Q1. The plot of normal distribution is symmetrically bell shaped, [illegible].

It satisfies the conditions of normal distribution –

(i) mean = median (bell shaped at centre)

(ii) ≈ 68% of the data falls within 1 S.D. of the mean

(iii) Asymptotic

(iv) Unimodal (one maxima)

Q2.

(a) Expectation value increases by the same amount by which each observation increased

we increased all grades by 1 mark, and E(grades) also increased by 1

But, variance remains unchanged.

(b) $f(x) = \frac{c}{x^3}$ $\quad$ $A = (-\infty, \infty) - (-1, 1)$

$$E(X) = \int_A x f(x)\,dx$$

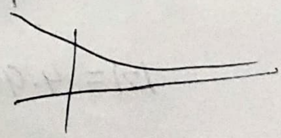
$$= \int_{-\infty}^{-1} x f(x)\,dx + \int_1^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{-1} \frac{c}{x^2}\,dx + \int_1^{\infty} \frac{c}{x^2}\,dx$$

$$= \left(\frac{-c}{x}\right)_{-\infty}^{-1} + \left(\frac{-c}{x}\right)_1^{\infty}$$

$$= c + c = 2c$$

$\therefore E(X) = \boxed{2c = \text{Expectation}}$



$$E(X^2) = \int_A x^2 f(x)\,dx$$

$$= \int_{-\infty}^{-1} x^2 f(x)\,dx + \int_1^{\infty} x^2 f(x)\,dx$$

$$= \int_{-\infty}^{-1} \frac{c}{x}\,dx + \int_1^{\infty} \frac{c}{x} = (\ln|x|)_{-\infty}^{-1} + (\ln|x|)_1^{\infty}$$

$=$

$$\text{variance} = E((x-\mu)^2) \qquad A = (-\infty,\infty) - (-1,1)$$

$$= \int_A (x-\mu)^2 f(x)\, dx$$

$$= \int_A \left(x^2 f(x) + \mu^2 f(x) - 2x\mu f(x)\right) dx$$

$$= \underbrace{\int_{-\infty}^{-1} \frac{c}{x}\, dx + \int_{1}^{\infty} \frac{c}{x}\, dx}_{\text{divergent integrals}} + \mu^2 f(x)\, dx - 2x\mu f(x)\, dx$$

$\Rightarrow$ variance is "infinite"

Now,

$$Y = aX + b$$

$$E(Y) = E(aX+b) = aE(X) + b$$
$$= 2ac + b$$

Var. (σ^2) $= E(Y^2) - (E(Y))^2 = E((ax+b)^2) - (E(ax+b))^2$

$$= E(a^2x^2 + b^2 + 2abx) - (aE(x)+b)^2$$

$$= (a^2E(x^2) + \cancel{b^2} + \cancel{2ab\, E(x)})$$
$$- (a^2E(x)^2 + \cancel{b^2} + \cancel{2abE(x)})$$

$$= a^2(E(x^2) - (E(x))^2) \quad (a^2 \text{ times the initial variance})$$
$$= a^2 \cdot \infty = \text{infinite}$$

H_0 : mean is 1.8m ($\mu = 1.8m$)

H_1 : $\mu \neq 1.8m$

i.e. $\mu < 1.8m$ or $\mu > 1.8m$ (two-tailed)

Q4. Given $\bar{x} = 1.73m$, $\mu = 1.8m$

$\sigma = 0.2m$ (known) , $n = 200$ [large sample]

so, we apply z-test

Null Hypothesis , H_0 : The sample has come from a population whose mean , μ is 100

i.e.

$H_0 : \mu = 100$

$H_1 : \mu \neq 100$ (Two-tailed test)

Test statistic →

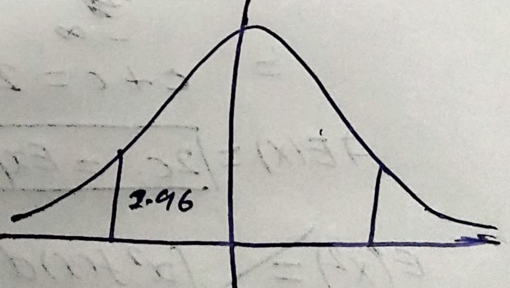
$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{1.73 - 1.8}{\sigma_{\bar{x}}}$$

where

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{200}} = 0.01414213$$

$\Rightarrow z = -4.949747$

$|z| = 4.949747$



Level of significance $= \alpha$

(i) for $\alpha = 5\%$

critical value (z_α) = 1.96

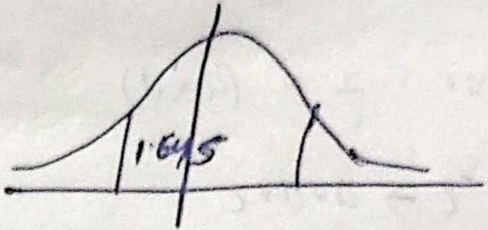
$|z| > 1.96$, ⇒ H_0 is rejected

or

Null Hypothesis is Rejected

(b) $\alpha = 10\%$

critical value (z_α)



$|z| > 1.645$

hence, null hypothesis is is rejected