

Circular Motion Problem Solving

Acceleration or a change in velocity is caused by a net force: Newton's 2nd Law $\vec{a} = \sum \vec{F} / m$
 An object accelerates when either the *magnitude* or the *direction* of the velocity changes

We saw in the last unit that an object speeds up or slows down when a net force acts in the same or opposite the direction of its motion (tangential to the motion)

$$\sum F_x = F_{netx} = ma_x \quad \text{and} \quad \sum F_y = F_{nety} = ma_y$$

In this unit, we examine the acceleration due to a change in *direction* of the velocity. An object changes direction or moves in a curve when a net force acts perpendicular to the direction of motion, towards the center of the curve. A net force perpendicular to the direction of motion is called a centripetal (center-seeking) force, F_c , and the acceleration it gives rise to is a centripetal acceleration, a_c . Centripetal acceleration is *a change in the velocity direction only* and it can be shown to be equal to v^2/r where v is the speed and r is the radius of the curve. The centripetal force is found by adding all the forces in the radial direction (perpendicular to the curve)

$$\sum F_r = F_c = ma_c = m(v^2 / r) = m\omega^2 r$$

Note that "Centripetal force" is just a fancy name for the radial component of the net force. It is not a new kind of force and is NOT drawn on force diagrams.

A net force could have both tangential and radial components; the component tangential to the direction of motion causes the object to speed up or slow down and the radial component causes the object to move in a curve.

In this unit, we focus on the acceleration associated with a change in direction only. In other words, **an object that moves with constant speed but continuously changes direction – UNIFORM CIRCULAR MOTION**. An object in uniform circular motion has constant speed, v , centripetal acceleration, a_c , and undergoes periodic motion with period T and frequency, f .

$$\left. \begin{array}{l} \text{tangential or linear speed: } v = d/\Delta t = 2\pi r/T = 2\pi r f \\ \text{angular speed: } \omega = \Delta\theta/\Delta t = 2\pi/T = 2\pi f \\ \text{centripetal acceleration: } a_c = v^2/r = \omega^2 r \end{array} \right\} v = \omega r$$

Problems:

1. The angular speed of a record player is 33 rotations per minute. It has a diameter of 12 inches (0.305m).

a) What is the period and frequency of rotation of the record?

$$f=0.55\text{Hz} \quad T=1.81\text{s}$$

b) What is the angular speed in rad/s

$$\omega=3.45\text{rad/s}$$

c) What is the tangential speed of the outer most part of the record?

$$v=0.53\text{m/s}$$

d) What is the tangential speed halfway out to the edge of the record?

$$v=0.265\text{m/s}$$



2. A 900 kg race car makes one lap around a track of radius 50. m in 11.0 s

a) What is the car's average speed?

$$v=28.6\text{m/s}$$

b) What is the car's angular speed?

$$\omega=0.57\text{rad/s}$$

c) What is the car's centripetal acceleration?

$$a_c=16.3\text{m/s}^2$$

d) What is the centripetal force acting on the car and which force provides it?

$$F_c=14.670\text{N}$$

3. a) A car goes around a curve at 20.0 m/s. If the radius of the curve is 50.0 m, what is the centripetal acceleration of the car? b) If the speed of the car were halved, how much different would a_c be to travel the same radius curve?

4. A speed skater goes around a turn with a 31 m radius. The skater has a speed of 14m/s and experiences a centripetal force of 460N. What is the mass of the skater?

5. Norman Neutron swings a rubber ball attached to a string over his head in a horizontal, circular path. The piece of string is 1.5 m long and the ball makes 120 complete turns each minute.

a) What is the average speed of the ball?

b) What is the ball's centripetal acceleration?

6. Calculate the centripetal acceleration of the Earth towards the Sun. ($r = 1.5 \times 10^{11}$ m)

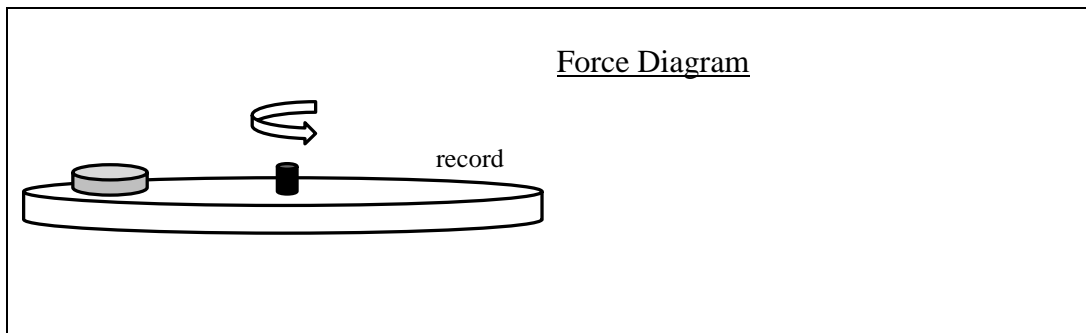
7. A 0.50 kg mass is attached to a string 1.0 m long and moves in a horizontal circle at a rate of 2.0 Hz.

a) Calculate the centripetal acceleration of the mass

b) Calculate the tension in the string.

8. A penny, placed on a spinning record, rotates at $33 \frac{1}{3}$ revolutions per minute when it is placed 10 cm from the center of the record (mass of penny = 2.5 grams).

- What is the tangential and angular speed of the penny as it moves around the center of the record?
- What is the magnitude of the centripetal force necessary to keep the penny moving in a circular path?
- How much frictional force is applied to the penny by the surface of the record?
- Draw a free body diagram showing all the forces acting on the rotating penny



- What is the minimum coefficient of static friction required to keep the penny from sliding off the record?

9. A Ferris wheel with a 38 m radius and tangential speed of 5.0 m/s has a 76 kg passenger riding it.

- What is the magnitude of the centripetal force acting on the Ferris wheel passenger at the top and at the bottom?

- Complete the freebody diagrams showing all the forces acting on the passenger at the top and at the bottom of the Ferris wheel. Label the forces. Indicate whether the apparent weight of the passenger (the normal force) is greater, equal to, or less than their true weight at the top and at the bottom.

Force Diagram for <u>top</u>	Force Diagram for <u>bottom</u>
<p style="text-align: center;">Label the forces</p> <div style="text-align: center; margin: 20px 0;"> <p>a) Top</p> </div> <p style="text-align: center;">Apparent wt _____ True wt</p>	<p style="text-align: center;">Label the forces</p> <div style="text-align: center; margin: 20px 0;"> <p>b) Bottom</p> </div> <p style="text-align: center;">Apparent wt _____ True wt</p>

c) How big is the normal force exerted on the passenger the top and at the bottom?

Note: The positive radial direction is taken as the direction towards the center of the circle. This is consistent with the direction of centripetal acceleration.

The **negative radial direction** is **away from the circle**.

NSL Application

$$\sum F_r = F_c = ma_c = (mv^2)/r$$

Start here with Newton's 2nd Law for the radial component of the net force. Use your freebody diagrams above to guide you in summing up the radial forces to find the centripetal force and the normal force.

d) How fast would the Ferris wheel have to go for the passenger to feel double their true weight at the bottom of the Ferris wheel?

e) At the speed found in part d) how heavy would the passenger feel at the top of the Ferris wheel (this is the normal force)?

10. Tarzan ($m = 85 \text{ kg}$) tries to cross a river by swinging from a 10-m-long vine. His speed at the bottom of the swing (as he just clears the water) is 8.0 m/s. Tarzan doesn't know that the vine has a breaking strength of 1000 N. Does he make it safely across the river? Justify your answer.

1a) $T=1.82\text{s}$, $f=0.55\text{Hz}$ b) $\omega=3.45\text{rad/s}$ c) $v=0.53\text{m/s}$ d) $v=0.265\text{m/s}$ 2a) $v=28.5\text{m/s}$ b) $\omega=0.57\text{rad/sec}$ c) $a_c=16.2 \text{ m/s}^2$ 2d) 14,580N 3) 8.0 m/s^2 4. 72.8kg 5a) 19 m/s b) 240 m/s^2 6) 0.0060 m/s^2 7a) 160 m/s^2 b) 80. N
8a) $v=0.35\text{m/s}$, $\omega=3.5\text{rad/sec}$ b) $F_c=0.00305\text{N}$ c) $f_s=0.00305\text{N}$ e) 0.124
9a) Top: $F_c = 50\text{N}$, Bottom: $F_c=50\text{N}$ b) TOP: F_N (apparent wt) = 695 N; BOTTOM: F_N (apparent wt) = 795 N d) 19.3 m/s
e) 0 N, weightless