Assignment-2

Pulak Deb Roy

23241078

Sec : 8

MAT 216

Qus no-1

$V = \{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in R\}$

$W = \{A \in V : A^2 = A\}$

for W to be a subspace of V, it needs to fullfill the following conditions.

① W contains 0 vectors ; $\vec{0} \in W$

if $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $\therefore A = A^2$ which belongs to W

$\therefore$ W meets this condition.

② closed under addition :

if $B, C \in W$ then $B+C \in W$

let, $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $B+C = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = D$

$\therefore B+C = D \notin W$

$\therefore$ W dosen't meet this condition.

③ Closed under multiplication.

if $B \in W$ the $cB \in W$ [c is a scaler]

let, $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $5B = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix} \notin W$

$\therefore$ W dosen't meet this condition as well.

$\therefore$ W is not a subspace of V.

Anu

Qus no. = 2(i)

To prove that, V is a subspace of R^2 we need to show that it satisfies the three conditions of a subspace.

Since A is a 2x2 matrix, we can compute $A \cdot 0 = 0$ where 0 is the 2 dimensional zero vector. So, 0 is an element of V and V is not empty.

Let x, y vector in V. So we have $Ax = 5x$ and $Ay = 5y$. We want to show that x+y is also in V.

$$Ax + Ay = 5x + 5y$$

$$\therefore A(x+y) = 5(x+y)$$

(x+y) is in V and V is closed under vector addition condition.

Similarly,
Let x be a vector in V and let c be a scalar. We have to show that cx is also in V.

$$c(Ax) = c(5x)$$

$$\therefore A(cx) = 5(cx)$$

Therefore, cx is also in V and V closed under scalar multiplication. Since V satisfy all the condition we can say V is a subspace of R^2.

Qus no - 2(ii)

To find the basis for V, we need to find a set of linearly independent vectors that span V. Since V is defined as the set of all vectors x such that $Ax = 5x$ we can rewrite this as $(A-5I)x = 0$

where I is the 2×2 Identity matrix.

So, we want to find the null space of matrix $A-5I$. Using row reduction,

$$A-5I = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} - 5\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 1 \\ 3 & -3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix} \begin{matrix} \rightarrow R_1' = -R_1 \\ \rightarrow R_2' = R_2 + 3R_1 \end{matrix}$$

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So the null space of $A-5I$ is spanned by the vector $(1,1)$ since any scalar multiple of this vector satisfies $(A-5I)x=0$

Therefore, a basis for V is $\{(1,1)\}$ and the dimention of V is 1.

Qus no - 3

Doing row echelon on matrix A,

$$A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix} \begin{matrix} \\ \rightarrow R_2' = R_2 - 3R_1 \\ \rightarrow R_3' = R_3 + R_1 \\ \rightarrow R_4' = R_4 - 2R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 0 & -14 & -14 & -14 & -28 \\ 0 & 4 & 4 & 4 & 8 \\ 0 & -5 & -5 & -5 & -10 \end{bmatrix} \begin{matrix} \\ \rightarrow R_2' = R_2/-14 \\ \\ \\ \end{matrix}$$

$$= \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 4 & 4 & 4 & 8 \\ 0 & -5 & -5 & -5 & -10 \end{bmatrix} \begin{matrix} \\ \rightarrow \\ \rightarrow R_3' = R_3 - 4R_2 \\ \rightarrow R_4' = R_4 + 4R_2 \end{matrix}$$

$$= \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The non-zero row vectors of R from a basis for the row space of R and hence the basis of row space of A.

Thess basis vectors are,

$$r_1 = [1 \quad 4 \quad 5 \quad 6 \quad 9]$$

$$r_2 = [0 \quad 1 \quad 1 \quad 1 \quad 2]$$

$\therefore$ So the basis of A, $R(A) = 2$

Now, keeping in mind that A and R may have different column spaces, we cannot find a basis for the column space of A directly form R.

The pivot columns of R vector are,

$$c_1' = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad c_2' = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

form a basis corresponding column vectors of A,

$$c_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \\ 2 \end{bmatrix} \quad c_2 = \begin{bmatrix} 4 \\ -2 \\ 0 \\ 3 \end{bmatrix}$$

$\therefore$ the basis of A, $C(A) = 2$

(Ans)

Ans no - 4

Doing reduced row echelon form on matrix A,

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix} \begin{matrix} \\ \rightarrow R_2' = R_2 - 2R_1 \\ \rightarrow R_3' = R_3 + R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & -7 & -7 & -4 \\ 0 & 7 & 7 & 4 \end{bmatrix} \begin{matrix} \\ \rightarrow R_2' = R_2/-7 \\ \rightarrow R_3' = R_3 + R_2 \end{matrix}$$

$$= \begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \rightarrow R_1' = R_1 - 4R_2 \\ \\ \\ \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 & -2/7 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$x_1 + x_3 - 2/7 x_4 = 0$$

$$x_2 + x_3 + 4/7 x_4 = 0$$

Here, x_1 and x_2 are dependent variable and x_3, x_4 are free variable.

So, $x_1 = -x_3 + \frac{2}{7}x_4$

$x_2 = -x_3 - \frac{4}{7}x_4$

Take, $x_3 = s$ and $x_4 = t$

$x_1 = -s + \frac{2}{7}t$

$x_2 = -s - \frac{4}{7}t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} \frac{2}{7} \\ -4/7 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ These vectors form a basis for the null space.

$\therefore$ nullity $(A) = 2$

(Ans)

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