

BRAC University
MAT-215
Practice Sheet # 7

1. Find the Laplace transformation of each of the following function:

(i) $3e^{-2t}$ (ii) $4t^3 - e^{-t}$ (iii) $7\sin 2t - 3\cos 2t$ (iv) $(t^2 + 1)^2$ (v) $(4e^{2t} - 2)^3$.

2. Evaluate each of the following:

(i) $\Lambda \{t^3 e^{-3t}\}$ (ii) $\Lambda \{5e^{3t} \sin 4t\}$ (iii) $\Lambda \{(t+2)^2 e^t\}$ (iv) $\Lambda \{e^{-t} (3\sinh 2t - 5\cosh 2t)\}$
(v) $\Lambda \{e^{-4t} \cosh 2t\}$ (vi) $\Lambda \{e^{2t} (3\sin 4t - 4\cos 4t)\}$.

3. Determine each of the following:

(i) $\Lambda^{-1} \left\{ \frac{12}{4-3s} \right\}$ (ii) $\Lambda^{-1} \left\{ \frac{2s-5}{s^2-9} \right\}$ (iii) $\Lambda^{-1} \left\{ \frac{23s-15}{s^2+8} \right\}$ (iv) $\Lambda^{-1} \left\{ \frac{1}{s^{3/2}} \right\}$
(v) $\Lambda^{-1} \left\{ \frac{s+1}{s^{4/3}} \right\}$ (vi) $\Lambda^{-1} \left\{ \frac{1}{s^4} \right\}$ (vii) $\Lambda^{-1} \left\{ \frac{1}{\sqrt{2s+3}} \right\}$.

4. Evaluate each of the following using partial fraction:

(i) $\Lambda^{-1} \left\{ \frac{6s-4}{s^2-4s+20} \right\}$ (ii) $\Lambda^{-1} \left\{ \frac{4s+12}{s^2+8s+16} \right\}$ (iii) $\Lambda^{-1} \left\{ \frac{2s^2-4}{(s+1)(s-2)(s-3)} \right\}$
(iv) $\Lambda^{-1} \left\{ \frac{5s^2-15s-11}{(s+1)(s-2)^3} \right\}$ (v) $\Lambda^{-1} \left\{ \frac{3s+1}{(s^2+1)(s-1)} \right\}$ (vi) $\Lambda^{-1} \left\{ \frac{2s^2-4}{(s+1)(s-2)(s-3)} \right\}$
(vii) $\Lambda^{-1} \left\{ \frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} \right\}$.

5. Solve the given differential equation:

(i) $Y'' - 3Y' + 2Y = 4e^{2t}$, $Y(0) = -3$, $Y'(0) = 5$
(ii) $Y'' + 9Y = \cos 2t$, $Y(0) = 1$, $Y(\pi/2) = -1$
(iii) $Y'' + 2Y' + 5Y = e^{-t} \sin t$, $Y(0) = 0$, $Y'(0) = 1$
(iv) $Y''' - 3Y'' + 3Y' - Y = e^t t^2$, $Y(0) = 0$, $Y'(0) = 0$, $Y''(0) = -2$.

Laplace Transformation of some elementary functions:

	$F(t)$	$L\{F(t)\} = f(s)$
1.	1	$\frac{1}{s}, s > 0$
2.	t	$\frac{1}{s^2}, s > 0$
3.	t^n $n = 0, 1, 2, \dots$	$\frac{n!}{s^{n+1}}, s > 0$
4.	e^{at}	$\frac{1}{s-a}, s > a$
5.	$\sin at$	$\frac{a}{s^2 + a^2}, s > 0$
6.	$\cos at$	$\frac{s}{s^2 + a^2}, s > 0$
7.	$\sinh at$	$\frac{a}{s^2 - a^2}, s > a $
8.	$\cosh at$	$\frac{s}{s^2 - a^2}, s > a $

Inverse Laplace Transformation

	$f(s)$	$L^{-1}\{f(s)\} = F(t)$
1.	$\frac{1}{s}$	1
2.	$\frac{1}{s^2}$	t
3.	$\frac{1}{s^{n+1}}, \quad n = 0,1,2, \dots$	$\frac{t^n}{n!}$
4.	$\frac{1}{s - a}$	e^{at}
5.	$\frac{1}{s^2 + a^2}$	$\frac{\sin at}{a}$
6.	$\frac{s}{s^2 + a^2}$	$\cos at$
7.	$\frac{1}{s^2 - a^2}$	$\frac{\sinh at}{a}$
8.	$\frac{s}{s^2 - a^2}$	$\cosh at$
9.	$\frac{1}{s^{n+1}}, n > -1$	$\frac{t^n}{(n+1)!}, \quad n > -1$