elapsed
Race Completion	Speed of runner	Constant		Censored Exp	Race Distance	Fixed	Delta	Finish time	Elapsed
Global Oil Discovery	Search a global area for oil	Exponential or Accelerated	Mean	Exp	Swept Volume	Mean	Exp	Oil Volume	Elapsed
Bathtub-shaped Reliability Curve	Applied stress to component	Becomes exponential	Mean	Exp	Accumulated Strain	Mean	Exp	Failure Rate	Elapsed
Rainfall	Preferential attachment of raindrop	Exponential	Mean	Exp	Critical Size	Mean	Exp	Amount	Steady State
GPS Acq	2D search for data	Quadratic	Mean	Exp	Data and Signal/Noise Ratio	Mean	Exp	Time to information threshold	Elapsed
Popcorn Popping	Heat content of kernel	Exponential above critical Temperature	Mean	Exp	Pressure buildup to Critical Point	Lower Bound + Mean	Shifted Exp	Time to pop	Elapsed
Dispersive Transport in Semiconductors	Electrons or Holes cross channel	Constant + Diffusional	Mean	Exp	Channel Width	Fixed	Delta	Current	Elapsed
Fokker-Planck	Concentration Diffusion Drift Law	Constant + Diffusional	Mean	Exp	Width	Fixed	Delta	Concentration	Elapsed
Porous Transport	Solute moves across media	Constant+ Diffusional	Mean	Exp	Width at breakthriugh	Fixed	Delta	Solute Flow	Elapsed
Heat Conduction	Heat moves from high temperature to low temperature	Diffusional	Mean	Exp	Temperature Measurement Point	Fixed	Delta	Temperature change	Elapsed
Labor Productivity	Diminishing return learning curve	Diffusional	Mean	Exp	Productivity Constraints	Lower and Upper Bound	Uniform	Productivity	Steady State
Chemical Reaction - CO2 Residence	Diffusion controlled motion	Diffusional	Mean	Exp	Distance	Mean	Exp	Impulse Response Concentration	Elapsed
1/f Noise	Markov Chain Switching	Constant	Symmetric Mean	Lorentzian in frequency domain	Super-statistics in rates	Lower and Upper Bound	Uniform	Noise rate	Steady State
Classical Reliability	Markov Failure rate, continuous wear per unit time	constant	Mean	Delta	Critical Point	Mean	Exp	Percentage failed	Elapsed

FIGURE 22-1. Table of entropic dispersion models

Still the ideas unfolded in their perfect array
Only hinting at what lay beyond them,
Hidden behind all the logic one finds without truth.

— Michael Nesmith

Physical Analogies for Dispersion

In many of the following case studies, we run across a simple construct that we refer to as dispersion; let us call this λ. What does the term λ really signify? A fairly good analogy, although not perfect, comes from the dynamics of an endurance race consisting of thousands of competitors of hugely varying skill or with different handicaps. If one considers that at the start of the race, the basic extent of the mob has a fairly narrow spread, roughly equal to the distance traveled. The value of λ over distance traveled describes the increase in spread or *dispersion* of the mob in relation to the average distance that the center of mass of the mob has traveled. Overall, we empirically observe enough stragglers that the standard deviation of the dispersive spread may to first-order match this average distance. The original analogy to what we worked out in Volume 1 comes about when we equate the endurance racers to a large group of oil prospectors seeking oil discoveries in different regions of the world. The dispersion term λ signifies that the same spread in skills would occur in the discovery cycle just like it does in an endurance race[1]. The more varied the difficulties that we as competitors get faced with, the greater the dispersion will become and a significant number of stragglers will always remain. The notion of stragglers then directly corresponds to the downside of a discovery profile — we will always have discovery stragglers exploring the nooks and crannies of inaccessible parts of the world for oil.

What follows are a few analogous formulations found in other domains that match the mathematical fundamentals of dispersive discovery.

Moore's Law

Moore's Law applies to the performance speedups we see in computer technology over the years, a rough doubling in speed-up every few years. We may want to pay close attention to how Moore's Law shakes out just out of curiosity and to see how a "soft landing" applies in that technology area. We know that Moore's Law has recently shown signs of abating. This could result from an abatement of technological progress as researchers start to give up on scaling techniques, which in the past has guaranteed speed increases as long as the research fabs could continue to reduce circuit dimensions. Or it could stem from a hard limit on the scaling itself, due to parasitics and losses as the electrical properties encounter quantum limits. I have a feeling that something similar to a "dispersive discovery" in the research growth advances will allow Moore's Law to continue to limp along, as researchers

1. Conversely we can relate the difficulty in prospecting for oil in certain cases with competitors forced to run through mud or while wearing cement boots

will continue to find niches and corners in the ultimately constrained and finite "volume" of semiconductor combinations available to us.

From Crystal Growth to Oxide Growth

The USGS has sponsored research into investigating crystal growth mechanisms [Ref 131]. I could find no explicit admission of the strong analogy to reservoir growth as dispersion plays a significant role in crystal growth rates. Although the time scale differs by orders of magnitude, the dispersion in growth rates of primordial reservoirs caused the fat-tail distribution of discovery sizes we see today — just as varying environmental growth conditions causes the distribution of crystalline particles.

We can consider the well-known characterization of silicon dioxide, SiO_2 as an example of dispersive growth. First characterized by Bruce Deal and Andrew Grove in the 1960's [Ref 329], a careful application of a diffusion-based oxide growth model partially enabled the semiconductor revolution. The Deal-Grove model works as a heuristic model in so far as a rigorous first-principles derivation does not exist.

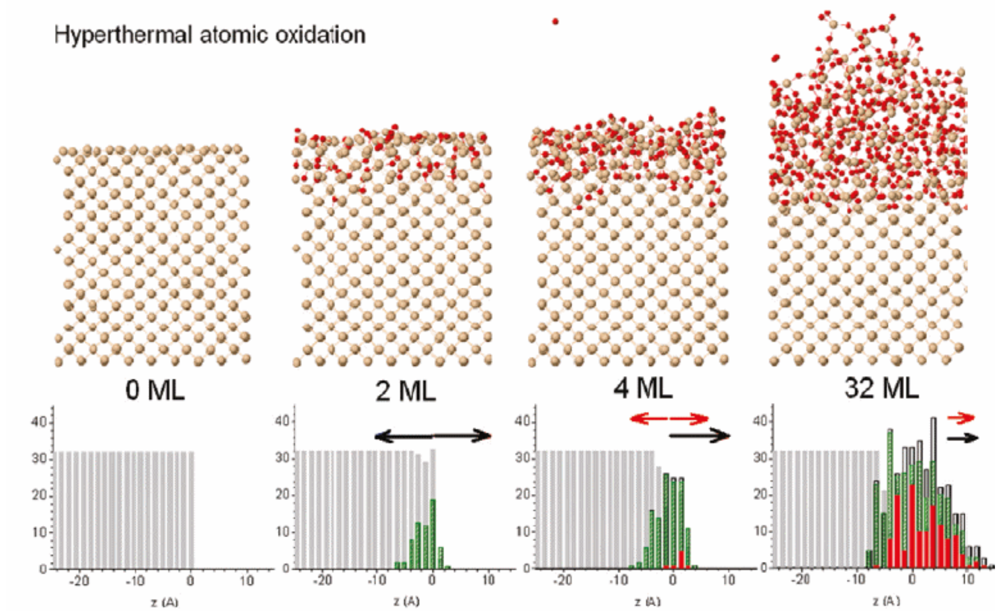


FIGURE 22-2. Chemical simulation of oxidation of silicon. A large degree of disorder is evidenced by the mixture of pure Si, SiO_x, and SiO_2 and as indicated in the histograms by light gray bars, green bars, and red bars, respectively. Black and red arrows indicate the growth direction of the oxidized and silica (SiO_2) layers, respectively (from [Ref 331])

The following derivation improves on the Deal-Grove model by assuming that the diffusion coefficient and location of the growing oxide layer is smeared by a maximum entropy amount; i.e. we can estimate the mean but we leave higher-order moments to vary to maximize the entropy.

Diffusion Solution. The standard approach for solving diffusion problems starts from the master diffusion equation (also known as Fokker-Planck):

$$\frac{\partial}{\partial t}C(t,x) = D\cdot\frac{\partial^2}{\partial x^2}C(t,x) \quad \text{(EQ 22-1)}$$

We can easily derive the solution of the response function if we think of the diffusion from a planar source outward. The kernel solution gives:

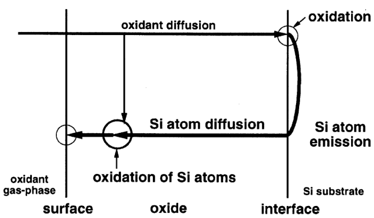


FIGURE 22-3. Mechanism for diffusion (from [Ref 330])

$$n(t,x|D) = \frac{1}{\sqrt{4\pi Dt}}\cdot e^{-x^2/4Dt} \quad \text{(EQ 22-2)}$$

We place an impulse of reactants at x=0 and want to watch the evolution of the concentration, n, with time. As the concentration drops, we assume that the diffused material from that amount contributes to the growth of the oxide layer.

Consider first that the kernel function represents a one-dimensional concentration profile, in units of number of atomic elements per thickness. The expression $n(t,x)$ describes how quickly the atomic concentration decreases from its initial value. So the accumulated concentration, representing the growth outward from the interface, is the spatial integral of the concentration density. Early on we assume that the region of the interface is spread over a width X.

$$N(t|X,D) = \int_0^X n(t,x|D)dx \quad \text{(EQ 22-3)}$$

The average flux of atoms outward from the interface, $J(t|X, D)$, is proportional to the gradient, and we apply the diffusion coefficient as the standard proportionality constant at X:

$$J(t|X,D) = D\cdot\frac{\partial}{\partial X}N(t|X,D) = \frac{\sqrt{D}}{\sqrt{4\pi t}}\cdot e^{-X^2/4Dt} \quad \text{(EQ 22-4)}$$

Next, we have an idea of a mean value for the diffusion coefficient, D, but don't know how much it varies. Lacking that information, we apply a maximum entropy estimate for the variance assuming a mean value D_0.

$$p_d(D) = \frac{1}{D_0} \cdot e^{-D/D_0} \quad \textbf{(EQ 22-5)}$$

We can then integrate the concentration across the diffusion probability density function, and after applying a few integration tricks, the solution reduces to:

$$J(t|X) = \int_0^\infty J(t|X, D)p_d(D)dD = \frac{1}{4\sqrt{t}} \cdot e^{-X/\sqrt{D_0 t}} \cdot \left(\sqrt{D_0} + \frac{X}{\sqrt{t}}\right) \quad \textbf{(EQ 22-6)}$$

We also need to consider that for a highly disordered layer, we should place a maximum uncertainty around the value of X.

$$p_x(X) = \frac{1}{x_0} \cdot e^{-X/x_0} \quad \textbf{(EQ 22-7)}$$

Once again we can apply a probability density function, this time to the flux, which marginalizes X according to the following integration:

$$J(t) = \int_0^\infty J(t|X)p_x(X)dX = \frac{D_0}{4} \cdot \left(\frac{1}{x_0 + \sqrt{D_0 t}} + \frac{x_0}{(x_0 + \sqrt{D_0 t})^2}\right) \quad \textbf{(EQ 22-8)}$$

As a last step, we need to integrate the average flux over time to arrive at the growing width, W, of the oxide layer:

$$W(t) = \int_0^t J(\tau)d\tau = \frac{1}{2}\sqrt{D_0 t} \cdot \frac{\sqrt{D_0 t}}{x_0 + \sqrt{D_0 t}} \quad \textbf{(EQ 22-9)}$$

The time integral of this flux is the accumulated concentration of material with a constant inflow of material (i.e. molecular or elemental oxygen) from the surroundings. By integrating the diffusional response, we can demonstrate how the step input transiently supplies reactants to the growing interface. The second factor is a suppresive effect due to disorder and for small t, compensates the lead term to provide a linear growth factor, which is the heuristic in the Deal-Grove growth law.

In summary, we applied a two step maximum entropy estimation process to model the disorder in the growing oxide layer. Without any knowledge about the distribution of D and X, apart that they must exist, we applied the following series of transforms:

$$n(t, x|D) \rightarrow N(t|X, D) \rightarrow J(t|X, D) \rightarrow J(t) \rightarrow W(t) \qquad \textbf{(EQ 22-10)}$$

This provides a diffusional response due to a continuously applied step concentration to model a growing thickness. For oxide growth, a step input of oxygen is supplied from one side of the interface, and the substrate supplies silicon atoms, see Figure 22-3 on page 389. The figure below provides a model fit to recent data from a set of SiO_2 growth experiments.

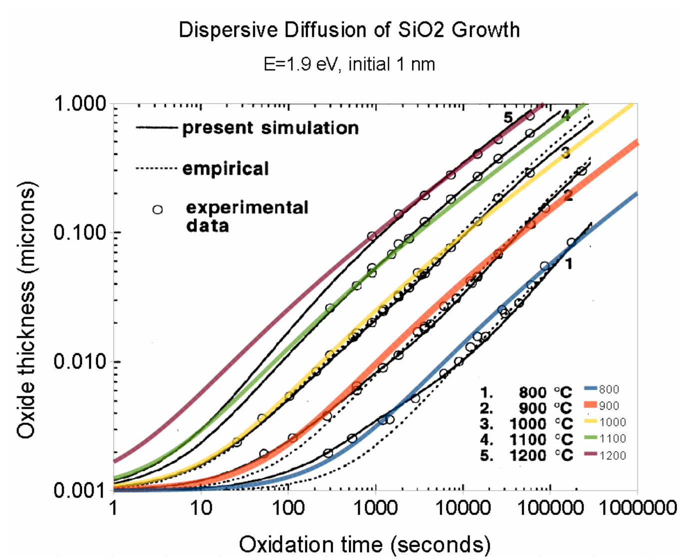
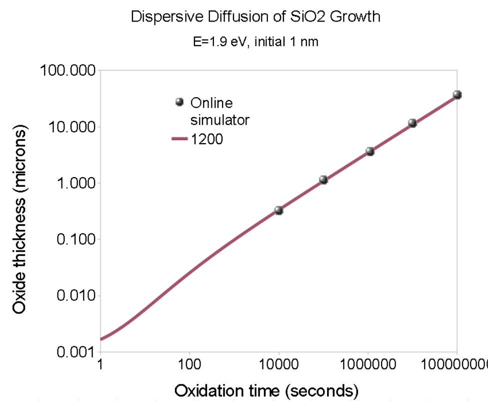


FIGURE 22-4. Comparison of dispersive diffusion model against the convention Deal-Grove nodel (empirical dotted) and a detailed simulation based on reaction kinetics. For silicon, an immediate oxide layer is formed which generates a baseline thickness of about 1 nanometer. (data and simulation from [Ref 330])

Over time, the response will attain a square root growth law, indicative of the Fick's law regime of what is often referred to as parabolic growth (somewhat of a misnomer). The larger the mean diffusion coefficient or the smaller the uncertainty level

x_0, the more quickly that the response will diverge from the short-term linear growth regime.[2]

FIGURE 22-5.
In the Fickian growth regime, the dispersive diffusion formulation follows a square root time dependence, and can be confirmed with an online SiO2 growth calculator.
http://www.cleanroom.byu.edu/OxideThickCalc.phtml



Interpretation. In the chapter heading "The Implications Why should you believe any of this?" , I placed a quote by the statistician George E.P. Box concerning the validity of models. Box's oft-quoted statement of *"All models are wrong, but some are useful"* not surprisingly misses some important context. We can find the original context in the book "Empirical Model-Building and Response Surfaces" by Box and Draper:

> "The fact that the polynomial is an approximation does not necessarily detract from its usefulness because all models are approximations. Essentially, *all models are wrong but some are useful.* However, the approximate nature of the model must always be borne in mind."[3]

In this case the context is to be careful in numerical computations so that the model numbers match the observations.

On the rest of the page, Box and Draper present a concise description of the differences between epistemic and aleatoric uncertainty, which is really the scope of the oxide growth modeling described in this section, and of environmental modeling in general, which covers the scope of this volume. Epistemic uncertainties are the systematic errors that one can introduce in a statistical model, while aleatoric errors are those that are fundamental in the natural behavior itself, be it noise or some other random effect.

2. Corrosion, mention in reliability section.
3. See screenshot of original page here: http://img59.imageshack.us/img59/269/allmodelsareapproximati.gif

So in this case, we can understand the basic mechanisms of oxide growth over many orders of magnitude via the parabolic Fickian diffusion law, but we may miss important details by how we numerically model the fundamental equations. To remedy this siituation, we treated the diffusion coefficient and the Si/SiO_2 interface location with the correct amount of aleatory uncertainty. Our epistemic uncertainty remains in the validity of the model we applied, and how accurately we can measure against the empirical observations.

So Box essentially described why we should be careful in numerical errors in statistical modeling, while many people have interpreted Box's quote to question the validity of using models in the first place. That is clearly an incorrect argument — all one has to consider is that all of mathematical engineering is based on models, and look at how far that has gotten us! The Deal-Grove model essentially allowed oxidation processes to become well characterized and predictable, which essentially revolutionized the integrated circuit manufacturing process. The dispersive diffusion model that I derived in this section should be taken in that spirit, a model that could become useful in characterizing a fundamental physical process.

Dispersive Transport

Few people in science and engineering seem to understand disorder. Where understanding exists, it often makes the difficult jump into topics such as anomalous diffusion as described by Levy flights and fractional random walks. Less complicated approaches can derive the observed physics behavior, *if we simply assume entropic disorder* as we have applied in Volume 1 and the previous oxide growth example.

If by the process of dispersion we want the particles to dilute as rapidly as possible, we need to somehow accelerate the rate or *kinetics* of the interactions. This becomes a challenge of changing the fundamental nature of the process, via a homogeneous change, or by introducing additional heterogeneous pathways that provide alternate pathways to faster kinetics. From this perspective, dispersion describes a mechanism to divergently spread-out the rates and dilute the material from its originally concentrated form. One can analogize in terms of a marathon race; the initial concentration of runners at the starting line rapidly disperses or spreads out as the faster runners move to the front and the slower runners drop to the rear. In a typical race, you see nothing homogeneous about the makeup of the runners (apart from their human qualities); the elites, competitive amateurs, and spur-of-the-moment entrants cause the dispersion. Whether we want to achieve a homogeneous dispersion or not, we have to account for the heterogeneous nature of the material. In other words, we rarely deal with pure environments so have to solve for much more than the limited variability we originally imagined. Generalizing from the rather artificial constraints of a marathon race, dispersion in other contexts (such as crystal growth or reservoir growth) results from an increase of disorder as a direct consequence of entropy and the second law of thermodynamics.

In terms of the spread in dispersion, we might often observe a tight bunching or a wide span in the results. The wider dispersion usually indicates a larger disorder, variability, or uncertainty in the characteristics — a "fat-tail" to the statistics so to speak. So when we introduce a dispersant into the system, we add another pathway and basically remove order (or introduce disorder) into the system. Dispersion may thus not accelerate a process in a uniform manner, but instead accelerates the differences in the characteristic properties of the material. This again describes an entropic process, and we have to add energy or find exothermic pathways to fight the tide of increasing disorder.

This seems like such a simple concept, yet it rarely gets applied to most scientific discussions of the typical disordered process. Instead, particularly in an academic setting, what one usually reads amounts to pontificating about some abnormal or anomalous kind of random-walk that must occur in the system. The scientists definitely have a noble intention — that of explaining a fat-tail phenomenon — yet they don't want to acknowledge the most parsimonious explanation of all. They simply do not want to consider heterogeneous disorder as described by the maximum entropy principle.

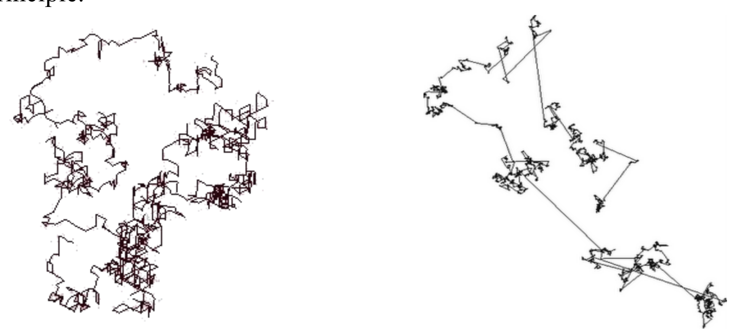


FIGURE 22-6. Difference between a classical random walk (left) and an anomalous random walk (right). The salient difference is that occasional long jumps (Levy flights) occur in the anomalous random walk. A much simpler approach admits that a heterogeneous nix of random walkers of different rates exists. This will give essentially the same observable outcome without resorting to arcane mathematical modeling.

The complicating factor in discussions about dispersion involves the intuitively related concept of *diffusion* and *convection* or *drift*. Diffusion also derives from the statistics of disorder and describes how particles can spontaneously spread out without a real driving force, apart from the uniform environment, for example from the thermal background. The analysis of a particle undergoing random walk leads directly to the concept of diffusion.

Random walk ideas seem to intrigue mathematicians and scientists because it places the concept of diffusion into a real concrete representation. In some sense everyone can relate to the idea of a particles bouncing around, but not necessarily to the idea of a gradient in concentration.

Convection and drift describe the motion of particles under an applied force, say charged particles under the influence of an electric field (Haynes-Shockley), or of solute or suspended particles under the influence of gravity (Darcy's Law). This essentially describes the typical constant velocity, akin to a terminal velocity, that we observe in a pure semiconductor (Haynes-Shockley) or a uniformly porous media (Darcy's).

Dispersion can effect both diffusion and drift, and that establishes the premise for the some of the analysis we will describe.

Climate change and Dispersants

This volume will also address some recent challenges in regards to environmental concerns.

Climate. To attach some significance to oil depletion, let us consider this analogy:

- Year-to-year oil discovery fluctuations act like the weather.
- The envelope of the slowing cumulative of oil production functions analogously to climate.

We can barely predict the weather (or near term oil discoveries) yet we have a good chance of getting the climate right (or long term production decline). We know that the availability of oil has long term implications for a productive economy, just as we know that the levels of CO_2 in our atmosphere has long term implications for climate change.

An even deeper analogy exists that links the two via the analysis of dispersion; in general I think the premise of "dispersion" of rates in oil depletion analysis have equivalents in climate change research. The same dispersion that fits into the dispersive discovery model and reserve growth confronts the climate change skeptics with a different problem to ponder. For climate change, you will find that not one single time constant rules over the temperature time series that the analysts pour over, as CO_2 has a mix of short and long-term residence times. Interestingly, the dispersion has positive effects on oil resources, in that it leads to greater reserves than we may currently believe, but the same effect has negative consequence as the longer time scales in GW analysis lead to lags in heating that we get tricked into not compensating for.

Do climate change models had approximately the same level of complexity as oil depletion models? I contend that scientifically modelling peak oil actually shows orders of magnitude less complexity than predicting global warming. Consider this analogy:

> Oil Depletion: An exercise in estimating the extraction of fluids from a container.
> Climate Change: An exercise in non-linear fluid dynamics of N-dimensionality.

As you can almost surmise, in many cases the dispersion math describing what happens to CO_2 as it enters the atmosphere essentially matches that of dispersive transport in semiconductors. A peculiar behavior in the transport that provides an initial photocurrent spike acts identically to the initially fast diffusive rate of CO_2. In other words, a fraction of charged carriers that can diffuse quickly to a recombination site (i.e. an electrode) act precisely the same as CO_2 that reacts quickly and removes itself from the atmosphere. Yet the long tails in the dispersion remain, both in the *disordered semiconductor*, and in the *disordered atmosphere*. The fat-tails will kill us in atmospheric CO_2 build-up, just like the fat-tails in amorphous semiconductors make it useless to use in a fast microprocessor or in a cellphone receiver.

Now put 2 and 2 together. No wonder many otherwise technically savvy people lack the knowledge to describe the CO2 buildup problem! Like the scientists and engineers who experiment with dispersive transport can't see the forest for the trees and thus can't come up with a simple derivation that a near layman can understand, the climate scientists also completely miss out on the obvious and have never come up with the equivalent Jaynes' "probability as logic" formulation. Impulse responses of electrical devices have a direct analogy to the impulse response of the atmosphere as it confronts a stimulus of CO2:

$$\text{ImpulseResponseCO2}(t) = \frac{1}{1+\sqrt{\frac{t}{T}}} \qquad \textbf{(EQ 22-11)}$$

Dispersants. As we will see in the following chapters, the dispersive transport also acts on solute in other medium. Credit the Gulf oil disaster with allowing the words dispersion and dispersants to enter our common vocabulary. In the context of the spill, the use of dispersants on the oil causes the potentially sticky coagulating oil to split apart into finer granularity drops and somehow make it more amenable to breaking down. Dispersion in terms of a chemical definition simply means spreading out particles in the medium, in this case seawater. So a dispersant breaks it up and dispersion scatters it about.

The BP team apparently wanted to break up the oil up so that it could easily migrate and essentially dilute its strength within a larger volume. So instead of allowing a highly concentrated dose of oil to impact a seashore or the ocean surface, the dispersants would force the oil to remain in the ocean volume, and let the vast expanse of nature take its course. Somebody in the bureaucratic hierarchy made the calculated decision to apply dispersants instead of the alternatives (including doing nothing). I can't comment on the correctness of that decision but this volume can expound on the topic of dispersion, a behavior that lay-people as well as scientists have often equal trouble understanding completely.

For a brief period during the crisis, as the media has forced us to listen to jargon-filled technical terms such as "top kill", "junk shot", and "top hat" used to describe all sorts of wild engineering fixes, we take a turn toward the more fundamental notions of disorder, randomness, and entropy to explain that which we cannot necessarily control. I believe that if we can understand concepts such as dispersion from first principles, we actually have a good chance of understanding how to apply it to a range of processes besides oil spill dispersal. In other words, well beyond this rather specific interpretation, we can apply the fundamentals to other topics such as greenhouse gases, financial market fluctuations, and oil discovery and production, amongst a host of other natural or man-made processes. Really, it turns into that fundamental a concept.

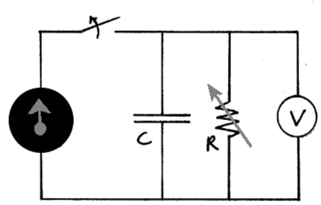
That just touches the tip of the iceberg and will apply models of dispersion extensively in the following chapters.

Physical analogies for the shock model

Along with physical analogies to dispersion used in discovery, we can find mathematical analogies to the oil shock model.

Resistive/Capacitive Circuit

We can cast the continuous oil shock model into a simple electrical RC (resistive/capacitive) circuit, with Kirchoff's law describing the differential equation.



RC Circuit Diagram

FIGURE 22-7. An Resistive/Capacitive (RC) circuit model which mimics the dynamics of a single phase in the oil shock model.

$$I(t) = C \cdot \frac{d}{dt}V(t) + V(t)/R(t) \tag{EQ 22-12}$$

I(t) acts as the forcing function (the black circle with the arrow pointing up), a source of current representing the tapped reserves. If you think of electrons as equivalent to oil molecules, then electron current becomes the amount of oil filling up our reserve capacity per unit of time and voltage *V(t)* represents cumulative charge of oil, more or less. Note how the capacity, *lo and behold*, turns into the capacitance *C* of the circuit. If the forcing function gets turned off (note the switch), then the amount drained off as oil production becomes the *V(t)/R(t)* term. Actually, 1/*R(t)* represents the reciprocal of resistance, i.e. conductance, of the circuit, which indicates the proportional amount bled from the reservoir capacity. The arrow through the resistor symbol makes it variable, as in a *potentiometer*. [4]

Cascaded Circuits

By tinkering some more with the schematic circuit analogy we can try to generalize it to the oil shock model, and came up with this:

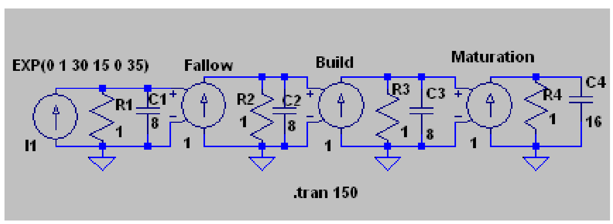


FIGURE 22-8. Staged RC circuits mimic the cascading of oil shock model phases (The staging reminds one of bucket brigade circuit used in flangers and reverbs.)

This essentially demonstrates the effects of stochastic latencies that occur in a peak oil model.

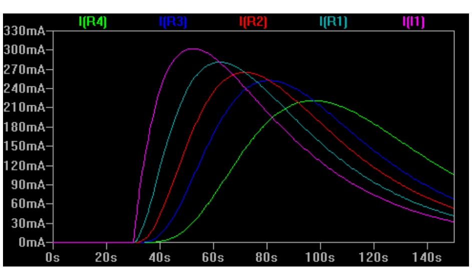


FIGURE 22-9. Solution of the cascaded RC circuit model using a popular CAD program showing the same behavioral dynamics as the oil shock model. Each successive stage shifts the curve to the right.

- Purple: Discovery forcing function
- Cyan: “Fallow phase” response, C1=8.
- Red: “Build phase” response, C2=8.
- Blue: “Maturation phase” response, C3=8.
- Green: Production output, C4=16.

4. Get yourself a good programmable current source with a storage oscilloscope measuring the shunted current through the resistor, and you can make your own Analog Peak Oil Simulation. Or you can run the circuit through a Spice simulator, or you can try solving the equations using Laplace transformations.

We live in a stochastic world. The best prediction you can make with only minimal prior knowledge takes into account only the currently occupied state. That basically captures the essence of a first-order Markov process or property, which can describe the salient effects of many different phenomenon, everything from random walk to oil depletion. The process involves lots of little stepped events that collectively accumulate in a fuzzified fashion. It inexorably leads you in a direction, but the direction remains governed by largely randomly distributed events.

"Markovian Process" *(Greg Graffin)*

You will all say that I am surely crazy
Only an unrepentant pessimist whose thoughts should be detained
But facts are sterile, not vulgar nor sublime
And they're not religion, they're for everyone
And signify the times
Today is a window, tomorrow the landscape
All you need to do is take a look outside
To know what we're bound to face
The level of disparity
The common man
The manner of destruction of the native land
The poverty of reprisal from all involved
And the scathing trajectory from the past
Markovian process lead us not in vain
Prove to our descendants what we did to them
Then make us go away

I used a freeware CAD tool / SPICE simulator[5] to create the schematic and generate the response curve. Read the x-axis coordinates as years instead of seconds, and it starts to make sense, i.e. 60s stands for the 19**60's**. The odd-looking schematic symbol connecting the RC meshes signifies a voltage to current generator which serves to isolate them, and by analogy to a stochastic system, makes them statistically independent.

Reliability Modeling

We can look at the first-order modeling of failure rates to gauge how reliably a given system will operate. This uses the same general dataflow and state transition diagrams as the shock model, and has an extended set of literature to describe its applicability. See textbooks on Markov modeling for reliability analysis.

The study of reliability also has some other parallels with oil depletion modeling. A discipline called "physics of failure", which aids in understanding individual cases of failure, serves in an analogous way to the characterization of individual oil fields and their depletion rates. But reliability modelers will typically generalize the failure physics and describe it in "rolled-up" terms of probability and statistics and come up with something practical. The analysts see the futility of accounting for the myriad of failure possibilities and instead opt for an ensemble approach based on collecting experimental and observational data and then associating that with a mean probability of failure per unit time (aka a *hazard function*). As in oil depletion modeling, all the variations smooth out and we can use the mean value which incorporates a built-in variance. So, we don't look at the science of reliability and how failures come about, but instead use the empirical data and use that to predict metrics such as mean-time-between-failures (MTBF).

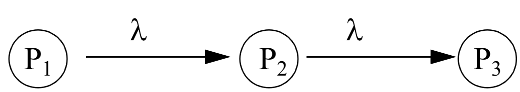


FIGURE 22-10. State diagram of a single component with a standby spare forming a fault-tolerant system. The probability flows from left to right. Initially, the system starts with a probability of one in the first state, but eventually the system ends up in the last state with a certainty of one.

TABLE 1.

State	Description
P_1	Primary component is operational
P_2	Standby component has been switched in and is operational
P_3	System failure

5. Many semiconductor companies give away SPICE simulators in the hopes that engineers will buy their discrete or IC components. Linear Technologies sells one called LTSPICE/SwitcherCADIII which I used. See also http://www.groups.yahoo.com/ltspice.

The above figure shows a typical state diagram for estimating the reliability of a fault-tolerant system. The various stages map the probability flow from a perfectly operational system to one with some degree of failure.

Like oil running out, eventually every system will fail. This makes the Markov model for reliability analysis a very analogous situation to oil depletion. Simply by replacing failure rates with depletion rates, we can use a similar analysis approach.[6]

Compartmental Models

The concept of compartmental models comes up primarily with respect to the data flow construction of the Oil Shock model. The literature on this has proven a bit spotty and discipline-specific. Some Stochastic Compartmental Models (SCM) refer to drug delivery models in the field of pharmacokinetics as well as to the global circulation models used for studying climate change. From the implicit assumption that any rate of extraction or flow is proportional to the amount available and nothing more (past and future history do not apply), SCM essentially describes the equation for the output compartment in a multi-compartment system interacting with the outside world. This also has some applicability in climate science where forcing functions, impulse responses, and convolutions become part of the analysis tool set.

This models the fluxes of material between distinct states, with parameters defining the proportional rates that transfer the amount of material among different stages of the material (data) flow graph. For example if one has N compartments linked in a unidirectional chain graph, the output goes as the weighted sum of N exponential decay terms, or as inputs to physics-based models that relate the constants to the properties of the systems.[7]

If nothing else, this analysis substantiates the assumptions used in the foundation of the Oil Shock model. The petroleum engineering concepts of *fractional flow* and *material balance* ([Ref 172]) also derives from the same compartment models of pharmacology. Fractional flow of oil underground also ties into diffusion and Darcy's law. To first-order, diffusion properties derive from concentration differentials and the flow stays proportional to the magnitude on the high side, so one can model diffusion through a series of stages separated by permeable membranes. Due to the slow nature of this effect, it likely plays into reserve growth.

6. We can also lump in risk analysis models such as practiced in the insurance/actuarial fields.
7. e.g. volumes of quantities, flux constants etc because under certain conditions of connectivity the two formulations are equivalent

Shocklet Data Flow

We can perform a few experiments that tie in the concepts of dispersion and compartment flow. In the following analysis, I ran the shocklet model through a commercial program, which has some practical benefit to those used to looking at a pure dataflow formulation of the model.

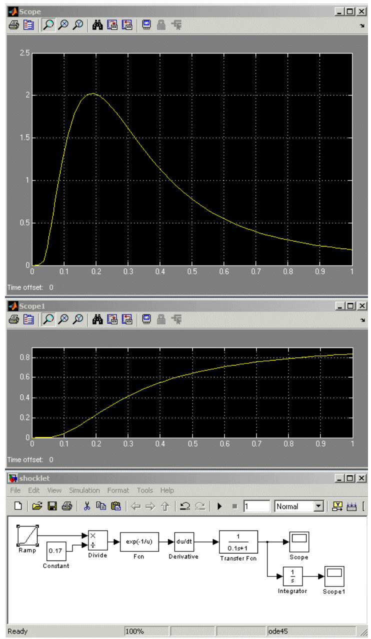


FIGURE 22-11.
Dataflow diagram and outputs showing a simulated model of a shocklet (bottom) dataflow diagram with dispersive stimulus input (middle) cumulative output (top) derivative of cumulative is the instantaneous value.

So that covers some of the physical analogies to the oil depletion models described in Volume 1. The subsequent chapters will describe in some depth applications of dispersion and data-flow.

CHAPTER 23 *Innovation and Evolution.*

How ideas spread

> *"One of the principle objects of theoretical research in any department of knowledge is to find the point of view from which the subject appears in its greatest simplicity."*
>
> — Josiah Willard Gibbs (1839–1903)

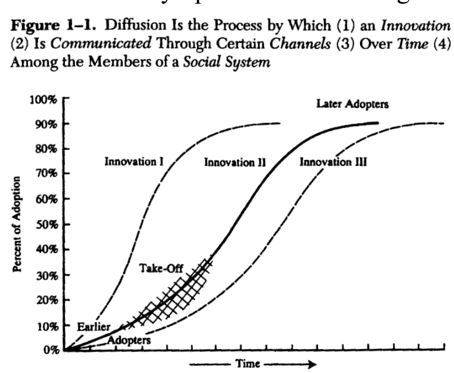
Diffusion of Information

The concept of "diffusion of innovations" fits well with the idea that we cannot predict behavior with any degree of certainty. When some idea with merit happens to come along, we rarely see instant acceptance, but rather a gradual uptake that gathers speed and then hits an asymptotic level.

> The rate of adoption is defined as: the relative speed with which members of a social system adopt an innovation

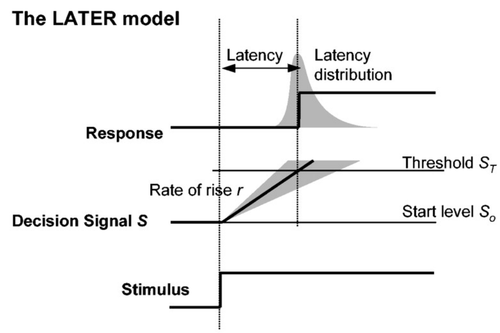
The rates of adoption set the stage for achieving the adoption level over time. The result ends up looking like a logistic curve with an asymptote of diminishing return.

FIGURE 23-1. From the book "Diffusion of Innovations" by Everett Rogers. A hypothetical innovation progresses through stages on the road to acceptance and final maturity.



Cognitive researchers have found that at the lowest and most innate level, humans demonstrate randomness in their response times. The randomness typically shows up as a dispersion in rates as shown in Figure 23-2 on page 404 [Carpenter et al]. The distribution in rates of response turns into a fat-tailed distribution of response times or latencies. This fundamental behavior turns up time and time again, as the uncertainty in rates translates into a distinctly recognizable asymmetric profile. As the diagram shows, even a uniform distribution of rates will project geometrically as an asymmetric curve. This becomes a fundamental and universal behavior of all rate dispersion models. And moreover, because these distributions have *stable* properties, under the effects of multiple convolutions, they retain the fat-tail in the time domain. This occurs in both physical and biological situations.

FIGURE 23-2. The LATER model of human response times by R.H.S. Carpenter [Ref 236]. By measuring visual responses, experiments have determined that the rates, ***r***, follow a random yet symmetric distribution. Taking the reciprocal of rate, the temporal latency generates an asymmetric skewed distribution. This establishes the significance of rates and not time as the fundamental stochastic variable.



Natural Language Growth

One pattern that has evaded linguists and cognitive scientists for some time relates to the quantitative distribution in human language diversity. Much like how plant and animal species diversify in a specific pattern, with very few species dominating within an ecosystem and relatively few species exceedingly rare, the same thing happens with natural languages. You find a few languages spoken by many people, and very few spoken rarely, with the largest number occupying the middle.

Consider a simple model of language growth whereby adoption of languages occur over time by dispersion. The cumulative probability distribution for the number of languages is:

$$P(n) = \frac{1}{1 + 1/g(n)} \quad \textbf{(EQ 23-1)}$$

This form derives from the application of the maximum entropy principle to any random variate where one only knows the mean in the growth rate and an assumed mean in the saturation level. I refer to this as entropic dispersion.

The key to applying entropic dispersion is in understanding the growth term ***g(n)***. In many cases ***n*** will grow linearly with time so the result will assume a hyperbolic shape. In another case, an exponential growth brought up by technology advances will result in a logistic sigmoid distribution. Neither of these likely explains fully the language adoption growth curve.

Intuitively one imagines that language adoption occurs in fits and starts. Initially a small group of people (at least two for arguments sake) have to convince other people on the utility of the language. But a natural fluctuation arises with small numbers as key proponents of the language will leave the picture and the growth of the language will only sustain itself when enough adopters come along and the law of large numbers starts to take hold. A real driving force to adoption doesn't exist, as ordinary people have no real clue as to what constitutes a “good” language, so that this random walk or Brownian motion has to play an important role in the early stages of adoption.

So with that as a premise, we have to determine how to model this effect mathematically. Incrementally we wish to show that the growth term gets suppressed by the potential for fluctuation in the early number of adopters. A weaker steady growth term will take over once a sufficiently large crowd joins the bandwagon.

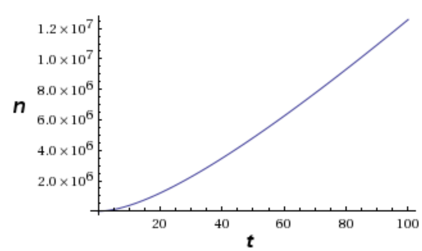
$$dn = \frac{dt}{c/\sqrt{n}+K} \tag{EQ 23-2}$$

In this differential formulation, you can see how the fluctuation term which goes as $1/\sqrt{n}$ suppresses the initial growth until it reaches a steady state as the ***K*** term becomes more important. Integrating this term once and we get the implicit equation:

$$2 \cdot C \cdot \sqrt{n} + K \cdot n = t \tag{EQ 23-3}$$

Plotting this for ***C***=0.007 and ***K***=0.000004, we get the following growth function.

FIGURE 23-3. Growth function assuming suppression during early fluctuations. Adoption builds slowly before a fluctuation-free level allows it to reach its stable rate.



This makes sense as you can see that growth occurs very slowly until an accumulated time at which the linear term takes over. That becomes the saturation level for an expanding population base as the language has taken root.

To put this in stochastic terms assuming that the actual growth terms disperse across boundaries, we get the following cumulative dispersion (plugging the last equation into the first equation to simulate an ergodic steady state):

$$P(n) = \frac{1}{1 + 1/g(n)} = \frac{1}{1 + \frac{1}{2 \cdot C \cdot \sqrt{n} + K \cdot n}} \quad \textbf{(EQ 23-4)}$$

I took two sets of the distribution of population sizes of languages (DPL) of the Earth's actually spoken languages from the references below and plotted the entropic dispersion alongside the data [Ref 216]. The first reference provides the DPL in terms of a probability density function (i.e. the first derivative of ***P***(***n***)) and the second as a cumulative distribution function. The values for ***C*** and ***K*** were the same. The fit works parsimoniously well and it makes much more sense than the complicated explanations offered up previously for language distribution.

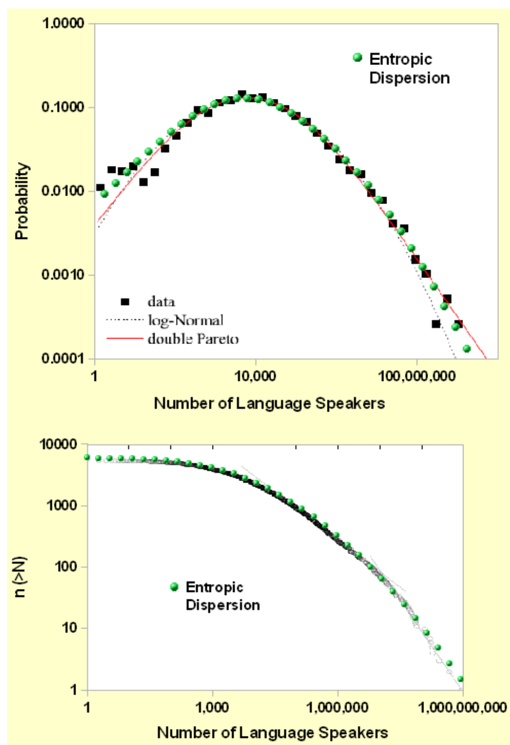


FIGURE 23-4. Language diversity (top) probability density function (below) cumulative. The entropic dispersion model in green. Data from [Ref 216][Ref 217]

In summary, the pieces to the puzzle assume dispersion according to the maximum entropy principle, and a suppressed growth rate due to fluctuations during the early adoption [Ref 218]. This gives two power law slopes in the cumulative; half in the lower part of the curve and unity in the higher part of the cumulative curve. This provides an example of a complex system reduced to simple terms.

Information and Crude Complexity

Summary Points: This section is meant to provoke alternative and creative thinking for what lies ahead.

- People become wary when you mention theory.
- Everyone talks about entropy without actually understanding it.
- Simplicity can come out of complexity.
- "Knowledge" remains a slippery thing.
- We think that science flows linearly as previous knowledge get displaced with new knowledge.
- Peak oil lies in this transition much like plate tectonics at one time existed outside of the core knowledge.
- We define knowledge by whatever the scientific community currently believes.

"Facts are not knowledge. Facts are facts, but how they form the big picture, are interconnected and hold meaning, creates knowledge. It is this connectivity, which leads to breakthroughs ..." [Ref 313]

Scientific theories get selected for advancement much like evolution promotes the strongest species to survive. New theories have to co-exist with current ones, battling with each other to prove their individual worth [Ref 211]. That may partly explain why the merest mention of "theory" will tune people out, as it will remind them of the concept of *biological* evolution, which either they don't believe in, or consider debatable at best. Generalize this a bit further and you could understand why they could also reject the scientific method. If we admit to this as a chronic problem, not soon solved, the idea of accumulating *knowledge* seems to hold a kind of middle ground, and doesn't necessarily cause a knee-jerk reaction like pushing a particular theory would.[1]

So, what kinds of things do we actually want to know? For one, I will assert that all of us would certainly want to know that we haven't unwittingly taken a sucker's bet, revealing that someone has played us. I suspect that many us want to avoid this kind of situation. In my mind, knowledge remains the only sure way to navigate the minefield of confidence schemes. In other words, you essentially have to *know* more than the next guy, and the guy after that, and then the other guy, *etc*. Science does a good job of addressing this as we constantly get fed the unconventional insights to explain our broader environmental or economic situation.

Ultimately we humans could consider knowledge as a survival tactic — which boils down to the adage of eat or be eaten. If I want to sound even more pedantic, I would suggest that speed or strength works to our advantage in the wild but does not translate well to our current reality. It certainly does not work in the intentionally complex business world, or even with respect to our dynamic environment, as we cannot outrun or out-muscle oil depletion or climate change without putting our thinking hats on.[2]

1. The TV pundit Chris Matthews regularly asks his guests to *"tell me something I don't know"*. That sounds reasonable enough until you realize that it would require beyond mind reading.

Let's look at the current state of the financial industry. In no way will conventional wisdom help guide us through the atypical set of crafty financial derivatives (unless you stay away from it in the first place). Calvin Trillin wrote in a NY Times piece that the prospect of big money attracted the smartest people from the Ivy Leagues to Wall Street during the last two decades, thus creating an impenetrable fortress of opaque financial algorithms, with the entire corporate power structure on board [Ref 212]. Trillin contrasted that to the good old days, where most people aiming for Wall St careers didn't know much and didn't actually try *too hard*.

> I reflected on my own college class, of roughly the same era. The top student had been appointed a federal appeals court judge — earning, by Wall Street standards, tip money. A lot of the people with similarly impressive academic records became professors. I could picture the future titans of Wall Street dozing in the back rows of some gut course like Geology 101, popularly known as Rocks for Jocks.

I agree with Trillin that the knowledge structure has become inverted; somehow the financial quants empowered themselves to create a world where no one else could gain admittance. And we can't gain admittance essentially because we don't have the arcane knowledge of Wall Street's inner workings. Trillin relates:

> "That's when you started reading stories about the percentage of the graduating class of Harvard College who planned to go into the financial industry or go to business school so they could then go into the financial industry. That's when you started reading about these geniuses from M.I.T. and Caltech who instead of going to graduate school in physics went to Wall Street to calculate arbitrage odds."
> "But you still haven't told me how that brought on the financial crisis."
> "Did you ever hear the word 'derivatives'?" he said. "Do you think our guys could have invented, say, credit default swaps? Give me a break! They couldn't have done the math."

If you can believe this, it appears that the inmates have signed a rent-controlled lease on the asylum and have created a new set of rules for everyone to follow. We have set in place a permanent thermocline that separates any new ideas from penetrating the BAU of the financial industry.

We need to contrast this to the world of science, where one can ague that we have more of a level playing field. In the most pure forms of science, we accept, if not always welcome, change in our understanding. And most of our fellow scientists

2. This of course presumes that we know anything in the first place. We often admit what we don't know and build from there. That becomes part of the scientific method; you use models of empirical data to understand nagging issues and stew over them for long periods of time. The stewing is usually over things you or I don't know. Of course, this makes no sense for timely decision making. If humans were the equivalent of Thompson's gazelles with laptop's cranking away on a model under a shady baobob tree on the Serengeti, we would quickly get eaten. Evolutionary strategies that include speed depend on the environmental context.

won't permit intentional hiding of knowledge. Remarkably, this happens on its own, largely based on some unwritten codes of honor among scientists. Obviously some of the financial quants have protected their knowledge, as Trillin's MIT and Caltech graduates do not seem to share their secrets too readily. By the same token, geologists who have sold their soul to the oil industry have not helped our understanding either.

Given all that, it doesn't surprise me that we cannot easily convince people that we can understand finance or economics or even resource depletion like we can understand other branches of science. Take a look at any one of the Wilmott papers featuring negative probabilities or Ito calculus, and imagine a quant using the smokescreen that "you can't possibly understand this because of its complexity". The complicated influence of the financial instruments, playing out in what Steve Ludlum calls the finance economy, does often provoke apathy [Ref 310]. Even the domain of resource depletion suffers from a sheen of complexity due to its massive scale — after all, the oil economy essentially circles the globe and involves everyone in its network.

Therein lies the dilemma: we want and need the knowledge but often find the complexity overbearing. Thus the key to applying our knowledge: we should not fear complexity, but embrace it. Something might actually shake out.

Complexity

The word complexity, in short order, becomes the sticking point. We could perhaps get the knowledge but then cannot breech the wall of complexity.

The tug-of-war between complexity and simplicity is illustrated well by the provocative book **"The Quark and the Jaguar: Adventures in the Simple and the Complex"** by the physicist Murray Gell-Mann. The economist Xavier Gabaix, who I believe has a good handle on why Zipf's law holds for population of cities, cites Gell-Mann and his explanation of power laws.

Gell-Mann contributes several nuggets of general advice. First, when a behavior gets too complex, certain aspects of the problem ***can*** become more simple. We can rather counter intuitively actually simplify the problem statement, and often the solution. Secondly, when you peel the onion, everything can start to look the ***same***. For example, the simplicity of many power-laws may work to our advantage, and we can start to apply them to map much of our current understanding[3]. As Gell-

3. Power laws are also fat-tail laws, which has importance wrt Black Swan theory.

Mann states concerning the study of the simple and complex in the preface to the book:

> It carries with it a point of view that facilitates the making of connections, sometimes between facts or ideas that seem at first glance very remote from each other. ([Ref 211] p. *ix*)

He calls the people that practice this approach “Odysseans” because they “integrate” ideas from those who “favor logic, evidence, and a dispassionate weighing of evidence”, with those “who lean more toward intuition, synthesis, and passion” (*ibid* p. *xiii*). This becomes a middle ground between a heuristic belief system approach and deeper analysis. He also cautions that at least some fundamental and basic knowledge underlines any advancements we will achieve.

> Specialization, although a necessary feature of our civilization, needs to be supplemented by integration of thinking across disciplines. One obstacle to integration that keeps obtruding itself is the line separating those who are comfortable with the use of mathematics from those who are not. (Gell-Mann p.15)

I appreciate that Gell-Mann does not treat the soft sciences as beneath his dignity and he seeks an understanding as seriously as he does deep physics. He sees nothing wrong with the way the softer sciences should work in practice, he just has problems with the current practitioners and their methods.

For now, I will describe how I use Gell-Mann and his suggestions as a guide to understand problems that have confounded me. His book serves as a verification blueprint for the way that I have worked out much of my analysis. Since Gell-Mann deals first and foremost in the quantum world, his ideas don't necessarily come out intuitively.

That becomes the enduring paradox — simplicity does not always relate to intuition. This fact weighs heavily on my opinion that cheap heuristics likely will not provide the necessary ammunition that we will need to make policy decisions.

BAU (business as usual) ranks as the world's most famous policy heuristic. A heuristic describes some behavior, and a simple heuristic describes it in the most concise language possible. So, BAU says that our environment will remain the same (i.e. “when NOT making a decision IS making a decision”). Yet we all know that this does not work. Things will in fact change. Do we simply use another heuristic? Let's try dead reckoning instead. This means that we plot the current trajectory and assume this will chart our course for the near future[4]. But we all know that that doesn’t work either as it will project cornucopian optimistic and never-ending growth.

Only the correct answer, not a heuristic, will effectively guide policy. Watch how climate change science works in this regard, as climate researchers don't rely on the Farmer's Almanac heuristics to predict climate patterns. Ultimately we cannot disprove a heuristic — how can we if it does not follow a theory? — yet we can replace it with something better if it happens to fit the empirical data. We only have to admit to our sunk cost investment in the traditional heuristic and then move on.

In other words, even if you can't “follow the trajectory” with your eye, you can enter a different world of abstraction and come up with a simple, but perhaps non-intuitive, model to replace the heuristic. So we get some simplicity but it leaves us without a perfectly intuitive understanding. The most famous example that Gell-Mann provides involves Einstein's reduction of Maxwell's four famous equations in complexity by half to two short concise relations. Einstein accomplishes this by invoking the highly non-intuitive notion of the space-time continuum. Gell-Mann specializes in these abstract realms of science, yet uses concepts such as “coarse graining” to transfer from the quantum world to the pragmatic tactile world, with the name partially inspired by the idea of a grainy photograph (Gell-Mann p.29). In other words, we may not know the specifics but we can get the general principles, like we can from a grainy photograph.

> Hence, when defining complexity it is always necessary to specify a level of detail up to which the system is described, with finer details being ignored.
> (Gell-Mann p.29)

The non-intuitive connection that Gell-Mann has triggered involves the use of probabilities in the context of disorder and randomness. Not all people understand probabilities, and in particular how we apply them in the context of statistics and risk[5], yet they don't routinely get used in the practical domains that may benefit from their use. How probabilities work in terms of complexity becomes consider-

4. The majority of people use fast and frugal heuristics to make day-to-day decisions, the so-called cheap heuristic that we all appreciate. We do not always require a computational model of reality to map our behaviors or understanding. As Nancy Cartwright noted:

> This is the same kind of conclusion that social-psychologist Gerd Gigerenzer urges when he talks about “cheap heuristics that make us rich.” Gigerenzer illustrates with the heuristic by which we catch a ball in the air. We run after it, always keeping the angle between our line of sight and the ball constant. We thus achieve pretty much the same result as if we had done the impossible—rapidly collected an indefinite amount of data on everything affecting the ball’s flight and calculated its trajectory from Newton’s laws.[Ref 311]

This points out the distinction between conventional wisdom and knowledge. A conventionally wise person will realize that he doesn't have to hack some algorithm to catch a ball. A knowledgeable person will realize that he can (if needed) algorithmically map a trajectory to know where the ball will land. So some would argue that, from the point of timely decision making, whether having extra knowledge makes a lot of sense. In many cases, if you have some common sense and pick the right conventional wisdom, it just might carry you in your daily business.

5. Except for sports betting of course.

ably more straightforward when presented in terms of entropy. And that presents a bit of a challenge since entropy ranks as a mostly nonintuitive concept.

Simplicity

Gell-Mann suggests that applying a simple model should not immediately raise suspicions that we have taken shortcuts in combatting complexity. Much of modeling involves building up an artifice of feedback-looped relationships (see the Limits to Growth system dynamics model for an example), yet that should not provide an acid test for acceptance. In actuality, the large models that work consist of smaller models built up from sound principles[6].

So I try to make simple connections to other disciplines, essentially the Odyssean thinking that Gell-Mann supports.

I would argue that the fundamental trajectory of oil depletion provides one potentially simplifying area to explore. For example, no one has explained exactly *why* the classical heuristic, i.e. Hubbert's logistic curve, often works. So I have merged that understanding with the fact that I can use it to also understand related areas such as:

1. Popcorn popping times
2. Anomalous transport
3. Network TCP latencies
4. Reserve growth
5. Component reliability
6. Fractals and the Pareto law

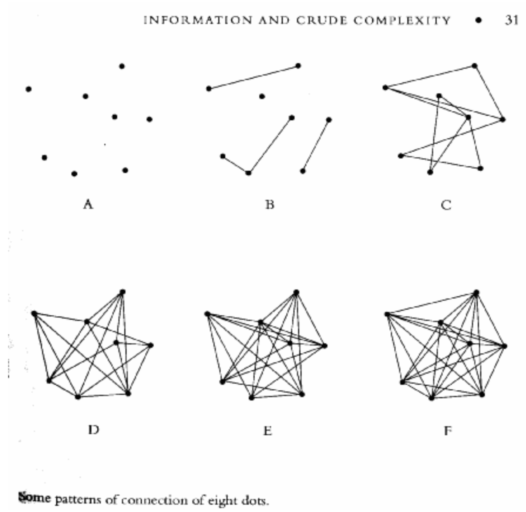
I collectively use these to support the dispersive discovery model[7].

Gell-Mann predicted in his book that this unification among concepts would occur if you continue to peel the onion. To understand the basics behind the simplicity/

6. Just ask Intel how they verify their microprocessor designs.
7. Yet it does bother me that no one has happened across this relatively simple probability formulation. You would think someone would have discovered all the basic mathematical principles over the course of the years, but apparently this one has slipped through the cracks.

complexity approach, consider the complexity of the following directed graphs of interconnected points.

FIGURE 23-5. Gell-Mann's connectivity patterns. Low connectivy and high connectivity graphs show low entropy because we can describe their structure more simply — (A) “no nodes connected” and (F) “all nodes connected”. The other graphs require more complex descriptions and thus a higher entropy measure.



Gell-Mann asks us which graphs we would consider simple and which ones we would consider complex. His answer relates to how compactly or concisely we can describe the configurations. So even though (A) and (B) appear simple and we can describe them simply, the graph in (F) borders on ridiculously simple, in that we can describe it as “all points interconnected”. So this points to the conundrum of a complex, perhaps highly disordered system, that we can fortunately describe very concisely. As humans, the fact that we can do some pattern recognition allows us to actually discern the regularity from the disorder.

However, what exactly the pattern means may escape us. As Gell-Mann states:

> We may find regularities, predict that similar regularities will occur elsewhere, discover that the prediction is confirmed, and thus identify a robust pattern: however, it may be a pattern that eludes us. In such a case we speak of an “empirical” or “phenomenological” theory, using fancy words to mean basically that we see what is going on but do not yet understand it. There are many such empirical theories that connect together facts encountered in everyday life.
> (Gell-Mann p.93)

That may sound a bit pessimistic, but Gell-Mann gives us an out in terms of always considering the concept of entropy and applying the second law of thermodynamics (the disorder in an isolated system will tend to increase over time until it reaches an equilibrium). Many of the patterns such as the graph labelled F in Figure 23-5 on page 413 have their roots in disordered systems. Entropy essentially quantifies the amount of disorder, and that becomes our “escape hatch” in how to simplify our understanding.

> *In fact, however, a system of very many parts is always described in terms of only some of its variables, and any order in those comparatively few variables tends to get dispersed, as time goes on, into other variables where it is no longer counted as order. That is the real significance of the second law of thermodynamics.* (Gell-Mann p.226)

One area that we can rigorously apply this formulation to has to do with the distribution of human travel times. Brockmann *et al* [Ref 213] reported in *Nature* a scalability study that provoked some scratching of heads[8]. The data seemed very authentic as at least one other group could reproduce and better it, even though they could not explain the mechanism[Ref 214]. The general idea, which I will describe later, amounts to nothing more than tracking individual travel times over a set of distances, and thus deriving statistical distributions of travel time by either following the cookie trails of paper money transactions (Brockmann) or cell phone calls (Gonzalez). This approach provides a classic example of a “proxy” measurement; we don't measure the actual person with sensors but we use a very clever approximation to it. Proxies can take quite a beating in other domains, such as historical temperature records used by climate scientists, but this set of data seems very solid.

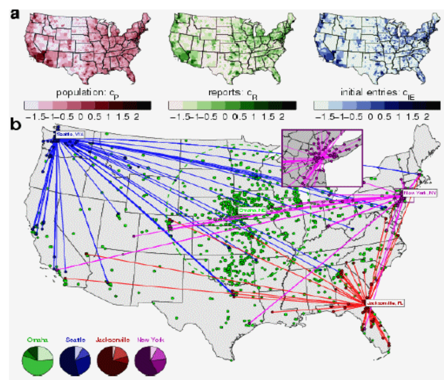


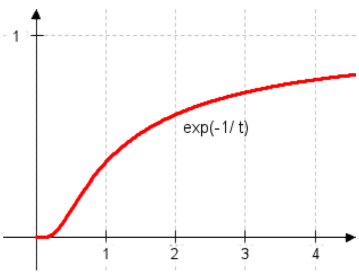
FIGURE 23-6. Human travel connectivity patterns, from Brockmann, et al [Ref 213]. This data gets reduced to histograms of travel times, distances, and speeds.

Note that this figure resembles the completely disordered directed graph shown by F in Figure 23-5 on page 413. This gives us some hope that we can actually derive a simple description of the phenomenon of travel times. We have the data, thus we can hypothetically explain the behavior. As the data has only become available recently, likely no one has thought of applying the simplicity-out-of-complexity principles that Gell-Mann has described.

8. One follow-on paper asked the questions “Do humans walk like monkeys?”.

So how to do the reduction[9] to first principles? Gell-Mann brings up the concept of entropy as ignorance. We actually don't know (or remain ignorant of) the spread or dispersion of velocities or waiting times of individual human travel trajectories, so we do the best we can. We initially use the hint of representing the aggregated travel times — the macro states — as coarse-grained histories, or mathematically in terms of probabilities.

> *Now suppose the system is not in a definite macrostate, but occupies various macrostates with various probabilities. The entropy of the macrostates is then averaged over them according to their probabilities. In addition, the entropy includes a further contribution from the number of bits of information it would take to fix the macrostate. Thus the entropy can be regarded as the average ignorance of the microstate within a macrostate plus the ignorance of the macrostate itself.* (Gell-Mann p.220)



In the way Gell-Mann stated it, I interpret it to mean that we can apply the Maximum Entropy Principle for probability distributions. In the simplest case, if we only know the average velocity and don't know the variance we can assume a damped exponential probability density function (PDF). Since the velocities in such a function follow a pattern of many slow velocities and progressively fewer fast velocities, but with the mean invariant, the unit normalized distribution of transit time probabilities for a fixed distance looks like the figure to the left. To me it actually looks very simple, although people virtually never look at exponentials this way, as it violates their intuition. What may catch your eye in particular is how slowly the curve reaches the asymptote of unity (which indicates a power-law behavior). If normal statistics acted on the velocities, the curve would look much more like a "step" function, as most of the transits would complete at around the mean, instead of getting spread out in the entropic sense.

Further since the underlying exponentials describe specific classes of travel, such as walking, biking, driving, and flying, each with their own mean, the smearing of these probabilities leads to a characteristic single parameter function that fits the data as precisely as one could desire. The double averaging of the microstate plus the macrostate effectively leads to a very simple scale-free law as shown by the blue and green maximum entropy lines I added in the figure below.

9. In terms of coarse graining, explaining the higher level in terms of the lower is often called "reduction".

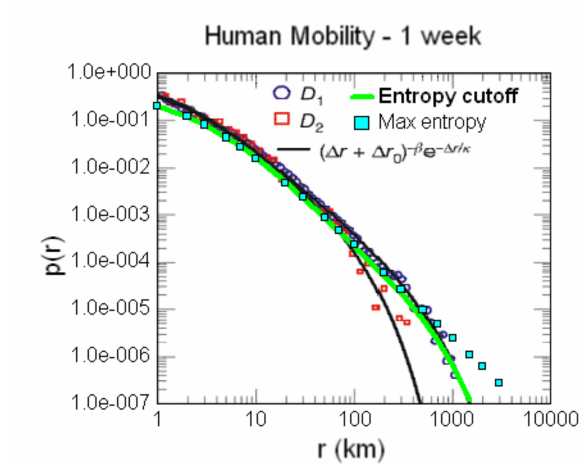


FIGURE 23-7. Dispersion of mobility for human travel. The green line indicates agreement with a truncated Maximum Entropy estimate, and the blue dots indicate no truncation

I present the complete derivation in a subsequent chapter. If you decide to read in more depth, keep in mind that it really boils down to a single-parameter fit — and this over a good 5 orders of magnitude in one dimension and 3 orders in the other dimension. Consider this agreement in the face of someone trying to falsify the model; they would essentially have to disprove entropy of dispersed velocities.

> *It has often been emphasized, particularly by the philosopher Karl Popper, that the essential feature of science is that its theories are falsifiable. They make predictions, and further observations can verify those predictions. When a theory is contradicted by observations that have been repeated until they are worthy of acceptance, that theory must be considered wrong. The possibility of failure of an idea is always present, lending an air of suspense to all scientific activity.* (Gell-Mann p.78)

Further, this leads to a scale-free power law that looks exactly like the Zipf-Mandelbrot law that Gell-Mann documents, which also describes ecological diversity (the relative abundance distribution) and the distribution of population sizes of cities, from which came the serendipitous reference that Gell-Mann made to Gabaix.

Since we invoke the name of Mandelbrot, we need to state that the observation of fractal self-similarity on different scales applies here. Yet Gell-Mann states:

> ***Zipf's law remains essentially unexplained****, and the same is true of many other power laws. Benoit Mandelbrot, who has made really important contributions to the study of such laws (especially their connection to fractals), admits quite frankly that early in his career he was successful in part because he placed more emphasis on finding and describing the power laws than on trying to explain them (In his*

> *book The Fractal Geometry of Nature he refers to his "bent for stressing consequences over causes.").* (Gell-Mann p.97)

Gell-Mann of course made this statement before Gabaix came up with his own proof for city size [Ref 215], and of course how it may apply to the variant for human travel. So we may have entered the realm of actually explaining many of these findings.

Barring the fact that it hasn't gone through a rigorous scientific validation, why does this formulation seem to work so well at such a concise level? Gell-Mann provides an interesting sketch showing how order/disorder relates to effective complexity, see the figure below. At the left end of the spectrum, where minimum disorder exists, it takes very little effort to describe the system. As in Gell-Mann's connectivity diagram, "no dots connected" describes that system. In contrast, at the right end of the spectrum, where we have a maximum disorder, we can also describe the system very simply — as in the "all dots connected" figure. The problem child exists in the middle of the spectrum, where enough disorder exists that it becomes difficult to describe and thus we can't solve the general problem easily.

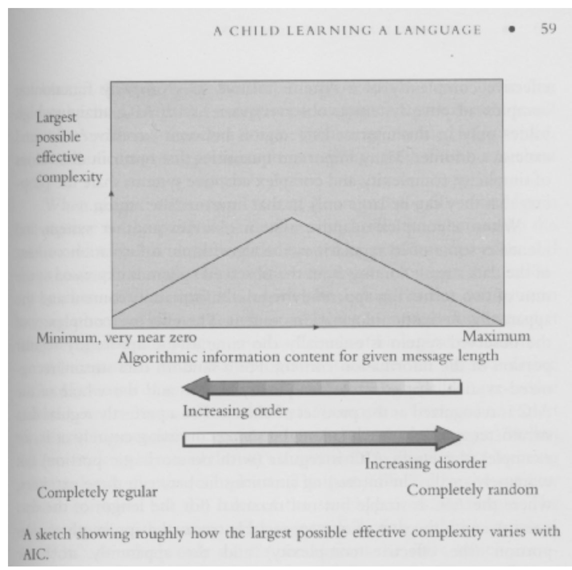


FIGURE 23-8. Gell-Mann's complexity estimator. "*the effective complexity of the observed system (can have) more to do with the particular observer's shortcomings than with the properties of the system observed*." (Gell-Mann p.56)

So in the case of human transport, we have a simple grid where all points get connected (we can't control where cell phones go) and we have a maximum entropy in travel velocities and waiting times. The result becomes a simple explanation of the empirical Zipf-Mandelbrot Law. The implication of all this is that through the use of cheap oil for powering our vehicles, we as humans have dispersed almost completely over the allowable range of velocities. It doesn't matter that we have one car that is of a particular brand and that an airliner is prop or jet, the spread in velocities while maximizing entropy is all that matters.

Acting as independent entities, we have essentially reached an equilibrium where the ensemble behavior of human transport obeys the second law of thermodynamics concerning entropy.

> *Entropy is a useful concept only when a coarse graining is applied to nature, so that certain kinds of information about the closed system are regarded as important and the rest of the information is treated as unimportant and ignored.* (Gell-Mann p.371)

Consider one implication of the model. As the integral of the distance-traveled curve in Figure 23-7 on page 416 relates via a proxy to the total distance traveled by people, the only direction that the curve can go in an oil-starved country is to shift to the left. Proportionally more people moving slowly means that fewer proportionally will move quickly — easy to state but not necessarily easy to intuit. That is just the way entropy works.

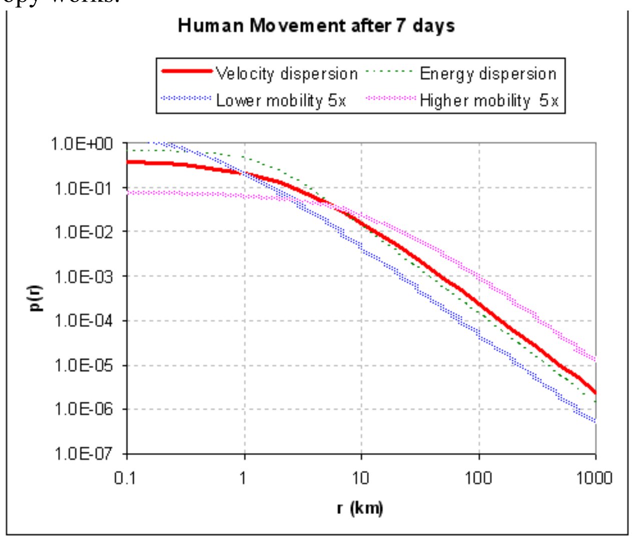


FIGURE 23-9. Assuming that human travel statistics follows the maximum entropy velocity dispersion model, a reduction in total travel will likely result in a shift as shown by the dotted blue curve.

But that does not end the story. Recall that Gell-Mann says all these simple systems have huge amounts of connectivity. Since one disordered system can look like another, and as committed Odysseans, we can make many analogies to other related systems. He refers to this process as “peeling the onion”. Figuratively as one can peel a particular onion, another layer can reveal itself that looks much like the surrounding layer. I took the dispersive travel velocities way down to the core of the onion in a forthcoming chapter explaining anomalous transport in semiconductors. Once understood, the anomalous behavior does not seem so anomalous anymore.

Like Gell-Man states, these simple ideas connect all the way through the onion to the core.

Scaling

The big onion that we peeled in Volume 1 concerned oil depletion. All the other models will indirectly support the main premise and thesis of disorder described by entropy. They range from the microscopic scale (semiconductor transport) to the human scale (travel times) and now to the geologic scale.

I assert that in the Popper sense of falsifiability, one must disprove all the other related works to disprove the main one, which amounts to a scientific form of circumstantial evidence, not quite implying certainty but substantiating much of the thought process. It also becomes a nerve-wracking prospect; if one of the models fails, the entire artifice can collapse like a house of cards. Thus the “air of suspense to all scientific activity” that Gell-Mann refers to.

So consider rate dispersion in the context of oil discovery. Recall that velocities of humans become dispersed in the maximum entropy sense. Well, the same holds for prospecting for oil. I suggest that like human travel, all discovery rates have maximum dispersion subject to an average current-day-technology rate.

A significant eye-opener occurred when I encountered Gell-Mann's description of depth of complexity. I consider this a rather simple idea because I had used it in the chapter “Finding Needles in a Haystack. How we discover oil” where I called it “depth of confidence”. It again deals with the simplicity/complexity duality but more directly in terms of elapsed time. Gell-Mann explains the depth of complexity by invoking the “monkeys typing at a typewriter” analogy. If we set a goal for the monkeys to type out the compleat works of Shakespeare, one can predict that due solely to probability arguments they would eventually finish their task. It would look something like the following figure with the depth *D* representing a crude measure of generating the complete string of letters that comprises the text.

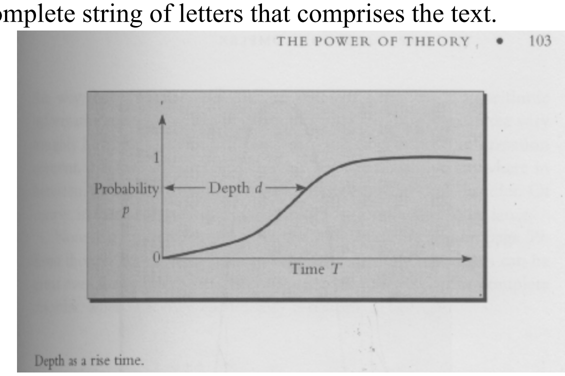
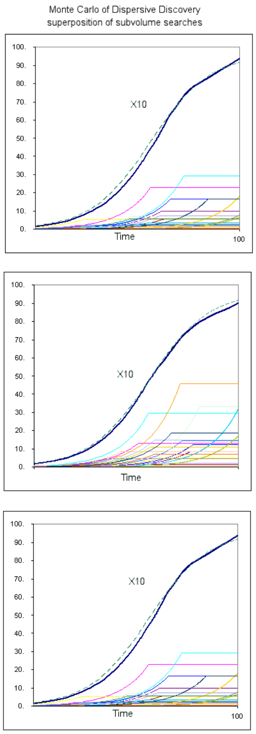


FIGURE 23-10. Gell-Mann's Depth (d) is the cumulative Probability (P) that one can gain a certain level of information within a certain Time (T).



No pun-intended, Gell-Mann coincidentally refers to *D* as a “crude complexity” measure; I use the same conceptual approach to arrive at the model of dispersive discovery of crude oil. The connection invokes the (1) dispersion of prospecting rates (varying speeds of monkeys typing at the typewriters) and (2) a varying set of sub-volumes (different page sizes of Shakespeare's works). Again, confirming the essential simplicity/complexity duality, the fact that we see a connectivity lies more in the essential simplicity in describing the disorder than anything else.

The final connection (3) involves the concept of increasing the average rate of speed of the typewriting monkeys over a long period of time. We can give the monkeys faster tools without changing the relative dispersion in their collective variability[10]. If this increase turned out as an exponential acceleration in typing rates (see Figure 23-14 on page 427), the shape of the *Depth* curve would naturally change. This idea leads to the dispersive discovery sigmoid shape — as our increasing prospecting skill analogizes to a speedier version of a group of typewriting monkeys. See the figure in the margin for a Monte Carlo simulation of the monkeys at work.

It doesn’t matter that we have one oil reservoir that has a particular geology and that this somehow deflects the overall trajectory, as we would have to if we considered a complete bottom-up accounting approach. I know this may stir up many of the geologists and petroleum engineers who hold to the conventional wisdom about such pragmatic concerns, but that essentially describes how a thinker such as Gell-Mann would work out the problem. The crude complexity suggests that we turn technology into a coarse grained “fuzzy” measurement and accelerate it to see how oil depletion plays out. So if you always thought that the oil industry essentially flailed away like various monkeys at a typewriter, you would approximate the reality more so than if you believed that they followed some predetermined and engineering analysis story-line, exemplified by the Verhulst-derived Hubbert Curve. So this model embraces the complexity inherent of the bottom-up approach, but ignoring the finer details and dismissing out of hand that determinism plays a role in describing the shape.

de Sousa gives a short explanation of how the deterministic Verhulst equation leads to the Logistic and it remains the conventional heuristic wisdom that one will find concerning the Hubbert Peak Oil curve [Ref 311]. However, Verhulst generated determinism does not make sense in a world of disorder and fat-tail statistics, as only stochastic measures can explain the spread in discovery rates. This becomes the mathematical equivalent of “not seeing the forest for the trees”. Pragmatically,

10. In marathon races, the dispersion in finishing times has remained the same fraction even as the winners have gotten faster

the details of the geology do not matter, just like the details of the car or bicycle or aircraft you travel in does not matter for modeling of human travel.

This approach encapsulates the gist of Gell-Mann's insights on gaining knowledge from complex phenomena. His main idea is the astounding observation that complexity can lead to simplicity. In an abstract sens, the following figure represents where I think some of the models reside on the complexity sphere.

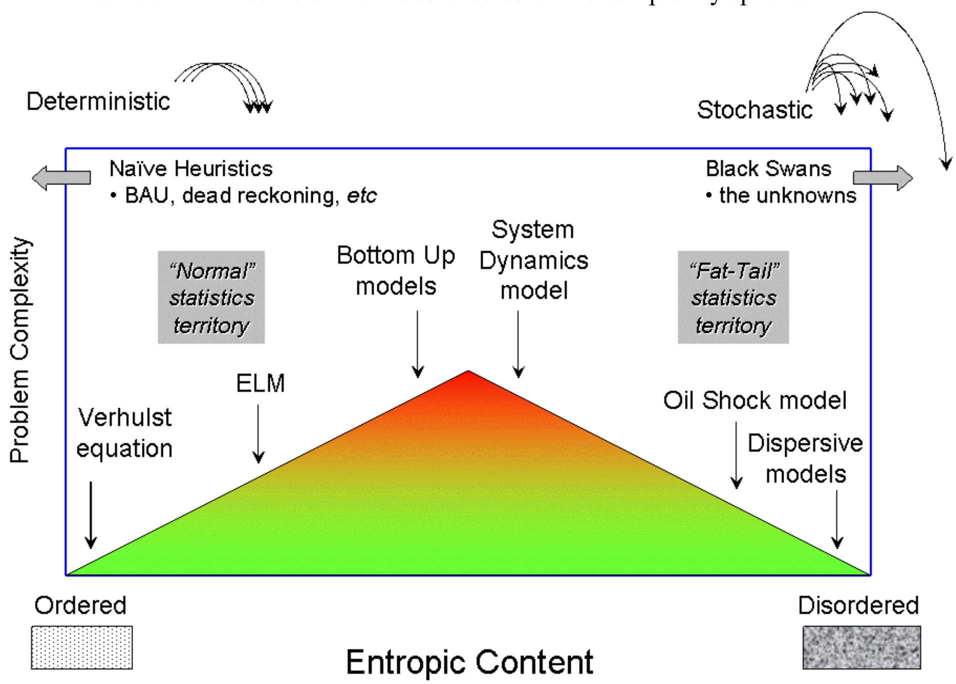


FIGURE 23-11. Abstract representation of our understanding of resource depletion. The less useful deterministic models align to the left while the stochastic models align to the right. Models with many variables yet few simplifying assumptions (such as MaxEnt) reside in the center. The center models may prove useful but their excess complexity makes them difficult to use.

Notice that I place the "Limits to Growth" System Dynamics model right in the middle of the meatiest complexity region. That of type model has perhaps too many variables and so will mine the swamps of complexity without adding much insight (or in more jaded terms, any insight that you happen to require). Many people assume that the Verhulst equation, used to model predator-prey relationships and the naive Hubbert formula of oil depletion, is complex since it describes a non-linear relation. However the Verhulst actually proves too limited, as it includes no disorder, and doesn't really explain anything but a non-linear control law. The only reason that it looks like it works is that the overly simplistic model has a fortuitous equivalence[11] to the simplified but more complex model, which exists as the dispersive discovery model on the other right-hand side of the spectrum. On the other hand, consider that the export land model (ELM) remains simple and starts to include real complexity, approaching the bottom-up models that many oil depletion analysts typically use.

Further to the left, I suggest that the naive heuristics such as BAU and dead reckoning don't fit on this chart. They assume an ordered continuance of the current state, yet one can't argue heuristics in the scientific sense as they have no formal theory to back them up[12]. The complementary effect way to the right suggests enough disorder that we can’t even predict what may happen, the so-called Black Swan theory proposed by Taleb.

On the bulk of the right side, we have other dispersive models. These all basically peel the onion, and follow Gell-Mann's suggestion that all reductive fundamental behaviors will show similarities at a coarse graining level.

This includes the variation referred to as the dispersive aggregation model for reservoir sizing outlined in Volume 1. You may ask if this is purely an entropic system, why would reservoirs become massive? For one nothing precludes order on a local scale:

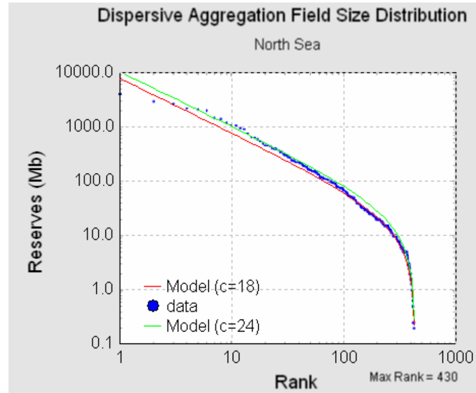
> Sometimes people who for some dogmatic reason reject biological evolution try to argue that the emergence of more and more complex forms of life somehow violates the second law of thermodynamics. Of course it does not, any more than the emergence of more complex structures on a galactic scale. Self organization can always produce local order. (Gell-Mann p.372)

Gell-Mann used the example of earthquakes and the relative scarcity of very large earthquakes to demonstrate how phenomenon can appear to “self-organize”. Laherrere has used a parabolic fractal law, a pure heuristic to model the sizing of reser-

11. See the derivation regaring Fermi-Dirac statistics from physics. This looks similar to the Logistic but comes about through a different mechanism.It has exactly the same functional form as the Hubbert Logistic sigmoid:
F(E) = 1/(1+exp((E-Ef)/kT)))
The "URR" in this case is 1 and Ef takes the place of time. Now, this has a specific derivation that has nothing to do with a deterministic outcome. If this was in fact deterministic, then physicists would have used this formula to derive it:
dF/dEf = -C*F*(1-F)
Try it out, the identity works, just like it works for the Hubbert curve.
Yet physicists don't do this. Why? Because it is patently ridiculous to pursue this line of reasoning as it doesn't explain anything. Physicists recognize an identity when they see one. Ef in fact is a statistical measure of energy and the Fermi-Dirac derivation comes from a detailed analysis of counting statistics. It doesn't come from the solution to a differential equation.
12. Excepting perhaps short-term Bayes. Bayesian estimates use prior data to update the current situation. BAU is a very naive Bayes (i.e. no change) whereas dead reckoning is a first order update, the derivative.

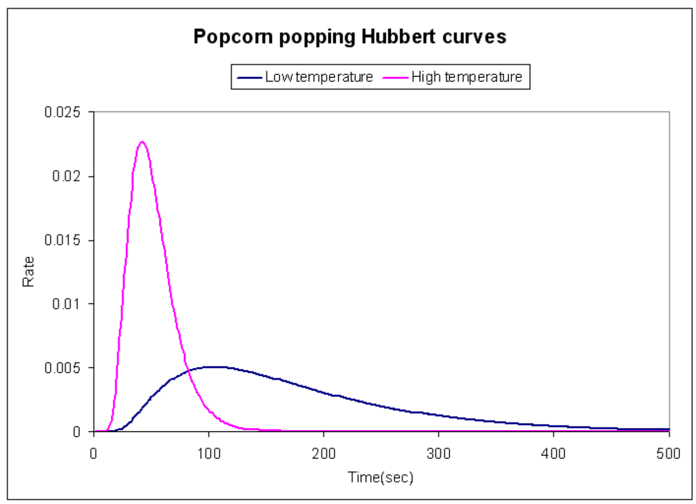
voirs (and earthquakes)[Ref 51], whereas I use a simple dispersive model to generate large reservoirs.

FIGURE 23-12. Dispersed velocities suggests a model of aggregation, much like Gabaix suggests for aggregation of cities [Ref 215]. Very few large reservoirs and many small ones, just as in the distribution of cities.



These dispersive forms all fit together very tightly. That essentially explains why I think we can use simple models to explain complex systems[13].

FIGURE 23-13. P opcorn popping kinetics follows the same dispersive dynamics as the discovery of oil ala Hubbert. High temperature obviously accelerates the popping rate.



Applying the Ideas

I found many other insights in Gell-Mann's book that expand the theme of this analysis and so seem worthwhile to point out[14].

13. I admit that I have tried to take this to some rather unconventional analogies, yet it seems to still work.

> *Running through the entire text is the idea of the interplay between the fundamental laws of nature and the operation of chance.* (Gell-Mann p.367)

The role of chance, and therefore probabilities, seems to rule above all else. Not surprising from a quantum mechanic.

Gell-Mann also has quite a few opinions on the state of multi-disciplinary research, with interesting insight in regards to different fields of study. He treats the problems seriously as he believes certain disciplines have an aversion to accommodating new types of knowledge.

And these concerns don't sit in a vacuum, as he spends the last part of the book discussing sustainability and ways to integrate knowledge to solve problems such as resource depletion.

> **The Informational Transition**
>
> *Coping on local, national, and transnational levels with environmental and demographic issues, social and economic problems, and questions of international security as well as the strong interactions among all of them, requires a transition in knowledge and understanding and in the dissemination of that knowledge and understanding. We can call it the informational transition. Here natural science, technology behavioral science, and professions such as law, medicine, teaching, and diplomacy must all contribute, as, of course, must business and government as well. Only if there is a higher degree of comprehension, among ordinary people as well as elite groups, of the complex issues facing humanity is there any hope of achieving sustainable quality.*
>
> *It is not sufficient for that knowledge and understanding to be specialized. Of course, specialization is necessary today But so is the integration of specialized understanding to make a coherent whole, as we discussed earlier. It is essential, therefore, that society assign a higher value than heretofore to integrative studies, necessarily crude, that try to encompass at once all the important features of a comprehensive situation, along with their interactions, by a kind of rough modeling or simulation. Some early examples of such attempts to take a crude look at the whole have been discredited, partly because the results were released too soon and because too much was made of them. That should not deter people from trying again, but with appropriately modest claims for what will necessarily be very tentative and approximate results.*
>
> *An additional defect of those early studies, such as Limits to Growth, the first report to the Club of Rome, was that many of the critical assumptions and quanti-*

14. The intent of referencing Gell-Mann heavily because it incorporates a popular science angle to the discussion. I consider Gell-Man close to Carl Sagan in this regard (w/o the "billions" of course). I essentially used the book as an interactive guide, trying to follow his ideas by comparing them to models that I had worked on. The main function of the book is to stimulate thought and discussion

> *ties that determined the outcome were not varied parametrically in such a way that a reader could see the consequences of altered assumptions and altered numbers. Nowadays, with the ready availability of powerful computers, the consequences of varying parameters can be much more easily explored.* (Gell-Mann p. 362)

Gell-Mann singles out geology, archaeology, cultural anthropology, most parts of biology for criticism, and many of the softer sciences, not necessarily because the disciplines lack potential, but because they suffer from some massive sunk-cost resistance to accepting new ideas. He gives the example of distinguished members of the geology faculty of Caltech *"contemptuously rejecting the idea of continental drift"* for many years into the 1960's (Gell-Mann p. 285).[15] And then Gell-Mann relates this story on practical modeling within the oil industry:

> *Peter Schwartz, in his book "The Art of the Lonq Wew", relates how the planning team of the Royal Dutch Shell Corporation concluded some years ago that the price of oil would soon decline sharply and recommended that the company act accordingly The directors were skeptical, and some of them said they were unimpressed with the assumptions made by the planners. Schwartz says that the analysis was then presented in the form of a game and that the directors were handed the controls, so to speak, allowing them to alter, within reason, inputs they thought were misguided. According to his account, the main result kept coming out the same, whereupon the directors gave in and started planning for an era of lower oil prices. Some participants have a different recollections of what happened at Royal Dutch Shell, but in any case the story beautifully illustrates the importance of transparency in the construction of models, As models incorporate more and more features of the real world and become correspondingly more complex, the task of making them transparent, of exhibiting the assumptions and showing how they might be varied, becomes at once more challenging and more critical.* (Gell-Mann p. 285)

Trying to understand why some people tend to a very conservative attitude, Gell-Mann has an interesting take on the word "theory" and the fact that theorists in many of these fields get treated with little respect.

> *"Merely Theoretical" — Many people seem to have trouble with the idea of theory because they have trouble with the word itself, which is commonly used in two quite distinct ways. On the one hand, it can mean a coherent system of rules and principles, a more or less verified or established explanation accounting for know facts or phenomena. On the other hand, it can refer to speculation, a guess or conjecture, or an untested hypothesis, idea or opinion. Here the word is used with*

15. This extends to beyond academics, as serious arguments exist about understanding the theory behind geostatistics and the use of a technique called "kriging" to estimate mineral deposits from bore-hole sampling (http://en.wikipedia.org/wiki/Talk:Geostatistics#.22Geostatistics_is_a_fundamentally_flawed_.E2.80.A6.22).

> *the first set of meanings, but many people think of the second when they hear "theory" or "theoretics".* (Gell-Mann p.90)

Unfortunately, I do think that this meme that marginalizes peak oil "theory" will gain momentum over time. Particularly, in terms of whether peak oil theory has any real formality behind it, as certainly no one in academic geology besides Hubbert[16] has really addressed the topic. Gell-Mann suggests that many disciplines simply believe that they don't need theorists.

In the field of economics, Barry Ritholtz has also recently suggested a more scientific approach [Ref 241], yet he doesn't think that modeling necessarily works in economics. He might well consider that economics and finance modeling overlooks dispersion completely. Taleb suggests that they should include fat-tails, as the amount of effort placed in applying normal statistics has proven out as a colossal failure. We get buried daily in discussions on how to best to generate a course-correction within our economy, balanced between a distinct optimism and a bleak pessimism. At least part of the pessimism stems from the fact that we think the economy will forever stay conveniently complex beyond our reach. I would suggest that simple models, ala *econophysics*, may help just as well and that it allows us to understand when expensive heuristics and complex models work against our best interests (i.e. when we have been played).

> *The "cost of information" addresses the fact that people may not know how to make reasonable free market decisions (for instance about purchases) if they don't have the necessary facts or insights.* (Gell-Mann p.325)

Above all, Gell-Man asks the right questions and provides some advice on how to move forward.

> *If the curves of population and resource depletion do flatten out, will they do so at levels that permit a reasonable quality of human life, including a measure of freedom, and the persistence of a large amount of biological diversity, or at levels that correspond to a gray world of scarcity, pollution, and regimentation, with plants and animals restricted to a few species that co-exist easily with mankind?* (Gell-Mann p.349)

> *We are all in a situation that resembles a fast vehicle at night over unknown terrain that is rough, full of gullies, with precipices not far off. Some kind of headlight, even a feeble and flickering one, may help to avoid some of the worst disasters.* (Gell-Mann p.366)

16. Who was a geophysicist and worked out of curiosity more than anything else.

We need to take this as a path forward to separating the simple from the complex.

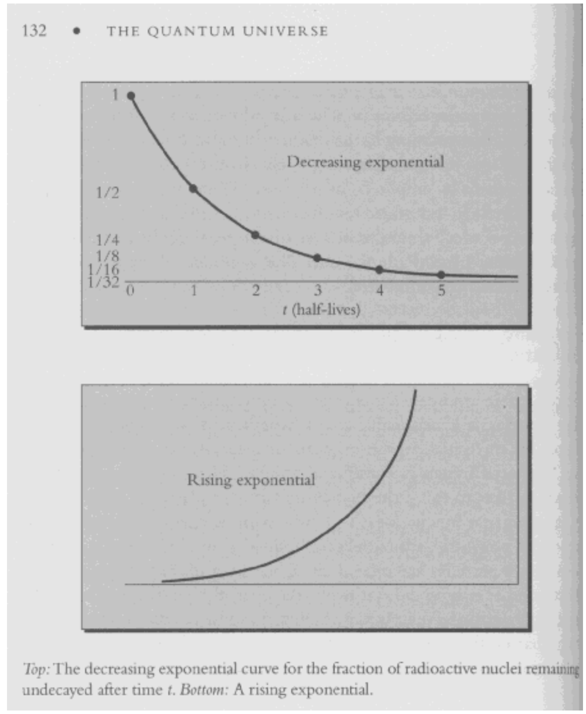


FIGURE 23-14. A damped exponential contains a maximum entropy amount of information, such as the decay of radioactive material. The rising exponential usually occurs due to a degree of feedback reinforcing some effect, such as technology advances.

Dispersion, Diversity, and Resilience

If we want to have any hope in controlling our destiny we have to understand our environment. In one sense, if we treat our environment as a control system, capable of responding to a stimulus, we need to understand not only its behavior, but how it will respond to the stimulus. One can ask: will it collapse in response to dwindling resources? Or will it rebound and stay resilient? For that we require a good model of the system. And of course, the simpler the model to describe, the better.

The system thinker and cyberneticist Ross Ashby summed it up with two seminal ideas. His simplicity criteria, the “Law of Requisite Variety” states, *“Variety absorbs variety, defines the minimum number of states necessary for a controller to control a system of a given number of states.”* Our capabilities thus become limited by the amount of information available to us. The second, the “Good Regulator theorem” goes *“every good regulator of a system must be a model of that system”*. In other words, to regulate any causal system, we should require a model of how the system will behave normally and how it will react to a disturbance. The two relate directly to the classical view of control theory, that of controllability and observability. That may sound a tad idealistic, but that's how an engineer would respond to a problem statement.

The seeming diverse complexity of a system such as the Amazon rain forest, remains in many ways simple to describe. Interestingly, we can actually understand how that system evolves and adapts, given that we have a simple-enough model to work with. To compare and contrast, we can also consider a model of oil resources and how we can understand a seemingly random distribution of reserves.

Further, to have any chance of controlling the behavior we need good observability via good measurements. A human mobility metric is just one example of this, one simple to model, which gives us a good understanding, and one that we can monitor in the future.

For these three cases, (1) biodiversity, (2) oil abundance, and (3) human mobility, I will describe a few simple models based on entropy principles (maximum entropy dispersion, the “entroplet”) and working with barest and most minimal information available to us. We will see how far that can take us. The discipline of complex and resilient systems remains wide-open for discussion.[17]

When I first approach a problem involving some type of disorder, I try to characterize the observed behavior according to whether it follows a predictable, unpredictable, or random/noisy process. I rarely use the categories of complex or chaotic. In my mind, if you do that, you give up some hope in solving the problem.

So we avoid conventional notions of complexity. As defined by the scientific establishment, complexity seems to have taken on a discipline of its own. Once some problem gets characterized as complex or chaotic, the big thinkers emerge from the Nonlinear Institute of Profundity, leaving the rest of us behind. As a more pragmatic approach, I wouldn't mind capturing a complementary world-view to overtly sophisticated models of complexity. I notice that the expert-level mathematical explanations invariably become hairy, while the popular explanations lack common sense or excessively rely on heuristics (see oil depletion analysis as a primary example of the latter). However, the most elegant approaches tend to apply the simplest patterns to concisely describe the complexity. I elaborated on this topic in the preceding section, so I won't try to explain the philosophy behind the “simplicity out of complexity” paradigm.

17. George Mobus had posted on Energy Flow, Emergent Complexity, and Collapse. As a response, I intentionally named this post Dispersion, Diversity, and Resilience. This doesn't mean that I disagree with his point-of-view, just that mine differs. [Ref 314]

Instead, I want to practically demonstrate where and how some simple and rather parsimonious explanations can go a long way to explaining why disordered, yet seemingly complex, systems have very intuitive explanations. And in keeping with the intent of this discussion, I will try to show how these system might become resilient against collapse. I believe that we often mistake complexity for garden-variety disorder (i.e. entropy) and that the diversity that arises due to strictly entropic arguments may also demonstrate some robustness. In the end, if we can understand how the diversity arises, then we may figure out how to achieve or at least maintain a good outcome.

As a muddying factor, we have to again consider that people's intuition fails when they try to deal with complexity or entropic disorder.[18] As you will see, the premise that I will outline essentially trivializes certain aspects of the "complex ecosystem". What we actually observe amounts to nothing more than our own filtered view on a disordered state. As the key point to retain, the observed disorder runs high enough so as to simplify the entire argument, both mathematically and conceptually.

In the previous section, I used the writings of Murray Gell-Mann to guide the narrative. He basically explained how seemingly complex systems often possess the simplest descriptions. To extend this context, I will use Edwin T. Jaynes research on probability and entropy to help justify the simplicity premise. According to Jaynes, we just scratch the surface of the practical applications of entropy if we consider it only as something that arises out of thermodynamics. Instead, if we treat entropy as a first-class measure of the disorder in a system, it can prove useful in many other scientific investigations.

Of course, we can make a connection to the oil depletion equation as well, as simplicity often proves contagious. The systems thinker John Gall (Systemantics) once said: *"A complex system that works is invariably found to have resulted from a simple system that worked"*.

I will go through a few cases of seeming complexity masquerading as random and disordered behavior. These share the approach of applying some basic ideas and common sense, and then working out the problems with similar math in all these analyses. So if you get stuck in understanding the principles in one of the explanations, something might spark your intuition in another. I apply the same math as a way to unify my understanding, as well as to substantiate the overall approach. As with most effective arguments, the more broadly we can apply the arguments, the more confidence we have in its generality and applicability. For many of the global

18. Although I consider the supporting explanations and rationalization for disordered systems rather simple (see the previous section), they do build on some math

problems we face, we don't have the benefit of a controlled experiment. The earth itself acts as both the test and the control. For that reason, if we can find unifying global behaviors, we gain confidence by the accumulation of these "proxy" explanations. As a side effect, you may end up finding quite a few interesting emergent results from the case studies.

The first case: Relative Abundance Distribution

In nature, the diversity of species gets reflected in the samples of various populations taken during scientific surveys. Scientists invariably find, and we get reminded quite often, that a few species predominate in their abundance while the majority of species have relatively sparse populations. Many species remain extremely rare or go undiscovered. This data typically gets plotted as a Relative Abundance Distribution (RAD) histogram.

As a result of these empirical observations — usually taken in some very diverse populations of a certain category of wildlife or plant — you will actually see relative counts of the most common species outnumber the rarest species by orders of magnitude. In sampling experiments, the rarest species may actually have counts of only 1.

Most recent research on this topic has concentrated on understanding the relative species abundance (RSA) of somewhat isolated ecosystems. Understandably, scientists approach it this way so that they can limit or control the set of measurable parameters and therefore understand the phenomena on a larger and more heterogeneous scale. This 2007 article in Nature, "Patterns of relative species abundance in rainforests and coral reefs"[Ref 219] , suggests that interactions among species don't have as large an effect as imagined .

> Abstract: *A formidable many-body problem in ecology is to understand the complex of factors controlling patterns of relative species abundance (RSA) in communities of interacting species. Unlike many problems in physics, the nature of the interactions in ecological communities is not completely known. Although most contemporary theories in ecology start with the basic premise that species interact, here we show that a theory in which all interspecific interactions are turned off leads to analytical results that are in agreement with RSA data from tropical forests and coral reefs. The assumption of non-interacting species leads to a sampling theory for the RSA that yields a simple approximation at large scales to the exact theory. Our results show that one can make significant theoretical progress in ecology by assuming that the effective interactions among species are weak in the stationary states in species-rich communities such as tropical forests and coral reefs.*

I have no problems with their assertions, only that the math that they invoke goes a bit overboard, and provides very little insight. Practically speaking, with just a few nods to maximum entropy, we can show agreement to the results in a few lines of derivation. With that simplicity, we get the benefits of a significant amount of extra insight. Also note that the authors state that they don't know the interactions; this uncertainty suggest that entropy arguments may work out well. In other words, maximum entropy provides an avenue for reasoning about an uncertain world.

To derive a universal RAD, we start out with a few assumptions.

We first assume that different species evolve as random processes that essentially fill up space. I propose a quantity that, for the lack of a better term, I call the adaptation level, $\boldsymbol{A}$. This can have the units of, for example, #organisms/acre so it maps to a parameter proportional to sampling some species.

We next use the Maximum Entropy Principle (**MaxEnt**) to describe the uncertainty in the time it takes for a species to reach an adaptation level.

$$p(t) = \frac{1}{t_0} \cdot e^{-t/t_0} \qquad \text{(EQ 23-5)}$$

$$P(A, r|t_0) = \int_{A/r}^{\infty} p(t)dt \qquad \text{(EQ 23-6)}$$

These are both probability distributions, the first a density function and the second a conditional cumulative probability[19].

The first expression essentially states that we don't know how long it takes to reach a given adaptation level, only that it has a mean time, $\boldsymbol{t_0}$. This mean time could reach millions of years, but for now the specific value doesn't matter. We just assume that the likelihood of times around that mean has a maximum entropy described by the exponential probability function, $\boldsymbol{p(t)}$. We then assume a rate, $\boldsymbol{r}$, that relates a time to reach a given adaptation level, $\boldsymbol{A = r*t}$. Every species that reaches the adaptation level has to evolve for a time $\boldsymbol{t=A/r}$, so the conditional cumulative probability is described as $\boldsymbol{P(A,r \mid t_0)}$ and derives to the value below.

$$P(A, r|t_0) = e^{-A/(r \cdot t_0)} \qquad \text{(EQ 23-7)}$$

We also have uncertainty in the adaptation level, $\boldsymbol{A}$, assuming only that it also has some mean value $\boldsymbol{A_0}$ with the same MaxEnt probability density function.

19. read as $P(A,r \mid t_0)$ = integral of $p(t)$ for all t such that t is greater than A/r

$$P(r|t_0, A_0) = \int_0^{\infty} P(A, r|t_0)dA \qquad \textbf{(EQ 23-8)}$$

This results in the cumulative adaptation rate function as follows:

$$P(r|t_0, A_0) = \frac{1}{1 + \frac{A_0}{r \cdot t_0}} = \frac{1}{1 + \frac{r_0}{r}} \qquad \textbf{(EQ 23-9)}$$

If we replace $\boldsymbol{A_0/t_0}$ with $\boldsymbol{r_0}$, we see that the above relation describes a set of species that evolve with a huge dispersion — high enough for it to describe a fat-tail distribution of rates. In other words, the large disorder in both the time scale and adaptation level generate a rate function that generates an even larger entropy in the evolution of various species. The two degrees of freedom in uncertainty gives it double the entropy of a single **MaxEnt** exponential probability density function. This uncertainty results in a very disordered system. I make the claim that diversification and growth of speciation possesses maximum entropy. However consumed energy aids in driving adaptation, it doesn't affect the probabilities, as entropy plays the lead role in generating the dispersion.

The relative abundance comes about when you consider that low values of r will lead to smaller relative population levels than higher values of $\boldsymbol{r}$. Since probabilities get invoked, we also see the effects of abundance as a combination of population size and rarity. In other words, abundance essentially relates the value of $\boldsymbol{r}$ (proportional to the size of the species population when evaluated over a period of time) to rarity, which states how often that size of population occurs. (To foreshadow a bit, the same argument holds in sizing oil reservoirs.)

At this point, I don't necessarily care how the authors' of the Nature article derived their own model, just that it has greater informational complexity than my derivation. Since the equation for $\boldsymbol{P(r \mid r_0)}$ contains only a single adjustable parameter, $\boldsymbol{r_0}$, it meets the Gell-Mann acid test for simplicity. When we transform $\boldsymbol{r_0}$ to an abundance, we use a proportionality constant, $\boldsymbol{k}$, and call the result an abundance dispersion factor, $\boldsymbol{D=k*r_0*time}$.

$$\textbf{CDF :} \qquad P(X) = \frac{1}{1 + \frac{D}{X}}$$

$$\textbf{PDF :} \qquad p(X) = \frac{D}{(D+X)^2} \qquad \textbf{(EQ 23-10)}$$

The basic normalized shape looks like the following histogram plotted on a double logarithmic scale (the log-normal heuristic is shown for comparison):

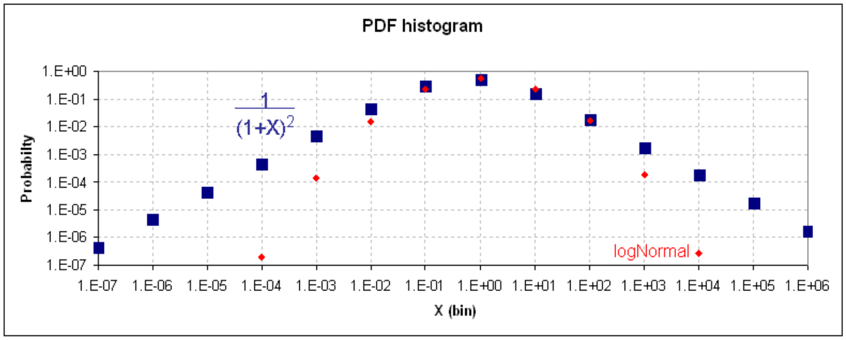


FIGURE 23-15. Probability Density Function (PDF) histogram (Preston Plot) of the entropic dispersion function, normalized to 1. Plotted per decade on a logarithmic scale, the function appears symmetric. The function is so ubiquitous and has such nice aggregating properties, I refer to it as an entroplet.

On this scale $\boldsymbol{D}$=1, and the term $\boldsymbol{X}$=abundance corresponds to the relative size of the population, while $\boldsymbol{p}(\boldsymbol{X})$ provides the abundance of that population in terms of a probability. Therefore, you can read it as either very small or very large $\boldsymbol{X}$ populations occur infrequently, with the peak frequency lying in between the extremes. However, since large populations consist of large numbers of organisms, they do show up more often in statistical samples. The rarest species may never show up in samples (the so-called *"Preston's veil"*) both because rare species have a small population and because that size population rarely occurs over the epochal time integration considered. Interestingly, no mean value exists for this PDF, which often happens for fat-tail distributions. In practice, this has little impact in a finite world, as we consider both finite time and space to generate limiting constraints.

That essentially describes the extent of the theory. Simple enough, so let's see how effectively a single parameter fit works with the observed data.

Results. Data of relative abundance usually gets plotted as a cumulative rank histogram (also known as a Whittaker plot) or as a Preston plot (which essentially describes the probability density function (PDF) as a logarithmically binned histogram).

I took data from the Nature article and applied the simple theory on Whittaker histograms first. The following diagrams contain a single-parameter fit to the data, shown as the RED lines. Each diagram corresponds to a different isolated tropical forest region and the RAD for sampled tree species within those regions. The BCI region has a dispersion factor of $\boldsymbol{D}$=23 while the Pasoh region has a factor of $\boldsymbol{D}$=14.

Since this function has scale-free properties, the dispersion factor really only shifts the location of the knee in the curve along the abundance axis. The BCI has a higher mode for the relative abundance than the Pasoh region, which could imply that the most common species adapted faster for the BCI region or that the BCI region evolved/adapted over a longer period of time. Importantly, we can't tell the difference because we have derived A_0/t_0 only as a ratio; in other words, we have lost the ability to separate the two effects.

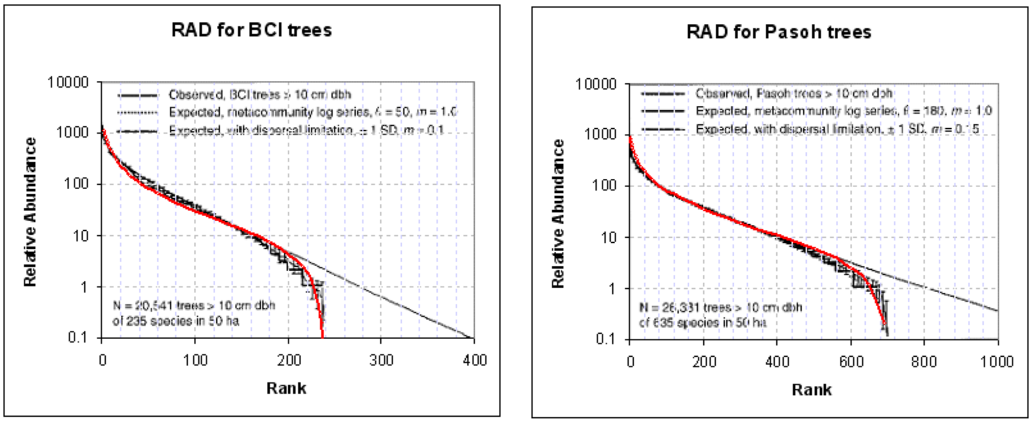


FIGURE 23-16. RAD histograms for isolated adaptation regions. The red lines shows the dispersive model. As defined with the local model, it can only shift up and down.

The movement of the mode becomes more apparent if graphed as a Preston plot. This bins the data on a more granular level, yet gives a view that more intuitively shows the most frequent population as a peak value. The same data shows up as BLUE dots below.

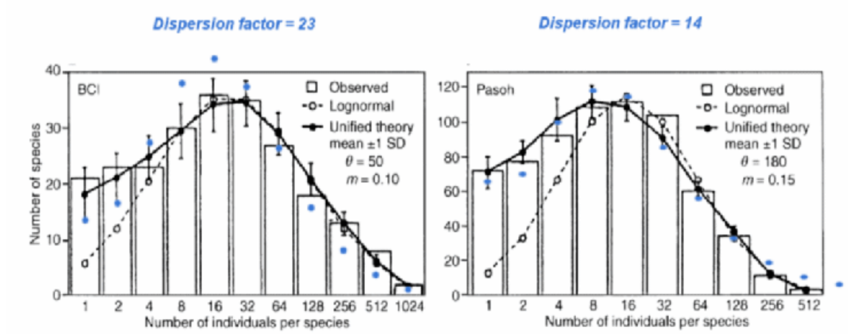
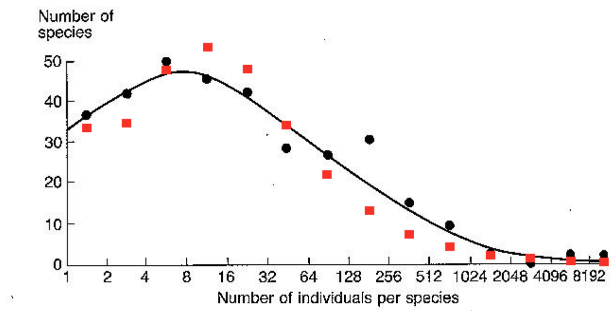
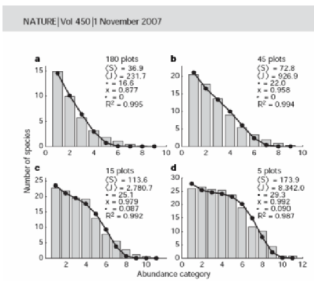


FIGURE 23-17. RAD histograms as Preston plots show the median position.

The general trend shows some universality. Plotted below in RED dots is the fit to a sampling of moth populations.

FIGURE 23-18. RAD histogram for moth population. Red dots show the dispersive model.





Remember, the factor ***D*** sets the peak position and provides the only adjustable parameter in the fit. Maximum entropy considerations alone set the width of the curve. For the moth RAD histogram shown above, the black line gives the log-normal fit. This may appear better than the dispersive fit, but the log-normal has three adjustable parameters available for tuning, and has little additional intuitive significance.

Not all of the RAD plots show a mode peak away from low abundance. For example, the adjacent figure reproduced from the Nature article generates data for some localized coral communities. From what I understand, these have gotten somewhat isolated from the larger metacommunity which understandably would show a larger diversity in the coral population. The isolation gives many low abundance species, binned as base 2 abundance categories between 0, 1 and 2 (2^0=1, 2^1=2, 2^2=4).

Qualitatively this also agrees with what the dispersive formulation says if we apply low values of the dispersion factor, ***D***. Again, these populations may not have evolved/adapted as fast or they may not have evolved/adapted over a long enough period. Without additional information, this is all that the maximum entropy principle can tell us.

If we plot a range of dispersion factors on the same binned max rank of samples, the results appear as below. Apart from low values of ***D***, varying the value of ***D*** simply shifts the distributions away from the origin. Each shape defines an entroplet, which lacks any parameter besides its position, becoming essentially scale-free.

On the horizontal logarithmic scale, the width looks like it remains constant, but it actually spreads out to accommodate a larger range of individuals per species.

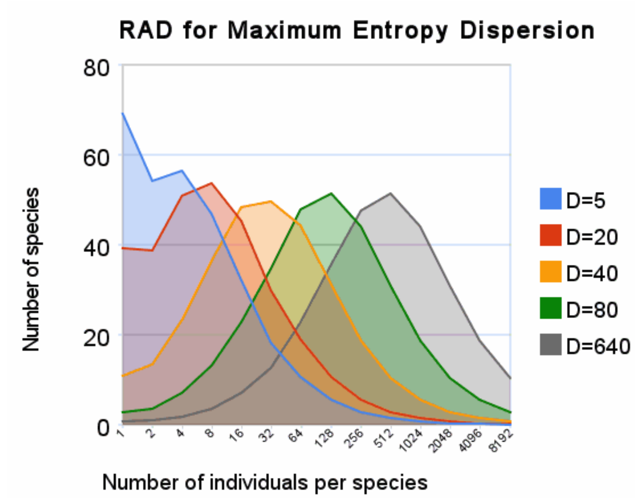


FIGURE 23-19. Dispersive model with various values of ***D***, the median abundance level.

We can further describe the extent of the metacommunity diversity. The histogram shown below to the left fits the dispersion of coral species on a more local scale while the plot on the right duplicates that result while also displaying the large shift (256 much larger than 66) on a metacommunity scale. So even though the dispersion factor changes by nearly a factor of four, the range in number of individuals per species agrees with the maximum entropy formulation.

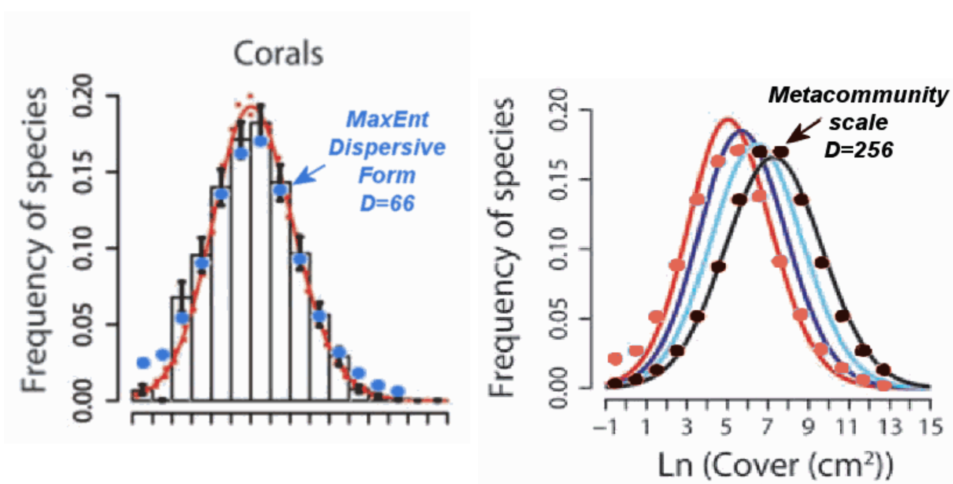


FIGURE 23-20. RAD histograms on coral communities. MaxEnt dispersion model is a single parameter fit.

The dispersive formulation will likely work on any general population simply because the relative abundance of species results from a principle of maximum disorder in the adaptation rates. With that uncertainty in place, the Maximum Entropy Principle guides us to the correct distribution.

This brings up an interesting situation. What happens when we try to apply this construct on a massively larger metacommunity scale? The tree diversity of the entire Amazon basin provides a situation that we can analyze in context. The distinction here is that the heterogeneous nature of the geography and geological events places the dispersion on a smeared time-scale. As you can see below, the histograms show a much broader shoulder than the isolated adaptation results.

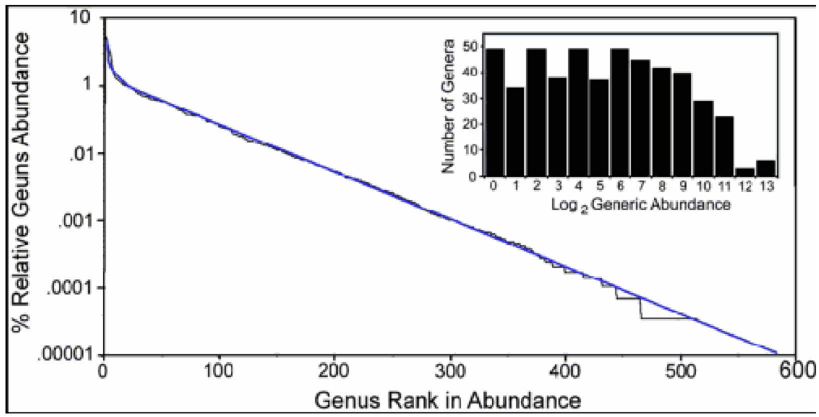


FIGURE 23-21. RAD histograms of trees in the Amazon basin. (Ignore the jump between 1 and .01 on the y-scale, as this looks like a typo)

To model the effect on a larger scale using **MaxEnt** dispersion, we have to consider a spread in time ranges. Unlike isolated regions, such as might happen on an island (e.g. BCI = Barro Colorado Island), adaptation did not start at one specific time in prior ecological history. Instead, due to a variety of factors, which can include mass extinctions, introductions, and climate change, the effective start time for adaptation ranges over a scale aligned with historical events.

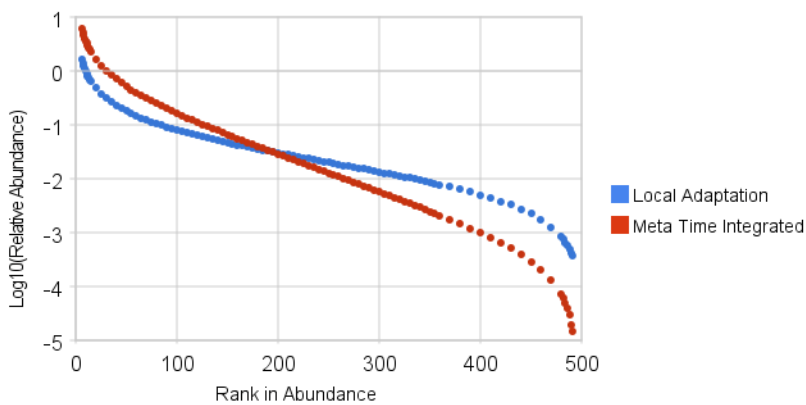
The time integration runs from the first significant event at a time ***t***+***T*** ago to the last significant event t ago (also the maximum entropy result for a range). The value of ***T*** denotes a very large ecological time scale in comparison to ***t***.

$$P(X) = \int_{t}^{t+T} k \cdot r_0 \cdot \frac{\tau}{(D + k \cdot r_0 \cdot \tau)^2} d\tau \qquad \textbf{(EQ 23-11)}$$

$$P(X) = C \cdot \ln\left(\frac{1 + \frac{X}{D_1}}{1 + \frac{X}{D_2}}\right) \quad \textbf{(EQ 23-12)}$$

This generates a logarithmic-shaped function that has a much more diverse spread in abundance levels compared to the isolated adaptation result. The values of $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ correspond to an epochal time range, and $\boldsymbol{C}$ is a constant that normalizes the result to 1 as $\boldsymbol{X}$ goes to infinity. Compare the red line below to Figure 23-21 on page 437.

FIGURE 23-22. RAD histograms of dispersive models with specific Local start time and smeared Meta start time. For Meta, D ranges from D_1=1.7 to D_2=3300. For Local, D is fixed.



If you compare the slopes of the plots, you can almost intuit what happened during the time integration. Each of the peak positions for various values of ***D*** gets spread out over time so that the linear superposition of the individual curves creates a broad almost flat-topped peak on the binned histogram as shown below. The interpretation for this uniformity makes intuitive sense: if you reach far back enough, every effective adaptation level would be achieved, split equally between each doubling octave. The effects of dispersion spreads out the impact over time so that each generation contributes evenly. In my book, that defines an existential entropy, conceptually similar to the flat white noise spectrum that forms the background radiation in the universe -- in other words, epochal noise as a form of ultimate disorder.

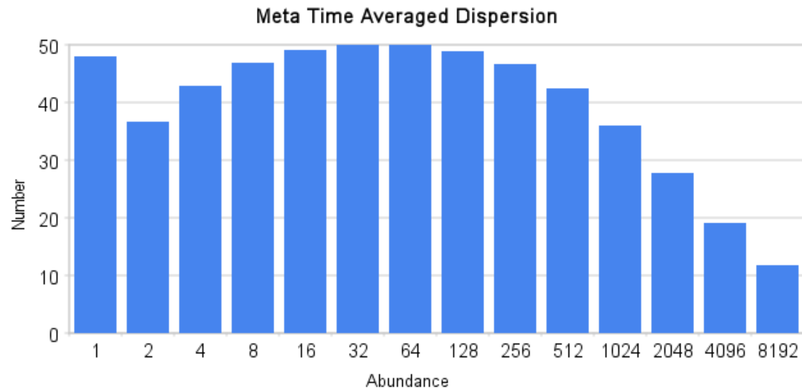
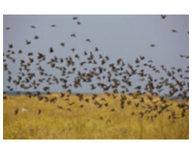


FIGURE 23-23. RAD histogram of time integrated metacommunity. Compare with the inset of Figure 23-21 on page 437

That basically explains the diversity of species according to entropic dispersion. Resilience occurs as a result of the shear diversity in the species. If one species becomes extinct, another one will likely take its place. It make take a long time, but it certainly will happen.

For an alternate derivation, go to the papers written by the authors of the *Nature* article and related papers[20]. They don't seem to mention entropy at all, preferring to use conventional combinatorial statistical arguments (exactly how you can derive entropy, by the way). So you can take the long way around the lake (their approach) or take the scientific shortcut across the water (my approach). I essentially ignored their overly sophisticated derivation because it lacked the simplicity that these problems should demonstrate. Go with the math that I have described and you can actually try out the models yourself and get the insight that I have reached. Recall that this model contains only a single parameter and, quite fundamentally, we have nothing to fear as the results derive from garden-variety disorder and the complexity essentially wilts away.



Application to Bird Survey Data

In North America, surveys on bird species abundance regularly occur and contain information specifically related to population fluctuation. The generally observed trend follows that many species exist in the middle of abundance and relatively small numbers of species exist at each end of the spectrum — few species exceed-

20. See for example these references: [Ref 222][Ref 223][Ref 225][Ref 226][Ref 227][Ref 229][Ref 230][Ref 231]

ingly common (i.e. starling) and few species exceedingly rare (i.e. whooping crane). Since the bird data comes from a large area in North America, the best fit followed a metacommunity growth model. The metacommunity adjustment impacts the knee of the histogram curve and broadens the Preston plot, effectively smearing over geological ages that different species have had to adapt.

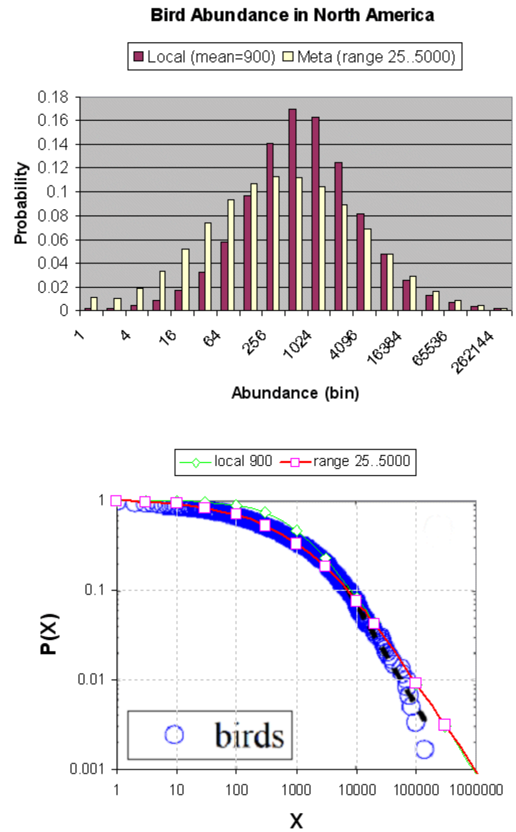


FIGURE 23-24. Preston plot (top) and rank histogram (bottom) of relative bird species abundance. This looks similar to other RAD models which describe a metacommunity distribution.

If we assume that the relative species abundance has a underlying model related to steady-state growth according to ***p***(***rate***), where rate is the relative advantage for species reproduction and survival, then this should transitively might apply to disturbances to growth as well. One paper actually tried to discern some universality in diverse growth papers, and it coincidentally used the bird survey data along with two economic measures of firm size and mutual fund size [Ref 233] [Ref 234].

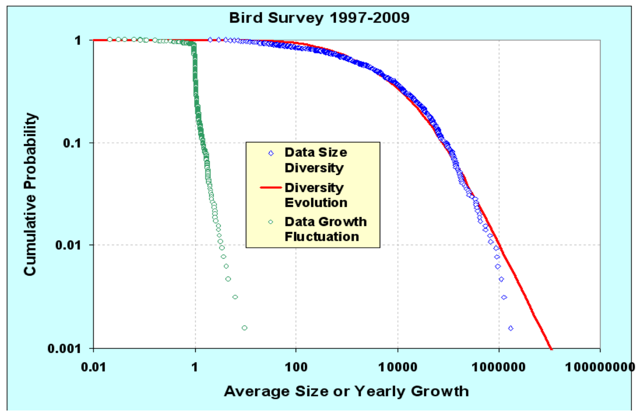
Using the USGS source data [Ref 235] from 1997 to 2009, I applied the same abundance distribution as outlined and came up with the fit below (see blue and red curves below, data and model respectively). That provided a sanity check, but

Schwarzkopf and Farmer indicated that the year-to-year relative growth fluctuations should also obey some fundamental behavior through the distribution of this metric:

$$RelativeGrowth(Year) = \frac{n(Year+1)}{n(Year)} \quad \text{(EQ 23-13)}$$

Sure enough, and for whatever reason, the "growth" in the surveyed data does show as much richness as the steady state averaged abundance distribution. The relative growth in terms of a fractional yearly change sits alongside the relative abundance curve below (in green). Notice right off the bat that the distribution of fractional changes drops off much more rapidly.

FIGURE 23-25. The red meta-model curve for the bird RAD data smears the median from 200 to 60000, indicating a meta community of species surveyed.



I believe that this has a simple explanation having to do with Poisson counting statistics. When estimating fractional yearly growth, we consider that the rarer bird species having the lowest abundance will contribute most strongly to fluctuation noise on year-to-year survey data. Values flipping from 1 to 2 will lead to 100% growth rates for example. (We ignore movements from 1 to 0 and 0 to 1 as these growths essentially become infinite)

I devised a simple algorithm that takes two extreme values (R greater than 1 and R less than 1) and the steady state abundance N for each species. The lower bound of:

$$R1 = R \cdot \frac{1-\sqrt{2/N}}{1+\sqrt{2/N}} \quad \text{(EQ 23-14)}$$

and the upper bound becomes:

$$R2 = R \cdot \frac{1 + \sqrt{2/N}}{1 - \sqrt{2/N}} \qquad \textbf{(EQ 23-15)}$$

The term $1.4/\sqrt{N}$ derives from Poisson counting statistics in that the relative changes become inversely related to the size of the sample. We double count in this case because we don't know whether the direction will go up or down, relative to R, a number close to unity.

This has much similarity to the model I used earlier in understanding language adoption. Small numbers of adopters experience suppressing fluctuations as $1/\sqrt{N}$

Expanding on the scale, the results of this algorithm are shown in the figure below.

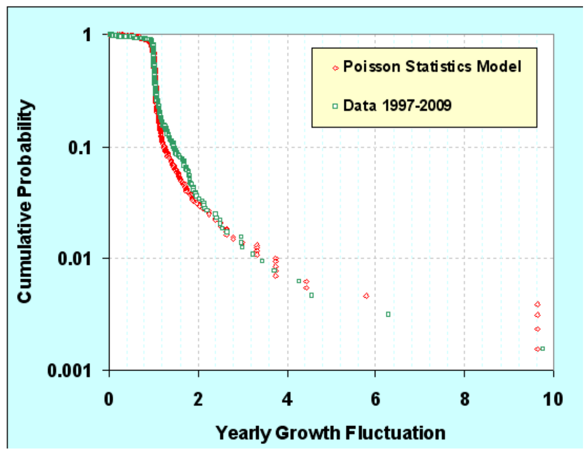
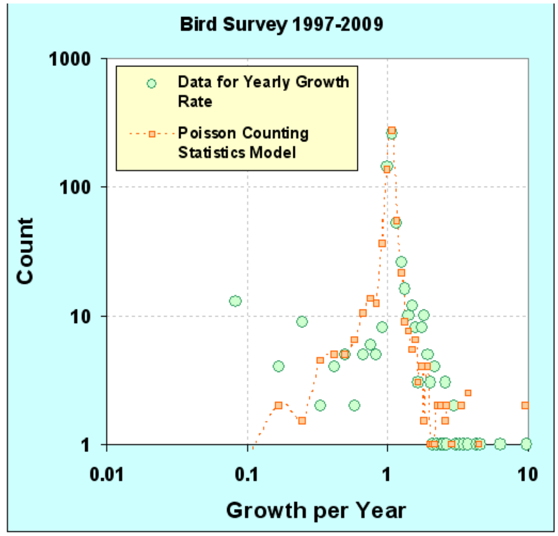


FIGURE 23-26. Model of yearly growth fluctuation in bird species abundance in terms of a cumulative distribution function. A value of unity indicates no change year-to-year.

Placing it in terms of a binned probability density function, the results look like the following plot. Note the high counts match closely the data simply because the $1/\sqrt{N}$ is relatively small. Away from these points, you can see the general trend develop even though the data is (understandably) obscured by the same counting noise.

FIGURE 23-27.
The probability density function of yearly growth fluctuations. Most species show little fluctuation but the rarer species show Poisson counting statistics noise in agreement with drawing from a bootstrapped sample.



As an essential argument to take home, consider that a counting statistics argument probably accounts for the yearly growth fluctuations observed. Before you make any other assertions, you likely have to remove this source of noise. Looking at the scatter plots, you can potentially see a slight bias toward positive growth for certain lower abundance species. This comes at the expense of lower decline elsewhere, except for some strong declines in several other low abundance species. This may indicate the natural ebb and flow of attrition and recovery in species populations, with some of these undergoing strong declines. I haven't done this but it makes sense to identify the species or sets of species associated with these fluctuations.

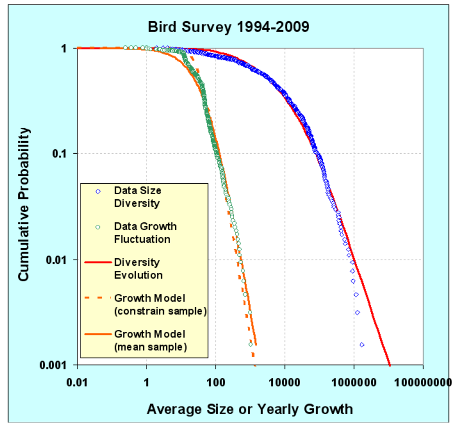
Two puzzling points also stick out. For one, I don't understand why Schwarzkopf and Farmer didn't immediately discern this effect. Their underlying rationale may have some of the same elements but it gets obscured by their complicated explanation. They do use a resampling technique (on 40+ years worth of data) but I didn't see much of a reference to conventional counting statistics, only the usual hand-wavy Levy flight arguments. They did find a power law of around-0.3 instead of the -0.5 we used for Poisson, so they may generate something equivalent to Poisson by drawing from a similar Levy distribution. Overall I find this violates Occam's razor, at least for this set of bird data.

Secondly, as the researchers point out, it seems that these differential growth curves have real significance in financial applications. More of the automated transactions look for short duration movements and I would think that ignoring counting statistics could lead the computers astray.

Mixed Bird Survey Data

As an aside, when I first pulled the data off the USGS server, I didn't look closely at the data sets. It turns out that the years 1994,1995,1996 were included in the data but appeared to have much poorer sampling statistics. From 1994 to 1996, the samples got progressively larger but I didn't realize this when I first collected and processed the data. Note the strange hitch in the data growth fluctuation curve below.

FIGURE 23-28. CDF of larger bird survay data sample. The yearly fluctuations appear larger.



At the time, I figured that the slope had a simple explanation related to uncertainties in the surveying practice.

Say the survey delta time has a probability distribution with average time — the T most likely related to the time between surveys:

$$p_t(time) = (1/T)e^{-time/T} \qquad \textbf{(EQ 23-16)}$$

then we also assume that a surveyor tries to collect a certain amount of data, ***x***, during the duration of the survey. We could characterize this as a mean, ***X***, or some uniform interval. We don't have any knowledge of higher order moments to we just apply the Maximum Entropy Principle

$$p_x(x) = (1/X)e^{-x/X} \qquad \textbf{(EQ 23-17)}$$

The ratio between these two establishes the relative rate of growth, ***rate*** = ***X/T***. We can derive the following cumulative quite easily:

$$P(rate) = \frac{T \cdot rate}{T \cdot rate + X} \qquad \textbf{(EQ 23-18)}$$

The yearly growth rate fluctuations of course turn out as the second derivative of this function. We take one derivative to convert:

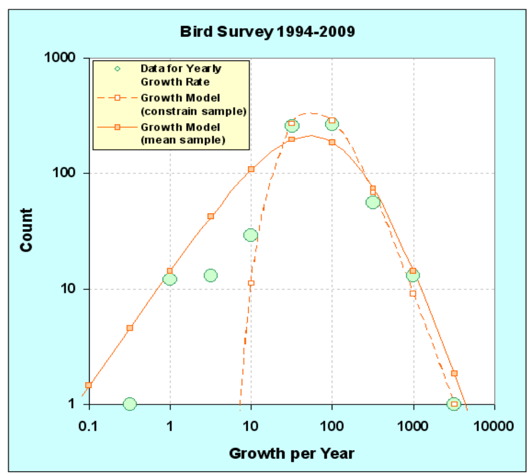
$$\frac{d}{drate}p(rate) = 2 \cdot \frac{T}{X \cdot (1 + rate \cdot T/X)^3} \quad \textbf{(EQ 23-19)}$$

On a cumulative plot as in Figure 23-28 on page 444, this shows a power-law of order 2 (see the orange curve). Near the knee of the curve, it looks a bit sharper. If we use a uniform distribution of $\boldsymbol{p_x(x)}$ up to some maximum sample interval, then it matches the knee better (see the dashed curve).

So the simple theory says that much of the observed yearly fluctuation may arise simply due to sampling variations during the surveying interval. Plotting as a binned probability density function, the contrast shows up more clearly in the figure below. In both cases we fit to $\boldsymbol{X/T} = 60$. This number is bigger than unity because it looks like every year the number of samples increases (I also did not divide by 15, the number of years in the survey).

But of course, the reason this maximum entropy model works as well as it does came about from real variation in the sampling techniques. Those years from 1994 to 1996 placed enough uncertainty and thus variance in the growth rates to completely smear the yearly growth fluctuation distribution.

FIGURE 23-29.
PDF of larger sample which had sampling variations. Note that this has a much higher width than the previous PDF. This was caused by changes in sampling procedure after the first few years.



Only in retrospect when I was trying to rationalize why a sampling variation this large would occur in a seemingly standardized yearly survey, did I find the real source of this variation. Clearly, the use of the Maximum Entropy Principle explains a lot, but you still may have to dig out the sources of the uncertainty.

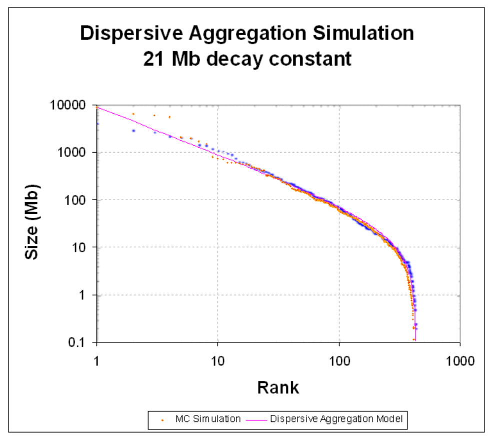
Can we understand the statistics of something as straightforward as a bird survey? Probably, but as you can see, we have to go at it from a different angle than that typically recommended and take care to consider only uniformly expressed data.

I especially harp on the math model because the math alone shows significant similarity to what happens when we consider the sizing and abundance of oil reservoirs. That serves as a complementary case.

An analogous behavior: the case of Oil Reservoir Size Distributions

I use essentially the same entropic dispersion formulation to describe the variation of reservoir sizes in the context of oil exploration. Instead of searching for living organisms and ranking the relative abundance, we sample geological formations and rank order the sizes of reservoirs we find. I have gone through the details of this approach in Volume 1 so won't repeat the details here. The fact that a dispersion form works just as well for oil as it does for species has to do with the disordered range in rates that go into reservoir formation. The figure below shows the agreement for entropic dispersion for North Sea reservoir data.

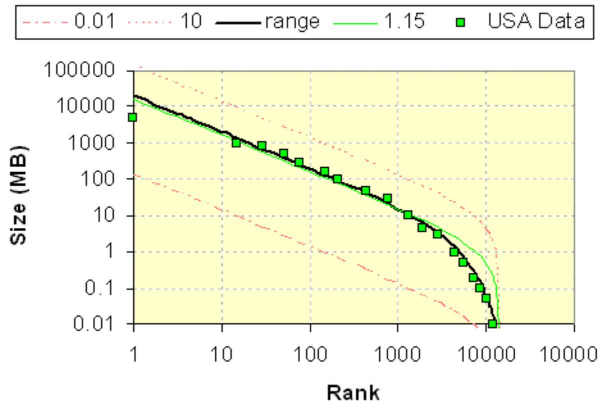
FIGURE 23-30.
Data of reservoir sizes for North Sea (blue dots) plotted alongside dispersive model.



I find it intriguing the similarity between the large population of a few species of living organisms and the large size of just a few oil reservoirs. The same MaxEnt math generates exactly the same fat-tail distributions.

Another interesting analogy in reservoir size distribution relates to how the dispersion factor varies significantly in different regions of the world. In particular, the USA has a dispersion factor that appears lower than elsewhere. Fitting the values for the USA, we see a value of around 1, whereas the North Sea has a value of 21 (see Figure 23-12 on page 423).[21]

FIGURE 23-31. Dispersion of reservoir sizes for USA. Time aggregation of entropic dispersion does not affect the tails of the distribution, only the shape of the knee, making it less sharp.



I use the same interpretation here as I use in species adaptation. If I assume that the isolated North Sea region "evolved" from a point in time long ago, then the single entroplet fits the data well. However, the entire USA shows a much more heterogeneous nature, partly due to its geographic area, and we can use a maximum entropy estimator to uniformly spread the entroplet functions over a range in start times (i.e. the MEP estimator for a fixed range is a uniform density). So if we apply this to the USA, the fit becomes better and the dispersion factor increases to better match that observed in other parts of the world (usually between 10 and 30). In other words, the North Sea acts as a localized community and the USA provides a metacommunity in the analogous ecosystem sense. The solid black line overlays the data points; we could interpret the range as occurring between 500 million years ago and 0.5 million years ago, with anything shorter than this time spilling beyond the **MaxRank** of 14,000 fields (from field data of 1986).

The flattening of the PDF that occurs in the Amazonian case for tree diversity also occurs in the USA for reservoir size diversity. The uniform time-shifted aggregation of the entroplets does not affect the tails but makes the mid-range of sizes more equally abundant. As a bit of insight, this likely accounts for the greater percentage of low volume stripper wells in the USA.

21. This could result from simply a definition discrepancy; the USA counts reservoir while other areas count fields.

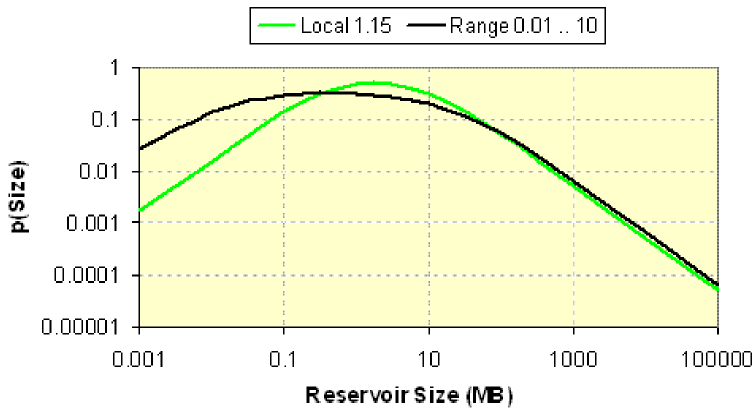


FIGURE 23-32. PDF of an reservoir sizing entroplet, and the time-shifted aggregation of a range.

The knee in Figure 10 becomes a flatter top, but the fat-tails remain unaffected.

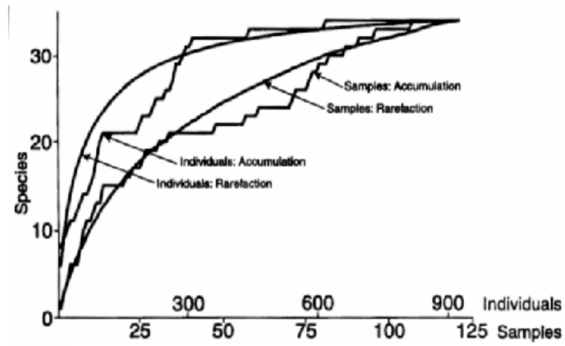
As one premise, the USA could have expended more effort in finding small reservoirs than other oil-producing countries, which may contribute to a lower range of values for ***D***. The geological rate of reservoir formation also could have progressed on average much more slowly here than elsewhere. Or, more likely according to this model, it has started from a more recent geological epoch, so it hasn't matured as much as the rest of the world. Whether this has consequences, or remains an inconsequential data point will have to wait until we get better data.

In a sense, the diversity of reservoir sizes around the world has effectively reached that of a set of biological species; the origin of oil formation in geological terms occurred long ago, but geological activity has likely allowed the movement of oil to effectively “restart” many times over epochs. Since I showed that the greater diversity of USA oil follows the pattern of Amazonian tree species, we should also find a more uniform density of oil reservoir sizes around the knee of the curve across the global spectrum.

However, unlike the resilience of a diverse biological population, no rebound effect occurs for oil depletion. Once we deplete oil, it doesn't come back. The size of the reservoir doesn't matter. Other smaller size reservoirs can take up the slack, but unlike the diversity of living organisms, dead organisms do not recover.

Many other analogs exist between species search and reservoir discovery. For example, shown below is the species equivalent of a creaming curve (from Species Abundance Patterns). The rate at which we find oil reservoirs has a close analogy to the rate at which we find species. In terms of the reserve growth issue, this has the same uncertainty in knowing when you have reached an asymptote.

FIGURE 23-33. Species equivalent of an oil creaming curve. As a region is sampled for species it reaches an asymptotic value similar to the creaming of new reserve additions.



The third case: Distribution of Human Travel

A final analogy has a purely human element. Human travel and mobility patterns in the USA show the same dispersion formulation with excellent agreement to recent data. We might imagine that human travel patterns would follow some complex behavior, yet if we simply assume that the delta X position changes and the delta T time changes each follow maximum entropy probability density functions, then the travel patterns reduce to a simple dispersive result (shown in Figure 23-7 on page 416).

The green curve generates a single parameter fit to measured cell-phone usage data. The expression shown in the inset is a heuristic developed by the authors of the original *Nature* article in which they describe a “magic” exponent, beta. That fits equally well but has no basis in fundamental understanding. The beta term shown happens to come close (1.75±0.15) with the entropic dispersion exponent of 2. Unfortunately, the academic discussions surrounding human mobility appear even more sophisticated than that for eco-system diversity and the simple understanding gets lost.

As an interesting experiment, the dispersive function can be generated via a Monte Carlo (MC) simulation drawing from two MaxEnt variants, delta A and delta T, and then dividing the two, resulting in a set of sampled rates. One such MC run looks like Figure 21-4 on page 383, with the analytical result overlaid. In the real world, the effect of reduced sample space generates the noise observed.

The resilience to human travel patterns in the face of dwindling oil supplies will become important in the future. In terms of the dispersive model, the value of D will likely shift to smaller values without changing the nature of the curve. We will still live in an entropic world, but the energy that allows us to move around easily will inhibit our resiliency.

The simple theory defined here certainly does not qualify as blind curve fitting. Although abstractly defined, the assumptions follow from scientifically valid premises. As Jaynes suggests, you use maximum entropy arguments when you face any degree of uncertainty in your numbers. If you have better numbers, you can use those. The Maximum Entropy Principle has a close relative in Bayes Rule in this respect. As it stands, like Bayes, the model helps to manage our uncertainty by including valid prior information.

Dispersion: The effects of disorder result in variation of adaptation rates as a form of dispersion. This has more to do with entropy than energy flow.

Diversity: The huge changes in relative abundance comes about from the dispersion. I do not consider this emergent complexity, instead it describes diversity predicated on disorder. One man's perceived complexity is another's effective simplicity.

Resilience: The significant diversity derived simply from disorder considerations leads to the possibility of resilience against potential collapse. Since species may not have as much interdependence as assumed, it seems intuitive that the diversity can act as a buffer against extinctions. If one species becomes extinct, another more slowly evolving species may take over.

Discussion. Using the same arguments as for species adaptation, we have arrived at similar results for human travel and reservoir sizing. The rather simple logical arguments should prove useful in any analytical context that proceeds under disordered, entropic conditions. In this regime you can't use deterministic models such as Lotka-Volterra, and you need instead to consider probabilities for all your measures. Jaynes had it right when he titled his final book as “*Probability Theory: The Logic of Science*”.

Besides Jaynes, the pioneers of fat-tail and fractal statistics have contributed some interesting insight, especially in pointing out that fat-tail statistics occur more often than common wisdom dictates. Taleb's admonition to not overuse Gaussian/normal statistics becomes very important when working under maximum uncertainty. As an example, the fact that we know the variance of a process, would suggest that we use a normal distribution, yet we have no knowledge of the standard deviation of any of our data sets. We barely have knowledge of the mean as it stands.

Gell-Mann has noted that Mandelbrot could likely have derived the dispersive model as it has a remarkable similarity to the discrete power-law Zipf-Mandelbrot heuristic. Mouillot [Ref 228] mentions Zipf-Mandelbrot in an abstract with reference to RAD. Yet Mandelbrot, and Taleb for that matter, appear to show some disdain for actually deriving any of these distributions, preferring instead to describe fractal or fat-tail behavior as heuristic functions. Although they have very good insight regarding uncertainty and randomness, they seem to prefer the world of descriptive rather than proscriptive statistics. Many of the diversity research efforts appear to think that modelling only involves heuristic fits of previously categorized statistical distributions to the data [Ref 224]. Only the authors of the Nature article have a model (the Neutral theory [Ref 220]) to base predictions on, yet that model will likely prove too intricate to get burned into our consciousness.[22]

22. Why no one else has previously formulated such a simple model as maximum entropy dispersion would require a separate discussion. It may exist somewhere in the literature but I have yet to find it. I realize that my arguments have some abstract concepts, such as equating uncertainty with entropy, that may take some getting use to. Suffice to say, Taleb may have captured the line: “We scorn the abstract, we scorn it with a passion.” from The Black Swan. As any software developer understands, the right abstraction often helps to clean up complexity. It just takes the right insight to get to that abstraction. The model also relates and has similarities to the Principle of Least Effect, which has both physics and information science origins.

CHAPTER 24

Reliability.

Entropy and how things break down

"There is nothing so practical as a good theory"

—Kurt Lewin, psychologist.

A consumption-oriented society ultimately depends on the eventual wear-out of goods we purchase. This obviously keeps people working by supplying us with replacement items. But this steady production also places demands on our resource base.If we had more durable and dependable products, we could conceivably conserve our energy and material resources, but at the expense of productive economic growth. These remain as trade-offs we will need to consider. Many of the creative interpretations surrounding reliability use ideas of entropy and disorder to understand when something will fail. Predicting the appearance of new oil reservoirs has much in common with predicting when something will fail; dispersion in the characteristic behaviors has much to do with the eventual outcomes.

Predictably Unreliable

During the summer of 2010, we have watched the unexpected and unpredicted blow-out of the Deepwater Horizon oil well (the ultra-rare 1 out of 30,000 failure according to conventional wisdom) and waited for the successful deployment of relief wells.

We also have the unnerving situation of knowing that something will eventually fail, but with uncertain knowledge of exactly when. Take the unpredictability of *popcorn popping* as a trivial example. We can never predict the time of any particular kernel but we know the vast majority will pop[1].

In a recent episode that I went through, the specific failure also did not come as a surprise. I had an inkling that an Internet radio that I frequently use would eventually stop working. From everything I had read on-line, the specific model had a power-supply flaw that would eventually reveal itself as a dead radio. Previous customers had reported the unit would go bad anywhere from immediately after purchase to a few years later. After about 3 years it finally happened to my radio and the failure mode turned out exactly the same as everyone else's — a blown electrolytic capacitor and a possible burned out diode.

The part obviously blew out because of some heat stress and power dissipation problem, yet like the popcorn popping, my interest lies in the wide range in failure times.

The radio failure in fact looks like the classic Markov process of a constant failure rate per unit time. In a Markov failure process, the number of expected defects reported per day equate proportionally to how many units remain operational. This turns into a flat line when graphed as failure rate versus time. Customers that have purchased the model will continue to routinely report the failures for the next few years, with fewer and fewer reports as that model becomes obsolete.

Because of the randomness of the failure time, we know that any failures should follow some stochastic principle and likely that entropic effects play into the behavior as well.

When the component goes bad, the unit's particular physical state and the state of the environment governs the actual process; engineers call this the *physics of failure*. Yet, however specific the failure circumstance, the variability in the component's parameter space ultimately sets the variability in the failure time.

So I see another way to look at failure modes. We can either interpret the randomness from the perspective of the component or from the perspective of the user. If the latter, we might expect that someone would abuse the machine more than another customer, and therefore effectively speed up its failure rate. Except for some occasional power cycling this likely didn't happen with my radio as the clock stays powered in standby most of the time. Further, many people will treat their

1. Wind is likewise unpredictable, yet we know that it will work at least part of the time without knowing precisely when, while we can only guess when a catastrophe with such safety-critical implications will occur.

machine gingerly. So we have a spread in both dimensions of component and environment.

If we look at the randomness from a component quality-control perspective, certainly manufacturing variations and manual assembly plays a role. Upon internal inspection, I noticed the radio needed lots of manual labor to construct. Someone posting to an online forum noticed a manually extended lead connected to a diode on their unit — not good from a reliability perspective.

This provides some background to my view of thinking about failures, a view which doesn't always match the conventional wisdom in reliability circles. In certain cases the result derives as expected, but in other cases the result diverges from the textbook solution.

Fixed wear rate, variable critical point: To model this to first-order, we assume a critical-point (***cp***) in the component that fails and then assume a distribution of the ***cp*** value about a mean. Maximum entropy would say that this distribution would approximate an exponential:

$$p(x) = \frac{1}{cp} \cdot e^{-x/cp} \quad \text{(EQ 24-1)}$$

The rate at which we approach the variable ***cp*** remains constant at ***R*** (everyone uses/abuses it at the same rate). Then the cumulative probability of failure is

$$P(t) = \int_0^{Rt} p(x)dx \quad \text{(EQ 24-2)}$$

This invokes the monotonic nature of failures by capturing all the points on the shortest critical path, and anything "longer" than the ***R*t*** threshold won't get counted until it fails later on. The solution to this integral becomes the expected rising damped exponential.

$$P(t) = 1 - e^{-\frac{Rt}{cp}} \quad \text{(EQ 24-3)}$$

Most people will substitute a value of τ for ***cp***/***R*** to make it look like a lifetime. This is the generally accepted form for the expected lifetime of a component to first-order.

$$P(t) = 1 - e^{-t/\tau} \quad \text{(EQ 24-4)}$$

So even though it looks as if we have a distribution of lifetimes, in this situation we actually have as a foundation, a distribution in critical points. In other words, I get the correct result but I approach it from a non-conventional angle.

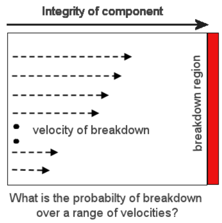


Figure described later.

Fixed critical point, variable rate: Now turn this case on its head and say that we have a fixed critical point and we have a maximum entropy variation in rate assuming some mean value, ***R***.

$$p(R) = \frac{1}{R} \cdot e^{-r/R} \quad \textbf{(EQ 24-5)}$$

Then the cumulative integral looks like

$$P(t) = \int_{\mathrm{cp}/t}^{\infty} p(r)dr : \quad \textbf{(EQ 24-6)}$$

Note carefully that the critical path in this case captures only the fastest rates and anything slower than the ***cp*/*t*** threshold won't get counted until later. The result derives to

$$P(t) = e^{-cp/Rt} \quad \textbf{(EQ 24-7)}$$

This has the characteristics of a fat-tail distribution because time goes into the denominator of the exponent, instead of the numerator. Physically, this means that we have very few instantaneously fast rates and many rates proceed slower than the mean.

Variable wear rate, variable critical point: In a sense, the two preceding behaviors act complementary to each other. So we can also derive ***P*(*t*)** for the situation whereby *both the rate and critical point vary.*

$$P(t) = \int_{0}^{\infty} P(t|r) \cdot p(r)dr \quad \textbf{(EQ 24-8)}$$

This results in the exponential-free cumulative, which has the form of an entroplet.

$$P(t) = \frac{\frac{Rt}{cp}}{1 + \frac{Rt}{cp}} = \frac{\frac{t}{\tau}}{1 + \frac{t}{\tau}} \quad \textbf{(EQ 24-9)}$$

Plotting the three variations side-by-side and assuming that $\tau = 1$, we get the following set of cumulative failure distributions. The full variant nestles in between the two other exponential variants, so it retains the character of more early failures (ala the *bathtub curve*) yet it also shows a fat-tail so that failure-free operation can extend for longer periods of time.

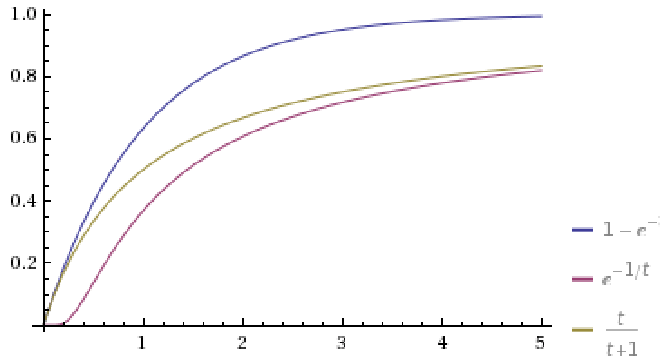


FIGURE 24-1. Three variants of the cumulative failure distribution.

To understand what happens at a more intuitive level we define the fractional failure rate (or hazard function) as

$$F(t) = \frac{\frac{dP}{dt}}{1 - P(t)} \qquad \textbf{(EQ 24-10)}$$

Analysts use this form since it makes it more amenable to predicting failures on populations of parts. The rate then applies only to how many remain in the population, and the ones that have failed drop out of the count.

Only the first case above gives a failure rate that approaches the Markov ideal of constant rate over time. The other two dip below the constant rate of the Markov simply because the fat-tail cumulative requires a finite integrability over the time scale, and so the rates will necessarily stay lower.

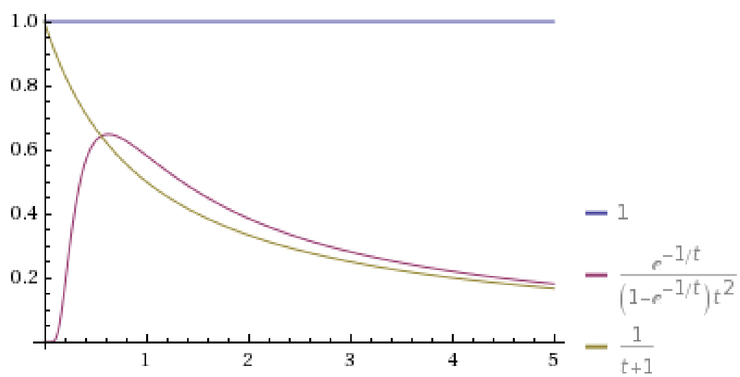


FIGURE 24-2. Three variants for the fractional failure rate (hazard function). The flat line corresponds to the classical Markov of a constant failure rate over time.

Shortly we will provide a full account of what happens when we generalize the first-order linear growth on the rate term, letting $\boldsymbol{R}=\boldsymbol{g}(\boldsymbol{t})$. The full variant ultimately gives $d\boldsymbol{g}/d\boldsymbol{t}\ /(1+\boldsymbol{g}(\boldsymbol{t}))$, so that if $\boldsymbol{g}(\boldsymbol{t})$ starts rising we get the complete bathtub curve.

If we don't invoke other time dependencies on the rate function $\boldsymbol{g}(\boldsymbol{t})$, we see how certain systems never show failures after an initial period. Think about it for a moment — the fat-tails of the variable rate cases push the effective threshold for failure further and further into the future.

In effect, normalizing the failures in this way explains why some components have predictable unreliability, while other components can settle down and seemingly last forever after the initial transient. Pandey agrees with the way I think about the general problem [Ref 242].

Enjoy your popcorn, it should have popped by now. You will find out why in the next section.

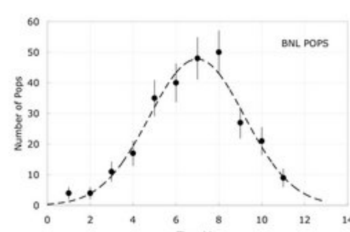
Popcorn Popping as a Dispersive Process

In the search for the perfect analogy to oil depletion that might exist in our experiential world, I came across the most mundane yet practical example that I have encountered so far — while microwaving some popcorn. It struck me that the dynamics of the popping in some sense captured the idea of simulated annealing, as well as reproducing the envelope of peak oil itself.

The fundamental question is: ***Why don't all the kernels pop at the same time?***

It took me awhile to lay out the groundwork, but I eventually came out with a workable model, the complexity of which mainly involved reaction kinetics. Unsurprisingly, the probability and statistics cranked out straightforwardly as it parallels the notion of dispersion that I have worked out in terms of the Dispersive Discovery Model in Volume 1. First, we define the basic premise with the aid of this figure:[2]

FIGURE 24-3. Popcorn popping kinetics. Each bin has a width of 10 seconds, and the first kernel popped at 96 seconds. So the overall width is quite large in comparison to the first pop time. Graph taken from Statistics with Popcorn [Ref 237]. The curve itself looks similar to a oil discovery peak.



In obvious ways, the envelope makes sense as experience and intuition tells us that a maximum popping rate exists which corresponds to the period of the loudest and densest popping noise. Certainly, if you stuck a thermometer in the popcorn medium you would find the average temperature rising fairly uniformly until it reaches some critical temperature.

Naively, you could them imagine everything popping at once or within a few seconds of one another — after all, water in a kettle seems to boil quite suddenly after it reaches a certain threshold. But from the figure above, the spread seems fairly wide for popcorn. The key to the range of popping times lies in the dispersive characteristics of the popcorn. This dispersion essentially shows up because of the non-uniformity among the individual kernels. Intuitively this may get reflected as variations in the activation barrier of the kernels or in the micro-variability in the medium.

Some may suggest that the temperature spread may occur simply because of the effects of the aggregation of the popcorn kernels interacting with each other as they pop. For example, one kernel popping may jostle the environment enough to effectively cool down the surrounding medium, thus delaying the effects of the next kernel. However, that remains a rather insignificant effect. I thought about doing the experiment myself until I ran across an impressively complete study executed by a team of food scientists. Measured painstakingly against temperature, the cereal scientists placed ***individual*** kernels in a uniformly heated pot of oil, and tabulated each kernel's popping time. They then plotted the cumulative times and determined the rate constants, trying to make sense of the behavior themselves (curiously, this experiment was only completed for the first time about 5 years ago).

> *"Kinetics of Popping of Popcorn", J. E. Byrd and M. J. Perona* [Ref 243]
> Anyone who has made popcorn knows that in a given sample of kernels, the kernels fortunately do not all pop at the same time. The kernels seemingly pop at random and exhibit a range of popping times (Roshdy et al 1984; Schwartzberg 1992; Shimoni et al 2001). The model described above does not explain this observation. ***It explains why popcorn pops, but has nothing to say about when a kernel will pop.*** The goal of this work was to use the methods of chemical kinetics to explain this observation. More specifically, we performed experiments in which the number of unpopped kernels in a sample was measured as a function of time at a constant bath temperature. This type of experiment has not been reported in the literature, and the data are amendable to the methods of chemical kinetics. In addition, we formulated a quantitative kinetic model for the popping of popcorn and have used it to interpret our results. The literature contains no kinetic model for the popping of popcorn.

2. High school and college science teachers like to use popcorn popping experiments as a way to introduce the procedures of statistical data collection and the scientific method.

If you read the article you note that the idea of a rate constant comes into play. In fact, you can think of the individual kernels obeying the laws of physics as they plot their own trajectory until they reach a critical internal pressure and ultimately pop. In other words, they pop at a rate that does not depend on their neighbors (since they have no neighbors in the experiment). I suggest that two mechanisms come into play with respect to the internal dynamics of the popcorn kernel. First, we have the mechanism of the starchy internal kernel which heats up at a certain rate and starts to build up in pressure over time. This happens at an average rate but with an unknown variance; let us say that has a maximum entropy such that the standard variance equals the mean; in other words the maximum entropy principle applies if we only know the mean.

Second, we have the average rate itself accelerating over time, so that the pressure both builds up and breaks down the underlying medium at a faster and faster pace. Finally, we have variations in the kernel shell (i.e. the thickness of the pericarp layer) which acts as an activation barrier, thus defining the effective exit criteria for the kernel to pop. However, this latter mechanism does not exhibit as large of a variance, as that would trigger more immediate popping and not as explosive an effect.[3] The overall process has analogies to the field of study known as *physics of failure* elaborated in the previous section; each popcorn kernel eventually fails to maintain its rigidity as it pops, and just like real failure data, this is dispersed over time. Physics of failure relies on understanding the physical processes of stress, strength and failure at a very detailed level, and I apply this at a level appropriate for understanding popcorn.

Given this premise, the math works out exactly to the approach formulated for Dispersive Discovery. I use the generalized version which applies the Laplace transform technique to generate a cumulative envelope of the fraction of unpopped popcorn over time=**t** and oil temperature=***T***.

$$P(t, T) = 1 - \frac{e^{-B/f(t, T)}}{1 + \frac{A}{f(t, T)}} \qquad \text{(EQ 24-11)}$$

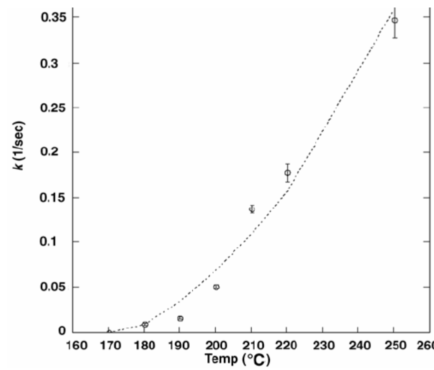
The term ***f*(*t*,*T*)** represents the mean value of the accelerating function, and the terms ***A*** and ***B*** reflect the amount of dispersion in the shell characteristics; if ***B*=1** and ***A*=0** the shell has a fixed breakthrough point and if ***B*=0** and ***A*=1** the shell has an exponentially damped breakthrough point (i.e. lots of weaker kernels). The latter set defines the complement of the logistic sigmoid, if ***f*(*t*,*T*)** accelerates exponentially.

3. A fixed exit criteria is like the finish line in a race. A random exit criteria is where the finish line varies.

$$f(t, T) = e^{R(T, t)} - e^{R(T, 0)}$$
$$R(t, T) = k(T - T_c)^2 \cdot t - c(T - T_c) \qquad \textbf{(EQ 24-12)}$$

The basic physics of failure is encompassed in the exponential term ***f*(*t*,*T*)** which states that the likelihood of failing increases exponentially over time, as the internal structure of the popcorn starts to compound its failure mechanisms. At $\boldsymbol{T = T_c}$ and below that temperature nothing ever pops. At higher temperatures than the critical temperature, the rate correspondingly increases according to a power law (see below).

FIGURE 24-4.
Rate constant for popcorn popping as a function of temperature. This obeys the Arrhenius Law. (From [Ref 243])



Temperature as related by the parameter ***k*** is the dial of the accelerator. The parameter ***c*** acts as the delay time for how long it takes the kernel to reach the equilibrium temperature of the oil. The set of data from the Byrd and Perona experiment is shown below, along with the temperature dependent dispersive model shown as the colored lines for ***A*** = 0.5, ***B*** = 0.5, ***c*** = 0.1/degree, $\boldsymbol{T_c}$ =170 degrees, and ***k*** = 0.00008/degree^2/second. The solid lines black lines are the fit to their model which essentially adjusts each parameter for each temperature set. I left the model minimally parametric over the temperature range and adjusted only the oil temperature for each curve — in other words, they all share the base parameters.

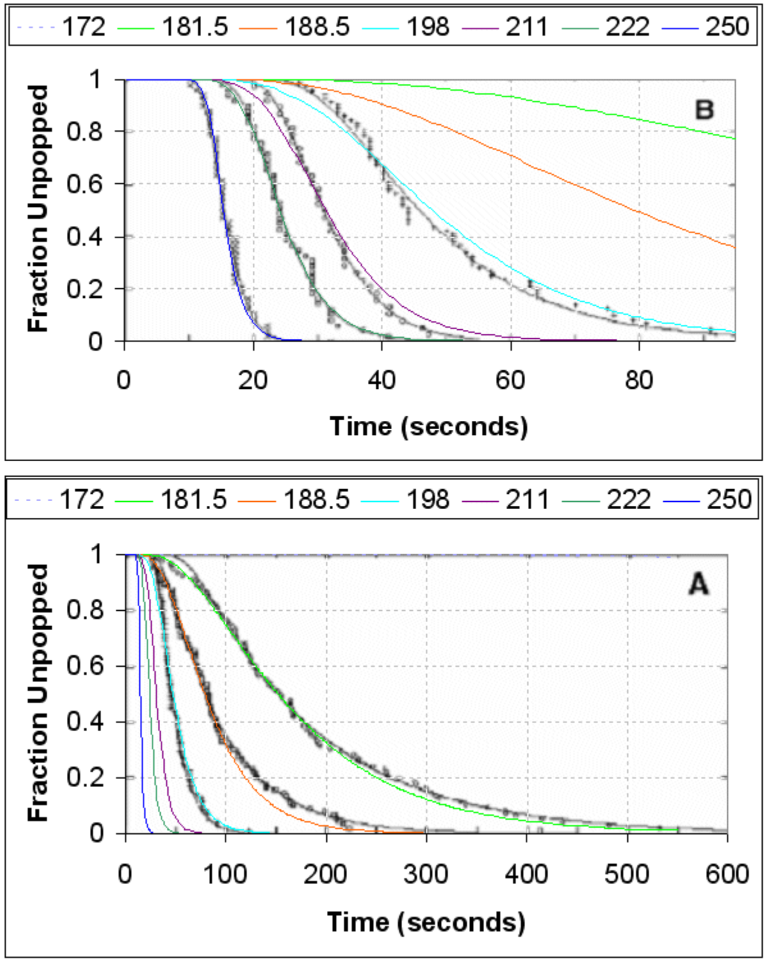
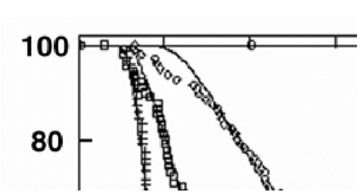


FIGURE 24-5. Measurements of the fraction of unpopped popcorn remaining as a function of time over a range in oil cooking temperatures (in degrees C). Two timescales are shown. These can be compared to the complementary sigmoid functions.

The authors of the study don't use the same formulation as I do because the theorists don't tend to apply the fat-tail dispersion math that I do. Therefore they resort to a first-order approximation which uses a Gaussian envelope to generate some randomness. They essentially do the equivalent of setting the ***B*** term to 1 and the ***A*** term to 0. This really shows up in the better fit at low temperatures at early popping times — see the green curve at ***T***=181.5 degrees in Figure 24-5 on page 462 which tends to flatten out at earlier times than their model.



Overall the fit works extremely well over the range of curves, as I contend that any deviations occur because of the limited sample size of the experiments. It gets awfully tedious to measure individual popping times and the fluctuations from the curve will surely arise. You can see this in a magnification of the low temperature data set shown in the inset figure. If you notice some of the 200 degree data points pop later than the 190 degree data points. These occur solely due to statistical probability artifacts due to the small sample size (between 50 and 100 kernels per experiment at the temperature measured). If they had at least 500 measurements per data set the fluctuations would have decreased by at least a factor of two. You can also observe that the statistical fluctuations show up very well if plotted as a frequency histogram in the figure below. Compare this set against Figure 24-3 on page 458, which smooths out the curve via larger bins.

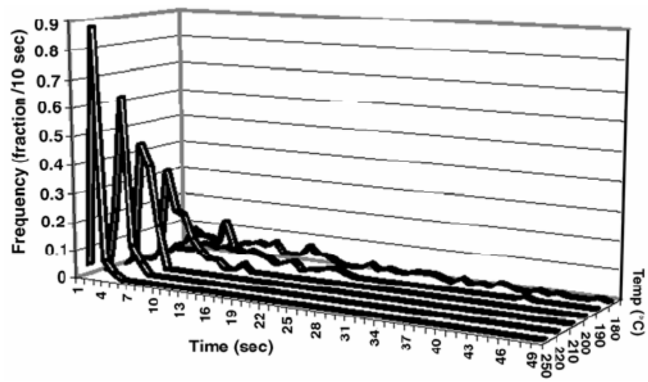


FIGURE 24-6. The set of "Hubbert Curves" for popping popcorn at several temperatures.

Each one of the curves gets transformed into something approaching a logistic curve as the temperature accelerates in temperature and then keeps rising. You can think of the individual pops as those kernels meeting the criteria for activation. The laggards include those kernels that haven't caught up, either intrinsically or because of the non-uniformity of the medium (as a counter-example, water is uniform and it mixes well as it boils).

Alternate title: *Solving the slippery nature of the bathtub curve.*

Failure is the Complement of Success

As I started looking at dispersion to explain the process of oil discovery, it seemed likely that it would eventually lead to the field of reliability. You see, for every success you have a failure. We don't actively seek a failure, but they lie ready to spring forth at some random unforeseen time. In other words, we can never predict when a failure occurs; just as when we look for something at random — like an oil reservoir, we will never absolutely know when we will find it.

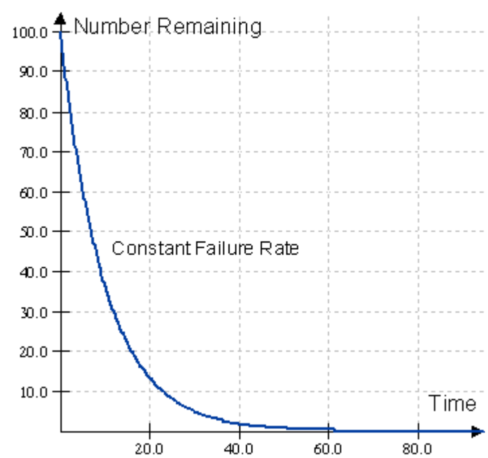
So the same dispersion in search rates leading to a successful oil find also leads to the occurrence — in that same parallel upside-down universe — of a failure. As we saw in the last section, what does the seemingly random popping of a popcorn kernel indicate but a failure to maintain its hard shell robustness? And by the same line of reasoning, what is a random discovery but a failure by nature to conceal its inner secrets from an intrepid prospector?

The Classic Failure Premise: The classic approximation for a random failure involves a single parameter, the failure rate ***r***. This gets derived at least empirically from the observation that if you have a pile ***N*** of working components, then the observed failure rate goes as:

$$\frac{d}{dt}N(t) = -r \cdot N(t) \qquad \textbf{(EQ 24-13)}$$

so the rate of loss relative to the number operational remains a constant throughout the aggregated lifetime of the parts. The solution to the differential equation is the classic damped exponential shown below:

FIGURE 24-7. The classical failure rate gives a damped exponential over time.



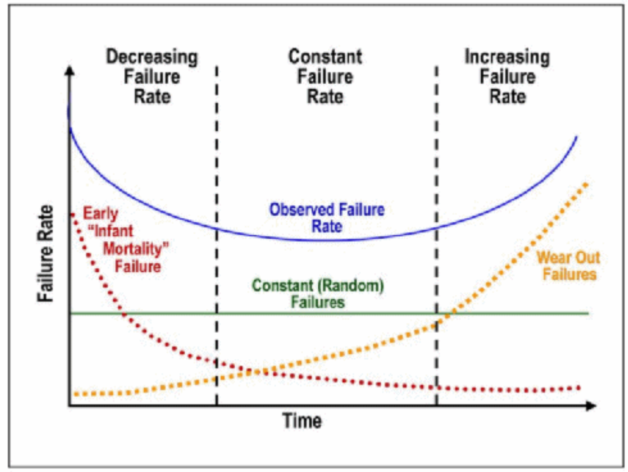
Now this works very effectively as a first-order approximation and you can do all sorts of reliability studies with this approximation. For example it matches the conditions of a Markov process, and the fact that it lacks memory means that one can solve large sets of coupled equations[4].

Deviation from the Classical Premise: However, in reality the classic approximation doesn't always hold. As often observed, the failure rate of a component does

4. This has application in the oil shock model

not remain constant if measured empirically over a population. Instead the shape over time ends up looking something like a bathtub.

FIGURE 24-8. The Bathtub Curve showing frequent early failures and late failures.



One can see three different regimes over the life-cycle of a component. The component can either fail early on as a so-called "infant mortality", or it can fail later on randomly (as a lower probability event), or eventually as a process of wear-out. Together, the three regimes when pieced together form the shape of a bathtub curve. Curiously, a comprehensive theory for this aggregated behavior does not exist (some even claim that a unified theory is impossible) and the recommended practice suggests one create an analysis bathtub curve in precisely a piece-wise fashion. Then the analyst can predict how many spares one would need or how much money to spend on replacements for the product's lifecycle.

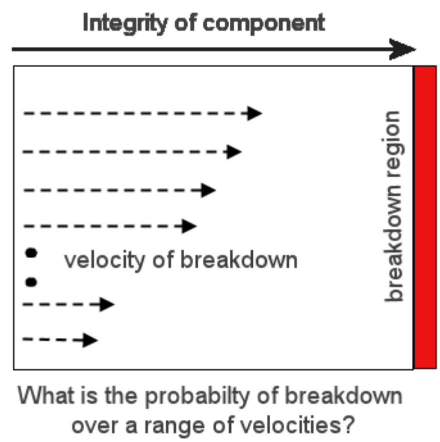
Although one can get by with that kind of heuristic, one would think that someone has unified a concept that *doesn't* require a piece-wise approximation. As it turns out, no one has really solved the problem of deriving the bathtub curve simply because they haven't set up the correct premise with a corresponding set of assumptions.

The Dispersive Failure Premise: Instead of going directly to the classic failure rate approximation, let's break the failure mechanism down into a pair of abstractions. First, recall the classic description of the irresistible force meeting the immovable object. Let's presume the battle between the two describes the life-cycle of a component. In such a situation we have to contend with modeling the combination of the two effects, as eventually the irresistible force of wear and tear wins out over the seemingly immovable object as its integrity eventually breaks down.

In other words, failure arises from a process governed by a time rate of change (of the irresistible force) which operates against a structure that maintains some sense of integrity of the component (the immovable object).

To set this up mathematically, consider the following figure. We scale the integrity of the component as a physical dimension; it could be a formally defined measure such as strain, but we leave it as an abstract length for the sake of argument. The process acting on this abstraction becomes a velocity; again this could be a real force, such as the real measure of stress[5]. Now, when something breaks down, the irresistible force has been applied for a certain length of time against the immovable object. The amount of time it takes to cover this distance is implicitly determined by the integral of the velocity over the time. However, due to the fact that real-life components are anything but homogeneous in both (1) their integrity and (2) the applied wear-and-tear, we have to apply probability distributions to their nominal values. Pictorially it looks like a range of velocities trying to reach the effective breakdown dimension over the course of time.

FIGURE 24-9. Abstraction for the time dependence of a failure occurrence. For a given sample, any one of a range of rates to breakdown can occur. This gets modelled as a probabilistic dispersion of rates.



Some of the trajectories will arrive sooner than others, some will arrive later, but a mean velocity will become apparent. This variation has an applicable model if we select an appropriate probability density function for the velocities, denoted by ***p(v)*** (and justified later for the integrity of the structure, a corresponding ***p(L)***). Then we can devise a formula to describe what fraction of the velocities have not reached the "breakdown" length. The probability of no breakdown as a function of time is the integral of $p(v)$ over time for those velocities not reaching the critical length, L.

5. See the next section

$$P(t) = \int_0^{L/t} p(v)dv \quad \text{(EQ 24-14)}$$

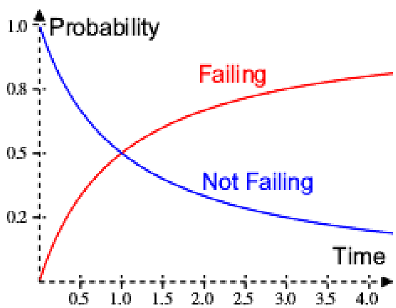
For the maximum entropy PDF of $p(v) = \alpha \cdot e^{-\alpha v}$ this mathematically works out as

$$P(t) = 1 - e^{-\alpha L/t} \quad \text{(EQ 24-15)}$$

for a set of constant velocities probabilistically varying in sample space. This becomes essentially a dispersion of rates that we can apply to the statistical analysis of failure. If we then apply a maximum entropy PDF to the set of ***L***'s to model randomness in the integrity of the structure

$$p(L) = \beta \cdot e^{-\beta L} \quad \text{(EQ 24-16)}$$

and integrate over ***L***, then we get



$$P(t) = 1 - \frac{1}{1 + \frac{\alpha}{\beta t}} \quad \text{(EQ 24-17)}$$

This has a hyperbolic envelope with time. The complement of the probability becomes the probability of failure over time. Note that the exponential distributions have disappeared from the original expression; this results from the alpha and beta densities effectively canceling each other out as the fractional term α/β. The α is a 1/*velocity* constant while the β is a 1/*length* constant so the effective constant is a breakdown time constant, $\tau = \alpha/\beta$.

$$P(t) = 1 - \frac{1}{1 + \frac{\tau}{t}} \quad \text{(EQ 24-18)}$$

The assumption with this curve is that the rate of the breakdown velocities remains constant over time. More generally, we replace the term ***t*** with a parametric growth term

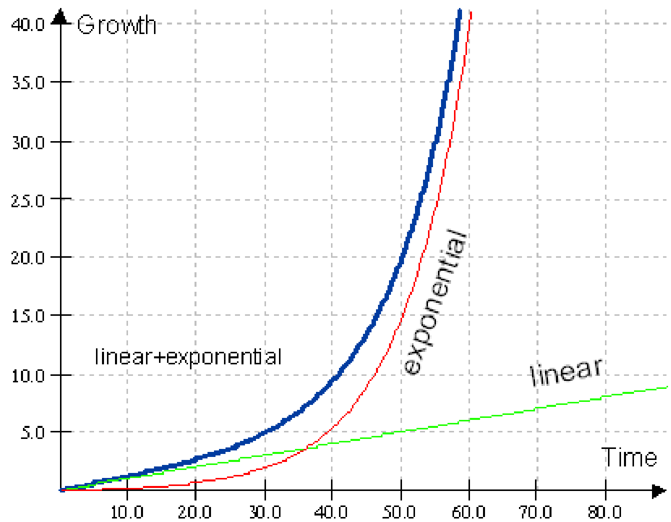
$$t \rightarrow g(t)$$

$$P(t) = 1 - \frac{1}{1 + \frac{\tau}{g(t)}} \quad \text{(EQ 24-19)}$$

If you think about the reality of a failure mode, we can conceivable suspend time and prevent the breakdown process from occurring just by adjusting the velocity frame. We can also speed up the process, via heating for example (as the popcorn example shows). Or we can imagine placing a working part in suspended animation, nothing can fail during this time so time essentially stands still. The two extreme modes roughly analogize to applying a fast forward or pause on a video.

A realistic growth term could look like the following figure. Initially, the growth proceeds linearly, as we want to pick up failures randomly due to the relentless pace of time. After a certain elapsed time we want to speed up the pace, either due to an accelerating breakdown due to temperature or some cascading internal effect due to wear-and-tear. The simplest approximation generates a linear term overcome by an exponential growth.

FIGURE 24-10. A linear into accelerating growth function written out as: g(t) = a*t + b*(e^{ct} -1).



This becomes a classic example of a parametric substitution, as we model the change of pace in time by a morphing growth function.

Now onto the bathtub curve. The failure rate is defined as the rate of change in cumulative probability of failure divided by the fraction of operational components left.

$$r(t) = -\frac{\frac{d}{dt}P(t)}{P(t)} \quad \text{(EQ 24-20)}$$

this results in the chain rule derivation

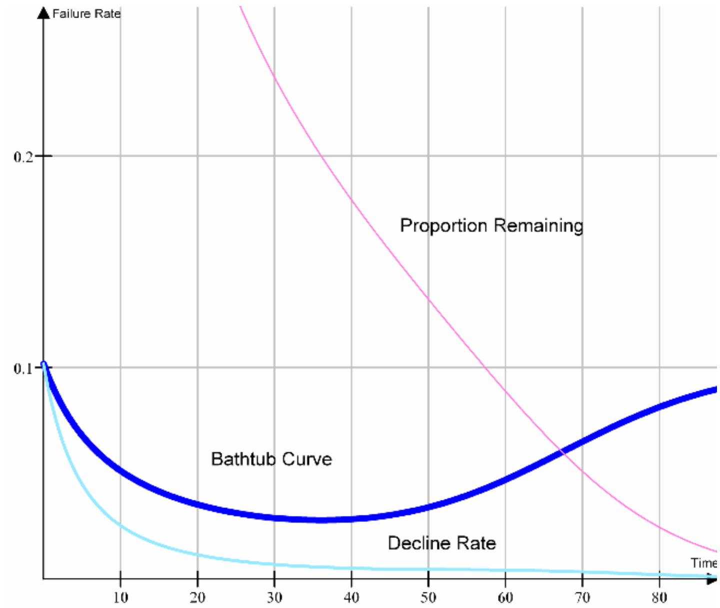
$$r(t) = \frac{\frac{d}{dt}g(t)}{\tau + g(t)} \quad \text{(EQ 24-21)}$$

for the **g(t)** shown above, this becomes

$$r(t) = \frac{a + b \cdot c \cdot e^{ct}}{\tau + a \cdot t + b \cdot (e^{ct} - 1)} \qquad \textbf{(EQ 24-22)}$$

which looks like the bathtub curve below for a specific set of parameters, ***a*=1**, ***b*=0.1**, ***c*=0.1**, τ **=10.0**.

FIGURE 24-11. Plotting the fractional failure rate or hazard function results in a bathtub-shaped curve.



The detailed shape will change for any other set but it will still maintain some sort of bathtub curvature. Now, one may suggest that we have too many adjustable parameters and with that many, we can fit any curve in the world. However, the terms ***a,b,c*** have a collective effect and simply describe the rate of change as the process speeds up due to some specific physical phenomena. For the popcorn popping example, this represents the accelerated heating and subsequent breakdown of the popcorn kernels starting at time ***t*=0**. The other term, τ , represents the characteristic stochastic breakdown time in a dispersive universe. For a failed (i.e. popped) popcorn kernel, this represents a roll-up of the dispersive variability in the internal process characteristics of the starch as it pressurizes and the dispersive variability of the integrity of the popcorn shell at breakdown (i.e. popping point). We use the maximum entropy principle to estimate these variances since we have no extra insight to the quantitative extent of this variance. As a bottom-line for the popcorn exercise, these parameters do exist and have a physical basis and so we can obtain a workable model for the statistical physics. I can assert a similar process occurs for any bathtub curve one may come across, as one can propose a minimal set of canonical parameters necessary to describe the transition point between the linear increase and accelerated increase in the breakdown process.

The keen observer may ask: whatever happened to the classical constant failure rate approximation as described in Equation 1? No problem, as this actually drops out of the dispersion formulation if we set $\boldsymbol{b}=\tau$ and $\boldsymbol{a}=\boldsymbol{0}$. This essentially says that the acceleration in the wear and tear process starts immediately and progresses as fast as the characteristic dispersion time τ. This is truly a zero-order approximation useful to describe the average breakdown process of a component.

So the question remains: why hasn't this rather obvious explanation become the accepted derivation for the bathtub curve? I can find no reference to this kind of explanation in the literature; if you read "*A Critical Look at the Bathtub Curve*" [Ref 202], from several years ago, you will find them throwing their hands up in the air in their attempt to understand the general bathtub-shaped profile.

Creep Failure

We can also look at a behavior that illustrates the early failure profile in a less abstract and perhaps more realistic manner. Due to dispersion, a linear rate of growth in the breakdown process will lead to an early spike in the failure (or hazard) rate. This becomes the characteristic leading downward slope in the bathtub curve.

The enhanced early probability of failure arises purely through the spread in the failures through disorder mechanisms just as in the popcorn popping experiment. This in general leads to a bathtub shape with an analogy to the life-span of the human body (infant mortality, maturity, old age). For mechanical equipment, the shape in (b) provides a more realistic portrayal according to some engineers.

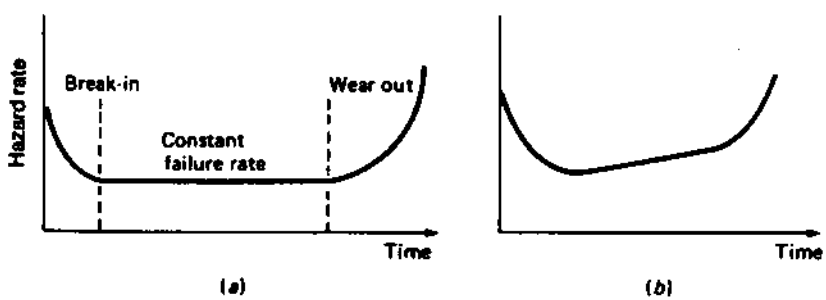


FIGURE 24-12. Empirically observed bathtub curves (a) electronics components (b) mechanical components

I earlier had abstracted away the velocity of the failure stimulus and integrity of the component to generate a parametric expression for the failure rate.

$$r(t) = \frac{\frac{\mathrm{d}}{\mathrm{d}t}g(t)}{\tau + g(t)} \qquad \textbf{(EQ 24-23)}$$

In a real situation the velocity/integrity abstraction might occur as a stress/strain pairing particularly in a mechanical component. In this case, a mechanism called creep can play a significant role in determining the failure dynamics. Creep happens to a load under a constant stress condition over a period of time. This leads to a curve as shown below, which demonstrates a relatively quick rise in strain (i.e. deformation) before entering a linear regime and then an exponential as the final wear-out mechanism becomes too great.

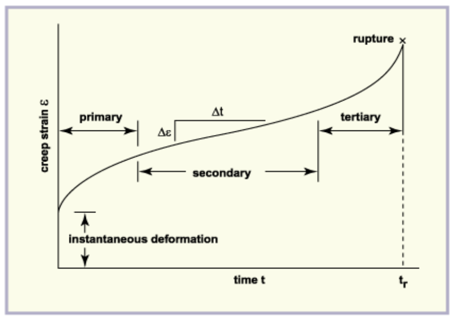


FIGURE 24-13. Creep curve which physically realizes the growth function of Figure 24-10 on page 468 (from http://www.ami.ac.uk).

Although the stress/strain relationship can get quite complex, to first-order, we can compare the curve in Figure 24-10 on page 468 to the general creep curve above. The monotonic increase in the abstract growth term, ***g***(***t***), remains the same in both cases, with both a linear and exponential regime noticeable in the middle (secondary) and late (tertiary) periods. The big difference lies in the early (primary) part of the curve, where due to elastic and plastic deformation[6] the growth increases rapidly before settling into the linear regime. This fast "settling-in" regime intuitively provides a pathway to an earlier failure potential than a purely accumulating process would.

One can approximate the general trend in the primary part of the growth either by a rising damped exponential or by a parabolic/diffusive growth that rises with the nth root of time. The following figure uses a combination of a square root and the exponential growth to model the creep growth:

$$g(t) = A\sqrt{kt} + B \cdot (e^{-ct} - 1) \qquad \textbf{(EQ 24-24)}$$

6. Particularly in a viscoelastic material

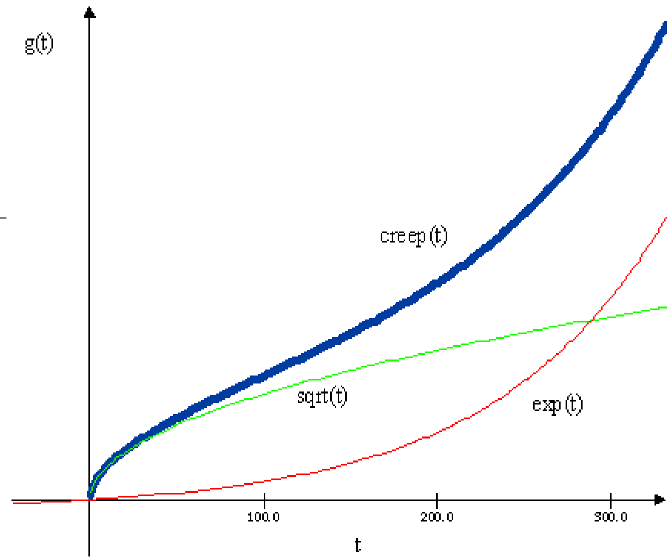


FIGURE 24-14. Modeled creep growth rate

Applying this growth rate to the dispersive failure rate, ***g*(*t*)**, we get the following bathtub curve.

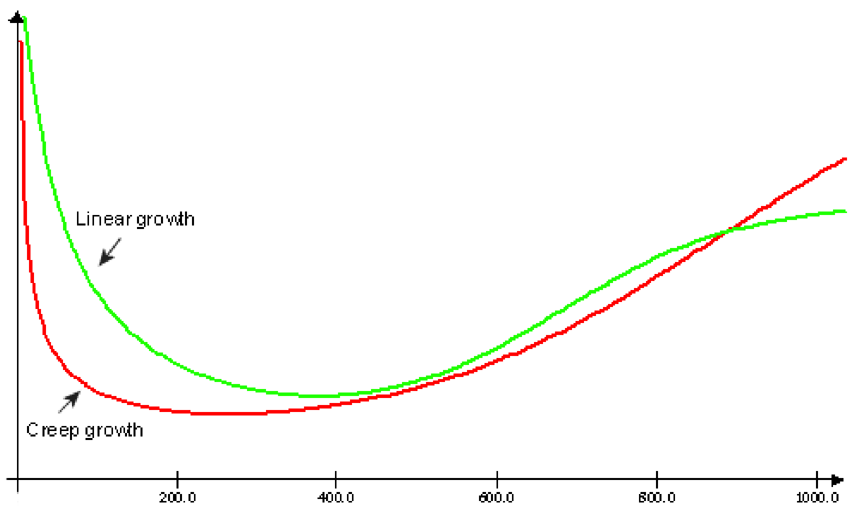


FIGURE 24-15. Bathtub curve for growth rate exhibiting physical creep has a sharper initial failure rate fall-off due to earlier deformation.

Clearly, the early part of the curve becomes accentuated relative to the linear growth mechanism, and a more asymmetric v-shape results, characteristic of the mechanical failure mode of (b) in Figure 24-12 on page 470.

Dealing with Built-In Obsolescence

The model of reliability in this chapter has some significance in an energy-constrained world. We specifically need to embrace uncertainty, and start to value resiliency[7]. Why must we accept products with built-in obsolescence that break down way too soon? Why can't we take advantage of the understanding that we can glean from failure dispersion and try to make products that last longer? Conservation of products could become as important as conservation of energy, if as things play out in a finite world and oil continues to become more and more expensive.

Curiously this analysis also relates to understanding oil discovery. As I stated at the outset, a failure essentially defines the flip-side of success. When we search for oil, we encounter initial successes around time=0 (think 1860). After that, as more and more people join the search process and we gain technological advances the accelerated search takes over. Eventually we find all the discoveries (i.e. failures) in a large region (or globally) and something approaching the classic logistic results. In this case, the initial downward slope of the oil discovery bathtub curve becomes swamped by the totality of the global search space. The mathematics of dispersive failures and the mathematics of dispersive discovery otherwise match identically.[8]

Thus you see how the popcorn popping statistical data looks a lot like the Hubbert peak, albeit on a vastly different time scale. In the inverted world of failure statistics, the popcorn popping experiment stands as an excellent mathematical analogy for dispersive discovery. If we consider the initial pops that we hear as the initial stirrings of discrete discoveries, then the analogy holds as the popping builds to a crescendo of indistinguishable pops as we reach peak. After that the occasional pop makes its way out. We effectively accelerated through peak.

7. **Resilience**: the capacity to absorb shocks to the system without losing the ability to function.
8. As a side observation, a significant bathtub curve could exist in a small or moderately sized discovery region. This may occur if the initial discovery search started linearly with time, with a persistent level of effort. If after a specific time, an accelerated search occurred the equivalent of a bathtub curve could conceivably occur. It would likely manifest itself as a secondary discovery peak in a region. So, in general, the smaller exploration regions show the initial declining part of the bathtub curve and the larger global regions show primarily the upswing in the latter part of the bathtub curve.

This happens with oil discovery as well, as the occasional discovery pop can occur well down the curve, as we have defined the peak in the early 1960’s.[9]

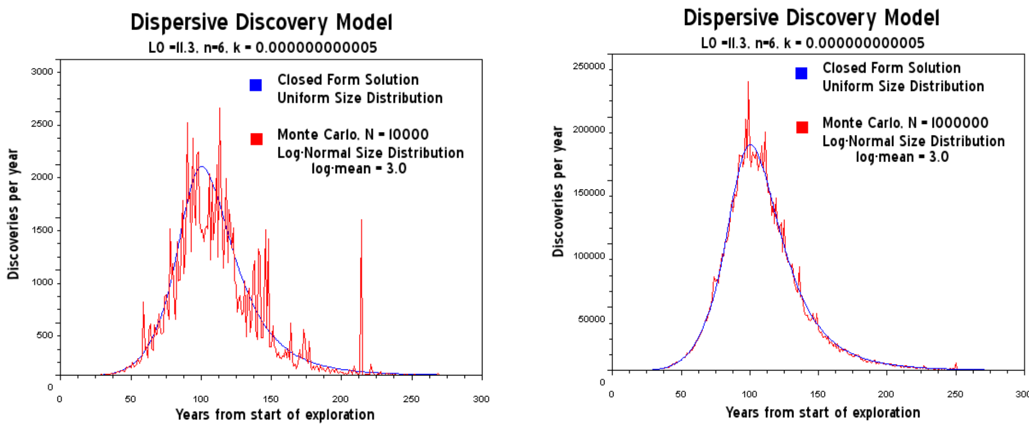


FIGURE 24-16. A discrete Monte Carlo simulation of dispersive oil discovery which shows both the statistical fluctuations and the sparseness of the finds down the tail of the curve. With more data sample the fluctuations diminish in size.

Bardi has suggested that we come up with a “mind-sized model” to described the Hubbert Peak [Ref 315]. This makes some sense as we still have the problem of trying to convince politicians and ordinary citizens of what constitutes the issue of peak oil and more generally peak oil.

Why we try to do this has some psychological basis. At one time, I came across an explanation of the three stages of learning. Understanding does not come about instantaneously, and instead progresses through various psychological stages. This explanation basically amounted to a cone of experience conceptual model, with a progression through these stages. The most basic amounts to a type of hands-on learning through first-hand experience and the most sophisticated to the levels of analogies and potentially mathematics for the sophisticated learner.

The fundamental problem with understanding peak oil is that the first stages occur on the most abstract level. We just cannot comprehend the global scope of the problem and can only reduce it to our day-to-day interactions and what we see at a local level. Therefore, only when we experience, or hear about something like gas station queues and price hikes do we make a connection to the overriding process at work. But these observations serve merely as side-effects and do not ultimately explain

9. So contrary to the occasional reporting of huge new oil discoveries, the days of sustained discoveries remain behind us and we are seeing the occasional pop of the popcorn.

the bigger picture of oil depletion. In Bardi's view, it does not represent a good mind's eye view of peak oil.

Bardi tried to do this by comparing oil depletion to a Lotka-Volterra (L-V) model of predator-prey interactions. This works on levels of learning, culminating in an abstraction that demonstrates the Hubbert Curve in mathematical curves. That would work perfectly... ***if*** it actually modeled the reality of the situation. Unfortunately the Lotka-Volterra model doesn't cut it and it will actually misrepresent our continued understanding of the situation. In other words, we cannot use Bardi's analysis as a tool for practicing depletion management. Our learning basically ends up in an evolutionary dead-end, and even though it effectively works on an emotional and psychological level to fill in our cone of experience, it misses the mark completely on mathematical correctness.

Other people try to make the symbolic connection by comparing the oil discovery process as searching through a pile of peanuts or cashews for edible pieces. This works on a qualitative level, but until we have some quantitative aspects, it ends up short on the symbolic mathematical level. I myself have used the analogy of finding needles in the haystack (see "Finding Needles in a Haystack. How we discover oil"), which brings in the formal concepts of dispersion. The idea of popcorn popping fills in more of the hands-on levels of our experience (who hasn't waited in agonizing expectation of the furtive last few pops?) and the experiments themselves demonstrate the math involved — the process of accelerating discovery as our reserve bag of popcorn inflates, followed by a peak in maximum discoveries, followed by a slow decline as the most stubborn kernels pop. The production cycle culminates as we consume the popped popcorn at a later date.

To compare this understanding against Bardi's Lotka-Volterra or of the Verhulst formulations shows the limitations of the predator prey model, as it simply does not take into account the stochastic environment of exploration. For example, what if we had applied an L-V formulation to the popcorn experiment, what would we get? Since it is deterministic, it would come out like a spike with the cumulative showing as a step function. All the L-V can do is work on the *mean value*, so that it misses the real dynamics of random effects. With L-V, **all the popcorn would pop at the same time**.

Moreover, the L-V model also assumes that some collective feedback plays a large role, as if the amount we have found or the amount remaining (as to in the Verhulst equation) determines the essentials of how depletion comes about. Yet, as the *single kernel* popcorn experiment bears out, the feedback term does not really exist. All we really have to understand is that certain stubborn pockets will remain and we need to keep accelerating our search if we want to maintain the Hubbertian Logistic-like shape. In other words, the natural shape does not decline as some intrinsic collective property. On an intuitive level, the popcorn kernels that have popped don't tell their buddy kernels to stop popping once the bag nears completion.

The L-V predator-prey model tells us that is the reality and Bardi wants us to believe that interaction happens. Yet only a good model fosters a better understanding of the situation, and if you go down the wrong path it only makes it worse. So in effect, with the correct mathematical model, we work out the popcorn dynamics from first principles and use that as a predictive tool for some future need.[10] This is the kind of mind-sized model that we have just worked out.

10. Or on a hands-on level, I would tell a politician that I could accurately gauge how long popcorn would take to pop and use that information to convince him that we could gauge future oil depletion dynamics. Unless he had never seen a popcorn popper in action he would say, yeah that would make sense — and then I could relate that to how it makes sense in how oil depletion dynamics plays out.

CHAPTER 25

Solar Energy

Dealing with disorder

"Often in physics, experimental observations are termed "anomalous" before they are understood."

— Richard Zallen, "The physics of amorphous solids", Wiley-VCH, 1998

Scientists have struggled with converting sunshine to electricity inexpensively for years. They realized early on that high-quality crystalline semiconductors had superior efficiencies to poorly fabricated materials, but that the high-quality materials also costed much more to produce. For the kinds of applications necessary to bring solar energy, such as expansive areas of photovoltaic (PV) grids, to widespread acceptance, this cost needs to come down, or the sub-par materials somehow improved.

Disorder in the PV material's crystallinity lay at the root of the problem, and we can use some of the same techniques that get used in analyzing liquid transport through porous material (a disordered medium by definition) to model some of the anomalous PV characteristics. This section navigates these waters with a different perspective than you will find in current textbooks. One take is quite simple and the other more fundamental, yet they arrive at nearly identical conclusions.

Dispersive Transport

To get the cost of photovoltaic (PV) systems down, we will have to learn how to efficiently use sub-optimal materials. By sub-optimal I mean that mass-produced PV materials will end up getting rolled or extruded or organically grown. Unless we perfect the process, most everything will turn out non-optimal. We already know the difference between clean-room cultivated single crystal semiconducting material and the defect-ridden and often amorphous materials that nature and entropy drives us to. For performance sensitive applications such as communications and computing we would only rarely consider disordered material as a candidate semiconductor. Certainly, the performance of these materials makes them unlikely candidates for high speed processing — yet for solar cell applications, they may serve us well. In the end, we just have to learn how to understand and deal with poor quality materials and devices. This analysis sets forth a baseline for the characterization of a maximally disordered semiconductor.

Background

The prehistoric 1949 Haynes-Shockley experiment first measured the dynamic behavior of charged carriers in a semiconducting sample. It basically confirmed the solution of the diffusion (Fokker-Planck) equation and it demonstrated diffusion, drift, and recombination in a conceptually simple setup.

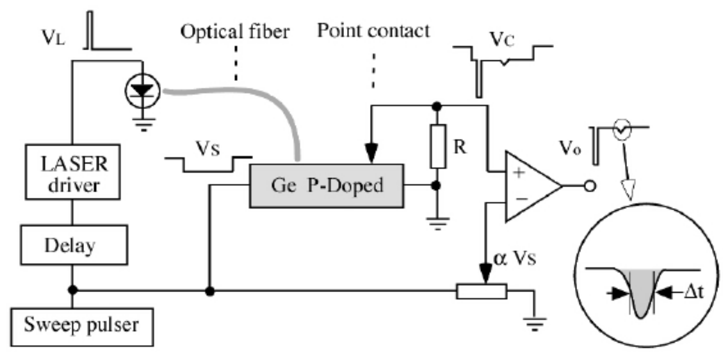


FIGURE 25-1. Apparatus for the Haynes-Shockley experiment (from labtrek.net). This measures drift and diffusion across a photo-induced semiconducting sample

This setup works according to theory for an ordered semiconductor with uniform properties but apparently gets a bit unwieldy for any disordered or non-uniform material sample. I inferred this as conventional wisdom since most scientists either punt or use heuristics partially derived from the inscrutable work of a select group of random-walk theorists [Ref 244].

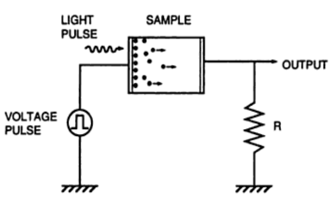
I had initially applied a very straightforward interpretation to the problem of carrier transport in disordered material. This dispersion analysis essentially set aside the Fokker-Planck formalism for a mean value approximation where I applied the Maximum Entropy Principle. In particular, I favor the MaxEnt solution because I can recite the solution from memory. It matches intuition in a conceptually simple way once you get into a disordered mind-set.

In the classical Haynes-Shockley experiment, a pulse gets injected at one electrode, and a nearly pure time-of-flight (TOF) profile results. The initial pulse ends up spreading out in width a bit, but the detected pulse usually maintains the essential Gaussian sigmoid shape. How much this profile smears out has to do with relative strengths of drift, diffusion, and dispersion. When this smearing of the dispersive transport reaches a certain magnitude depending on the material, they refer to it as anomalous behavior.

Why is dispersion anomalous?

If this question sounds familiar after reading Volume 1, it should, as we have seen similar bewilderment concerning reserve growth[1].

> Often in physics, experimental observations are termed "anomalous" before they are understood. Once theory succeeds in explaining and illuminating the observations, they are no longer "anomalous" and instead come to be regarded as "obvious". A crucial paper can trigger such an "anomalous => obvious" transition, and in the present case that key role was played by a 1975 paper by Scher and Montroll. That landmark paper has become basic to our understanding of a striking characteristic of carrier motion (now called dispersive transport) which is a common occurrence in amorphous semiconductors, though foreign to our experience with crystals. — *Richard Zallen, "The physics of amorphous solids" [Ref 248]*



The term anomalous in scientific code-speak essentially means "dunno". We have to admit that we don't understand lots of things, largely due to issues of complexity, observability, or just too much noise. Yet that doesn't prevent us from trying to extract a fundamental meaning of some strange behavior that we observe.[2]

The applied mathematicians Scher and Montroll originally tried to explain the concept of anomalous behavior of photo-conductivity in amorphous semiconductors.

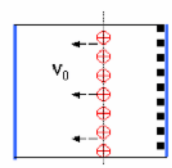
1. Or "enigmatic" as some have referred to the oil situation, see "The Reserve Growth. How estimates of oil evolve"
2. This touches on the nature of theoretical and experimental research and illustrates how a fundamental idea can take quite a circuitous route before it lodges in a remotely related application area. The acceptance of the original idea tends to create a momentum that makes it difficult to dislodge from the conventional wisdom and impenetrable to anyone but the cognoscenti.
This traverses the scale of applicability from statistical mechanics to environmental geology picking up arcs of connectivity along the way. I also buried a valuable nugget in here, presenting a surprisingly powerful analytical result that has laid dormant for over 30 years, perhaps even 20 years prior to that, and has huge implications for the analysis of solar cells, MOS technology, and quantum electronics.

Scientists had long understood the complementary non-anomalous behavior in non-amorphous materials. To set the stage see the schematic figure to the right.

In a crystalline semiconductor with a contact electrode at each end, a pulse of light incident at one electrode will, upon the effect of an electric field or potential drop between the electrodes, generate an almost immediate flow of current across the load lasting as long as the transit time of the photoinduced carriers. The carriers, either electrons or holes depending on the polarity of the electric field bias relative to the absorbing electrode, will drift from the site

Experimentalists consider the behavior well characterized; the mobility of the carriers at temperature, the strength of the electric field, and the contact separation, ***d***, determine the transit time, t_T, of the output current pulse. Because the carriers scatter against lattice imperfections, the speed does not continuously accelerate but instead achieves a bounded drift velocity, v_0.

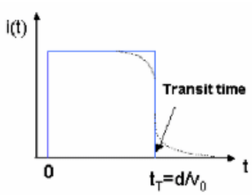
This drift mobility has an intuitive real-world analog — think in terms of a drag coefficient for the analogous situation of a falling body under gravitational forces, which eventually achieve what we call terminal velocity. For all practical purposes, the equivalent terminal velocity occurs almost immediately in a semiconductor (a short relaxation or quenching time) and sets the bound for the transit time duration.



(a)

Ideally, the pulse looks like a perfect square wave with temporal duration t_T. In terms of a mathematical expression, the current behaves as

$$I(t) = K \cdot [u(t) - u(t - t_T)] \qquad \textbf{(EQ 25-1)}$$



(b)

where ***u***(***t***) is the unit step (or Heaviside operator) operator with magnitude ***K***.

Because of carrier diffusion, the actual drop-off in current has rounded leading and trailing edges as the charged carrier pulse spreads out a bit into a Gaussian packet as it propagates. This relatively innocuous but well-understood form of dispersion, known as diffusion, occurs from random walk excursions as the carrier makes its way across the transit width. In the ideal case, the diffusion constant varies linearly with the mobility (or drag for particle systems) according to the Einstein relation.

For high mobilities and small contact separations, the amount of diffusion that occurs does not appreciably round the pulse edges. The prized high mobility solid-state material allows device manufacturers to fabricate ultra-high speed photodetectors as the sharp transition and short transit time generates an excellent and well-characterized frequency response.

This class of semiconductor has an ordered structure due to the crystalline lattice structure and it has properties such as carrier mobility which remain uniform through the sample. Such behavior shows little dispersion, either through diffusion or disorder. The narrower the distribution of velocities, the sharper the transition. Scientists have generally understood this for years and no one raises the spectre of anomalous behavior.

The truly anomalous behavior observed occurs in amorphous versions of certain semiconductors. The narrow pulse of carriers seen in an ordered sample now shows a huge spread in its concentration profile as it makes its way between the contacts. Obviously dispersion plays some role in this behavior, as it goes by the name “dispersive transport”. Scientists had known about this “anomalous dispersion” since 1957 but it took nearly two decades before Scher and Montroll presented the solution to the problem mathematically [Ref 247].

The following two figures give a qualitative view of the dispersive transport that occurs in a disordered semiconductor. The first figure describes what we think happens internally and the second figure provides a view of the observable result. The next two figures illustrate a couple of experimental results of widely studied amorphous (a-As_2Se_3) and organic materials (TNF-PVK) ([Ref 248][Ref 249][Ref 250]). [3]

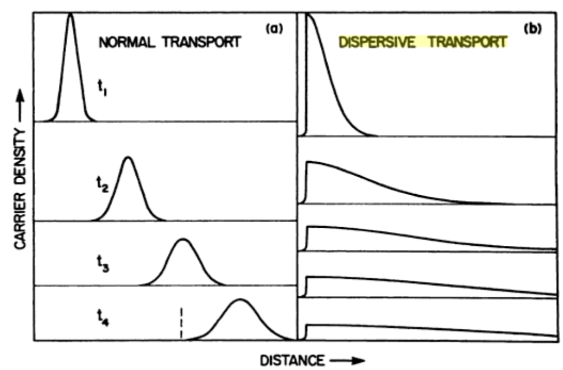


FIGURE 25-2. (a) Normal transport shows a drifting packet of carriers. As long as the packet travels, we can detect a current proportional to the amount of carriers active. As they reach the opposing contact, the current rapidly declines to zero. The Gaussian shaped packet widens slightly as it travels due to diffusion about the mean. A few extra fast carriers reach the far contact sooner than the bulk of the carriers, while a few stragglers take up the rear. (b) In dispersive transport, the velocity of the carriers varies over a wide range so that the original narrow impulse of carriers quickly spreads out as it drifts and diffuses across the width. This gives a long tail to the photo-response profile. The stragglers keep arriving in this “fat tail” world, with progressively fewer in number as though they had joined and tried to complete a long race to the finish line (from [Ref 246])

3. On a log-log plot, as long as the orders of magnitude scale is maintained, one can fit a curve simply by translation

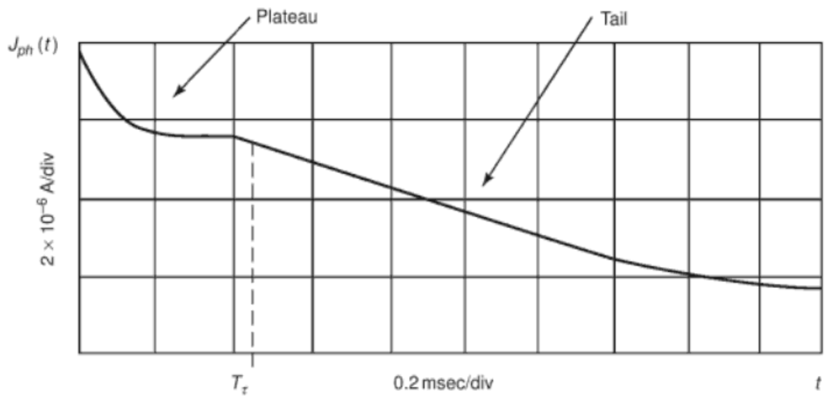


FIGURE 25-3. The typical measurement in the photoresponse current starts with a spike followed by a soft plateau, then a shoulder or transition region, followed by the ubiquitous long tail[Ref 249]. Shapes of the current profile taken over a range of experimental conditions show invariance in the general shape with respect to the electric field and specimen thickness. Importantly, this profile does not follow from the expected spread of the Gaussian packet. Scale invariance or universality manifests itself in statistics if one can first transform the ordinates into dimensionless quantities while conserving the moments. According to [Ref 250], transport of holes and electrons near the absorption region electrode contributes to the initial spike, which has no impact on the longer tail due to the complementary carrier type (this transient spike is also known as prompt transport). On a log-log plot, as long as the orders of magnitude scale is maintained, one can fit a curve simply by translation.

FIGURE 25-4. Typical experimental Time-Of-Flight curve shows a set of superimposed measurements from an early Scher-Montroll behavior which exhibited the “universality” property of the scaling across different measurement conditions (the applied voltage in this case).

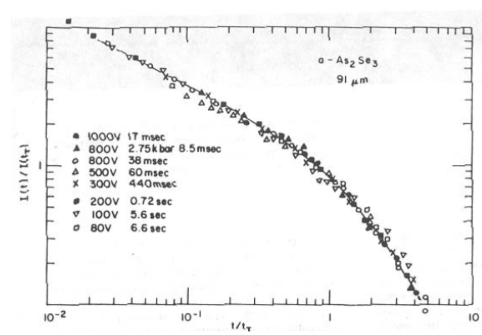
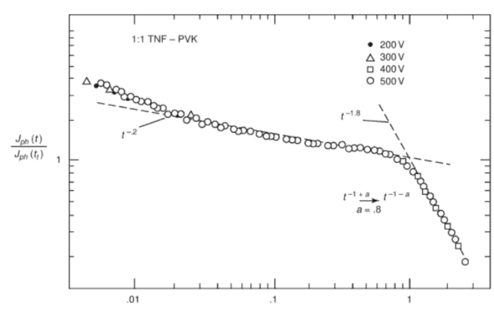


FIGURE 25-5. The TOF curve of an organic semiconductor illustrates the characteristic knee that Scher and Montroll had predicted; the two slopes differ but must sum to -2 according to their theory.



The anomaly in the title of the Scher-Montroll paper referred to the fact that no one previously could formally explain the long tails in the response (so called "time of flight") measurements. Other researchers clearly had an inkling that it had something to do with the high amounts of disorder leading to greater amounts of diffusion and dispersive spread than in an ordered material. Amorphous materials naturally have many inhomogeneities, defects, and carrier traps that can lead to varying delays in transit time. Scher and Montroll derived a statistical formulation of random walk called the *Continuous Time Random Walk* (CTRW) that they then applied to the experimental results.

Most experimentalists around that time got good results using the CTRW formulation so that it has become fairly well accepted in semiconductor circles for the last 30 years. The math gets fairly hairy in spots, and soon experimentalists began to simply use the empirical sloped lines to get at the Scher-Montroll disorder parameter (α). High values of alpha indicate more order and low values more disorder ($0 < \alpha < 1$).

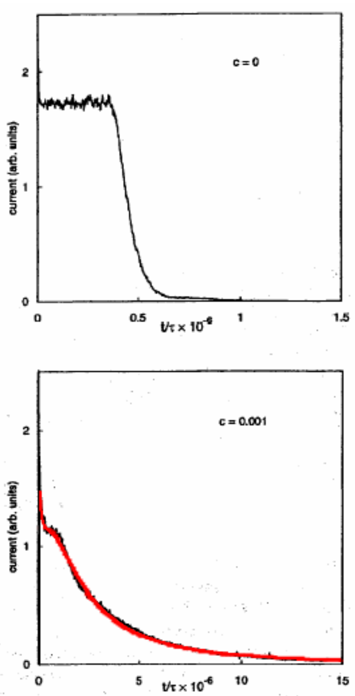


FIGURE 25-6.
Ordered transport above and dispersive below.

Research continues in understanding dispersive transport as new electronic materials come on line. Physicists have had a long-standing interest in disorder, as the finding of an order/disorder transition easily classifies as a type of "holy grail" discovery, certain to elicit recognition from their colleagues. As the figure to the right shows, one can add a controlled amount of disorder to a sample and observe the results of diffusive transport. The upper TOF trace shows linear transport while the lower trace shows the effects of dispersive transport. In the latter case the disorder comes in the way of the intentional introduction of impurities, apparently forming electronic traps which slow down the carrier motion as it traverses the width of the sample.

Generally I have noticed that the basis for much of the current research has to do with finding some novel aspect of dispersion relating somehow to material properties. In reality, the rather mundane effect of randomness due to heterogeneity likely plays a far more important role. For many of these materials, we simply can't control the distribution of defects and traps and the disorder evolves into a garden variety randomness, with which we have a single mean rate, say average drift velocity, to characterize the behavior.

If you look at the curve to the inset, the red line shows my simple assumption for maximum disorder. I had noticed this same shaped curve in studies of dispersive discovery for oil in Volume 1, and had a hunch that I could use the same formulation in the semiconductor case. After all, as I assert that dispersion is just dispersion, and I have enough experience dealing with semiconductor physics that I didn't expect any gotchas. As for the ideas of Scher and Montroll, I turn their formulation upside down and don't even consider a random walk premise, as this leads to overly complex math [Ref 244][Ref 251] [4]. Follow the logic of the next section for a more formal analysis, which brings in the Fokker-Planck formulation.

Adding Disorder: Fokker-Planck for Disordered Systems

For the time-of-flight for a disordered system, the "intuitive" Maximum Entropy solution looks like:

$$q(t) = q \cdot N \cdot e^{-\frac{w}{(\sqrt{(\mu E t)^2 + 2Dt})}} \quad \text{(EQ 25-2)}$$

This essentially states that the expected amount of charge accumulated at one end of the sample (at a distance ***w***) at time ***t***, follows a maximum entropy probability distribution. The varying rates described by μ and ***D*** disperse the speed of the carriers so that a broadened profile results from the initial pulse spike.

The equation above formed the baseline for the interpretation I described initially. Yet let's go through the complete analysis as well.

For completeness, we can apply the basic diffusion laws. If I can produce an equivalent solution by applying the Maximum Entropy Principle directly to the Fokker-Planck (F-P) equation, then this would give a better foundation for the "inspection" result above.

The F-P diffusion equation gets expressed as a partial differential equation with a conservation law constraint:

$$\frac{\partial}{\partial t}n(x, t) = -\frac{\partial}{\partial x}[D_1(x, t)n(x, t)] - \frac{d^2}{dx^2}[D_2(x, t)n(x, t)] \quad \text{(EQ 25-3)}$$

In this case $D_1 = \mu$ (carrier mobility) and $D_2 = D$ (diffusion coefficient), and $n(x,t)$ is the carrier concentration. Ignoring carrier recombination, the solution in one-dimension looks like:

4. The fact that the actual argument derives from such a simple premise, I have to seriously consider why no one has picked up on this before. I actually question some of the belief systems inherent in these fields of study and assert the likelihood of a sunk cost effect getting in the way of a fundamental understanding.

$$n_0(x, t) = \frac{A}{\sqrt{4\pi Dt}} \cdot e^{\frac{(x + \mu Et)^2}{4Dt}} \quad \textbf{(EQ 25-4)}$$

This of course works for well-ordered semiconductors, but ***D*** and μ will likely vary for disordered material. I made the standard substitution via the Einstein Relation for

$$D = V_t \cdot \mu \quad \textbf{(EQ 25-5)}$$

where ***Vt*** = β ***/q*** stands for the chemical or thermal potential at equilibrium (usually β equals ***kT*** where ***k*** is Boltzmann's constant and ***T*** is absolute temperature). At equilibrium, the stochastic force of diffusion exactly balances the electrostatic force ***F*** = ***qE***.

From the basic physics, we can generate a maximum entropy density function for ***D***

$$p(D) = \frac{1}{D_0} \cdot e^{-D/D_0} \quad \textbf{(EQ 25-6)}$$

then

$$n(x, t) = \int_0^{\infty} p(D) \cdot n_0(x, t) dD \quad \textbf{(EQ 25-7)}$$

This looks hairy but the integral comes out straightforwardly as (ignoring the constant factors)

$$n(x, t) = \frac{1}{\sqrt{t \cdot (4D + t \cdot (E\mu)^2)}} \cdot \frac{e^{-xR(t)}}{R(t)} \quad \textbf{(EQ 25-8)}$$

where

$$R(t) = \sqrt{\frac{1}{Dt} + \frac{E}{(2 \cdot V_t)^2}} - \frac{E}{2V_t} \quad \textbf{(EQ 25-9)}$$

If we evaluate this for carriers that have reached the drain electrode at ***x=w***, the total charge collected ***n*** is:

$$Q(t) = q \cdot n(t) = \frac{q \cdot N}{\sqrt{t \cdot (4D + t \cdot (E\mu)^2)}} \cdot \frac{e^{-wR(t)}}{R(t)} \quad \textbf{(EQ 25-10)}$$

The measured current is the mean value rate of change of charge from 0 to ***w***.

$$I(t) = \frac{1}{w}\int_0^w \frac{\mathrm{d}}{\mathrm{d}t}Q(t) \quad \text{(EQ 25-11)}$$

At this point we want to generalize the solution. This makes it potentially applicable to notions of transport physical transport of material in porous matter. This would include the motion of oil underground, CO2 in the air, and perhaps even spilled oil at sea.

In the one-dimensional model of applying an impulse function of material, the concentration ***n*** will disperse according to the following equation:

$$n(x, z) = \frac{z + \sqrt{zL + z^2}}{\sqrt{zL + z^2} \cdot 2} \cdot e^{-2\frac{x}{z + \sqrt{zL + z^2}}} \quad \text{(EQ 25-12)}$$

where the force ***F=qE***

$$\begin{aligned} z &= \mu F t \\ L &= \beta / F \end{aligned} \quad \text{(EQ 25-13)}$$

The term ***z*** takes the place of a time-scaled distance, which can speed up or slow down under the influence of a force ***F*** (i.e. gravity, or electric field for a charged particle). The characteristic distance ***L*** represents the effect of the stochastic force β (aka Boltzmann's constant) and ties in the diffusional aspects of the system. The specific parameterization of the exponential results in the fat-tail observed.

Intuitively one may not have to go through the trouble of solving the FPE, simply because intuition would suggest that the dispersive envelope would cancel out most of the details of the diffusion term. In the intuitive dispersive transport model of the first equation, the dispersion would at most follow the leading wavefront of the drifting diffusional field as $\sqrt{Lz + z^2}$ or even as $\sqrt{Lz} + z$.

One can imagine that the diffusion term would follow as the square root of time according to Fick's first law and that drift would follow time linearly, with only an idea of the qualitative superposition of the terms in my mind.

Entropic Dispersion Analysis

As I implied earlier, we can use an inspection technique to come up with a non Fokker-Planck derivation to the dispersive transport problem. I started to look at this problem because I had a nice intuitive way of modeling dispersive behavior in oil production and figured that I could try applying the general dispersive model to dispersive transport. And that I could do it much more simply than the approach by Scher and Montroll. At certain places in their seminal papers and review articles, I find passages that amount to "*... and then a miracle occurs*" and knew that this meant some messy first-principles work had gone missing. The way I turned their model on its head basically amounted to working in the *rate domain*, corresponding to velocities, instead of the *time domain* The latter derives from the classical work used to describe everything from Brownian motion to large scale diffusion. The former relates to a more or less pragmatic view of the world which relies on entropy considerations instead of the statistics of hopping over energy barriers with small probabilities.

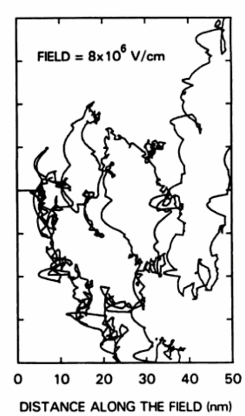


FIGURE 25-7. A Levy random walk

As a basic premise, I use the Maximum Entropy Principle to select a stochastic rate probability density function (PDF or more precisely PMF for probability mass function) in which I can then derive dispersive transport.[5] that corresponded to random-walk hopping (see figure).[Ref 254]

> One way to choose the "right" distribution p is by using the principle of maximum entropy. This principle states that the least biased probability assignment is that which maximizes the system entropy subject to the constraints supplied by the available information [Ref 253].

For the constrained system of interest, all we really know is the mean carrier transport velocity. If we don't know the higher order moments, MaxEnt says to use a damped exponential as the PDF to maximize entropy. In a general sense, this maximizes the amount of disorder that exists in this quasi-equilibrium system. It says that many slow carriers exist, with an exponentially diminishing supply of fast carriers. For the fixed geometry shown in the schematics at the top, the normalized expression for the time dependence of dispersed current reaching the far contact derives as follows:

For a pulsed light source, the entire impulse response equation boils down to a simple charge conservation problem. We know that charge builds up as the photons excite the carriers, but we only know the mean rate of the dispersive terms and we let the Maximum Entropy Principle figure out the rest. The concentrations build up as the following form, with $\boldsymbol{g}(\boldsymbol{t})$ acting as the transport growth term across a region of width $\boldsymbol{w}$:

5. One can use the Laplace transform to characterize the dimension of the disorder

$$C(t) = C_0 \cdot g(t) \cdot (1 - e^{-w/g(t)}) \quad \text{(EQ 25-14)}$$

This essentially describes the integral over a PDF of normalized velocities

$$p(v) = \frac{1}{v_0} \cdot e^{-\frac{v}{v_0}} \quad \text{(EQ 25-15)}$$

for carriers that have swept through a portion of the transport layer, anywhere within the interval form ***x***=0 to ***x***=***w***.

$$C(t|x) = \int_{v > \frac{x}{t}}^{\infty} p(v) dv = e^{-\frac{x}{v_0 t}} \quad \text{(EQ 25-16)}$$

We then make the subtle replacement of $v_0 t \to g(t)$ to generalize the transport position from a linear term to an arbitrary growth function. The

$$C(t) = C_0 \int_0^w C(t|x) dx = C_0 \cdot g(t) \cdot (1 - e^{-w/g(t)}) \quad \text{(EQ 25-17)}$$

Crucially the formulation maintains the moments of the distribution. If the velocity distribution becomes dispersed as a damped exponential then the cumulative position distribution of a particle/carrier also advances by a damped exponential. We then make a guess to the growth term as follows:

$$g(t) = \sqrt{2 \cdot D \cdot t} + \mu \cdot E \cdot t \quad \text{(EQ 25-18)}$$

where ***D*** is the diffusivity, ***w*** is the active width, ***u*** is the charge mobility, and ***E*** is the electric field strength (***E*** = Voltage/***w***). The total number of excited carriers is $\boldsymbol{C_0}$, and this number provides the maximum amount of current that gets collected. Common to all stochastic probability problems, the conservation of probability becomes a strong constraint.

The current derives as the amount that we can detect moving through the transport region:

$$I(t) = \frac{\mathrm{d}}{\mathrm{d}t} C(t) = C_0 \cdot \frac{\mathrm{d}}{\mathrm{d}t} g(t) \cdot \left(1 - e^{-w/g(t)} \cdot \left(1 + \frac{w}{g(t)}\right)\right) \quad \text{(EQ 25-19)}$$

Note the d$\mathbf{g}(\mathbf{t})$/d$\mathbf{t}$ term (with the $\frac{1}{\sqrt{t}}$ term below) keeping only the drift term.

$$\frac{\mathrm{d}}{\mathrm{d}t}g(t) = \frac{1}{2} \cdot \sqrt{\frac{D}{t}} + \mu \cdot E \qquad \textbf{(EQ 25-20)}$$

Plotting the original fitted curve trace with the extra chain-rule term, we can actually see the initial transient due to the $\frac{1}{\sqrt{t}}$ term. In the normalized case, I show the response profile below superimposed on the figure from Kao [Ref 249]. At a subjective level, it follows the qualitative plateau/decline behavior quite well.

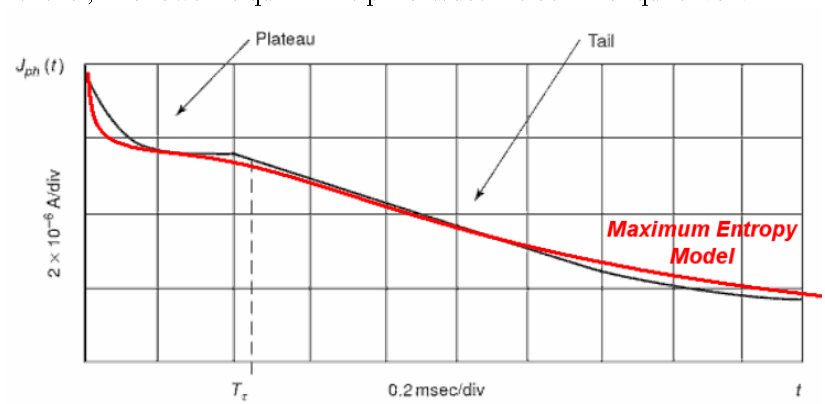


FIGURE 25-8. The maximum entropy dispersion for time of flight according to the simplified equation

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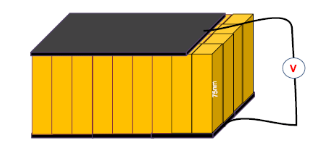


FIG. 1. (Color online) Schematic description of a thin film device "broken" into many mobility pathways.

The key to understanding physics is to keep it simple, but not too simple[6]. In fact the maximum entropy formulation that I had used previously in the dispersion analysis for oil field sizes, discovery, and reserve growth, I retain in this analysis. Also known as the "method of least information", it essentially relies on using common sense in not trying to under- or over-estimate the variance of the dispersive spread. In one sense, the interpretation I make looks similar to the schematic at the left. I assume the equivalence of *multiple mobility pathways* through the device.

Alternatively we can use a growth expression involving time as shown below [Ref 255]

$$\langle x \rangle = \sqrt{Dt + (v_0 t)^2} \qquad \textbf{(EQ 25-21)}$$

6. As attributed to Einstein

This essentially incorporates the concurrent diffusion component along with the drift component of the velocity as a root-mean-square estimate. The drift velocity v0 relates to the electric field by $\boldsymbol{v}_0 = \mu \boldsymbol{E}$, where μ is the carrier mobility and $\boldsymbol{E}$ is the electric field strength. Think of this as a case where we can maintain moments of the distributions across dimensions, $\langle \boldsymbol{t} \rangle / \boldsymbol{t}_T = \langle \boldsymbol{x} \rangle / \boldsymbol{w}$.

Qualitatively the constant (drift) velocity drops as $1/\boldsymbol{t}^2$ while diffusional velocity drops by $1/\boldsymbol{t}$. I am not certain whether the formulation by Scher and Montroll take this into account. They simply say that long-range correlations go as $1/t^{-\alpha-1}$ when they set up their CTRW model. I believe this step links my exponentially damped rate dispersion to their long range time correlations.

As one might expect, the derived entropic FPE solution borrowed from a little of each of these guesses for an RMS-like value, essentially averaging between the two:

$$\frac{\mu E t + \sqrt{4Dt + (\mu E t)^2}}{2} = \frac{z + \sqrt{zL + z^2}}{2} \quad \textbf{(EQ 25-22)}$$

So the solution to the dispersive FPE form for a disordered system turns out entirely intuitive, and one can almost generate the result from inspection. The difference between the intuitive entropic dispersion derivation and the full FPE treatment amounts to a variable factor of 2 in the FPE solution. You can see this by comparing the two approaches for the case of $\boldsymbol{L}$=1 and unity width for the dispersive transport current model.

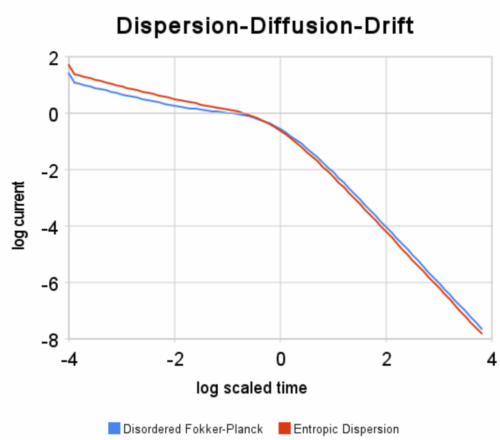


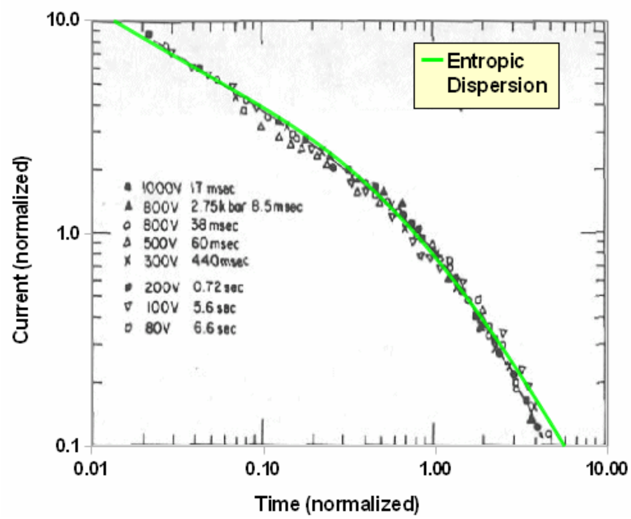
FIGURE 25-9. Differences between the original entropic dispersive model and the fully quantified FPE solution will converge as *L* gets smaller

The simple entropic dispersive expression and the Fokker-Planck result obviously differ in their formulation, yet the two show the same asymptotic trends. For an arbitrary set of parameters, one can't detect a practical difference. Use whichever you feel comfortable with as the overall profile won't change too much.[7]

Case Studies

If we go back and look at Scher and Montroll's original motivating data set, one can see how immediately practical the simplified expression becomes.

FIGURE 25-10. A plot of transit times for amorphous arsenic triselinide (a-As_2Se_3) with the simple entropic dispersion model. This was the original set of data points that Scher and Montroll used to develop their CTRW model.



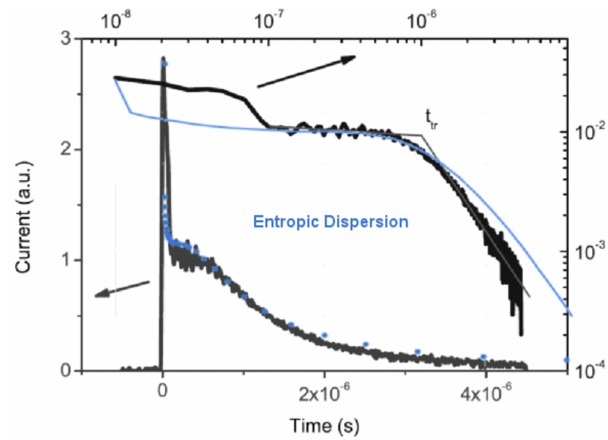
Organic Semiconductor Applications

The photocurrent profile displayed below came from Andersson [Ref 252] analyzing the transport in a specific organic semiconducting material, the polymer APFO.

The collected current profile looks like the following

7. The dynamics of the carrier profile in the animated GIF linked below. http://img8.imageshack.us/img8/3825/concentration.gif. The initial profile starts with a spike at the origin and then the profile broadens as the mean starts drifting and diffusing to the opposing contact. You don't see much from this perspective as it looks completely like mush. Yet, when plotted on a log-log scale, it does take on more character.

FIGURE 25-11. Typical photocurrent trace showing the initial diffusional spike, a plateau for relatively constant collection from the active region, and then a power-law tail produced from the entropic drift dispersion.



The blue line drawn through the set of traces follows the entropic dispersion formulation.The upper part of the curve describes the diffusive spike while the lower part generates the fat-tail due to the drift component (this shows an inverse square power law in the tail).

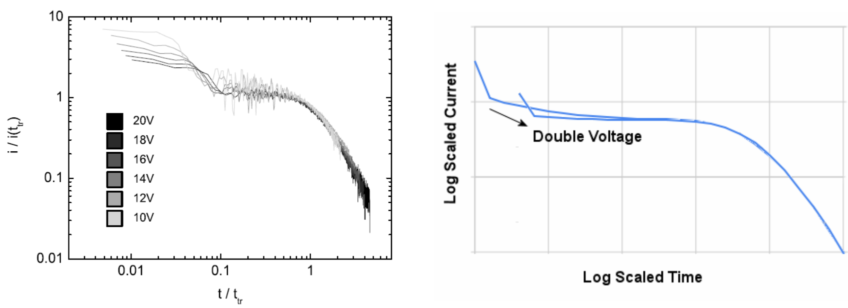


FIGURE 25-12. Universal profile generated over a set of applied electric field values. For this set, scaling of transit time with respect to the applied field holds, indicative of a constant mobility. However, carrier diffusion causes the initial transient and this does not scale, as the electric field has no effect on diffusion, as shown in the right set of blue curves.

Most scientists when discussing this shape have either (1) referred to Scher/Montroll and the vague heuristic α, (2) dismissed these features, or (3) labelled them as uninteresting. Andersson follows suit:

> *At best this transient, as the high α value indicates, might be possible to evaluate in a meaningful way with a bit of error and at worst it is of no use. Either way the amount of material and effort required is rather large compared to the usefulness of the results. APFO-4 is also the polymer that, among the investigated, gives*

> *the"nicest" transients. The conclusion from this is that if alternative measurement techniques can be used it is not worthwhile to do TOF* [Ref 252].

Not to dismiss the hard work that went into Andersson's experiment, but I would beg to differ with his assessment of the worthiness of the approach. When characterizing a novel material, every measurement adds to the body of knowledge, and as the interpretation of the aggregation of data becomes more cohesive, we end up learning much more of the internal structure. As I have learned, if someone does not understand a phenomena, they tend to dismiss it

By their very nature, disordered systems contain a huge state space and we really can't afford to throw out any information. Witness the organic semiconductor material that Montroll and Scher originally referred to in 1975 as following their CTRW model. As fitted in the figure below, the simplified entropic dispersion model does an arguably better fit by fine tuning essentially a singe parameter.

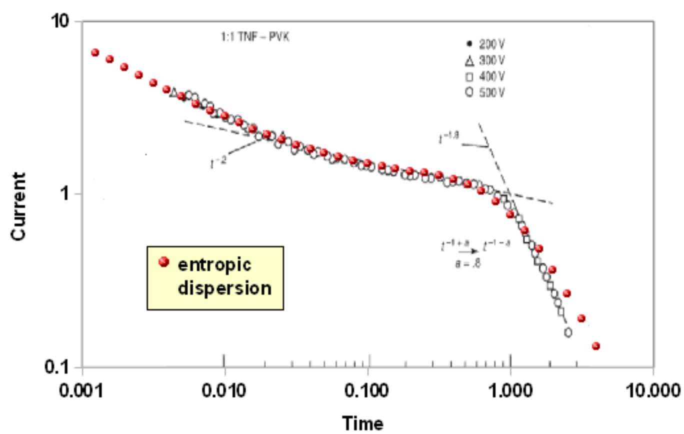


FIGURE 25-13. The other example that Scher and Montroll referred to as following their CTRW model is the organic semiconductor TNF-PVK. The simplified model does a better job of explaining the fine structure, especially in terms of the concave nature of the plateau.

Which brings up another interesting set of TOF experiments that I dug up. These also deal with organic semiconducting materials — the polymers with the abbreviations ANTH-OXA6t-OC12 and TPA-Cz3d. The following figures show the TOF results for various applied voltages. I superimposed the entropic dispersion equation form as the red line with the derived mobility in the caption below each figure. The research team had applied the Scher & Montroll Continuous Time Random Walk (CTRW) heuristic as indicated by the intersecting sloped lines. The CTRW model clearly fails in this situation as the slopes need quite a bit of creative interpretation. Note that we don't observe the diffusive spike; I integrated the charge from 10% to 100% of the width instead of 0% to 100%.

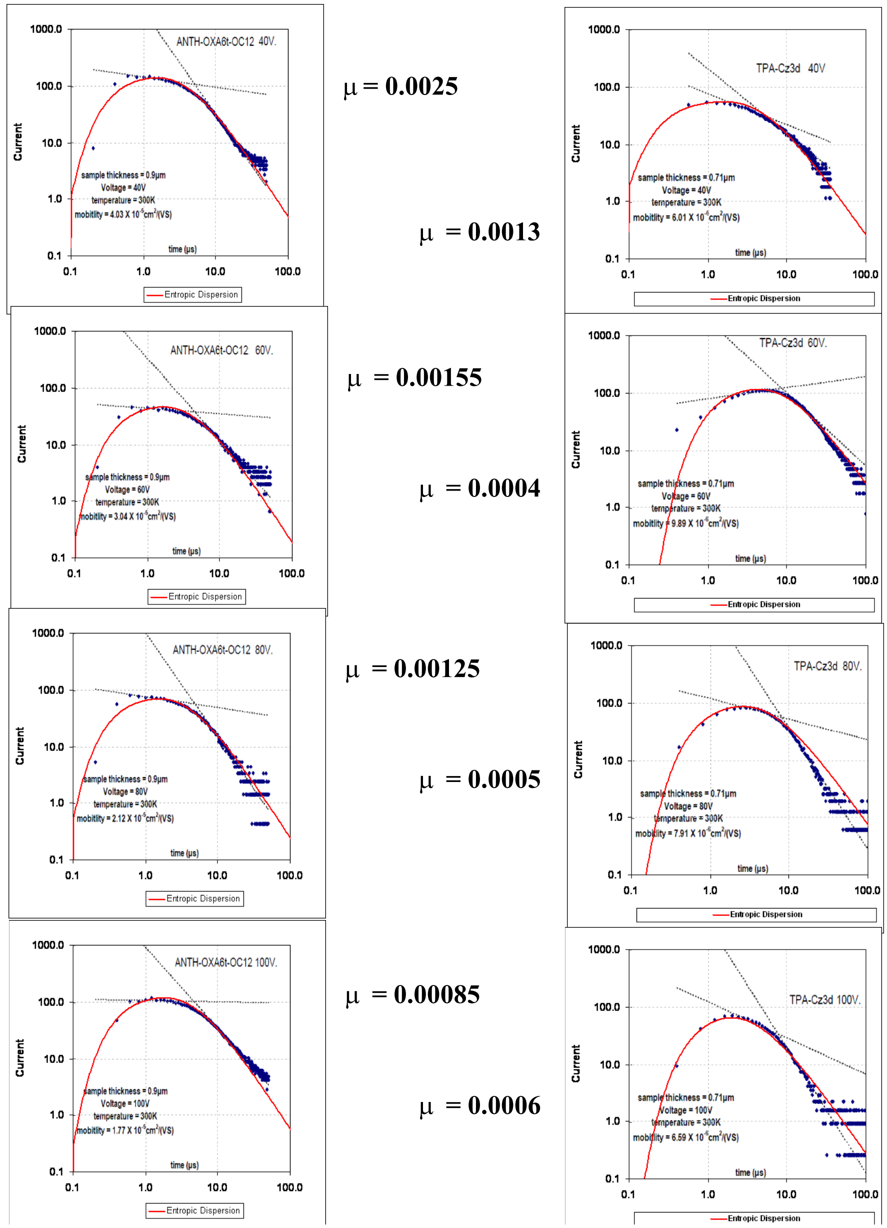


FIGURE 25-14. Sequence of time-of-flight experiments for an organic semiconductor left ANTH-OXA6t OC12, right TPA-Cz3d

The number of papers I find, especially when dealing with organic semiconductors, that cannot apply the Scher/Montroll theory indicates that the CTRW theory lacks generality. In other words, it works poorly for describing suboptimal material. I will also say the theory has some very serious flaws, including the claim that an $\alpha = 1$ defines a non-dispersive material. How could a power-law of -2 be anything but dispersive?

Transport in a-Si:H

Amorphous semiconductors have a huge influence on the solar cell and photovoltaic industry. In general, it costs much less to manufacture amorphous materials as the fabrication facilities do not have to follow as strict a material process. Unfortunately the performance characteristics of the amorphous silicon in comparison to its crystalline brethren leaves lots of room for improvement. Although not as important for solar cells, the photo-response time for an incident light stimulus shows the long tails characteristic of diffusive transport.

The fact that the novel entropic dispersion formulation works on any disordered material makes it much more general. Several years ago Scher wrote a popular article for *Physics Today* extolling the wonders of his theory, and how it seemed to fit a variety of disordered systems [Ref 245]. He mentioned how well it fit amorphous silicon based on the number of orders of magnitude that his piece-wise line segments matched. Well, the entropic dispersion does just as well:

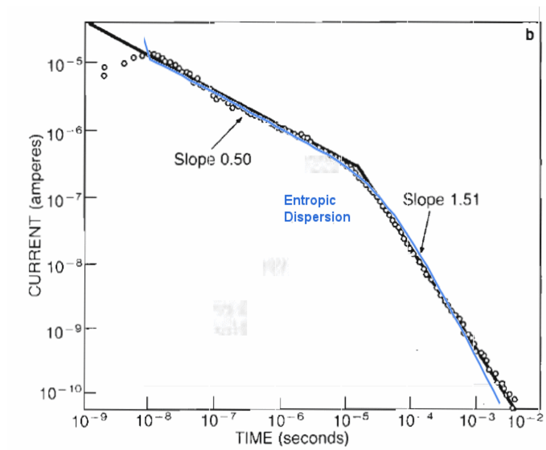


FIGURE 25-15. Time-of-flight measurement demonstrating the knee-in-the-curve separating the diffusion and drift regimes for a disordered semiconductor.

And nothing mysterious about that slope of 0.5; that results from the diffusion having a square root dependence with time.

For any one pathway, the advance in the particles motion has a diffusive component as well as a drift component. To discriminate between the effects of drift and diffusion one can vary the electric field. [Ref 259] provides a comprehensive study of the photo-response of amorphous hydrogenated Silicon (a-Si:H). This material was undoped and the investigation looked specifically at hole carriers. It leans toward questioning the applicability of Scher and Montroll's original formulation in terms of an alternate model that they formulate.

Take a close look at the figure below with data from[Ref 259], and observe how well the curve matches all the inflection points, and works over several orders of magnitude.

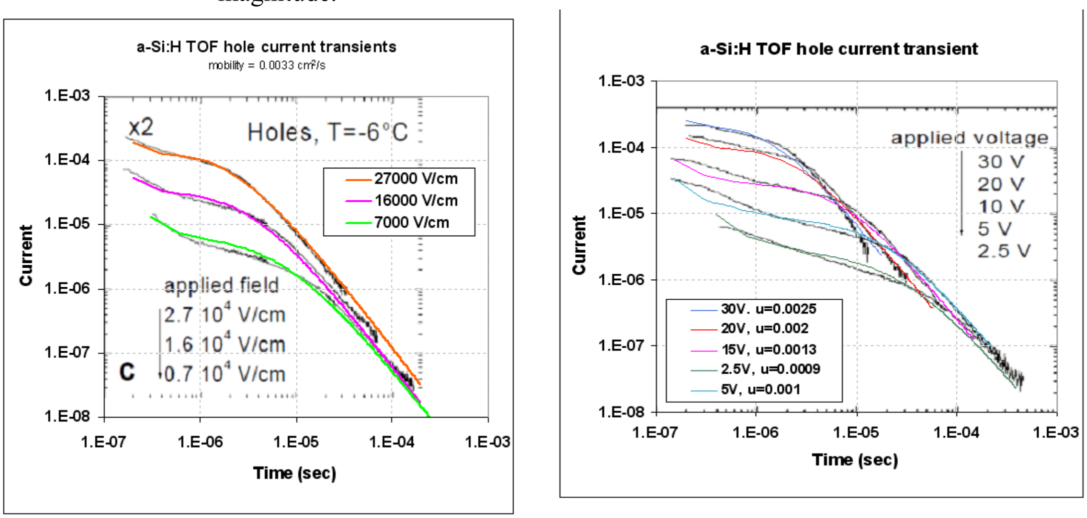


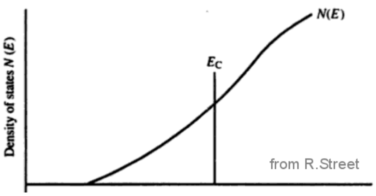
FIGURE 25-16. Dispersive transport which includes a term to describe the initial transient. Note the agreement of the dispersive transport model at short durations. Left curve fits a fixed *average* mobility sample. For the right curve, the average mobility depends on applied electric field strength.

You can work the fit in a more elaborate fashion; the more time you spend with better values of width and electric field, the better an estimate you can make of the average mobility, μ , and diffusivity, $\boldsymbol{D}$. Suffice to say, no extra factors play into the equations. For the curve fits to the above left, I used the coefficients as stated in the legend and the table below.

Mobility (u)	Width (w)	Diffusion constant (D)	Temperature (T)	Current Scaling (C)	Electric Field (E)
0.0033 cm2/V/s	2.4 microns	0.69 * μ	264	1.2e-13	varies as in figure (V/cm)

A few interesting issues surface as a result of this particular analysis.

(1) For one, in this case the relation between mobility and diffusion constant does not obey the Einstein relation but this rarely happens in non-ideal and disordered materials as the energy states get sufficiently smeared across the bandgap. The general Einstein relation relates the diffusion constant ***D*** to the energy distribution of the carrier states. [Ref 256]

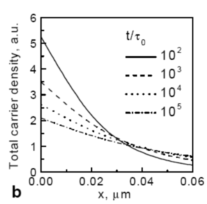


$$D = \frac{\mu}{q} \cdot \frac{N(E_c)}{\frac{\mathrm{d}}{\mathrm{d}E}N(E_c)} \qquad \textbf{(EQ 25-23)}$$

As the energy distribution follows an exponential Maxwell-Boltzmann distribution, the derivative retains the exponential, and the kT term drops out, leading to the conventional Einstein relation $D = \mu\, kT/q$

Diffusion exists in the absence of an electric field and so thermal energy acts as the only stimulus to allow a carrier to move to an adjacent site. For a narrow variation of E_c around the Boltzmann distribution[8], the relation $D = \mu\, /kT/q$ holds as an invariant, but as E_c spreads out — and in the maximum entropy case of a large variance knowing only the mean — the diffusion constant tracks E_c more than it does temperature, T. I worked this out and $D = \mu\,(kT+E_c)/q$ in that case. Since Ec typically exceeds the statistical value of thermal energy, kT, we will see a higher diffusivity than one would expect from an ordered solid (see [Ref 247]).

(2) Also, the initial transient spike having to do with fast diffusing carriers occurs at very short time frames. Because of this short time span, the transient waveform may become much more sensitive to capacitive effects. This means that the shape of this transient does not hold as much importance as the total charge underneath that part of the curve.



(b) carrier densities in a material with exponential DOS.

The authors apply their own model to the results and suggest that the dispersion is wider than gaussian as the figure to the left margin shows, yet they also curiously indicate that is a gaussian non-dispersive transport.

Much of the confusion arises from the original Scher-Montroll formulation which demarcates the curves into ordered or non-disordered instead of what I would like to see — a dispersed diffusion-dominated regime versus a dispersed drift-dominated regime.

8. The Maxwell-Boltzmann is an approximation to the actual Fermi-Dirac distribution at higher temperatures.

The upshot of the good agreement of my fundamental model with the results means that any smart electrical engineer can start using the simple formulation right now, and should that engineer want to calculate frequency response or impulse response of an amorphous material device, they just have to use these equations, either the Fokker-Planck derived ones or the entropic dispersion approximations. They can do FFT or Laplace transforms or anything they want since they have an analytical result which they can plop into their notebook or spreadsheet or Matlab and work out. I guarantee no one would want to mess with the Montroll-Scher result as it gets way too unwieldy and I dare say that no one actually understands it. I consider this simplicity a huge benefit. (The only caveat: you need a disordered material to apply this to.... but, of course, that goes with the premise.)

As I said before, no one in the semiconductor industry seems to use this simple dispersive formulation, preferring to hand-wave and heuristically account for the fat-tails of the transient. Importantly, this particular impulse response function both explains the behavior seen, and derives from the most simple particle counting statistics (i.e maximum entropy randomness), so it likely serves as the most canonical model for dispersive transport in disordered materials.

Many experimental results show the knee in the curve of Scher and Montroll, but with usually not much dynamic range. I looked at a few material studies done fairly recently to see how well the simple theory works.

Transport in SiO2

For verifying any theoretical formulation, you usually want to match the behavior to as wide a dynamic range as experimentally feasible. The larger the dynamic range in the measured quantities, the more confidence that you have in its worth or value.

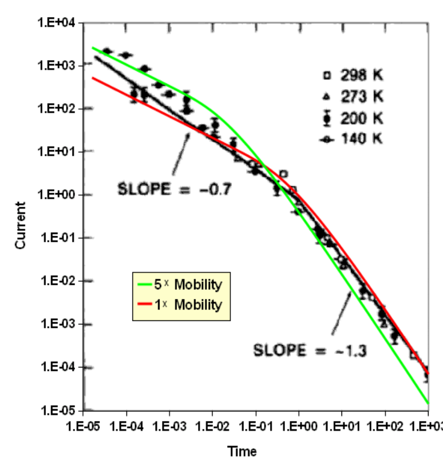
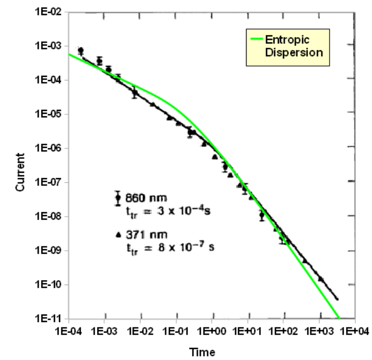


FIGURE 25-17. Dispersive transport via the maximum entropy model compared to SiO2 measurements. This shows mainly diffusion with the drift catching up at longer times.

FIGURE 25-18. The effect of changing the width of the transport layer. The Montroll-Scher knee does not show up prominently.



The case of carrier transport across SiO2 insulating layers for MOS devices provides some cases of amazing dynamic range, up to 8 orders of magnitude in current [Ref 250]. The fundamental idea remains the same in this situation as the photoresponse experiment, although a different form of ionizing radiation supplies the pulse of carriers — in this case holes become the charge carrier instead of electrons. Otherwise, the same diffusive transport occurs, with the authors trying to explain the results by applying the same unwieldy Scher-Montroll formulation. As a side note, these kinds of measurements need a delicate touch as the dose of the radiation can actually effect the field due to space charge formation[9].

In any case the fits to the data using the simple diffusive transport model works over a large dynamic range in ordinates. The sharp bend near the top indicates the potential start to the plateauing, and one can observe that some of the pairs of data indeed do flatten out. The other gradual bend indicates the transition between diffusion transport and drift transport. The universality of this bend does not scale perfectly as drift does depend on the electric field whereas the diffusion doesn't. And as we will see in the next example, the temperature may not play a big role in deviations from universality.

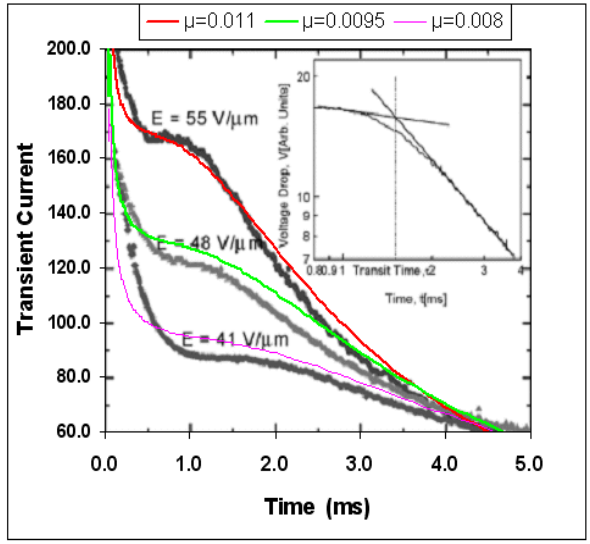
Quantum Dots

Scientists have looked to unique materials including a variety of organic semiconductors in the hope of creating structures suitable for quantum dot devices. This paper [Ref 260] provides a few time-of-flight curves in terms of a completely dif-

9. I did some experimental research on a related topic years ago where I tried to force dopant concentrations via ion bombardment into a growing junction and the bias of the junction alone pulled the mobile dopants from one side of the junction to the other. The key is that even though you see weird stuff happen, you can always explain it via some rather elementary considerations.

ferent material system. These TOF's appear to obey the same simple maximum entropy model for dispersive transport as you can see below.

FIGURE 25-19.
TOF traces taken at different applied electric fields. The original diagram did not have dimensions on the axis so I adapted the scaling based on the inset. The simple dispersive transport model is shown varying with electric field and a slight variation in mobility.



Of course good agreement means that the disorder in the systems has to agree with the maximum entropy model. Nothing precludes different diffusion mechanisms or even further disorder, implying even fatter tails than 1/***t***. Some systems likely exist with a mix of order and disorder, such as crystalline semiconductors with many defects. In that case, one could conceivably separate out the effects.

The Unification of Diffusion and Drift with Dispersion

Solving the Fokker-Planck equation (FPE) under maximum entropy conditions provides the fundamental unification between dispersion, diffusion and drift. For fans of Taleb and Mandelbrot, this shows directly how “thin-tail” statistics become “fat-tail” statistics without resorting to fractal arguments. The Fokker-Planck equation shows up in a number of different disciplines. Really, anything having to do with diffusion or drift has a relation to Fokker-Planck. Thus you will see FPE show up in its various guises: Convection-Diffusion equation, Fick's Second Law of Diffusion, Darcy's Law, Navier-Stokes, Shockley's Transport Equation, Nernst-Planck; even something as seemingly unrelated as the Black-Scholes equation for finance has applicability for FPE (where the random walk occurs as fractional changes in a metric).

Because of its wide usage, the FPE tends to take the form of a hammer, where everything it applies to acts as the nail.[10] Since the solution of FPE results in a probability distribution, it gives the impression that some degree of disorder prevails in the system under study. I find this understandable since the concept of diffusion implies an uncertainty exactly like a random walk shows uncertainty. In other words, no two outcomes will turn out exactly the same. Yet, in mathematical terms, the measurable value associated with diffusion, the diffusion constant ***D***, has a fixed value for random motion in a homogeneous environment. When the parameters actually change, you enter in the world of *stochastic differential equations*; I won't descend to deeply into this area, only to relate this as a basic concept. The diffusion and mobility parameters have a huge variability that we have yet adequately accounted for in many disordered systems.

For that reason, the FP equation really applies to ordered systems that we can characterize well. Not surprisingly the ordinary solution to FPE gives rise to the conventional ideas of normal statistics and thin-tails.

So for phenomenon that appear to depart from conventional normal diffusion, as the so-called anomalous diffusion, we have two distinct camps and corresponding solution paths to choose from. The prevailing wisdom suggests that an entirely different kind of random walk occurs (Camp 1). No longer does the normal diffusion apply, giving rise to normal statistics; instead we get the statistics of fat-tails and random walk trajectories called Levy flights to concretely describe the situation (see Figure 25-7 on page 487). The mathematics quickly gets complicated here and most of the results get cast into heuristic power-laws. It takes a leap of faith to follow these arguments.

The question comes down to whether we wish to ascribe anomalous diffusion as a strange kind of random walk (Camp 1) or simply suggest that heterogeneity in diffusional and drift properties adequately describes the situation (Camp 2). I take the stand in the latter category and stand pretty much alone in this regard. Find some academic research article on anything related to anomalous diffusion and very few will accept the most parsimonious explanation — that a range of diffusion constants and mobilities explain the results. Instead the researcher will punt and declare that some abstract Levy flight describes the motion.

Above all I would rather think in practical terms, and simple variability has a very pragmatic appeal to it, and understandability to boot.

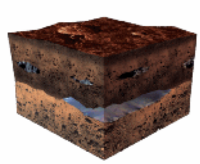
10. You don't see this more frequently than in finances, where Black-Scholes played the role of the hammer

CHAPTER 26

The Earth.

Randomness in ground and topography

"We scorn the abstract, we scorn it with a passion."
— N.N. Taleb



Like the sun, the earth's core will remain a remain a reliable source of energy for the foreseeable future, pending any galactic calamities. But since the earth gives up its heat energy grudgingly and at its own pace, we have to show patience in extracting the heat and adapting it for practical uses. This section simplifies some of the discussion relating to practical limits on increasing the thermal extraction rates. Entropy once again rules, and we have to learn to deal with it.

But first, I will go through the derivation of what I consider a very overlooked and simple argument having to do with the transport of materials in porous media — much as what you would find in tracing a contaminant though a groundwater basin. Or what may happen if you "frac" for natural gas and open up new pathways to a drinking water aquifer. Or how oil will migrate to a reservoir over time, feeding the production output of a stripper well for years. Or what happens if you spill oil in a waterway.

Solute Transport through Porous Media

Hydrologists study the movement of water through the ground, gaining knowledge from measurements of carefully constructed and controlled experiments. To do the measurements in a fresh and uncharacterized location could take quite a bit of preparation and effort, so anything that they can do in the analysis realm will help in understanding unexpected variations in flow rates. With that knowledge, they can anticipate how water will flow or drain in any particular area.

Obviously, oil has many of the same fluid properties as water and fundamentally obeys the same flow laws. The dynamics of water and oil flow has huge ecological significance when we consider the effect of chemical and oil spills. Furthermore it can help answer the question of how underground hydraulic fracturing, intended to loosen up fossil fuel deposits, could also lead to contamination of aquifers.

Yet looking through the literature, we see very little concise analysis of the effects of flow in disordered environments. A large watershed, underground reservoir, or aquifer has natural variability built in and researchers struggle to make analytical sense of the flow data. They perform massive simulations with complicated configurations and detailed physical models, and attempt to calibrate against the empirical data. Many seem to claim success, but other researchers such as Lange[Ref 258]and Haag [Ref 257] say the results show little sensitivity to the specifics of a model. So, perhaps we need to step back and consider how we can use entropy to simplify the analysis, particularly if the complexity of the environment verges on disorder.

Putting this another way, we can try to reason about variations in gravel, fractures, rock, and soil up until we realize that it all amounts to a pile of entropic rubble. And in making that assumption, we can leverage some very simple principles and actually create what amounts to universal behaviors that we can use to describe the observations. What would normally occupy an entire PhD thesis due to its potential complexity, we can effectively reduce and thus describe very concisely[1].

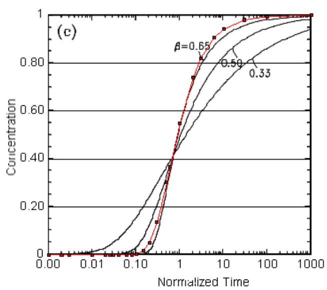
Dispersive Transport in Porous Media.

I got sidetracked into the dispersive transport behavior of carriers in disordered solid-state materials as I searched for ideas that might substantiate the oil depletion models that I had worked on. I have long asserted that much of the behavior of oil, from reservoir sizes, to oil discovery, and on to reserve growth has as a basis the effects of dispersion. Just as the disorder in amorphous semiconductors causes a dispersion in carrier velocities, as we saw in the last chapter, so too does the randomness and disorder in aspects of the macro process. We essentially derived a new math shorthand to describe oil, and generalized this derivation to apply equally well to a field totally removed from the macroscopic. The math essentially went from the macroscopic to the microscopic, and now back again to substantiate the original conjecture.

1. This concise reduction is typical of behavior governed by entropic complexity

Whether the randomness of dispersion has to do with varying velocities in the drift of oil over eons or the variance of human search efforts, these all lead to the same fundamental formulation for dispersive analysis. Moreover, any chaotic or complex behavior gets smoothed out by the filter of dispersion. That has become a recurring theme in the analysis.

Recall again the two scientists Scher and Montroll, who originally formulated the CTRW theory to explain dispersive carrier transport, which we laid out in the previous chapter. They essentially worked as applied mathematicians and had gained quite a bit of recognition for their ideas. Since their original research in 1975, Scher[2] continued applying the same formalism to other application areas including … you guessed it … the transport of materials underground via porous structures.



And this defines pretty much the same life-cycle that petroleum operates under. So Scher essentially transitioned from the microscopic world of semiconductors to the macroscopic world of the earth. In this century Scher worked as a consultant for a group of geologists and environmental scientists that use the CTRW theory to explain the way that contamination and other solutes spread over time via diffusive transport. Since then, others have used CTRW theory in the context of geological materials, see [Ref 263][Ref 264]

As I noted earlier, I have problems with the CTRW theory in that at a certain step in the derivation, the authors invoke the legendary "and then a miracle occurs" argument into the proof. This turns into the essential observation that long-range correlations go as $\frac{1}{t^{\alpha}}$, where α serves as a disorder parameter.

In our formulation we introduce disorder by invoking the maximum entropy principle on the variance of velocities. In this case, the inverse time power-law behavior naturally takes over and an integral exponent depending on the mean velocity from the specific type of motion occurring — either diffusion (t^{-1}) or drift (t^{-2}). If a combination of the behaviors occurs, just solve the classical equations of motion assuming Fick's Law and calculus, and the characteristic dispersive formula appears, just as for dispersive transport in amorphous semiconductors.

2. Montroll held advanced research director positions at IBM Research, Institute for Defense Analysis, and Office of Naval Research.

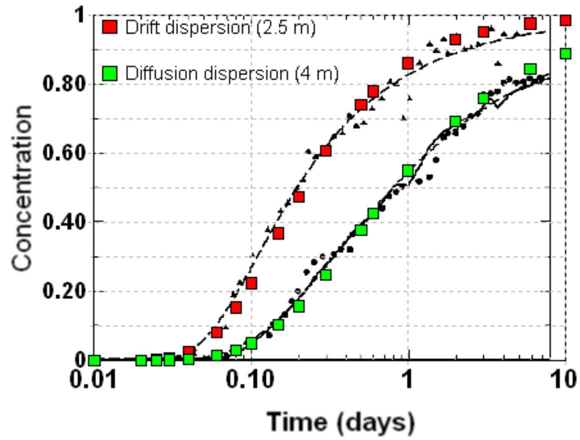


FIGURE 26-1. Application of the dispersive transport to the motion of solute. This experiment showed a transition as the solute migrated over time. Analysis of Tracer Test Breakthrough Curves in Heterogeneous

FIGURE 26-2. Uranine dye moving downstream in Fisher Creek after injection to trace the destination of the water as it disappears. [Ref 268])

Double Breakthrough

1. Consider a contaminant that enters an aquifer in a single dose
2. Predict how long it will take to pass by a downstream location
3. How do you solve this problem?

A large scale experiment typically looks like this scenario to the left.

And you get a result that looks like the following figure. Intuitively, one would expect that the concentrated dose will disperse as it travels downstream and that the original concentration will spread out in time. The red curve that goes through the data gives you a feel for what I will derive via a simple model.

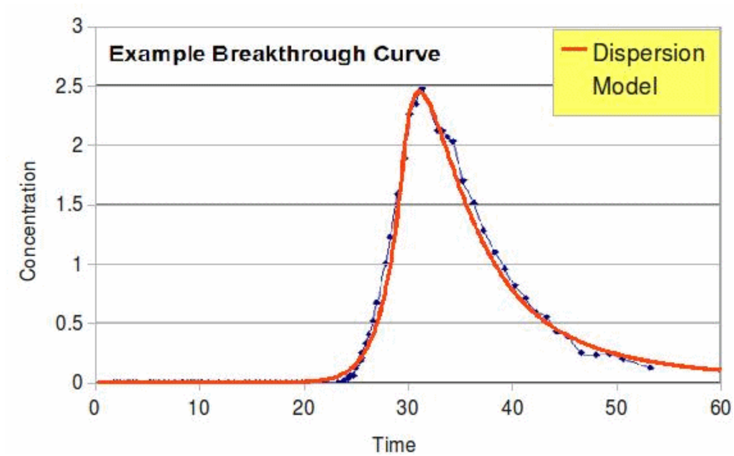
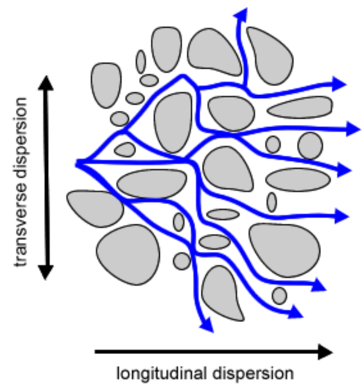


FIGURE 26-3. Random breakthrough curve example

As a main premise, I assume that disorder plays a big role in providing a variety of pathways from source to sink. One can imagine that some paths might occur on the main waterway, providing a maximum speed or path of least resistance. Other paths may follow obstructions or diversions which will either slow down or speed up the flow from the main path.

FIGURE 26-4. The illustration shows some of the causes of pore-scale dispersion. Solute traveling more tortuous pathways between sediment grains will move more slowly than that moving along more direct pathways. Diverging pathways will also cause the contaminant to spread perpendicular to the aquifer flow direction.



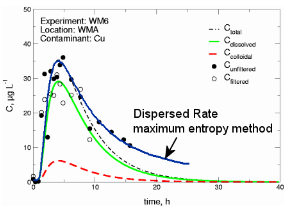
Most of the solute transport measurements use something called *breakthrough curve* analysis. Measuring breakthrough curves enable a researcher to estimate the amount of dispersion occurring in a flow of solute (or contamination or whatever) in a media. This has an analog to the time-of-flight measurements used in photo-response studies described earlier.[3]

For a nondispersive flow, the breakthrough curve looks like a unit step where the tracer material is detected abruptly at a specific time at a certain point downstream. But due to randomness and variability in the media due to pore structures (for example), the dispersion smears the breakthrough curve over a broad time window.

A very simple model for a breakthrough curve involves solving the equation for a maximum entropy spread in velocities (for a given mean velocity) at a specific distance L. Given the average time taken is $\boldsymbol{T=L/v}$, and a random variate would take time = $\boldsymbol{t}$, then the breakthrough curve looks like $\exp(\boldsymbol{-L/vt})$. If you plot this curve it looks like what some people refer to as a *reciprocal exponential*. It doesn't follow the classic exponential because the time parameter goes in the denominator. That happens because we deal with rates, and not time as the stochastic variant.

3. One significant difference is that solute does not induce a charge of current that we can measure. Instead the solute is measured directly as a concentration density.

Next, we must realize that an idealized breakthrough curve assumes a *fixed* separation, $\boldsymbol{L}$, in a very controlled experimental environment. In reality, the distance $\boldsymbol{L}$'s become spread out over space and for a maximum entropy PDF of $\boldsymbol{L}$, an uncontrolled cumulative breakthrough curve will have a temporal behavior that looks like $1/(1+\boldsymbol{L}/\boldsymbol{vt})$, where $\boldsymbol{L}$ becomes the mean separation, or $e^{-\boldsymbol{L}/\boldsymbol{vt}}$ if we have a fixed $\boldsymbol{L}$. This looks exactly like the formula for enigmatic reserve growth in oil discoveries that I derived before in “The Reserve Growth. How estimates of oil evolve” .



I rationalize using a maximum entropy estimator for the dispersion because the effects could occur due to a range in factors. So, in a sense, variability overrules complexity and if we can concentrate on understanding the mean value, we have a very simple way to characterize the system. Take a look at the figure in the margin from a hydrogeology experiment [Ref 262]. Granted, I do not know anything about the particulars of the particular experiment, yet I assert that I can do a better job of fitting to the results solely because I do not place a bias on my estimator. I simply apply maximum entropy to randomize the effect, making the only assumption the mean transport rate.

I see some indication that Scher and his colleagues have at least considered this simple premise. From the following extract, note that they imply that some sort of ensemble average acts as a precondition to further analysis. In other words, they make the presupposition that geology is random and uncontrollable, just like amorphous semiconductors and human processes.

> The point average of v and D can be very sensitive to small changes in the local volume used to determine the average. Conversely, if one fixes the volume to a practical pixel size (e.g., 10 m3) the use of a local average v and D in each volume can be quite limited, i.e., the spreading effects of unresolved residual heterogeneities are suppressed [e.g., Dagan, 1997]. We will return to this issue in a broader context in section 4. It essentially involves the degrees of uncertainty and its associated spatial scales. We start, at first, with an ensemble average of the entire medium and discuss the role of this approach in the broader context. [Ref 261]

The Fokker-Planck equations used for electrical charge transport can actually apply directly as solutions to Darcy’s law when it comes to describing the flow of material in a disordered porous media[4]. Yet we should not act surprised by this result. The actions of multiple processes acting concurrently on a mobile material will generally result in a universal form governed by maximum entropy. It doesn't matter if we model carriers in a semiconductor or particles in a medium, the result will largely look the same. In a hydraulic conductivity experiment, Lange treated the

4. To the petroleum engineers, hydrologists, and geologists who think they own the solution to this problem, that is rarely the case when it comes to fundamental physics.

breakthrough curve of a trace element through a natural catchment as a FPE convection-dispersion model, and came up with the same results independent of the fractionation of the media [Ref 258].

By applying the simple dispersion model (blue curve below) to Lange's results, one sees that an excellent fit results with the fat-tail exactly following the *hyperbolic decline* that reservoir engineers often see in long-term flow behavior.!

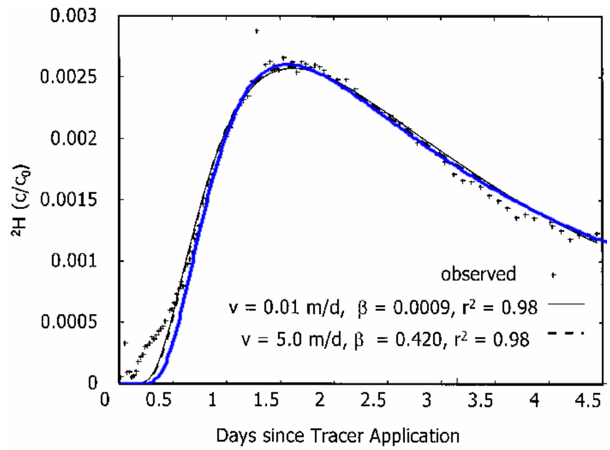


FIGURE 26-5. Breakthrough curve of a traced material showing results from an entropic dispersion model in blue. The discrepancy at small times gets resolved in a later analysis.

Moreover, the amount of diffusion that occurs appears quite minimal. For this case, adding a greater proportion of diffusion does not improve the overall fit of the curve. Just as in the semiconductor case, the shape has a significant meaning when analyzed from the perspective of maximum entropy. Nothing complicated about this other than admitting to the fact that heterogeneous disordered systems appear everywhere and we have to use the right models to characterize their behavior.

The details of this particular experiment are described in two papers [Ref 257][Ref 258]. The authors have mixed feelings about the applicability of modeling *bio-geochemical* systems and speculate whether we should use any kinds of models for "ecological risk assessment". They point out that ecological systems obviously can adapt under certain circumstances and no amount of physical modeling can predict which way the system will go. Will spilled oil decompose faster as the environment adapts around it? Will that make dispersion less relevant? They leave the interpretation open.

Still the work of modeling the physical process alone has enormous value as Haag and Kaupenjohann point out:

> *Despite not being a 'real' thing, "a model may resonate with nature" (Oreskes et al. 1994) and thus has heuristic value, particular to guide further study. Corresponding to the heuristic function, Joergensen (1995) claims that models can be*

> *employed to reveal ecosystem properties and to examine different ecological theories. Models can be asked scientific questions about properties. According to Joergensen (1994), examples for ecosystem properties found by the use of models as synthesizing tools are the significance of indirect effects, the existence of a hierarchy, and the 'soft' character of ecosystems. However, we agree with Oreskes et al. (1994) who regard models as "most useful when they are used to challenge existing formulations rather than to validate or verify them". Models, as 'sets of hypotheses', may reveal deficiencies in hypotheses and the way biogeochemical systems are observed. Moreover, models frequently identify lacunae in observations and places where data are missing (Yaalon 1994).*
>
> *As an instrument of synthesis (Rastetter 1996), models are invaluable. They are a good way to summarize an individual research project (Yaalon 1994) and they are capable of holding together multidisciplinary knowledge and perspectives on complex systems (Patten 1994).*
>
> *While models as a product may have heuristic value, we would like to emphasize also the role of the modeling process: "[...] one of the most valuable benefits of modeling is the process itself. These benefits accrue only to participants and seem unrelated to the character of the model produced" (Patten 1994). Model building is a subjective procedure, in which every step requires judgment and decisions, making model development 'half science, half art' and a matter of experience (Hoffmann 1997, Hornung 1996). Thus modeling is a learning process in which modelers are forced to make explicit their notions about the modeled system and in which they learn how the analytically isolated components of a system can be 'glued' (Paton 1997). As modeling mostly takes place in groups, modeling and the synthesis of knowledge has to be envisaged as a dynamic communication process, in which criteria of relevance, the meaning of terms, the underlying concepts and theories, and so forth are negotiated. Model making may thus become a catalyst of interdisciplinary communication.*
>
> *In the assessment of environmental risks, however, an exclusively scientific modeling process is not sufficient, as technical-scientific approaches to 'post-normal' risks are unsatisfactory (Rosa 1998) and as the predictive capacity and operational validity of models (e.g. for scenario computation) is in doubt. The post-normal science approach (Funtowicz & Ravetz 1991, 1992, 1993) takes account of the stakes and values involved in environmental decision making. Following a 'post-normal' agenda, model development and model validation for risk assessment should become a trans-scientific (communication) task, in which "extended peer communities" participate and in which nonequivalent descriptions of complex systems are made explicit, negotiated, and synthesized. In current modeling practice, however, models are highly opaque and can rarely be penetrated even by other scientists (Oreskes, personal communication). As objects of communication, models still are closed systems and black boxes.*

We need to really take up the charge on this as our future depends on understanding the role of entropy in nature. For too long, we have not shown the intellectual curiosity to model how much oil we have underground, what size distribution the reservoirs take, and how fast that they can empty[5], even though some perfectly acceptable models can describe this statistically, using dispersion no less!

Disordered Behavior

We can elementarily describe the process of disordered flow. In a well-drained watershed, a primary stream will provide a well-defined maximum speed as well as many alternate pathways that move more slowly.

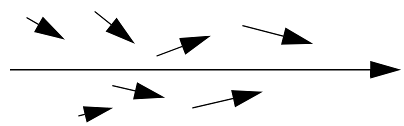


FIGURE 26-6. Velocity vectors describing flow of particles in liquid through a watershed or porous material. The actual structure does not matter much as long as it shows disorder.

If a well-defined primary stream does not exist, but we can deduce that an average flow does occur, the Maximum Entropy Principle would suggest that the probability density of the velocity flow would approximately follow an exponential distribution where v_0 indicates the average flow:

$$p(v) = \frac{1}{v_0}e^{-v/v_0} \quad \textbf{(EQ 26-1)}$$

This dispersive effect essentially describes the concept of disordered drift, disordered advection, or disordered convection, terms which essentially identify the same behaviors but which appear in different disciplines. The distinguishing feature between the forms lies in the driving force and the particle responding to the force — gravity for liquid or solute and an electric field for charged particles.

Additionally, the concept of diffusion describes the field-independent motion of the solute, as it spreads from regions of high concentration to regions of low concentration.

The Fokker-Planck equation provides the general form for describing diffusion and drift.

5. See the Macondo Gulf oil disaster.

$$\frac{\partial}{\partial t}n(x, t) = -\mu F \cdot \frac{\partial n}{\partial x} + D \cdot \frac{\partial^2 n}{\partial x^2} - \frac{n}{\tau} \quad \textbf{(EQ 26-2)}$$

The last term describes loss of particles to the environment either through environment capture or carrier charge recombination.

In one dimension, the solution to Fokker-Planck assuming a delta function stimulus at time t=0 is the following:

$$n(x, t) = \frac{1}{\sqrt{4\pi D t}} \cdot e^{\frac{-(x-\mu F t)^2}{4Dt}} \cdot e^{-t/\tau} \quad \textbf{(EQ 26-3)}$$

This does work well for a homogeneous or uniform environment, but what happens if we allow both the diffusion coefficient and mobility coefficient to vary? In the case of electrical conduction, the two coefficients likely vary in unison according to the Einstein relation D/μ=kT/q. In the last chapter, I solved this assuming a common probability distribution for D and μ. But in the more general case, one can imagine that the two coefficients will vary independently. For the maximum entropy condition of assuming only knowledge about the average values of the coefficients, we can actually solve this analytically.

MaxEnt Solution to Fokker-Planck

First let us assume that D has a mean value D_0 and some probability density function. Then we can integrate the solution over all possible values of D

$$n(x, t) = \int_0^\infty p(D) \cdot n(x, t|D)dD \quad \textbf{(EQ 26-4)}$$

For the maximum entropy condition, the diffusion coefficient varies as:

$$p(D) = \frac{1}{D_0} e^{-D/D_0} \quad \textbf{(EQ 26-5)}$$

Solving this requires an amazingly useful integration identity,

$$\int_0^\infty \frac{e^{-Ax-\frac{B}{x}}}{\sqrt{x}} = \sqrt{\frac{\pi}{B}} e^{-2\sqrt{AB}} \quad \textbf{(EQ 26-6)}$$

so the result drops out as the symmetrically pleasing

$$n(x, t) = \frac{e^{-|x - \mu Ft| / \sqrt{D_0 t}}}{2\sqrt{D_0 t}} \qquad \textbf{(EQ 26-7)}$$

For the drift or convection term we can substitute an effective velocity to the mobility-force term:

$$v = \mu F$$

$$n(x, t|v) = \frac{e^{-|x - vt| / \sqrt{D_0 t}}}{2\sqrt{D_0 t}} \qquad \textbf{(EQ 26-8)}$$

and then consider the maximum entropy distribution of velocities:

$$n(x, t) = \int_0^{\infty} p(v) \cdot n(x, t|v) dv$$

$$n(x, t) = \int_0^{\infty} \frac{1}{v_m} e^{-v/v_m} \cdot \frac{e^{-|x - vt| / \sqrt{D_0 t}}}{2\sqrt{D_0 t}} dv \qquad \textbf{(EQ 26-9)}$$

Because of the analytically integrable exponential form, this should be trivial to solve, but the absolute value requires us to keep track of the slope discontinuities and solve it in two parts, one part for $x > vt$ and for $x < vt$. When worked out, the result turns out only slightly complicated, although certainly not something you will find referenced in any textbooks or reference books:

$$n(x, t) = \frac{e^{-x/\sqrt{D_0 t}}}{2 v_m \sqrt{D_0 t} \cdot \left(\frac{1}{v_m} - \sqrt{\frac{t}{D_0}}\right)} - \frac{e^{-x/v_m t}}{\left(\frac{1}{v_m}\right)^2 - \frac{t}{D_0}} \qquad \textbf{(EQ 26-10)}$$

If we set D_0 arbitrarily close to zero, then the concentration profile reduces to a much simpler expression:

$$n(x, t) = \frac{e^{-x/v_m t}}{v_m t} \qquad \textbf{(EQ 26-11)}$$

We will get back to this later through an alternate derivation. For now, we should note the effect of the diffusion coefficient on the tail behavior. The following figure shows the difference in the concentration profile with and without diffusion ($x = v_m = 1$). This has a fairly broad concentration dispersion, largely because of the maximum entropy spread in velocities.

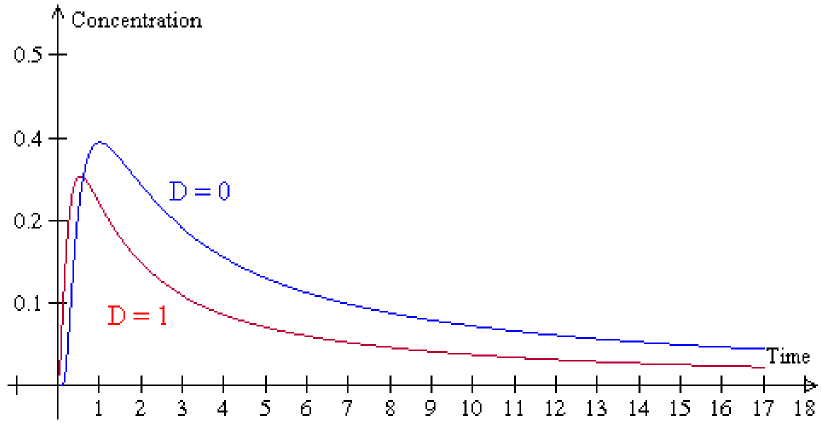


FIGURE 26-7. Model concentration profiles of a breakthrough curve with and without a diffusive component. The diffusion allows certain particles to reach the destination more quickly.

Interestingly, the tail behavior on rescaling does not noticeably change. At long times the drift component will necessarily overcome the fast diffusers.

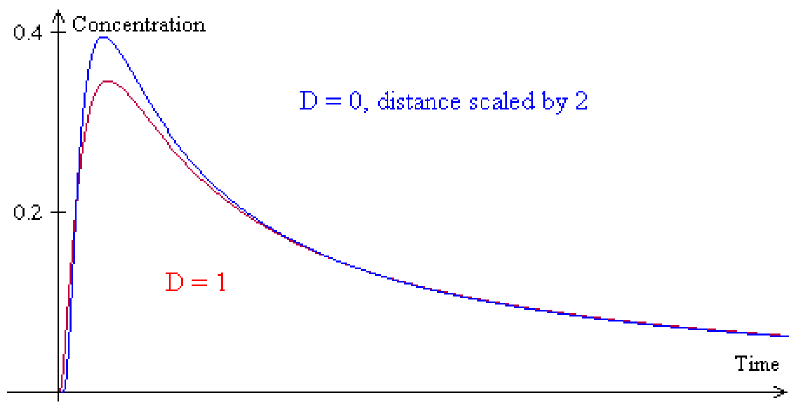
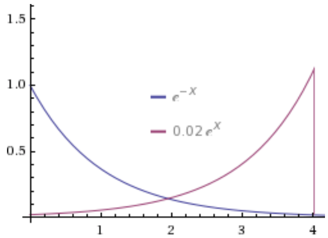


FIGURE 26-8. Model concentration profiles of a breakthrough curve with and without a diffusive component. The distance is rescaled to align the peaks and the differences become less distinct.

We can also integrate the disordered formulation of the Fokker-Planck equation to account for velocity profiles other than the MaxEnt declining exponential. In many situations, the maximum entropy constraints should consider more than just an average velocity. For the consideration of a primary flow, we can imagine that a maximum velocity would occur and all other velocities would range below this maximum value.

$$p(v) = ke^{\alpha v} \qquad v < v_0$$
$$p(v) = 0 \qquad v > v_0$$
(EQ 26-12)



In this model, the coefficient α describes the sharpness of the velocity profile, which approaches a delta function at $v = v_0$ for large α. The solution requires an explicit piecewise evaluation

$$x > v_0 t \qquad n(x,t) = K \cdot \frac{e^{\alpha x\sqrt{t/D}/t - \alpha v_0} \cdot \left(e^{v_0 \cdot (\alpha - \sqrt{t/D})} - e^{x \cdot (\alpha - \sqrt{t/D})/t}\right)}{t \cdot (\sqrt{t/D} - \alpha)}$$
$$x < v_0 t \qquad n(x,t) = K \cdot \frac{e^{-\alpha v_0 - x/\sqrt{Dt}} \cdot \left(e^{v_0 \cdot (\alpha + \sqrt{t/D})} - 1\right)}{\sqrt{Dt} \cdot (\sqrt{t/D} + \alpha)}$$
(EQ 26-13)

Although complicated, this reproduces much of the fat-tail behavior one would expect from dispersion. Even with a strong primary flow component, the dispersive element does appear in what experimental analysts refer to as the classical break-through curves.

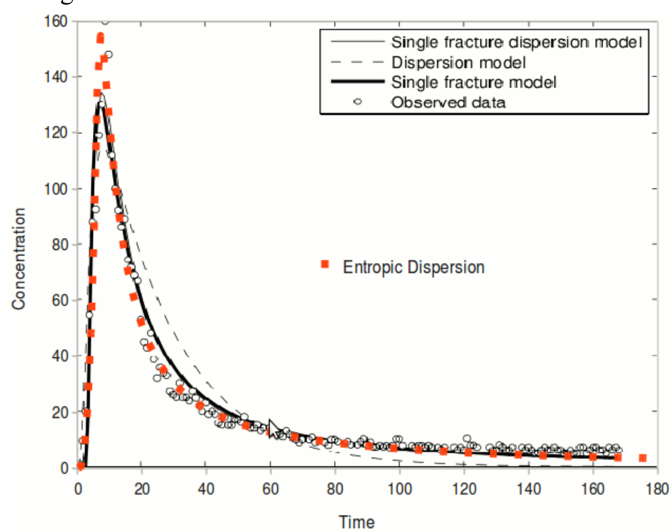


FIGURE 26-9. The following data is from Mathias [Ref 265]. It shows the amount of tracer solute measured at a downstream location from the injection point.

The tail of the data points lies a bit higher than the model would suggest. The following figure uses a log scale where you can see the measurement noise.

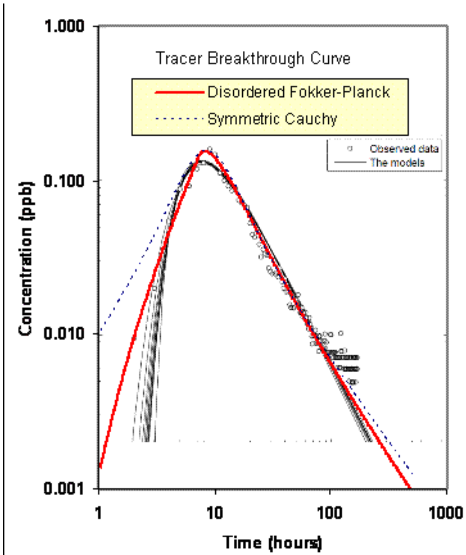


FIGURE 26-10. Same data and model plotted on a logarithmic scale.

In general, the literature shows models with too much complexity and we can get much better agreement with simple entropy arguments.

Breakthrough Curve Simplified

As a relatively simple concept, the breakthrough experiment introduces a tracer material (which could represent a contaminant or a solute) at some point in the watershed, and then the analyst collects measurements of solute concentration downstream at some point X. The deconvolution of the measured concentration, while allowing for diffusion and drift will tell us how long the solute takes to travel the course. If analyzed correctly, this will also generate a velocity distribution, and thus a direct indication of the amount of disorder in the watershed's pathways.

Based on the principle of least action, only one route will reach the destination in the fastest time, following the path of gravity and winding around any obstacles. This will give a maximum velocity $v_0 = X/t_s$, where t is the shortest time measured for the fastest particle detected.

If we disallow diffusion and recombination, the Fokker-Planck equation actually reduces to the following expression for a delta stimulus:

$$n(x, t) = \int_0^{\infty} p(v) \cdot \delta(x - vt) \cdot dv \qquad \textbf{(EQ 26-14)}$$

for the solute current flow, which we need for rate measurements, it reduces to

$$J(x, t) = \int_0^{\infty} vp(v) \cdot \delta(x - vt) \cdot dv \tag{EQ 26-15}$$

We can solve these equations quite trivially and thus use them to provide some sanity checks to the full Fokker-Planck equation and perhaps use it for more complicated velocity profiles. Using the properties of the delta function

$$\begin{aligned} n(x, t) &= (1/t) \cdot p(x/t) \\ J(x, t) &= (x/t^2) \cdot p(x/t) \end{aligned} \tag{EQ 26-16}$$

This remarkably simple formulation allows us to directly apply the distribution of velocities and through a parameter transformation, determine the expected downstream concentration. This would also work for the electrical current analog to first order if we didn't consider diffusion (which is problematic due to the Einstein relation). However, since diffusion works as a second order effect in a watershed with a strong primary flow, we can get away with this approximation.

We can see how this works in a typical application. First, let us assume that a strong primary flow exists with maximum velocity v_0. Then make the model that deviations from the primary velocity occur as a drag term, v_{drag}.

$$v = v_0 - v_{\mathrm{drag}} \tag{EQ 26-17}$$

We can approximate the drag term by either of two ways. Initially, we can say that the drag itself has an average value and then an entropic distribution about this point.

$$p(v_{\mathrm{drag}}) = \frac{1}{v_d} e^{-v_{\mathrm{drag}}/v_d} \tag{EQ 26-18}$$

alternately, we can derive the drag velocity as a ratio distribution from the changes in path over the changes in time over many intervals.

$$p(v_{\mathrm{drag}}) = \frac{p(\Delta x)}{p(\Delta t)} \tag{EQ 26-19}$$

This leads to a scale-free approximation to the drag velocities:

$$p(v_{\mathrm{drag}}) = \frac{v_d}{(v_d + v_{\mathrm{drag}})^2} \tag{EQ 26-20}$$

We can then invert this value, and come up with a probability distribution for the drift component

$$p(v) = \frac{v_d}{(v_d + v_0 - v)^2} \quad \textbf{(EQ 26-21)}$$

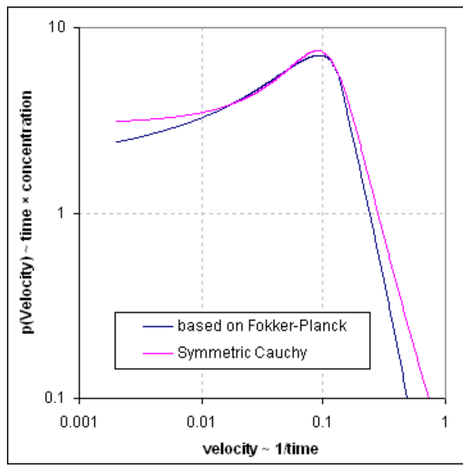
The profile of this distribution for $v_d \ll v_0$ shows an accelerating increase from a finite probability for small velocities to a sharp maximum when $v = v_0$. We can readily convert this to a concentration profile that can measure in a breakthrough curve experiment.

$$n(x, t) = (1/t) \cdot p(x/t)$$

$$n(x, t) = \frac{v_d/t}{\left(v_d + v_0 - \frac{x}{t}\right)^2} = \frac{v_d \cdot t}{((v_d + v_0) \cdot t - x)^2} \quad \textbf{(EQ 26-22)}$$

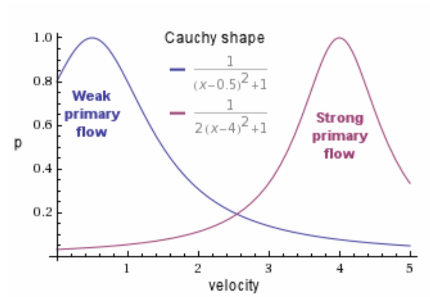
This has the characteristics of a Cauchy or Lorentzian profile which goes as a symmetric $1/t^2$ shape

FIGURE 26-11. Full Fokker-Planck model and Cauchy approximation



Again, this derivation ignores the effects of diffusion, but as we showed in the exact solution to the Fokker-Planck for a disordered system, diffusion has a narrow range of impact. Diffusive spread can start out strong by broadening the profile quickly, but the drift component sustains the shape for longer times.

FIGURE 26-12.
Velocity profile showing dispersion around the primary flow



With some finite probability, a particle can leave the system. The last term in the Fokker-Planck equation represents this possibility with a time constant τ. We can easily add this term to our Fokker-Planck solution by inspection. We start with our proposed modification, which involves a multiplicative exponential with the time constant τ.

$$n(x, t|\tau) = n(x, t) \cdot e^{-t/\tau}$$

$$\frac{\partial}{\partial t}n(x, t|\tau) = \frac{\partial}{\partial t}(n(x, t)) \cdot e^{-t/\tau} + n(x, t) \cdot e^{-t/\tau} \cdot (-1/\tau) \qquad \textbf{(EQ 26-23)}$$

$$\frac{\partial}{\partial t}n(x, t|\tau) = 0 + n(x, t|\tau) \cdot (-1/\tau)$$

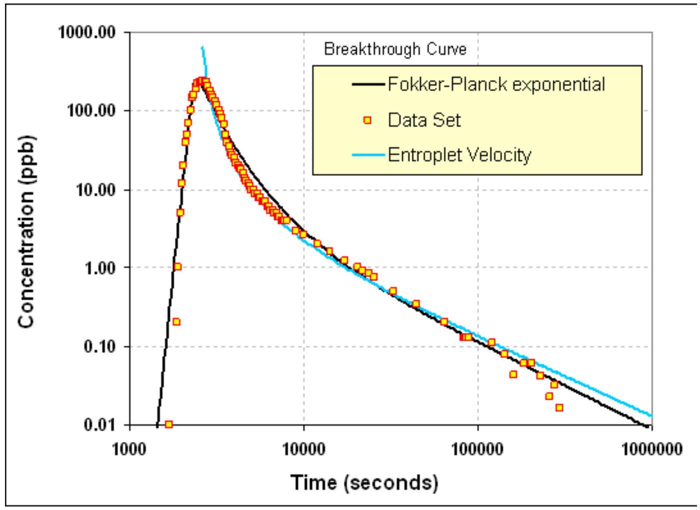
Yet, the Fokker-Plank equation adds a $n(x, t)/\tau$ factor to account for the particle loss so our proposed multiplicative term exactly balances the Fokker-Planck equality. This has significance in that we can actually estimate the particle loss time constant from the residuals of the curve fit. Our fat-tails will not go on forever, and this likely corrects for the difference between experiment and the fit of the entropic drift.

Experiment

We can plot the previous equation and compare it to the measurements from a breakthrough curve experiment. The exact nature of the experiment shouldn't matter as long as it travels through a disordered environment, such as a natural watershed or through mixed porous or fractured earth.

The first set of data comes from Benson [Ref 266]. Importantly, only two degrees of freedom control the fit of the curve. The first controls the peak concentration time and the second does the density scaling. This turns it into a very simple and potentially universal fit to any entropic flow.

FIGURE 26-13. Flow measurements from Benson et al. alongside Fokker-Planck and skewed entropic dispersion. [Ref 266]



We can also invert the concentration profile and generate the velocity profile.

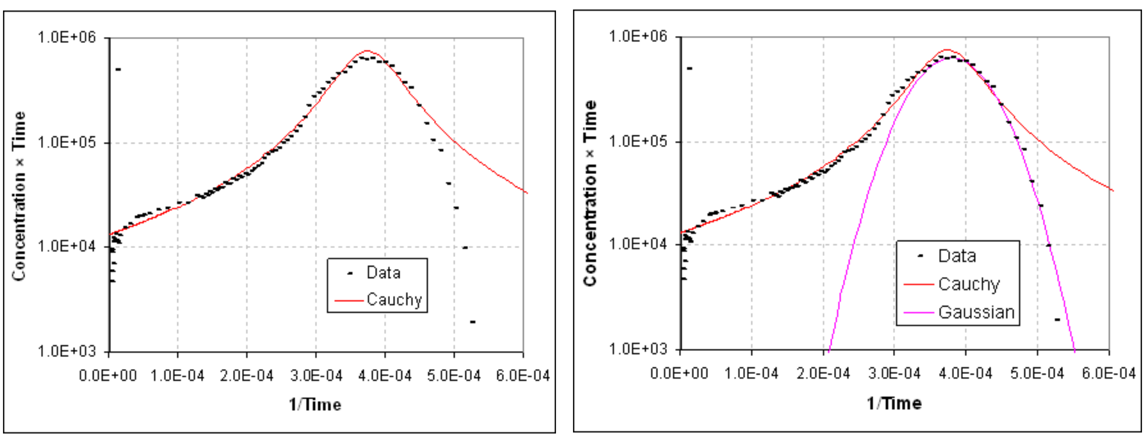
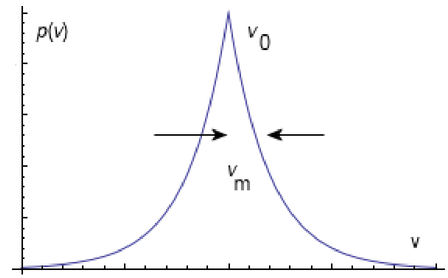


FIGURE 26-14. (left) Inversion of time scale allows one to recover the velocity distribution. (right) Plotted alongside a gaussian demonstrates the skewness of dispersion with diffusion likely responsible for the leading (fast velocity) edge.

Another set of data comes from a very carefully controlled laboratory experiment by Borgne[Ref 267]. I model this as a main path that has a mean velocity v_0 and the other paths have probabilities that range below this, with some mean deviation v_m

from v_0. A distribution that maximizes entropy while holding to these two minimal constraints looks like the following graph.

FIGURE 26-16. MaxEnt velocity distribution for absolute mean deviation



Next we apply the Maximum Entropy Principle to generate a velocity distribution as shown in the figure above:

$$p(v) = \frac{1}{v_m} e^{-|v - v_0| / v_m} \qquad \text{(EQ 26-24)}$$

No other distribution has a higher entropy given that mean and an absolute deviation from the mean, so it ranks as the least biased estimator for that set of constraints. (Note that this does not describe the normal or Gaussian distribution as that requires a second-moment, i.e variance, constraint. It turns out that the mean deviation distribution, also known as the Laplace, is actually a smeared Gaussian where we have MaxEnt uncertainty in σ-squared. So Laplace entropy is higher than the Gaussian entropy)

To generate a concentration at some downstream location ***x***.

$$n(x, t) = \frac{1}{v_m t} e^{-|x / (v_m t) - v_0 / v_m|} \qquad \text{(EQ 26-25)}$$

The experiment relied on a customized apparatus for making precise measurements of the contaminant, a fluorescent dye called uranine. The value of this particular experiment lies in the large dynamic range of the resultant data. The concentration runs over 4-orders of magnitude and the time scale 2-orders. Their own model, although generating a good fit to the data, needed a numerical calculation to solve, violating my assertion that we can model via simpler mechanisms.

The following figure allows for the wide dynamic range by plotting the concentration (also known as a breakthrough curve) on a log-log scale. The red triangles fit the Maximum Entropy dispersion model, ***n***(***x***,***t***), for a fixed value of x and a value of $v_m/v_0 = 0.18$. By inverting the concentration we can get the probability distribution of velocities in the bottom figure; on a semi-log plot a symmetric two-sided exponential looks like a perfect isosceles triangle. Based on the outstanding fit and symmetric distribution I find it blatantly obvious that entropic mechanisms generate the dispersion observed. You won't get this parsimonious a fit from such a simple model — with essentially a single parameter v_m/v_0 — unless it has some real merit.

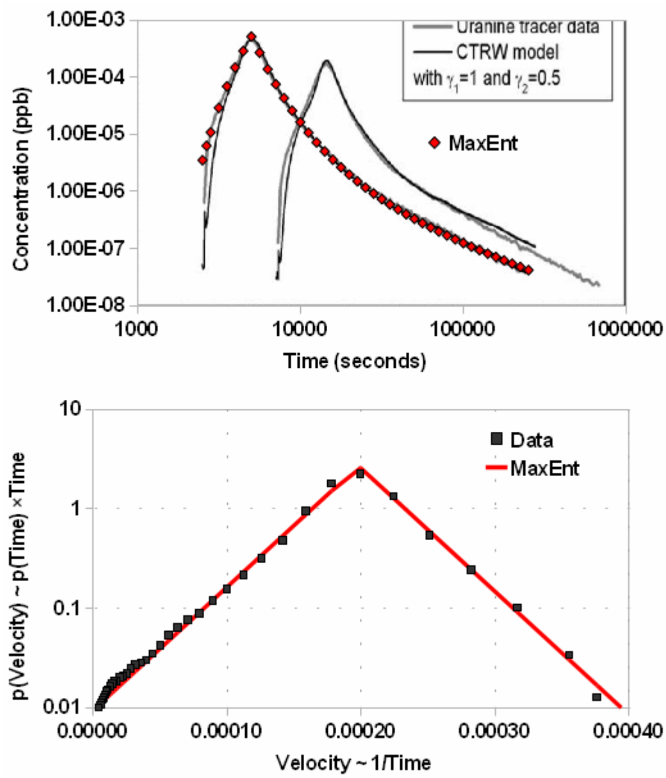
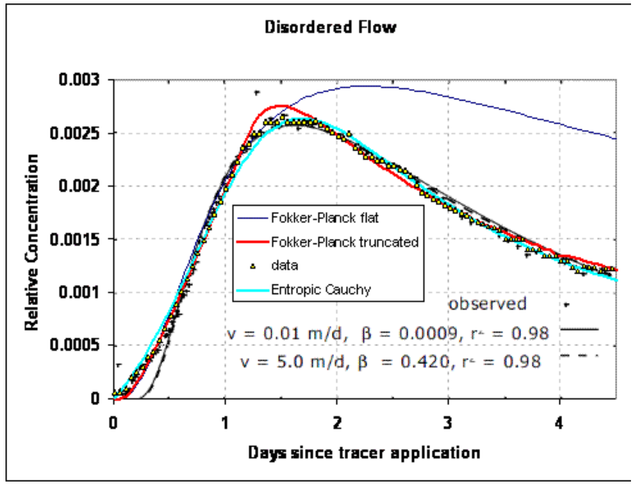
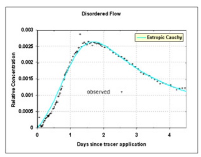


FIGURE 26-17. Break through curve (top) and measured velocity distribution (bottom) for fluorescent dye tracer experiment.

Finally we go back to the maximally disordered data. The data from Lange [Ref 258] fits a Cauchy velocity profile best.

FIGURE 26-18. Data from Lange provides an example of an extremely disordered path for solute flow. The Cauchy velocity distribution generates a high entropy.





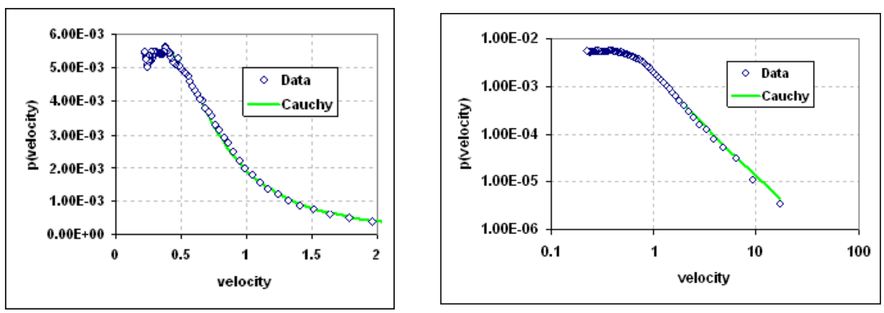


FIGURE 26-19. Cauchy velocity profiles on linear (left) and log (right) scales.

The simplicity of the model also points out how readily fat-tail effects emerge from entropic disorder. The power law drop-off obeys a 1/time behavior that certainly has consequences in terms of how long a contaminant will remain in a groundwater basin. Velocity dispersion with a mean MaxEnt constraint will always lead to a power-law drop-off in time. The following figure demonstrates how the velocity profiles compare over the different experimental configurations. These do not have to yield symmetric profiles, and in particular the Benson data drops off faster on the high velocity side, but these do demonstrate the first-order characteristics to the breakthrough curves. They range from the highly disordered Lange and Mathias data to the less disordered Benson and Borgne experimental data.

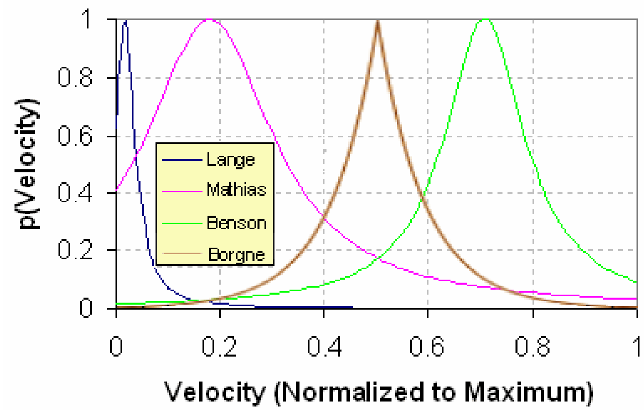


FIGURE 26-20. Flows for different referenced studies plotted on the same velocity scale.

I would suggest that the detailed physical modeling of these kinds of porous structures provides less insight than we would think. The underlying disorder caused by the multitude of pathways maximizes the entropy of the system, which in turn will tend to obscure the physics that researchers try to unearth (so to speak). In other words, you could not model a more complex system given those constraints if you tried. Nature will always win out with entropy in its back pocket. We can try but we need to model the disorder first[6].

Thermal Entropic Dispersion

As we learn how to extract energy from disordered, entropic systems such as amorphous photovoltaics and wind power, we can really start thinking creatively in terms of our analysis. Most of the conventional thinking goes out the window as considerations of the impact of disorder requires a different mindset.

We just solved the Fokker-Planck diffusion/convection equation for disordered systems and demonstrated how well it applied to transport equations; we gave examples for both amorphous silicon photocurrent response and for the breakthrough curve of a solute.

Both these systems feature some measurable particle, either a charged particle for a photovoltaic or a traced particle for a dispersing solute.

6. If you pose this kind of problem to a research geologist or hydrologist, prepare for a numerical model or the results from a piece of commercial software. This material rarely gets treated with consideration of logic and elementary first-principles considerations.

Similarly, the conduction of heat also follows the Fokker-Planck equation at its most elemental level. In this case, we can monitor the temperature as the heat flows from regions of high temperature to regions of low temperature. In contrast to the particle systems, we do not see a drift component. In a static medium, not abetted by currents (as an example, mobile ground water) or re-radiation, heat energy will only move around by a diffusion-like mechanism.

We can't argue that the flow of heat shows the characteristics of an entropic system — after all temperature serves as a measure of entropy. However, the way that heat flows in a homogeneous environment suggests more order than you may realize in a practical situation. In a perfectly uniform medium, we can propose a single diffusion coefficient, ***D***, to describe the flow or flux. A change of units translates this to a thermal conductivity. This value inversely relates to the R-value that most people have familiarity with when it comes to insulation.

For particles in the steady state, we think of Fick's First Law of Diffusion. For heat conduction, the analogy is Fourier's Law. These both rely on the concept of a concentration gradient, and functionally appear the same, only the physical dimensions of the parameters change. Adding the concept of time, you can generalize to the Fokker-Planck equation (i.e Fick's Second Law or the Heat Equation respectively).

Much as with a particle system, solving the one-dimensional Fokker-Planck equation for a thermal impulse you get a Gaussian packet that widens from the origin as it diffuses outward. See the picture below for progressively larger values of time. The cumulative amount collected at some point, ***x***, away from the origin results in a sigmoid-like curve known as an *complementary error function* `or` **erfc**.

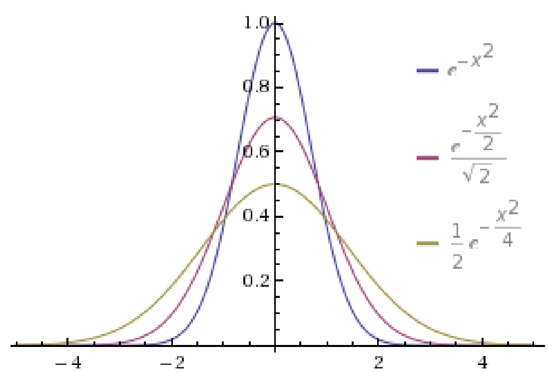


FIGURE 26-21. Broadening of a concentration profile due to diffusion. As the diffusion coefficient ***D*** increases, the width of the peak increases.

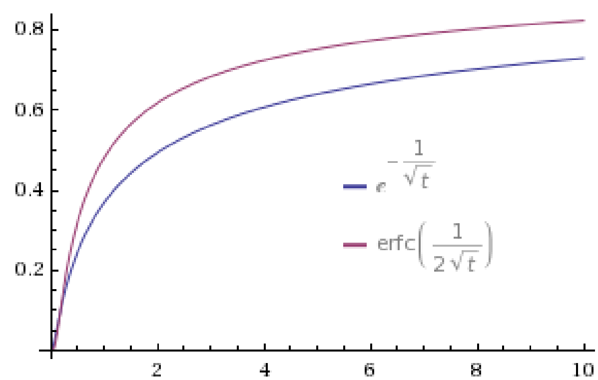
Yet in practice we find that a particular medium may show a strong amount of non-uniformity. For example, the earth may contain large rocks or pockets which can radically alter the local diffusivity. The same thing occurs with the insulation in a dwelling; doors and windows will have different thermal conductivity than the walls. The fact that reflecting barriers exist means that the *effective* thermal conductivity can vary[7]. I see nothing radical about the overall non-uniformity concept, just an acknowledgment that we will quite often see a heterogeneous environment and we should know how to deal with it.

Previously, I solved the Fokker-Planck equation for a disordered system assuming both diffusive and drift components. In that solution I assumed a maximum entropy (MaxEnt) distribution for mobilities and then tied diffusivity to mobility via the Einstein relation. The solution simplifies if we remove the mobility drift term and rely only on diffusivity. The cumulative impulse response to a delta-function heat energy flux stimulus then reduces to:

$$T(x, t) = T_1 \cdot e^{-x/(\sqrt{Dt})} + T_0 \qquad \textbf{(EQ 26-26)}$$

No **`erfc`** in this equation (which by the way makes it useful for quick analysis). I show the difference between the two solutions in the graph below. The uniform diffusivity form (red curve) shows a slightly more pronounced knee as the cumulative increases than the disordered form (blue curve) does. The fixed ***D*** also settles to an asymptote more quickly than the MaxEnt disordered ***D*** does, which continues to creep upward gradually. In practical terms, this says that things will heat up or slow down more gradually when a variable medium exists between yourself and the external heat source. Because of the variations in diffusivity, some of the heat will also arrive a bit more quickly than if we had a uniform diffusivity.

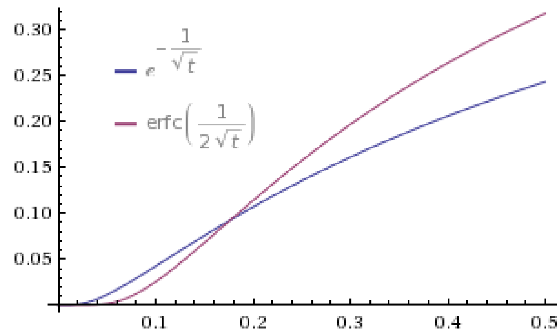
FIGURE 26-22. Difference between heat flow of ordered and disordered medium (for a one-dimensional distance *x*=1 and a scaled diffusivity of *D*=1)

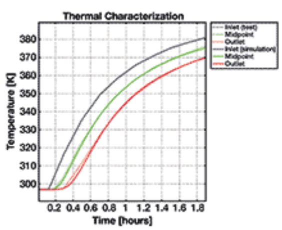


7. Similarly this arises in variations due to Rayleigh scattering in observations of wind and in wireless power levels, See “Rayleigh Fading, Wireless Gadgets, and a Global Context” on page 572..

For small times the differences appear in the figure below. Overall the differences appear a bit subtle compared to what would occur if a convective field applied to the system.

FIGURE 26-23. Scaled version of Figure 26-22 on page 526 for small times





The subtlety has as much to do with the fact that diffusion already implies disorder, while the MaxEnt formulation simply makes the fat-tails fatter than the normal diffusivity generates. The extra disorder essentially disperses the heat more efficiently — some gets to its destination faster and a sizable fraction later.

Which brings up the question of how we can get some direct evidence of this behavior from empirical data. With drift, the dispersion becomes much more obvious, as systems with uniform mobility with little disorder show very distinct knees (ala photocurrent time-of-flight measurements or solute breakthrough curves for uniform materials). Adding the MaxEnt variation makes the fat-tail behavior very obvious, as you would observe from the anomalous transport behavior in amorphous semiconductors. You can also drive the drift by varying the drift field, either by gravity for a solute or an electric field for charged particles. With diffusion alone, the knee automatically smears, as you can see from the figure to the left for a typical thermal response measurement.

Heat Exchanger Evidence

Much of the interesting engineering and scientific work in characterizing thermal systems comes out of Europe. This paper [Ref 269] investigating earth-based heat exchangers contains an interesting experiment. As a premise, they wrote the following, where incidentally they acknowledge the wide variation in thermal conductivities of soil:

> The thermal properties can be estimated using available literature values, but the range of values found in literature for a specific soil type is very wide. Also, the values specific for a certain soil type need to be translated to a value that is representative of the soil profile at the location. The best method is therefore to measure

directly the thermal soil properties as well as the properties of the installed heat exchanger.

This test is used to measure with high accuracy:

• The temperature response of the ground to an energy pulse, used to calculate:
— the effective thermal conductivity of the ground
— the borehole resistance, depending on factors as the backfill quality and heat exchanger construction

• The average ground temperature and temperature - depth profile.

• Pressure loss of the heat exchanger, at different flows.

The authors of this study show a measurement for the temperature response to a thermal impulse, with the results shown over the course of a couple of days. I placed a solid red and blue line indicating the fit to an entropic model of diffusivity in the figure below. The mean diffusivity comes out to ***D***=1.5/hr (with the red and blue curves ± 0.1 from this value) assuming an arbitrary measurement point of one unit from the source. This fit works arguably better than a fixed diffusivity as the variable diffusivity shows a quicker rise and a more gradual asymptotic tail to match the data.

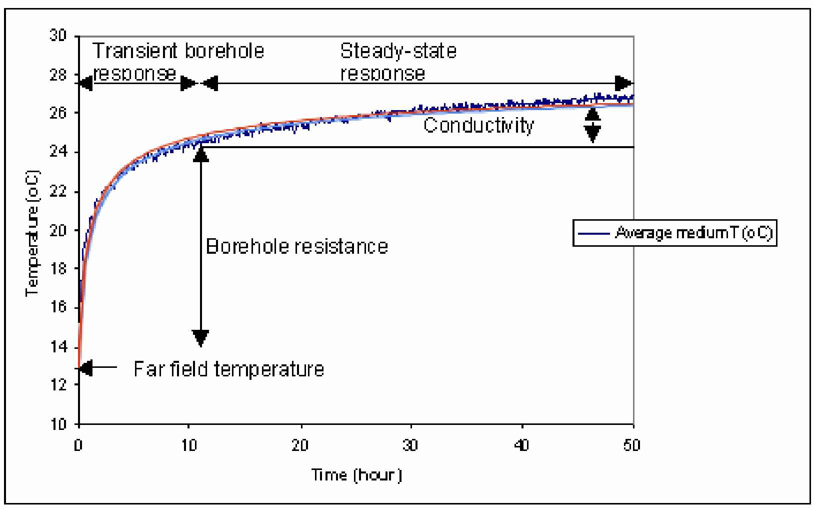


FIGURE 26-24. Behavior of thermal conductivity data matched against a a dispersive model.

This experiment measures the temperature of a buried sensor situated at some distance below the surface after we apply an impulse of thermal energy. The physics solution to this problem considers the heat kernel function as the impulse response or Green's function for that variation of the master equation. However we do not know the diffusion coefficient to any degree of precision. The earthen material that the heat diffuses through shows heterogeneous disorder, and at best we can guess a mean value for the diffusion coefficient. By inferring through the maximum entropy principle, we can say that the diffusion coefficient has a PDF with an exponentially distributed with a mean value D.

We then work the original heat equation solution with this smeared version of D, and then the kernel simplifies to a *exp*() solution. But we also don't know the value of x that well and have uncertainty in its value. If we give a Maximum Entropy uncertainty in that value, then the solution simplifies to

$$\frac{1}{2} \cdot \frac{1}{x_0 + \sqrt{Dt}} \qquad \text{(EQ 26-27)}$$

where x_0 is a smeared value for x.

The following figure is an alternate fit to the experimental data. There are two parameters to the model, an asymptotic value that is used to extrapolate a steady state value based on the initial thermal impulse and the smearing value which generates the red line. The slightly noisy blue line is the data, and one can note the good agreement.

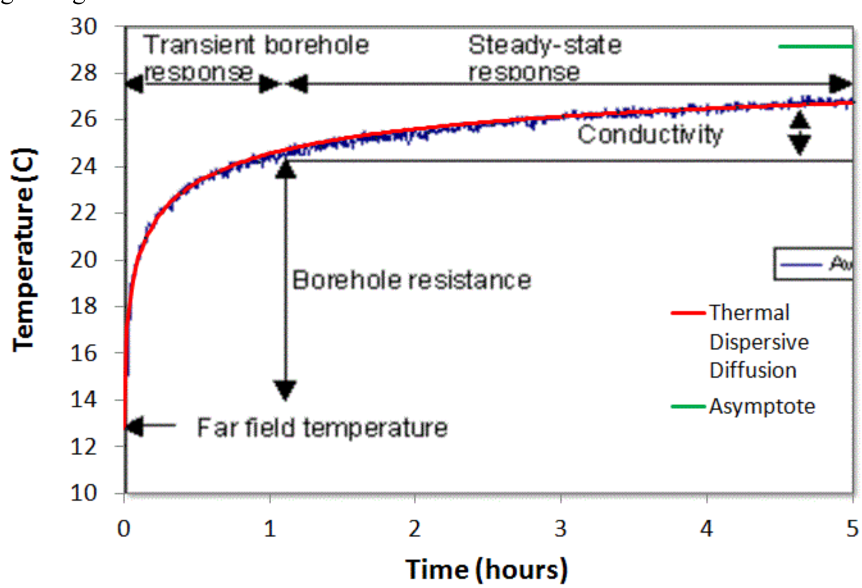


FIGURE 26-25. Fit of thermal dispersive diffusion model (red) to a heat impulse response (blue).

Notice the long tail on the model fit. The far field response in this case is the probability complement of the near field impulse response. In other words, what diffuses away from the source will show up at the adjacent target. By treating the system as two slabs in this way, we can give it an intuitive feel.

The transient thermal response tells us much about how fast a natural heat exchanger can react to changing conditions. One of the practical questions concerning an exchanger's utility arises from how quickly the mechanism will adapt to temperature changes. Ultimately this has to do with extracting heat from a material showing a natural diffusivity and we have to learn how to deal with that law of nature. Much like we have to acknowledge the entropic variations in wind or cope with variations in CO2 uptake, we have to deal with the variability in the earth if we want to take advantage of our renewable geothermal resources.

Back to Oil

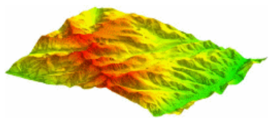
As a very general technique we can apply the equivalent of breakthrough analysis across many domains. The usual problem remains that different application domains use different terminology. In volume 1, I did not use breakthrough analysis terminology because no one does controlled experiments when they look for or extract oil. After all, oil exploration remains a commercial enterprises and oil prospectors get what they can, while they can, and don't necessarily ponder any deeper meaning. Yet, I view the over-riding dispersion analysis as a very general concept and I simply apply the same technique in oil depletion by making the analogy to dispersion in human-aided discovery search rates. The fact that it also occurs for physical processes such as contaminant flow in groundwater, carrier transport in amorphous semiconductors, or heat dispersion should not surprise anyone.

Over 50 years have lapsed since the day that Hubbert first sketched a Logistic curve to model oil depletion, and I think the science establishment has had a mental block concerning the dispersion issue throughout this time. As we demonstrated in Volume 1, we can easily and simply explain the dynamics of the oil production lifecycle by using closely related ideas from dispersion analysis.[8]

It takes a bit of intuition to determine the situations where disorder rules and where it does not. If you can appropriately apply the arguments you can often start to understand the mysterious "anomalous" dynamics. In much the same way that we can understand the dynamics of the Hubbert curve via dispersion so to can we understand the transient of an amorphous semiconductor time-of-flight experiment by applying dispersion. As a bottom-line, we often can use fundamental concepts of

8. Just apply an accelerating electric field to the TOF experiment and I guarantee the output will start looking like a Logistic.

disorder to understand the dynamics of these behaviors. As a result, absolutely nothing about any of these empirical observations would I consider anomalous. It all depends on your perspective.



What about Topography?

Entropy makes its mark everywhere. Take the case of modeling topography. How can we model and thus characterize disorder in the earth's terrain? Can we actually understand the extreme variability we see?

If we consider that immense forces cause upheaval in the crust then we can reason that the energy can also vary all over the map, so to speak. The process that transfers potential energy into kinetic energy to first order has to contain elements of randomness. To the huge internal forces within the earth, generating relief textures equates to a kind of brownian motion in relative terms — over geological time, the terrain amounts to nothing more than inconsequential particles to the earth's powerful internal engine.

In a related sense the process also resembles the pressure distribution in the earth's atmosphere, a classic application of maximum entropy that we can re-apply in the case of modeling terrain slope distributions.

Premise. We take the terrain slope ***S*** as our random variable (defined as *rise*/*run*). The higher the slope, the more energetic the terrain. Applying Maximum Entropy to a section of terrain, we can approximate the local variations as a MaxEnt conditional probability density function:

$$p(S|E) = \frac{1}{cE}e^{-S/cE} \qquad \textbf{(EQ 26-28)}$$

where ***E*** is the local mean energy and ***c*** is a constant of proportionality. But we also assume that the mean ***E*** varies over a larger area that we are interested in, as in the superstatistical sense of applying a prior distribution.

$$p(E) = k \cdot e^{-k \cdot E} \qquad \textbf{(EQ 26-29)}$$

where ***k*** is another MaxEnt measure of our uncertainty in the energy spread over a larger area.

The final probability is an integral over the marginal distribution consisting of the conditional multiplied by the prior:

$$p(S) = \int_0^{\infty} p(S|E) \cdot p(E) dE \qquad \textbf{(EQ 26-30)}$$

This integrates as a *BesselK* function of the zero order, K_0, available on any spreadsheet program[9].

$$p(S) = \frac{2}{S_0} \cdot K_0(2\sqrt{S/S_0}) \qquad \textbf{(EQ 26-31)}$$

The average value of the terrain slope for this distribution is simply the value $\boldsymbol{S_0}$.

Now we can try it on a large set of data. I downloaded all the digital elevation model (DEM) data for the 1 degree quadrangles (aka blocks/tiles) in the USA from the USGS web site[10].

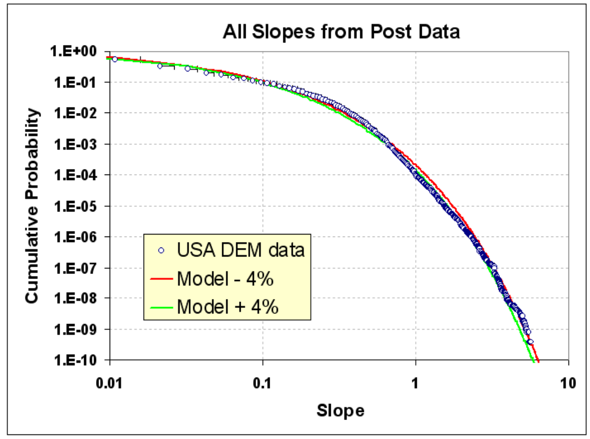
This consists of post data at approximately 92 meter intervals (i.e. a fixed value of *run*) at 1:250,000 scale for the entire USA. I concentrated on the lower 48 and some spillover into Canada. From individual DEM files and calculate the slopes between adjacent posts and came up with an average slope (rise/run) of 0.039, approximately a 4% grade or 2.2 degrees pitch. I take the absolute values of all slopes so that the average is not zero.

The cumulative plot of terrain slopes for all 5 billion calculated slope points appears on the following chart. I also added the cumulative probability distribution

9. See this article http://mobjectivist.blogspot.com/2010/10/stock-market-as-econophysics-toy.html for a similar derivation in an unrelated field. The slope in this case is fractional returns in the stock market.
10. http://dds.cr.usgs.gov/pub/data/DEM/250/ (nearly 1000 files on the server.)

of the BesselK model with the calculated average slope as the single adjustable parameter.

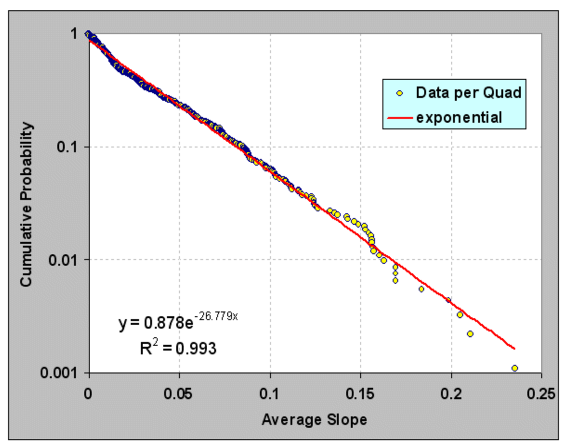
FIGURE 26-26. CDF of USA DEM data and the BesselK model with a small variation in S0 (+/-4% about the average 0.037 rise/run) demonstrating sensitivity to the fit.



This kind of agreement does not just happen because of coincidence. It occurs because random forces contribute to maximizing the entropy of the topography. Enough variability exists for the terrain to reach an ergodic limit in filling the energy-constrained state space.

As supporting evidence, it turns out that we can generate a distribution that maps well to the prior by estimating the average slope from the conditional PDF of each of the 922 quadrangle blocks and then plotting this aggregate data set as another histogram (see Figure 26-27 on page 533).

FIGURE 26-27. Generation of the prior distribution by taking the average slope of each of the nearly 1000 quadrangles. The best fit generates a value of S0 (1/27=0.037) close to that used in the previous figure.



Practically speaking, we see the variability in slopes expressed at the two different levels. The entire USA at the integrated (BesselK model) level and the aggregated regions at the localized (exponential prior) level. These remain consistent as they agree on the single adjustable parameter S_0.

The modeled distribution has many practical uses for analysis, including transportation studies and planning. Obviously, vehicles traveling up slopes use a significant amount of energy and you might like to have a model to base an analysis on without having to rely on the data by itself [11]. (As a caveat, I did not include any of the spatial correlations that must also exist and might prove useful as well)

Perusing the recent research, I couldn't find anyone that had previously discovered this simple model [Ref 293] [Ref 294] [Ref 295] [Ref 296]. Not that they haven't tried, coming up with a good slope distribution model seems to amount to a mini Holy Grail among geophysicists.

If someone wants to generate Monte Carlo statistics for the BesselK model without having to do the probability inversion, the algorithm turns out surprisingly simple. Draw two independent random samples from a uniform [0.0 .. 1.0] interval, apply the natural log to each, multiply them together, and then multiply by the S_0 scaling constant.

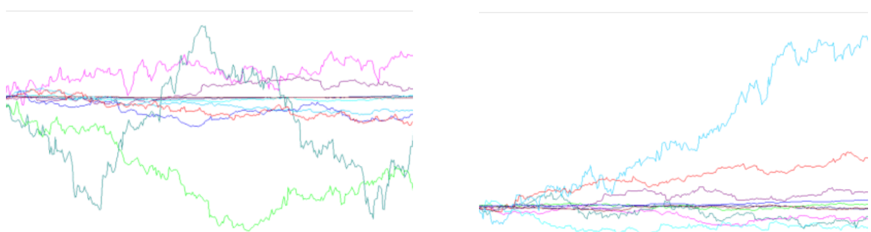
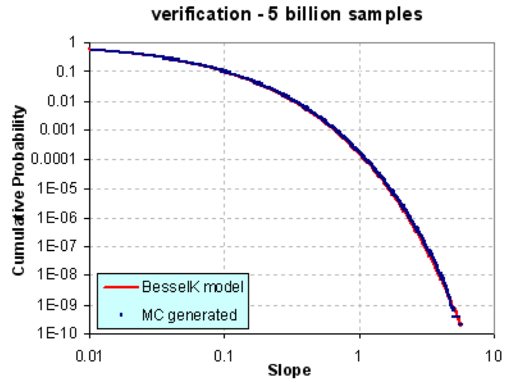


FIGURE 26-28. Examples of some random-walk realizations drawing from a two-level model follow. The flatter regions occur more often reflecting the regional data.

11. See "Dispersion Statistics in Human Travel" on page 547.

This random draw algorithm will give the following cumulative if done 5 billion times, which is the same size as the real USA DEM data sample.

FIGURE 26-29. Generation of the BesselK model via Monte Carlo.



The only statistical noise is at the 10^{-9} level, same as in the DEM data set.

The Stretched Exponential

Interesting that this BesselK distribution does not give as fat a tail as the power-law, which in contrast does occur for earthquake magnitudes (See “Earthquakes” on page 583.). The BesselK function as applied for topography actually follows somewhat closely a s*tretched exponential* with a $e^{-\sqrt{2S}}$ dependence in the leading terms. The cumulative of a stretched exponential turns into a Weibull distribution. The Weibull provides an alternative to the fat-tail we used in reliability analysis (See “Reliability. Entropy and how things break down” on page 453.) and arises not surprisingly in radar ground-clutter statistic and the Weibull in wind dispersion analysis described in the next section

Laherrere and Sornette describe the stretched exponential further in [Ref 51]. Sornette details the general derivation of the stretched exponential in [Ref 271] noting that it fills the gap between a power law and the exponential distribution. Both the power-law and the stretched exponential can come about from superposition of exponentials but the actual result depends on how the exponential arguments play together during the integration step. We leave that as an exercise inferred from the integrations described in this text.

CHAPTER 27

Wind Energy.

More disorder

"Crunch all you want, we'll make more."

— old Doritos ad

Relying on wind energy as a future energy source brings up the notion of *predictable unpredictability*. We realize that wind does not blow all the time, yet that consumers demand for energy (primarily electrical) remains constant over time. Unless we can reduce that variability, through energy storage or geographic distribution, wind will have that mark against it. This section explains with some very elementary considerations of entropy why wind will remain a random phenomenon that we will have to learn to deal with. Its enduring persistence makes it an unending source of energy if we can predictably harness it.

Wind Dispersion, the Renewable Hubbert Curve

Most critics of wind energy points to the unpredictability of sustained wind speeds as a potential liability in widespread use of wind turbines. Everyone can intuitively understand the logic behind this statement as they have personally experienced the variability in day-to-day wind speeds.

However comfortably we coexist with the concept of "windiness", people don't seem to understand the mathematical simplicity behind the wind speed variability. Actually the complexity of the earth's climate and environment contributes to this simplicity as it generates more states for the system to exist within, which increase the likelihood of variability[1].

Let me go through the derivation of wind dispersion in a few easy steps. I start with the premise that every location on Earth has a mean or average wind speed. This speed has a prevailing direction but assume that it can blow in any direction.

Next we safely assume that the kinetic energy contained in the aggregate speed goes as the square of its velocity.

$$E \sim v^2 \qquad \text{(EQ 27-1)}$$

This comes about from the Newtonian kinetic energy law ½mv^2 and it shows up empirically as the aeronautical drag law (i.e. wind resistance) which also goes as the square of the speed. (Note that we can consider E as an energy or modified slightly as a power, since the energy is sustained over time)

As usual, I apply the Principle of Maximum Entropy to the possible states of energy that exist and come up with a probability density function (PDF) that has no constraints other than a mean value (with negative speeds forbidden).

$$p(E) = k \cdot e^{-kE} \qquad \text{(EQ 27-2)}$$

where $\boldsymbol{k}$ is a constant and $1/\boldsymbol{k}$ defines the mean energy. This describes a declining probability profile, with low energies much more probable than high energies. To convert to a wind dispersion PDF we substitute velocity for energy and simplify:

$$p(v)dv = p(E)dE$$
$$p(v) = p(E) \cdot \frac{\mathrm{d}E}{\mathrm{d}v} = 2cv \cdot e^{-cv^2} \qquad \text{(EQ 27-3)}$$

This gives the empirically observed wind speed distribution, showing a peak away from zero wind speeds and a rapid decline of frequency at higher velocity. I would consider this as another variation of an entropic velocity distribution. Many scientists refer to it as a Rayleigh or Weibull distribution. The excellent agreement in the figure below between the model and empirical data occurs frequently.

1. I suggest that wind variability also could prove useful to understand the dispersion of airborne particulates, such as what occurred in the aftermath of the 2010 Icelandic volcano.

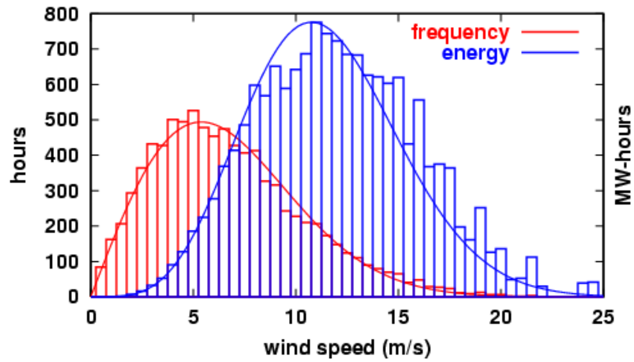


FIGURE 27-1. Wiki entry for Wind Power states: *"The Weibull model closely mirrors the actual distribution of hourly wind speeds at many locations. The Weibull factor is often close to 2 and therefore a Rayleigh distribution can be used as a less accurate, but simpler model."* Distribution of wind speed (red) and energy (blue) for all of 2002 at the Lee Ranch facility in Colorado. The histogram shows measured data, while the curve is the Rayleigh model distribution for the same average wind speed.

Many engineers use the heuristic family of Weibull curves. The Rayleigh comes out as the simpler model because it derives from first principles and any deviation from the quadratic exponent works as a refinement. In the following curve, the value of 1.95 works for all practical purposes the same as 2.

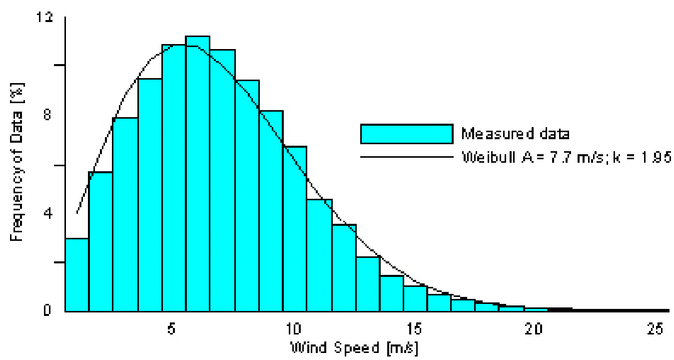


FIGURE 27-2. Weibull distribution includes the Rayleigh as a subclass. The Weibull allows a parameterized exponent while Rayleigh it is set to 2. In this Weibull case 1.95 lies arbitrarily close to 2.

Contrary to other distributions, the wind PDF does not qualify as a fat-tail distribution. This becomes obvious if you consider that the power-law only comes about from the reciprocal measure of time, and since we measure speed directly, we invoke the entropic velocity profile directly as well.

So the interesting measure relates to the indirect way that we perceive the variations in wind speed. Only over the span of time do we detect the unpredictability and disorder in speed — whether by gustiness or long periods of calm.

We can then pose all sorts of questions based on the entropic wind speed law. For example, how long would we have to wait to generate an accumulated amount of energy?

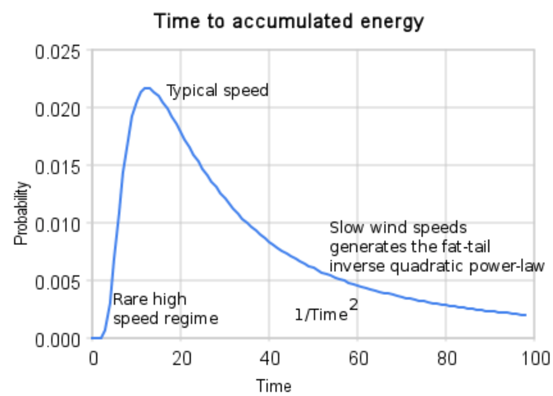
We can answer this analytically by simply equating the steady-state wind speed to a power and then integrating over all possibilities of the distribution that meet the minimum accumulated energy condition over a period of time.

The naive answer is trivial with the time PDF turning into the following fat-tail power-law:

$$p(t|E > cv^2) = cv^2 \cdot e^{-cv^2}/t^2 \qquad \textbf{(EQ 27-4)}$$

This equation corresponds to the following graphed probability density function (where we set an arbitrary $\boldsymbol{E}$ of $\boldsymbol{cv}^2 = 25$). It basically illustrates the likelihood of accumulating a specific energy goal within a given ***Time***.

FIGURE 27-3. A derived distribution from the data set which plots time it takes to accumulate a set energy level in the context of a variable wind source.



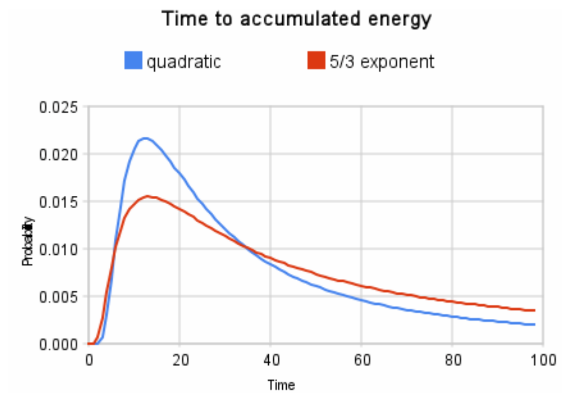
Because of the scarcity of high wind speeds and the finite time it takes to accumulate energy, one observes a short ramp-up to the peak. Typical wind-speeds round out the peak and the relatively common slow speeds contribute to the long fat-tail.

But since power has an extra velocity factor (Power = Drag*velocity, see Betz' law), it takes longer to integrate the low power values and the exponent changes from a quadratic value of 2 to a value of 5/3, via a chain rule.

$$p(t) = \frac{2}{3} \cdot d \cdot e^{-d/t^{2/3}}/t^{5/3} \qquad \textbf{(EQ 27-5)}$$

As you can see in the graph below, the fat-tail becomes fatter and the ramp-up a bit sooner for roughly the same peak value.

FIGURE 27-4. A variation of the accumulated energy whereby an added drag term will reduce the efficiency and therefore the likelihood of accumulating energy quickly.



That gives us the power-law and a shape that looks surprisingly close to the time depletion curve for an oil reservoir. In fact, since probabilities have such universal properties, the curvature of this profile has the same fundamental basis as the Hubbert oil depletion profile. The huge distinction lies in the fact that wind energy provides a *renewable* source of energy, whereas oil depletion results in a dead-end.

So the hopelessness of the Hubbert curve when applied to peak oil turns to a sense of optimism when you realize that wind power generates a case of a *Renewable Hubbert Curve*. In other words, anytime you spin-up the wind-turbine with the goal of achieving a desired energy level you can always obtain a mini Hubbert cycle, *if you have patience*, just like you need patience with a train schedule (cue foreshadowing).

subtitle: *Wind is entirely predictable in its unpredictability*

Wind Energy Dispersion Analysis

The previous maximum entropy derivation assumed only a known mean of wind energy levels (measured as power integrated over a fixed time period). From this simple formulation, one can get a wind speed probability graph. Knowing the probability of wind speed, you can perform all kinds of interesting extrapolations — for the prior example, we can project the likelihood of how long it would take to accumulate a certain level of energy.

Canada

One can find abundant data on wind speed distribution in various internet archives.[2] One retrieved data set consisted of about 36,000 sequential hourly measurements in terms of energy (kilowatt-hours) for Ontario. The following chart

2. Len Gould volunteered a spreadsheet of Ontario wind speed data.

shows the cumulative probability distribution function of the energy values. This shows the classic damped exponential function, which derives from either the Maximum Entropy Principle (probability) or the Gibbs-Boltzmann distribution (statistics). It also shows a knee in the curve at about 750 KWh, which comes from a regulating governor of some sort designed to prevent the wind turbine from damaging itself at high winds.

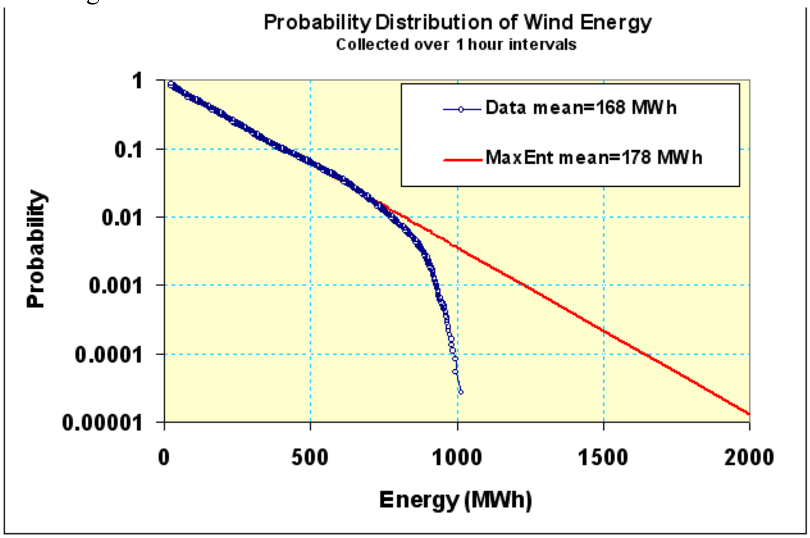
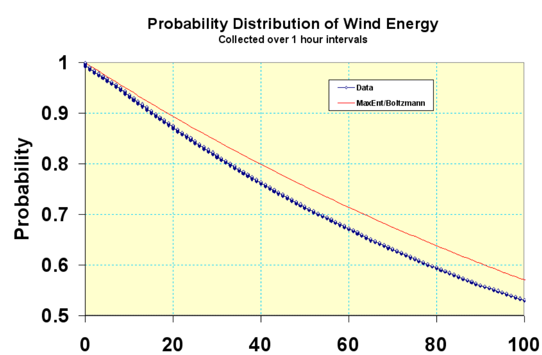


FIGURE 27-5. Probability distribution function of wind speeds in Ontario, Canada. This follows the Rayleigh distribution according to the straight line fit. Deviation from this curve at higher wind speeds results from an automated cut-out of the generator to prevent damage to the turbine or blades. Although these will provide high energy density, their rarity prevents them from contributing much to the cumulative.

I also charted the region around zero energy to see any effect in the air flow transfer regime (which should be strong near zero). In this regime the probability would go as sqrt($\boldsymbol{E}$)*exp(-$\boldsymbol{E}/\boldsymbol{E_0}$) instead of exp(-$\boldsymbol{E}/\boldsymbol{E_0}$).

FIGURE 27-6.
Blow-up of previous figure showing slight divergence at smaller wind energies



Yet only a linearized trend appears courtesy of the Taylor's series expansion of the exponential around ***E***=0. Remember that this data consists of a large set of independent turbines. You might think that because of the law of large numbers that the distribution might narrow or show a peak. Instead, the mixture of these turbines over a wide variation in the wind speed provides a sufficiently disordered path so that we can apply the maximum entropy principle.

With a gained confidence in the entropic dispersive model, we can test the prior nagging question behind wind energy — How long do we have to wait until we get a desired level of energy?

I generated a resampled set of the data (only resampled in the sense that I used a wraparound at the 4 year length of the data to create a set free from any boundary effects). The output of the resampling essentially generated a histogram of years it would take to reach a given energy level. I chose two levels, ***E(T)***=1000 MW-hrs and ***E(T)***=200 MW-hrs.

I plotted the results below along with the predetermined model fit next to the data.

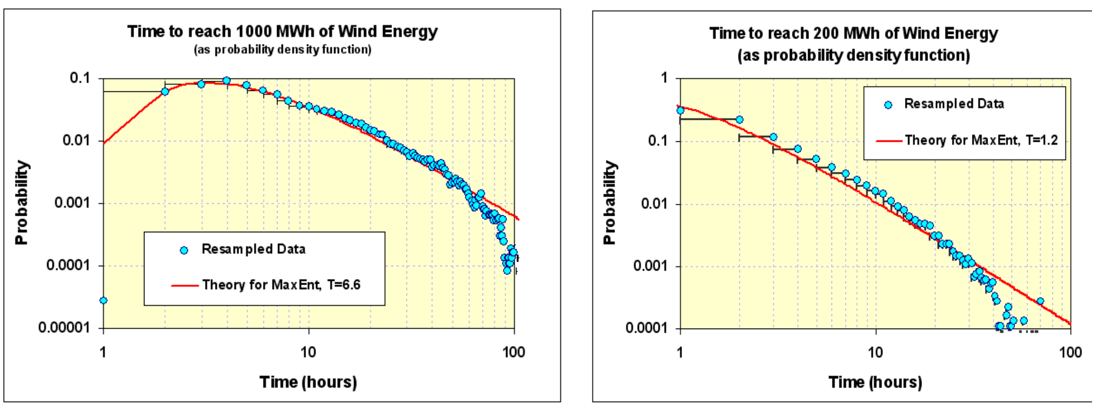


FIGURE 27-7. Derived distributions for time to reach a certain energy level. The longer it takes to reach a certain level, the greater the likelihood that the curve will match the predicted levels.

The model described previously predicts the behavior applied to the two sets of data:

$$p(t|E > E(T)) = T \cdot \frac{e^{-T/t}}{t^2} \qquad \textbf{(EQ 27-6)}$$

where ***T*** is the average time it will take to reach ***E(T)***. From the exponential fit in the first figure, this gives ***T***= 200/178 and ***T***=1000/178, respectively, for the two charts. As expected we get the fat-tails that fall off as $1/\boldsymbol{t}^2$ (not $1/\boldsymbol{t}^{1.5}$ as the velocity flow argument would support).

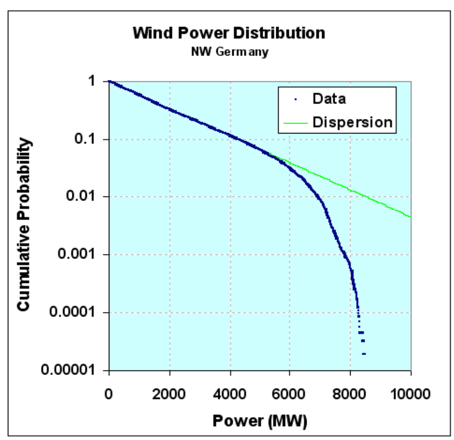
The models do not work real effectively at the boundary conditions, simply because the wind turbine limiting governors prevent the accumulation of any energy levels above 1000 MWh level; this occurs either in a short amount of time or at long times as a Poisson process of multiple gusts of lower energy[3]. That said, any real deviations likely arise from short-duration correlations between wind energy measurements spaced close together. We do see this as the lower limit of $\boldsymbol{E}(200)$ shows more correlation curvature than $\boldsymbol{E}(1000)$ does. Wind speeds do change gradually so these correlations will occur; yet these seem minor perturbations on the fundamental entropic dispersion model, which seems to work quite well under these conditions.

As a bottom-line, this analysis tells us what we already intuited. Because of intermittency in wind speed, it often takes a long time to accumulate a specific level of energy. Everyone knows this from their day-to-day experience dealing with the elements. However, the principle of maximum entropy allows us to draw on some rather simple probability formula so that we can make some additional estimates for long-term use.

Germany

By adding more data to our knowledge on wind dispersion, we can observe how dispersion in wind speeds has a universal character. I picked up the previous data set from several years worth of output from Ontario. This set hails from northwest Germany and consists of wind power collected at 15 minute intervals.

FIGURE 27-8.
Wind variations for Germany. The curve has all the same characteristics as that for Ontario, demonstrating the universal behavior in wind speed variability.

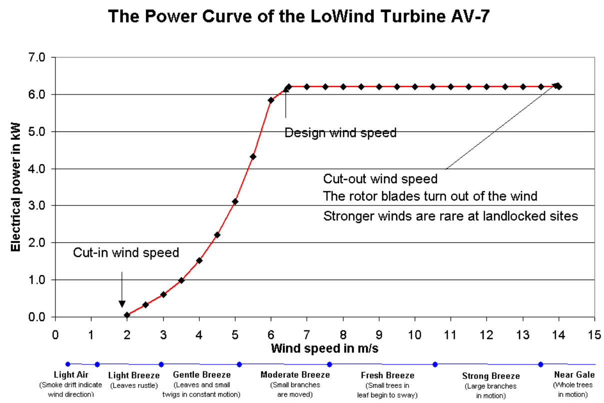


3. Gustiness contributes to energy losses as the inertia of the blades can't adjust quickly enough to account for increases in energy. This may balance out the further gains we should have according to the velocity flow argument

Note that the same entropic dispersion holds as for Ontario. Both curves display the same damped exponential probability distribution function for frequency of wind power (derived from wind speed). We also see the same qualitative cut-out above a certain power or wind energy level. Once again, note that we don't gain much by drawing from these higher power levels as they occur more sporadically than the nominally rated wind speeds at the upper reaches of the curve. The following figure gives an explanation for the cutout above the "max" wind speed.

FIGURE 27-9. Turbine Specification

Power regulation: pitch regulated with variable speed
Operating data
Rated power: 3,000
kW Cut-in wind speed: 3 m/s
Rated wind speed: 12 m/s
Cut-out wind speed: 25 m/s



Too many people get the idea that the sporadic nature of wind confronts us with some kind of problem or issue to deal with. I instead suggest that we will have to get used to a different way of thinking about wind. The entropic dispersion of wind acts much like a variation of the Carnot cycle. In the Carnot cycle of engine efficiency, we have to live with a maximum level of energy conversion based on temperature differences of the input and output reservoirs. With wind, the earth's environment and atmosphere provides the temperature-like differences which leads directly to the variability over time.

So we can easily imagine achieving very high usage efficiency as long as we acknowledge the entropic characteristics of the wind. This amounts to a law of nature. We need to talk about efficiencies within the constraints of the physical laws just as with the Carnot cycle. We will observe intermittency as a result of entropic dispersion and we have to get used to it. We should not call it a fundamental "problem", as we cannot change the characteristics of entropy (apart from adding energy, and that just moves us back to square one).

Consider this from a different prospective: certain people would suggest that the fundamental problem with farming derives from the intermittent nature of the rain. With farming, we adapt — likewise with wind energy. Instead of a problem, we need to call it an opportunity and deal with the predictable unpredictability. The derivation essentially becomes the equivalent of a permanent weather forecast. We should realize that the windspeed will eventually pick up and not to fret too much about it[4].

If that still bothers someone, we should always remember that electrical power as it stands right now comes from distributed sources. Considering that wind shows variability, distributing power from turbines in geographically separated areas will help regulate the flow, as different regions will ebb and flow in wind speed.[5]

4. Weathermen perform a useless function in this regard. Only something on the scale of massive global warming will likely effect the stationary results.
5. Again, both dispersive discovery and the entropic wind dispersion model use the same set of ideas from probability. The analysis can see through the complexity and discover the underlying elegance and intuitive power of simple entropy arguments, one a non-renewable resource and the second renewable.

CHAPTER 28

Travel

Dispersion in human mobility

I played the Red River Valley
He'd sit in the kitchen and cry
Run his fingers through seventy years of livin'
And wonder, "Lord, why has every well I've drilled gone dry?"
We were friends, me and this old man
We's like desperados waitin' for a train
Desperados waitin' for a train
He's a drifter, a driller of oil wells
He's an old school man of the world

— Guy Clark

Without a doubt, our reliance on petroleum for travel outweighs many of its other uses. Certainly, air travel would become virtually impossible without such a convenient, *concentrated* form of energy. And air travel remains the best way to get from point **A** to point **B** fast and usually on-time. It boils down to the essential fact that we may need to trade-off speed for predictability. This section brings up the analysis of human travel in terms of statistical data — how much of the time we spend in various modes of travel and how dependable the modes measure up. Once again the concept of entropy can simplify the discussion.

Dispersion Statistics in Human Travel

This discussion centers around an intriguing paper on the statistics of human travel published in *Nature* magazine [Ref 198]. In the past, most researchers have found it hard to collect data on the topic. Yet with the use of clever techniques, such as using the internet or cellular network records, we suddenly have a trove of information to weed through. In the first case, the researchers collected indirect travel information by compiling the spread of money, via the bill tracking system **http://www.whereisgeorge.com**. This becomes a so-called "proxy" measure for human travel data.

I can't argue the basic premise behind the approach. The authors assume that people disperse throughout a geographic region and then make the claim that you can essentially track their motion (indirectly) by following the trajectory of the money they carry. They also make the correct interpretation that dispersion via random walk mechanisms plays a big role in the spread. That part seems quite reasonable. And the utility of understanding this phenomena holds great promise. For example, one can use it to understand the implications of reducing the travel overhead as we enter an energy-scarce era, as well as understanding the dynamics of pandemics. Yet, considering how important the concept is and the prestige of the journals that publish the work, they miss some of the important and simple conceptual mathematics behind the phenomena. By that I mean they made the result far too complex and opaque.

I assert that if they had used the correct formulation of dispersion, the agreements to their premise would have shown a very simple universality; yet they invoke some sophisticated notion of Levy flights and other esoteric models of stochastic mathematics to derive an overly complex result. Eventually they come up with a scaling law exponent which they affix with the value 0.59 and 1.05. They claim these particular numbers holds some notion of "universality" and claim that this results in some new form of "ambivalent" process. This seems a bit pretentious and in what follows we will derive some actual, much more practical models.

The statistics of what they want to model stems from sets of collected data from time-stamped geographical locations. The figure below shows typical vector traces of travel across the USA.

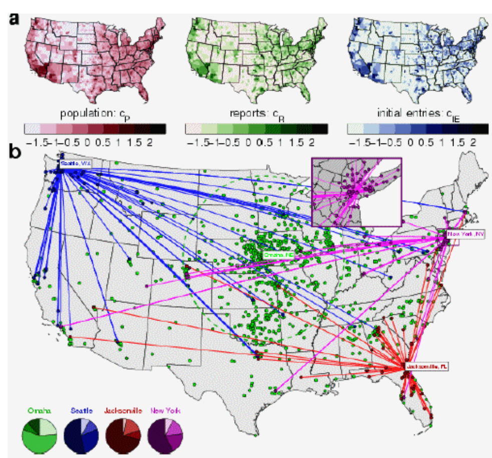


FIGURE 28-1. Typical bill travel traces due to Brockmann

The collected information appears very comprehensive and the fact that everyone uses money, attests to the fact that the approach should show little bias with minimal sample error. The sophisticated random-walk model they use derives from the same Scher-Montrose Continuous Time Random Walk (CTRW) model that we saw other researchers apply to the physics in the last two chapters[1]. Instead of such overt statistical physics formality, I would set up the mathematical premise straightforwardly. We have a dispersion of velocities described by a maximum entropy probability distribution function (PDF).

$$p(v) = \alpha \cdot e^{-\alpha v} \qquad \textbf{(EQ 28-1)}$$

The PDF above describes a mean velocity with a standard variance equal to the mean. This places all moments as finite values and becomes an intuitive minimally biased estimate considering no further information is available. We next assume that the actual distance traveled happens over a period of time.

The authors first describe a PDF of money traversing a distance ***r*** in less than 4 days.

$$p(r|t < 4\text{days}) = \text{probability that bill travelled r distance in under 4 days} \qquad \textbf{(EQ 28-2)}$$

This becomes very easy to solve. We only need to assume that the distance traveled gets uniformly distributed across the interval — another maximum entropy estimator for that constraint.

So for any one time, we express the cumulative distance traveled at any one time by the following cumulative distribution function (CDF), with ***r*** scaled as a distance:

$$P(r, t) = e^{-\alpha(r/t)} \qquad \textbf{(EQ 28-3)}$$

The term α has the units of time/distance. As we haven't considered the prospects of waiting time variation yet, this point provides a perfect place to add that to the mix.

Intuitively, money does not constantly stay in motion, but instead may sit in a bank vault or a piggy bank for long periods of time. So we smear the term α again with a maximum entropy value.

$$p(\alpha) = \beta \cdot e^{-\beta \cdot \alpha} \qquad \textbf{(EQ 28-4)}$$

But we still have to integrate this over all smeared values, with respect to $P(r,t)$. This gives the equation

1. The CTRW model applied to the solar cell photoresponse problem on a microscopic scale is similarly overly complicated

$$P(r, t) = \frac{\beta}{\beta + \frac{x}{t}} \quad \textbf{(EQ 28-5)}$$

To generate the final PDF set, we take the partial derivative of this equation with respect to time, **t**, to get back the temporal PDF, and with respect to **r**, to get the spatial PDF (ignoring any radial density effects).

$$\frac{\partial}{\partial t}P(r, t) = \frac{\beta \cdot r}{(\beta \cdot t + r)^2}$$
$$\frac{\partial}{\partial r}P(r, t) = \frac{\beta \cdot t}{(\beta \cdot t + r)^2} \quad \textbf{(EQ 28-6)}$$

So we can generate a PDF for ***r*** for a specific value of T=4 days and then fit the curve to a value for β. So β becomes the average velocity/waiting time for a piece of paper money (the bank note the authors refer to) between the times it makes trips from place to place. The fact that time acts as the dispersive parameter and this goes in the denominator of ***velocity***=***r***/***t***, implies that the fat tails appear in both the spatial dimension *and* the temporal dimension.

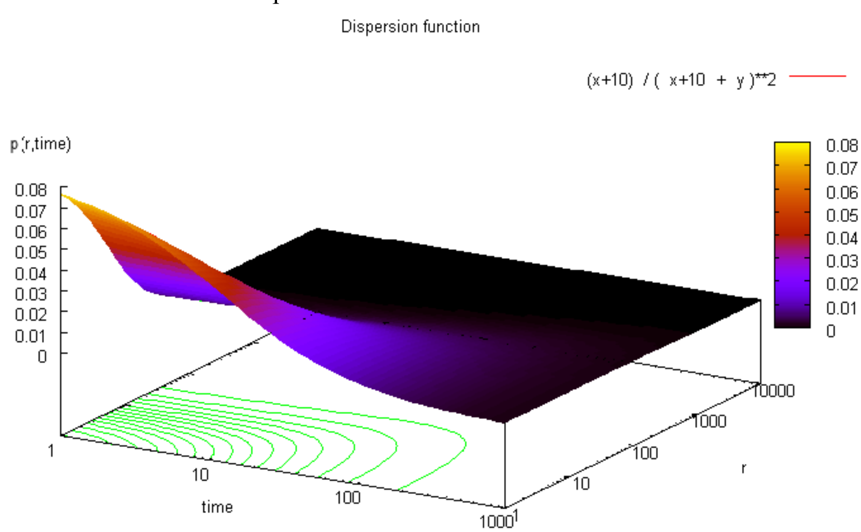
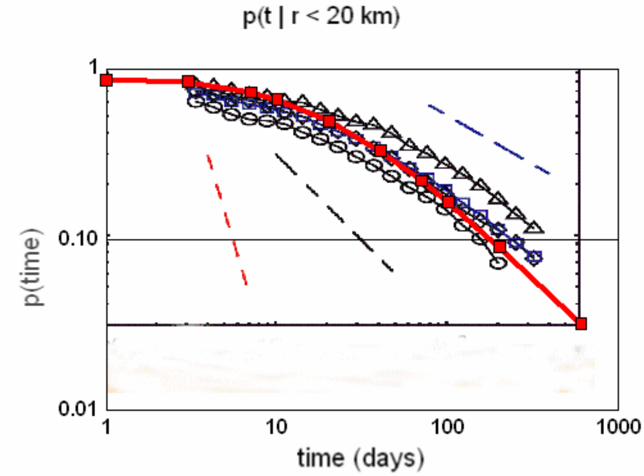


FIGURE 28-2. Contour profile in spatial/temporal dimensions for dispersion of money. High levels and hot colors are high probability.

To verify the model, the authors plot the results in two orthogonal dimensions. First against time:

$$P(t|r < 20\text{km}) = \text{Probability that bill travelled <20kmin given time} \quad \textbf{(EQ 28-7)}$$

The constrained curve fits are shown below with β set to 1 kilometer per day. The various sets of data refer to 3 classes of initial entry locations (metropolitan areas, intermediate cities, and small towns).



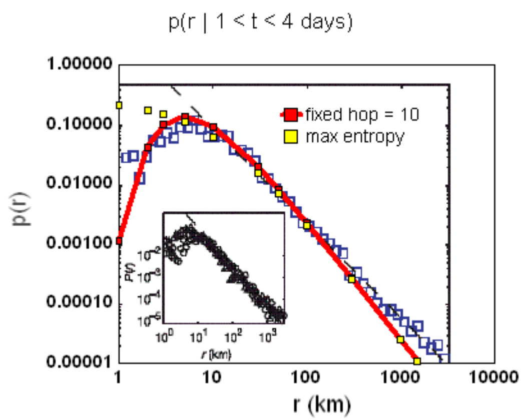


FIGURE 28-3. Profiles of binned probability along two orthogonal axis. The red lines are fits for dispersion with beta = 1.

The short-distance spatial curve has some interesting characteristics. From the original paper, a plot of normalized probabilities shows relative invariance of distance traveled with respect to time for short durations of less than 10 days.

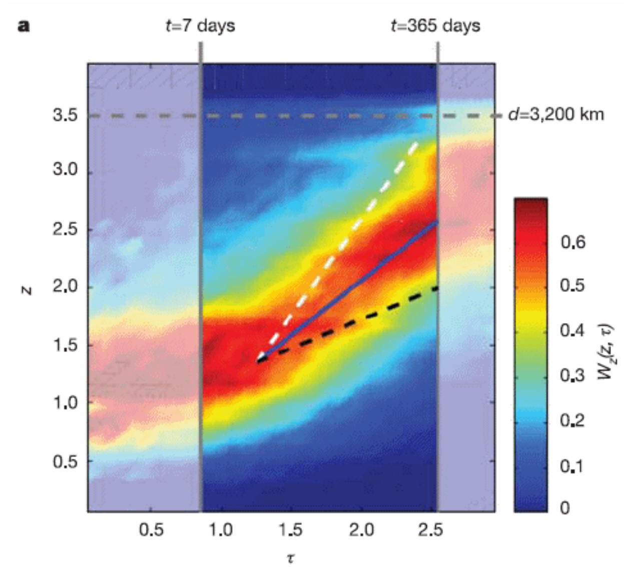
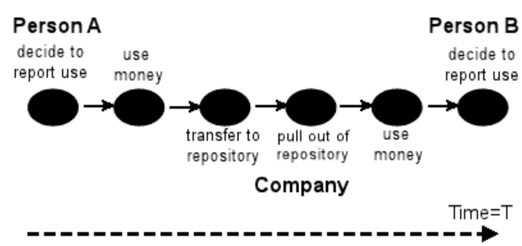


FIGURE 28-4. Contour profile from Brockmann.

Time appears bottle-necked for around 10 days on the average. This is understandable as the waiting time between transactions can contain 5 distinct stages. As most of these transactions may take a day or two at the minimum, it is easy to conceive that the total delay is close to T=10 days between updates to the bill reporting web site. So this turns into a weighted hop invariant to time but scaled to reflect the average distance that the money would actually travel.

FIGURE 28-5. Latency at short time intervals has to go through 5 processing steps. This state flow behaves similarly to what occurs in oil processing, ala the Oil Shock model in Volume 1.



If we plot our theory on a similar color scale, it looks like the 2-dimensional profile shown next:

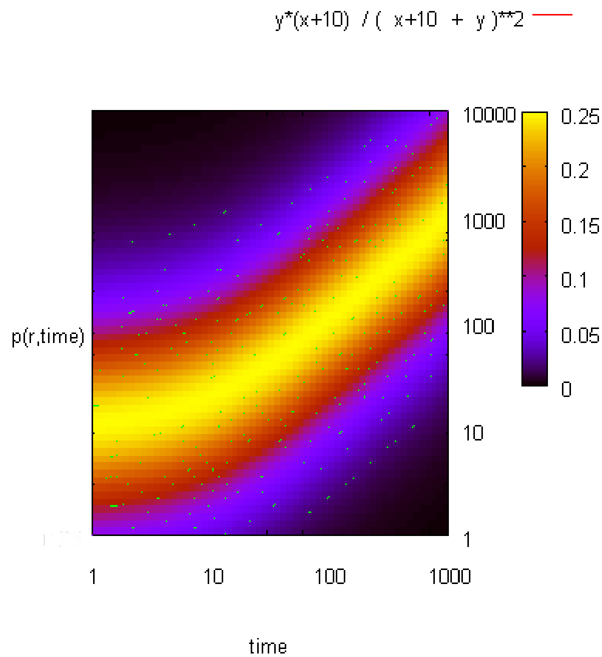


FIGURE 28-6. Contour plot of scaled probability values. This shows good agreement with the Brockmann data in Figure 28-4 on page 552.

Note that these look different than the first contour color scale in Figure 28-2 on page 550 since the authors multiplied the probability values by ***r*** to maintain a similar dynamic range across all values. Otherwise the probability values would diminish at large ***r*** or ***t***.

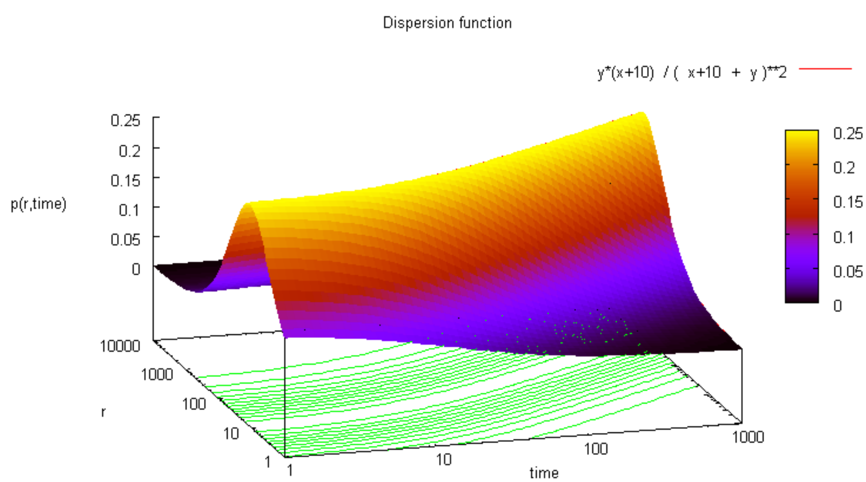


FIGURE 28-7. Alternate perspective of previous figure. Note that the scaling by r keeps the dynamic range intact, in comparison to Figure 28-2 on page 550

The bottom-line is that the actual model remains simple and both CTRW and the scaling-law exponent do not come into play. Brockmann also curiously called the process “ambivalent”, which seems to imply uncertainty. I consider it the opposite of ambivalent and the solution remains rather straightforward. They perhaps did not consider Occam's razor, in that the simplest explanation works the best. Consider this as a typical oversight when it comes to understanding dispersion.

Another article in Nature supports the human mobility patterns observed with a different proxy mechanism [Ref 199]. The model seems to match the observed trends even more precisely and further reinforces the fundamental idea of entropic dispersion of travel velocities. Instead of using a paper money tracking system as in the previous Brockmann article, the authors (Gonzalez et al) used public cell-phone calling records — this seems to perform more directly rather than the indirect mechanism of proxy records of bill tracking to monitor human mobility.

> *Given that money is carried by individuals, bank note dispersal is a proxy for human movement, suggesting that human trajectories are best modeled as a continuous time random walk with fat tailed displacements and waiting time distributions.*
>
> *...*
>
> *Contrary to bank notes, mobile phones are carried by the same individual during his/her daily routine, offering the best proxy to capture individual human trajectories.*
>
> *...*
>
> *Individuals display significant regularity, as they return to a few highly frequented locations, like home or work. This regularity does not apply to the bank notes: a bill always follows the trajectory of its current owner, i.e. dollar bills diffuse, but humans do not.*

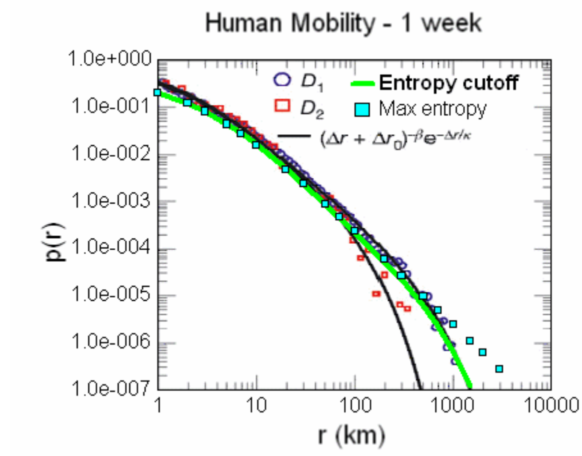
Even though the proxy records give the same general fat-tail trends, the essential problem with the bank note process remains the transaction process. The very likely possibility exists that a dollar bill exchanges hands among three unique individuals at a minimum between reporting instances, yet the cell-phone records an individual at more randomized and therefore less deterministic intervals (refer to Figure 28-5 on page 552). So I don’t expect the average rates of travel to agree between the two datasets.

The following fit uses the same model as I used previously, with data sampled at one week intervals. Notice that the data fits the Maximum Entropy Dispersion (the green curve) even better than bill tracking at 10 day intervals.

$$\frac{d}{dr}P(r) = \frac{\beta \cdot t}{(\beta \cdot t + r)^2} \quad \textbf{(EQ 28-8)}$$

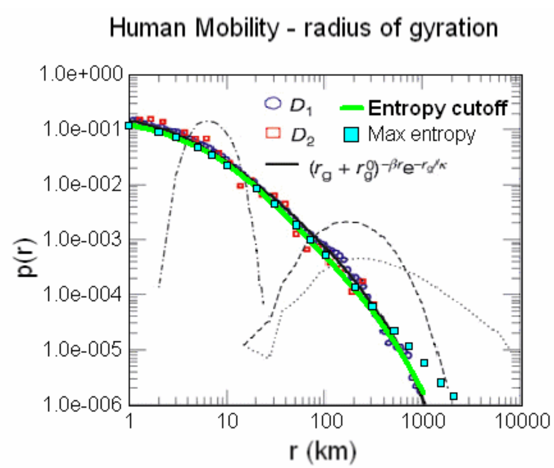
The value of β for this data set is 0.36 instead of 1 for the bill dispersion data set. I placed a cutoff on the dispersion by preventing a smearing into faster rates of 400 km/day and above, but this seems fairly reasonable as the model otherwise works over 5 orders of magnitude. It actually works so well that it detects a probability offset in the original data calibration; the probability PDF should sum to one over the entire interval yet the Gonzalez data exhibits a bias as it creeps up slightly over the normalized curve. This shift appears real as their own heuristic function (when I took the time to plot it) also shows a normalization bias.

FIGURE 28-8. Human mobility data from Gonzalez with the entropic dispersion model plotted alongside. Entropic dispersion has a fixed power-law so that it can only move up-and-down on this scale.



Another figure that Gonzalez plots mines the data according to a different sampling process, yet the general trend remains.

FIGURE 28-9. Human mobility collected according to a different measurement ruleset. This looks close to the previous figure as the model is scale-invariant.



Moreover, the researchers almost got the Maximum Entropy dispersion function right by doing a blind curve fit, but ultimately could not explain it. Instead of the predicted powerlaw exponent of -2, they use -1.75. Yet since they do not normalize their curve correctly with the beta parameter that a probability distribution requires, they think this value of -1.75 holds some significance. Instead the -1.75 power law is likely an erroneous fit and -2 works better — while not violating Occam's law and performing more parsimoniously.

Entropy always wins out on these phenomena and it really tells us how (in the sense of having no additional information, i.e. a Jaynesian model of entropy) people will statistically use different forms of transportation. The smearing occurs over such a wide range because people will walk, bicycle, residentially drive, freeway drive, or take air transportation. The entropy of all these different velocities serves to generate the dispersion curve that we empirically observe. The fact that it takes such little effort to show this with a basic probability model truly demonstrates how universal the model remains. That a single parameter indicating a median value of dispersive velocity can map over several orders of magnitude[2] fits again well within Occam's Rule[3].

Ultimately, I puzzle on why the researchers in the field seem to flail about trying to explain the data with non-intuitive heuristics and obscure random walk models.

Gonzalez at al have gotten tantalizingly close to coming up with a good interpretation, much closer than Brockmann in fact, yet they did not quite make the connection. If I could tell them, I would hint that their random walk is random but the randomness itself is not randomized. That explains so many phenomena yet they can't quite grasp the fundamental underlying concept.

As far as using the simple model for crafting future policy in regards to optimizing human travel in an energy constrained environment, I say why not. It essentially features a single parameter and covers the entire range of data. In terms of potential usage, one apply it for modeling the propagation of infectious diseases (a public health concern) or trying to minimize travel (an energy policy view).

As a caveat, I definitely ignored the diffusional aspects and the possibility of random walk in two-dimensions, yet I believe these largely get absorbed in the entropic smearing and the USA is not as much of a random two-dimensional stew as one may imagine. But as with all these simple dispersion arguments, I get the feeling that somehow this entire analysis approach has been unfortunately overlooked over the years.

---

2. With a second-order correction at the end-points having to do with the constrained physical breadth of the USA and how fast people can ultimately travel in a short period of time.
3. A complexity metric called Aikake Information Criteria (AIC) actually penalizes a model for having too many parameters, which is a variant of Occam's Rule put into practice.

Dispersion and Train Delays

Since nothing I write about contains any particularly difficult or obscure math, I thought to present this post as a word problem.

Say you reside in England. Consider taking a train from point **A** to point **B**. You have an idea of how long the trip will take based on the current train schedule but have uncertainty on its latency (i.e. possible time delay). The operators of the trains only tell you at best the fraction that arrive within a few minutes of their scheduled time. In other words, you have no idea of the fatness of the tails, and whether you will get delayed by tens of minutes on a seemingly short run.

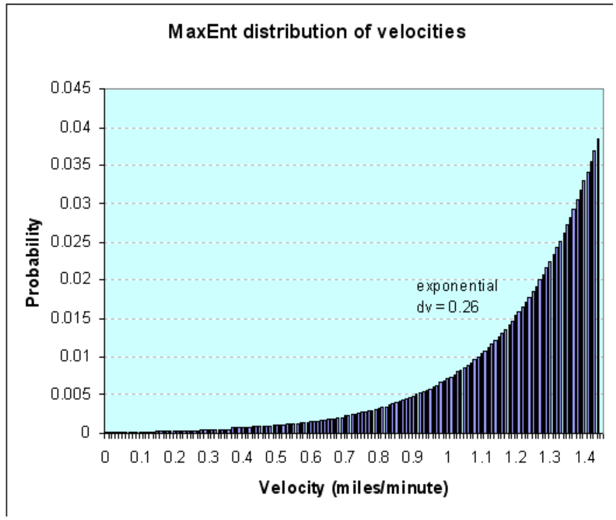
How would you derive the probability of a specific lateness of time ***dt*** based only on the knowledge of the distance ***X*** between point **A** and **B**, the maximum train speed ***Vm***, and an average train speed (same as distance ***X*** divided by the average trip duration ***t***)?

I won't solve this problem in anyway remotely that I would consider a classical approach. Instead I will make an assumption based on the principle of maximum entropy with the constraint of least amount of available information. So let us see how close we can come to the empirical distribution of train delay times observed based on assuming very limited information.

First of all, I have rather limited experiences travelling by train in England. I do know that the speed of a train can vary quite a bit as I have experienced the crawl from Heathrow to Victoria Station. I also know that the train has some maximum speed that it won't exceed. You realize this when you notice that the train rarely arrives early. So the average train speed and maximum train speed provides a pair of constraints that we can use for estimating a train *velocity probability density function* (PDF).

I assert via maximum entropy (MaxEnt) arguments that the train velocity (or speed for purists) PDF likely looks like this:

FIGURE 28-10. Maximum Entropy estimation of velocities for a train along a route.



In accordance with the constraints, MaxEnt predicts an exponential profile up to the maximum value, which ends at 1.44 miles/minute in the histogram. I would describe it as a dispersive velocity profile following a reverse damped exponential[4]:

$$p(v) = \frac{1}{\Delta v} \cdot e^{(v - v_m)/(\Delta v)} \qquad \textbf{(EQ 28-9)}$$

where $\boldsymbol{v}$ ranges from 0 to $\boldsymbol{v_m}$. The factor Δv relates to the average speed by

$$\Delta v = v_m - X/t \qquad \textbf{(EQ 28-10)}$$

The smaller that Δv becomes, the closer that the average speed approaches the maximum speed. Since we don't know anything more about the distributions of speeds, MaxEnt suggests that this exponential “approximation” will work just fine.

The only remaining step we need to do involves some probability theory to relate this velocity distribution to a time delay distribution. As the most straightforward approach, we determine the cumulative probability of velocities that will reach the destination in time $\boldsymbol{T}$. This becomes the integral over $\boldsymbol{p(v)}$ from a just-in-time speed $\boldsymbol{X/T}$ to the maximum speed $\boldsymbol{v_m}$:

4. The damping takes place for smaller velocities, see the next chapter for a similar behavior describing TCP statistics

$$P(t < T) = \int_{X/T}^{v_m} p(v)dv \qquad \textbf{(EQ 28-11)}$$

This results in the cumulative probability distribution function (upper-case P):

$$P(T) = 1 - e^{\frac{X/T - v_m}{\Delta v}} \qquad \textbf{(EQ 28-12)}$$

To turn this into a probability density function, we simply need to take the derivative with respect to ***T***.

$$p(T) = \frac{v_m \cdot T_m}{\Delta v \cdot T^2} \cdot e^{-\Delta t \cdot v_m/(\Delta v \cdot T)} \qquad \textbf{(EQ 28-13)}$$

We can also cast it in terms of the time delay by

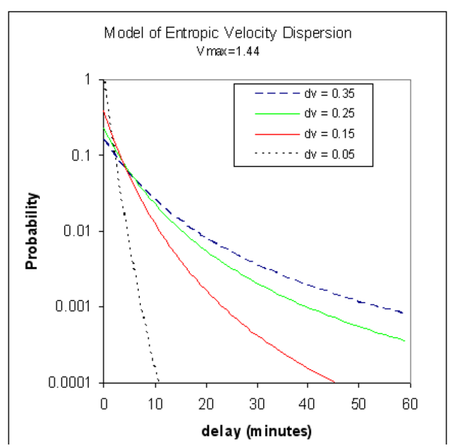
$$T = \Delta t + X/v_m \qquad \textbf{(EQ 28-14)}$$

so

$$p(\Delta t) = \frac{X}{\Delta v} \cdot \frac{e^{-\Delta t \cdot v_m/(\Delta v \cdot (\Delta t + X/M))}}{\left(\Delta t + \frac{X}{v_m}\right)^2} \qquad \textbf{(EQ 28-15)}$$

This might seem like a complicated expression but all of the parameters are well known but one, Δv. And even in this case, we can estimate Δv from some data. The following plot illustrates how the PDF changes with Δv. Note that as Δv becomes small, the probability of arriving on time becomes closer to 1 (e.g. the Swiss or German train system)

FIGURE 28-11.
Model of train delays for different values of Δv.



By definition, we have a fat-tail probability distribution because the density follows off as an inverse square power law.

So we need some data to check how good this distribution works. The topic of train scheduling arose from a paper [Ref 201] by an expert in the field of superstatistics [Ref 200]. Superstatistics may work as an alternate technique to solve the problem but as used by Briggs and Beck, it requires a couple of arbitrary parameters to fit the distribution to. This actually points to the difference between solving a problem as I am trying to do, versus blindly employing arbitrary statistical functions as Briggs and Beck attempt.

In any case, the authors did the hard work of collating statistics of over 2 million train departures covering 23 train stations in the British Rail Network.

Most of the statistics look generally similar but I take the route from Reading to London Paddington station as a trial run.

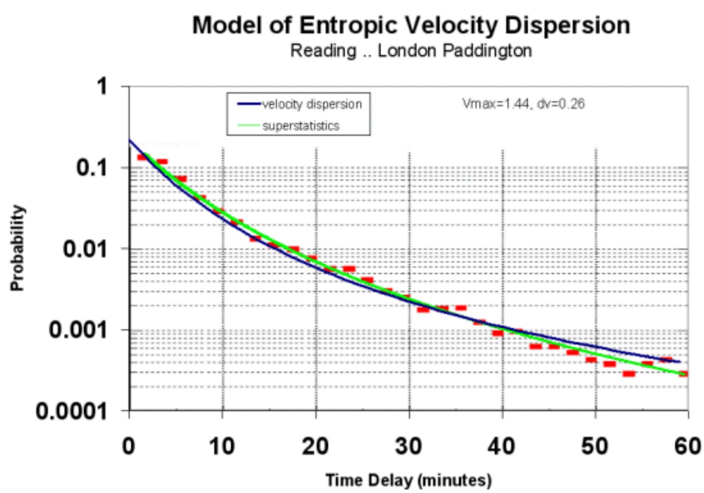


FIGURE 28-12. Train delay model of probability density function fit to data collected from the Reading to London Paddington route.

Both the superstatistics approach and my entropic dispersion solution fit the data remarkably well. One can see clearly that the profile neither follows Poisson statistics (which would give a straight line) nor does it follow normal Gaussian statistics (which would show an upside-down parabolic shape on a semi-log graph).

The shape of the tail points to the real fat-tail probabilities that occur on British rail lines — as we can see long delays do occur and the power law likely comes from the principle of entropic dispersion.

So to recap, the values I used stem from real observables:

X = 36 miles (exact)
$\mathbf{v_m}$ **= 1.44** miles/minute (from the schedule, see below)
Δv **= 0.26** miles/minute (from the curve fit or alternatively from the average latency)

FIGURE 28-13. Table of estimated arrival times reported for the route. Obviously the estimated times are gathered from the data collected for that route over the years.

Leaving	From	To	Arriving	Duration
05:57	Reading [RDG]	London Paddington [PAD]	**06:24**	0h 27m
06:02	Reading [RDG]	London Paddington [PAD]	**06:36**	0h 34m
06:16	Reading [RDG]	London Paddington [PAD]	**06:54**	0h 38m
06:22	Reading [RDG]	London Paddington [PAD]	**07:00**	0h 38m
06:30	Reading [RDG]	London Paddington [PAD]	**07:09**	0h 39m

I stated earlier that most people would approach this problem from the perspective of Poisson statistics using *time* as the varying parameter. Briggs and Beck do this as well, but they use another layer of probability, called *superstatistics* to “fatten” the tail. Although I appreciate the idea of superstatistics (see the description of hyperbolic discounting of an example of a doubly integrated distribution function), I believe that entropic dispersion of velocities gives a more parsimonious explanation.

This approach shows consistency with the data on human travel yet it provides a more narrow focus on the application of entropic dispersion.

Marathon Dispersion

I originally began my studies of dispersion outside of the oil extraction realm by looking at dispersion in the finishing times for marathon races [Ref 129]. The dispersion results from human variability but due to (self) censoring in the population of runners, we never see complete dispersion like we would in oil exploration and discovery.

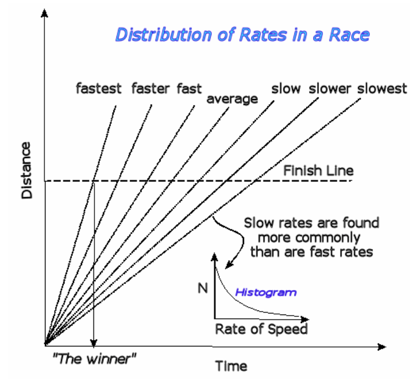


FIGURE 28-14. Runner's dispersion in foot speed leads to a "reciprocal" variation in time.

The figure on the right shows how the finishing times get dispersed in time due to variations in the running speeds of the competitors. This shows a limiting max speed and truncated slow speeds as the very slow runners would rather not punish themselves for long periods of time, resulting in censorship of the velocity data over slower velocity ranges. The following chart with numbers that I collected from the finishing times of a typical Honolulu marathon show this truncation quite clearly. If all *potential* runners, comprised mostly of those not so physically fit, the tails of finishing times would extend for a long while.

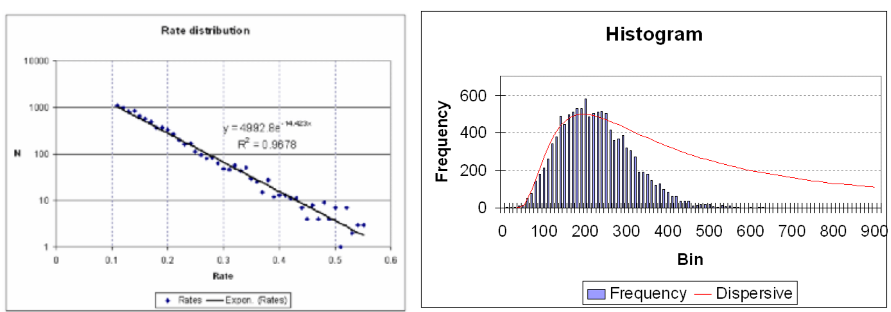
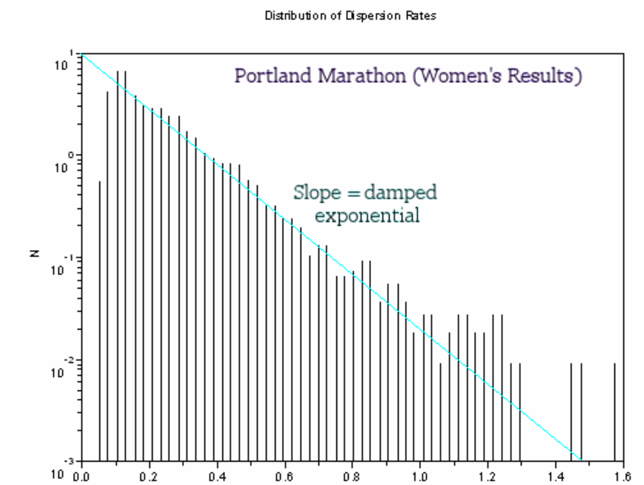


FIGURE 28-15. Speed of racers show an exponential dispersion of rates (left) but censoring of finishing times from a completely dispersive model occurs as slow runners choose not to finish or choose not to race (right).

Taking a different marathon (Portland) with a bit more lax finishing time cut-off, one can see that the maximum entropy rates of speed follow more closely the damped exponential. A possible bulge also occurs at the fastest times, but the possibility of prize money for the top several finishers may explain this, thus adding to another piece of information preventing this from becoming the most optimal example of maximum entropy in practice. The missing pieces of information and constraints that we know exist yet cannot accurately quantify prevent us from applying the exponential distribution effectively.

FIGURE 28-16. Portland marathon results plotted as reciprocal of finishing time, proportional to the speed of runners.



Verifying Dispersion

Maximum entropy modeling works to the extent that you can only model based on observable measurements you can make. These generate the moments and constraints for the probability distributions. Any other information you may want to include can lead to unwarranted assumptions. So maximum entropy provides a way of dealing with limited information.

So if we did have more information, then we would get different results. Marathon finishing times gives a good example of this behavior. Humans show a huge variation in athletic abilities and you should see this in the distribution. Hard constraints exist in the maximum speed of a runner but not necessarily in the minimal speed of a runner. But then you add in other constraints such as the artificial cut-off time in a specific race, and it modifies the finishing time distribution in the tails. All the people that could race at the slower speeds won't enter the race because they know that they won't make the cut-off time. We also see a bulge in the top runners due to the lure of prize money, thus artificially attracting from outside of a uniform sampling pool. So we can *know* about these things in the information sense and thus adjust the model and get different results, in keeping with the maximum entropy principle

On the other hand, when we start piling up the constraints and information, it becomes harder and harder to model entropy this way, unless the combination of constraints also starts to become random. Then we can again start to reason about the situation. As the best example of this situation, consider the distribution of human travel times that we discussed earlier in the chapter. The maximum entropy principle applied to this problem gives very parsimonious agreement with the data. Most of these scale-free probability distributions occur because the dimensionality of the problem disappears. So a short time resulting from a slow rate operating over a short distance looks the same as a very fast rate operating over a larger distance. We essentially got rid of the constraints of the marathon finishing times by just describing a situation where everyone in the entire country wants to get somewhere else, but neither the finish line nor the cut-off time gets specified. Even though the problem looks way more complex, the solution drops out very straightforwardly — a great example of the counterintuitive notion of greater complexity leading to simpler results that we fore-shadowed in “Innovation and Evolution. How ideas spread” .

CHAPTER 29

Communications

Entropy in data

"We thought we were getting something for nothing,
But we were getting nothing for everything."

— Wendell Berry

The advent of network and mobile devices has transformed the way that we communicate with one another. This has relieved some of the burden on our petroleum-based transportation infrastructure, from shipping information via land and air to one based on electronics. This has also become quite a reliable means of transmitting information. Yet, even digital communications show the sometimes subtle influences of entropy and disorder. It doesn't take much effort to find this if you know where to look. This section describes some of these issues, which like oil depletion has to do with garden-variety variability as much as anything else.

Network Dispersion

To give a hint as to how natural dispersion works, we take an example from a physical process, that of network round-trip time (RTT) dispersion of TCP/IP network packets. Computer scientists know that these network packets have non-guaranteed delivery times, routinely caused by collisions and other latencies. The following figure provides a typical example of packet dispersion, caused by slight differences in transmission rates of messages (taken from SLAC experiments[1])

1. The SLAC=Stanford Linear Accelerator requires a huge amount of data collection and fast performance [Ref 128]

.

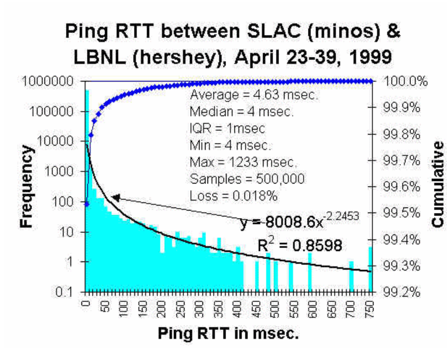


FIGURE 29-1.
Results from SLAC experiment showing a packet latency histogram. The solid black line is a heuristic fit.

The equation showing maximum entropy in rates (akin to the rates used in disordered drift) gives the cumulative $P(t) = e^{-T/t}$ or the PDF as follows:

$$p(t) = T \cdot e^{-\frac{T}{t}} / t^2 \qquad \textbf{(EQ 29-1)}$$

matches the general shape of the curve drawn on a semi-log scale in the figure below. I assumed a simple dispersion of rates and fixed round-trip distance, using ***T***=50 microseconds as the mean latency:

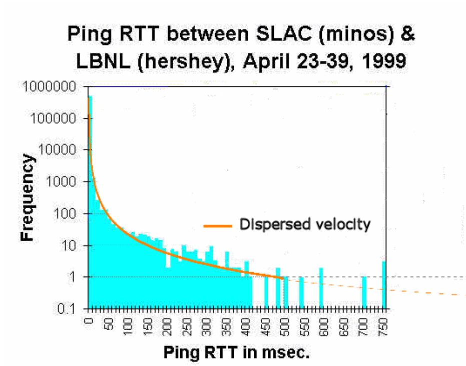


FIGURE 29-2.
Fit to SLAC latency histogram using a network dispersion model. The packet velocities are distributed as a damped exponential but the point-to-point distances are fixed.

Although the authors of the this study chose to use a heuristic fit, the phenomena also has some basis in what researchers refer to as “asymptotic dispersion rate” and “average dispersion rate”. If we say that the maximum entropy in rates drops off from a perfect “collisionless” path then the following distribution may apply better:

$$p(r) = k \cdot e^{rT} \qquad r < r_{max} \qquad \textbf{(EQ 29-2)}$$

To solve this for a probability flow, we use the following formulation.

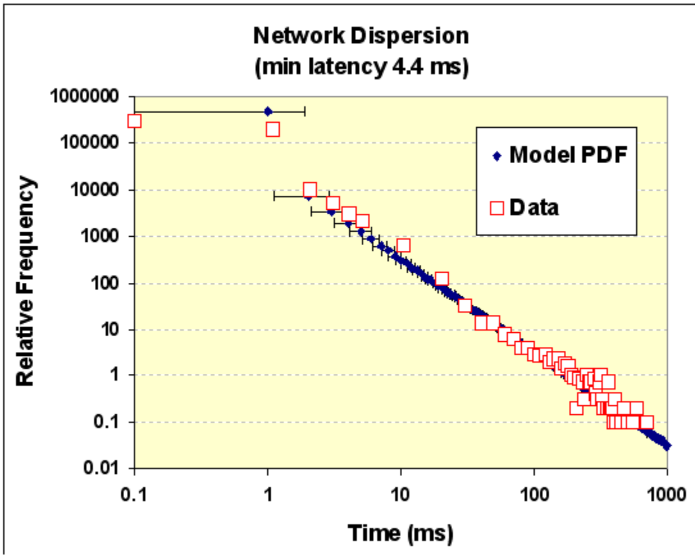
$$p(t) = \int_0^{\infty} r \cdot p(r) \cdot \delta(x - rt) dr \quad \textbf{(EQ 29-3)}$$

This gives the result

$$p(t) = \frac{k}{t^2} \cdot e^{\frac{T}{t}} \qquad t > \frac{x}{r_{max}} \quad \textbf{(EQ 29-4)}$$

This gives a simple distributed latency caused by a maximally dispersed network transport rate,. The equation above gives the $1/t^2$ slope shown below adjusted for the log density along the horizontal axis.

FIGURE 29-3. On a log-log scale, the PDF of the RTT times reveals a slope of -2. Other research has shown the fat-tail to resemble a Cauchy distribution [Ref 317]



Some uncertainty exists in the minimum measurable round-trip time at around 1 ms, but the rest of the curve agrees with the simple entropic dispersion discussed elsewhere in this volume.

"1/f" Noise

It turns out the routine observation of a certain class of noise spectrum has much to do with dispersion of processes. Noise that shows much more prevalence at low frequencies than at high frequencies comes about from a superposition of processes that essentially act at different rates. The random superposition of these processes produces the so-called “1/f” noise observed in many different real-world applications[2]. Many engineers and scientists perceive that *1/f* noise has some mysterious origin — as in not very well understood

> Do we understand 1/*f* noise? My impression is that there is no real mistery (*sic*) behind 1/*f* noise, that there is no real universality and that in most cases the observed 1/*f* noises have been explained by beautiful and mostly *ad hoc* models. [Ref 191]

The analogy to dispersion in latencies, as described by TCP/IP experiments, remains tenuous only because latencies arise from real space (i.e *time*) considerations whereas we measure 1/*f* noise in the frequency space (i.e. 1/*time*). If we relate low frequencies to slow processes and the more rarely occurring high frequencies to fast rates than we can analogize to the familiar power-law distribution that occurs in real space. A key finding in 1/*f* research leads to the observation that the actual distribution in rates does not matter too much and the fat Pareto-like tails come out just the same [Ref 191].

The electronic noise pioneer van der Ziel [Ref 192] first made the superposition argument in 1950 as he postulated that energetic electrons would exhibit motion over a range of velocities due to the way Boltzmann’s Law works in semiconductors.[3] He indirectly suggested that time constants show a PDF envelope as shown in the following relationship:

$$\rho(\tau) \cdot d\tau = d\tau / \tau \quad \textbf{(EQ 29-5)}$$

This actually shows a much broader distribution than we use in Eq 4-1, but it gives the same result in the observed fat tail. Machlup [Ref 270] later suggested that this has greater applicability than just in the semiconductor physics realm, and made connections to earthquake and thunderstorm fractal noise sources as long as the above scale-invariant relationship held.

From this launching point, researchers who study 1/*f* noise in depth have a seeming handle on how to go about unifying several different heuristics: from the mechanisms of fractal processes, to the observation of log-normal distributions, to the Pareto rule [Ref 271]. Montroll [Ref 190] ties some of the loose ends together, showing in particular how the log-normal distribution fits in given a highly dis-

2. Whether they show true noise or not; this is noticed in the distribution of audio frequencies in popular music recordings and in a variety of radio programs [Ref 191]
3. I studied under Van der Ziel in college. As part of a term paper, I handed in what I thought was a novel approach to deriving a 1/f law by superposing a train of varying telegraph noise distributions. I asked whether it merited publishing, and he said no, but it re-emerges here.

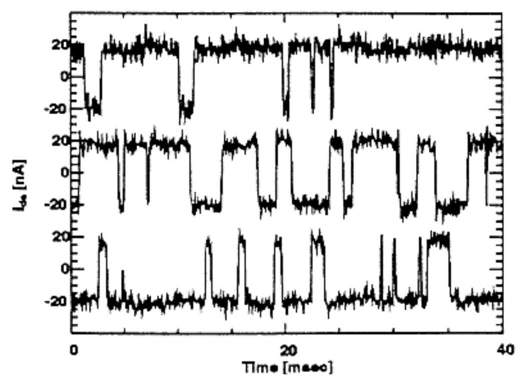
persed distribution. This also falls under the category of "long memory" time series analysis. In what follows, we give an elementary derivation of 1/*f* noise.

Telegraphing Monkeys, Entropy, and 1/f Noise. The observed behavior known as *1/f noise* seems to show up everywhere. They call it *1/f* noise (also known as flicker or pink noise) because it follows an inverse power law in its frequency spectrum. It shows up both in microelectronic devices as well as emanating from deep space. Its ubiquity gives it an air of mystery and the physicist Bak tried to explain it in terms of self-organized critical phenomena. I find no need for that level of contrivance, as ordinary entropic disorder will work just as well.

With that as an introduction, I have a pretty simple explanation for the frequency spectrum power law based on a couple of maximum entropy ideas.

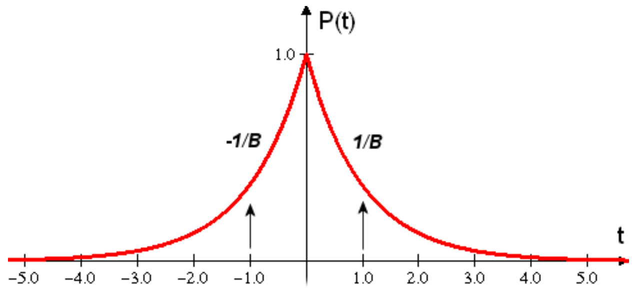
Something called random telegraph noise (RTS) (or burst or popcorn noise) can occur for a memory-less process. One can describe RTS by simply invoking a square-wave that has a probability of ***B*** to switch states at any ***dt*** time interval. This turns into a temporal Markov Chain kind of behavior and the typical noise measurement looks like the following monkeys typing-at-a-telegraph trace and sounds like popcorn popping in its randomness. It pretty much describes an ordinary Poisson process.

FIGURE 29-4. Random telegraph noise



The Markov Chain pulse train as described as above has an autocorrelation function that looks like a two-sided damped exponential. The correlation time equals 1/***B***.

FIGURE 29-5. Autocorrelation of random telegraph noise.



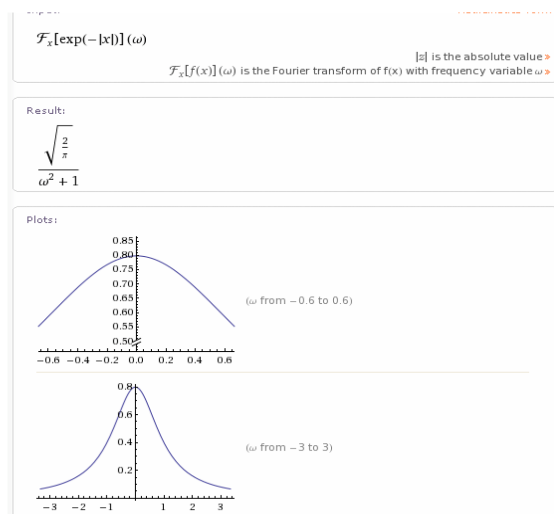
The autocorrelation (related to a *self convolution*) of a stochastic measure has some interesting properties. In this case, it does have maximum entropy content for the two-sided mean of 1/***B*** — **MaxEnt** for the positive axis and **MaxEnt** for the negative axis. In other words this is the maximum entropy distribution given the constraints of knowing only the absolute value of a mean |1/***B***|.

In addition, the Fourier Transform of the autocorrelation gives precisely the frequency power spectrum. This comes out proportionately to:

$$S(w) = \frac{\sqrt{2/\pi}}{B^2 + w^2} \qquad \textbf{(EQ 29-6)}$$

where ***w*** is the angular frequency. The figure below shows the ***B***=1 normalized result.

FIGURE 29-6. Demonstration of Fourier transform of semi-Markov exponential to the Cauchy/ Lorentzian power-law frequency spectrum.



Now consider that this result only gives one spectrum of the many Markov switching rates that may exist in nature. If we propose that ***B*** itself can vary widely, we can solve for the superstatistical RTS spectrum.

Suppose that ***B*** ranged from close to zero to some large value ***R***. We don't have a mean but we have these two limits as constraints. Therefore we let maximum entropy generate a uniform distribution for ***B***.

To get the final spectrum, we essentially average the RTS spectrums over all possible intrinsic rates:

$$\tilde{S}(w) = \int_0^R S(w|B)dB \qquad \textbf{(EQ 29-7)}$$

This generates the following result

$$\tilde{S}(w) = \frac{\operatorname{atan}(R/w)}{w} \qquad \textbf{(EQ 29-8)}$$

If ***R*** becomes large enough then the arctan converges to a constant $\pi/2$ and reduces to the $1/f$ spectrum if we convert $w=2\pi f$.

$$\tilde{S}(f) \sim 1/f \qquad \textbf{(EQ 29-9)}$$

If we reduce ***R*** in the limit, then we get a regime that has a $1/f$ component and a $1/f^2$ above the ***R*** transition point, where it reverts back to a telegraph noise power-law.

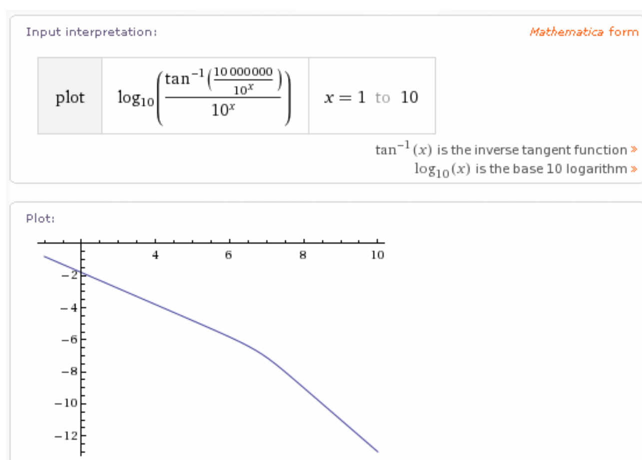


FIGURE 29-7. The transition between power-law regimes due to superposition of Cauchy functions.

Milotti eliminated some of the mystery behind 1/***f*** noise in his review paper [Ref 191]. He takes a slightly different tact but comes up with the same result that I have above, essentially challenging Bak's theory and said that no universality stands behind the power-law, just some common sense.

As an even simpler first order derivation of *1/f* noise, consider the following maximum entropy argument. If the noise represents electromagnetic radiation, then one can perhaps generate an even simpler derivation. The energy of a photon is ***E*(*f*)=*h**f** where ***h***=Plank's constant and ***f*** is frequency. According to maximum entropy, if energy radiation remains uniform through the frequency spectrum, then we can only get this result if we apply a 1/***f*** probability density function:

$$E(f) \times p(E(f)) = h \cdot f \cdot (1/f) = \text{constant} \qquad \textbf{(EQ 29-10)}$$

Rayleigh Fading, Wireless Gadgets, and a Global Context

The intermittent nature of wind power has a fundamental explanation based on entropy arguments (see the previous chapter). It turns out that this same entropy-based approach explains some other related noisy and intermittent phenomena that we deal with all the time. The obvious cases involve the use of mobile wireless gadgets such as WiFi devices, cell phones, and global positioning system (GPS) navigation aids in an imperfect (i.e. intermittent) situation. The GPS behavior has the most interesting implications which I will get to in a moment.

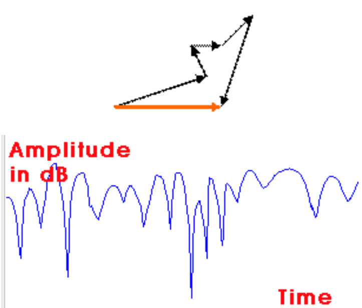


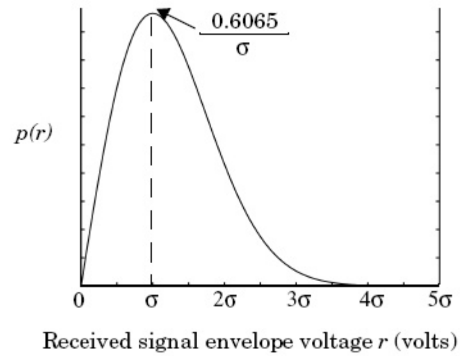
FIGURE 29-8. Random vector sum to give the amplitude shown

First of all, consider that we often use these wireless devices in cluttered environments where the supposedly constant transmitted power results in frustrating fade-outs that we have all learned to live with. An example of Rayleigh fading appears to the left margin. You can find some signal interference-based explanations for why this happens, originating via the same intentional phase cancellations that occur in noise-cancelling headphones. For the headphones, the electronics flip the phase so all interferences turn destructive, but for wireless devices, the interferences turn random, some positive and some negative, so the result gives the random signal shown.

In the limit of a highly interfering environment the amplitude distribution of the signal shows a Rayleigh distribution, the same observed for wind speed.

$$p(r) = 2kr \cdot e^{-kr^2} \qquad \textbf{(EQ 29-11)}$$

FIGURE 29-9. The Rayleigh Distribution of signal strength.

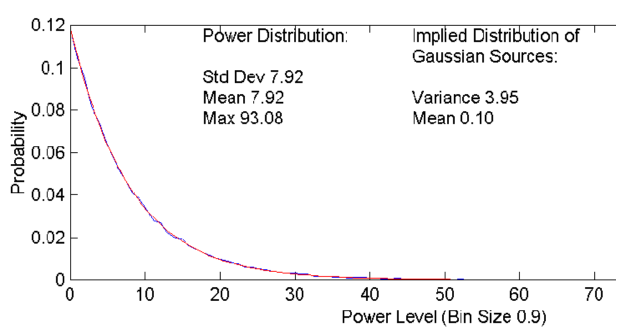


Our knowledge of the situation reduces to that of knowing only the average power level of the signal. In that case, we can use Maximum Entropy Principles to estimate the amplitude from the energy stored in the signal, just like one can derive it for wind speed. So, as a starting premise, if we know the average power alone, then we can derive the Rayleigh distribution.

The following figure shows the probability density function of the correlated power measured from a GPS signal. Since power in an electromagnetic signal relates to energy as a flow of constant energy per unit time, then we would expect the energy or power distribution to look like a damped exponential, in line with the maximum entropy interpretation. And sure enough, it does exactly match a damped exponential as shown in Figure 29-10 on page 573.

$$p(E) = k \cdot e^{-kE} \qquad \textbf{(EQ 29-12)}$$

FIGURE 29-10. Not e that the standard deviation equals the mean, a clear indication of a damped exponential. (from [Ref 320])



Yet since power (***E***) is proportional to Amplitude squared ($\boldsymbol{r}^2$), we can derive the probability density function by invoking the chain rule.

$$p(A) = p(E) \cdot \frac{\mathrm{d}E}{\mathrm{d}r} = e^{-kr^2} \cdot \frac{\mathrm{d}}{\mathrm{d}r}(r^2) = 2kr \cdot e^{-kr^2} \quad \text{(EQ 29-13)}$$

which precisely matches the Rayleigh distribution, implying that Rayleigh fits the bill as a Maximum Entropy (MaxEnt) distribution. So too does the uniformly random phase in the destructive interference process qualify as a MaxEnt distribution, which will range from 0 to 360 degrees (which gives an alternative derivation of Rayleigh). So all three of these distributions, Exponential, Rayleigh, and Uniform all act together to give a rather parsimonious application of the maximum entropy principle.

The most interesting implication of an entropic signal strength environment relates to how we deal with this power variation in our electronic devices. If you own a GPS, you experience this when trying to acquire a GPS signal from a cold-start. The amount of time it takes to acquire GPS satellites can range from seconds to minutes, and sometimes we don't get a signal at all, especially if we have tree cover with branches swaying in the wind.

Explaining the variable delay in GPS comes out quite cleanly as a fat-tail statistic if you understand how the GPS locks into the set of satellite signals. The solution assumes the entropy variations of the signal strength and integrating this against the search space that the receiver needs to lock-in to the GPS satellites.

Since the search space involves time on one axis and frequency in the other, it takes in the limit ~$\boldsymbol{N}^2$ steps to decode a solution that identifies a particular satellite signal sequence for your particular ***unknown*** starting position[4]. This gets reduced because of the mean number of steps needed on average in the search space. We can use some dynamic programming matrix methods and parallel processing (perhaps using an FFT) to get this to order $\boldsymbol{N}$, so the speed-up for a given rate is $\boldsymbol{t}^2$. So this will take a stochastic amount of time according to MaxEnt of:

$$P(t|R) = e^{-cRt^2} \quad \text{(EQ 29-14)}$$

However because of the Rayleigh fading problem we don't know how long it will take to integrate our signal with regard to the rate $\boldsymbol{R}$. This rate has a density function proportional to the power level distribution:

4. If you already know your position and have that stored in your GPS, the search time shrinks enormously. This is the warm or hot-start mode that is currently used by most manufacturers. The cold-start still happens if you transport a "cold" GPS to a completely different location and have to re-acquire the position based on unknown starting coordinates.

$$p(R) = k \cdot e^{-kR} \quad \textbf{(EQ 29-15)}$$

then according to the rules for marginal distributions the conditionals line up to give the probability of acquiring a signal within time ***t***:

$$P(t) = \int_0^{\infty} P(t|R) \cdot p(R) dr \quad \textbf{(EQ 29-16)}$$

this leads to the entropic dispersion result of:

$$P(t < T) = \frac{1}{1 + \left(\frac{T}{a}\right)^2} \quad \textbf{(EQ 29-17)}$$

where ***a*** is an empirically determined number derived from ***k*** and ***c***. I wouldn't consider this an extremely fat tail because the acceleration of the search by quadrature tends to mitigate very long times.

Data Analysis. I collected data from a GPS project that has a goal to speed up wild-fire response times by cleverly using remote transponders [Ref 272]. They published a good chunk of data for cold-start times as shown in the histogram below. Note that the data shows many times that approach 1000 seconds. The single parameter entropic dispersion fit (***a***=62 seconds) appears as the blue curve, and it fits the data quite well:

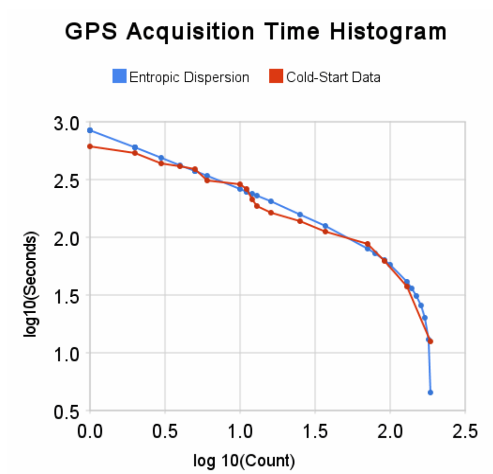


FIGURE 29-11. Fit of GPS acquisition times to a dispersive model.

Dealing with Disorder. The GPS acquisition problem raises an intriguing possibility in how we manage and deal with disorder. Interesting how we can sharpen the tail in a naturally entropic environment by applying an accelerating technology (see dispersive effects of oil discovery in Volume 1). Put this in the context of a diametrically opposite situation where the diffusion limitations of CO_2 slow down the impulse response times in the atmosphere, creating even bigger fat-tails which will inevitably lead to climate change.

If we can think of some way to accelerate the CO_2 removal, can we shorten the response time, just like we can speed up GPS acquisition times or speed up oil extraction. Or should we have just slowed down oil extraction to begin with? Ultimately, those kinds of questions come up when we start to comprehend the impact of disorder on our environment.

CHAPTER 30

Environment.

Disorder around us

"Tell me something I don't know"

— Chris Matthews

Our environment shows great diversity in the size and abundance in natural structures. Since we extract oil from random structures in our environment, it stands to reason that many of the same mechanisms leading to oil formation could also reveal themselves in more familiar natural phenomena. Take the size distribution of lakes as an example.

Lakes

Freshwater lakes accumulate their volume in a manner analogous to the way that an underground reservoir accumulates oil. Over geologic time, water drifts into a basin at various rates and over a range in collecting regions. As lakes capture most of their volume through water drainage, one can imagine that the rate of influx plays a factor in how large a lake can become. This behavior gets described in "The Facts in the Ground. Where do we find oil reservoirs?" and the Maximum Entropy prediction of the size distribution leads to the following expression:

$$P(\text{Size}) = \frac{1}{1 + \frac{a}{\text{Size}}} \qquad \textbf{(EQ 30-1)}$$

Surveys of lake size show the same reciprocal power law dependence, with the exponent usually appearing arbitrarily close to one. In Figure 30-1 on page 578, the data plotted on a ranked plot clearly shows this dependence over several orders of magnitude.

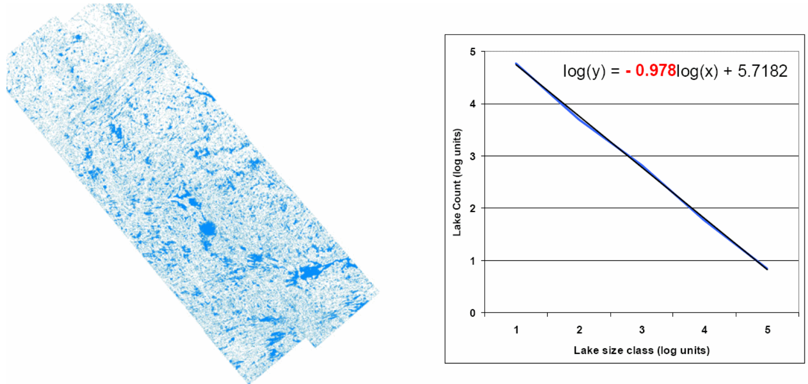
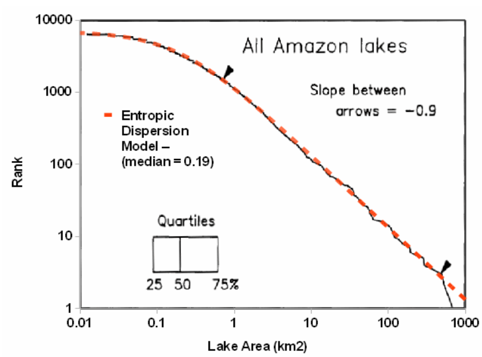


FIGURE 30-1. Northern Quebec lakes map and size distribution adapted from [Ref 321]

More revealing, in Figure 30-2 on page 578, we can observe the bend in the curve that limits the number of small lakes in exact accordance to the equation shown above. The agreement with such a simple model suggests that a universal behavior links the statistics between environmental phenomena as seemingly distinct as those of lakes and oil reservoirs.

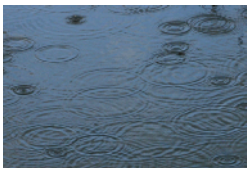
FIGURE 30-2. Amazon lake size distribution from “Estimation of the fractal dimension of terrain from Lake Size Distributions” [Ref 322]



This provides other intuitive clues to how to think about reservoir sizing. Consider the fact that very few freshwater lakes reach gigantic portions, the Great Lakes serving as a prime example. Similarly, the rare occurrence of “super-giant” reservoirs follow from the same principles. We clearly won’t find any new huge freshwater lakes, while the future occurrence of super-giant oil reservoirs remains very doubtful just from the statistics of oil reservoirs found so far. Finding substantial numbers of super-giant reservoirs would result in deviations from the size distribution plot, making it very unlikely.

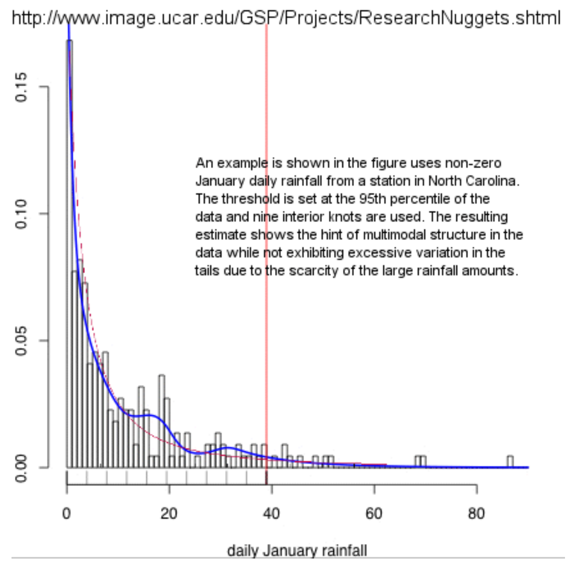
The rest of this chapter covers other environmental models that arise from equally simple and straightforward arguments.

Rainfall



Entropic dispersion also occurs in daily rainfall data. I happened across research work at the National Center for Atmospheric Research under the title "Extreme Event Density Estimation"[1]. The researchers there seem to think the following graph has some mysterious structure. It basically displays a histogram of daily rainfall in January at a station in North Carolina.

FIGURE 30-3.
Rainfall distribution in North Carolina showing occasional extreme values which indicate a relatively weak fat-tail.



On first glance, this data doesn't appear highly dispersed as the tail stays fairly thin, yet if take a look at this fit, we can understand the extreme value aspects:

1. Much more of the study of rainfall data and behaviors such as flooding comes under the category of *extreme value analysis*. EVA typically concentrates on the tails of the probability distributions, using a piece-wise approach to analyze empirical data. Our analysis differs in that we treat the entire distribution as a whole, while not making the distinction between extreme events and the expected situation. As an aside, public data for rainfall is hard to come by and so the data sets are not as comprehensive as one would like.

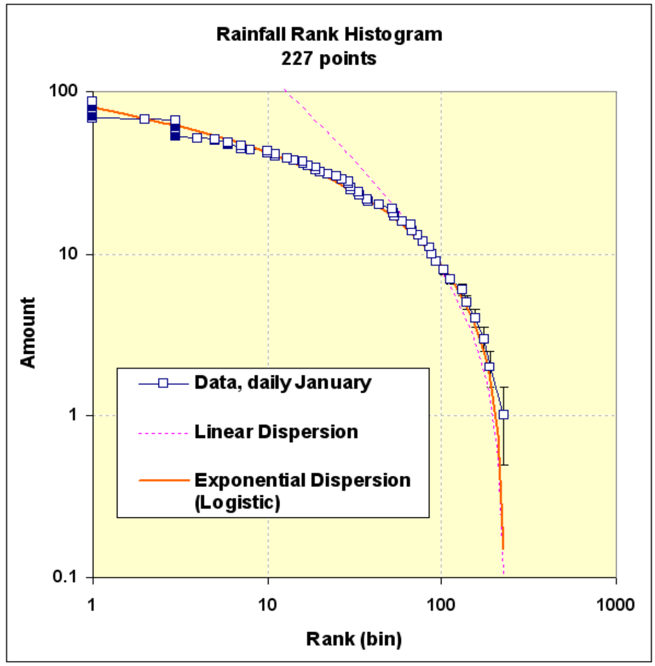


FIGURE 30-4. Plotted as a rank histogram indicates that the tail appears closer to an exponential.

You need to understand first that a critical point exists for rain to fall. The volume and density at which nature decides it reaches this critical point has much to do with the rate at which a cloud develops in its intensity and payload. If we assume that clouds develop by some sort of preferential attachment, then the uncertainties at which the preferential attachment process increases with time balanced against the uncertainties in the critical point contribute to the entropic dispersion:

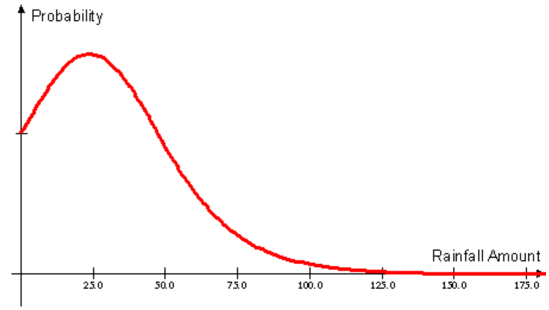
$$p(x) = \frac{r}{(r+g(x))^2} \quad \textbf{(EQ 30-2)}$$

The term $g(x)$ essentially measures the preferential attachment accelerated growth rate:

$$g(x) = k \cdot (e^{ax} - 1) \quad \textbf{(EQ 30-3)}$$

This has the mechanism for preferential attachment since $dg/dx = a * (g(x)+k)$, which describes exponential growth plus a linear term. When plotted, we get the curve shown plotted along with the data points, where r=1, a=1/17 years and k=2 (dimensions in mm). Coincidentally, this is actually just the logistic sigmoid function; we never see the characteristic peak or inflection point since it starts off well up the towards the halfway point of the cumulative. See the following graph where I used ***r***=10 to accentuate the sigmoid peak.

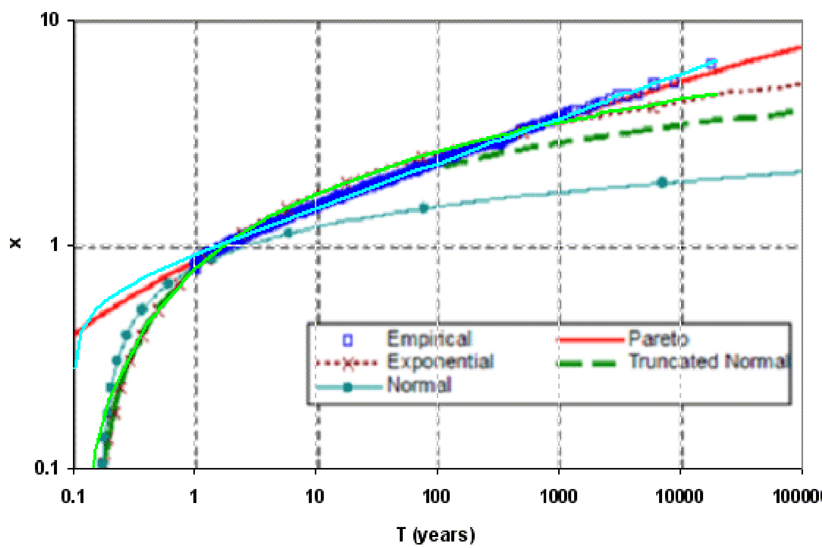
FIGURE 30-5.
Growth function which produces a logistic. The actual distribution used to model rainfall starts on the downslope.



In spite of the good fit, this curve has a limited locality: consisting of one rainfall station in North Carolina. What happens when we look at the distribution of rainfall on a global scale?

The global curve below does not work as well for the exponential growth (bright green) but it does work very well for the power-law growth model (cumulative growth ~ x^5, bright turquoise)[2]. The values shown here are in inches of rainfall, not millimeters as in the previous figure.

FIGURE 30-6.
Rainfall collected on a much larger scale indicates that the growth function is slower than an exponential and reduces to a power-law. From [Ref 273] which attempts an understanding of the distribution via entropy arguments.

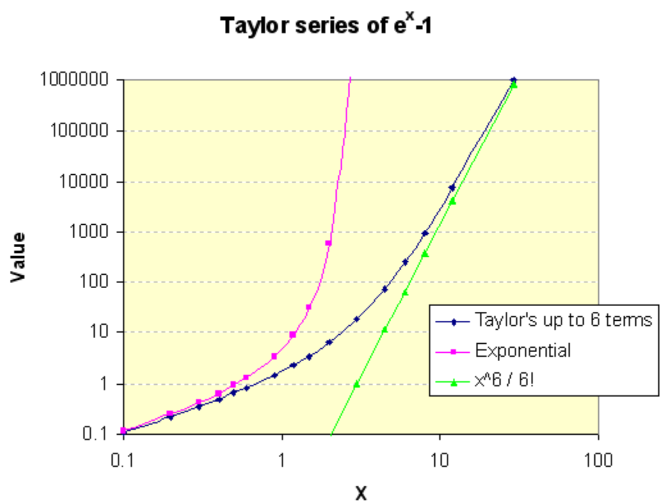


2. Note that this curve uses hydrologist's lingo, where the rate of return in years T equates to a histogram bin. The rate of return analogizes to principle such as the "100 year storm"

Since we can expand an exponential in terms of a Taylor series and see a sum of power terms, it makes some sense that the t^5 term may emulate the exponential growth (or vice versa), or perhaps generate the limiting trend. Exponential growth eventually moderates and the 5th power may provide the major effect along the curve over the remainder of the fat tail. Note that the cumulative power is 6 in the model because we measure rainfall linearly while growth of water content goes as volumetric density, cubed as a radial growth, so a high power law is not inconceivable.

The graph below shows an exponential morphing into a x^6 dependence by limiting the Taylor series expansion of the exponential to 6 terms.

FIGURE 30-7. How an exponential growth function merges into a power law by taking a finite number of Taylor's Series elements. Exponential growth cannot continue indefinitely.



As the arguments for oil discovery (also a power law, as uncertain acceleration in technology along an uncertain volume) emulate rain strength (uncertain acceleration in cloud/droplet growth along an uncertain critical volume/density), I consider this a further substantiation of the overall entropic dispersion formulation.

Yet I've got to wonder: do climate scientists think this way? The hydrology researcher Koutsoyiannis, who identified the power-law dependence in the previous figure, seems to have followed this path [Ref 273]. His multiple Markov chain remains a bit unwieldy as he can only generate a profile via simulation, yet that may not matter if we can use a power-law argument directly as I did for dispersive discovery. This needs a deeper look as it exposes a few remaining modeling details in a comprehensive theory.

Earthquakes

What causes the relative magnitude distribution in earthquakes?
In other words, why do we measure many more small earthquakes than large ones? And why do the really large ones happen only occasionally enough to classify as Mandelbrotian gray swans?

Of course many physicists want to ascribe it to the properties of what they call *critical phenomena* and some researchers makes the claim for a universal model of earthquakes [Ref 274].

> *Because only critical phenomena exhibit scaling laws, this result supports the hypothesis that earthquakes are self-organized critical (SOC) phenomena* [Ref 274].

I consider that a strong assertion because you can also read it as implying that scaling laws would never apply to a noncritical phenomenon.

In just a few steps I will show how garden-variety disorder will accomplish the same thing.

First the premises

1. Stress (a force) causes a rupture in the Earth resulting in an earthquake.
2. Strain (a displacement) within the crust results from the jostling between moving plates which the earthquake relieves.
3. Strain builds up gradually over long time periods by the relentlessness of stress.

This build-up can occur over various time spans. We don't know the average time span, although one must exist, so we declare it as a dispersive Maximum Entropy probability distribution around τ

$$p(t) = \frac{1}{\tau} \cdot e^{-t/\tau} \qquad \textbf{(EQ 30-4)}$$

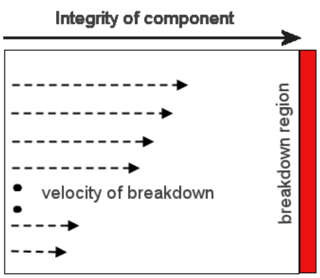
Next the cumulative probability of achieving a strain (x) in time T is the integral of $p(t)$ for all t such that t is less than x/v.

$$P(x, v|T) = \int_0^{x/v} p(t)dt \qquad \textbf{(EQ 30-5)}$$

The term x/v acts as an abstraction to indicate that x changes linearly over time at some velocity v. This results in the conditional cumulative probability:

$$P(x, v|T) = 1 - e^{-x/(vT)} \quad \text{(EQ 30-6)}$$

At some point, the value of x reaches a threshold where the accumulated strain caused by stress breaks down. This looks similar to the breakdown of a component's reliability, see adjacent figure reproduced from "Reliability. Entropy and how things break down" . We don't know this value either — but we know an average exists, which we call X — so by the Maximum Entropy Principle, we integrate this over a range of x.



What is the probabilty of breakdown over a range of velocities?

$$P(v|T, X) = \int_0^{\infty} P(x, v|T) \cdot \frac{1}{X} \cdot e^{-x/X} \cdot \mathrm{dx} \quad \text{(EQ 30-7)}$$

This results in:

$$P(v|T, X) = \frac{X}{vT + X} \quad \text{(EQ 30-8)}$$

So we have an expression that has two unknown constants given by X and T and one variate given by a velocity v (i.e. the stress). Yet, since the displacement x grows proportionally as $v*T$, then rewrite this as

$$P(v|T, X) = \frac{X}{x + X} \quad \text{(EQ 30-9)}$$

This gives the cumulative distribution of strains leading to an earthquake. The derivative of this cumulative is the density function which has the power-law exponent 2 for large x.

If we plot this as a best fit to California earthquakes in the article referenced above, we get the following curve, with the green curve showing the entropic dispersion:

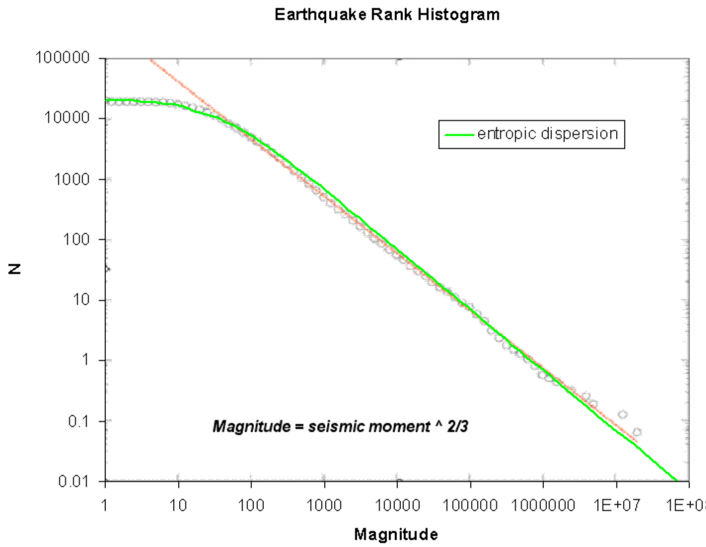


FIGURE 30-8. California earthquake magnitude rank histogram. Using the energy content instead of the seismic moment reveals that a simple dispersion fits the data.

This becomes another success in applying entropic dispersion to understanding disordered phenomena. Displacements of faults in the vertical direction contribute to a potential energy that eventually will release. All the stored potential energy gets released in proportion to the seismic moment. The magnitude measured follows a 2/3 power law since seismometers only measure deflections and not energy. The competing mechanisms of a slow growth in the strain with an entropic dispersion in growth rates, and an entropic (or narrower) distribution of points where the fault will give way.

The result leads to the inverse power-law beyond the knee and the arguably good fit to the data. So we have an example of a scaling law that arises from a non-critical phenomenon according to the conventional definition. It becomes more like the failure of a part that we described earlier.[3]

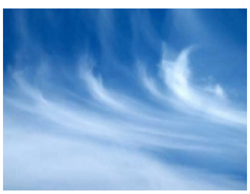
3. Research physicists have this impulsive behavior of having to discover a new revolutionary law instead of settling for the simple parsimonious explanation. One belief says that *self-organized critical phenomena* associated with something akin to a phase transition causes earthquakes. In my opinion, explaining things away as arising simply from elementary considerations of randomness and disorder within the Earth's heterogeneous crust and upper mantle won't win any Nobel prizes. The originator of self-organized criticality apparently had a streak of arrogance: *A sample of Prof. Bak's statements at conferences: After a young and hopeful researcher had presented his recent work, Prof. Bak stood up and almost screamed: "Perhaps I'm the only crazy person in here, but I understand zero - I mean ZERO - of what you said!". Another young scholar was met with the gratifying question: "Excuse me, but what is actually non-trivial about what you did?"* Was it possible that other physicists quaked in their boots at the prospect of ridicule for proposing the rather obvious? We must always look for the simple explanation first.

Laherrere and Sornette have also incorporated the parabolic fractal model based roughly on an inverse power law (which Laherrere also used to model reservoir size distributions) to observations such as the aforementioned distribution in earthquake magnitudes [Ref 51]. I believe the linear growth rate and dispersion links the phenomena.

Disordered Growth in Ice Crystals

Although on a totally different size scale (microns) and time scale (hours) than that of oil reservoir growth (million barrels and eons/ages), the essential behavioral notions behind the two cases of growth remain much the same.

I suggest that in the most disordered environments, the role of entropy overrides other factors enough so that some simple dispersion arguments can explain the size distribution completely.



Take as an example the formation of ice crystals in a cirrus cloud. Depending on the surrounding temperature, a crystal nucleates on some foreign particle and then starts growing. The atmospheric conditions have enough variety that the growth rate will disperse to the maximum entropy amount given a mean rate value. The end state for volumetric growth will also show the same amount of variation. Put these two factors together and we end up with the same size distribution as we have for oil reservoir sizes.

$$p(x) = \frac{S}{(S+x)^2} \quad \textbf{(EQ 30-10)}$$

where $\boldsymbol{x}$ is the size variate and $\boldsymbol{S}$ is the mean size. This becomes the entroplet form of a probability density function. Plotting this on a logarithmic size scale, the envelope looks the following. I roughly plotted how the dispersed velocity components play into the aggregation of the full profile (note that this serves as a mirror image of the other way of thinking about growth, that of varying end-states).

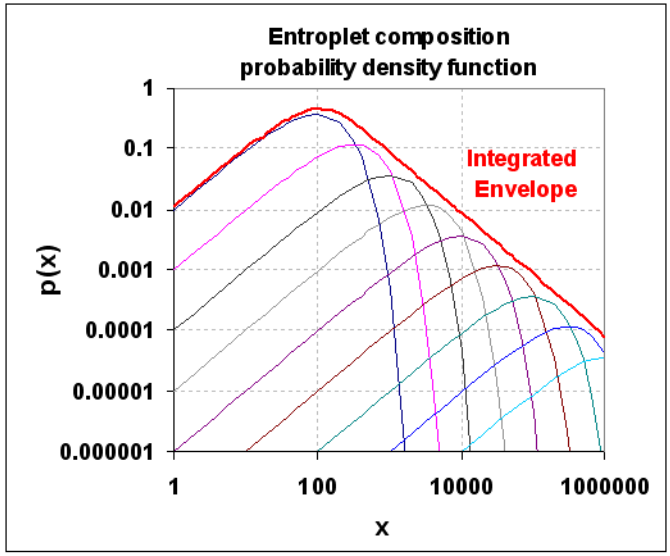


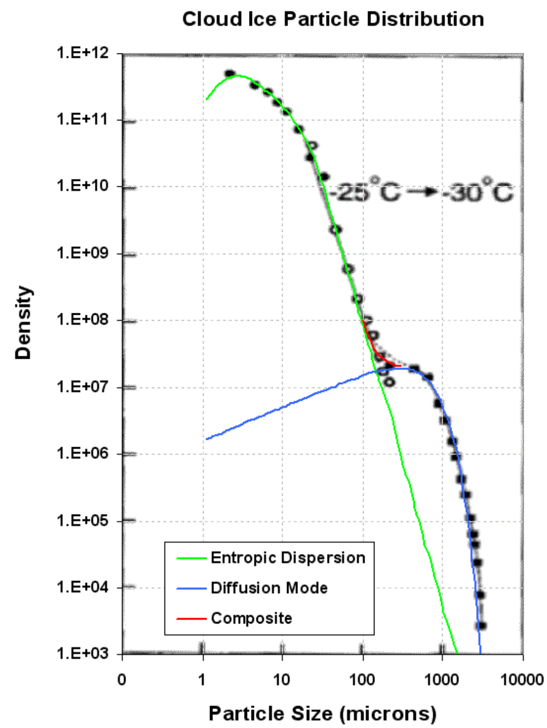
FIGURE 30-9. Superposition of exponential density profiles leads to a entropic dispersion envelope.

In some sense, the entroplet aggregates the non-dispersed rate functions which show individually much steeper exponential declines.

Graphically, these individual curves do not stand-out on their own since the entropic disorder smooths and disperses the curves efficiently. Keep this in mind for a moment.

The following particle size distribution (PSD) graph shows measurements taken from high altitude cloud experiments [Ref 275]. The size gets measured along a single length dimension and the density of the particles takes the place of a probability. I assume that they have plotted the density as a mass function along the *x*-axis. I convert this to a volumetric size growth problem, integrate the growth for a time duration corresponding to the cloud formation period, and plot the entropic dispersion model result below. The value for S is 2 and the integration scale is 16.

FIGURE 30-10.
From [Ref 275], "*Cloud ice particle number density n(D) vs. the long dimension of particles as observed at the temperature range of -25× C and -30× C [from Platt, 1997]. It shows a bimodal structure in the ice crystal distribution with the second peak at ~500 microns*"



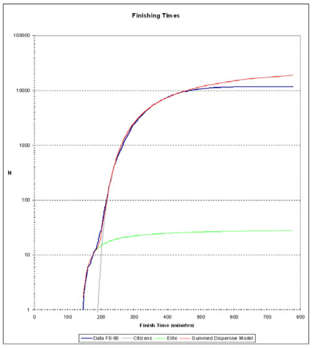
By smearing over start time we can essentially broadens the peak but retain the power law tail. Convert the linear growth to a time varying parameter (***x***=***kt***) and then treat the growth as an evolving pattern which starts from multiple points in time as the cloud develops.

$$P(x) = \int_T^{T+x/k} \frac{1}{1+\frac{S}{kt}} dt \qquad \textbf{(EQ 30-11)}$$

This is a cumulative so take the derivative with respect to x to get the PDF.

$$p(x) \approx \frac{1}{S+x} - \frac{1}{S+kT+x} \qquad \textbf{(EQ 30-12)}$$

As described in "Innovation and Evolution. How ideas spread" , this broadening also applies to species diversification and for oil field growth. The only complicating factor in this analysis is that oil reservoir is a volume, yet crystal sizes get reported as a length and we have to convert that to a volume. This means the derivative has to include a chain rule to convert the volume x to a length parameter L, $x \sim L^3$ generates $dx/dL \sim L^2$.



The data fits the entropic dispersion model nicely (green line), but notice at low density that an extra mode shows up as the blue line. This clearly has a sharp exponential drop so likely has a non-dispersive origin. In terms of the higher density entropic model, this stands out as an ordered nucleation regime in the midst of a sea of disordered ice crystal growth modes. Compare to the first figure again and one can see how a distinct peak could occur.

Why or how *physically* can this bump occur? One can understand this in the context of a completely unrelated analysis, that of marathon finishing times. In a race composed of ordinary citizens, mixed in with some elite runners (bribed by prize money), the elites will form a low density bump in a histogram of finishing times.

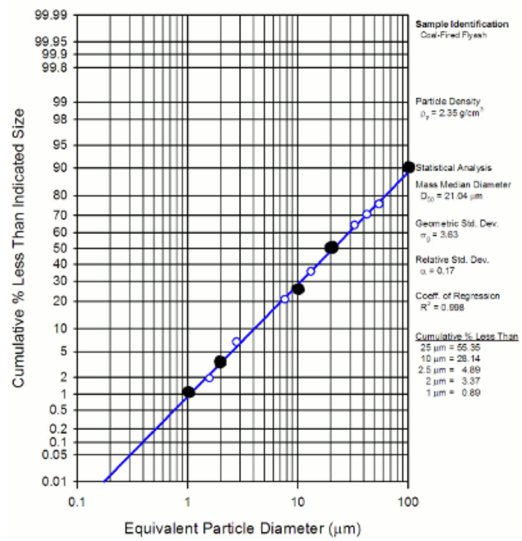
The incentives and training and good genetics of the elites separate them from the recreational athlete enough so that they generate a statistically measurable deviation from the trend. Anyone who follows sports understands how this can happen.

I assert that an ice crystal growth model could show this same behavior. Some unknown nucleation process has provided an optimal growth environment for these crystals to deviate from the entropic distribution. Hypothesizing, this could take the form of a catalyst or an accommodating growth substrate. With a power tail of -3/2 this might well have a strong diffusive growth component. However, the nuclei occur rarely enough so they do not drown out the much more common random or spontaneously occurring growth centers. It thus shows up as a clear non-dispersive growth mode in a sea of non-uniformity.

On a micro-level, we do have a population of reproducible structured shapes to bind against — as the airborne particulate world shows some uniformity in its density. I would venture that nothing like this would ever happen on the scale of oil reservoirs, black swans notwithstanding. On the scale of oil reservoirs, no two substrates will ever have a common origin, so that the entropic trends will dominate.

This original analysis may prove of some help to those looking at cloud-based climate change forcing functions, or particle size distributions of volcanic ash (see the figure below). It definitely opens up some possibilities to thinking in a different way. Researchers apply a log-normal fit to the data — yet one that uses an entropic dispersion formulation with the appropriate volume/diameter exponent often can work just as well. Below, I use a root 1/2 dispersive growth rate on volume which may indicate a diffusion-controlled rate.

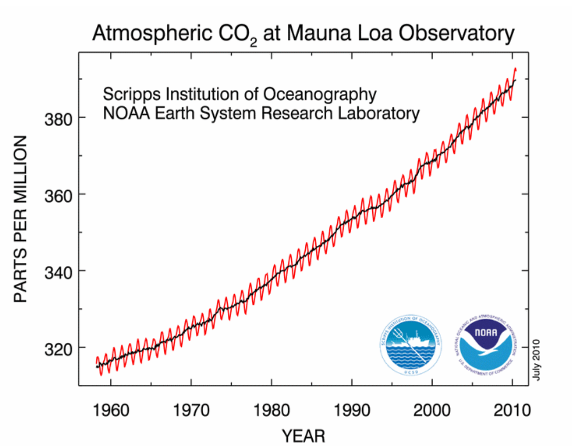
FIGURE 30-11. Wiki data for volcanic ash size distribution.



Fossil Fuel Emissions and CO_2 levels

In case you missed the salient premise behind climate change, increased levels of CO_2 form a greenhouse effect that can lead to global warming. I would rate the graph below of the concentration of atmospheric CO_2 measured at Mauna Loa as one of the most famous charts in the annals of science, rivalled only by its close kin, the "hockey stick" graph[4]:

FIGURE 30-12. The classic and frightening atmospheric CO2 build-up.



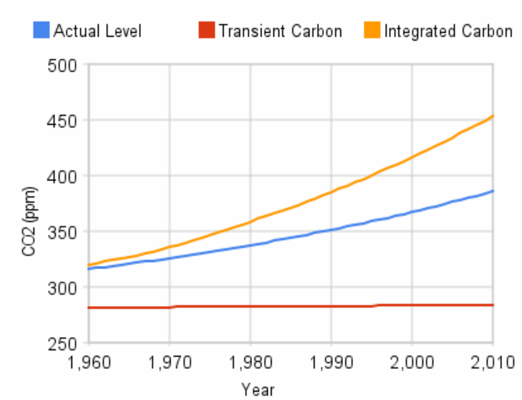
From just a technical perspective, it has an interesting composition — a committed research team that has collected data for some 50 years, measurements showing very little noise, the fascinating periodic cycle due to seasonal variations, and Al Gore to present it.

I don't think many people realize how easy one can derive this curve. You only need a historical record of fossil fuel usage, a few parameters and conversion factors, and the knowledge of how to do a convolution. Since the Oil Shock model uses convolutions heavily, doing this calculation fits in well with an oil production profile. And since oil production leads to CO_2 emissions, you can get the rest of the picture.

So the way I view it, the excess CO_2 production becomes just another stage in the set of shock model convolutions, which model how fossil fuel discoveries transition into reserves and then production as described in Volume 1. The culminating step in oil usage becomes a transfer function convolution from fuel consumption to a transient or persistent CO_2 (depending on what you want to look at). Add in the other hydrocarbon sources of coal and natural gas and you have a starting point for generating the Mauna Loa curve.

The Recipe. First of all, we can roughly anticipate what the actual CO_2 curve will look like, as it will lie somewhere between the two limits of immediate recapture of CO_2 (the fast transient regime hovering just above the baseline) and no recapture or sequestering (the persistent integrated regime which keeps accumulating). See the figure below.

FIGURE 30-13. The actual CO2 levels fall between the constraints of immediate uptake (red curve) and persistent inertness (orange curve). The latter results from an accumulation or integration of carbon emissions.



4. The sketch of Hubbert's Peak is unfortunately an also-ran in this contest

Although this transient can show very long persistence and a very fat tail, as I will eventually get to, we only need an average rate to generate the initial rise curve[5].

So the ingredients:

1. Conversion factor between tons of carbon generated and an equivalent parts-per-million volume of CO_2. This is generally accepted as 2.12 Gigatons carbon to 1 ppmv of CO_2. Or ~7.8 *Gt* CO_2 to 1 via purely molecular weight considerations.
2. A baseline estimate of the equilibrium CO_2, also known as the pre-industrial level. This ranges anywhere from 270 ppm to 300 ppm, with 280 ppm the most popular (although not necessarily definitive).
3. A source of historical fossil fuel usage. The further back this goes in time the better. I have three locations: one from the Wikipedia site on atmospheric CO_2, one from the Carbon Dioxide Information Analysis Center at Oak Ridge National Labs, and another from the NOAA site.[6]
4. A probability density function (PDF) for the CO_2 impulse response. If you don't have the actual PDF, use the first-order reaction rate exponential function, $\boldsymbol{R}(\boldsymbol{t})=\exp(-\boldsymbol{kt})$.
5. A convolution function, which you can do on a spreadsheet with the right macro.

The convolution of carbon production $\boldsymbol{P_c}(\boldsymbol{t})$ with the impulse response $\boldsymbol{R}(\boldsymbol{t})$ generates $\boldsymbol{C}(t)$:

$$C(t) = k \cdot \int_0^t P_c(t-x) \cdot R(x)dx + L \qquad \text{(EQ 30-13)}$$

Multiplying the result by a conversion factor $\boldsymbol{k}$; then adding this to the baseline $\boldsymbol{L}$ generates the filtered Mauna Loa curve as a concentration in CO_2 parts per million.

The sticky part. If you follow climate science research, you may often read about different estimates for the "CO_2 Half-Life" of the atmosphere. This becomes the impulse response function, $\boldsymbol{R}(\boldsymbol{t})$, we just described. Unfortunately, the value for this quantity has elicited much debate. I have heard numbers as short as 6 years and others as long as 100 years or more.

5. The oscillating part decomposes trivially as a seasonal response, and we can safely add that in later.
6. http://en.wikipedia.org/wiki/Carbon_dioxide_in_Earth%27s_atmosphere and http://www.noaanews.noaa.gov/stories2009/20090421_carbon.html

> *ClimateProgress.org -- Strictly speaking, excess atmospheric CO2 does not have a half-life. The distribution has a very long tail, much longer than a decaying exponential.*
> *As an approximation, use 300-400 years with about 25% 'forever'.*
> *....*
> *ClimateProgress.org -- David is correct. Half-life is an inappropriate way to measure CO2 in the atmosphere. The IPCC uses the Bern Carbon Cycle Model. See Chapter 10 of the WGI report (Physical Basis) or http://www.climate.unibe.ch/~joos/OUTGOING/publications/hooss01cd.pdf*

This issue has importance because CO_2 latency and the possible slow sequestering time has grave implications for rebounding from a growing man-made contribution of CO_2 to the atmosphere. A typical climate sceptic response will make the claim for a short CO_2 lifetime:

> *Endangerment Finding Proposal*
>
> *Lastly; numerous measurements of atmospheric CO2 resident lifetime, using many different methods, show that the atmospheric CO2 lifetime is near 5-6 years, not 100 year life as stated by Administrator (FN 18, P 18895), which would be required for anthropogenic CO2 to be accumulated in the earth's atmosphere under the IPCC and CCSP models. Hence, the Administrator is scientifically incorrect replying upon IPCC and CCSP -- the measured lifetimes of atmospheric CO2 prove that the rise in atmospheric CO2 cannot be the unambiguous result of human emissions.*

The model of the carbon cycle describes the detailed mass balance of carbon between the atmosphere and the earth's surface. The surface can be either land or water, it doesn't matter for argument's sake. We know that the carbon-cycle between the atmosphere and the biota is relatively fast and the majority of the exchange has a turnover of just a few years. Yet, what should really interest us is the deep exchange of the carbon with slow-releasing stores. This process is described by diffusion and that is where we can use a master diffusion equation to represent the flow of CO_2.

We can't debate this as it describes the flow of particles; they call it the master equation because it invokes the laws of probability and in particular the basic random walk that just about every physical phenomenon displays.

The origin of the master equation is best described by considering a flow graph and drawing edges between compartments of the system. This is often referred to as a compartment or box model. The flows go both ways and are random, and thus model the random walk between compartments.

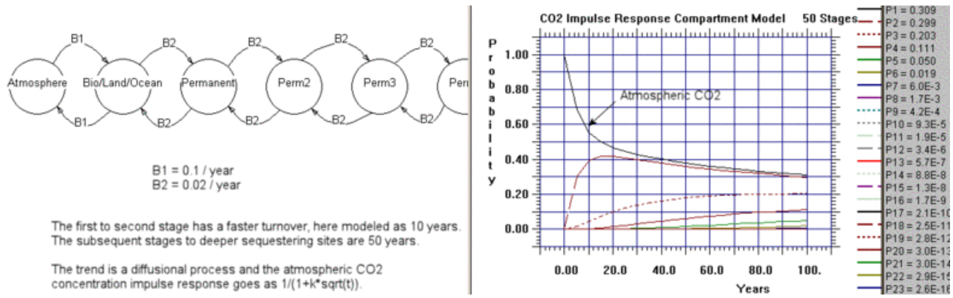


FIGURE 30-14.

The above chart is a Markov model consisting of 50 stages of deeper sequestering with each slab having a constant but small hop rate. The single interface between the atmosphere and the earth has a faster hopping rate corresponding to faster carbon cycling.

First thing one notices about the graph is that after a rapid initial fall-off it only slowly decreases over time. The physical meaning is that, due to diffusion, the concentration randomly walks between the interface and deeper locations in the earth. The fat-tail time dependence is a classic trait of all random walks and you can't escape seeing this if you have ever watched nature in action. That is just the way particles move around.

Not knowing a lot about the specific chemistry involved but understanding that CO_2 reaction kinetics has much to do with the availability of reactants, I can imagine the profile may vary, particular as a function of location and altitude. CO_2 at higher altitudes would have fewer reactants to interact with while reactivity with the land and sea to deep sequestering can also vary.

Deriving the Fat-Tail in CO_2 Persistence

So what happens if we have a dispersed rate for the CO_2 reaction?

Say the CO_2 mean reaction rate is $\boldsymbol{R}$=0.1/year (or a 10 year half-life). Since we only know this as a mean, the standard deviation is also 0.1. Placing this in practical mathematical terms, and according to the Maximum Entropy Principle, the probability density function for a dispersed rate $\boldsymbol{r}$ is:

$$p(r) = \frac{1}{R} \cdot e^{-r/R} \qquad \text{(EQ 30-14)}$$

One can’t really argue about this assumption, as it works as a totally unbiased estimator, given that we only know the global mean reaction rate.

So what does the tail of reaction kinetics look like for this dispersed range of half-lifes? Assuming the individual half-life kinetics act as exponential declines then the dispersed calculation derives as follows

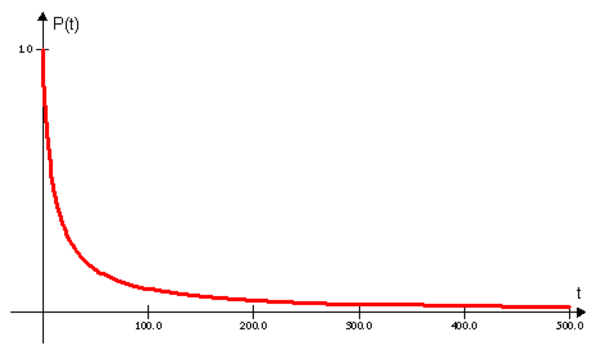
$$P(t) = \int_0^{\infty} p(r) \cdot e^{-rt} dr \qquad \text{(EQ 30-15)}$$

This expression when integrated gives the following simple expression:

$$P(t) = \frac{1}{1 + Rt} \qquad \text{(EQ 30-16)}$$

which definitely gives a fat-tail as the following figure shows (note the scale in 100's of years). I can also invoke a more general argument in terms of a mass-action law and drift of materials; this worked well for oil reservoir sizing. Either way, we get the same characteristic entroplet shape.

FIGURE 30-15. Drift (constant rate) Entropic Dispersion



For the plot above, at 500 years, for ***R***=0.1, about 2% of the original CO_2 remains. In comparison for a non-dispersed rate, the amount remaining would drop to e^{-50} or ~**2 10^{-20}%** !

Now say that ***R*** holds at closer to a dispersed mean of 0.01, or a nominal 100 year half-life. Then, the amount left at 500 years sits at 1/(1+0.01*500) = 1/6 ~ 17%.

In comparison, the exponential would drop to exp(-500/100) = 0.0067 ~ 0.7% Also, 0.7% of the rates will generate a half-life of 20 years or shorter. These particular rates quoted could conceivably result from those volumes of the atmosphere close to the prime sequestering sites.

Now it gets interesting ... Climatologists refer to the impulse response of the atmosphere to a sudden injection of carbon as a key indicator of climate stability. Having this kind of response data allows one to infer the steady state distribution. The IPCC used this information in their 2007 report.

> *Current Greenhouse Gas Concentrations*
>
> *The atmospheric lifetime is used to characterize the decay of an instanenous pulse input to the atmosphere, and can be likened to the time it takes that pulse input to decay to 0.368 (l/e) of its original value. The analogy would be strictly correct if every gas decayed according to a simple expotential curve, which is seldom the case.*
>
> ...
>
> *For CO2 the specification of an atmospheric lifetime is complicated by the numerous removal processes involved, which necessitate complex modeling of the decay curve. Because the decay curve depends on the model used and the assumptions incorporated therein, it is difficult to specify an exact atmospheric lifetime for*
>
> *CO2. Accepted values range around 100 years. Amounts of an instantaneous injection of CO2 remaining after 20, 100, and 500 years, used in the calculation of the GWPs in IPCC (2007), may be calculated from the formula given in footnote a on page 213 of*
>
> *that document. The above-described processes are all accounted for in the derivation of the atmospheric lifetimes in the above table, taken from IPCC (2007).*

Notes:

[a] The CO_2 response function used in this report is based on the revised version of the Bern Carbon cycle model used in Chapter 10 of this report (al. 2001) using a background CO_2 concentration value of 378 ppm. The decay of a pulse of CO_2 with time t is given by

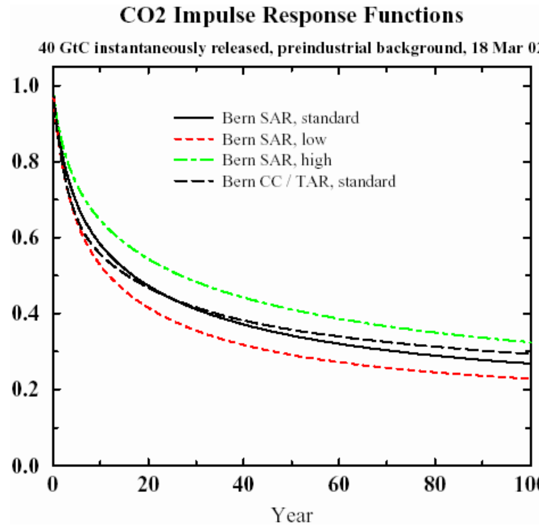
$$a_0 + \sum_{i=1}^{3} a_i \cdot e^{-t/\tau_i}$$

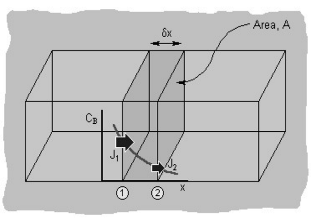
Where $a_0 = 0.217$, $a_1 = 0.259$, $a_2 = 0.338$, $a_3 = 0.186$, $\tau_1 = 172.9$ years, $\tau_2 = 18.51$ years, and $\tau_3 = 1.186$ years.

The following graph shows impulse responses from several sets of parameters using the referenced Bern IPCC model[7]. This result shows an unusual asymptotic trend to a constant baseline, and the model parameters reflect this. For a system at equilibrium, the impulse response decay should go to zero. I believe that it physically does, but that this model completely misses the fact that it eventually should decay completely. In any case, the tail shows a huge amount of "fatness", easily stretching beyond 100 years, and something else must explain this fact.

7. found in "Parameters for tuning a simple carbon cycle model" http://unfccc.int/resource/brazil/carbon.html

FIGURE 30-16.
IPCC Model for Impulse Response





If you think of what happens within the earth's crust, the migration of CO_2 from active carbon-cycle regions to deeper sequestering regions can only occur via the process of diffusion. We can write a simple relationship for Fick's Law diffusion as follows:

$$\frac{d}{dt}G(t) = D \cdot \frac{C(0) - C(x)}{G(t)} \qquad \textbf{(EQ 30-17)}$$

This states that the growth rate d$\boldsymbol{G}(t)$/dt remains proportional to the gradient in concentration it faces. As a volume gets swept clean of reactants, $\boldsymbol{G}(t)$ gets larger and it takes progressively longer for the material to "diffuse" to the side where it can react. This basically describes oxide growth as well which we covered in Volume 1.

The outcome of Fick's Law generates a growth law that goes as the square root of time, $\sqrt{t}$. According to the dispersion formulation for cumulative growth, we simply have to replace the previous linear drift growth rate with the diffusion-limited growth rate.

$$P(t) = \frac{1}{1 + R\sqrt{t}} \qquad \textbf{(EQ 30-18)}$$

or in an alternate form where we replace the probability $\boldsymbol{P(t)}$ with a normalized response function $\boldsymbol{R(t)}$:

$$R(t) = \frac{a}{a+\sqrt{t}} \qquad \text{(EQ 30-19)}$$

At small time scales, diffusion can show an infinite growth slope, so using a finite width unit pulse instead of a delta impulse will create a reasonable picture of the dispersion/diffusion dynamics.

Diffusion Solution. We can easily derive the solution of the response function if we think of the diffusion from a planar source outward. The diffusion kernel resulting from solving the Fokker-Planck equation (sans drift term) is:

$$C(t, x|D) = \frac{1}{\sqrt{4\pi D t}} \cdot e^{-x^2/4Dt} \qquad \text{(EQ 30-20)}$$

We place an impulse of concentrate at x=0 and want to watch the evolution of the concentration with time. We have an idea of a mean value for the diffusion coefficient, D, but don't know how much it varies. The remedy for that is to apply a maximum entropy estimate for the variance assuming a mean value D_0.

$$p_d(D) = \frac{1}{D_0} \cdot e^{-D/D_0} \qquad \text{(EQ 30-21)}$$

So then we can apply this to the kernel function:

$$C(t, x) = \int_0^\infty C(t, x|D)p_D(D)dD \qquad \text{(EQ 30-22)}$$

This actually comes out quite clean

$$C(t, x) = \frac{1}{2\sqrt{D_0 t}} \cdot e^{-x/\sqrt{D_0 t}} \qquad \text{(EQ 30-23)}$$

This gives us a result that shows a singularity at t=0 for x=0. In practice, the value of x is not precise, so that we can also place an uncertainty around the value of x.

$$p_x(x) = \frac{1}{x_0} \cdot e^{-x/x_0} \qquad \text{(EQ 30-24)}$$

Once again we can apply this to the concentration, marginalizing x out of the picture:

$$C(t) = \int_0^\infty C(t|x)p_x(x)dx \quad \text{(EQ 30-25)}$$

This integral is very straightforward

$$C(t) = \frac{1}{2} \cdot \frac{1}{x_0 + \sqrt{D_0 t}} \quad \text{(EQ 30-26)}$$

which is precisely the equation, apart from normalization, estimated from the dispersional formulation applied to Fick's law described earlier

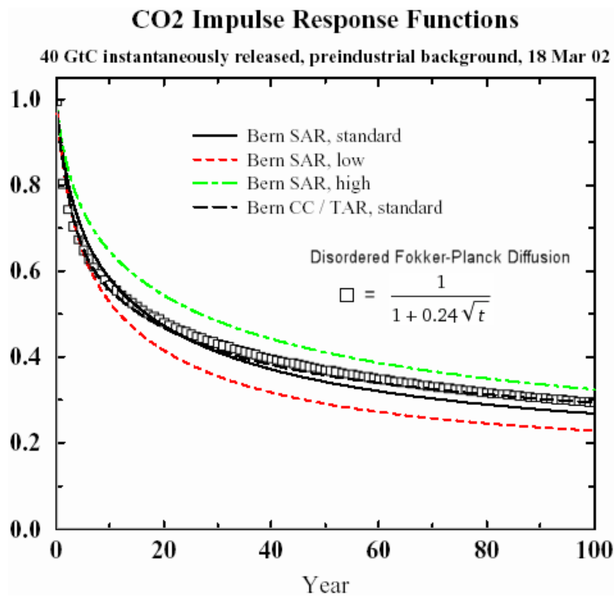


FIGURE 30-17. Entropic Dispersion with diffusional growth kinetics describes the CO2 impulse response function with a *single* parameter *a*. The square of this number describes a characteristic time for the CO2 concentration lifetime.

Remarkably, this simple model reproduces the IPCC model accurately, with the appropriate choice of the disordered diffusion coefficient. The analytically calculated points lie right on top of the lines in the figure above, which actually makes it hard to see the excellent agreement. I find it very useful that the model essentially boils down to a single parameter of entropic rate origin (while both diffusion and dispersion generates the shape).

You don't see it on this scale, but the tail will eventually reach zero, but at a rate asymptotically proportional to the square root of time. In 10,000 years, it will reach approximately the 2% level (i.e. $2/\sqrt{10000}$. This defines the concept of a lengthy CO_2 *adjustment time*.

Two other interesting observations grow out of this most parsimonious agreement.

First of all, why did the original IPCC modelers from Bern not use an expression as simple as the entropic dispersion formulation? Instead of using a three-line derivation with a resultant single parameter to model with, they chose an empirical set of 5 exponential functions with a total of 10 parameters and then a baseline offset. That makes no sense unless their model essentially grows out of some heuristic fit to measurements from a real-life carbon impulse[8]. I can only infer that they never made the connection to the real statistical physics.

Secondly, the simple model really helps explain the huge discrepancy between the quoted short lifetimes by climate sceptics and the long lifetimes stated by the climate scientists. These differ by more than a magnitude. Yet, just by looking at the impulse response in the preceding figure, you can see the fast decline that takes place in less than a decade and distinguish this from the longer decline that occurs over the course of a century. This results as a consequence of the mechanism of random walk diffusion and that of entropy within the carbon-cycle, leading to a large dispersion in reaction rates, and the rates limited by diffusion kinetics as the CO_2 migrates to sequestering sites.

The fast slope evolving gradually into a slow slope has all the characteristics of the "law of diminishing returns" characteristic of diffusion, with the accurate fit occurring because I included dispersion correctly and according to maximum entropy principles. Note that the previous section on cloud ice crystal formation kinetics shows this same parsimonious agreement to the data.

Think of it this way: if this simple model didn't work to describe the CO_2 adjustment time kinetics, one would have to explain why it failed or didn't apply.

Production Emissions Stimulating CO_2 levels

So we see how a huge fat tail can occur in the CO_2 impulse response. What kind of implication does this have for the long term?

8. perhaps data from paleoclimatology investigation of an ancient volcanic eruption

A disconcerting trend for one, and that brings us to the point that the point that climate scientists have made all along. With a fat-tail, one can demonstrate that a CO_2 latency fat-tail will cause the responses to forcing functions to continue to get worse over time.

As this paper notes [Ref 303] and we will derive, applying a stimulus to a fat-tail generates a non-linear impulse response which will look close to the Mauna Loa curve shape. Not surprisingly but still quite disturbing, applying multiple forcing functions as a function of time will not allow the tails to damp out quickly enough, and the tails will gradually accumulate to a larger and larger fraction of the total. Mathematically you can work this out as a convolution and use some neat techniques in terms of Laplace or Fourier transforms to prove this analytically or numerically.

As a first sanity check let's assume that the incoming stream of new CO_2 from fossil fuel emissions is called ***P(t)***. This becomes the forcing function. Then we can describe the system evolution by the equation

$$c(t) = P(t) \otimes r(t) \qquad \textbf{(EQ 30-27)}$$

where the operator $\otimes$ is not a multiplication but signifies convolution. For the forcing function ***P(t)*** we use a growing power law.

$$P(t) = kt^N \qquad \textbf{(EQ 30-28)}$$

where ***N*** is the power and ***k*** is a scaling constant.This roughly represents the atmospheric emissions through the industrial era if we use a power law of ***N***=4.

So all we really want to solve is the convolution of ***P(t)*** with the fat-tail ***r(t)***. By using Laplace transforms on the convolution expression, the answer comes out surprisingly clean and concise. Ignoring the scaling factor :

$$c(t) \sim t^{N+\frac{1}{2}} \qquad \textbf{(EQ 30-29)}$$

With that solved, we can now answer the issue of where any "missing CO2" resides. This is an elementary problem of integrating the forcing function, ***F(t)***, over time and then comparing the concentration, ***c(t)***, to this value. Then this ratio of ***c(t)*** to the integral of ***F(t)*** is the amount of CO_2 that remains in the atmosphere.

Working out the details, this ratio is:

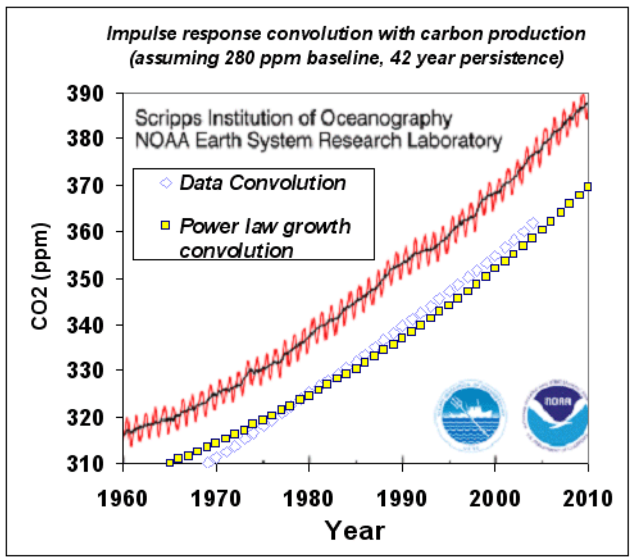
$$q \cdot \frac{\sqrt{\pi}}{t} \cdot \frac{(N+1)!}{(N+1/2)!} \qquad \textbf{(EQ 30-30)}$$

Plugging in numbers for this expression, q=1, and N=4, then the ratio is about 0.28 after 200 years of growth. This means that 0.72 of the CO_2 is going back into the deep-stores of the carbon cycle, and 0.28 is remaining in the atmosphere. If we choose a value of q=2, then 0.56 remains in the atmosphere and 0.44 goes into the deep store. This ratio is essentially related to the effective diffusion coefficient of the carbon going into the deep store.

This essentially explains the "25% forever" in the ClimateProgress comment. Diffusion and dispersion of rates essentially prohibit the concentrations to reach a comfortable equilibrium. The man-made forcing functions keep coming and we have no outlet to let it dissipate quickly enough.[9]

We can now try to apply these numbers to the full response recipe. I used $\boldsymbol{R}(\boldsymbol{t})=\exp(-\boldsymbol{t}/\boldsymbol{T})$, where $\boldsymbol{T}$=42 years and $\boldsymbol{L}$=1280 ppm baseline and data from Figure 30-19 on page 603 for $\boldsymbol{P_c}(\boldsymbol{t})$, giving the following fit.

FIGURE 30-18. Convolution ala the Shock Model of the yearly carbon emission with an impulse response function. An analytical result from a power-law (N=4) carbon emission model is shown as a comparison.

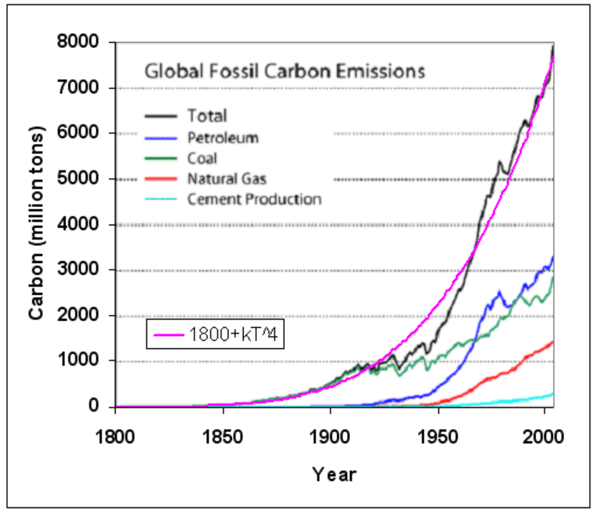


I also applied a curve fit model of the carbon generated, which followed a *Time*^{4} acceleration, and which had the same cumulative as of the year 2004. You can see

9. We also need to consider the CO2 saturation level in the atmosphere. The CO2 concentration may asymptotically reach this level and therefore stifle the forcing function build-up, but I imagine that no one really knows how this could play out.

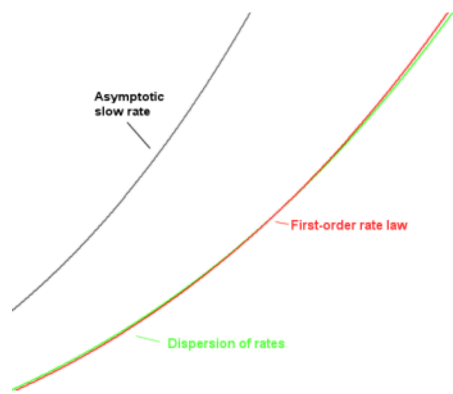
subtle differences between the two which indicates that the rate function does not completely smooth out all the yearly variations in carbon emission. So the two convolution approaches show some consistency with each other, but the fit to the Mauna Loa data appears to have a significant level shift. I will address this in a moment.

FIGURE 30-19. Carbon emission data used for model. A power-law starting in the year 1800 generates a smoothed idealized version of the curve useful for generating a closed-form expression.



The precise form of the impulse response function, other than the average rate selected, does not change the result too much. I can make sense out of this since the strongly increasing carbon production wipes out the fat-tails of slower order reaction kinetics (see the figure below). In terms of the math, a $\boldsymbol{Time}^4$ power effectively overshadows a weak $1/\sqrt{Time}$ or $1/\boldsymbol{Time}$ response function. However, you will see start to see this tail *if and when* we start slowing down the carbon production. This will give a persistence in CO_2 above the baseline for *centuries*.

FIGURE 30-20. Widening the impulse response function by dispersing the rates to the maximum entropy amount, does not significantly change the curvature of the CO2 concentration. Dispersion will cause the curve to eventually diverge and more closely follow the integrated carbon curve but we do not see this yet on our time scale.



Once we feel comfortable doing the convolution, we can add in a piecewise extrapolated production curve and we can anticipate future CO_2 levels. We need a fat-tail impulse response function to see the long CO_2 persistence in this case (unless 42 years is long enough for your tastes).

The Loose End. If you look at the fit in Figure 30-18 on page 602, you can obviously see an offset of the convolution result from the actual data. This may seem a little puzzling until you realize that the background (preindustrial) level of CO_2 can shift the entire curve up or down. I used the background level of 280 ppm purely out of popularity reasons. More people quote this number than any other number. However, we can always evaluate the possibility that a higher baseline value would fit the convolution model more closely. Let's give that a try.

The following figure shows a different CO_2 data set which includes the Mauna Loa data as well as earlier proxy ice core data. Based on the levels of CO_2, I surmised that the NOAA scientist that generated this graph subtracted out the 280ppm value and plotted the resultant offset. I replotted the data convolution as the dotted gray line.

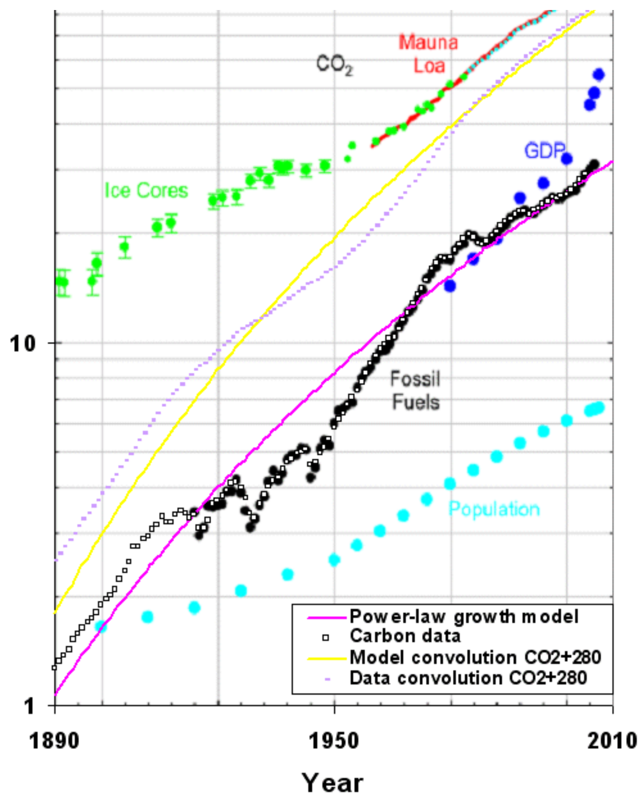


FIGURE 30-21. The CO2 data replotted with extra proxy ice core data, assuming a 280ppm baseline (pre-industrial) level. The carbon production curve is also plotted. You can clearly see that the convolution of the impulse response results in a curve that has a consistent shift of between 10 and 20 ppm below the actual data.(from [Ref 327])

Note that the curve consistently shows a shift 14ppm below the actual data (note the logscale). This indicates to me that the actual background CO_2 level sits 14ppm above 280ppm or at approximately 294ppm. When I add this 14ppm to the curve and replot, it looks like:

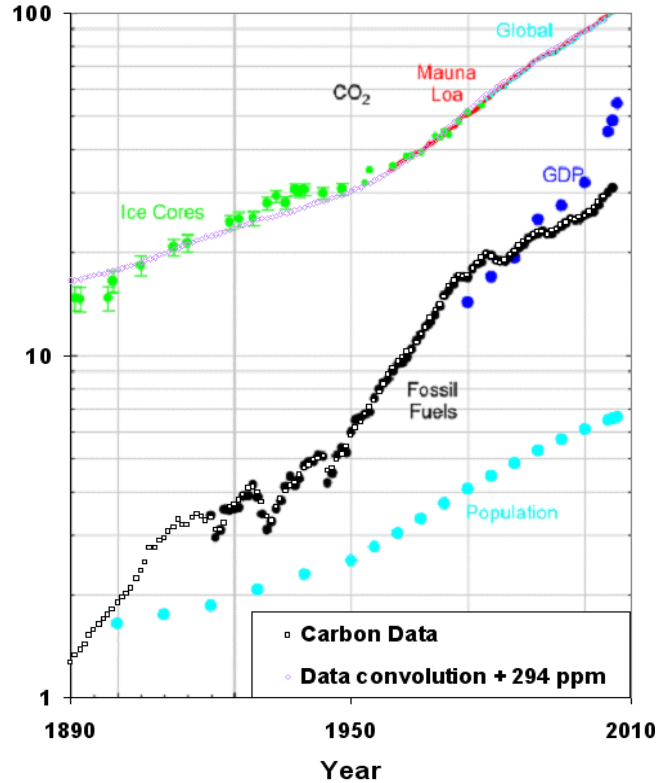


FIGURE 30-22. The convolution model replotted from Figure 30-21 on page 604 with a baseline of 294ppm CO2 instead of 280. Note the generally better agreement to the subtle changes in slope.

Although the data does not go through a wide dynamic range, I see a rather parsimonious agreement with the two parameter convolution fit.

Just like in the oil shock model, the convolution of the stimulus with an impulse response function will tend to dampen and shift the input perturbations. If you look closely at the figure above, you can see faint reproductions of the varying impulse, only shifted by about 25 years. I contend that this “delayed ghosting” comes about directly as a result of the 42-year time constant I selected for the reaction kinetics rate. This same effect occurs with the well known shift between the discovery peak and production peak in peak oil modeling. Even though King Hubbert himself pointed out this effect years ago, no one else has explained the fundamental basis behind this effect, other than through the application of the shock model. That climate scientists most assuredly use a convolution approach as well points out a potential unification between climate science and peak oil theory. Rutledge of CalTech has looked at this connection closely, particularly in relation to future coal usage.

Bottom Line. To believe this model, you have to convince yourself that 294 ppm marks the real background pre-industrial level (not 280), and that 40 years works as a pretty good time constant for CO_2 decomposition kinetics. Everything else follows from first-order rate laws and the estimated carbon emission data.

Some other forcing inputs could easily make up the 14 ppm offset from the generally accepted 280 ppm. For example, changing agriculture and forestry patterns, and other human modifications of the biota could alter the forcing function during the 200+ year time-span since the start of the industrial revolution. Although recyclable plant life should eventually become carbon neutral, the fat-tail of the CO_2 impulse response function means that sudden changes will persist for long periods of time. A slight rise from time periods from before the 1800's coupled with an extra stimulus on the order of 500 million tons of carbon per year (think large-scale clearcutting and tilling from before and after this period) would easily close the 14 ppm CO_2 gap and maintain the overall fit of the curve.

However, we would need to apply the fat-tail response function, $g/(g+\sqrt{t})$, to maintain the offset for the entire period as the clear-cutting has subsided. So what actually happens if we use the fat-tail response for the entire convolution? A commenter pointed this useful insight:

> Comment by *EoS*:
>
> *I don't think it is useful to think of an average CO2 lifetime. That implies a lumped linear model with only a single reservoir, hence an exponential decay towards equilibrium. In reality there are lots of different CO2 reservoirs with different capacities and time constants. So any lumped model had better use several reservoirs with widely varying time constants at a minimum, or else it will get the time behavoir seriously wrong.*

It turns out that the variation or dispersion in reaction rates *makes very little difference in the slope on the climb up*. That is fundamental and I addressed that in Figure 30-20 on page 603. The reason for this is very simple mathematics — the climb up in CO_2 arises by power laws on the order of N>3 or by exponential increases. That comes directly from accelerating fossil fuel usage. In contrast the reaction rates of CO_2 have exponents that are negative or have inverse power laws of very low order, the so-called fat-tail distributions. When you put these together, the power law increase essentially crushes the long-tails and all you see are the average value of the faster kinetics. I put in the analytical solution so you can see this directly in the convolution results.

Alternately, apply a simple convolution of accelerating growth [exp(***at***)] with a first-order reaction decline [exp(-***kt***)] and you will see what I mean. You get this:

$$C(t) = \frac{e^{at} - e^{-kt}}{a+k} \qquad \textbf{(EQ 30-31)}$$

The accelerating rate ***a*** will quickly overtake the decline term ***k***. If you put in a spread in ***k*** values as a distributed model, the same result will occur, substantiating the inexorable incline. Climate scientists should realize this as well since they have known about the uses of convolution in the carbon cycle for years [Ref 276].

The following figure uses fossil fuel data from the ORNL Carbon Dioxide Information Analysis Center over the last 260 years, and we apply the diffusion fat-tail response function shown in the inset.

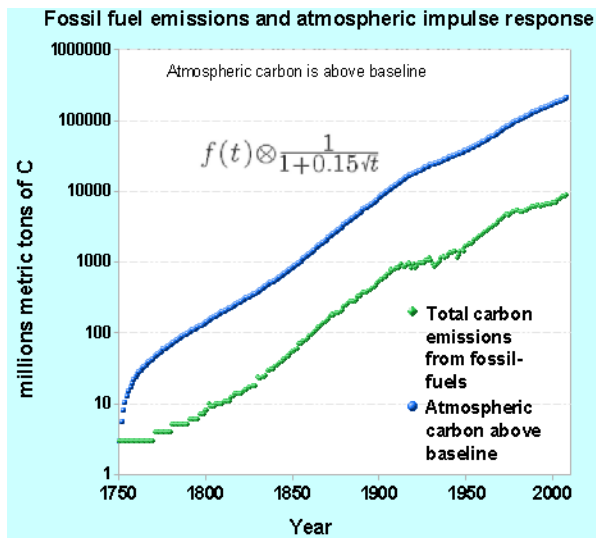


FIGURE 30-23. The result of fossil fuel convolution assuming the diffusional impulse response function.

Scaled and overlaid on the estimated atmospheric CO_2 levels, one can again see the generally good agreement in the figure below:

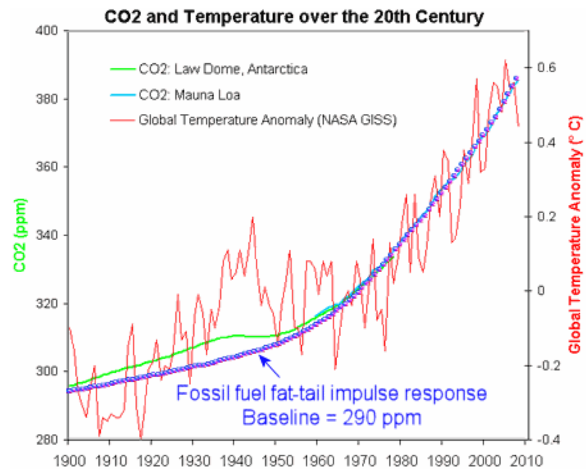


FIGURE 30-24. Fossil fuel response convolution over the last century

Yet, if we were to stop burning hydrocarbons today, then we would see the results of the fat-tail decline. Again, I think the climate scientists understand this fact as well but that idea gets obscured by layers of computer simulations and the salient point or insight doesn't get through to the layman. This is understandable because these are not necessarily intuitive concepts.

This following figure models CO_2 uptake if we abruptly stop growing fossil fuel use after the year 2007. We don't simple stop using oil and coal, we simply keep our usage constant.

Up to that point in time a dispersive (i.e. variable) set of rate kinetics will be virtually indistinguishable from a single rate. And you can see that behavior as the curves match for the same *average* rate. But once the growth increase is cut off, the dispersive/diffusive kinetics takes over and the rise continues. With the first-order kinetics the growth continues but it becomes self-limiting as it reaches an equilibrium. This works as a plain vanilla rate theory with nothing by the way of feedbacks in the loop. When we include a real positive feedback, that curve can even increase more rapidly.

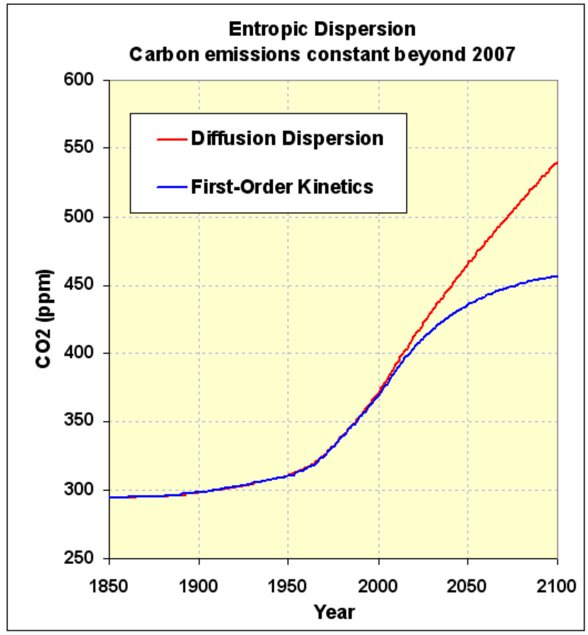


FIGURE 30-25. Extrapolation of slow kinetics vs. fat-tail kinetics

Recall that this analysis carries over from studying dispersion in oil discovery and depletion. The rates in oil depletion disperse all over the map, yet the strong push of technology acceleration essentially narrows the dispersed elements so that we can get a strong oil production peak or a plateau with a strong decline. In other words, if we did not have the accelerating components, we would have had a long drawn out usage of oil that would reflect the dispersion. That explains why I dislike the classical derivation of the Hubbert Logistics curve, as it reinforces the opinion of peak oil as some "single-rate" model.

In fact just like climate science, everything gets dispersed and follows multiple pathways, and we need to use the appropriate math to analyze that kind of situation.

Climate scientists understand convolution, but peak oil analysts don't, except in the context of the shock model. That basically outlines why I want to share these ideas with climate scientists and unify the concepts. It will help both camps, simply by dissemination of fresh ideas and unification of the strong ones. Doing this exercise has turned into an eye-opener for me, as it didn't really occur to me how straightforward one can derive the CO_2 results. Of course, this simple model does not take into possible positive feedback effects such as CO_2 outgassing from increasing temperature [Ref 304] and see figure below, yet it does give one a nice intuitive framework to think about how hydrocarbon production and combustion leads directly to atmospheric CO_2 concentration changes and ultimately climate change.

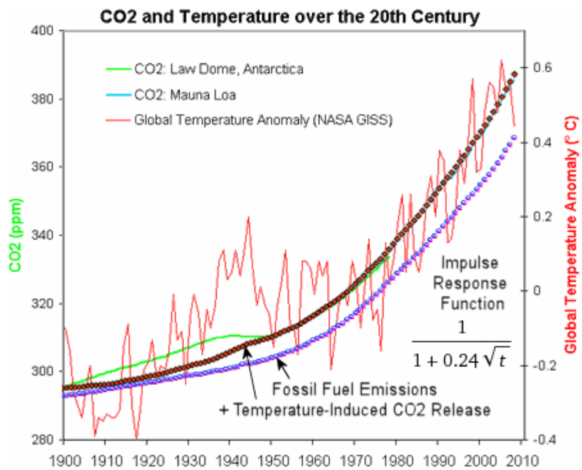


FIGURE 30-26. Alternative response incorporating possible positive feedback from temperature increase (see [Ref 328]). Extra CO_2 comes from outgassing in the ocean which can multiply the effect of fossil fuel emission.

Authors [publication year]	Residence time
Based on natural carbon-14	(years)
Craig [1957]	7 +/- 3
Revelle & Suess [1957]	7
Arnold & Anderson [1957] inc. living and dead biosphere	10
(Siegenthaler,1989)	4-9
Craig [1958]	7 +/- 5
Bolin & Eriksson [1959]	5
Broecker [1963] recalc. by Broecker & Peng [1974]	8
Craig [1963]	5-15
Keeling [1973b]	7
Broecker [1974]	9.2
Oeschger et al. [1975]	6-9
Keeling [1979]	7.53
Peng et al. [1979]	7.6 (5.5-9.4)
Siegenthaler et al. [1980]	7.5
Lal & Suess [1983]	3-25
Siegenthaler [1983]	7.9-10.6
Kratz et al. [1983]	6.7
Based on Suess Effect	
Ferguson [1958]	2 (1-8)
Bacastow & Keeling [1973]	6.3-7.0
Based on bomb carbon-14	
Bien & Suess [1967]	>10
Münnich & Roether [1967]	5.4
Nydal [1968]	5-10
Young & Fairhall [1968]	4-6
Rafter & O'Brian [1970]	12
Machta (1972)	2
Broecker et al. [1980a]	6.2-8.8
Stuiver [1980]	6.8
Quay & Stuiver [1980]	7.5
Delibrias [1980]	6
Druffel & Suess [1983]	12.5
Siegenthaler [1983]	6.99-7.54
Based on radon-222	
Broecker & Peng [1974]	8
Peng et al. [1979]	7.8-13.2
Peng et al. [1983]	8.4
Based on solubility data	
Murray< (1992)	5.4
Based on carbon-13/ carbon-12 mass balance	
Segalstad (1992)	5.4

FIGURE 30-27. Segalstad pulled together all the experimentally estimated *residence times* for CO2. http://www.co2web.info

Do we believe that the dispersion of CO_2 lifetimes actually exists? Applying Bayesian reasoning to the uncertainty in the numbers that people have given, I would think it likely. Uncertainty in people's opinions usually results in uncertainty (i.e. dispersion) in reality (see the sidebar). By collecting the statistics for the equivalent rates, it turns out that the standard deviation approximately equals the mean (0.17/year) — this supports the idea that the uncertainty in rates found by measurement matches the uncertainty found in nature, thus giving the entropic fat tail.

The following paper addresses many of the uncertainties underlying climate change: "*The shape of things to come: why is climate change so predictable?*"

> *The framework of feedback analysis is used to explore the controls on the shape of the probability distribution of global mean surface temperature response to climate forcing. It is shown that ocean heat uptake, which delays and damps the temperature rise, can be represented as a transient negative feedback. This transient negative feedback causes the transient climate change to have a narrower probability distribution than that of the equilibrium climate response (the climate sensitivity). In this sense, climate change is much more predictable than climate sensitivity. The width of the distribution grows gradually over time, a consequence of which is that the larger the climate change being contemplated, the greater the uncertainty is about when that change will be realized. Another consequence of*

> *this slow growth is that further efforts to constrain climate sensitivity will be of very limited value for climate projections on societally-relevant time scales. Finally, it is demonstrated that the effect on climate predictability of reducing uncertainty in the atmospheric feedbacks is greater than the effect of reducing uncertainty in ocean feedbacks by the same proportion. However, at least at the global scale, the total impact of uncertainty in climate feedbacks is dwarfed by the impact of uncertainty in climate forcing, which in turn is contingent on choices made about future anthropogenic emissions.[Ref 302]*

In some sense, the fat-tails may work to increase our certainty in the eventual effects — we only have uncertainty in the *when* it will occur. Analysts tend to think that fat-tails only expose the rare events. In this case, they can reveal the inevitable That knotty and long CO_2 residence time combined with fossil fuel combustion leads to the inevitable rise that we will describe in the next section.

I contend that entropy and disorder in physical processes plays such a large role that it ends up controlling a host of observations that we have covered so far. Unfortunately, most scientists don't think in these terms; they still routinely rely on deterministic arguments alone. Which gets them in the habit of using heuristics instead of the logically appropriate stochastic solution.

Which leads me to realize that the first two observations have the unfortunate effect of complicating the climate change discussion. *I don't really know, but might not climate change skeptics twist facts that have just a kernel of truth?* Yes, certainly *"some"* of the CO_2 concentrations may have a half-life of 10 years, but that misses the point completely that variations can and do occur. I am almost certain that sceptics that populate sites such as ClimateAudit.org see that initial steep slope on the impulse response and convince themselves that a 10 year half-life must happen, and then decide to use that to challenge climate change science. Heuristics give the skilled debater ammo to argue their point any way they want.

I can imagine that just having the ability to argue in the context of a simple entropic disorder can only help the discussion along, and relying on a few logically sound first-principles models provides great counter-ammo against the sceptics.

Waste Half-Life

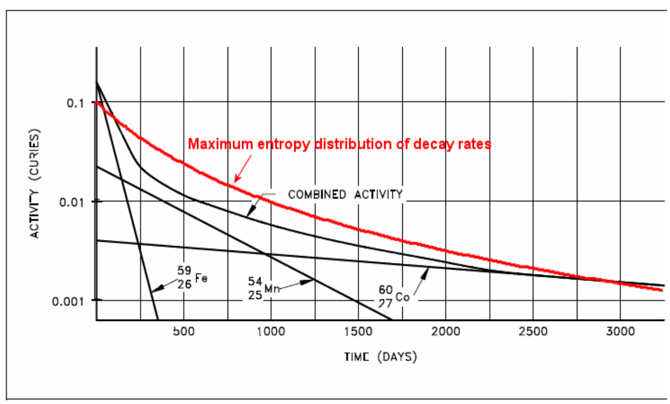
The disastrous Gulf Oil Spill of 2010 had everyone thinking about the half-life of the leaking crude oil and the expanding slick. First of all, the oil will biodegrade over time. We don't have the situation as in CO_2 where a sizable fraction will wander around the atmosphere trying to find a suitable location to react and form solutes.

Most of the oil will stay on the surface where it will get plenty of attention from aerobic microoganisms. Some of the oil will sink into the ocean and find anaerobic conditions at the bottom and essentially become inert or wash up on shore as sticky globs. Also the composition of crude oil includes many different hydrocarbons, some of which biodegrade at much slower rates, due to their molecular structure [Ref 305].

So I imagine that we can't calculate the half-life of the spilled oil in terms of a single rate constant, ***k***. This kind of first-order kinetics would likely show an exponential decline, which proceeds pretty quickly once you get past the half-lifetime, 1/***k***. Instead we will get a mix of various rates, with the fast rates occurring initially and the slower rates picking up the slack.

Radioactive waste-dumps also show a mix of decay constants [Ref 306]. Nominally, radioactive material will show a single Poisson emission rate, leading to an exponential decline over time. But when the different radioactive materials get combined, the Geiger counter will pick up this mixture of rates, and the decline will turn from an exponential to a fat tail distribution. See the red curve below.

FIGURE 30-28. Mix of Iron-56,Cobalt-60, and Manganese-54



A maximum entropy mix of decay rates (where a high decay rate indicates a potentially more energetic state) will generate the following half-life decline profile:

$$P(t) = \frac{1}{1 + k \cdot t} \quad \textbf{(EQ 30-32)}$$

where ***k*** is the average of the individual rates. This looks exactly the same as the hyperbolic decline of reservoirs that we covered in Volume 1.

For the derivation, we assume that we have a mean energy E_0 and then a probability density function will show many small energies and progressively fewer high energies.

$$p(E) = \frac{1}{E_0} \cdot e^{-E/E_0} \qquad \text{(EQ 30-33)}$$

but the decomposition rate R depends on E, so that

$$P(t) = \int_0^\infty P(t|E) \cdot p(E) dE \qquad \text{(EQ 30-34)}$$

$$P(t|E) = e^{-rEt} \qquad \text{(EQ 30-35)}$$

$$P(t) = \frac{1}{1 + rE_0 t} \qquad \text{(EQ 30-36)}$$

As you can see, the combined activity shows a much larger equivalent half-life since the tail has so much meat in it. In the limit of a full dispersion of rate constants, the average half-life will actually slowly diverge as the log of infinity. However, it never reaches this because the slowest decay rate will eventually dominate and that will not diverge.

In any case, this gives a good qualitative description of a random composition waste dump. If I make the same MaxEnt assumption for crude oil and assume that the most energetic oil (by the bond strength of the hydrocarbon) will likely prove the most difficult to decompose, then the half-life may also show a similar kind of fat-tail as that of a waste dump. It looks like benzene breaks down much slower than diesel oil for example.

As usual, disordered natural phenomena show many of the same dispersive characteristics, driven largely by maximizing entropy.

Environmental Prospects

I always look for analogies between physical systems. This often leads to dead ends but sometimes you uncover some interesting parallels that actually add to the knowledge-base of information and ideas for ***both*** systems.

As I worked out the problem of CO_2 dispersion in the atmosphere, I went back and revisited the work I did on dispersive transport in semiconductors. Essentially the same math gets used on both analyses, with the same fundamental goal in mind — that of trying to characterize the annoyingly sluggish response from an input stimulus.

For the climate case, the poor response comes from CO_2 molecules wandering around aimlessly trying to find a good resting place. For the disordered semiconductor, the carrier of electricity (the electron or hole) encounters so many trapping states and scattering centers, that it effectively takes much longer for the charge to cross a region. It does have the advantage of the assist of an electric field, but the low effective transport rate makes an amorphous semiconductor such as hydrogenated amorphous silicon (**a-Si:H**) marginally useful for any time-sensitive applications — yet eminently usable as a photo-voltaic.

Still, knowing the physical characteristics helps to understand the nature of the material, and could unlock some secrets beneficial to future applications of material such as polycrystalline or amorphous silicon, or any disordered semiconductor. In the future, we will make mass quantities of this material for the PV industry and we won't have the luxury of single crystal material.

The fact that dispersive transport does have the help of an electric field, makes it amenable to experimentation. By applying various electric fields, one can distinguish between a *drift* component and a *diffusive* component (of the photoelectric current, for example). With no electric field, any photo-generated carriers will wander around until they recombine. This can take relatively long times, especially in comparison to a piece of single crystal silicon. As the electric field increases, the carriers get swept out faster and the diffusion plays less of a role.

The fact that the atmosphere has no drift role apart from turbulent diffusion, means that CO2 plays the analogous part of a electronic device with generated carriers, but nowhere to remove them (alas, we have no electrodes attached to the atmosphere). So, I wanted to get a bit of insight by looking at the carrier transport problem, and as a goal, perhaps find a way to increase the removal of CO_2 by something equivalent to an electric field, and particularly to ask if this could reduce the CO_2 mean residence time.

CHAPTER 31

Econophysics.

Understanding economic behavior

"Investment occurs when you have a stable economy, when you can foresee what things are going to do in the future…it's very critical that we get the uncertainties out of the system"

— Alan Greenspan

One definition states economics as the study of how the forces of supply and demand allocate scarce resources. Predicting human behavior in this context makes economics as a science difficult beyond belief. Recent evidence suggests that to predict economic activity accurately you need to use all the psychological tricks and ideas from game theory that you can apply [Ref 278]. Unless economic progress gets tied directly to constrained resources, it becomes a zero-sum game among the participants seeking to maximize their own wealth from a virtual pile of debt. It has taken ages and will take even more time to bring economics into a credible scientific discipline, yet certain topics in economics lend themselves to a straightforward analysis similar to that we have used for oil depletion. This could prove valuable should the majority of economists fully realize the impact of finite constraints on their worldview.

"The different branches of science may seem so far apart only because we lack the common method on which they grow and which holds them together organically [...] The statistical concept of chance may come as dramatically to unify the scattered pieces of science future [...] We are on the threshold of another scientific revolution. The concept of natural law is changing".

— Jacob Bronowski

So even though we cannot easily pin down human and group decision making, we can identify the objects acted on — quantities dependent on constrained resources — and thus have a solid chance of understanding certain aspects of economics. I really think we can make some progress understanding this and I think it has great importance for planning.

Yakovenko who has studied income and wealth distribution said this about the discipline of *econophysics*:

> *Econophysics distances itself from the verbose, narrative, and ideological style of political economy*
>
> ...
>
> *The econophysicist Joseph McCauley proclaimed that "Econophysics will displace economics in both the universities and boardrooms, simply because what is taught in economics classes doesn't work".* (referencing P.Ball, 2006, "Econophysics: Culture Crash," Nature 441, 686–688). [Ref 284]

I consider the research important but realize too that econophysics occupies a transition zone of science and has not yet started a paradigm shift. The economists don't completely appreciate the discipline because econophysicists don't always defer to the legacy of their classical work; the statisticians don't because they often deal with unwieldy fat-tail models; and the physicists ignore the discipline because they don't consider economics a science. Despite the lack of interest and threatened by the sunk costs of the status quo, Yakovenko makes a strong case by describing the history behind econophysics, reminding us that Boltzmann and company had suggested this field from the start, regrettably getting buried over the ensuing years.[1]

> "*Today physicists regard the application of statistical mechanics to social phenomena as a new and risky venture. Few, it seems, recall how the process originated the other way around, in the days when physical science and social science were the twin siblings of a mechanistic philosophy and when it was not in the least disreputable to invoke the habits of people to explain the habits of inanimate particles.*"

Curiously, the fractal and fat-tail proponents also seemed to have dismissed the field:

> *A long time ago, Benoit Mandelbrot (1960, p 83) observed: "There is a great temptation to consider the exchanges of money which occur in economic interaction as analogous to the exchanges of energy which occur in physical shocks between gas molecules." He realized that this process should result in the exponential distribution, by analogy with the barometric distribution of density in the atmosphere. However, he discarded this idea, because it does not produce the Pareto power law, and proceeded to study the stable Levy distributions.*

1. Concerning the origins of economic theory in the 19th century, Nadeau asserts that not only does the formation of classical economics rely on odd analogies to physics, but that the analogies that the early economists chose came out of physics theories that nobody had verified yet [Ref 282]. They basically consisted of vague notions on the balance of energy existing in the "ether", in other words not the concrete Newtonian approach that everyone agreed on, but ideas of the more abstract mysterious origins of electrical or magnetic energy that started to gain traction around that time. And to top it off, many of the early economists had strong religious beliefs and thought god had something to do with the economy (i.e. the idea of the invisible hand). This all occurred slightly before physicists like Maxwell figured out the practical formulation that has served us well. Nadeau implied that the only perturbation to the original ideas came by way of Keynes and others who allowed that we could subtly guide economies, and this provided a not-so-invisible hand.

In spite of its early origins, econophysics-type approaches help to explain macroeconomics in a fresh way, separated from the conventional focus on exchangeable resources and whatever else that regular economists deem important. I will begin by transitioning gradually into the subject beginning with a common-day topic.

The Web as Microeconomic Activity

As a micro-econophysics exercise, we consider the case of a set of web sites where the goal of the site is to increase the connectivity of the site (the in-degree and out-degree) and also to improve the strength of the site. The former describes a static measure, essentially counting links, while the latter describes the throughput of the site, corresponding to a dynamic connectivity with the rate of users clicking through as the measure.

Many of the web profit-making schemes work off of these two measures, either via click-through advertising techniques or simply to attract potential customers.

We first realize that the skill-level of web-site operators ranges over orders of magnitude. You will find high-flyers who use every single technology and work at a break-neck pace. At the other end of the spectrum you will find operators who barely put effort into their site. Ultimately, we don't know anything more than some average rate of productivity, $\boldsymbol{R}$, exists in building the "popularity" of a web site.

According to the Maximum Entropy Principle we can make an unbiased estimate of the dispersion of the rate, given knowledge of only the average:

$$p(r) = \frac{1}{R} \cdot e^{-r/R} \qquad \textbf{(EQ 31-1)}$$

The population of sites will generate a cumulative distribution that will disperse according to these rates operating over some time interval. For a specific time the cumulative will follow:

$$P(t|w) = \int_{w/t}^{\infty} p(r)dr = e^{-w/(Rt)} \qquad \textbf{(EQ 31-2)}$$

where $\boldsymbol{w}$ is the work that a web site will likely expend during its lifetime. We don't know this so we leave it as a marginal variate.

Next we model that the growth of the network links actually follows a learning curve path over time. This essentially describes a situation where initially you can make fast progress in generating links but that eventually this rate will reach a level of diminishing returns.

Let's derive a growth function from first principles. Say that the differential growth goes as:

$$dg = \frac{k}{g+\mu}dt \quad \textbf{(EQ 31-3)}$$

This says that the incremental achievement per unit time goes inversely proportionally to the growth level already achieved by the site. The factor of μ exists to prevent infinite growth initially. The solution to this differential formulation is:

$$\frac{1}{2}g^2 + \mu \cdot g = k \cdot t \quad \textbf{(EQ 31-4)}$$

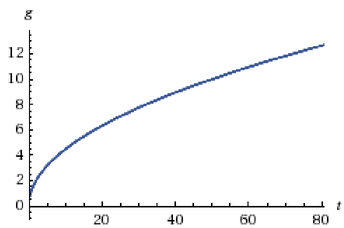


FIGURE 31-1.
Learning curve

Which gives the growth learning curve to the left. Since the result for ***g***(***t***) is implicitly set, we need to adroitly make the observation that our population is equally distributed over time, the so-called ergodic assumption. We notice that the ***R*t*** term grows in proportion to ***k*t*** so that we can interchange the two freely. After the substitution of ***R*T*** for ***k*t*** the cumulative becomes:

$$P(g|w) = e^{-w/\left(\frac{1}{2}g^2 + \mu \cdot g\right)} \quad \textbf{(EQ 31-5)}$$

This is a conditional distribution based on the overall work, ***w***, expended. We also lack any information on this value so that we can remove the marginal probability by integrating over a range of ***w***

$$p_w(w) = \frac{1}{W} \cdot e^{-w/W}$$

$$P(g) = \int_0^{\infty} P(g|w) \cdot p(w)dw = \frac{1}{1 + \frac{W}{\frac{1}{2}g^2 + \mu \cdot g}} \quad \textbf{(EQ 31-6)}$$

Link Studies. A number of research efforts provide ranked statistical data for static and dynamic web links. Over the years the data has definitely become more comprehensive; early efforts had orders of magnitude less information than the heavily trafficed network of today. I chose a set from a web ranking study with the widest dynamic range of user traffic I could find [Ref 285]. The following figures show the data with fitted models for both in-degree and in-strength statistics. The in-strength statistics have a wider dynamic range as user traffic adds a multiplier factor over static links. The left-most column shows in-degree distributions with the original exponent of -2 and the lowest log-likelihood value of -2.15. The right-most column shows in-strength distributions with the original exponent of -2 and the lowest log-likelihood value of -1.8. Note that the in-degree statistics have values of c=0, indicating that initial links have no extra startup cost (i.e. an infinite slope occurs initially because the first links come easily).

Otherwise the deviation from the assumed learning curve power-law of 2 may have little significance (±10%). We know that dispersion exists in the real world of web development economics and we know that eventually every site reaches a point of diminishing returns. The fact that the probabilities range over 14 orders of magnitude with general agreement over every decade strongly suggests that something similar to this model must reflect the aggregate statistical situation.

in-degree	**in-strength**
power=2, W=1.3, μ =0	power=2, W=3.4, μ =4.5
power=2.15, W=1.3, μ =0	power=1.8, W=3, μ =7.4

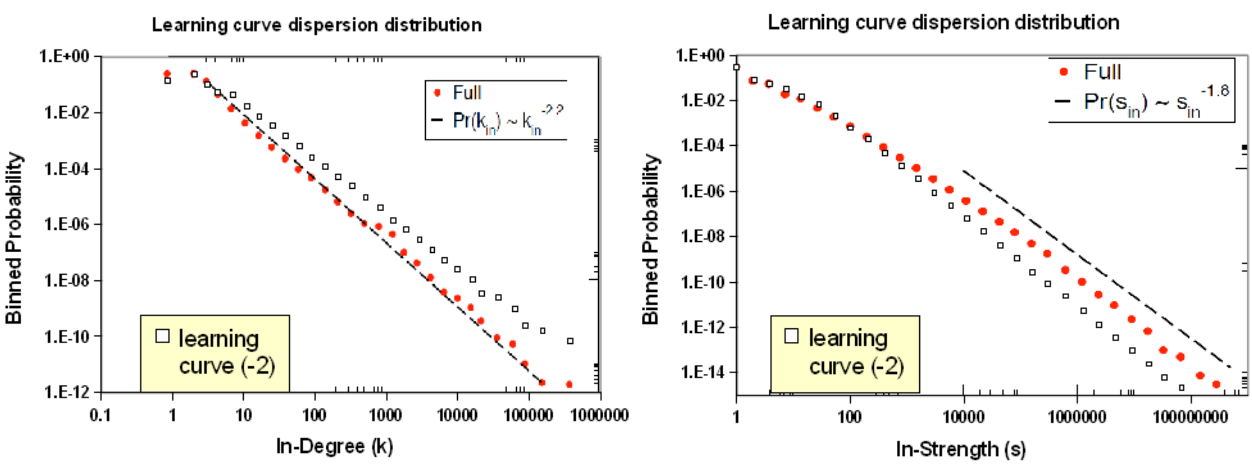


FIGURE 31-2. Web site link connectivity (left) and link strength (right) for the original learning curve model.

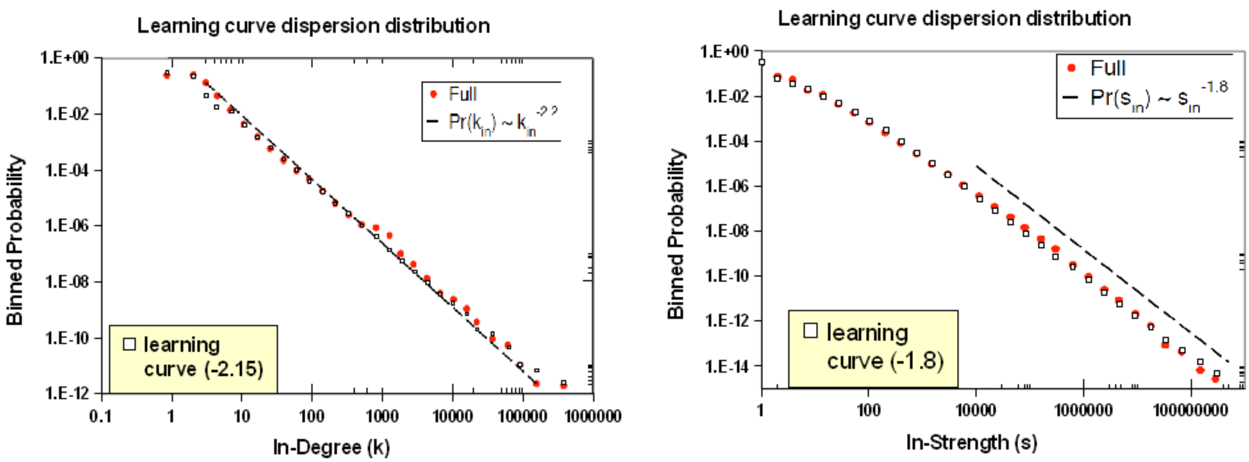


FIGURE 31-3. Web site link connectivity (left) and link strength (right) for the learning curve model with better fitting new exponents.

Before the web existed another kind of link network allowed scientists to make substantial progress in their research endeavors. The citation index of scientific publications essentially created the equivalent of a static network of connected research topics. In peer-reviewed research, the cream rises to the top and the most heavily cited papers inevitably pointed to the most cutting-edge findings and highest scientific impact.

Since deep research tends to have a narrow focus, the spread of citations will certainly follow a learning curve path. With limited difficulty, one can get at least a few citations after the publication of a paper, but thereafter further citations follow on merit alone. Like web links, the spread follows the same Fickian diffusion arc of diminishing returns, with the dispersion primarily controlling the range in quality of the research itself. This shows the near evolutionary behavior described in "Innovation and Evolution. How ideas spread" in action but with a deep learning curve influence.

The following plots show a CDF and PDF for a comprehensive set of citations from the Institute of Scientific Information [Ref 292]. For parameters W=250, c=30, and a power-law of 2, the dispersive learning curve models demonstrate good agreement and a fairly stiff initial learning curve slope, due to the large relative value of c. Both web linkages and scientific citations appear to follow the same fundamental learning curve behavior.

Could we extract human productivity information in the same manner?

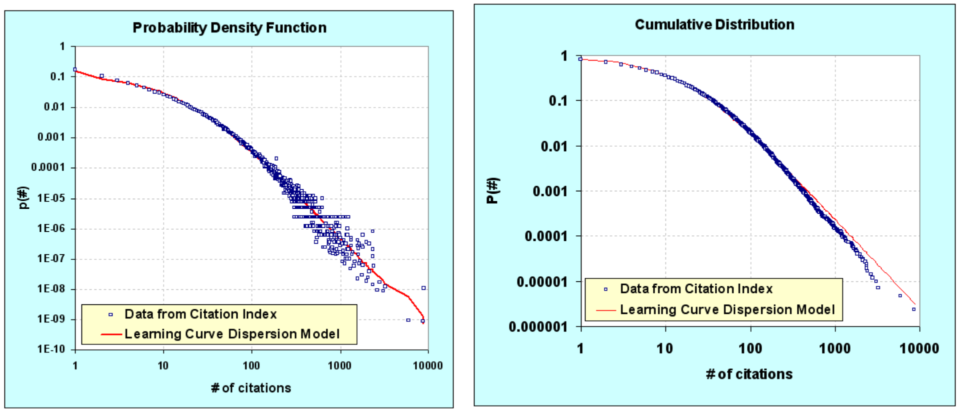


FIGURE 31-4. Scientific citation cumulative (left) and probability density (right) distributions showing excellent agreement with a dispersive learning curve model.

Extracting a Learning Curve in Labor Productivity

The previous section on web link statistics leads directly to a similar pattern concerning human productivity. A comprehensive study recently done on Japanese labor statistics covered a million firms and over 15 million workers, with goal of creating a set of sample probability distributions describing among other things, worker productivity[Ref 283]. The individual productivity derives from proxy measures of corporate revenue and profit. Enough variety exists in the set of Japanese labor pool for the statistics to reach an ergodic level. This essentially means that enough variation exists in the ways that the laborers operate that they will visit all the productivity states possible considering the constraints. That partly explains why we can gain confidence that some deep fundamental stochastic behavior may explain the results, in much the same way that the web link statistics play out, with the sheer numbers drowning out the noise. This demonstrates a pure way of applying econophysics.

Toward that path, we create a corresponding model of labor productivity that invokes similar math with the same intuitive concept of a learning curve for individual productivity. In practice, a learning curve can exist where a worker can pick up much of the rudimentary skills very quickly, yet to get that last level of productivity will often take much more time[2].

Let's call the labor productivity ***C*(*t*)** and note that it has some minimum (***Max***) and maximum level (***Min***) based on the basic minimum requirements of the company and on the maximum technically achievable productivity.

Then over the course of time, a new hire may see productivity rise according to this simple relationship:

$$dC(t) = \frac{k}{C(t) + \mu} dt \qquad \textbf{(EQ 31-7)}$$

This mathematical relationship says that the increment of productivity (***dC***) per unit of time (***dt***) is inversely proportional to the productivity accumulated so far. In this case productivity equates to skill level. Thus, the worker can learn the early skills very easily, where the accumulated skills have not risen to a high level. (again the parameter μ prevents the learning curve rate from going to infinity initially). However, as time advances and these skills accumulate, the growth of new skills starts to slow and it reaches something of an asymptotic limit. We can consider this a law of diminishing returns as the weight of the accumulated skills starts to weigh down the worker.

Rearranging the equation, we get:

$$(C + \mu) \cdot dC = k \cdot dt \qquad \textbf{(EQ 31-8)}$$

We can integrate this to get the result:

$$\frac{1}{2}C^2 + \mu C = kt + \text{constant} \qquad \textbf{(EQ 31-9)}$$

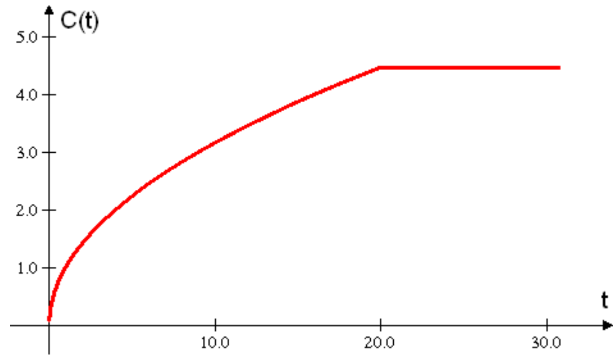
This results in the same constraint relationship that I used in the previous section in determining cumulative link growth according to a dispersion formulation. The equation if given constrained limits (both *Max* and *Min* values) has a solution according to the basic quadratic formula — which I used for a Monte Carlo simulation. Otherwise we simply use the ergodic view that all the various values of ***t*** will get visited over time, and all the possible constant values will show up according to maximum entropy.

The following figure shows a single instance of a labor productivity learning curve. This has a minimum level and a maximum level at which productivity clamps to. Imagine a set of many of these curves, all with different quadratic slope and maximum constraint and that turns into the statistical mechanics of the labor productiv-

2. Also see Fick's diffusion equation or a random walk, which shows the same quasi-asympotic properties. See "Diffusional Growth" on page 158.

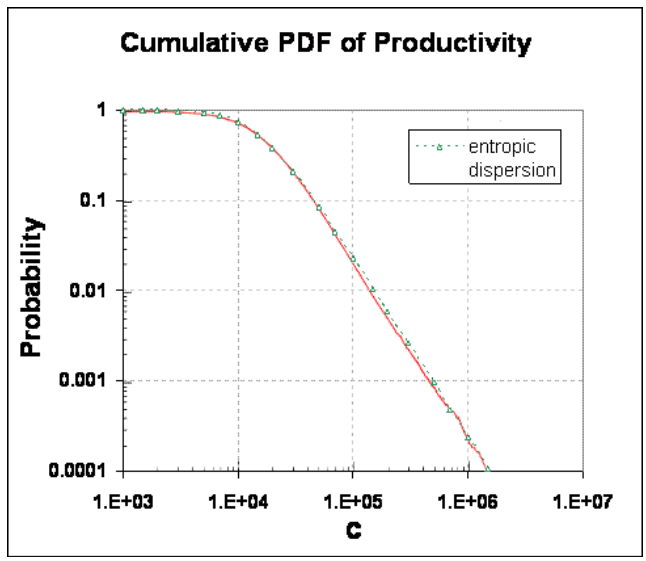
ity distribution function for $\boldsymbol{P}(\boldsymbol{C})$ that we see aggregated over all possible learning times.

FIGURE 31-5.
A labor productivity learning curve that gets asymptotically truncated at some level.



which leads to this density function and the excellent fit to the data when entropic dispersion gets applied.

FIGURE 31-6.
Shows the rolled up Japanese firm data along with the single parameter fit to the entropic dispersion model. This is the cumulative density function over several orders of magnitude.



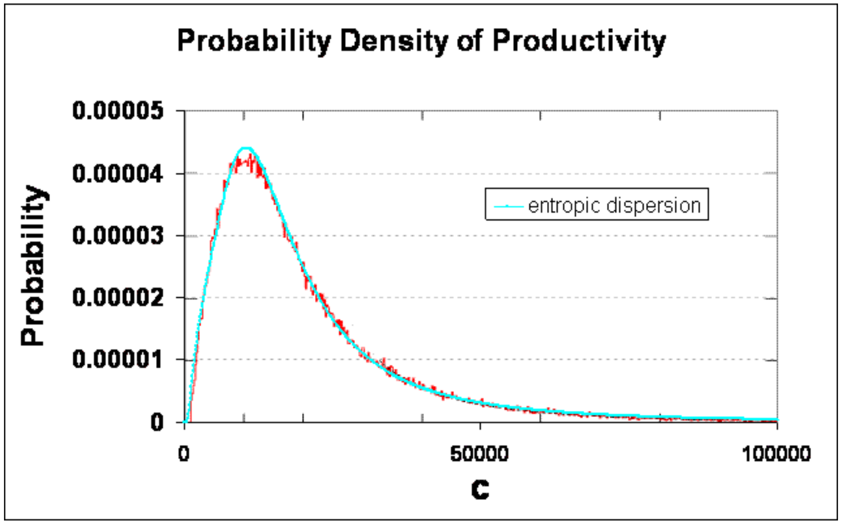


FIGURE 31-7. Probability density function fit to the low productivity values close to the knee of the curve above. A lack of real noise comes from a huge 15 million point sample.

Based on the strong fit observed, this essentially gives an econophysics explanation to a learning curve model for labor productivity. All workers go through a learning curve that shows a minimum proficiency and a maximum productivity that constrains the levels; in between we see the quadratic solution growth which shows up as the inverse power law of 2 in the labor productivity distribution function.

If we modify the labor productivity model that constrains a a uniform density between *Min* and *Max* values with one that shows a maximum entropy range of productivities, we come up with a cumulative productivity as follows:

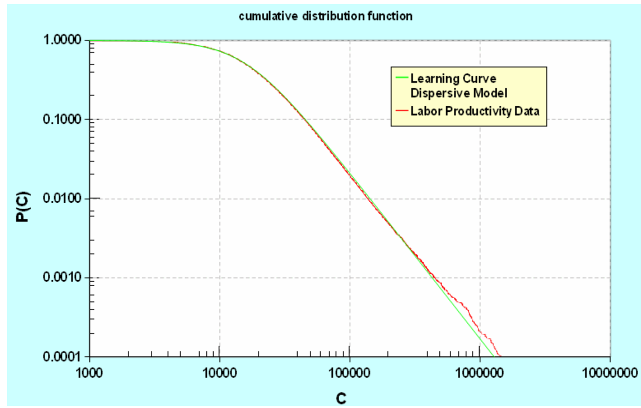
$$P(C) = \frac{1}{1 + \frac{W}{\frac{1}{2}C^2 + \mu \cdot C}} \quad \textbf{(EQ 31-10)}$$

This works out to give exactly the same result as we used to describe web links and scientific citations. The general corresponding probability density function, with an arbitrary power-law exponent ***a*** replacing the value 2, derives to

$$p(C) = \frac{W \cdot (C^{a-1} + \mu)}{\left(W + \frac{1}{a}C^a + \mu \cdot C\right)^2} \quad \textbf{(EQ 31-11)}$$

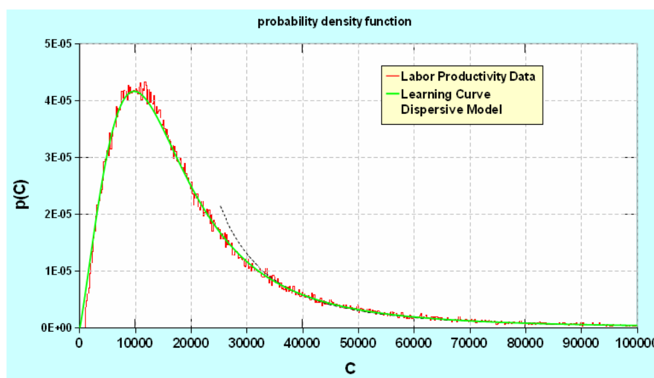
The fit using this model assuming an power-law of ***a***=2.1 instead of 2 and a value of μ =0 and W= 3.2e8 generates equally good agreement to the point that it becomes hard to distinguish the data from the model. A μ =0 indicates that minimal startup costs exist and the typical worker becomes immediately productive.

FIGURE 31-8. Unbelievably good fit for the maximally disordered learning curve dispersive model to the CDF. Slight variations exist down the slope where counting statistics play a larger role for the rarer high productivities.



When μ =0 and power-law of 2, the mean value for C is $\pi\sqrt{W/2}$ which works out above 20,000 productivity units (¥ in this case). Interestingly no second moment exists for this fat-tail distribution.

FIGURE 31-9. An even more unbelievably good fit for the learning curve dispersive model to the PDF. This is essentially a 1 and 1/2 parameter fit. The value for *W* aligns the peak and slight variations around the power law of 2 adjusts the fat-tail slope.



Shahili has stated that having a simple model makes it robust and better at prediction [Ref 288]. Since few parameters get used for fitting the model to the data, likely the same few parameters will get used in the future. You thus don't have to worry about scores of parameters changing as it confronts new data sets. And this data does change from year-to-year [Ref 283], suggesting that fluctuations around the power-law of 2 may occur indicating yearly variations in the learning curve difficulty. Based on the simplicity of the model and the relentless avoidance of assuming anything more than a mean value, the principle of maximum entropy tells us that this behavior does indeed fit reality. Nothing else can come conceivably close.

That gives us some valuable insight to the variability in the individual level of economic productivity expected. We can next try to scale the ladder and test to its applicability to the *firm* level.

The Firm Size Entroplet

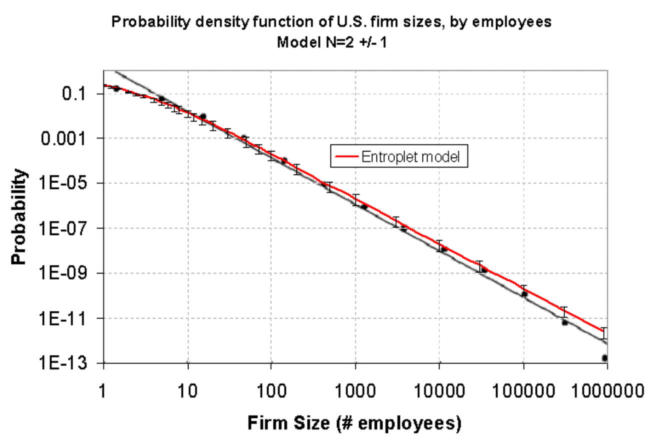
The statistical growth of a sampled set of company sizes (measured in terms of number of employees) should follow an entropic dispersion. The growth of an arbitrary firm behaves much like the adaptation of a species as I describe in the earlier section "Dispersion, Diversity, and Resilience" . The two avenues for growth include a maximum entropy variation in time intervals (***T***) and a maximum entropy variation in innovation or preferential attachment of employees to large firms (***X***). The combination of these two stochastic variants leads to the entroplet function.

Data from Axtell parsimoniously supports the model [Ref 286]. The probability density function has to normalize to 1 and we have one free parameter (N) with which to fit the data.

$$p(\text{Size}) = \frac{N}{(\text{Size} + N)^2} \tag{EQ 31-12}$$

The figure below shows the best fit to the data superimposed on Axtell's Zipf law straight line. The model suggests that the characteristic dispersed firm size is N=2 employees. Axtell obtained a regression fit for an exponent of 2.059, which agrees well with the MaxEnt value of 2. The entroplet also works better for the small firm data, where it looks like Zipf's law should truncate. The data in the large size tail region suffers from relatively poor statistics due to the low frequency of occurrence.

FIGURE 31-10. PDF of firm size using the entroplet model, compared against [Ref 286].



I will give a quick outline[3] of another proof for deriving the entroplet that differs from the one I used for species adaptation (which used a CDF instead of a PDF). We want to find the pdf of two random variables $R = X/T$ where X and T remain independent, each exponentially distributed, with the mean scaled to unity. Then the PDF is

$$p(R) = \int_0^{\infty} t \cdot p(t \cdot R|t) \cdot p(t) dt \qquad \text{(EQ 31-13)}$$

where we assume that t ranges over T and $R=X/t$ becomes a scaled version of X. This basically states that we have placed an uncertainty in the two values and then turn the reciprocal into a multiplicative factor to make the conditional probability trivial to solve over all possible values of the random variables.

$$p(R) = \int_0^{\infty} t \cdot e^{-tR} \cdot e^{-t} dt \qquad \text{(EQ 31-14)}$$

which for $r > 0$ reduces to the normalized function

$$p(R) = \frac{1}{(1+R)^2} \qquad \text{(EQ 31-15)}$$

To use this for general modeling, we denormalize the values of unity with the parameter N, and change the rate R to a proportional *Size* (the ergodic limit approximation).

$$p(\text{Size}) = \frac{N}{(\text{Size} + N)^2} \text{ same} \qquad \text{(EQ 31-16)}$$

These econophysics case studies have arguable merit because the math model invariably fits the data to a tee. One can ask why the firm size distribution doesn't follow the same learning curve model as it does for the individual laborer. I believe that it has more to do with the growth potential of a firm rather than an individual; a firm can always add more workers or merge with other firms to sustain at least a quasi-linear growth rate. These become the mutations that an adapting species would also need to invoke to survive.

In the words of Joseph McCauley, the realm of econophysics occupies the world *"where noise rather than foresight reigns supreme"*, with the noise referring to the huge state space that the observables can occupy [Ref 287]. In this case, we have no idea how the growth of an arbitrary company will proceed; yet we know the aver-

3. Adapted from Leon-Garcia, p.273 [Ref 280].

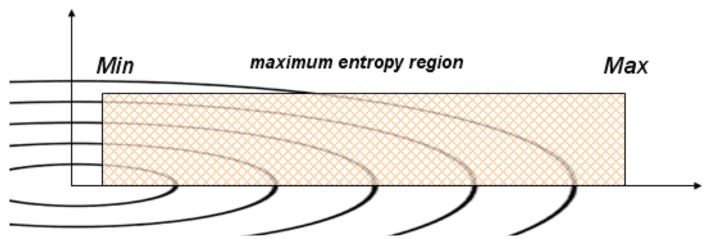
age rates of growth and other metrics. This gives us enough of a toehold so that we can estimate the entire distribution from those numbers with the help of our statistically noisy friend, entropy (c.f. connection of entropy with economics [Ref 318]).

Ergodicity

The labor productivity model that I used to generate the observed distribution involved the application of an inverse non-linearity via a differential equation. The actual model did not come close to a chaotic form but it serves as a useful discussion point. The nature of this model at least contributes to the potential for the economic states to reach an ergodic limit as the individual agents run through complete trajectories with each one possessing a different average rate.

This can also occur similarly to how a complex expression invoked on a calculator can allow one to generate a pseudorandom distribution.[4] In the parlance of attractors, other more complex equations naturally translate into a scenario where you can imagine all sorts of possible trajectories to occur:

FIGURE 31-11. Trajectories of a chaotic attractor allow all possible states of a system to be reached.



Unfortunately, this does not help too much with intuition. Yes one can execute a bunch of random calculations and note how easily various states accumulate, but the complex or non-linear math can get in the way.

The term *ergodicity* refers to the uniformity and fairness in occupation of a system of probability-based states. As another way to think about it, an ergodic Tic-Tac-Toe distribution would gain enough statistics such that the average measured occupancy of squares of an averaged game's end would equal the probability of some theoretically predicted occupancy.

4. One of the first mathematical experiments I remember working in school had to do with random number generation. At the time, hand-held calculators still held some excitement and to take advantage of this, the instructor gave the students an assignment to come up with their own uniform random number generator by creating some complicated algorithm on the calculator. The student could then easily test his results. As I recall, I felt a certain amount of pride in the clever way that I could get results that appeared random. I don't think I understood what pseudo-random meant at the time. In retrospect, I probably had enough knowledge to realize the power of the inverse trig functions and how they could generate numbers between zero and one, and of the idea of truncating to the decimal part of the number (to get a value between 0 and 1). I am sure that my classroom random number generator would not pass any quality tests, but it worked well enough for learning purposes.

One can get to this state either by capturing statistics over a long period of time or having some process that doesn't have statistical runs that break the *stationarity* principle. One often sees confusion between the adjectives stationary and ergodic. The first has more to do with suggesting that a particular snapshot in time does not differ from another snapshot at some later time (i.e. independent events). The latter makes certain that the process has enough variety in its trajectories to visit all the possible states.

Generating a psedu-random distribution via “complex” calculator functions can gave one an idea of how easy a model or system can enter into a very disordered state, and one of almost “uniform randomness”. It really does not take much in terms of a combination of linear or non-linear calculations before one can obtain a well-travelled set of trajectories and thus an ergodic distribution that works effectively to meet the maximum entropy principle.

Income Disparity

The discussion of labor productivity economics leads one to consider how income distributions might relate as modeled behaviors. Income distribution distinguishes itself from productivity in that a certain proportion of a worker’s income can come from “non-productive” means; by that I mean mechanisms such as compounding interest where the individual doesn’t actually have to do any “work” in the physical sense. The econophysics researcher Yakovenko discovered that historical income distributions had much to do with this so-called fast income growth [Ref 284]. As we have done in our analyses so far, such as in species adaptation (See “Dispersion, Diversity, and Resilience” on page 427.), we can apply very similar mathematics and use entropy to argue for large dispersive effects on measurable quantities.

Earlier we showed how the two channels of species adaptation work. One of the channels involves variations in time, in which species adapt via maximum entropy over slow periods. The other channel involves variations in adaptation levels themselves; this abstraction provides a kind of “short-circuit” to a slower evolution process in which small changes can provide faster adaptation. If the first case acts as a $\Delta time$ and the second acts as a ΔX then $\Delta X/\Delta time$ generates a velocity distribution leading to the observed relative abundance distribution as the *entroplet* or entropic dispersion function. That works very well in several other fat-tail power-law applications, such as the preceding firm size distribution and in the model for dispersive discovery of oil described in Volume 1, which includes an acceleration factor akin to compound growth.

So the intent next is to show how this primitive adaptation model works in the context of human adaptation — in particular, in the greedy sense of making as much money as possible.

Yakovenko implies that the adaptation channel of time remains a strong driver on the lower wage earners. If we only assume this as a variant, then according to maximum entropy, the income distribution drops off as exp($-v/v_0$) where v indicates income velocity, and ends up as proportional to a relative income, if time drives the velocity, *Income* = v * *time*. The cumulative becomes, where *t*=*time*:

$$P\left(\frac{\text{Income}}{t}\right) = e^{-\text{Income}/(t \cdot v_0)} \qquad \textbf{(EQ 31-17)}$$

This works for a portion of the income curve, primarily consisting of the low income classes. Yet it does not generate the observed Pareto power-law for the higher income part of the distribution. To get that we need the other fast channel for dispersive income growth.

Humans can't mutate on command (and obviously don't have the diversity in geological formations) so they lack the same fast channels that exist for other fat-tail power laws. To get the fast channel, we can hypothesize the possibility of *income that builds on income*, i.e. *compound interest growth*. This has some similarity to the ideas of preferential attachment, where a large volume or quantity will attract more material (or generate more mutations in species if a population gets large). Yakovenko calls it multiplicative diffusion (also see [Ref 284]):

> *This is known as the proportionality principle of Gibrat (1931), and the process is called the multiplicative diffusion (Silva and Yakovenko, 2005).*
>
> ...
>
> *Generally, the lower-class income comes from wages and salaries, where the additive process is appropriate, whereas the upper-class income comes from bonuses, investments, and capital gains, calculated in percentages, where the multiplicative process applie*s [Ref 284]

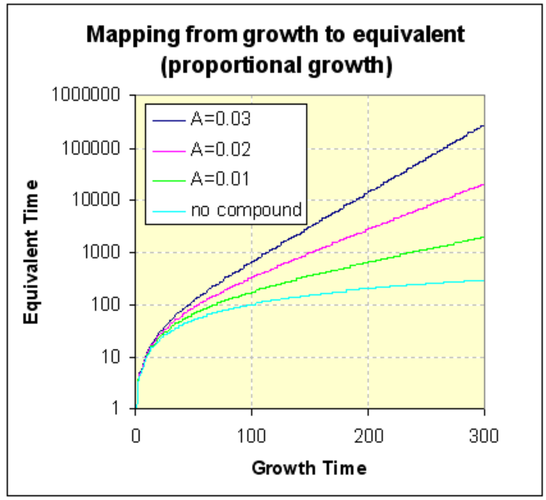
The simplest variant of compound growth is the following equation:

$$\frac{d}{dt}g(t) = A \cdot g(t) + 1 \qquad \textbf{(EQ 31-18)}$$

This has the solution

$$g(t) = 1/A \cdot (e^{At} - 1) \qquad \textbf{(EQ 31-19)}$$

FIGURE 31-12. Compound growth starts immediately and will effect all income streams in a proportional amount.



Since we consider income growth as a relative or proportional process, the growth factor $g(t)$ should fit directly into the income velocity expression (31-17)

$$t = 1/A \cdot \ln(g(t) \cdot A + 1) \quad \textbf{(EQ 31-20)}$$

If we plug this into the cumulative *Income* distribution we get

$$P(\text{Income} = g) = e^{-\frac{A}{v_0 \ln(A \cdot g(t) + 1)}} \quad \textbf{(EQ 31-21)}$$

This turns into a case where time essentially gets compressed during the accelerated compound growth phase: Yet this form alone does not match the observed stratification of income classes and in particular the position of the inflection point observed. The distribution may work (at least conceptually) if *all* income classes continuously stored away part of their wages as compound savings, yet we know that lower income classes do not save much of their wages at all.

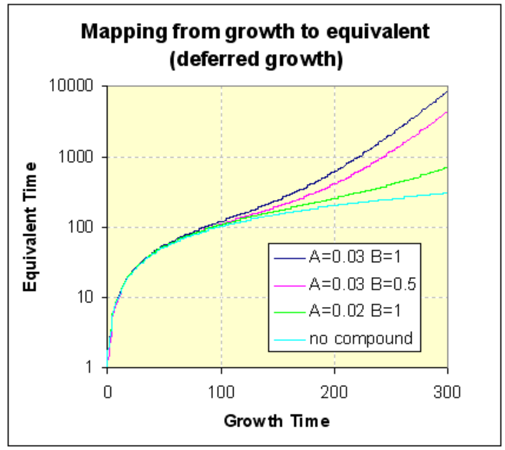
The growth equation which discounts low-incomes effectively looks like the following:

$$\frac{d}{dt}g(t) = A \cdot (g(t) - t) + 1 \quad \textbf{(EQ 31-22)}$$

Early growth gets compensated by the linear growth term t, and the parametric deferred growth form looks like:

$$g(t) = t + B \cdot e^{At} \quad \textbf{(EQ 31-23)}$$

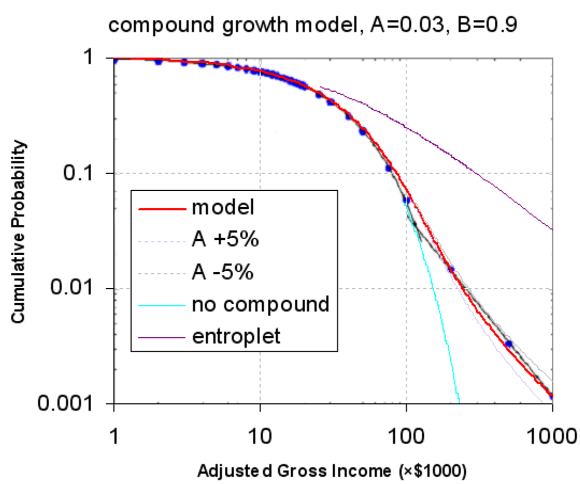
FIGURE 31-13. Deferred growth has to overcome the linear term and then will grow according to the savings rate. Lower fractional savings (B) or lower compound interest (A) will defer the growth to the future.



Inverting the equation to obtain the time mapping requires a parametric substitution. I replace the running time scale with the substitution: $t + B*\exp(A*t)$, yet maintain the cumulative with a linear growth. This has the effect of compressing time during the intense compounding growth period.

See the red line in the figure below which assumes a deferred growth fit with the two parameters shown: B=0.9 and A=v0=0.03..

FIGURE 31-14. Income distribution in the USA. The measure income distribution fits between two-degree of freedom entropic dispersion (top curve, entroplet) and one-degree of freedom dispersion (bottom curve, exponential). The introduction of compound growth can magnify the dispersive effects.



The exponential term, A, essentially borrows from the average income growth, v_0, as a zero-sum game and builds up the compounding interest. I could have used an alternate exponential factor for A differing from v_0, but in the steady-state economy money has to come from somewhere, so that having the interest rate track the average income growth dispersion makes sense (one less parameter to gain parsimony).

That leaves one other free parameter besides A; the term B refers to a proportional savings rate. If a person saves money relative to his non-compounded growth, it will provide the initial value for B. If B remains close to zero, it implies no savings; whereas a high value implies a large fraction goes into savings for compounding growth.

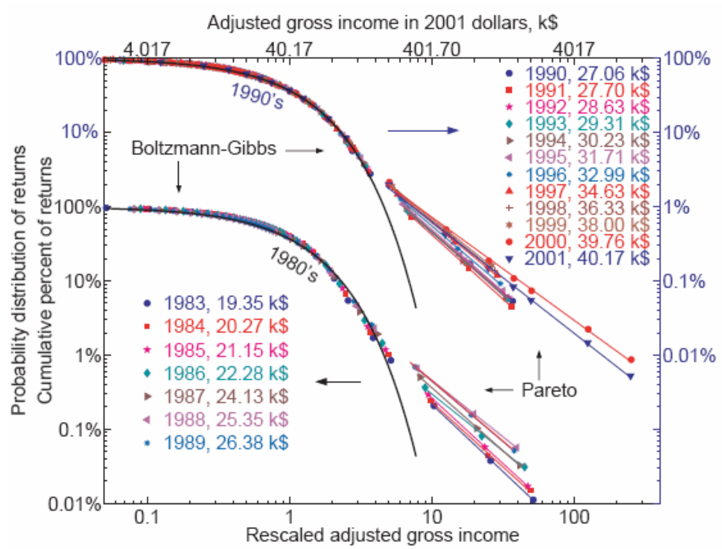


Fig. 2

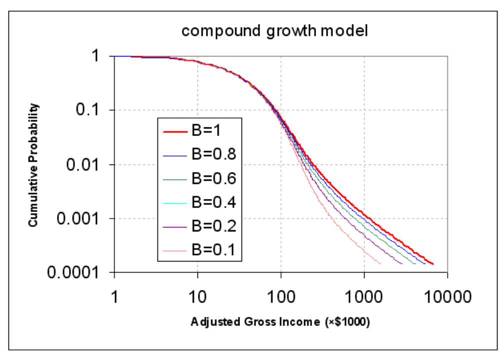


FIGURE 31-15. Variations of savings fraction, *B*, accounts for most of the yearly fat-tail fluctuations observed. (top) Plot from Yakovenko, recessions have lower investment savings than boom periods. (bottom) Model variations in *B*.

The combination of A and B generates the fast growth channel so that we can duplicate the fast growth needed for the observed income dispersion.

Modifying B moves the fat-tail up and down on the histogram. The meaning of B compares to the Gini factor or coefficient used to quantify the disparity between the income classes. Whereas modifying A, if needed, serves to flatten or steepen the fat-tail portion (according to a sensitivity analysis this has a larger effect the further right you go on the tail). If either of these disappears, it reduces to the exponential tail steep/narrow dispersion.

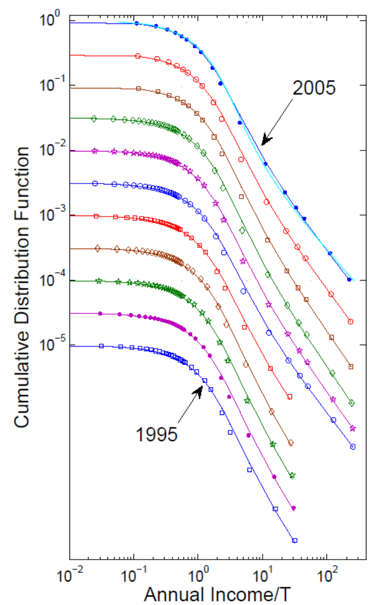


FIGURE 31-16.
From [Ref 289], a family of income distribution curves. The inflection point at high incomes shows the greatest amount of variability.

When Gregory Chaitin wrote that *"a theory has to be simpler than the data it explains, otherwise it does not explain anything", he paraphrased what Leibniz stated in 1686 [Ref 297]*

The top curve to the right shows how effectively this procedure works on wide dynamic range data.

Alternate View. The so-called "80/20" rule where 20% of the people make 80% of the total income comes from the application of the Pareto rule. The high income part of the curve maps to a model with a stochastic "well of knowledge" or "depth of wisdom". We don't actually reach a specific point at which we determine our salary, but instead grow that salary over a range of years that we would willingly try to advance. For a dispersion with a damped exponential well, this leads to a straight line on the log-log plot, which matches that of the high income part of the curve. The low income part of the curve has a more rapidly declining profile, indicative of a income earner that trains for a fixed period of time and becomes satisfied with the salary at that point. This period may match the length of time for a high school education or a trade school job. On the other hand, for the higher-income earner, the training may become a life-long process, so that the goal-posts continuously expand until they reach retirement age. To first order, this effort relates to education time, continuing education, and overtime worked, leading to increased wages with experience.

So one can try to separate the two segments by summing two distinct distributions, i.e. a low-income exponential distribution and a high-income continuous learning power-law (see the figure below).

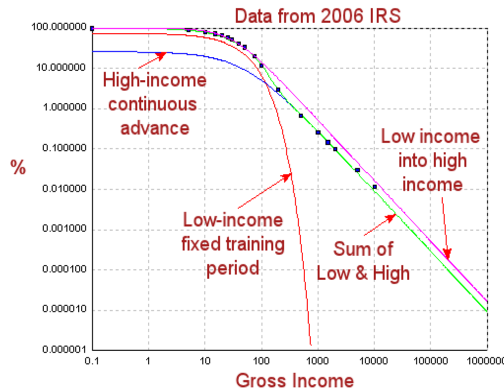


FIGURE 31-17. Data from IRS on USA yearly income shows a Pareto rule decline. Assuming a dispersion of income increases over a wage-earners life-time, a dispersive growth model fits the data arguably well.

The decline rate under the conditions of a variable academic stint plus a job maturation period, similar to a learning curve, should lead to a slope of ~ $1/t^2$, close to that observed (usually above $1/t^{1.5}$). Dispersion would indicate that only a few billionaire salaries would exist, and you can see that in the extrapolated curve. Other than that, the difficulty remains in absolutely aligning and correlating time effort with an average level of salary.

Discussion. Either approach explains a way entropy can drive income distributions and likely some part of wealth disparity. However, the latter mechanism does not work as well in demonstrating the strong inflection point where income fluctuates strongly from year-to-year, indicating a strong influence due to the financial markets. The compound growth also seems to exhibit better understandability and we can also use that to accommodate the fact that that business opportunities can also lead to compound growth, thus breaking through asymptotic learning curve barriers. By making the transition continuous, you begin to understand that people's behaviors in terms of money may show that a continuous morph from low to high income.

Income differs from productivity in that much of the income can come from mechanisms totally unrelated to actual effort expended, thus defeating the law of diminishing returns caused by learning curve constraints (i.e. compound interest *can* violate the laws of physics to some degree).

> *The fact that income distribution consists of two distinct parts reveals the two-class structure of the American society.*
>
> *About 3% of the population belonged to the upper class, and 97% belonged to the lower class.*
>
> *... nations or groups of nations may have quite different Gini coefficients that persist over time due to specific historical, political, or social circumstances. The Nordic economies, with their famously redistributive welfare states, have G in the mid-20%, while many of the Latin American countries have G over 50%, reflecting entrenched social patterns inherited from the colonial era.* [Ref 284]

Money and the effects of compounded interest acts to disperse the wealth of individuals by artificially speeding up the evolution or adaptation of our species. Look at the case of Mexico; I recall noting that a few of the wealthiest people on the planet live there, which largely comes from oil income and whatever compounded interest their investments have gained. On the other hand, the Nordic countries tax the wealthy before they can tuck away their investments and the dispersion and disparity in incomes drops way down.

Which adaptation route should we follow? Using the proxy of compounded growth income does allow us to approach the natural relative abundance distribution of other biological species. Yet this comes at the expense of a rather artificial shortcut. The income distribution curve appears very precariously positioned and we will see large swings anytime we enter recessionary periods. The wealthy at the top appear to have enormous sensitivities to marginal rates of savings and the income growth at the bottom. The worker bees seem to show more stability. Propping up the wealthy through whatever debt-financing schemes one can find will likely keep the fat-tail on income from collapsing. In reality, a fat-tail can only exist in an infinite wealth (i.e. resource) world. Yet, and alas, money does not obey many of the laws of physics and any fundamental understanding remains illusory in the context of constantly adapting capitalistic economic policies.

Do we need more data on income and wealth to really nail our understanding?

> *Despite imperfections (people may have accounts in different banks or not keep all their money in banks), the distribution of balances on bank accounts would give valuable information about the distribution of money. The data for a large enough bank would be representative of the distribution in the whole economy. Unfortunately, it has not been possible to obtain such data thus far, even though it would be completely anonymous and not compromise privacy of bank clients.*[Ref 284]

If we want to control our destiny, we have to understand the statistical dynamics. As long as people remain infatuated with deterministic outcomes and neglect to include entropic dispersion, the econophycists will have the field to themselves.

The mainstream economists desperately trying to avoid sinking the costs of their intellectual investments apparently resist the use of these methods. Growth can't keep going unencumbered and we need to start paying attention to what the models can tell us.

Population Center Dispersion

What Paul Krugman says in the margin serves as a useful piece of advice. I would agree that the mechanism of mathematics provides the necessary *insight* to our understanding. Unless you work the math, the insight may never come, simply because it captures and retains the bookkeeping and juggling of ideas for our overtaxed brains. Often times you would never know if certain effects canceled out, compounded, or came out negligible unless you took the time to formalize the arguments.

"In the economic geography stuff, for example, I started with some vague ideas; it wasn't until I'd managed to write down full models that the ideas came clear. After the math I was able to express most of those ideas in plain English, but it really took the math to get there, and you still can't quite get it all without the equations."
— P. Krugman

Take a look at one of Krugman's books on economics and geography. One area of interest he had concerns the size and spatial distribution of city and town populations, and specifically the organization of edge cities. Many had observed a general behavior of very few large populations and progressively more smaller cities. This has the moniker of Zipf's Law — a heuristic that seems to match the data from a rank histogram of the USA:

$$P(Size) = \frac{1}{Size^n} \qquad \text{where} \qquad n = 1 \qquad \textbf{(EQ 31-24)}$$

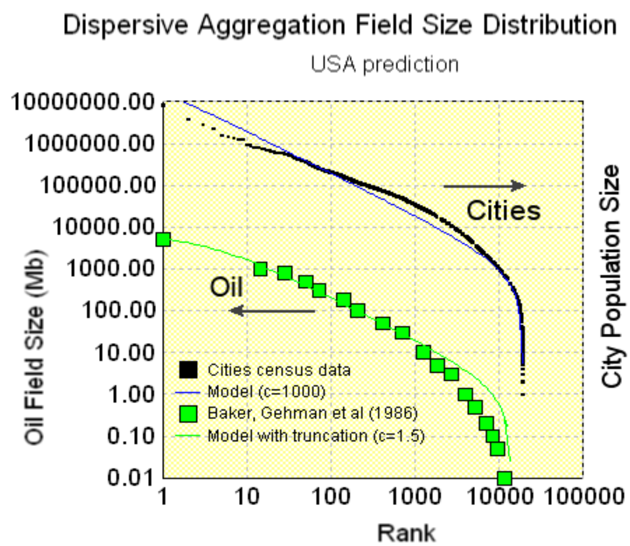
Granted some people might consider this completely intuitive from the start and require no insight beyond this point to make more elaborate arguments. Or you can take this empirical result and try to understand why it occurs by diving further into the math ([Ref 291][Ref 215]). Krugman had invoked a preferential attachment argument due to Herbert Simon to explain the historical city population growth [Ref 290].

An alternate understanding derives from a straightforward application of dispersion to growth rates of the measure of interest. Consider the abstract equivalency of large cities to super-giant oil reservoirs. For the case of oil reservoirs, a maximum entropy dispersion of material drift velocity during its formation can generate such a dispersion (see "The Facts in the Ground. Where do we find oil reservoirs?"). The solution to this leads to a probability distribution function

$$P(\text{Size}) = \frac{1}{\left(1 + \frac{c}{\text{Size}}\right)} \qquad \textbf{(EQ 31-25)}$$

This looks something like the Zipf-Mandelbrot variation which includes a constant limiting term to prevent a singularity at the origin (in ecological terms we showed this as also similar to a. See “The first case: Relative Abundance Distribution” on page 430.).

FIGURE 31-18. Power-law histogram of oil field size distribution laid alongside USA city population size histogram. (oil from the Baker and Gehman data))



We can illustrate the similarity between the distribution of city population sizes (from USA census data) with the distribution of discovered oil reservoir sizes in the USA. See to the right for a ranked histogram of the two sets of measures. I fit a dispersive aggregation model for each set of data with the single parameter **c** describing the characteristic aggregation size. Most interesting in the aligned data set is the multilevel correspondence between the number of cities and the number of reservoirs at varying densities. First note that the number of very large cities and very large reservoirs is comparable, about 9 cities (not urban areas) over a million people and about 13 or 14 oil reservoirs over a billion barrels in recoverable reserves. From there, the ratio holds relatively steady, for every city of a specific size numbered in thousands, you find an oil reservoir with that same size in million barrels that you can “attach” to that city. So you find that around 200 cities have 100K or more in population and about 200 reservoirs have 100 MB or more in oil. This works for awhile until the number of small cities starts to overtake the number of small reservoirs as counted by Baker [Ref 20].

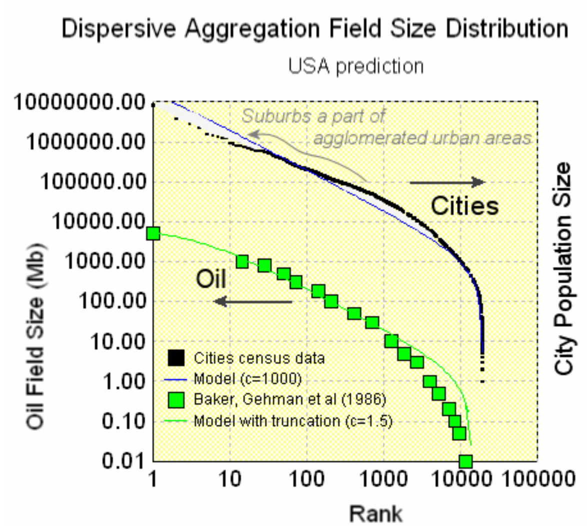
Applying a sanity check to the $1000\times$ ratio, means that each "reigning" citizen of a large city will use about 1000 barrels (100 MB/100K) of natively-supplied oil over the course of time. I use the term "reigning" to indicate that each city's population has followed a slowly growing equilibrium that will asymptotically reach some carrying capacity with births/immigration balanced by deaths/emigration, whereas oil has a finite limit not at all related to a carrying capacity. So reigning essentially means a steady-state citizen, and that equivalent person has used 1000 barrels or 42,000 gallons of American-sourced unrefined oil over the past 150 years.

Laharrere has also previously worked the distribution of population size distributions which used urban aggregates instead of city population sizes [Ref 51]. The model fit departs from the data within certain regions of the profile. It really comes down to distinguishing between major metropolitan areas and large cities. Ranking by major metropolitan and you find that we have 50 **major metropolitan areas** greater than 1 million, but only 10 **cities** greater than 1 million. The greater New York City metro region has between 21 to 22 million people and Los Angeles has between 12 and 13 million, which puts the first two ranked points above the chart area.

So you can imagine filling adjusting the rest of the curve by borrowing population centers of less than 100,000, and adding to the cities to model the major metropolitan regions. These essentially constitute the "suburbs" of any large city, with most suburbs in the USA falling between 10,000 and 100,000 in population size. Correspondingly, to do this correctly, it would require someone to categorize a few thousand additional cities to find out if they belong to the previously categorized major metropolitan areas.

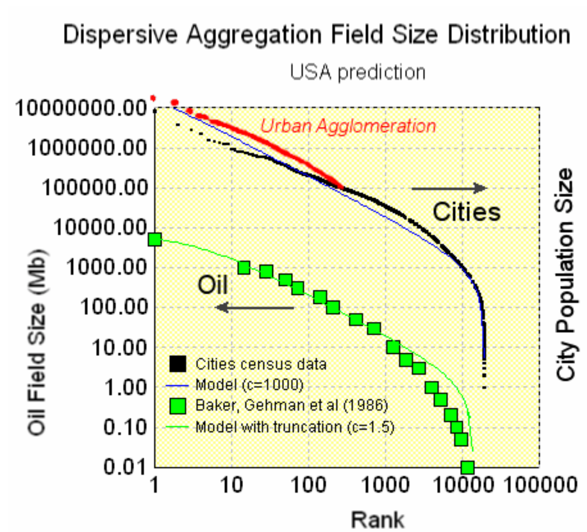
On a log-log histogram, and with this shape of entroplet curve, the areas between the curve within the fat-tail have approximately equal population, so that you can imagine the city data shifting from one region to the other, as in the white patches of the curve below.

FIGURE 31-19.
View of how suburbs get incorporated into urban centers, which may act as a more natural dispersive aggregation measure.



Laherrere rationalized this approach in his paper and you can see the results in the figure below, where the red set of data indicates the effects of urban agglomeration. Unfortunately, he did not continue that below areas of 100,000 in population.

FIGURE 31-20.
Urban center agglomeration may agree with entropic dispersion better than counting cities alone.



Urban agglomeration likely follows dispersion patterns better than a city population distribution does because cities form political boundaries which have nothing to do with the actual physical process of growth; preferential attachment would occur to the region and not the city.

So for now, I can't do much more than show approximately how the shift occurs. It would take quite a bit of data rearrangement to correctly classify as a true isolated city as opposed to an agglomerated metro region.

Interestingly, the same process likely occurs for oil reservoir accounting. Whether a large reservoir can include additional adjacently situated "satellite" reservoirs has to do more with oil companies accounting practices more than anything else. And I have no control over that so we do the best we can with the available data.

I find this exercise useful, if for nothing else, that it can give people a feel for how many reservoirs that we have remaining. Think of how many sizable cities we have, and that amounts to how many equivalently-sized reservoirs that we have to essentially feed those cities. Eventually, all these reservoirs will turn into ghost towns as they deplete and become shut-in while the cities will remain. Most people do not have a feel for what 300 million people means yet they can start to comprehend thousands of reservoirs and cities, and the significance of those numbers. It also helps to put the "Drill, Baby, Drill" mantra into context: think in terms of how many reservoirs we would need to discover to replace the ghost reservoirs that will crop up — essentially one for every city if we want to depend on a captive native resource.[5]

Why we can't finish stuff: Bottlenecks and Pareto

Why does the law-making process take much longer to finish than expected? Why does a project schedule consistently overrun? (*Why does it take so long to write a book?*) These questions pertain to microeconomic business decisions, where we can apply some of the same ideas that we used in other disciplines.

In any individual case, you can often pinpoint the specific reason for an unwanted delay. But you will find (not surprisingly) that a single reason won't universally explain all delays across a sample population. And we don't seem to learn too quickly. For example, if we did learn from what we did wrong in one case, ideally we should no longer do such a poor job the next time we tried to get something done. Yet, inevitably we will face a different set of bottlenecks in the process next time we try. I will demonstrate that these invariably derive from a basic uncertainty

5. As an influential economist, Krugman's awareness [Ref 316] of the real problems of constrained resources that this country (and the world) faces remains important. Economists have the habit of theoretically deferring to the substitution of one resource for another as soon as we reach a constraint.

in our estimates of the rate it takes to do some task. The explanation that follows belongs under the category of fat-tail and gray swan statistics.

As far as I can tell, no one has treated the analysis quite in this fashion, even though it comes from some very basic concepts. This has implications for how we look at bottom-line human productivity and how effectively we can manage uncertainty.

The classical (read *old-fashioned*) development cycle follows a sequential process. In the bureaucratic sense it starts from a customer's specification of a desired product. Thereafter, the basic stages include requirements decomposition, preliminary design, detailed design, development, integration, and then test. Iterations and spirals can exist on individual cycles, but the general staging remains — the completion of each stage creates a natural succession to the next stage, thus leading to an overall sequential process. For now, I won't suggest either that this works well or that it falls flat. This just describes the way things typically get done under some managed regiment and the sequence shows up across business and politics [Ref 281].

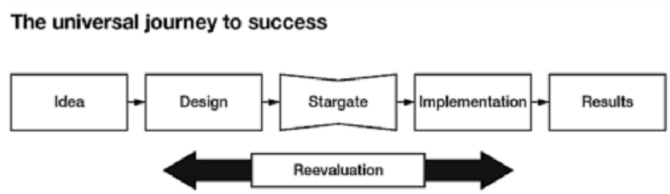


FIGURE 31-21. A project roadmap consists of five stages. The stage called Stargate refers to the transition between virtual and real. [Ref 281].

If done methodically, one can't really find too much to criticize about this process. It fosters a thorough approach to information gathering and careful review of the system design as it gains momentum. Given that the cycle depends on planning up front, someone needs to critically estimate each stage's duration and lay these into the project's schedule.

Only then can the project's team leader generate a bottom-line estimate for the final product delivery. Unfortunately, for any project of some size or complexity, the timed development cycle routinely overshoots the original estimate by a significant amount. Everyone has ideas of why this happens, but we lack a quantitative spin on the analysis.

The premise of this analysis is to put the cycle into a probabilistic perspective. We thus interpret each stage in the journey in stochastic terms and see exactly how it evolves. We don't have to know exactly the reasons for delays, just that we have uncertainty in the range of the delays. Economists often touch on this subject when they consider the ideas of exponential and hyperbolic discounting.

Uncertainty in Rates. We first assume that we don't have extensive amounts of knowledge about how long a specific stage will take to complete. At the very least, we can estimate an average time for each stage's length. If only this average gets considered, then we can estimate the aggregated duration as the sum of the individual stages. That becomes the bottom-line number that management and any customers will have a deep interest in, but it does not tell the entire story. In fact, we need to instead consider a range around the average, and more importantly, we have to pick the right measure to take as the average. As the initial premise, let's consider building the analysis around these two points:

1. We have limited knowledge of the spread around the average.
2. Use something other than *time* as the correct metric to evaluate.

I suggest we should use *rate* as a progress measure instead of using *time* as the variant for each stage. The variant takes the shape of a probability density function.

But what happens if we lack an estimate for a realistic range in rates? Development projects do not share the same relative predictability of a foot-race, and so we have to deal with the added uncertainty of project completion. We do this by deriving from the principle of maximum entropy (MaxEnt). This postulates the idea that if we only have knowledge about some constraint (say an average value) then the most likely distribution of values within the constraint turns into the distribution that maximizes the entropy. This amounts to the same thing as maximizing uncertainty and it works out as a completely non-speculative procedure in that we can introduce as preconditions only the information that we know. Fortunately, the maximum entropy for a distribution parametrized solely by a single mean value is well known; we used this extensively throughout this volume.

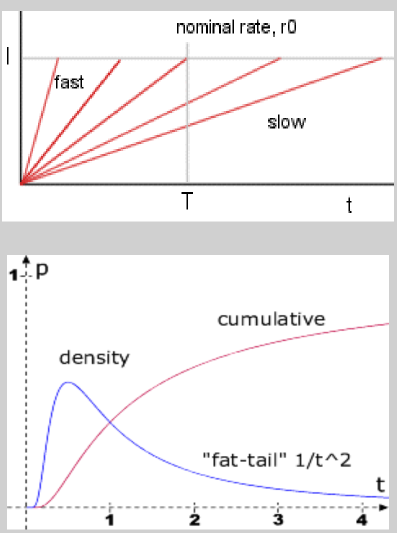


FIGURE 31-22.
Fat-Tail as a cumulative (below) draws from asymmetry the inverted variate (above).

To derive a MaxEnt PDF, we assume that over the course of time, we have an idea of an average rate, r_0, but we have no idea of how much it will vary around that value. So we use MaxEnt to provide us an unbiased distribution around r_0, matching our uncertainty in the actual numbers.

$$p(r) = \left(\frac{1}{r_0}\right) \cdot e^{-r/r_0} \qquad \textbf{(EQ 31-26)}$$

This weights the rates toward lower values and r_0 essentially indicates our expectation of a mean rate of completion.

Over an ensemble of r values, the probability of meeting a schedule, specified by an amount of work W needed to get done in time t becomes:

$$P(t) = e^{-W/(t \cdot r_0)} \qquad \textbf{(EQ 31-27)}$$

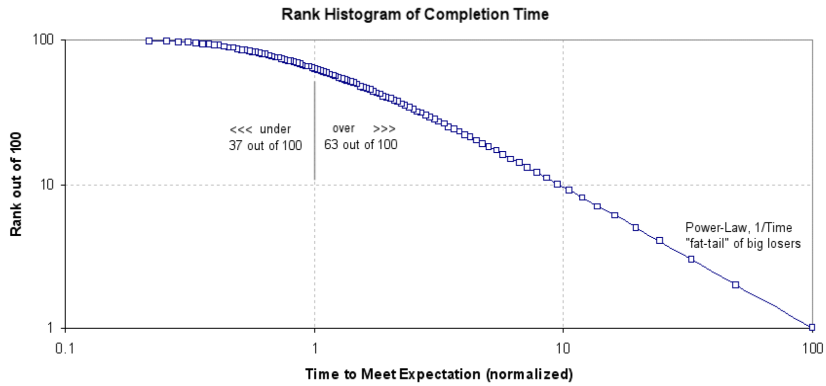


FIGURE 31-23. Indication of how often long completion times will occur. as a histogram

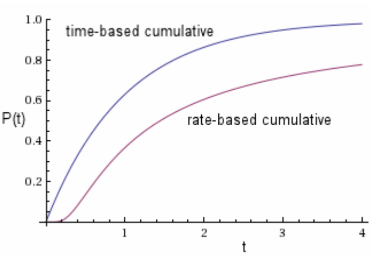


FIGURE 31-24. Comparison of exponential time-based and fat-tail rate-based

With just a single stage one can see how a rate-based may extend the period to completion. In the figure to the left, a time-based PDF treats short durations as more common while the rate-based model treats slow rates as more common. Given that the rate goes as the reciprocal of time, this leads to slower uptake.

If we string several of these PDF's together to emulate a real staged process, then we can understand how the rate-based spread can cause the estimates to diverge from the expected value. In the following graphs, we chain five stochastic stages together with each stage parameterized by the average value time units. The time-based function is simply a Gamma distribution of order 5. This has a mean of 5*10 and a mode (peak value) of (5-1)*10. Thus, the expected value corresponds closely to the sum of the individual expected stage times. In contrast, the staging of the *rate-based* distributions does not sharpen much (if any), and the majority of the completed efforts extend well beyond the expected values. This turns the highly predictable critical path into a problematically bottlenecked process.

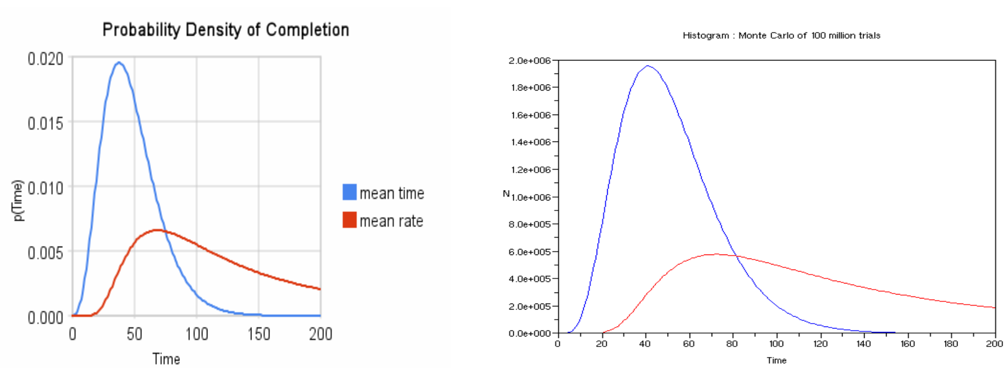


FIGURE 31-25. Convolution of five PDF's assuming a mean time and mean rate

The time integration of the PDF's gives the cumulative distribution function (CDF); this becomes equivalent to the completion probability for the project depending on the scheduled completion date. You can see that the time-based estimate has a more narrow envelope and reaches a 60% success rate for meeting the original scheduled goal of 50 time units. On the other hand, the rate-based model has a comparatively very poor completion probability, achieving only a 9% success rate for the original goal of 50 time units. By the same token, it will take 150 time units to have the same 60% confidence level that we had for the time-based model, about 3 times as long as what we desired.

The reason for the divergence has to do with the fat-tail power laws in the PDF of the rate-based curves. Our original single stage cumulative success probability clearly diverges as we add more stages, and it will just get worse as we add more. This leads to a *stable* distribution with the $1/t^2$ fat-tails.[6]

Implications for Completion Times. This gives us an explanation of why scheduled projects faced with uncertainty seldom meet their deadlines. Now, can we do much about it?

We have few options to choose from. We can either (1) get our act together and remove all uncertainties and perhaps obtain funding for a zipper manufacturing plant or some other project with more certainty, or (2) we can try to do tasks in parallel.

For case 1, we do our best to avoid piling on extra stages in the sequential process unless we can characterize each stage to the utmost degree. The successful chip companies in Silicon Valley, with their emphasis on formal design and empirical material science processes, have the characterization nailed. They can schedule each stage with certainty and retain confidence that the resultant outcome stays within their confidence limits.

For case 2, if you can find tasks that don't depend on each other, then the project leader simply has to split apart the work and allocate to different teams. Ideally they work in parallel and the longest task time determines the completion time. This is often easier said than done, as many projects require serial tasks.

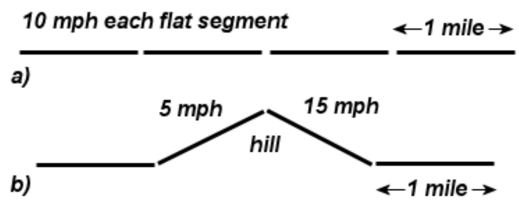
The blame does not lay completely on the variances in productivity, as the peculiar properties of rates also play a factor. One rate example that I personally relate to contrasts riding a bike in flat versus hilly country. If I set a goal of traveling

6. The Cauchy distribution is also $1/t^2$ and it occurs from multiple self- convolutions.

between points **A** and **B** in the shortest amount of time, I know from experience how the rates affect my progress.

You would think that the fast pace in going down hills will counter-balance the slower pace in going up hills and you might think you could keep up the same rate as on flat terrain. Not even close, as the slow rates going up-hill absolutely slows you down in the long run. That results from the mathematical properties of working with rates. For the figure below, I set up an example of a x=4 mile course consisting of four 1-mile segments. On flat ground (a) I can cover the entire course in T=24 minutes if I maintain a constant speed of r=10 mph ($T = x/r = 4/10*60$).

FIGURE 31-26. Construction of a velocity profile according to time segments. Hilly routes always result in the longest travel time



For the hilly course (b), one segment becomes steep enough that the constant rate drops to 5 mph. Work out the example, and you will find that the time it takes to cover the course will exceed 24 minutes for any finite value of speed going down the backside of the hill. For a 15 mph downhill, the extra time amounts to 4 minutes. Only an infinite downhill speed will match the flat course in completion time. This might seem non-intuitive until you do the math and you realize how much slow rates can slow your overall time down. And that jives exactly with the agonizing learned behavior that comes with the physical experience. The mismatch between the obvious physical intuition we get from riding the bike course and the lack of intuition we get by looking at the abstract problem formulation extends to other domains.

But then you look at a more rigidly designed transportation system such as bus schedules. Even though a bus line consists of many segments, most buses routinely arrive on schedule. The piling on of stages actually improves the statistics, just as the Central Limit Theorem would predict. The scheduling turns out highly predictable because the schedulers understand the routine delays, such as traffic lights, and have the characteristics nailed.

On the other hand, software development efforts and other complex designs do not have the economics nailed. At best, we can guess at programmer productivity (a rate-based metric, i.e. lines of code/day) with a high degree of uncertainty, and we wonder why we don't meet schedules. For software, we can use some of the same concurrent development tricks and skip around the class hierarchy if stuck. But pesky debugging can really kill the progress as it effectively slows down a programmer's productivity.

The legislative process also has little by way of alternatives. Since most laws follow a sequential process, they become very prone to delays. Consider just the fact that no one can ascertain the potential for filibuster or the page count of the proposed bill itself. Actually reading the contents of a bill could add so much uncertainty to the stage that the estimate for completion never matches the actual time. No wonder that no legislator actually reads the bills that quickly get pushed through the system. We get marginal laws fool of loopholes as a result of this. Only a total reworking of the process into more concurrent activities will accelerate this process.

And then the Pareto Principle comes in. If we look again at the first figure, you can see another implication of the fat over-run tail. Many business people have heard of the 80/20 rule, aka the Pareto Principle, where 80% of the time gets spent on 20% of the overall scheduled effort. In the figure below, I have placed a few duration bars to show how the 80/20 rule manifests itself in rate-driven scheduling.

Because this curve shows probability, we need to consider the 80/20 law in probabilistic terms. For a single stage, 80% of the effort routinely completes in the predicted time, but the last 20% of the effort, depending on how you set the exit criteria, can easily consume over 80% of the time. Although this describes only a single stage of development and most people would ascribe the 80/20 rule to variations within the stage, a general class equivalence holds.

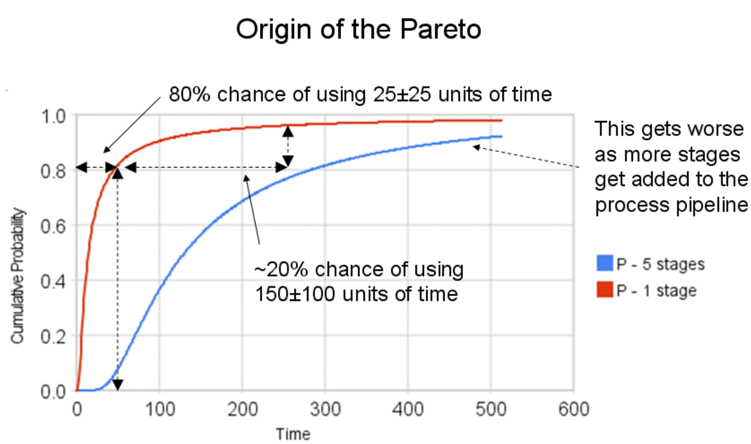


FIGURE 31-27. Normalizing time ratios, we on average spend less than 20% of our time on 80% of the phase effort, and at least 80% of our time on the rest of the effort. This is the famous Pareto Principle or the 80-20 rule known to management.

In terms of a development effort, the long poles in the 80/20 tent relate to cases of prolonged debugging efforts, and the shorter durations to where a steady development pace occurs.

These rules essentially show the asymmetry in effort versus time and quantitatively depends on how far you extend the tail. A variation of the Pareto Principle describes the 90-9-1 rule.

- 100 time units to get 90% done
- 1000 time units to get the next 9% done
- 10000 time units to get the “final” 1% done

In strict probability terms, nothing ever quite finishes and business decisions will determine the exit criteria. This truly only happens with fat-tail statistics and again it only gets worse as we add stages. The possibility exists that the stochastic uncertainty in these schedule estimates don’t turn out as bad as I suggest. If only we can improve our production process to eliminate potentially slow productivity paths through the system, this analysis will become moot. That may indeed occur, but our model does describe the real development world as we currently practice it quite well. Empirically, the construction of complex projects often take much longer than originally planned and projects do get canceled because of this delay. Bills don't get turned into laws, and the next great idea never gets completed.

The Bottom Line. The lengthy delays that this model predicts arise entirely from applying an uncertainty to our estimates. In other words, without further information about the actual productivity rates we can ultimately obtain, an average rate becomes our best estimate. A rate derived application of the maximum entropy principle thus helps guide our intuition, and to best solve the problem, we need to characterize and understand the entropic nature of the fundamental process. For now, we can only harness the beast of entropy, we cannot tame it. Taleb said this in his book *The Black Swan*:

> Let's say a project is expected to terminate in 79 days, the same expectation in days as the newborn female has in years. On the 79th day, if the project is not finished, it will be expected to take another 25 days to complete. But on the 90th day, if the project is still not completed, it should have about 58 days to go. On the 100th, it should have 89 days to go. On the 119th, it should have an extra 149 days. On day 600, if the project is not done, you will be expected to need an extra 1,590 days. *As you see, the longer you wait, the longer you will be expected to wait.*

In the context of the staged process described, I can definitely see this happening. As each stage misses its cut-off date, the delays keep building up. The way Bayes rule works, the earlier probabilities get discounted as you reach a new milestone, and the weight of the future fat-tail data factors more prominently in the new estimate.

This same dispersion math figures into reserve growth and oil creaming curves covered in Volume 1. So the rule of diminishing returns on long-term payouts plays into resource depletion as well. Once a reservoir reaches a certain level of depletion and the creaming curve starts flattening out, the estimated time to reach that same level of relative depletion will keep growing. The operator has no choice and makes a decision to shut-in the reservoir at that point. Again, since project scheduling and oil reservoir estimates possess the same level of uncertainties in early projections, it makes sense that no one really understands exactly how long these operations will take to play out. Thus, you see businesses take a “wait and see” attitude on when to shut down. The best economic decision often comes down to making a decision and declaring the project as “complete”.

The key to making progress in a post-oil world is to treat the transition process seriously and transform it from a bureaucratic process to war-time economics. Remember that during the major wars, projects did complete, as our future depended on it.

Insurance Payouts

A conceptual actuarial algorithm would start with a probabilistic model of the insurable incidents and then try to balance the policy owner's potential payouts against the reserves built up from the incoming premiums. As an objective, an insurance company wants to always keep its head above water, by keeping the balance positive:

$$\text{Balance} = \text{Reserve} - \text{Payouts} \tag{EQ 31-28}$$

For the earthquake model (See “Earthquakes” on page 583.), a simple expected pay-out scheme would multiply the strength of the earthquake (S) against the probability of it occurring $p(S)$. In other words, the larger the earthquake, the more the damages and the more the payout.

$$\frac{\mathrm{d}}{\mathrm{d}t}\text{Payout} = \text{UnitPayout} \times \text{RateOfEarthquakes} \times \int_0^{Size} p(S) \cdot S ds \tag{EQ 31-29}$$

The entroplet for the earthquake:

$$p(S) = \frac{c}{(c+S)^2} \tag{EQ 31-30}$$

leads to an indefinite integral that will diverge as a logarithm if *Size* extends to infinity:

$$\frac{d}{dt}\text{Payout} = c \times (\ln(c + Size) + c/(c + Size)) \qquad \textbf{(EQ 31-31)}$$

An infinite payout won't happen due to physical constraints but it does demonstrate precisely why the fat-tail of low-probability events have such an impact on an actuarial algorithm. They essentially drive the time-averaged payout to a surprisingly large number over a policy-owner's lifetime. (In contrast, a thin-tail probability distribution such as a damped exponential will pay-out only on the average earthquake size and not the maximum)

Should the insurer's balance ever go negative, a reinsurance company picks up the rest of the payout. The whole artifice sits precariously close to a Ponzi scheme, and the insurers optimally hope to push the payout as far in the future in possible, praying that the big one won't happen. In California, earthquake insurance does not come automatically with a homeowner's policy. This also explains why insurance companies typically don't offer flood insurance. The 100-year floods occur too often. In other words, the “fat-tail” probability distributions for floods and insurance generate too high an occurrence for supposedly rare events and the companies would end up losing if they offered only affordable premiums to make up their monetary reserve. The alternative of higher-priced premiums would only attract a fraction of the population.

I don't know if I can yet model all the extraneous game theoretic aspects, but if you had an insurance policy against earthquakes, the insurance actuary would consider this model quite useful. They couldn't tell you when the earthquake would happen, but they could reasonably predict the size in terms of a probability (essentially related the slope of the above curve). The problem comes in when you consider the risk of the insurance company (or the reinsurance company) not covering the spread in the face of a calamity. It happened to AIG and the financial collapse of 2008 forced the government to bail them out.

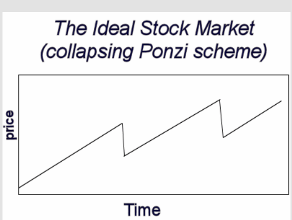
This essentially explains why fat-tail distributions can wreak havoc on our predictions. If we know that a given data set follows a thin distribution such as a Normal or Exponential, a large outlier will not affect the results. But for a fat-tail distribution, when a “gray swan” outlier occurs, it will act to sway the expected value considerably, especially if we did not previously account for its possibility. This happens because if we want to keep the mean to a finite value, we have to place limits on our integration range. Yet once we find a data point outside this range, we have to update our average with this knowledge, and this will push the average up. This does not happen with the thin-tail functions because any new data always stays within range and it will only update the mean if you treat it as a Bayesian update of the entire data set. So the trade-off lies between keeping a large enough range to accommodate gray swans versus keeping the range small so as not spook people with large premiums.

Again, this should remind us that with catastrophic accidents we never want to see the fat-tails and gray swans. Yet, and in stark contrast, if we have a goal of finding more oil, a gray swan becomes a *desired* outcome. Finding another super-giant that will force us to update our fat-tail statistics keeps the cornucopians fueled with optimism. The finite hope does exist, and that may forever prevent us from facing reality. This merging of resource constraints with economics keeps econophysics, ecological economics, environmental economics, or whatever you want to call it, an interesting research area to keep track of.

The Market as a Dependent Variable

I find it tempting to try to understand other aspects of the FIRE[7] economy using econophysics, in particular in how it relates to volatile investments, speculation, financial derivatives, and real estate. Certainly these have elements of uncertainty in them and they would benefit from similar econophysics analyses. Yet, going in this direction would also push the envelope of applicability for this volume.

Looking at the stock market during volatile periods, I have little doubt, however naive, that the professional profiteers earn their keep by keying on sudden drops in valuation. I usually hear it referred to as "profit taking". The amateur investors chug along the upslope and earn money gradually over the long term, but never respond quickly enough to the "market correction dips" and so predictably and routinely contribute to the transfer of wealth from the *hoi polloi* to the *oilogopoly*. The changes do not follow a normal Gaussian distribution but instead a fat-tail Cauchy distribution which can show wild swings.



"*Markets tend to drift upward or cascade down. You get slow rises and dramatic falls.*"

— Emanuel Derman
[Ref 159]

Since the investor class has virtually invented this collapsing Ponzi scheme to making money (perhaps more properly *extracting money*), it would not surprise me if resource consumption, when left to purely market forces, follows this same profile. Unfortunately, or fortunately, depending on you point of view, the collapse can only happen **once**. Oil, unlike money, does not get recycled.

Figure 17-8 on page 343 shows the collapse in an oil depletion context. The "one-shot" drop essentially demonstrates that given powerful enough extractive techniques, big oil may just create the biggest, longest Ponzi scheme ever encountered.

7. FIRE=Finance, Insurance, and Real Estate

With but one twist. The rebound can’t depend on the sucker class getting lured back in, because oil molecules don’t have gullibility genes. It will end in The Overshoot Point and we will, for the long term, have to get use to different levels of productivity and sustainability. The uncertainties in the system have everything to do with estimating how much cheap oil we can continue to find and process. Until we know this with more certainty, the wild ride will continue.[8]

At the same time, the analytic battle will continue on several fronts. We have an economic theory that rides a roller coaster as it tries to follow the market. We have the quants on Wall Street who have caused many people to distrust any math. We have chaos believers who immediately put up red flags warning that dangers lurk in any analysis. We have other mathematicians that blow smoke by creating impressive formalisms that show purity but lack any connection to practicality. And finally we have a somewhat MEGO[9] public that can't get interested.

I stand by the side of using practical probability and statistics and don't get cowed by the nay-sayers that say we can’t and will never understand any of this.

8. The oil companies will eventually run out of easily accessible petrol, but by that time, they will own everything in site from earlier bubble bursting exercises, the cashout from Wall Street earnings into hard currencies providing all the leverage necessary.
9. MEGO = my eyes glaze over

CHAPTER 32

Epilogue: A Historical Perspective of Peak Oil

"The ominous logic of the Hubbert curve."
"Nobody fools the drill."

— George Pazik

The physicist David L. Goodstein (who has also written about peak oil) said that "*Physics, I think, should never be taught from a historical point of view — the result can only be confusion or bad history — but neither should we ignore history*" [Ref 173]. Given that individuals have always raised dire fears about looming shortages throughout history, we really needn't rehash these accounts as it doesn't add much to the technical discussion and largely rested primarily on empirical guess work anyways. At a minimum we could rewind to the date that oil depletion started to become reported and only study the arguments that weny beyond mere speculation.

From the textbook "*Environmental Geology*" by P.T.Flawn published in 1970 [Ref 175]:

> About 3 billion barrels of crude petroleum and 2.5 trillion cubic feet of natural gas, helium, and carbon dioxide are produced from U.S. wells each year. The domestic annual ground water withdrawal is on the order of 6 million acre feet. All of these withdrawals can be expected to increase, except perhaps crude oil. Although the U.S. retains the capacity to increase production of crude oil from known deposits, and analysis of the reserve situation suggests that this country is close to the inflection point on the total resource curve and that production of domestic crude oil will decline if present conservation practices are continued (Hubbert, 1962, 1966, 1967). New discoveries in Alaska and offshore, together with improved secondary recovery practices, make it difficult to predict with exactitude just when the decline will begin. However, the decline in production of domestic crude oil will

> force development of alternative sources of liquid fuel, namely tar sands, shale oil, and coil, and thus increase large-scale open-pit mining.

That excerpt gives an academic perspective borne out by subsequent events, and it acknowledged the work of M.King Hubbert, who alone yielded some clout in industry circles. Apart from Hubbert, oil industry geologists and petroleum engineers have largely remained silent about the concept of peak oil.

Hamblin wrote a geology text called "The Earth's Dynamic Systems" in 1975 and he related definite concerns about our resources, stating [Ref 279]:

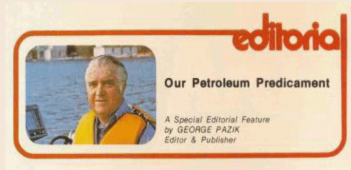
> *We basically know the extent of our mineral resources and the rates of our consumption. It is not difficult to project how long they will last.*

So even 35 years ago we could read technical material with strong claims that we could predict the future of our oil reserves. Yet as we look at our current predicament, a few more valid theories of economics and of resource depletion dynamics developed in the meantime certainly wouldn't have hurt.

The rest of this chapter reprints a classic "Peak Oil" editorial from 1976 further extolling the work of Hubbert, published in a popular (and my favorite) fishing magazine from the 1970's called *Fishing Facts*. Because of my tunnel-vision for outdoors activities at the time (Hubbert coincidently was a fishing fanatic as well), I can't say whether other typical newsstand magazines featured such pessimistic outlooks. In retrospect, I think it took some courage for the publisher to write something with an edge to it within a magazine that ostensibly catered to a rather conservative audience. This editorial first grabbed my eye as a youth and it has stayed in the back of my mind ever since. Pazik had written and continued to write strong editorials on conservation and environmental pollution after this editorial. However, this piece served as a bellwether piece, and educators such as Albert Bartlett[1] have referenced it long after its initial publication. Compared to what Pazik did over 30 years ago, today's media have only begun to catch up. Note that he does not actually call out "Peak Oil" by name but does talk about the concept of peaking.

1. http://www.albartlett.org/articles/art_forgotten_fundamentals_references.html

Our Petroleum Predicament



A Special Editorial Feature by GEORGE PAZIK Editor & Publisher, Fishing Facts, November 1976

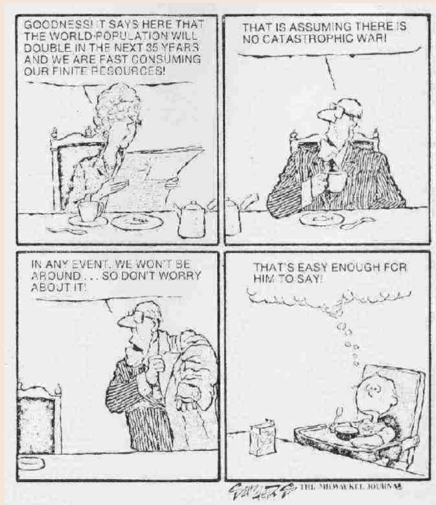
FORWARD. If you and I could get away on a fishing trip for a few days, we would probably talk about many things. Our talk would not just be confined to fishing; we would probably discuss many other things which concern us. Since there is no way I could go fishing with 200,000 people, I use these editorials each month as an opportunity to visit with you. I don't confine my editorials to fishing, either, but have always covered a number of subjects which concern us as fishermen and as fellow human beings.

This editorial is written in early September. It should reach you at just about the time of the third anniversary of the Arab Oil Embargo and, perhaps, just before the American presidential election. Although a recent poll shows that 55% of us don't believe there ever was an "Energy Crisis", all of us will have to admit that the Arab embargo changed the world.

No wonder we're confused. We have been confronted with an almost daily barrage of stories about the so-called "Energy Crisis". One day we're going to burn garbage for fuel, another day some "expert" proposes a space station to gather the sun's energy. Each day seems to bring yet another proposal, but one certain proposal never stops coming through: LET THE PRICE GO UP AND WE'LL GET ALL THE OIL AND NATURAL GAS WE NEED. That one must be on a broken record because we hear it over and over again. It lends credence to a popular belief that the Energy Crisis was artificially created to line the pockets of the petroleum industry and the oil-producing nations. I sense that all of us, Canadians and Americans alike, share an uneasy feeling that all is not well. We suspect that we are not being told the truth, the whole truth and nothing but the truth.

I have learned some startling facts about the so called "Energy Crisis" which will shock and dismay most people. I believe it is my obligation to pass these facts on to you for your own careful consideration. I have been as careful as I know how to be in assembling the facts, figures, judgements and opinions contained in the Special Editorial Feature that follows. It has been particularly difficult to get statistics which are all for the same time periods. In some cases they don't exist, therefore I've tried to be conservative in the way I use these statistics.

I am not a reporter. I do not claim to be impartial. I have no obligation to give equal space to all sides of every issue and to be "fair" to all sides. My only obligation is to be fair to you. I carefully identify my sources of information, I quote others accurately, and I clearly state which are my opinions. However, just by sifting through the millions of words which have been written *about the "Energy Crisis" and choosing which I will use, I have already expressed* my *opinions by making those choices. If there are mistakes, they are unintentional but they are* my *responsibility.*



Three years ago when the Arab Oil Embargo hit us, I knew I would want to write an editorial on it. I started to save hundreds of clippings from newspapers and magazines. I began attending lectures, seminars and meetings; some private, some public. I read numerous books and scientific papers and made voluminous notes. Always I felt that I needed a little more *solid information before I could write the kind of editorial the subject deserved. Two and* a *half years went by and I still felt that I didn't have the whole story. Suddenly things began to fall into place and I realized I was ready at last.*

You know, over thirty years ago I graduated from the University of Wisconsin with a *degree in Metallurgical Engineering* as my *major course of study and with Mining Engineering* as my *minor study. I never entered the field of engineering because the Army wanted* me *to be* a *gunslinger in World War II and I never went back to it. Little did I dream thirty years ago that* my *studies in physics, chemistry, mathematics, excavating, are processing, metal extraction, etc., would be* so *useful to* me *in* a *study of Energy for an editorial I would write for* a *magazine I headed up* as *Publisher and Editor. I would never attempt to work* as *an engineer today, but those courses of thirty years* ago *sure helped* me *understand the many complexities of our energy-producing systems.*

You will note that I chose the title, "Our Petroleum Predicament" rather than the term "Energy Crisis" we have all heard until we're sick of it. I don't think "Energy Crisis" is correct. I chose the word Petroleum because it includes both oil and natural gas, which supply 75% *of our energy, and these are the substances with which we have* a problem. *Our coal is not threatened, neither is water power. The word "Crisis" implies an immediate situation whose outcome will determine whether good or bad consequences will follow. "Crisis" is also used to describe the turning point of* a *disease when it becomes apparent whether the patient will live or die. Using the word "Crisis" implies that something came up suddenly which can and must be cured with* a *quick-fix remedy or all is lost.* You *can see why the term "Energy Crisis" does not correctly describe our present problems.*

"Our Petroleum Predicament", however, does exist, as *I will prove to you in this Special Editorial Feature. The Arabs didn't invent it or cause it. It's been coming on us ever since World War*

II. Nobody did it to us, we did it to ourselves. There is no quick fix, no one answer, just many possible answers, none of them quick, cheap or easy.

As *fishermen we have to be concerned. Plentiful supplies of petroleum are essential to our sport* as *we now know it.* As *human beings we must also be concerned. Finally,* as *citizens of the United States and Canada, we have to know what is happening to us.*

This is not a *message of impending doom but* a *promise of the great opportunity that awaits us if we start acting wisely, NOW.* It is a promise of a golden age; a future that is ours for only the effort to bring it about. *To get there, however, will require us to change the way we feel about* a *great many things in our lives. If* we *fail to make these changes,* a *cataclysm awaits us.*

Let me *try to tell you about it.*

FIRST, THE CONCLUSIONS. There are many people who are too busy to read lengthy articles. They'll glance at the first few paragraphs and then flip through to the end in order to read the conclusions. To make it easy for such readers, and maybe add an extra touch of interest for the rest, I've listed immediately below some of the-most important facts and conclusions presented in this Special Editorial Feature.

- **In the early stages of the 1973-74 Arab Oil Embargo, President Richard M. Nixon went on national television to tell the country about Project Independence, which was his answer to "the energy crisis":** "Let us set as our national goal, in the spirit of Apollo, with the determination of the Manhattan Project, that by the end of this decade we will have developed the potential to meet our energy needs without depending on any foreign energy sources. Let us pledge that by 1980, under Project Independence, we shall be able to meet America's energy needs from America's own energy resources." **Project Independence was a politically-inspired slogan at best, a dangerously misleading doctrine at worst. There is no way that this nation can develop the energy to meet our needs without depending on** any **foreign energy sources by 1980, by 1985, and probably not even within this century.**
- Project Independence was based on four basic premises: (1) Major expansions in nuclear energy. (2) The development of oil from oil shales. (3) Vastly increased production of coal. (4) Large, new oil and natural gas discoveries as a result of intensive exploratory drilling. None of these are happening now or are likely to happen by 1980, 1985, or by any other date in the near future.
- We are now using about 6 billion barrels of oil per year, yet at its peak in 1970, U.S. production of oil was only 3.245 billion barrels, it slipped to 3.210 billion in 1971 and to 3.108 billion in 1972. [2] It's continuing to go down every year. What makes us think we could DOUBLE our production of this diminishing resource so as to achieve 6 billion barrels a year and thus be "Independent"?

- Our frantic efforts to dig up all our oil and natural gas can best be described as a policy to "deplete America first".' How this adds to our national security escapes people of only average intelligence. Perhaps it involves some secret principle known only to a privileged few but which cannot be shared with the masses.
- The present U.S. national energy policy can best be described by the title of that pleasant old song: "Drifting and Dreaming".
- U.S. production of petroleum reached a peak in 1970 and has gone down every year since.[2] It is not likely to ever rise again to its former heights unless Alaskan oil can be brought in soon enough, in large enough amounts, in which case it could peak slightly again and then go down again to ultimate depletion. [8]
- By the most optimistic predictions, Alaska's Prudhoe Bay oil field will produce about 10 billion barrels of oil, but it will take a half century or more to get it all out.[2] That's not quite enough oil, totally, to fill U.S. present needs for two years.
- There have been no major new discoveries of oil in Alaska since the discovery of the Prudhoe Bay oil field in 1968, [8] a matter of great disappointment to many in the industry.
- Since Alaskan oil has been planned for shipment to the west coast, (where it isn't needed), half of it or more may go instead to Japan,3 which is exactly what some observers have been predicting for at least five years. [8]
- It is estimated that drilling for oil off the Atlantic Coast may produce 2 to 4 billion barrels; the Pacific Coast may produce 2 to 5 billion barrels4 Just as with all oil fields, it will take about a half century to get it all. (We are using about 6 billion barrels of oil per year at present.)
- In 1945, it required 51 new-field wildcat wells to make 1 profitable discovery of oil. By 1965, despite the use of the latest improved technology, it had dropped to 137 to 1.2 If we believe that this ratio will greatly improve in the future, we must assume that the oil companies drilled their worst prospects first.
- About 80% of the oil that will ever be produced from the lower 48 states was already discovered by 1971.[2]
- The United States is now importing a little over 40% of its oil. Another Arab Oil Embargo would make a shambles of our economy.
- North America never had more than 15% of the world's total petroleum (includes oil and natural gas). The Canadian share is not yet half gone, but the American share is more than half gone. [2]
- The United States now probably has less than 5% of the world's crude oil reserves.[2]
- America is using oil faster than any nation on earth. Guess who's going to run out first?
- Those who think the Arabs and the Russians don't know all this are the same people who believe in Santa Claus and the Easter Bunny.

- We can buy all the oil we want to buy from "our friends". All we have to do is to pay their price and meet their terms. You guess if the terms will get easier and the price will get lower as our own supplies get closer to running out?
- Petroleum has so many wonderful and varied uses in the petrochemical industry: rubber, plastics, synthetic fibers, pharmaceuticals, fertilizer, etc., that simply burning it up for its heat alone can be described as an atrocity, a chemical crime.[5]
- U.S. production of natural gas peaked in 1973 and has gone down every year since. Proved reserves have declined for the past 5 years. [2]
- About 75% of all the natural gas that will ever be produced in the lower 48 states was already discovered by 1975. [2]
- Natural gas is already in such short supply that it is not longer a question if some users will be cut off but only who will be first and second.
- By 1974 estimates, each 20c per 1,000 cubic feet raise in price of natural gas would give the industry approximately an additional 56 billion dollars on estimated proved reserves; plus an approximate additional 70 billion on the gas still likely to be discovered in the future. That's a total of 126 billion dollars.[2]
- By 1974 estimates, each $1 price raise on a barrel of petroleum liquids produced in the United States would give the industry approximately an additional 45 billion dollars for their estimated proved reserves; plus an additional sum of approximately 62 billion dollars on the recoverable petroleum liquids likely to be discovered in the future. That comes to an additional 107 billion dollars.[2] (126 for natural gas plus 107 for petroleum liquids equals 233 billion dollars for the simple little price raises used in this example.)
- Perhaps the petroleum industry is entitled to more money because of inflation. However, as to the claim that they will then "find all the new gas and oil we need," let's slow up and think. In this poker game they will be getting hundreds of billions of dollars more whether they find additional oil and gas or not. How about some hard evidence to show that all that "new" gas and oil really exists?[8]
- Japan is one of the world's most heavily industrialized nations. Why is it that 210 million Americans squander as much energy each year as 107 million Japanese consume totally?6
- From 1940 thru 1974, the gas mileage delivered by American automobiles has gone in only one direction: DOWN. When Detroit blames government-mandated emission controls and safety features (which only started in earnest in 1971) for the bad mileage of their gas-guzzling monsters, they are neglecting their sorry record of the preceding 31 years. We've had 35 years of waste, in all, because Detroit sold and we bought: "longer", "lower", "wider", "heavier", "faster" and "more powerful".
- Until May 1975, the highest and most erroneous estimates of our reserves of oil and natural gas for the preceding 14 years came from the official governmental agency entrusted with that job: The U.S. Geological Survey, of the Department of the Interior. By all their esti-

mates during that period we had ample oil and natural gas reserves to last us thru the end of this century and beyond. Why was this sorry situation allowed to continue for 14 years? Who was responsible? Are they still giving us their "expert" advice?

- Former Secretary of the Interior for eight years, Stewart Udall has written: "Having helped lull the American people into a dangerous overconfidence, I felt a moral duty to admit my own errors and to expose the wildly optimistic assumptions that had misled the country. It was clear to me that an enormous energy balloon of inflated promises and boundless optimism had long since lost touch with any mainland reality".[7]
- For the past 20 years a world-renowned geologist has been trying to tell us about our declining petroleum reserves. In 1956 he predicted that domestic crude oil production would peak between 1965-70. It happened in 1970. In 1961, he warned that domestic natural gas production would peak in about 1977. It happened in 1973. He worked for the U.S. Geological Survey from 1963 thru August 1976. Why was he ignored?
- Petroleum is a unique resource without equal. It is found in huge natural fields, not narrow seams covered by tons of earth. It is pumped not mined. It is concentrated chemical ~energy and a basis for a myriad products. There are no meaningful substitutes for petroleum in sight on this planet for the rest of this century. [7]
- Those who believe that some miracle of technology is going to magically produce a cheap, simple substitute for petroleum are probably the same people who still believe that storks bring babies.
- The only SURE way to avoid a catastrophic disruption of our lives is to change from the most wasteful users of energy on earth to one of the stingiest. We need to get off the exponential growth for growth's sake joyride we've been on since World War II. That joyride was based on cheap oil, something we'll never see again. This means changing our way of life, our national outlook as a people. Some people may do this voluntarily but the bulk of us want to make sure that everyone is making the same sacrifices. That means that energy conservation will have to be mandated by law. We could do it now, cheerfully, or do it later when there is no longer a choice because disaster is upon us. Either way we'll do it.
- We must go to the sun for our future energy. The sun's energy is virtually unlimited and good for at least a billion years. It is environmentally ideal and free for the taking. The technology exists today to harness the sun to produce electricity, either from heat or directly from sunlight by photo voltaic cells. The electricity can then be converted to chemical energy in the form of hydrogen from the hydrolysis of water and shipped by pipeline to wherever it is needed. Upon burning, hydrogen produces water. When we combine limitless clean energy from the sun with a new lifestyle which no longer insists that "more is better" and that "there is never enough", we will be in on the dawning of a new age upon the earth. [8]
- If any politician had guts enough to tell you all these things, would you vote for him?

ASIDE FROM THAT. There's a very, very sick joke that goes:

"Aside from 'that', Mrs. Lincoln, how did you enjoy the play at Ford's Theatre?"

A woman's husband, President of the United States, has just been shot by an assassin. The whole course of history has been altered. Yet someone is asking the lady how she enjoyed the play they were watching when her husband was shot! There is no "aside" from THAT! No one could brush aside an event of such tragic significance, or could they?

We do. We do it all the time. It's part of human nature. We try to shove aside unpleasant things or unpleasant facts with which we do not want to agree,

You know, in ancient times it was the custom to kill a messenger who brought bad news. Today we are not quite that barbaric.

We refuse to discuss the bad news and just ignore the messenger and everything he said. That's the "aside from" technique.

The other technique we use is to attack the messenger personally, try to destroy his character, question his sanity, or seek to find something in his personal life which will allow us to discredit him.

In no case do we deal with the bad news he brought. We prefer not to deal with bad news until il comes knocking at the door to deal with us. Will we ever change?

Well, aside from that...

THE LONELY PROPHET. The crowd gathered slowly in the convention hall for the first morning of the convention, as it does for nearly all conventions. It was a time for greeting old friends, backslapping and handshaking, and for some a time to clear heads still buzzing from partying the night before. San Antonio, Texas was playing host to about five hundred petroleum engineers in town to attend a three day meeting of the Production Division, Southern District of the American Petroleum Institute. Petroleum engineers are men who spend a lot of time in the field; hardy, rugged men in a he-man's industry.

There was nothing of special significance expected to come from this three day meeting which began on Wednesday, March 7, 1956. The first speaker of the morning was expected to give a broad-brush review of world energy resources. For many of those in attendance it was expected to be a re-hash of things they already knew. The United States was the world's greatest oil producer, the world's greatest oil consumer, and the U.S. petroleum industry was one that didn't even know the word "Can't". It was the greatest can-do industry of the world's greatest can-do country. If America was a world superpower, the American petroleum industry was its superindustry. Every new discovery was described as "vast", our petroleum reserves were always described as “boundless”, and the outlook was never anything less than “fabulous”. And so forth.

Petroleum geologists, engineers, and corporate officials all shared an intuitive judgement that went something like this: “We’ve been in the oil business in this country now for almost a hundred years. We’ve made a lot of money, had a lot of fun, and the future looks just great. It’s taken us almost a hundred years (97) to produce about 53 billion barrels of oil and we know that we have two or three times that much still to go. We won’t run out of oil during our lifetime or even during the lifetime of our children. Our grandchildren might have a problem, but with the new “Atoms For Peace” program announced by President Eisenhower last year, even our grandchildren should have no great problems with energy. Besides, we have at least a five hundred year supply of coal in this country.”

The meeting got underway quite close to schedule and the first speaker was introduced. His name was M. King Hubbert, a native of Texas who had taught at a number of universities and for the last thirteen years had worked as a geologist with Shell Oil of Houston. Hubbert was fifty-two at the time; not a big, imposing man in size or appearance and not the kind of forceful speaker that brings an audience to a fever pitch. He was always addressed as “Dr. Hubbert”, because he was the holder of several degrees and a true man of science. He was no armchair scientist, however, he had also won his spurs in the field. He began to speak in an evenly modulated voice that soon won his audience’s attention with the force of his words and logic rather than with the force of his lungs.

He was committing heresy, right up there on the platform in front of their eyes. Contrary to the unwritten but well-established rules of conduct for petroleum industry figures, he started to point out to his audience that American petroleum reserves were not as vast as they had always assumed them to be and that American oil production (for the lower 48 states and continental shelves) could be expected to reach its peak in another ten to fifteen years and then drop down each year on its way to depletion. He committed yet another unpardonable sin by reminding his audience that we were already importing oil equal to about 20% of our production...dirty words in the petroleum industry. Some members of the audience were visibly upset, but they listened in silence, unable to combat the irrefutable logic of the man on the platform, but disturbed with him for saying it. It was truth that no one wanted to hear, but truth they could not deny. He just wasn’t supposed to be saying such things!

His words also had serious financial implications for the industry. The date at which production peaks or culminates is the dread date at which an industry which has always had production increases of 5% or 10% a year will now begin to have production decreases of about the same amounts each year. It is the date at which creditors, suppliers, stockholders, etc. start to get wary. There is a big difference in running an industry whose production can be counted on to steadily increase or one which will steadily decrease. This is a date to be avoided at all costs and to be shoved as far into the future as possible.

The pre-printed version of Hubbert's paper distributed at the meeting made the following statements:

> *"According to the best currently available information, the production of petroleum and natural gas on a world scale will probably pass its climax within the order of a half a century, while for both the United States and for Texas, the peaks of production may be expected to occur within the next 10 or 15 years.*
>
> *"Assuming this prognosis is not seriously in error, it raises grave policy questions with regard to the future of the petroleum industry. It need not be emphasized that there is a vast difference between the running of an industry whose annual production can be counted on to increase on the average 5 to 10 percent per year and one whose output can be depended upon to decline at that rate. Yet, in terms of the production of natural gas and crude oil, this appears to be what the petroleum industry in the United States is facing."*

(When the paper was published, after Shell Oil Company censors had finished with it, the statement above was deleted and replaced with the following: *"the culmination for petroleum and natural gas in both the United States and Texas should occur within the next few decades.")*

At the finish of his talk, Hubbert was applauded politely and the conference went on to the next speaker, but the industry had been stung. Hubbert was soon to learn how upset Shell officials were with his talk and how they wished it had never happened. They knew they could do little to suppress a talk given before 500 petroleum engineers so they had to content themselves with the minor censoring job shown above. His mathematics and his eye-opening production curve had to stand as they were. Hubbert was not fired by Shell, incidentally, he continued to work for them until the end of 1963 when he retired at the mandatory retirement age of sixty.

The industry never forgot what he had done to them and got very busy over the next 5 or 6 years discovering tremendous quantities of oil, (on paper), in order to shove that dreaded day of culmination as far into the future as possible. After all, the industry position had always been contained in the following public relations statement:

"The United States has all the oil it will need for the foreseeable future."

Hubbert was considered an outcast, an incurable pessimist, an oddball. No other industry figure supported him. They did their best to ignore him and forget him, even though they were busy "discovering" oil on paper for the next 18 years. In fact, it can be truthfully said that his harmless little drawing of 1956 did more to increase (on paper at least) the petroleum resources of the United States within the next five years than the combined exploratory and productive efforts of the petroleum industry had been able to accomplish during the preceding century!

Aside from that...

M. King Hubbert. M. KING HUBBERT, research geophysicist, U.S. Geological Survey, was born in central Texas. After two years at a junior college in his native state, he received his scientific education during the 1920's from the University of Chicago, obtaining B.S., M.S. and Ph.D. degrees in geology, physics and mathematics.

During and immediately following this period, he also: worked in Texas, New Mexico and Oklahoma as an oil geologist for Amerada Petroleum Corporation; taught geology and geophysics for a decade at Columbia University; spent summers in geophysical exploration for minerals with the Illinois State and U.S. Geological Surveys; and spent a year and a half in Washington during World War II on mineral-resource studies with the Board of Economic Warfare.

The following 20 years (1943-1963) were with Shell Oil and Shell Development Companies in Houston as research geophysicist, associate director of exploration and production research, and chief consultant (general geology). During his last two years with Shell, Dr. Hubbert was on loan for one quarter of each year as Visiting Professor of Geology and Geophysics at Stanford University. Upon retirement from Shell at the end of 1963, he assumed dual positions as research geophysicist with the U.S. Geological Survey (three-quarters of each year), and Professor of Geology and Geophysics at Stanford. After 1968 the Stanford position was relinquished.

Dr. Hubbert is a member of some half-dozen scientific and engineering societies, including: The Geological Society of America (President, 1961); the American Association of Petroleum Geologists (Associate Editor and twice Distinguished Lecturer); Society of Exploration Geophysicists (Honorary Life Member and former Editor); Society of Petroleum Engineers of AIME (Distinguished Lecturer); American Association for the Advancement of Science; American Academy of Arts and Sciences; and National Academy of Sciences (Committee on Natural Resources, advisory to President John F. Kennedy, 1961-1962; Chairman, Division of Earth Sciences, National Research Council, 1963-1965; Committee on Resources and Man, National Research Council, 1966-1970).

Dr. Hubbert's scientific work has been of a general, rather than of a specialistic, nature. It has included: geophysical exploration for oil and gas, and other minerals; petroleum geology and engineering; structural geology and the physics of earth deformation; physics of underground fluids, including the motion of ground water, entrapment of petroleum under hydrodynamic conditions and fluid behaviour in petroleum reservoir engineering. It has also embraced continuing studies for more than 4 decades of the world's mineral and energy resources, and the significance of their exploitation to human affairs. His active interests have also included the philosophy and history of science and its bearing on the education of scientists. He has had some 65 technical papers published, and is author or co-author of several texts.

For his work in geophysics, Dr. Hubbert was awarded the Arthur L. Day Medal of The Geological Society of America in 1954. For his contributions to petroleum engineering, he received the AIME's Anthony F. Lucas Gold Medal Award in 1971.

On October 6, 1972, Dr. Hubbert was awarded the degree of Doctor of Science *honoris causa* from Syracuse University. He also accepted an invitation from the Board of Regents of the University of California system to serve as a Regents' Professor on the Berkeley campus during the Spring Quarter, of 1973.

Dr. Hubbert retired from the U.S. Geological Survey on August 31, 1976. He is now much in demand at seminars on energy problems and has scheduled a European trip for the first half of 1977 where he has been invited to address scientific groups in a number of countries.

THE 'HARMLESS' LITTLE HUBBERT CURVE. One could be clever and say of that eventful day in San Antonio twenty years ago that "Hubbert really threw them a curve". Well, he did, and it was a very special kind of curve, one which was to grow in importance and respect for these last twenty years. Let me explain it to you in my fashion, not so much in the words of Hubbert the scientist but in the words of an editor of a fishing magazine.

According to standard geological theory, petroleum (which includes crude oil and natural gas) was formed over a period of 600 million years of geologic history. Coal was formed over some 300 million years. Their source is believed to be plant and animal life that did not die and then decay in the presence of oxygen as normally all plants do, but instead, plant life that somehow became covered with silt, mud, water, etc., in which oxygen was not present. Thus the plants did not decay but through 600 million years of heat and pressure from overlying layers of sediments, were converted into petroleum. Such plant deposits are still being accumulated today in swamps and bogs, but very slowly. To all intents and purposes, the amount" of petroleum that might "be formed during man's short history on this planet can be considered negligible. The same holds true for coal. These are the so-called fossil fuels.

Basically, our extracting petroleum and/or coal from the earth amounts to drawing checks on a bank account that will never receive any more deposits.

The total amount in the account is fixed. The more we take out, the less is left. The faster we take it out, the sooner our account will be empty.

In 1929, geologist Donald Foster Hewett of the U.S. Geological Survey addressed the American Institute of Mining Engineers, delivering what Hubbert has called "one of 'he more important papers ever written by a member of the U.S. Geological Survey",[18] entitled "Cycles in Metal Production". Hewett had made a trip to Europe in 1926 during which he visited 28 mining districts, believing that many of the problems that harassed Europe mining industries in 1926 would also face us in this country at a later date. Hubbert pointed out the correlation with fossil fuels: *".... like metals, the exploitation of fossil fuels in a given area must begin at zero, undergo a more or less continuous increase, reach a culmination and then decline, eventually to a zero rate of production."* [18]

Taking off on Hewett's earlier work, Hubbert used integral calculus to produce a curve that mathematically would illustrate the birth, middle years, and finally the death of an exhaustible resource, Fig. 1. Never mind the integral calculus part, this simple curve can be read by any grade school child. The production rate in billions of barrels of crude oil per year is shown on the vertical axis. The horizontal axis shows time in years. The curve illustrates that the production of crude oil started slowly, (about 1859), grew gradually for awhile, but by 1956 was growing very rapidly. Sooner or later, production reaches a peak from which it never recovers and then production continues to slide down and down, rapidly at first and then more slowly toward the end. Actually, it could be possible to have several peaks a short time apart before production slides forever downward, and actual production will never go completely to zero.

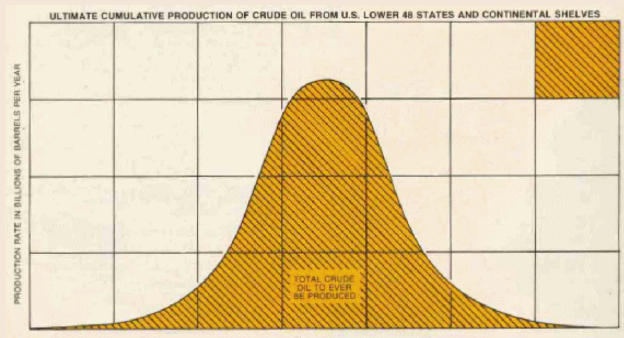


FIGURE 1. The Hubbert Production Cycle Curve of an exhaustible resource, shown here for Crude Oil, as adapted b Howard L. Baumann of Fishing Facts Magazine, from Hubbert 1974 report to U.S. Senate Committee.[2]

Note the square in the upper right hand corner. This is to illustrate the fact that each square on a drawing of this type equals a definite quantity of oil. For example, if we chose the vertical scale for each line to show 1 billion barrels per year production and each horizontal line to show 20 years, one square would equal 20 billion barrels of oil.

If you can make a pretty good estimate of the total amount of crude oil likely to ever be produced, obviously ALL of that crude oil HAS TO FIT UNDERNEATH THE CURVE. As an example, if you estimated 180 billion barrels as the total, 180 divided by 20 billion barrels per square would equal 9 squares which must fit underneath that curve. (Remember, this is a theoretical curve used to illustrate the principles involved, the actual production curve comes later.)

In preparing for his 1956 paper before the petroleum engineers, Hubbert gathered together the best estimates available at that time for the ultimate total amount of crude oil to be produced from the lower 48 states and adjacent continental shelves. His own estimate was 150 billion barrels. Only a month before, however, Wallace E. Pratt, retired vice president of the Standard Oil Company of New Jersey had released the results of a survey conducted among 25 of the men Pratt considered to be the best-informed in the industry. Pratt's own estimate for crude oil came to about 145 billion barrels, *and reported that of* 23 *replies* the highest of all, for 200 billion barrels, came from DeGolyer and McNaughton of Dallas. Also in February, Pogue and Hill, released their study which was for 165 billion barrels.

Consequently, Hubbert estimated the total amount of crude oil ultimately to be produced from the 48 states and adjacent continental shelves would lie between 150 and 200 million barrels.

He then plotted the curve shown in Fig. 2. Note that the solid portion underneath the first part of the curve shows the amount of crude oil *actually produced* to that date. The lighter shaded portion underneath the curve shows the known reserves of crude oil. The dotted lines with no oil shown underneath represent predictions for the future. Note that one prediction is given by the broken-line curve showing 150 billion barrels as the ultimate total of crude oil, the other prediction is given by the broken-line curve showing 200 billion barrels as the ultimate total of crude oil to be produced.

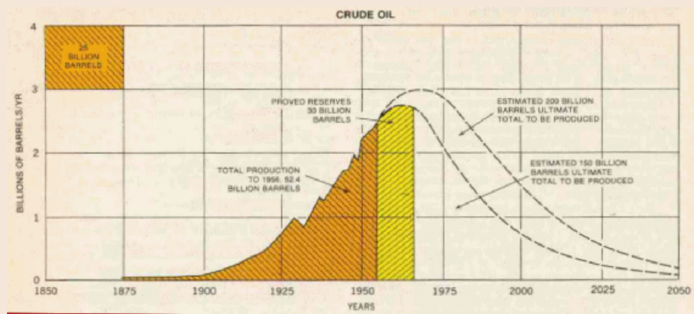


FIGURE 2. Hubbert's 1956 prediction of future production of crude oil in the lower 48 United States and adjacent co shelves. Adapted by Howard L Baumann of Fishing Facts Magazine, from Hubbert 1974 report to U.S. Senate Commit

Look at these curves carefully, note that they show a peak production for the 150 billion barrel curve at about 1965 and the peak production for the 200 billion barrel curve only about 5 years later in about 1970.

Now imagine the alarm this simple curve caused in the petroleum industry twenty years ago. It showed that the industry-wide production of U.S. crude oil would peak somewhere within 10 or 15 years, (1965-70) and would go steadily down from there, not likely to ever rise again!

Note that a whopping one-third increase from 150 to 200 billion barrels of crude oil only postponed the dread "day of reckoning" when production would peak... by only about five years! That meant that if that dreadful date was to be postponed very far into. the future, estimates of 2, 3, 4 & 5 times as much ultimate total production had to be obtained. (That is exactly what followed in the next five years!)

No wonder "The 'Harmless' Little Hubbert Curve" had such an effect on the petroleum industry, and has even more significance today.

THE OMINOUS LOGIC OF THE HUBBERT CURVE. At first glance, you might be tempted to look at the Hubbert Curve and think: "How interesting. Really clever. Wonder what's for supper tonight?"

You do not have to be a wizard in mathematics to understand the inescapable truths of the Hubbert Curve. Any man or woman can easily understand the *meanings* of that curve which could be used for the production of oil, natural gas, coal, copper, zinc, aluminum, tin, lead, gold, silver, etc.

The key is to remember that each square on the drawing equals a certain number of units ... in our case, billions of barrels of oil. Once we have settled on a reasonably good estimate of all the oil that will ever be produced by the U.S. 48 states and adjacent continental shelves, (and that number in September of 1976 seems to be about 170 billion barrels of crude oil), we can quickly count how many squares that equals on the drawing. If each square were picked to equal 25 billion barrels, as was shown in Fig. 2, then it will take about 6-3/4 squares to fit underneath the final Hubbert Curve for today. (The actual number would be 6.8 squares).

The first part of the curve is already fixed by known production of crude oil to the date of 1956 on Fig. 2. That's 524 billion barrels. That fits exactly under the curve to 1956. The proved reserves in 1956 amounted to 30 billion barrels. That is also shown under the curve. 524 plus 30 equals 824 barrels that are known, fixed, set. 170 billion minus 824 billion equals 87.6 left to be discovered and to fit under the curve. That's equal to just 3-1/2 more squares that can be made to fit under the final curve. Knowing that oil extraction slows down and stretches out over a long period of time from a depleting oil reservoir field, we know that the final 3-1/2 squares under our final curve must be stretched out into the future. It would be pretty hard to make the final portion of the curve for 170 billion barrels look much different than the two curves shown on Fig. 2 for 150 billion barrels and for 200 billion barrels. It would be pretty hard to make the final 170 billion barrel curve not fall somewhere between the other two.

What does this mean? It means we are very limited as to what we can do about our production of crude oil. Obviously if we made a superhuman effort to find it and pump it out faster, we could make the curve rise again, perhaps, but any raise above the curve as shown in Fig. 2 would be taking oil away from under the future part of the curve and then it would have to drop off even faster. It's inescapable that our oil reserves are like a checking account which will never receive another deposit. The faster you draw it out, the larger the checks you write, the sooner the account will be empty!

Now you know the ominous significance of the Hubbert Curve and why the petroleum industry tried to obtain vastly larger estimates of the ultimate total production. In effect, the only way to postpone the awful day of peaking followed by shrinking production was to try to add more money to the account. (With the aid of hindsight obtained from looking at it from September of 1976, their deposits to this country's crude oil checking account were paper money, and worth no more than the paper they were written on. But that story comes later.)

Fig. 3 shows what happened. In the five years following Hubbert's 1956 predictions, based on estimates of between 150 and 200 billion barrels of crude oil ever to be produced in the lower 48 states, estimates by others came in at 204, 250, 372, 391, 400, and zoomed up to 590 in 1961. Two years after that, even the 590 estimate was humbled by one for 658 billions of barrels of oil. Note on Fig. 3 that the three highest estimates of all came from men of the U.S. Geological Survey of the Department of the Interior. (Knowing that these were all thrown out the window by a U.S. Geological Survey Study in 1975, makes you wonder "who can you trust?")

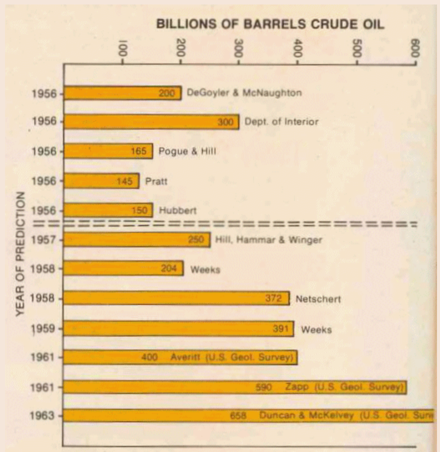


FIGURE 3. Estimates of ultimate total of crude oil to be produced from 48 United States and adjacent continental shelves. Prepared by Howard L. Baumann of Fishing Facts Magazine from data furnished by Hubbert.[2]

Although the 1963 estimate was made by Duncan and McKelvey of the U.S. Geological Survey, it still is not clear to me whether or not the Survey recognized it as being their official estimate. (McKelvey was later to become director of the Survey, a position he still holds today.) So let's put that one aside and examine the implications of the estimate for 590 billion barrels of crude oil. See Fig. 4. Here the Hubbert Curves are drawn for 150, 200, and 590 billion barrels of oil as estimates of the ultimate total production. You will notice that this whopping big increase only shoves the dreaded date of peak production and decline another 25 years into the future, predicted to take place in about 1995.

If the 590 figure were correct, we would have now in 1976, another 19 years to find suitable substitutes for crude oil. As we know now, we don't. Production actually did peak in 1970 and has been going down ever since. If we could somehow raise it up again it could only quickly fall again - and then go down even faster than before. That should be easy enough to understand because it's an inescapable truth. If you understand it, however, you are part of a very, very small minority in this country.

THE REASONS FOR OUR CHOICE OF WORD SO FAR. You surely must have been wondering why I keep stressing CRUDE Oil and the U.S. 48 STATES AND ADJACENT CONTINENTAL SHELVES. You know that oil has been discovered in Alaska, why don't I talk about it? Maybe you've heard the term PETROLEUM LIQUIDS and you're wondering why I keep specifying crude oil?

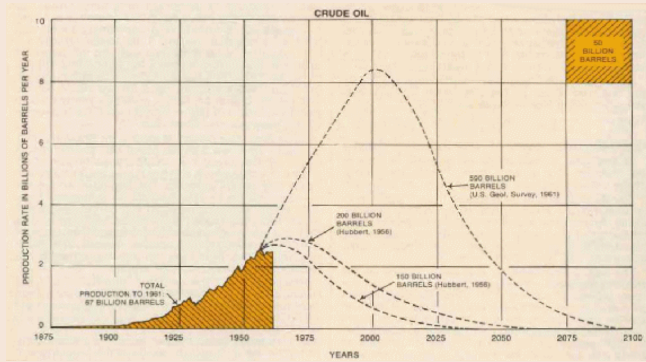


FIGURE 4. **Comparison of complete cycles of U.S. crude oil production based on estimates of 150, 200, and 590 billion barrels ultimate total to be produced. Adapted by Howard L. Baumann of Fishing Facts Magazine from Hubbert 1974 report to Senate Committee [2]**

In the discovery and exploitation of natural gas, there is usually found a relatively small amount of NATURAL GAS LIQUIDS. In today's statistics of the petroleum industry, crude oil and natural gas liquids are sometimes added together and classed as "petroleum liquids". In general, a figure for petroleum liquids will consist of about 85% crude oil and 15% natural gas liquids. The reason I keep sticking to crude oil alone thus far in this Special Editorial Report is because the statistics for crude oil alone have been kept from the beginning of petroleum exploration. Also, crude oil is the foundation of the petroleum industry. In order to have the most meaning, I've been sticking to crude oil statistics. It avoids a lot of confusion. We'll add petroleum liquids in later on in this report.

The discovery of oil in Alaska's Prudhoe Bay occurred in 1968. It looks like a 10 billion barrel field. How much additional oil it holds can only be estimated by methods which work sometimes and do not work at other times. We'll talk about Alaska also a little later in this report.

I thought I'd drop these comments in at this time, however, so that you don't wonder whether or not I'm aware of Alaskan oil and the term "petroleum liquids".

NOBODY FOOLS THE DRILL. The petroleum industry has been employing increasingly sophisticated methods of finding oil and estimating how much is down there, and getting as much of it out as possible. Contrary to some of the TV commercials you've seen which might give you the impression that *suddenly* we're doing all these wonderful modern things NOW to get oil, big strides were made in petroleum technology in the 20 years from 1945 to 1965. Although new improvements are always being made, the biggest strides were probably made in that 20 year period.

Despite all of the wonderful tools the petroleum geologist and the petroleum engineer now has at his disposal, THE ONLY TOOL THAT DISCOVERS OIL OR GAS IS THE DRILL!

Let that one sink in for just a minute. *The drill provides the moment of truth in all exploration for petroleum.* Let me help you understand the significance of that statement.

It's practically a professional requirement for a petroleum geologist to be an incurable optimist. It costs a lot of money to drill a new hole. New oil fields are discovered only by new-field wildcat wells. These are holes that are drilled well away from existing oil fields. That's exploratory drilling and is different from the drilling done to tap known oil fields for further production. It's important to understand the difference. The average cost of a new-field wildcat well in 1972 was $96,000.[2]

In order to break even, it is essential that such a well discover a million barrels of oil or more, or the equivalent in natural gas. In order to get such a well drilled, a petroleum geologist has to put together all of his data from all of his instruments, geologic maps, regional geology, subsurface geology, seismic map, gravity data, and so forth, and convince the boss that if a well is drilled in such and such a place it will discover a million barrels of oil or more. Obviously, that's playing poker with high-priced chips.

In 1945, it required 51 new-field wildcat wells to make one profitable discovery of oil. By 1965, it required 137![2] Yes, it's getting harder and harder to find oil.

When gas and oil are taken together, the numbers will change. As of 1972, the latest year for which such information was available was 1965. In 1965, a total of 6,175 new-field wildcat wells were drilled to find 97 significant (profitable) discoveries of oil or gas.[2] That's 1 in 64, for oil and gas combined. "There is little doubt that most, if not all, of these 97 successful wells were drilled in response to professional recommendations of petroleum geologists, or their counterparts, petroleum geophysicists; but who recommended the drilling of the 6,078 failures?"[2] (Hubbert, in 1974 report to U.S. Senate Committee.) The same petroleum geologists and geophysicists, of course.

Now that you understand something of the odds the oil companies face, even with the best professional guidance of their geologists, you can truly understand the significance of the statement:

THE ONLY TOOL THAT DISCOVERS OIL OR GAS IS THE DRILL!

The drill discovers crude oil (or natural gas) in the amounts that are present and in the amounts that the technology of the time allows to be recovered from that particular oil or gas field. *Each new oil field is credited to the year in which it was discovered.* For example, the big oil field of East Texas was discovered in 1930. It is credited to that year because it was the exploratory drilling of that year which found it. Today, 46 years later, the East Texas field is still producing oil from reserves that were found in 1930.

Accurate records are kept of all the exploratory drilling for oil (or gas), and the total number of feet of exploratory drilling runs into almost 2 billion feet. Now, if you can figure out the number of barrels of oil that are discovered per foot of exploratory drilling, you will have an undeniable measurement of how you're doing in this high-priced poker game of exploring for oil or gas.

In November, 1967, *The American Association of Petroleum Geologists Bulletin* printed a scientific paper prepared by Dr. Hubbert. It was entitled, "Degree of Advancement of Petroleum Exploration in United States."

Dr. Hubbert had laboriously researched the exploratory oil drilling figures and discoveries all the way back to 1859 when oil was first discovered in this country by "Colonel" E.L. Drake on August 27, 1859 at Titusville, Pennsylvania, at a depth of 70 feet. He grouped the oil discoveries, not by years, but by each 100 million feet of drilling.

Fig. 5 is the result of some of his research updated thru 1972. Note that the number of barrels of crude oil discovered per foot of exploratory drilling for the first 100 million feet of drilling covered 60 years and produced oil at an average rate of about 240 barrels per foot of drilling. During the second 100 million foot interval, it dropped to about 161 barrels per foot.

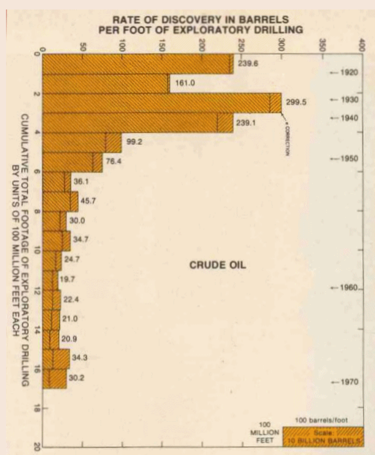


FIGURE 5. Average discoveries of crude oil per foot for each 100 million feet of exploratory drilling in the U.S. 48 states and adjacent continental shelves. Adapted by Howard L. Baumann of Fishing Facts Magazine from Hubbert 1971 report to U.S. Senate Committee.[2]

Third interval of 100 million feet of drilling covered the 1930 accidental discovery of the East Texas Field, (biggest in the U.S. 48 states), and covered the period from 1938 to 1937. The discovery rate in this interval jumped to a record never attained again, about 300 barrels per foot.

The following 100 million foot interval saw the rate drop to 239. Then it kept dropping to 99, 76, and by the 15th interval, as low as 21 barrels/foot. This was followed by a slight reversal to 34 and to 30 barrels per foot for the last two intervals.[2, 14]

Study Fig. 5 carefully, for it is important that you understand all that it tells us. The dates are at the top and a little hard to follow because they show when the actual drilling was done and are unevenly spaced because more drilling was done in some time intervals than in some others.

Note that roughly 540 million feet of exploratory drilling took place between 1859 and 1950, that's 91 years. Note that roughly 750 million feet of exploratory drilling took place from 1950 to 1960, a period of 10 years; note that roughly 470 million feet of exploratory drilling took place in the 10 years between 1960 and 1970. (These figures are rough calculations based on reading Fig.5.)

You will see that a great amount of drilling took place in 1950-1960, but that the number of barrels discovered per foot kept dropping. In the 10 years between 1960 and 1970, drilling dropped by about 38% from the previous decades but perhaps we can understand why. The number of barrels of oil per foot of exploratory drilling continued, overall, to drop. Perhaps you are one of those suspicious persons who believes that the oil companies refuse to look for more oil. If so, let me point out to you that 470 million feet of drilling is a lot of hole … and you can see that there were increasing numbers of dry ones. Compare the 470 million feet of drilling in the 10 years from 1960-1970 with the 540 million feet drilled during the first 91 years of oil's history in this country.

> "… the period from about 1945 to 1965, during which this drastic decline occurred (in discovery rate of barrels per foot), was also the period of the most intensive exploratory activity, and of research and development of exploratory and production techniques, in the history of the petroleum industry." [2] (Hubbert in 1974 report to U.S. Senate Committee.)

Now you can more fully appreciate the significance of the statement that the president of an oil company can issue orders to his men to drill more holes in any particular year, but it is beyond his power to order them to discover more oil or *gas* per foot of hole. Nobody fools the drill!

THE DRILL CAN BE USED TO TELL US HOW MUCH OIL OR GAS IS LEFT. In Fig. 6, you will find that Dr. Hubbert has used drilling figures in another way. He shows us that the pattern of declining production rates in barrels of oil per foot closely fits a mathematical curve, which he uses to project that the ultimate total production of crude oil in the U.S. 48 states and adjacent continental shelves will be about 172 billion barrels. In Fig. 6 he also shows that the 590 billion barrel estimate of 1961 by A.D. Zapp of the U.S. Geologic Survey can be proved to be an overestimate of 418 billion barrels.

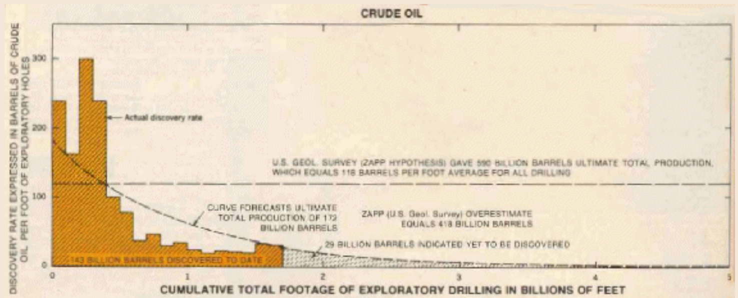


FIGURE 6. Estimation of ultimate total crude oil production for the U.S. 48 states and adjacent continental shelves; by comparing actual discovery rates of crude oil per foot of exploratory drilling against the cumulative total footage of exploratory drilling. A comparison is also shown with the U.S. Geol. Survey (Zapp Hypothesis) estimate. Adapted by Howard L. Baumann of Fishing Facts Magazine, from Hubbert 1974 report to U.S. Senate Committee[2]

WE'RE NOT GOING TO GET MUCH MORE CRUDE OIL. In Fig. 7, Dr. Hubbert shows the latest projection of the now-famous Hubbert Curve which he prepared at the request of Senator Henry Jackson, Chairman of the U.S. Senate Committee on Interior and Insular Affairs. This curve is only part of a large study of "U.S. Energy Resources, A Review As Of 1972". Part I, a monumental report of 267 pages was prepared entirely by Dr. Hubbert, submitted to the Committee in late 1973 and printed and released by the Committee in June, 1974. It is the most comprehensive report available by Dr. Hubbert, and is available for $2.15, from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402, Serial No. 93-40 (92-75), Part I. You might be interested in sending for a copy for yourself.

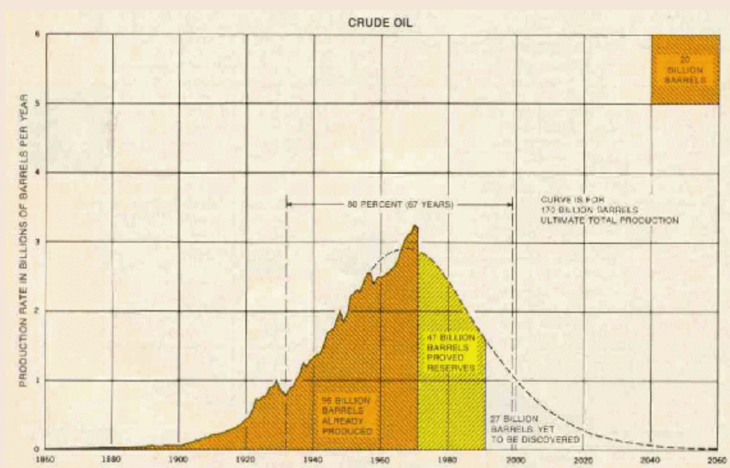


FIGURE 7. Complete cycle of crude oil production in U.S. 48 states and adjacent continental shelves as of 1971. Adapted by Howard L. Baumann of Fishing Facts Magazine, from Hubbert 1974 report to U.S. Senate Committee.[2]

By now you should be an old hand at reading the Hubbert Curve. You will see that he has settled on a final value (as of 1971), of 170 billion barrels of crude oil ultimate total ever to be produced from the U.S. 48 states and adjacent continental shelves. You will also note from Fig. 7, that most of our crude oil has already been produced and/or discovered and that not much remains to be discovered and produced. From our study of the Hubbert Curve and our understanding of its relentless mathematical logic, you will see that there really isn't much we can do to drastically change that pattern. (No matter what some of the oil companies are saying!)

THE STORY ON NATURAL GAS. Figure 8 is the first Hubbert Curve on natural gas and dates back to 1961. Just as in the first Hubbert Curve on crude oil in 1956, the figures for the future are somewhat tentative, but within limits. Statistics on proved reserves have only been kept since 1945. He estimated the peak in proved discoveries was in 1961, the peak in proved reserves at about 1969, and the dreaded peak in the rate of production about 1977. What later happened in fact was that the industry withdrew natural gas from our natural gas checking account much faster than Hubbert had anticipated in 1961 and caused our balance to speed faster toward zero. The dread peak in production (and decline every year thereafter) actually took place in 1973. Fig. 9 is the Hubbert Curve as of 1972 for natural gas showing that the peak of production is very near. As with the final 1971 Hubbert Curve for crude oil, you can easily understand that most of our natural gas has been produced and/or discovered, (some is in proved reserves, of course) and that the curve shows production has to drop off very sharply from the peak which was actually reached in 1975. Not much can be done to alter that which we see is going to happen. All of the natural gas that will ever be found must fit underneath the curve.

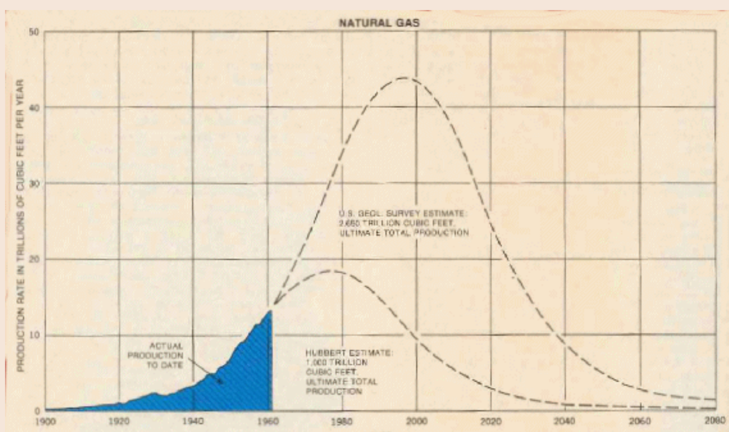


FIGURE 8. Comparison of predicted cycles of natural gas production for U.S. 48 states and adjacent continental shelves. based on estimates of 1961 for ultimate total production. as adapted by Howard L. Baumann of Fishing Facts Magazine. from Hubbert 1974 report to U.S. Senate Committee.[2]

In Fig. 10, Dr. Hubbert applies the same analysis he used for crude oil in developing data from exploratory drilling. You will note the same steady drop that we saw in the figures for crude oil. Obviously, we are taking natural gas from a diminishing supply. There's not all that much money left in our natural gas checking account.

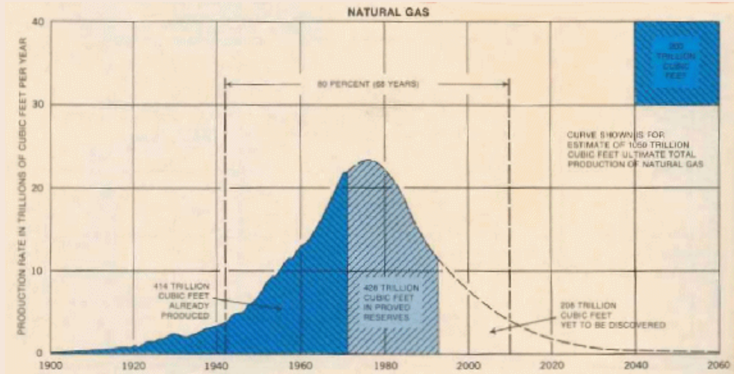


FIGURE 9. 1972 estimate of complete cycle of natural gas production in U.S. 48 states and adjacent continental shelves. Adapted by Howard L. Baumann of Fishing Facts Magazine, from Hubbert 1974 report to U.S. Senate Committee. [2]

The American Gas Association, through their spokesmen in Washington and in the press, have been telling us: "Give us a decent price for our natural gas and we'll get production back up to that 1973 level."

My personal reaction is they ought to show the American people some cold, hard data to prove that all that "new" natural gas is really there, waiting to be found. The news from the drill says otherwise!

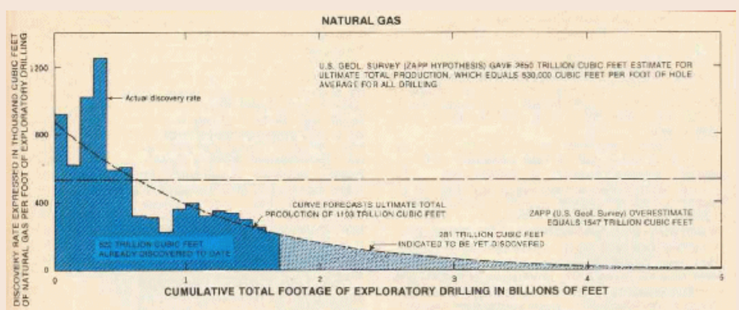


FIGURE 10. Estimation of ultimate total natural gas production for the U.S. 48 states and adjacent continental shelves; by comparing actual discovery rates of natural gas per foot of exploratory drilling against the cumulative total of exploratory drilling. A comparison is also shown with the U.S. Geol. Survey (Zapp hypothesis) estimate. Adapted by Howard L. Baumann of Fishing Facts Magazine from Hubbert 1974 report to U.S. Senate Committee [2]

THIS IS NOT A WITCH HUNT. Perhaps it would be right to point out at this place in the editorial that I am not looking for "robber barons", "crooks", "rip-off artists", etc. as some critics have called members of the petroleum industry in an attempt not to believe that our country is faced with an "energy crisis". Oil companies take big risks, they make big money, and they play for keeps. The president of any corporation will make a profit for that company or his replacement will. The petroleum industry has been reluctant to admit that our oil reserves are dropping, but some of them are changing even that during the past year or two.

What manipulations take place in the marketplace are not known to me, I am only trying to give you an understanding of some of the facts of life in the exploration and production of oil and gas so that you will understand that our country is faced with a true Petroleum Predicament. Are the petroleum companies trying to make all the money they can out of it? I don't *know* and neither do you, but my guess at an answer is, "Of course. Doesn't every company try to make all the money it can at ANY time and ALL THE TIME?"

The petroleum industry doesn't need me to defend them, but neither will I attack them without facts.

The facts of petroleum production in the United States are simply that it's getting harder and harder to find more oil and gas ... especially in the insane amounts that we are gobbling up!

It is not my purpose in this editorial to get into the subject of whether or not the petroleum industry represents a cartel, whether or not they operate in violation of anti-trust laws, whether or not they should be "broken up", or any of the other charges and counter-charges that are being made by critics of the industry. Congress is giving careful study to these questions, I believe, and indeed they should.

BIG, BIG MONEY IS INVOLVED. In the principal Conclusions listed at the beginning of this editorial, I quoted some figures prepared by Dr. Hubbert in his 1974 report to the U. S. Senate Committee on Interior and Insular Affairs *on the total cost* to *the American people* of a $1 per barrel price raise on oil and a 20c raise on a 1,000 cubic feet of natural gas, based on 1974 estimates. The combined total for those simple price raises came to $233 billion. Actually, price raises amounting to several times those examples have taken place since 1974. You can see that the amount of money involved is out of sight.

The petroleum industry has mounted one of the greatest lobbying campaigns in history I am told, to press Congress and The White House for more money, which *they say* they need, and which, *they say,* will enable them to go out and "find all the natural gas and oil that we need". Based on my observations, the American Gas Association has been especially active in this campaign.

The oil industry seems to be quite realistic, no longer do we get the wildly optimistic statements of 1973 & 1974. On the contrary, some of the major oil companies, to their credit, have been quite candid about our dwindling resources of crude oil. Not so with the natural gas people, however, They are promising us-that "if you allow us to get a fair price for our natural gas, we can afford to go out and bring gas production back up to the peak of 1973 by 1985". I personally heard an official of the American Gas Association make that kind of statement here in Milwaukee last March. (I sat next to Dr. Hubbert at that meeting, and he put his head between his hands and said softly, "No, no, no.")

About eight months earlier, a significant article had appeared in the *Milwaukee Journal* on July 11, 1975,21 that read as follows:

> "NATURAL GAS PAST PEAK, FPC WARNS"
>
> *"Washington, D.C.-The Federal Power Commission staff, in its gloomiest report yet, said Friday that the nation's natural gas production had already reached its peak and had begun* to *decline.*
>
> "*In the report, the FPC's Bureau of Natural Gas said that no matter what is done, natural gas production in the US apparently will never again meet demand.*
>
> *"The bureau warned energy policy-makers* to *develop plans and policies keyed* to *the possibility that the nation may indeed be experiencing the early effects of* a *resource being pushed toward exhaustion:*
>
> *"The staff takes issue with statements that there are as yet huge undiscovered natural gas supplies that will pull the nation out of the present shortage. Such estimates range from* 568 *trillion cubic feet to* 1,450 *trillion. The nation uses about* 22 *trillion cubic feet* a *year.*
>
> "The report said more recent estimates were that 400 trillion cubic feet or less remained undiscovered."

Five months later, another significant warning appeared in the *Milwaukee Journal* on December 14, 1975,[21] by Paul G. Hayes of the Journal Staff:

> "NATURAL GAS DIP TERMED IRREVERSIBLE"
>
> "A *top federal energy expert said here Tuesday that there probably was no way that the US could ever reverse its declining production of natural gas.*
>
> "*Our analysis is that conventional US gas production has reached its peak and will be declining for the indefinite future,' said Frank C. Allen, chief of the Bureau of Natural Gas of the Federal Power Commission.*

My personal reaction to this campaign for more money is this: If the industry is entitled to certain price increases due to increased operating costs and high risks in exploration, Congress should grant these increases. However, as to the promises about getting us all we want, hold on a minute.

Is the American Gas Association trying to tell us that they honestly drilled all their worst holes first? The drilling records show that gas produced per foot of exploratory drilling has been in a general decline for the past 20 years, despite all the increases in technology. (See Fig. 10). Has the industry saved their best holes to drill NOW, after 20 years of declining discoveries per foot of hole? *I do not understand.*

The oil industry today seems to be talking more sense, in my opinion. They seem to be saying, in effect, "Look, it's getting tougher and more costly all the time to get more oil. We are in business to sell all the oil we can, but we have to have more money if we are going to undertake these expensive projects. We'll find you more oil if you give us more money."

I think that Congress should tell both of them: "Show us the hard evidence that all that "new" oil and gas is really out there to be found."

When anyone says they are going out to get us *vast* new quantities, I say, "Tell it to the drill".

In 1492, Columbus discovered America. After his famous voyage, and for perhaps the next 400 years, men kept discovering more new islands and even some new continents, adding them to the maps of the known world. No one would argue today that there are great numbers of new islands and continents out there waiting to be discovered. Sooner or later, as we looked long enough and hard enough, we discovered them all. After all, their number was fixed, there were just so many.

In a similar fashion; the amount of crude oil and natural gas on this continent were fixed hundreds of millions of years ago. The continental United States is already the most heavily searched, drilled, poked-into-and-explored (for oil) piece of real estate on earth. By every sign we are getting close to discovering the major portion of whatever is left undiscovered. (You know, there weren't many new islands or continents remaining to be discovered in, say 1890.)

THE ODDS ARE BETTER AT LAS VEGAS. As I pointed out earlier, drilling to find oil in the U.S. is becoming an ever more risky business. Let me give you an example. In 1953, Ralph Miller of the U.S. Geological Survey presented to the Senate Committee on Interior and Insular Affairs the Survey's best estimates for oil to be found in the Gulf of Mexico off of Louisiana and Texas. These were *good estimates* based on a careful geological comparison between the offshore areas and similar areas onshore. Since the same formations of oil-bearing sediments which extended out under water were already known to be good producers of oil on land, the projections were based on solid data. (The U.S. Geological Survey, in 1953, had not yet started the wild overestimating game they played from 19611975.)

Ralph Miller presented estimates of 9 billion barrels of oil to be recovered off the Texas shore and 4 billion barrels off the Louisiana shore. After more than 20 years of intensive exploration, about 5 billion barrels have been discovered off of *Louisiana* and almost NONE off of *Texas!*

A September 12, 1976 article in the *Milwaukee Journal* quotes a United Press International story from New York City. Bids were being taken for leasing some 876,000 acres of underwater land on the Atlantic continental shelf. Estimates as to how much crude oil the shelf contains range from 400 million to 14 billion barrels. (Keep those figures in mind, we'll bump into them again).

Larry Lindahl, Getty Oil's Director of Exploration, talks about the "Destin Dome" venture of several years ago:

> *"Exxon Corp. paid $221 million for a single tract of land off the coast of Panama City, Florida that was called Destin Dome."*
>
> *"After that they put in $800 million more for drilling and research and came up with nothing, not a drop of oil to show for their billion bucks."*

Only ten days before this article appeared in the *Milwaukee Journal,* I had interviewed Dr. Hubbert for 5-1/2 hours on September 3rd, 1976, in the study of his home in Washington, D.C. Dr. Hubbert described the Destin Dome as a very large anticlinal structure of the type known to normally contain large amounts of oil. This was so large, that if it were full of oil; it would be an oil field comparable to the Middle East. (The world's largest known to date.) Three companies, including Exxon and Mobil combined to pay a bonus for the lease, which was said to have been the highest bonus ever paid. These companies went and drilled 7, deep, dry holes. It was one of the costliest fiascos in the history of the American oil industry.

These same sure-fire, reliable methods (?) have been used to estimate the total amount of oil we hope to get out of Alaska. At this very early stage of the game, Alaska is still virgin territory. Alaska's Prudhoe Bay has been estimated to hold 5-10 billion barrels, discovered in 1968. To the disappointment of many in the industry, no major new discoveries have been made in Alaska for the past eight years.

More on Alaska: Newsweek of August 16, 1976,[16] reports:

> "ON-AGAIN, OFF-AGAIN OIL"
>
> *"The Alaska pipeline has more headaches in store for Washington energy officials. For one thing, they must decide soon whether to risk a new environmental battle by relaxing construction standards (waiving the mandate to X-ray all pipe welds, for instance) in order to make sure the line begins pumping North Slope oil by 1977, the scheduled opening date. In addition, they may have to scale down forecasts that oil will flow from the Slope at a 2-million-barrel-a-day pace. With no new discoveries recorded since the original 1968 strike, one top oil man says, the area's reserves will not support production higher than 1.4 million barrels a day."*

Even Las Vegas doesn't have a table where players can bet and lose as much as a billion dollars. And, even Vegas wouldn't dare run a game with the odds now not uncommon in the oil industry!

THE WILD YEARS FOR CRUDE OIL ESTIMATES. In Fig. 3, we looked at the way estimates of crude oil ultimate total production shot up in the 5-7 years after Hubbert's precedent-shattering predictions of 1956. The key in any of these estimates is the total amount of oil expected to ever be produced, what we call the ultimate total production. The importance of that figure is that it's good for as many years as you want to use it. Each year more oil is produced, each year the proved reserves change, so that by a simple process of addition and subtraction you can quickly come up with the amount of oil you still expect to discover in the future. Not all estimates, however, were put into that neat little format. In order to arrive at the figures listed below, some additional arithmetic had to be done. (Between that and the problem of finding exact dates, we increase our chances of making slight errors; which, though inconsequential in importance, might lead to charges of "sloppy work" by people who don't want to believe that we have an energy problem in the first place! That's the old "kill the messenger" habit, of course.)

Let's go back now, and take a look at some of estimates for ultimate total production of crude oil that were made in the period following the 1956 prediction by Hubbert.

Date	Ultimate Total Production	Estimate By
1956	150-200 billion barrels	Hubbert
1961	407 -507 billion barrels	Senate Committee on Interior & Insular Affairs (Lasky)
1962	885-1,000 billion barrels (by inference)	McKelvey, (U.S. Geological Survey) letter to National Academy of Science of July 20,1962: "... Zapp's estimate of 590 billion barrels is still a conservative estimate because it allows for only 50 percent recovery, whereas 75-85 percent is probably reasonable to expect for ultimate."
1963	650	Duncan & McKelvey (U.S.Geological Survey)
1965	billion barrels (slight variations from year to year but all	McKelvey & Duncan (U.S. Geological Survey)
1966	in this range)	McKelvey & Duncan (U.S. Geological Survey)
1967		McKelvey & Duncan (U.S. Geological Survey)

1965	400 billion barrels	Hendricks. (U.S. Geological Survey)
1969	168 billion barrels	Hubbert, Energy Resources in "Resources and Man" Chapter 8, National. Academy of Science"
1971	432 billion barrels on 60% recovery	National Petroleum Council American Association of Petroleum Geologists (Ira H. Cram)22
1972	420-2250 billion barrels	Theobald, Schweinfurth & Duncan, U.S. Geological Survey Circular 650 [11]
1972	No limit, apparently	Vincent E. McKelvey, Director, U.S. Geological Survey in his McKinstrey Memorial Lecture, "Mineral Resource Estimation and National Policy", presented at Harvard University in February of 1971 and published in 1972. Concluding statement:"

"Personally, I am *confident that for millennia to come we can continue to develop the mineral supplies needed to maintain* a *high level of living for those who now enjoy it and to raise it for the impoverished people of our own country and the world.* My *reasons for thinking* so *are that there is* a *visible undeveloped potential of substantial proportions in each of the processes by which we create resources and that our experience justifies the belief that these processes have dimensions beyond our knowledge and even beyond our imagination at any given time".*

When you examine the "boomer" estimates of superabundance the United States was receiving for crude oil, (estimates for natural gas were equally high in every case where they accompanied the crude oil estimates), it is no wonder that official Washington and the country felt supremely confident that we had enough gas and oil to last us through the rest of this century and well into the next one!'

THE GATHERING CLOUDS OF DOUBT. The swaggering overconfidence of the sixties and early seventies started to come under a cloud in 1971. Pressures mounted in Congress for an explanation as to why United States Geological Survey figures were so high in contrast to the figures estimated by Hubbert, who was highly rated in scientific circles and growing in stature in both the public and political sectors. Some of the oil companies were privately circulating statements that their own figures were considerably below those of the U.S. Geological Survey.

In September of 1971, Sen. Henry M. Jackson wrote a letter to the Department of the Interior requesting that Hubbert prepare an update of his 1962 report for the National Academy of Science and subsequent scientific papers including "Resources and Man" which had been published during the interim.

Interior bowed to Jackson's request, of course, and' Hubbert was assigned to the task. He was promised extra secretarial help and a technical assistant. On November 1, 1971, however, Hubbert "lost" the one secretary he had enjoyed, due to a budget tightening order from the White House. For about two years he would send out official letters in longhand. No technical assistant was forthcoming, either, although some time later he would obtain the services of a college student as a part-time assistant.

Hubbert thus had to begin the two-year task of preparing Part I of the report, "U.S. Energy Resources, A Review as of 1972", a Background Paper Prepared at the Request of Henry M. Jackson, Chairman, Committee on Interior and Insular Affairs, United States Senate.[2] He prepared the 267 page report in longhand and his wife typed it all for him at their home. The Report was finished in late fall of 1973 and transmitted to the Senate Committee, which released it in June of 1974.

On March 26, 1974, the U.S. Geological Survey of the Department of the Interior handed out a News Release entitled: "U.S. Geological Survey Releases Revised U.S. Oil and Gas Resource Estimates,"[13] Earlier in the month, this report had beer) summarized by Vincent E. McKelvey, Director, U.S. Geological Survey in an appearance before the Senate Committee on Interior and Insular Affairs.

I have assembled figures from that News Release for you as follows:

(in billions of barrels)	High Range	Low Range
Total U.S. production thru 1972, of crude oil and natural gas liquids.	115.27	115.27
Measured Reserves	48.3	48.3
Indicated and inferred reserves	25	45
Undiscovered recoverable resources	200	400
Crude oil in billions of barrels	388.57	608.57
Using an estimated 85% for crude oil alone, gives ultimate total production estimates as shown.	330.28	517.28

THE BUBBLE BURSTS. Hubbert's estimates, as of 1972, for Energy Resources of the United States, were published by the Senate Committee on Interior and Insular Affairs in June of 1974.2 His estimates were as follows:

Estimates of Ultimate Total Amounts of Crude Oil, Natural Gas Liquids, and Natural Gas to be Produced in the Entire United States and Bordering Continental Shelves:

	Lower 48 States	Alaska	Total U.S
Crude Oil in billions of barrels	170	43	213
Natural Gas Liquids, in, billions of barrels	34	5	39
Total Petroleum liquids, (also known as hydrocarbon liquids) in billions of barrels	204	48	252

In March of 1974, a letter from John Moody, Executive Vice President of Mobil Oil to McKelvey, Director of U.S. Geological Survey, was circulated all over Washington, pointing out that some of McKelvey's figures were as much as ten times those of Mobil. This led to a gathering of a panel for the National Academy of Science, and the subsequent issuance of a report in February of 1975 entitled, "Mineral Resources and the Environment".[10]

Their estimate for ultimate total production of crude oil was 247 billion barrels, compared to Hubbert's of 213 billion barrels, making them about 16% higher than Hubbert. (The National Academy of Science does no direct research itself, it reviews and passes judgement on the work done by all the various parties in a given line of scientific work.)

Pressures on the U.S. Geological Survey were so great by September of 1974 that they finally set up a Resource Appraisal Group on a crash basis to conduct a totally independent review of estimates for U.S. oil and gas' and to produce an appraisal by June of 1975.

The group was co-chaired by Betty Miller of Sun and Harry Thomsen of the Survey's Denver office. Their findings were that the ultimate total production of crude oil for the entire United States will lie between 218.12 billion barrels (with a 95% probability) and 295.12 billion barrels (with a 5% probability), published in U.S. Geological Survey Circular 725, June 19754 Their figures, compared to Hubbert's estimates of 213 billion barrels, show a remarkably close agreement. When you consider that three completely separate sources, trying to estimate what lies beneath millions of square miles of land and water, could come out so close to one another ... it's a good sign that at last this country was back on the right track.

Circular 725 ended a 15-20 year period in the history of the U.S. Geological Survey which can only be classified as a sorry episode.

Now the bubble has burst. Our big, long spree based on cheap and plentiful petroleum is over. North America never had more than 15% of the world's petroleum, and the U.S. share is now more than half gone.[2] We are using petroleum faster than any nation on earth. No amount of money is going to find oil that isn't there. We have now reached the peak from which we can clearly see the way down the slope.

OIL FROM THE ATLANTIC SHELF: GOING GOING ALMOST GONE. A good illustration of the senseless ballooning of so-called scientific estimates is the brief, recent history of the chances of getting oil from the Atlantic continental shelf. In 1963, news stories appeared about "hot new" oil finds on the Atlantic shelf. The experts at the U.S. Geological Survey of the Department of the Interior announced that 48 billion barrels were there to be taken. In early 1974, Secretary of the Interior, Rogers C. B. Morton announced that there were 200 billion barrels of oil to be found in the offshore continental shelves of the United States. In March of 1974, the experts of the U.S. Geological Survey announced that the 200 billion barrels was now more likely to be 100 and that the number for the Atlantic shelf had shrunk from 48 to 15. In June of 1975, the Resource Appraisal Group, in the Survey's Circular 725, corrected that to a much lower estimate of only 0.2 billion barrels with a 95% probability and 6 billion with a 5% probability. Stating it in a different manner, they estimated 2 billion with a 75% probability to as high as 4 billion with a 25% probability.

Current stories in the press now talk about 200 million to 14 billion barrels for the Atlantic shelf. From 48 to 15 to 2 - 4 to 0.2 - 14! Somewhere we lost 46.6 billion barrels of oil. No wonder we're running short!

PETROLEUM - THE WONDER CHEMICAL BUILDING MATERIAL. *The Milwaukee Journal* of January 26, 1975,[24] reports:

> *Richard Perry of Union Carbide Corp. noted that about* 93% *of all carpeting, 70% of all women's and children's clothing. - including stockings and panty hose - and more than 40% of men's clothing is manufactured from man made fibers derived from petrochemicals.*

Also derived from petrochemicals are detergents, plastic food wraps and bags, aspirin, antibiotics, solvents for dry cleaning, household and appliance paints, vinyl wall coverings and floor tile and refrigerants for air conditioners, freezers and refrigerators.

Here are some more of the things made from petrochemicals in a list furnished by Phillips Petroleum:

> "Everything from plastics: toys, fishing poles, boats, buttons, battery cases, bowling balls, school and stadium seats, golf clubs, chairs, house building materials, garden hose, pocketbooks, surgical materials, tableware, parts for radios, automobiles, furniture, ice coolers.

> *"The biggest use of synthetic rubber is for tires. Tire-type synthetic rubber has been improved tremendously through the years. More and more kinds of synthetic rubber are being "tailor made" for a growing number of uses other than in tires. Some of these are for wire and cable coatings, adhesives, shoe soles and heels, flooring, footwear, and so on.*

We don't have the space to tell you more of the many uses for petroleum and natural gas, but doesn't it seem to be a shame just to burn it for heat? Now you know why Dr. Ralph E. Lapp, nuclear physicist, has described the burning of petroleum as "an atrocity, a chemical crime".[5] This is the wonderful stuff that we waste more of than any nation on earth.

THE RISE & FALL OF AMERICAN OIL. Stewart Udall was Secretary of the Interior for eight years under Kennedy and Johnson. He was the boss man of Interior during the time when oil predictions were running wild. He got "taken in" by it all, too, and he freely admits it in a powerful, hard-hitting book that tells it like it really is in this country and what we face. He was joined by Charles Conconi and David Osterhout in writing *The Energy Balloon.* I suggest you buy or borrow a copy and read it. I think it will disturb you. He says, "Having help lull the American people into a dangerous overconfidence, I felt a moral duty to admit my own errors and to expose the wildly optimistic assumptions that had misled the country. It was clear to me that an enormous energy balloon of inflated promises and boundless optimism had long since lost touch with any mainland reality."

Here's an important page from the book, read it and cry. Some day we can explain to our children why we allowed it to happen:

U.S. PETROLEUM MILESTONES.

Pioneer Period.

1859 First U.S. oil well drilled
1905 High tide of Rockefeller's Standard Oil Trust: petroleum supplies only 10% of U.S. energy
1925 U.S. produces 71% of the world's oil
1930 Conditions of glut: oil sells for 10c a barrel
1945 U.S. uses 4-1/2 million barrels of oil a day while supplying 70% of allied war needs.

Golden Age of Oil.

1949 U.S. is an oil-exporting country
1953 U.S. oil companies account for about half of world oil production
1954 U.S. pumps half of the world's oil from domestic fields and consumes nearly all of it in this country
1955 U.S. has 20% of the estimated world crude oil reserves

Maturity and Decline.

1961 U.S. proven reserves reach a peak and begin to decline
1970 U.S. production peaks and begins to decline

1973 U.S. imports 38% of the oil it uses
1973 U.S. annually consumes about 30% of the world oil supply
1973 U.S. has only 5% of proven world reserves

OIL FROM 'ALTERNATE SOURCES'. You read about "oil from shale", right? You heard about 1,000 billion barrels of oil out west? Don't get excited, it's going to stay there. Dr. Hubbert told the Senate Committee on Interior and Insular Affairs it wouldn't work, three years ago this month.

It really sounds simple. You "simply" dig up such enormous quantities of shale (1.88 million tons a day,) that it's equal to digging a Panama Canal every week. You crush it fine and heat to 1,100 degrees in a retort to boil off the oil locked in the rock. Then you get rid of the rock. Only now it's turned caustic and has increased in bulk by 20% to 33%. So you back-fill the leftovers, called tailings, into the hole you dug it out of. Since you still have a lot left over, you dump it into the empty scenic canyons of the west. To do this you need to grab off 89% of the undeveloped water of Colorado and Utah and half of Wyoming's. Oh yes, and you turn the Colorado River system into alkaline salts which means you wreck the agriculture in Colorado, Arizona and southern California. What will this get you? 1-1/2 million barrels of oil a day out of the 17 million per day that the U.S. is using!

A news item in the Milwaukee Journal of August 29, 1976,[25] says that the last of the oil shale development companies, Standard Oil, Gulf, Shell and Ashland, have walked away from the projects in Colorado and Utah, asking the Department of the Interior to release them from paying any more on their leases. Standard and Gulf have already paid $126 million of the $210 million they bid, and Shell and Ashland have paid about $70 million of the $117.8 million they bid. You have to admit they tried, really tried and they spent a big buck to make it work, but it won't. Oil would have to go to a price of $20 a barrel to make the economics work and nothing would be worth the damage done to the West by such a project.

Oil from tar sands? Don't keep your engine idling while you wait. One plant in Alberta, Canada has been in operation since 1966 at a rate of 45,000 barrels a day. They can't make money at that rate, should have a plant 2-1/2 times as large. That would be 112,500 barrels a day. That's 7/10 of 1 percent of what we burn each day, and this oil belongs to Canada. Shell of Canada has just pulled out of the project after dropping a bundle of dough. It's just tough, really tough, trying to find new oil.

Stewart Udall says that he had 8 years' experience with the problems of oil from shale and oil from tar sands. He makes the flat-out statement that:

> "This century would see no real substitutes for petroleum on this planet". THAT IS OUR PETROLEUM PREDICAMENT! Aside from that...

THE MONSTER OF EXPONENTIAL GROWTH. Suppose you put money into the bank at 7% compound interest. In 10 years it will double. A system of exponential growth is one in which the entire system grows by a certain amount each year, such as the money example above. To get the doubling time in years, you simply divide the number 70 by the rate of annual growth. Thus, a 5% compound interest rate would double your account in (70 divided by 5 which equals) 14 years.

Remember the "chain letter"? You simply ask two of your friends to join the chain letter scheme and then they each ask two friends, and so on. That doubles each time, right? Well, in just 28 doublings you'd exceed the present population of the United States (222 million), and in 32 doublings you'd exceed the 4 billion population of the world! Gee, you'd think you could find 28 people to take your letter, wouldn't you?

A French riddle goes something like this: You have a pond in which you place a water lily. Every day the water lilies will double in numbers until on the 30th day, your pond is completely full. You're not going to notice it until your pond is only half full, then you're going to do something about it. When will your pond be half full? On the 29th day! Okay, a small dam separates your pond from a huge lake which combined with your pond to form a lake would be 32 times bigger than your pond. Now, you tear out the dam and combine the two together. How much more time will you gain before your new kingsize pond fills up with water lilies? Five days!

An ancient Persian king was fascinated by the game of chess. One of his court attendants brought the king a present of a beautifully hand-carved, ebony chess board inlaid with mother of pearl, gold, and precious stones. The king was delighted.

> "What can I give you to show my appreciation?", the King asked him.
>
> "A few grains of wheat, your majesty. Just one grain for the first square on the board, 2 grains for the second square, 4 grains for the third square, and so forth. That's all, your majesty", the court attendant replied.
>
> "It shall be done", said the King, and sent his servants to his storehouse for a bag of grain.

When it came to the 19th square, it called for 524,288 grains of wheat and the King's storehouse was empty. The King called one of his Wise Men and they conferred for a minute. Suddenly the King ordered the attendant dragged from the room and had his head cut off. What caused his sudden change of heart toward the man he had promised anything only a short time before?

The Wise Man had told the King that when they reached the 64th square, which was 64 doublings, it would require the entire world's annual wheat crop for the next 2,000 years to fill the board!

That's exponential growth, it's a monster that sneaks up on you. Suddenly the numbers get so big they can't be managed. Time runs out fast, as with the water lilies. Ultimately the numbers become impossible, as with the grains of wheat. 64 doublings of one automobile would cover the earth with cars 72 miles deep!

We've been on an exponential growth kick in this country since the turn of the century, and especially since the end of World War II. If "more is better", then still more should be even better than that. There is never enough. The Gross National Product must grow 4-5-6-8% each year, so must autos, so must electric appliances. We doubled our registration of cars in the sixties. Electricity has doubled every 10 years since who remembers when? Energy doubles every fourteen or fifteen years, while the population will double in about 75 years. More, more, more.

We've been on a high speed joyride for the last thirty years. Cheap and plentiful oil was the vehicle. It is behind all of our "progress" of which we are so proud. "More is better. There is never enough", that's the slogan of this country; to say otherwise is to be un-American.

We've had about as many doublings as we can take in most of our systems. And now the cheap, plentiful oil that lubricated our joyride has come to an end.

We can still get oil, all we want, from "our friends". The price and terms, however, are getting steeper. **Cheap oil is over, we blew it!**

CATACLYSM OR A GOLDEN AGE - OUR CHOICE. In the last hour of my Washington interview with Dr. Hubbert, I asked him what kept him going for 20 years, trying to give people a message they didn't want to hear? How did it feel to give the country a vital message when you were 52 and have to wait until you are almost 70 before they start to listen? He didn't hesitate a moment, he looked directly into my eyes and started softly to speak. I turned the tape recorder closer to him so I wouldn't miss a word:

> "It's a problem of trying to educate the public to the state we're in. We must view it over a long time span - this phase we're in of exponential growth is about over. We're entering into something new.
>
> "It could be a cataclysm, but it doesn't have to be.
>
> "It need not even be an energy crisis. The transition from exponential growth over to a stabilized state or non-growth, need not be a state of intellectual decay. In fact, it could easily be a Renaissance - an intellectual renaissance - a golden age. Because with our technology and with adequate supplies of energy, we ought to have a lot of leisure. And the proper use of this leisure can bring us an intellectual renaissance, if we can get beyond the old 'if you don't work you don't eat' business.
>
> "We now have an exponential growth culture that at the present time doesn't even know how to cope with a state of non-growth. But the cultural adjustments that must be made and can be made, could easily lead to a flowering of civilization whereby we would look at the mental state we're in now as The Dark Ages - culturally.
>
> "I'm not looking for culprits because we're all culprits. Every last one of us is doing things that shouldn't be done in a rational society because we haven't known any alternative."

Dr. Hubbert stopped for a minute in deep thought, then he started talking again:

> "We can only continue to use oil as long as it lasts. We should be looking for other sources of energy. There's only one that's big enough, it's free, and good for at least a billion years. That's the sun. We must move into solar energy.
>
> "The technology exists today to convert solar energy directly to electricity. But you can't store it overnight and it's difficult to transmit over long distances without great losses.
>
> "We can convert that electricity into chemical energy by the hydrolysis of water. You get oxygen which passes off into the atmosphere and hydrogen which you can then ship by pipeline to wherever it's needed. The only cheaper way to move a gas is by tanker. We can then burn the hydrogen locally at the power plant to generate electricity. The product of combustion is water, it's all ideal environmentally. Hydrogen is difficult to handle, but we learned how to handle natural gas, we can learn this.
>
> "There's a Professor John O'Mara Bockris at Flinders University in South Australia,.a leader in electrochemistry, who has the whole thing worked out.

He just recently released this book, "Energy - The Solar-Hydrogen Alternative", I got my copy several months ago.

Dr. Hubbert paused, so I asked him a question, "How about the economists saying that if we let the price of gas and oil rise far enough we'll get all the gas and oil we want?"

> "That is orthodox economic theory. It has no validity whatever when you apply it to a diminishing supply. No price on earth is going to create oil that isn't there. To make a statement like that requires no evidence or knowledge to make it.
>
> "The idea of price elasticity can only be applied to a substance in abundant supply, not in diminishing supply.
>
> "Yes, we can improve our recovery factor of oil and gas, and we have been doing that very slowly. Progress comes hard now because we're already quite advanced in extracting oil and gas.
>
> "The claim of being able to supply all the gas and oil we need if the price goes high enough is hokuspokus. The only thing it will accomplish for sure is to put hundreds of billions of dollars of extra money into the coffers of the industry for oil and gas already discovered, whether they produce any additional gas and oil or not. It's that kind of a poker game.
>
> "It is my judgement that we could run the country better with almost certainly a third less energy than we're using now, by utilizing the technology that we now know how to use. The matter of energy waste implies that we could do the same as we're doing now with a lot less energy."

Dr. Hubbert stopped talking, I reached over and turned off the tape recorder. Nothing more needed to be said.

EPILOGUE: When I finally pushed the typewriter away it was 7 AM. I glanced out the basement window, it was light outside. I could hear Bernice come down the steps to the kitchen to prepare breakfast. Our Carol was still asleep.

I had worked all night in order to finish this editorial. After three years of preparation and research it was finally done. It would have been easier to write a book than to try to cut down all my material to fit this editorial.

Soon I'll be out on the freeway, part of the gas guzzling crowd. Our company switched to small economical cars a year ago, but today the big ones are selling like hot cakes again. America has forgotten all about The Energy Crisis and the Arab Oil Embargo. **It's waste as usual.**

I wonder about Dr. Hubbert. It's three years ago since he submitted that report to the Senate Committee. Nothing has happened. Nothing has changed. The National Academy of Science and the U.S. Geological Survey have both confirmed Hubbert's range of estimates. In effect, the Survey took away about two-thirds of our gas and oil (on paper).

The administration did nothing. Congress did nothing.

Will we kill the messenger this time - or just ignore him? Is anybody out there listening? Well, aside from that...

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See also
http://mobjectivist.blogspot.com/2007/03/common-ground.html
http://mobjectivist.blogspot.com/2005/02/wisdom-from-u-of-colorado.html

{President) Carter's (1977) speech drew a strong reaction from the Saudis and the oil industry. Think tanks soon emerged - many whose names are today familiar - to suggest there was really no energy problem, and they led the charge to establish a permanent right-wing media in the US.

— Thom Hartmann, 5/3/2005
CommonDreams.org

The Opposing View

Once the 1970's and early 1980's oil crisis ran its course, views like Pazik's stayed dormant for years, and contrarian voices such as Julian Simon gained credibility. Ecologists such as Paul Ehrlich and Jeremy Rifkin became marginalized. At the height of the recovery, the following excerpt seemed the conventional wisdom.

> Cato Policy Report, March/April 1998
> Julian Simon Remembered: It's A Wonderful Life
> by Stephen Moore
>
> … Today, many of Julian Simon's views on population and natural resources are so triumphant that they are almost mainstream. No one can rationally look at the evidence today and still claim, for example, that we are running out of food or energy. But those who did not know Julian or of his writings in the 1970s and early 1980s cannot fully appreciate how viciously he was attacked—from both the left and the right. Paul Ehrlich once snarled that Simon's writings proved that "the one thing the earth will never run out of is imbeciles." A famous professor at the University of Wisconsin wrote, "Julian Simon could be dismissed as a simpleminded nut case, if his ideas weren't so dangerous."
>
> To this day I remain convinced that the endless ad hominem attacks were a result of the fact that—try as they would—Simon's critics never once succeeded in puncturing holes in his data or his theories. What ultimately vindicated his theories was that the doomsayers' predictions of global famine, $100 a barrel oil, nuclear winter, catastrophic depletion of the ozone layer, falling living standards, and so on were all discredited by events. For example, the year 2000 is almost upon us, and we can now see that the direction in which virtually every trend of human welfare has moved has been precisely the opposite of that predicted by Global 2000. By now Simon and Kahn's contrarian conclusions in The Resourceful Earth look amazingly prescient.
>
> …
>
> Two weeks before Julian died, I was driving through central Iowa and was surprised and delighted to find gasoline selling for 89 cents a gallon. I hadn't seen gas prices that low since before the OPEC embargo in the early 1970s. I instantly thought of Julian. It was one of those little real-world events that confirm that he was right all along.

Today the view have see-sawed back again.

An Alternate Take

Hubbert did in the Soviet Union?

An interesting anecdote regarding the potentials of oil depletion modeling suggest that understanding this topic *well* can ultimately influence behavior [Ref 74].

> It seems that the CIA took Hubbert's methodology seriously and applied it to the USSR (Anonymous 1977). This report predicted that Soviet oil production would peak in the early 1980's. In fact there were two peaks, the first in 1983, at 12.5 million barrels per day and the second in 1988 at 12.6 barrels per day. Since then production has declined steadily. It seems likely that the Reagan administration, which took office in 1981, bearing in mind the economic havoc produced when US production peaked in 1981, followed by the Arab oil embargo and the "oil crisis" of 1973-74 and the deep recession that followed, decided to use the "oil weapon" to destabilize the USSR. Reagan embarked on a major military buildup, putting the Soviet Union under pressure to keep up. Meanwhile, declining prices after 1981 forced the USSR to pump more oil to supply its clients in Eastern Europe and to sell in world markets for hard currency. Then in 1985 Reagan persuaded Saudi Arabia to flood the world markets with cheap oil. Again, the USSR had to increase output to earn hard currency. This led to the second peak in 1988. Two years later the USSR imploded (Heinberg 2004) pp 40-41. [Ref 74]

In terms of the oil shock model, the implosion caused a reverse shock which has since stabilized.

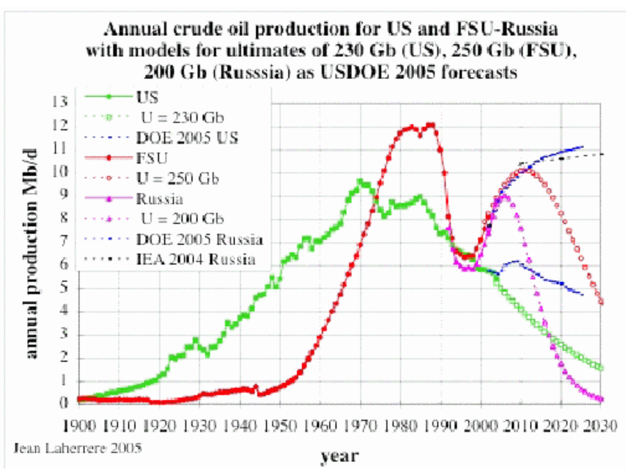

FIGURE 32-1. Dynamics of oil production in the Soviet Union as it disbanded and projections for the future (from Laherrere).

Which brings us to asking a pretentious question: Can we generate a better model than the logistic curve, donate it to the intelligence agencies, and thus help them out in putting an end to oil-funded terrorism?

Hugh Hewitt interviewing Iraqi propagandist Ahmed Chalabi:
AC: *I think Iraq is the only country in the world now that can actually produce **8 million barrels of oil a day from here until the end of the century**. Iraq is rich. Iraq also has a very, very competent and smart population. We have high levels...*(Call dropped)
HH: *Dr. Chalabi, when we got cut off, you were saying that Iraq is a rich country, capable of producing **8 million barrels of oil a day from now until the end of the century**. What's the implication of that for Iraq ten years from now, and for America's role there?*.

If the USA actually bought this propaganda in the run-up to the Iraqi war, then they received bad advice. Generating 8 million barrels a day for the next almost 100 years will never happen in Iraq.

Oil painting "oilman" courtesy of Gene Gould.

Or will the foundation of a comprehensive oil depletion model get used to make a killing in the oil futures market instead? We basically have no way of controlling such outcomes, as the psychology of the marketplace has no pre-ordained trajectory of its own. Alas, the trajectory of non-renewable resource depletion remains an invariant and it will impact us all at some point.

CHAPTER 33

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"If you want to learn how to build a house, build a house. Don't ask anybody, just build a house."

— Christopher Walken

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Web Sites

- Energy Bulletin — http://www.energybulletin.net
- GraphOilogy — http://graphoilogy.blogspot.com
- Peak Energy (Australia) — http://peakenergy.blogspot.com
- Peak Oil message board — http://peakoil.com
- ASPO International — http://www.peakoil.net
- Gristmill — http://gristmill.grist.org

- R-Squared Energy Blog — http://i-r-squared.blogspot.com
- Peak Energy (USA) — http://peake.blogspot.com
- RIGZONE — http://www.rigzone.com/data
- Resource Insights — http://resourceinsights.blogspot.com
- Azimuth Project — http://www.azimuthproject.org

Acronyms

- AAPG American Association of Petroleum Geologists
- ac acre
- AGA American Gas Association
- AGI American Geological Institute
- AGU American Geophysical Union
- ANWR Arctic National Wildlife Refuge
- API American Petroleum Institute
- ASPE American Society of Petroleum Engineers
- ASPO The Association for the Study of Peak Oil and Gas
- atm atmosphere
- Bbbl billion barrels
- bbl barrels
- BBOE billion barrels of oil equivalent
- Bcf billion cubic feet
- BLM Bureau of Land Management (DOI)
- BOE barrels of oil equivalent
- bopd barrels of oil per day
- Btu British thermal unit
- CNG Compressed Natural Gas
- DHS Department of Homeland Security
- DOE Department of Energy
- DOI Department of the Interior
- DOS Department of State
- DOT Department of Transportation
- DWOP Deep Water Operations Plan

• EA	Environmental Assessment
• EDF	Environmental Defense Fund
• EIA	Energy Information Administration (DOE)
• EIS	Environmental Impact Statement
• EPA	Environmental Protection Agency
• ESA	Endangered Species Act
• FERC	Federal Energy Regulatory Commission
• FMV	Fair Market Value
• FOIA	Freedom of Information Act
• FWS	Fish and Wildlife Service (DOI)
• FY	Fiscal Year
• GCED	Gulf Coast Environmental Defense
• GIS	Geographical Information System
• GOM	Gulf of Mexico
• GSA	Geological Society of America
• ha	hectare (2.471 acres)
• HSA	Homeland Security Act
• IAA	Interagency Agreement
• IBLA	Interior Board of Land Appeals
• IEA	International Energy Administration
• IEEE	Institute of Electrical and Electronic Engineers
• IOGCC	Interstate Oil and Gas Compact Commission
• IPAA	Independent Petroleum Association of America
• ISO	International Organization for Standardization
• LNG	liquefied natural gas
• LOOP	Louisiana Offshore Oil Port
• LOA	Letter of Agreement
• LOS	Law of the Sea
• LPG	liquefied petroleum gas
• m	meter
• Mcf	thousand cubic feet
• Mcfgpd	thousand cubic feet of gas per day
• md	Measured Depth
• MEFS	Minimum Economic Field Size

- MER Maximum Efficient Rate
- MMbbl million barrels
- MMS Minerals Management Service
- MPR Maximum Production Rate
- MOU Memorandum of Understanding
- NEP National Energy Plan
- NEPA National Environmental Policy Act
- NGL natural gas liquid
- NGPL natural gas production liquids
- nmi nautical mile
- NOAA National Oceanic and Atmospheric Administration (DOC)
- NPC National Petroleum Council
- NPV Net Present Value
- NRDA Natural Resource Damage Assessment
- NRDC Natural Resources Defense Council
- OCS Outer Continental Shelf
- OIG Office of Inspector General
- OMB Office of Management and Budget
- ORA Office of Regulatory Affairs
- PLNG Pressurized Liquefied Natural Gas
- POTUS President of the United States
- SBA Small Business Administration
- SPE Society of Petroleum Engineers
- Tcf trillion cubic feet
- TVD true vertical depth
- UCRR undiscovered conventionally recoverable resources
- UERR undiscovered economically recoverable resources
- URR ultimately recoverable resources
- USGS United States Geological Survey

Oil Depletion Analysts

1. M. King Hubbert
2. Colin Campbell

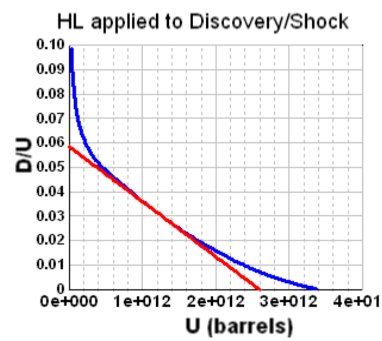
3. Jean Laherrere
4. Samuel Foucher
5. Dudley Stark
6. Rembrandt Kopelaar
7. Jeffrey Brown
8. Bernard. Michel
9. Marcel Schoppers
10. Robert L. Hirsch
11. Roger Bentley
12. Frederick Robelius
13. Ken Deffeyes
14. Jan Skrebowski
15. Ali Samsam Bakhtiari
16. Buzz Ivanhoe (world discovery data)
17. David Goodstein
18. Kjell Aleklett
19. Robert Rapier

Other Models

Several novel depletion analysis approaches exist which are referenced though various web sites.

- Particle Filtering
  http://mobjectivist.blogspot.com/2006/03/particles-on-brain.html
  How to Track an Oil Production Curve
  http://graphoilogy.blogspot.com/2006/02/how-to-track-oil-production-curve.html
- The Generalized Bass Model
  http://graphoilogy.blogspot.com/2006/01/stochastic-bass-model.html
  World Oil Depletion Models: Price Effects Compared with Strategic or Technological Interventions; Technological Forecasting and Social Change, (in press). GUSEO, R. et al. (2006).
- Loglet Analysis
  http://www.theoildrum.com/story/2006/9/3/113719/7594
  Logistic Growth and Substitution: The Mathematics of the Loglet Lab Software Package. Technological Forecasting and Social Change 61(3):247:271, 1999. Perrin S. Meyer, Jason W. Yung and Jesse H. Ausubel

- Hybrid Shock Model (HSM)
  http://graphoilogy.blogspot.com/2007_04_01_archive.html
  I like the HSM for a few largely pragmatic reasons. For one, it couples the empirical Logistic model to the rate driven Shock Model. Foucher's mathematical connection to the Shock Model provides insight on how to bridge the gap between the models. It shows the critical link from the shock model's perturbed extraction rate to an equivalent Logistic extraction rate. And I would think that for others used to looking only at the Logistic model, it provides the path to injecting some intuitive first principles from the land of empiricism.
  Secondly the HSM allows one to inject an arguably more robust predictive capability to the base shock model. Foucher has rightly stated that the original shock model does not extrapolate extraction rate beyond the currently available discovery and production data, whereas the HSM at least suggests a Logistic-like growth beyond the current point in time. This causes a natural rounding in the HSM while the base shock model can start dropping off quickly if the extraction rate is extrapolated as a constant. With a good discovery model like Dispersive Discovery, I notice that the natural rounding returns, so the unified discovery/shock model eliminates that issue. And a reserve growth maturation phase also results in a rounder peak.
- The Hubbert Linearization
  http://www.theoildrum.com/node/2357
  Although Hubbert first mentioned this and Deffeyes perfected it, Robert Rapier has played the devil's advocate in seriously looking at the shortcomings of the Hubbert Linearization heuristic. I side with Rapier in many of these cases as the dispersive discovery and oil shock models do not always linearize. In the adjacent figure, the use of HL will actually undershoot the URR estimate by about 25% (if the model proves correct).



- Export Land Model
  http://en.wikipedia.org/wiki/Export_Land_Model
  The geologist Jeffrey Brown has pioneered this analysis.

Nomenclature

Unless otherwise indicated, lower case letters denote probability densities or praobabilities. Otherwise, upper case letters denote cumulative probabilities.

$p(x|y)$ = conditional probability density of x given known y

$p(x, y)$ = joint probability density of x and y

$E(x) = \langle x \rangle$ = expected value of x

$\bar{x}$ = arithmetic mean of a set of numbers

U = cumulative (ultimate) estimate

r = rate function (fraction/time)

P = cumulative production if in barrels, instantaneous if bls/yr

R = cumulative reserve

t = time

k = arbitrary constant, i.e. Boltzmann constant

$D(t)$ = cumulative discovery

λ = dispersed measure (length or volume) variate

Q = cumulative measure

L = container size (usually length)

V = container size (volume)

C = constant

$P(x)$ = cumulative probability distribution

T = temperature

D = diffusivity

μ = mobility

$C(t, x)$ = concentration

τ = time constant

υ = velocity

$n(x, t)$ = concentration

$g(t)$ = growth function

W = width

CHAPTER 34

Code

"Like Cindy Crawford, I have done quite a bit of modeling in my career."

— Robert Rapier

Code Snippet #1

This is GNAT code suitable for compiling with a GNU GCC compiler. It will generate a comma separated value output which a spreadsheet can read.

Triangular discovery stimulus convolved with unperturbed shock model

```
with Text_IO;
with Ada.Numerics.Elementary_Functions;

procedure Conv is
   type Flt is array(Natural range) of Float;

   function Get (Arr : Flt; I : Integer) return Float is
   begin
      if I Arr'Last then
         return 0.0;
      else
         return Arr(I);
      end if;
   end;

   function Convolve (A, B : in Flt) return Flt is
      Total : constant Natural := A'Length+B'Length;
```

```
      C : Flt(0..Total);
      V : Float;
   begin
      for J in 0..Total loop
         V := 0.0;
         for I in 0 .. J loop
            V := V + Get(A, I) * Get(B, J-I);
         end loop;
         C(J) := V;
      end loop;
      return C;
   end;

   function Triangle_Window (L : Natural) return Flt is
      R : Flt(0..L-1);
      Half : constant Natural := L/2;
   begin
      for I in 0..L-1 loop
         if I Half then
            R(I) := Float(I);
         else
            R(I) := Float(L - I);
         end if;
      end loop;
      return R;
   end;

   function Exponential (L : Natural; Alpha : Float) return Flt is
      use Ada.Numerics.Elementary_Functions;
      R : Flt(0..L-1);
   begin
      for I in 0..L-1 loop
         R(I) := Alpha * exp(-Alpha*Float(I));
      end loop;
      return R;
   end;

   W : constant Flt := Triangle_Window(130);
   E : constant Flt := Exponential(300, 0.02);
   R : constant Flt := Convolve (W, E);
   Year : constant Natural := 1900;
begin
   for I in R'Range loop
      Text_IO.Put_Line(Integer(Year+I)'Img & "," &
         Get(W,I)'Img & "," & R(I)'Img);
   end loop;
end;
```

Code Snippet #2

Triangular discovery stimulus convolved with perturbed shock model

```
with Text_IO;
procedure Shock_Model_Continuous is

   type Shock is record
      Year : Float; -- Year of shock
      Rate : Float; -- Rate of extraction
   end record;
   type Shocks is array (Natural range ) of Shock;

   Triangle_Width : constant := 87.0; -- Triangle window
   Volume : constant := 2400.0; -- In BBls

   S : constant Shocks :=
        ((1944.0, 0.060), -- Start of data
        (1974.0, 0.060), -- Start of oil embargo
        (1974.5, 0.051), -- End of oil embargo
        (1980.0, 0.051), -- Start of Iran hostage crisis
        (1984.0, 0.034), -- End of recession
        (1990.0, 0.034), -- Start of Gulf War
        (1992.0, 0.030), -- End of recesssion
        (2100.0, 0.030));

   -- Make the shock a continuous function
   -- by interpolating between points in the list
   function Interpolate_Shocks (Year : Float) return Float is
      K : Integer := S'Last - 1;
   begin
      for J in S'First + 1 .. S'Last loop
         if S (J).Year > Year then
            K := J - 1;
            exit;
         end if;
      end loop;
      return S (K).Rate +
             (S (K + 1).Rate - S (K).Rate) * (Year - S (K).Year) /
             (S (K + 1).Year - S (K).Year);
   end Interpolate_Shocks;

   -- Creates a symmetric window for dicovered reserves
   function Triangle_Window (Year, Start, Length : Float)
      return Float is
      Y : constant Float := Year - Start;
      Half : constant Float := Length / 2.0;
   begin
      if Y Length then
         return 0.0;
```

```
      end if;
      if Y Half then
         return Y / Half;
      else
         return (Length - Y) / Half;
      end if;
   end Triangle_Window;

   C : Float := 0.0; -- Reserve
   R : Float; -- Extraction Rate
   P : Float; -- Production Rate
   DT : constant Float := 0.01; -- 1/100th of year
   Start : constant Float := S (S'First).Year;
   Finish : constant Float := S (S'Last).Year;
   Time : Float := Start;
   K : constant Float := Volume / Triangle_Width * 2.0;
   V : Float := 0.0; -- Volume of oil extracted
begin
   loop
      R := Interpolate_Shocks (Time);
      C := C +K * Triangle_Window (Time, Start, Triangle_Width) * DT
         - C * R * DT;
      P := R * C / 365.0; -- BBls/day
      V := V + R * C * DT;
      if Time > Float (Integer (Time)) - DT / 2.0 and
         Time Float (Integer (Time)) + DT / 2.0
      then -- Print output only once per year
         Text_IO.Put_Line
         (Integer'Image (Integer (Time)) & "," & P'Img & "," & V'Img);
      end if;
      Time := Time + DT;
      exit when Time > Finish;
   end loop;
end Shock_Model_Continuous;
```

Code Snippet #3

This code compiles with the GCC gnatmake compiler provided with most recent vintages of Linux (or downloadable for Windows). To build, save as file "shock_model.adb", then `gnatmake shock_model` to make the executable. Spreadsheets read the CSV-format: "year", "BBls/day", "cumulative BBls"

Triangular discovery stimulus convolved with perturbed shock model, improved

```
with Text_IO;
with Ada.Numerics.Elementary_Functions;
```

```
procedure Shock_Model is

   subtype Years is Natural range 1944 .. Natural'Last;

   type Shock is record
      Year : Years; -- Year of shock
      Rate : Float; -- Prod. or depletion rate (fraction / year)
   end record;
   type Shocks is array (Years range <>) of Shock;

   Width : constant := 87; -- Triangle window
   Volume : constant := 2400.0; -- In BBls

   S : constant Shocks :=
      ((Years'First, 0.06), -- Starting Year
      (1974, 0.048), -- OPEC
      (1980, 0.035), -- Iran +
      (1983, 0.029), -- Recession
      (1991, 0.026), -- Gulf I
      (2150, 0.0)); -- Ending Year

   -- General purpose array of floats indexed by year
   type Flt is array (Natural range <>) of Float;

   -- Safe array get function, truncates values out of range
   function Get (Arr : Flt; I : Integer) return Float is
   begin
      if I in Arr'Range then
         return Arr (I);
      else
         return 0.0;
      end if;
   end Get;

   -- Takes 2 arrays and returns the convolution array
   function Convolve (A, B : in Flt) return Flt is
      Total : constant Natural := A'Length + B'Length;
      C : Flt (0 .. Total);
      V : Float;
   begin
      for J in 0 .. Total loop
         V := 0.0;
         for I in 0 .. J loop
            V := V + Get (A, I) * Get (B, J - I);
         end loop;
         C (J) := V;
      end loop;
      return C;
   end Convolve;

   -- Creates a symmetric window, truncated left by Start
   -- The prior accumulated data is provided by Impulse
```

```
function Triangle_Window
    (Width : in Natural;
     Start : in Natural := 0;
     Impulse : Float := 0.0) return Flt is
    R : Flt (0 .. Width - 1 - Start);
    Half : constant Natural := Width / 2;
    Offset : Natural;
begin
    for I in R'Range loop
        Offset := I + Start;
        if Offset Half then
            R (I) := Float (Offset) / Float (Half); else
            R (I) := Float (Width - Offset) / Float (Half);
        end if;
    end loop;
    R (0) := R (0) + Impulse;
    return R;
end Triangle_Window;

-- An exponential array populated to length
function Exponential
    (Length : in Natural; Alpha : in Float) return Flt is
    use Ada.Numerics.Elementary_Functions;
    R : Flt (0 .. Length - 1);
begin
    for I in 0 .. Length - 1 loop
        R (I) := Alpha * Exp (-Alpha * Float (I));
    end loop;
    return R;
end Exponential;

-- The summed difference between two arrays up to Width
-- This is used to renormalize when an Alpha rate changes
function Leftover
    (A, B : in Flt; Width : in Natural) return Float is
    V1 : Float := 0.0;
    V2 : Float := 0.0;
begin
    for J in 0 .. Width - 1 loop
        V1 := V1 + Get (A, J);
        V2 := V2 + Get (B, J);
    end loop;
    V1 := V1 + 0.5 * Get (A, Width);
    V2 := V2 + 0.5 * Get (B, Width);
    return (V1 - V2);
end Leftover;

-- Recursive formula to re-convolute on successive rate shocks
-- Returns production curve in array
function Gen
    (S : in Shocks; -- Input stimulus
     Left : in Float; -- Pre-history reserve level
```

```
         Curve : in Flt; -- The starting curve
         TS : in Natural) -- TimeSpan of results
        return Fltis
    begin
        if S'Length = 1 then -- returns on completion
            return Curve;
        else
            declare
                W1 : constant Natural := S (S'First).Year - Years'First;
                W2 : constant Natural := S (S'First+1).Year - Years'First;
                T : constant Flt := Triangle_Window (Width, W1, Left);
                R : constant Flt :=
                     Convolve (T, Exponential (TS, S (S'First).Rate));
            begin
                return Gen
                    (S => S (S'First + 1 .. S'Last),
                     Left => Leftover (T, R, W2 - W1),
                     Curve => Curve & R (1 .. W2 - W1),
                     TS => TS);
            end;
        end if;
    end Gen;

    Time_Span : Natural := S (S'Last).Year - S (S'First).Year;
    R : constant Flt := Gen (S, 0.0, (0 => 0.0), Time_Span);
    Total : Float := 0.0;
    K : constant Float := Volume / Float (Width) * 2.0;
begin
    for I in R'Range loop
        Total := Total + K * R (I);
        Text_IO.Put_Line
            (Integer (Years'First + I)'Img & "," & -- Year
             Float'Image (K * R (I) / 365.0) & "," & -- Per Day
             Float'Image (Total));
    end loop;
end Shock_Model;
```

Code Snippet #4

Prints to standard out “Year”, “ModelResults”, “BPData”

World discovery stimulus ASPO convolved with perturbed shock model, compared to BP production data

```
with Text_IO;
with Ada.Numerics.Elementary_Functions;

procedure ASPO is
```

```
   type Flt is array (Natural range ) of Float;

   -- Safe array retrieval function
   function Get (Arr : Flt; I : Integer) return Float is
   begin
      if I in Arr'Range then
         return Arr (I);
      else
         return 0.0;
      end if;
   end Get;

   -- Discretized convolution function
   function Convolve (A, B : in Flt) return Flt is
      Total : constant Natural := A'Length + B'Length;
      C : Flt (0 .. Total);
      V : Float;
   begin
      for J in 0 .. Total loop
         V := 0.0;
         for I in 0 .. J loop
            V := V + Get (A, I) * Get (B, J - I);
         end loop;
         C (J) := V;
      end loop;
      return C;
   end Convolve;

   -- Discretized exponential function
   function Exponential (L : Natural; Alpha : Float) return Flt is
      use Ada.Numerics.Elementary_Functions;
      R : Flt (0 .. L - 1);
   begin
      for I in 0 .. L - 1 loop
         R (I) := Alpha * Exp (-Alpha * Float (I));
      end loop;
      return R;
   end Exponential;

   -- Data transcribed from ASPO discovery curve
   -- http://wolf.readinglitho.co.uk/mainpages/discoveries.html
   Strikes : constant Flt := -- GBBls/year starting year 1932
(9.0, 4.0, 4.5, 3.0, 5.0, 5.5, 42.0, 42.5, --1930's
52.0, 18.0, 16.0, 5.0, 3.0, 7.4, 7.6, 7.0, 49.0, 53.0, --1940's
54.0, 19.5, 16.0, 26.0, 20.0, 28.0, 24.0, 35.0, 40.0, 40.5, --1950's
42.0, 44.0, 50.0, 41.0, 50.0, 48.5, 47.5, 32.0, 30.5, 30.4, --1960's
29.5, 40.5, 37.0, 38.5, 24.0, 26.5, 31.0, 37.0, 38.0, 36.0, --1970's
26.0, 24.0, 20.0, 20.0, 22.0, 21.0, 20.5, 18.0, 16.2, 17.5, --1980's
16.5, 20.0, 17.0, 16.0, 9.0, 8.5, 9.0, 9.0, 9.5, 14.0, --1990's
18.0, 17.5, 13.0, 9.0, 9.0); --2004

   Avg_Fallow : constant Flt := Exponential (200, 0.125); -- 8 years
```

```
Avg_Build : constant Flt := Exponential (200, 0.125);
Avg_Mature : constant Flt := Exponential (200, 0.125);
Discovery : constant Flt :=
Convolve (Strikes,
          Convolve (Avg_Fallow,
                 Convolve (Avg_Build, Avg_Mature)));
-- Safe function for reading forcing function
function Discovery_Window (Year, Start : Float) return Float is
   Y : constant Float := Year - Start;
begin
   if Y 0.0 then
      return 0.0;
   end if;
   return Discovery (Integer (Y));
end Discovery_Window;

-- British Petroleum data
BP_data : array (1965 .. 2500) of Integer := -- in million Bls/day
(1965 => 31803,
 1966 => 34568,
 1967 => 37118,
 1968 => 40436,
 1969 => 43633,
 1970 => 48061,
 1971 => 50844,
 1972 => 53666,
 1973 => 58463,
 1974 => 58617,
 1975 => 55824,
 1976 => 60412,
 1977 => 62713,
 1978 => 63331,
 1979 => 66049,
 1980 => 62946,
 1981 => 59533,
 1982 => 57296,
 1983 => 56598,
 1984 => 57683,
 1985 => 57468,
 1986 => 60461,
 1987 => 60785,
 1988 => 63160,
 1989 => 64051,
 1990 => 65470,
 1991 => 65288,
 1992 => 65788,
 1993 => 66046,
 1994 => 67116,
 1995 => 68103,
 1996 => 69895,
 1997 => 72158,
 1998 => 73586,
```

```
 1999 => 72333,
 2000 => 74950,
 2001 => 74828,
 2002 => 74443,
 2003 => 77054,
 2004 => 80260,
 others => 0);

type Shock is record
   Year : Float; -- Year of shock
   Rate : Float; -- Rate of extraction
end record;
type Shocks is array (Natural range ) of Shock;

S : constant Shocks :=
((1932.0, 0.070), -- Start of data
 (1974.0, 0.070), -- Start of oil embargo
 (1974.5, 0.065), -- End of oil embargo
 (1980.0, 0.065), -- Start of Iran hostage crisis
 (1983.5, 0.044), -- End of recession
 (1990.0, 0.044), -- Start of Gulf War
 (1992.0, 0.042), -- End of recesssion
 (2001.0, 0.042), -- Last good year!
 (2003.0, 0.046), -- Running Out??? Why is extraction going up?
 (2100.0, 0.046));

-- Make the shock a continuous function
-- by interpolating between points in the list
function Interpolate_Shocks (Year : Float) return Float is
   K : Integer := S'Last - 1;
begin
   for J in S'First + 1 .. S'Last loop
      if S (J).Year > Year then
         K := J - 1;
         exit;
      end if;
   end loop;
   return S (K).Rate +
          (S (K + 1).Rate - S (K).Rate) * (Year - S (K).Year) /
          (S (K + 1).Year - S (K).Year);
end Interpolate_Shocks;

C : Float := 0.0; -- Reserve
Rate : Float; -- Extraction Rate
P : Float; -- Production Rate
DT : constant Float := 0.01; -- 1/100th of year
Start : constant Float := S (S'First).Year;
Finish : constant Float := S (S'Last).Year;
Time : Float := Start;
V : Float := 0.0; -- Volume of oil extracted
Data : Float := 0.0;
```
```
begin
```

```
    loop
       -- Integration of discovery window against extraction rate
       Rate := Interpolate_Shocks (Time);
       C := C + Discovery_Window (Time, Start) * DT - C * Rate * DT;
       P := Rate * C;
       V := V + Rate * C * DT;
       if Time > Float (Integer (Time)) - DT / 2.0 and
          Time Float (Integer (Time)) + DT / 2.0
       then -- Print output only once per year
          if Time >= Float (BP_data'First) then
             -- BP data only available beyond a certain date
             Data := Float (BP_data (Integer (Time))) * 365.0 / 1.0e6;
          end if;
          Text_IO.Put_Line (Integer'Image (Integer (Time)) & "," &
             P'Img & "," &Data'Img & "," & Float'Image (V));
       end if;
       Time := Time + DT;
       exit when Time > Finish;
    end loop;
end ASPO;
```

Code Snippet #5

A generalized command-line driven program that takes input discovery data (+ other parameters) and which then generates production curves from the date of first discovery to the present time (and on out into the future). The code basically refactors the specific code from other code snippets, to get a better separation of data from the algorithms.

The program reads the input parameters from either a set of files (for yearly data) or from the term following the option (for fixed constants). It doesn't matter what units you use as long as they remain consistent with the other ones. The program sends the output data to the shell terminal.

`-d`

File containing discovery data (continuous years, one entry per line)

`-s`

File containing shock data (only years with discontinuities), two entries per line, _year_ + (space) + _extraction rate that year_. The program interpolates between entries, so to prevent shocks, give it only two lines (start and finish times) with the same extraction rate on each line.

`-p`

Optional file containing production data (continuous years, one entry per line).

The program doesn't do anything with this data; it only carries it over to make it convenient for comparison to the estimated production.

-1

The *1/e* rate for remaining fallow (i.e. 1 over the average time between discovery and construction)

-2

The *1/e* rate for finishing construction (i.e. 1 over the build time)

-3

The *1/e* rate for reaching maturing (i.e. 1 over the startup time)

-t

Timespan for complete simulation

Typical invocation:

shockmodel.exe -1 0.2 -2 0.2 -3 0.2 -t 600 -d strikes.dat -p prod.dat -s shocks.dat > output

General Shock Model Program

```
with Text_IO;
with Ada.Numerics.Elementary_Functions;
with GNAT.Command_Line;
with Ada.Float_Text_IO;

procedure ShockModel is
   type Flt is array (Natural range ) of Float;
   type Flt_Access is access all Flt;

   Expansion_Factor : constant := 10; -- Year subtics
   Time_Span : Integer := 600;

   -- Safe array retrieval function
   function Get (Arr : Flt; I : Integer) return Float is
   begin
      if I in Arr'Range then
         return Arr (I);
      else
         return 0.0;
      end if;
   end Get;

   -- Discretized convolution function
   function "*" (A, B : in Flt) return Flt is
      Total : constant Natural := A'Length + B'Length;
```

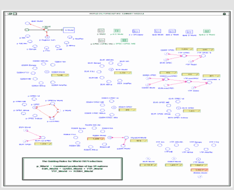
```
   C : Flt (0 .. Total);
   V : Float;
begin
   for J in 0 .. Total loop
      V := 0.0;
      for I in 0 .. J loop
         V := V + Get (A, I) * Get (B, J - I);
      end loop;
      C (J) := V;
   end loop;
   return C;
end "*";

-- Discretized exponential function
function Exponential (L : Natural; Alpha : Float) return Flt is
   use Ada.Numerics.Elementary_Functions;
   R : Flt (0 .. L - 1);
begin
   for I in 0 .. L - 1 loop
      R (I) := Alpha * Exp (-Alpha * Float (I));
   end loop; return R;
end Exponential;

-- Fills in data between discrete years
function Expander (Arr : Flt) return Flt is
   R : Flt (0 .. Arr'Length * Expansion_Factor - 1);
begin
   for I in R'Range loop
      R (I) := Arr (I / Expansion_Factor);
   end loop;
   return R;
end Expander;

-- Safe function for reading forcing function
function Discovery_Window
   (Discovery : Flt; Year, Start : Float) return Float is
   Y : constant Float := Year - Start;
begin
   if Y 0.0 then
      return 0.0;
   end if;
   return Discovery (Integer (Y) * Expansion_Factor);
end Discovery_Window;

type Shock is record
   Year : Float; -- Year of shock
   Rate : Float; -- Rate of extraction
end record;
type Shocks is array (Natural range ) of Shock;
type Shocks_Access is access all Shocks;

-- Make the shock a continuous function
```

```
-- by interpolating between points in the list
function Interpolate_Shocks (Year : Float; S : Shocks) return Float is
   K : Integer := S'Last - 1;
begin
   for J in S'First + 1 .. S'Last loop
      if S (J).Year > Year then
         K := J - 1; exit;
      end if;
   end loop;
   return S (K).Rate +
      (S (K + 1).Rate - S (K).Rate) * (Year - S (K).Year) /
       (S (K + 1).Year - S (K).Year);
 end Interpolate_Shocks;

-- Strikes into Mature Discovery
Strikes : Flt_Access;
Discovery : Flt_Access;
Phases : Flt (1 .. 3) := (others => 0.0);

-- Generates the mature discovery by repeated convolution
procedure Phasing is
   Avg_Fallow : constant Flt :=
    Exponential (Time_Span, Phases (1)/Float (Expansion_Factor));
   Avg_Build : constant Flt :=
    Exponential (Time_Span, Phases (2) / Float (Expansion_Factor));
   Avg_Mature : constant Flt :=
    Exponential (Time_Span, Phases (3) / Float (Expansion_Factor));
begin
   Discovery := new Flt'(Expander (Strikes.all) * Avg_Fallow *
                  Avg_Build * Avg_Mature);
end Phasing;

S : Shocks_Access;
Production : Flt_Access; -- To compare against, optional

procedure Compute is
   C : Float := 0.0; -- Reserve
   Rate : Float; -- Extraction Rate P : Float; -- Production Rate
   DT : constant Float := 0.01; -- 1/100th of year
   Start : constant Float := S (S'First).Year;
   Finish : constant Float := S (S'Last).Year;
   Time : Float := Start;
   V : Float := 0.0; -- Volume of oil extracted
   Data : Float := 0.0;
   PIndex : Integer;
begin
   loop
      -- Integration of discovery window against extraction rate
      Rate := Interpolate_Shocks (Time, S.all);
      C := C +
           Discovery_Window (Discovery.all, Time, Start) * DT -
           C * Rate * DT;
```

```
        P := Rate * C;
        V := V + Rate * C * DT;
        if Time > Float (Integer (Time)) - DT / 2.0 and
           Time Float (Integer (Time)) + DT / 2.0
        then -- Print output only once per year
           PIndex := Integer (Time - Start);
           if PIndex in Production'Range then
              Data := Production (PIndex);
           else
              Data := 0.0;
           end if;
           Text_IO.Put_Line
             (Time'Img & "," & P'Img & "," & Data'Img);
        end if;
        Time := Time + DT;
        exit when Time > Finish;
     end loop;
  end Compute;

  function Get_Flt_Data (File : in String) return Flt_Access is
     FT : Text_IO.File_Type;
     function Get_Data (Data : in Flt) return Flt is
        Value : Flt (0 .. 0);
     begin
        Ada.Float_Text_IO.Get (FT, Value (0));
        Text_IO.Skip_Line (FT);
        return Get_Data (Data & Value);
     exception
        when Text_IO.End_Error =>
           return Data;
     end Get_Data;
  begin
     Text_IO.Open (FT, Text_IO.In_File, File);
     return new Flt'(Get_Data (Flt'(1 .. 0 => 0.0)));
  end Get_Flt_Data;

  function Get_Shock_Data (File : in String) return Shocks_Access is
     FT : Text_IO.File_Type;
     function Get_Data (Data : in Shocks) return Shocks is
        Pair : Shocks (0 .. 0);
     begin
        Ada.Float_Text_IO.Get (FT, Pair (0).Year);
        Ada.Float_Text_IO.Get (FT, Pair (0).Rate);
        Text_IO.Skip_Line (FT);
        return Get_Data (Data & Pair);
     exception
        when Text_IO.End_Error =>
           return Data;
     end Get_Data;
  begin
     Text_IO.Open (FT, Text_IO.In_File, File);
     return new Shocks'(Get_Data (Shocks'(1 .. 0 => (0.0, 0.0))));
```

```
    end Get_Shock_Data;

begin
    -- Command line options
    loop
        case GNAT.Command_Line.Getopt ("1: 2: 3: t: d: s: p:") is
            when ASCII.NUL =>
                exit;
            when '1' =>
                Phases (1) := Float'Value (GNAT.Command_Line.Parameter);
            when '2' =>
                Phases (2) := Float'Value (GNAT.Command_Line.Parameter);
            when '3' =>
                Phases (3) := Float'Value (GNAT.Command_Line.Parameter);
            when 't' =>
                Time_Span := Integer'Value (GNAT.Command_Line.Parameter);
            when 'd' =>
                Strikes := Get_Flt_Data (GNAT.Command_Line.Parameter);
            when 's' =>
                S := Get_Shock_Data (GNAT.Command_Line.Parameter);
            when 'p' =>
                Production := Get_Flt_Data (GNAT.Command_Line.Parameter);
            when others =>
                raise Program_Error; -- cannot occur_
        end case;
    end loop;
    Phasing;
    Compute;
exception
    when GNAT.Command_Line.Invalid_Switch =>
        Text_IO.Put_Line ("Invalid Switch " &
                    GNAT.Command_Line.Full_Switch);
    when GNAT.Command_Line.Invalid_Parameter =>
        Text_IO.Put_Line ("No parameter for " &
                    GNAT.Command_Line.Full_Switch);
end ShockModel;
```

Dr. Richard Duncan used a commercial piece of software, Stella, from ISEE Systems, to forecast future trends in oil production. Since the vendor markets Stella as a system's modeling tool which ostensibly solves linear and non-linear dynamics, I thought Duncan may have gone the extra mile and actually done something above and beyond using it as simply a charting and bubble diagramming tool.



The company also provides a sample model for Oil Price Dynamics. Based on my own evaluation of a 30-day trial version, I believe that Stella, with enough fiddling around, may turn out as potentially useful to someone as a depletion modeling tool. However, with a single-seat license costing around **$1800**, I will pass on it.

Other than using my own custom-programmed algorithms, I have had better success modeling oil depletion using a free-ware electrical circuit simulator than my rudimentary attempts at using the Stella tool. The tool Modelica may prove useful as

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