Assignment - 3

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Sec : 06

Question 1

0 points possible (ungraded)

[MZK 1]

Chain Rule (Partial Derivative):

Compute $\frac{dz}{dt}$ while $z = xe^{xy}$, $x = t^2$, $y = t^{-1}$

(Write your final answer in terms of t)

Submit

Question 2

0 points possible (ungraded)

[SAN 3] Radioactive isotope Carbon-14 decays at a rate proportional to the amount present. If the decay rate is 12.10% per thousand years and the current mass is 135.2 mg, what will the mass be 2.2 thousand years from now?

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Question 3

0 points possible (ungraded)

[All 3] According to the ideal gas law, the pressure, temperature, and volume of a gas are related by $P = \frac{kT}{V}$, where k is a constant of proportionality. Suppose that V is measured in cubic inches (in^3), T is measured in kelvins (K) and that for a certain gas the constant of proportionality is $k = 10 in \cdot lb/K$ (a) Find the instantaneous rate of change of pressure with respect to temperature if the temperature is 80 K and the volume remains fixed at $50\ in^3$. (b) Find the instantaneous rate of change of volume with respect to pressure if the volume is $50\ in^3$ and the temperature remains fixed at $80\ K$.

Submit

Question 4

0 points possible (ungraded)

[AQD 4] Find the n-th Taylor polynomials for $f(x) = \ln$ about $x = e$ and express it in sigma notation.

Submit

Qustion: 1

$$z = xe^{xy}, \quad x = t^2, \quad y = t^{-1}$$

$$\frac{\partial z}{\partial x} = (x)(e^{xy}) + (e^{xy})(x)$$

$$= (x)(ye^{xy}) + e^{xy}$$

$$= e^{xy}(xy+1)$$

$$\frac{dx}{dt} = (t^2)$$

$$= 2t$$

$$\frac{\partial z}{\partial y} = (x)(e^{xy}) + (e^{xy})(x)$$

$$= x^2 e^{xy}$$

$$\frac{dy}{dt} = (t^{-1})$$

$$= -t^{-2}$$

$$\therefore \frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$= e^{xy}\{(xy+1)(2t) + (x^2 e^{xy})(-t^{-2})\}$$

$$= e^{t}(t^2 + 2t)$$

(Ans)

Question: 2

$A = 135.2$

$K = -12.10\% = -0.121$ (because of decay)

$t = 2.2$

As we know,

$$x(t) = Ae^{kt}$$

$$= 135.2 e^{-0.121 \times 2.2}$$

$$= 103.6$$

(Ans)

Problem 3

As we know, $P = \frac{KT}{V}$

whene k is constant.

(a) Given, $T = 80$, $K = 10$ in.lb/K

volume nemains constant, $50\ in^3$

$$\frac{\partial P}{\partial T} = \frac{\partial}{\partial T}\frac{KT}{V}$$

$$= \frac{K}{V} \cdot \frac{\partial}{\partial T}(T)$$

$$= \frac{K}{V}$$

$$\left.\frac{\partial P}{\partial T}\right|_{(V=50)} = \frac{10}{50}$$

$$= 0.2$$

(Ans)

(b) $K = 10$ in. lb/K

$V = 50 \text{ in}^3$

$T = 80K$ which remains constant.

$$\frac{\partial V}{\partial P} = \frac{\partial}{\partial P} \frac{KT}{P}$$

$$= KT \frac{\partial}{\partial P}\left(\frac{1}{P}\right)$$

$$= KT \cdot \left(-\frac{1}{P^2}\right)$$

$$= -\frac{KT}{P^2}$$

$$= -\frac{KT}{\left(\frac{KT}{V}\right)^2}$$

$$\therefore \frac{\partial V}{\partial P} = -\frac{V^2}{KT}$$

$$\left.\frac{\partial V}{\partial P}\right|_{(T=80,\, V=50)} = -\frac{(50)^2}{10 \times 80}$$

$$= -3.125$$

(Ans)

Question: 4

$f(x) = \ln(x)$ where $x_0 = e$

$f'(x) = \frac{1}{x}$

$f''(x) = -\frac{1}{x^2}$

$f'''(x) = \frac{2}{x^3}$

again, $f(e) = \ln e$

$f'(e) = \frac{1}{e}$

$f''(e) = -\frac{1}{e^2}$

$f'''(e) = \frac{2}{e^3}$

Tylor Polynomial,

$$T_{(x)} = f(x_0) + f'(x_0)(x - x_0) + f''(x_0)\frac{(x - x_0)^2}{2!}$$

$$+ f'''(x_0)\frac{(x - x_0)^3}{3!} + \cdots + f^{(n)}(x_0)\frac{(x - x_0)^n}{n!}$$

$\text{—} (1)$

$T_{(x)} = T_{(e)}$ — from ①,

$$T_{(e)} = f(e) + f'(e)(x-e) + f''(e)\frac{(x-e)^2}{2!} + f'''(e)\frac{(x-e)^3}{3!}$$

$$+ \cdots\cdots$$

$$= 1 + \frac{1}{e}(x-e) + \left(-\frac{1}{e^2}\right)\frac{(x-e)^2}{2!} +$$

$$\frac{2}{e^3}\frac{(x-e)^3}{3!} + \cdots\cdots$$

$$= 1 + \frac{(x-e)}{e} - \frac{1}{2!}\cdot\frac{(x-e)^2}{e^2} + \frac{2}{3!}\cdot\frac{(x-e)^3}{e^3} + \cdots$$

$$+ \cdots \frac{(-1)^{n+1}}{n}\cdot\frac{(x-e)^n}{e^n}$$

$$T_e = 1 + \sum_{n=1} \frac{(-1)^{n+1}}{n}\frac{(x-e)^n}{e^n}$$

(Ans)