Lecture Note (PHY-110)

Math and Natural Science Department

Brac University

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Chapter 1

2D Motion

We have introduced the concepts of position, velocity and acceleration to describe motion in one dimension; however we live in a multidimensional universe. In order to explore and describe motion in more than one dimension, we need to understand clearly how we can define those physical quantity in higher dimensional universe.

1.1 Position, Velocity, Acceleration in 2D

Now let's consider we want to locate a particle in 2 dimension. So, we need to extend our ideas of position, velocity, acceleration to higher dimension. Let's

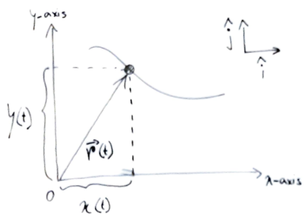

Figure 1.1: Location of a particle in 2 D

consider the diagram shown above in (fig-1.1), a particle is moving along the curve line. At any time t we can locate the particle with the position vector $\vec{r}(t)$. Now as it's a vector quantity, we can decompose the two components x and y along the two axis. In this way we can treat each direction independently. In Cartesian coordinates, the position vector $\vec{r}(t)$ with respect to some choice of origin for the object at time t is given by,

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} \tag{1.1}$$

We can define now the average velocity for a time interval Δt as,

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} \tag{1.2}$$

$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} \tag{1.3}$$

If we take the limiting value $\lim_{\Delta t \to 0}$, we can get the instantaneous velocity,

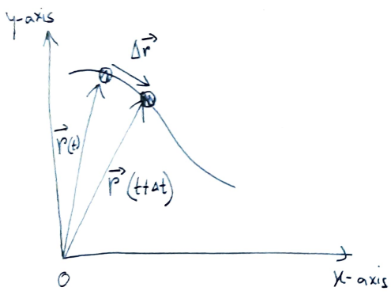

Figure 1.2: Average velocity

$$v(\vec{t}) = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{i} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\hat{j}$$

$$v(\vec{t}) = \frac{dx(t)}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$v(\vec{t}) = v_x(t)\hat{i} + v_y(t)\hat{j}$$

where $v_x(t) = dx(t)/dt$ and $v_y(t) = dy(t)/dt$ denote the x and y components of the velocity respectively.

The acceleration vector $\vec{a}(t)$ is defined in a similar fashion as the derivative of the velocity vector,

$$\vec{a}(t) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j} \tag{1.4}$$

we can find the magnitude at any given time t as,

$$|\vec{a}(t)| = \sqrt{a_x^2 + a_y^2}$$

1.2 Projectile Motion

Consider the motion of a body that is released at time $t = 0$ with an initial velocity v_0 . To describe this motion, first choose a coordinate with the positive y-axis in

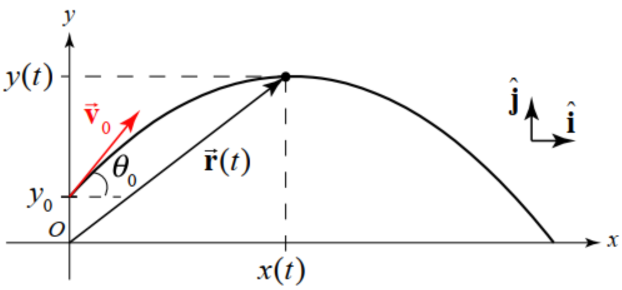

Figure 1.3: Projectile motion

the upward vertical direction and the positive x-axis in the horizontal direction in the direction that the object is moving horizontally. And choose the origin at the ground immediately below the point the object is released. In the figure above shows, our coordinate system with the position of the object $\vec{r}(t)$ at time t , the initial velocity $\vec{v}_0$, and the initial angle θ_0 with respect to the horizontal, and the coordinate functions x(t) and y(t) . Let's consider the initial conditions,

1.2.1 Initial conditions

We can decompose the initial velocity $\vec{v}_0$ in the two components in Cartesian coordinate system.

$$\vec{v}_0 = v_{x,0}\hat{i} + v_{y,0}\hat{j} \tag{1.5}$$

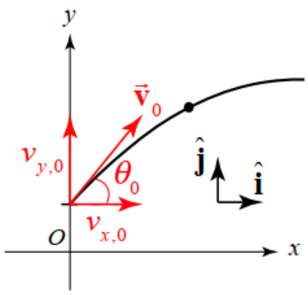

Figure 1.4: A vector decomposition of the initial velocity

In most case of the description of the flight of a projectile includes the statement, “a body is projected with an initial speed v_0 at an angle θ_0 with respect to the horizontal.” The components of the initial velocity can be expressed in terms of the initial speed and angle according to,

$$v_{x,0} = v_0 \cos\theta_0, \tag{1.6}$$

$$v_{y,0} = v_0 \sin\theta_0 \tag{1.7}$$

As the initial speed is the magnitude of the initial velocity, we have that

$$v_0 = \sqrt{v_{x,0}^2 + v_{y,0}^2} \tag{1.8}$$

and the angle θ_0 as,

$$\theta_0 = \tan^{-1}\frac{v_{y,0}}{v_{x,0}} \tag{1.9}$$

The initial position vector is usually given as,

$$\vec{r_0} = x_0\,\hat{i} + y_0\,\hat{j} \tag{1.10}$$

Note that the trajectory in Figure 1.3 has $x_0 = 0$, but this will not always be the case.

1.2.2 Equation of motion

We begin by neglecting all forces other than the gravitational interaction between the object and the earth. This force acts downward with magnitude $F_g = mg$, where m is the mass of the object and $g = 9.8ms^2$

In vector decomposition the only force acting on the body is,

$$\vec{F_g} = -mg\hat{j} \tag{1.11}$$

As from Newton's mechanics,

$$\vec{F_{total}} = m\vec{a} \tag{1.12}$$

Because we are modeling the motion with only one force, we have that,

$$\vec{F}_{total} = \vec{F_g}$$

As, it's a vector quantity,

$$ma_x\hat{i} + ma_y\hat{j} = -mg\hat{j}$$
$$\implies a_y = -g \quad \text{and} \quad a_x = 0$$

We see that the acceleration is a constant and is independent of the mass of the object. Notice that $a_y < 0$. This is because we chose our positive y -direction to point upwards. The sign of the y -component of acceleration is determined by how we choose our coordinate system. As, there are no horizontal forces acting on the object, we conclude that the acceleration in the horizontal direction is also zero. So we have,

$$a_x = 0 \tag{1.13}$$

Therefore the x-component of the velocity remains unchanged throughout the flight of the object. Now we need to consider the x and y component separately to find expressions for the x - and y -components of velocity and position:

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t')dt' = 0 \tag{1.14}$$

$$\implies v_x(t) = v_{x,0} = v_0 \cos\theta_0, \tag{1.15}$$

$$v_y(t) - v_{y,0} = \int_{t'=0}^{t'=t} a_y(t')dt' = -gt \tag{1.16}$$

$$\implies v_y(t) = v_{y,0} - gt = v_0 \sin\theta_0 - gt \tag{1.17}$$

In the same way, the position of the body can be found as,

$$x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t')dt' = \int_{t'=0}^{t'=t} v_{x,0}dt' = v_{x,0}t \tag{1.18}$$

$$x(t) = x_0 + v_{x,0}t = x_0 + v_0 \cos\theta_0 t \tag{1.19}$$

$$y(t) - y_0 = \int_{t'=0}^{t'=t} v_y(t')dt' = \int_{t'=0}^{t'=t} (v_{y,0} - gt)dt' = v_{y,0}t - \frac{1}{2}gt^2 \tag{1.20}$$

$$y(t) = y_0 + v_{y,0}t - \frac{1}{2}gt^2 \tag{1.21}$$

$$y(t) = y_0 + v_0 \sin\theta_0 t - \frac{1}{2}gt^2 \tag{1.22}$$

The complete set of vector equations for position and velocity for each independent direction of motion are given by,

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} \tag{1.23}$$

$$\vec{r}(t) = (x_0 + v_0 \cos\theta_0 t)\hat{\imath} + (y_0 + v_0 \sin\theta_0 t - \frac{1}{2}gt^2)\hat{\jmath} \tag{1.24}$$

$$\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} \tag{1.25}$$

$$\vec{v}(t) = v_0 \cos\theta_0 \hat{i} + (v_0 \sin\theta_0 - gt)\hat{j} \tag{1.26}$$

$$\vec{a}(t) = a_x\hat{i} + a_y\hat{j} = a_y\hat{j} = -gt\hat{j} \tag{1.27}$$

1.2.3 Orbit equation

Let's consider the orbit equation of the body undergoing projectile motion. in order to find this,, we begin with the the x -component of the position in equation (1.19) and write t as a function of position x(t).

$$x(t) = x_0 + v_{x,0}t \tag{1.28}$$

$$t = \frac{x(t) - x_0}{v_{x,0}} \tag{1.29}$$

We then substitute Eq. (1.29) into Eq. (1.22) yielding,

$$y(t) = y_0 + v_0 \sin\theta_0 (\frac{x(t) - x_0}{v_{x,0}}) - \frac{1}{2}g(\frac{x(t) - x_0}{v_{x,0}})^2 \tag{1.30}$$

A little algebraic simplification yields the equation for a parabola:

$$y(t) = y_0 + \tan\theta_0 x(t) - \frac{1}{2}g(\frac{x(t)^2}{v_{x,0}^2}) \tag{1.31}$$

The graph of y(t) as a function of x(t) is shown in Figure 1.5.

The velocity vector is given by,

$$\vec{v}(t) = v_x\hat{i} + v_y(t)\hat{j} \tag{1.32}$$

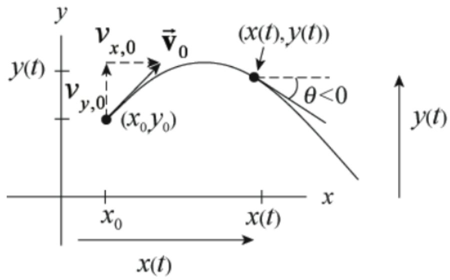

Figure 1.5: The parabolic orbit

Let θ be the angle that the velocity vector forms with respect to the positive x -axis. Then,

$$\theta = \tan^{-1} \frac{v_y(t)}{v_x(t)} \tag{1.33}$$

Notice that although we can determine the angle of the velocity, we cannot determine how fast the body moves along the parabolic orbit from our graph of y(x) ; the magnitude of the velocity cannot be determined from information about the tangent line.

1.2.4 Exercise Problem 1: Time of Flight and Maximum Height of a Projectile

A person throws a stone at an initial angle $\theta_0 = 45°$ from the horizontal with an initial speed of $v_0 = 20 m.s^{-1}$. The point of release of the stone is at a height $d = 2$ m above the ground. You may neglect air resistance.

a) How long does it take the stone to reach the highest point of its trajectory?

b) What was the maximum vertical displacement of the stone? Ignore air resistance.

1.2.5 Exercise 2: Hitting the Bucket

A person is holding a pail while standing on a ladder. The person releases the pail from rest at a height h_1 above the ground. A second person, standing a horizontal distance s from the pail, aims and throws a ball the instant the pail is released in order to hit the pail. The person releases the ball at a height h_2 above the ground, with an initial speed v_0 , and at an angle θ_0 with respect to the horizontal. Assume that v_0 is large enough so that the stone will at least travel a horizontal distance s before it hits the ground. You may ignore air resistance.

(a) Find an expression for the angle θ_0 that the person aims the ball in order to

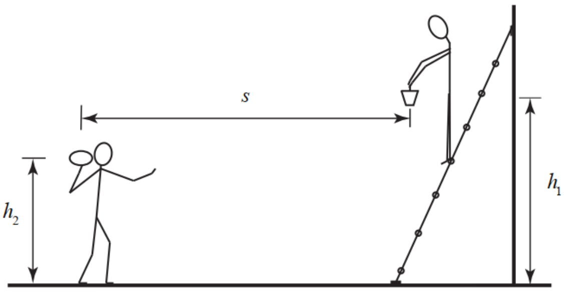

Figure 1.6: Exercise 2

hit the pail. Does the answer depend on the initial velocity?

b) Find an expression for the time of collision as a function of the initial speed of the ball v_0 , and the quantities h_1 , h_2 , and s .

c) Find an expression for the height above the ground where the collision occurred as a function of the initial speed of the ball v_0 , and the quantities h_1 , h_2 , and s .

1.3 Some exercise from Resnick- Halliday Walker problems

CHAPTER 4: MOTION IN 2 AND 3 DIMENSIONS

•11 The position $\vec{r}$ of a particle moving in an xy plane is given by $\vec{r} = (2.00t^3 - 5.00t)\hat{i} + (6.00 - 7.00t^4)\hat{j}$, with $\vec{r}$ in meters and t in seconds. In unit-vector notation, calculate (a) $\vec{r}$, (b) $\vec{v}$, and (c) $\vec{a}$ for $t = 2.00$ s. (d) What is the angle between the positive direction of the x axis and a line tangent to the particle's path at $t = 2.00$ s?

•14 A proton initially has $\vec{v} = 4.0\hat{i} - 2.0\hat{j} + 3.0\hat{k}$ and then 4.0 s later has $\vec{v} = -2.0\hat{i} - 2.0\hat{j} + 5.0\hat{k}$ (in meters per second). For that 4.0 s, what are (a) the proton's average acceleration $\vec{a}_{avg}$ in unit-vector notation, (b) the magnitude of $\vec{a}_{avg}$, and (c) the angle between $\vec{a}_{avg}$ and the positive direction of the x axis?

••16 The velocity $\vec{v}$ of a particle moving in the xy plane is given by $\vec{v} = (6.0t - 4.0t^2)\hat{i} + 8.0\hat{j}$, with $\vec{v}$ in meters per second and t (> 0) in seconds. (a) What is the acceleration when $t = 3.0$ s? (b) When (if ever) is the acceleration zero? (c) When (if ever) is the velocity zero? (d) When (if ever) does the speed equal 10 m/s?

•26 A stone is catapulted at time $t = 0$, with an initial velocity of magnitude 20.0 m/s and at an angle of 40.0° above the horizontal. What are the magnitudes of the (a) horizontal and (b) vertical components of its displacement from the catapult site at $t = 1.10$ s? Repeat for the (c) horizontal and (d) vertical components at $t = 1.80$ s, and for the (e) horizontal and (f) vertical components at $t = 5.00$ s.

••27 ILW A certain airplane has a speed of 290.0 km/h and is diving at an angle of $\theta = 30.0°$ below the horizontal when the pilot releases a radar decoy (Fig. 4-33). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700$ m. (a) How long is the decoy in the air? (b) How high was the release point?

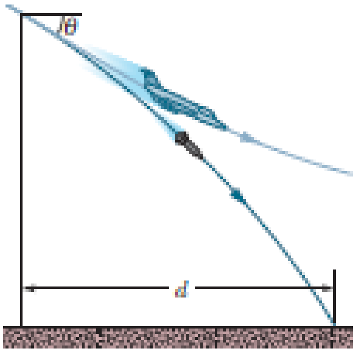

Figure 4-33 Problem 27.

••28 GO In Fig. 4-34, a stone is projected at a cliff of height h with an initial speed of 42.0 m/s directed at angle $\theta_0 = 60.0°$ above the horizontal. The stone strikes at A, 5.50 s after launching. Find (a) the height h of the cliff, (b) the speed of the stone just before impact at A, and (c) the maximum height H reached above the ground.

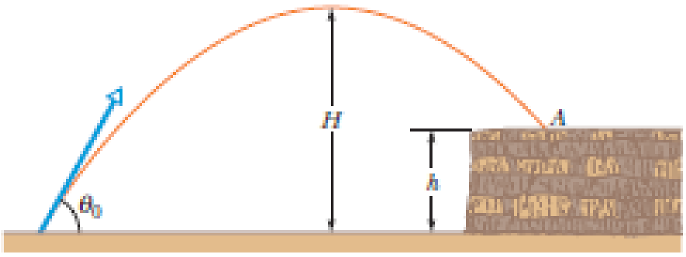

Figure 4-34 Problem 28.

••32 GO You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (Fig. 4-35). The wall is distance $d = 22.0$ m from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?

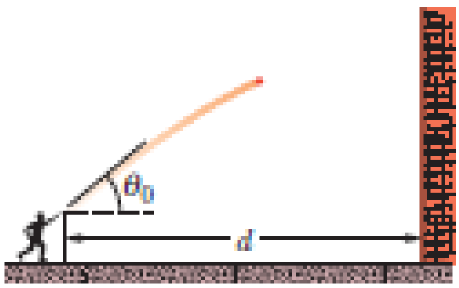

Figure 4-35 Problem 32.

••43 ILW A ball is shot from the ground into the air. At a height of 9.1 m, its velocity is $\vec{v} = (7.6\hat{i} + 6.1\hat{j})$ m/s, with $\hat{i}$ horizontal and $\hat{j}$ upward. (a) To what maximum height does the ball rise? (b) What total horizontal distance does the ball travel? What are the (c) magnitude and (d) angle (below the horizontal) of the ball's velocity just before it hits the ground?