Assignment 2

Q1. Given $\bar{x} = 535$, $\sigma = 96$, $n = 50$, $\mu_0 = 500$

$H_0 : \mu > 500 = \mu_0$ (modified increases yield)

$H_0 : \mu \le 500 = \mu_0$

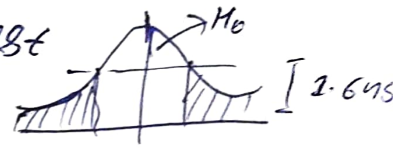
Test statistic →

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{|535 - \mu|}{96/\sqrt{30}} ? \frac{35}{(96/\sqrt{50})} = 0.3645833 \times \sqrt{50} = 2.577993$$

$z > 0.3645833$

Region for acceptance of null hypothesis:

$H_0 : \mu > \mu_0 \Rightarrow$ upper-Tailed test



For $\alpha = 0.05 \Rightarrow z \ge 1.645$

(5% significance) $\because z = 2.577993$

$\Rightarrow H_0$ is accepted

or

null hypothesis is accepted

Hence, at 5% significance the modified process increases the yield

Q2. $\hat{y_i} = a_0 + a_1 x$

$$\begin{bmatrix} n & \sum_{i=1}^{n}(x_i) \\ \sum_{i=1}^{n}(x_i) & \sum_{i=1}^{n}(x_i^2) \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{n}(y_i) \\ \sum_{i=1}^{n}(x_i y_i) \end{Bmatrix}$$

$A'x = b$

$$\begin{bmatrix} 20 & 1023.0599 \\ 1023.0599 & 64071.90056 \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{Bmatrix} 190 \\ 12494.87904 \end{Bmatrix}$$

$$\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{Bmatrix} -2.593307392 \\ 0.2364532019 \end{Bmatrix}$$

$a_0 = -2.593307392$

$a_1 = 0.2364532019$

$$R^2 = 1 - \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - \bar{y})^2} = 0.950110619$$

Q5.
(a) the variables,
a linear regression involves fitting with a (linear power) 1st degree curve

$$\hat{y} = ax + b$$

(b)

$$\begin{bmatrix} \sum_i 1 & \sum_i x_i & \sum_i x_i^2 \\ \sum_i x_i & \sum_i x_i^2 & \sum_i x_i^3 \\ \sum_i x_i^2 & \sum_i x_i^3 & \sum_i x_i^4 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} \sum_i y_i \\ \sum_i (x_i y_i) \\ \sum_i (x_i^2 y_i) \end{bmatrix}$$

$$\begin{bmatrix} n & \sum x & \sum x^2 \\ \sum x & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum (xy) \\ \sum (x^2 y) \end{bmatrix}$$

$$n\beta_0 + (\sum x)\beta_1 + (\sum x^2)\beta_2 = (\sum y) \quad \text{— (I)}$$

$$(\sum x)\beta_0 + (\sum x^2)\beta_1 + (\sum x^3)\beta_2 = \sum (xy) \quad \text{— (II)}$$

$$(\sum x^2)\beta_0 + (\sum x^3)\beta_1 + (\sum x^4)\beta_2 = \sum (x^2 y) \quad \text{— (III)}$$

solving these three eqns we can get a general solution for $\beta_0, \beta_1, \beta_2$

for $\beta_2 = 0$

$$\begin{bmatrix} n & \sum x \\ \sum x & \sum x^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \end{bmatrix}$$

$$\Rightarrow \sum x\, n\beta_0 + (\sum x)^2 \beta_1 = (\sum y)(\sum x)$$

$$n\sum x \beta_0 + n\sum x^2 \beta_1 = n\sum (xy)$$

$$\beta_1 = \left((\sum x)(\sum y) - n\sum (xy)\right) / \left((\sum x)^2 - n\sum x^2\right)$$

$$B_1 = \frac{(\Sigma x)(\Sigma y) - n(\Sigma(xy))}{(\Sigma x)^2 - n(\Sigma x^2)}$$

$$nB_0 = \frac{\Sigma y - \Sigma x B_1}{1}$$

$$= \frac{-(\Sigma x)^2(\Sigma y) + n\,\Sigma(xy)\cdot\Sigma x}{(\Sigma x)^2 - n(\Sigma x^2)} + \Sigma y$$

$$= \left(\frac{-\cancel{(\Sigma x)^2(\Sigma y)} + n\,\Sigma(xy)\cdot\Sigma x + \cancel{(\Sigma y)\cdot(\Sigma x)^2} - n(\Sigma x^2)(\Sigma y)}{(\Sigma x)^2 - n(\Sigma x^2)} \right) / n$$

$$= \frac{n\,\Sigma xy\cdot\Sigma x - n\,\Sigma x^2\cdot\Sigma y}{(\Sigma x)^2 - n(\Sigma x^2)}$$

$$\Rightarrow B_0 = \frac{\Sigma xy\cdot\Sigma x - \Sigma x^2\cdot\Sigma y}{(\Sigma x)^2 - n(\Sigma x^2)}$$