Chapter - 1

1. In the classes, we discussed three forms of floating number representations as shown below,
(1) Standard/General/Lecture Note Form, (2) Normalized Form, (3) Denormalized Form
Now, let's consider **$\beta = 2$, m = 4 and $-3 \le e \le 4$.** Based on these, answer the following:
   - **a)** What are the maximum and minimum numbers that can be stored in the system by these three forms defined?
   - **b)** How many numbers can be represented in the system by these three forms?
   - **c)** If the above system has negative support, find the maximum and minimum numbers that can be stored by the three forms defined.
   - **d)** If the above system has negative support, find how many numbers can be represented by the three forms.
   - **e)** For each of the above three forms, find how many numbers will have $\delta = 0$?
   - **f)** Show that, for e = -2, all the numbers have equal spacing.

2. In the IEEE standard (1985) for double-precision (64 bit) arithmetic, if the exponent bias were to be altered to exponentBias = 600, what would the smallest and largest number be? Note that the conditions for representing ± inf and ± 0 are still maintained as before.

3. For a **14 bit** system, if β=2, **sign bit**=1 bit, and **fraction**=9 bit, what will be the **Non-negative Smallest** and **Largest** possible number that can be represented using the **Normalized** form of the floating point representation after **Exponent Biasing**?

4. Consider the real number x = 1.42 and y = 1.34 where **$\beta = 2$, m = 4 and $-2 \le e \le 3$** for Standard form.
   - **a)** Find fl(x) and fl(y).
   - **b)** Find the rounding errors for both cases.
   - **c)** Find fl(x*y).
   - **d)** What will be machine epsilon (ϵ_m) value for the above system?

5. Consider the quadratic equation $x^2 - 16x + 5 = 0$. Explain how the loss of significance occurs in finding the roots of the quadratic equation if we restrict to 5 significant figures. Discuss how to avoid this and find the roots.

<u>**Chapter - 2**</u>

1. Consider the following nodes:

Sl.	x	Y
1	-1	2.2
2	0	10.6
3	1	17.0
4	2	22.4

a) What will be the degree of interpolating polynomial for the above nodes?
b) How many lagrange bases will be there for the interpolating polynomial of the above nodes? Find them.
c) Find the interpolating polynomial using lagrange method.
d) Find Pn (3.2).

2. Consider the data points (-1, 8), (0, 4) and (1, 16). Using Newton's divided difference method,
a) Find the coefficients a_k for k = 0, 1, 2.
b) Write the interpolating polynomial using the above method.
c) If we add another node (2, 24), what will be the interpolating polynomial then?
d) Find Pn (-0.9).

3. Compute the upper bound of interpolation error for $f(x) = 2sin(x) - 3cos(x)$ where $x \in \{-\pi/4, 0, \pi/4\}$ within [-1, 1]. Consider up to 4 significant figures.

4. Consider the function $f(x) = e^{x} - x^{3}ln(x)$. Find the upper bound of interpolation error where $x \in \{2, 3, 4\}$ within [1.6, 2.3]. Consider up to 4 significant figures.

5. Consider the following dataset:

x	$f(x)$	$f'(x)$
0.1	-0.62	3.46
0.2	-0.30	3.15

a) If we want to construct a 3 degree polynomial using the above information, which method should be used?

b) Compute the bases of the method you chose in (a).
c) Write the polynomial and find the value at x = 2.

6. Consider the data points (-1, 9), (0, 3) and (1, 18). Using Vandermonde matrix method,
   a) What will be the degree of the polynomial constructed using above information?
   b) Find the interpolating polynomial using the above information.
   c) Find Pn(3).

7. Consider the function $f(x) = e^{-3x^2}$ which shows Runge Phenomenon when it is interpolated by equally spaced nodes.
   a) What is the name of the way to overcome this problem?
   b) Calculate the five nodes using the way you mentioned in (a).

8.

Consider the Runge function $f(x) = e^{-\frac{1}{x^2}}$ which shows Runge phenomenon/divergence behavior when $f(x)$ is interpolated by the equally-spaced nodes over some interval. To overcome this divergence, we may interpolate $f(x)$ with Chebyshev nodes.

(a) (2 marks) Exactly what property of Chebyshev nodes allows us to overcome the Runge phenomenon?

(b) (3 marks) Find six Chebyshev nodes over the interval $[-2.5, 2.5]$.

(c) (5 marks) Find the interpolating polynomial using the Lagrange Form with the first 3 nodes found in part-(b), and simplify the expression of the interpolation polynomial to the standard form.

Chapter - 3

1. Consider the function $f(x) = e^{x} + x\ln(x)$. Now answer the following:
   a) Find numerical derivative $f'(3)$ using Forward, Backward and Central Difference methods with step size, $h = 0.1$. Keep up to 5 significant figures.
   b) Find the truncation error for the three methods in the above case. Keep up to 4 significant figures.
   c) Compute the upper bound of truncation error for Forward, Backward and Central Difference methods when $x = 2$ and $h = 0.2$. Keep up to 3 decimal places.

2. Deduce an expression for D^{1}_{h} from D_h by replacing h by h/5 using the Richardson Extrapolation method.

3. Deduce an expression for D^{1}_{h} from D_h by replacing h by 2h/3 using the Richardson Extrapolation method.

4. Consider the function $f(x) = x^{3}\cos(x) - e^{x} + \sin(x) - \ln(x)$. Now answer the following:
   a) Compute $D^{1}_{0.2}$ at $x = 3.4$ using Richardson Extrapolation method up to 4 significant figures.
   b) Compute $D^{2}_{0.2}$ at $x = 3.1$ using Richardson Extrapolation method up to 4 significant figures.