Chapter 2

Electric Field

The idea of the electric field as a measure of the strength of the influence of a system of charge is introduced. Two different ways of visualizing the electric is discussed. The dipole is introduced and it is shown that the electric field of a dipole falls off as $1/r^3$*.*

2.1 The Electric Field

Consider an isolated point charge q fixed at some point in space. Now, if we bring another charge Q in its vicinity and place it at the point p then the force that the second charge feels due to the first charge is given by Coulomb's law

$$\mathbf{F} = k\frac{qQ\hat{\mathbf{r}}}{r^2}, \tag{2.1}$$

where $\mathbf{r}$ is the vector joining the charge q to the point p where we have placed the second charge Q. As usual, $\hat{\mathbf{r}}$ is the unit vector in the $\mathbf{r}$ direction and r is the magnitude. Note that we can rewrite this equation as

$$\mathbf{F} = Q\mathbf{E}(p) \tag{2.2}$$

where

$$\mathbf{E}(p) = k\frac{q\hat{\mathbf{r}}}{r^2}. \tag{2.3}$$

We can change the value of the charge Q and regardless of the its value the force on it would still be given by the mathematical expression (2.2) as long as the original charge q remains fixed in its original position. This suggests that the quantity defined in (2.3) is a property of the point p in space that encodes the strength of the charge q. The quantity $\mathbf{E}(p)$ is known as the *electric field* at the point p due to the point charge q. We call the charge q the *source* of the electric field. Due to the superposition principle we can easily generalize the definition of the electric field when more than one source is present.

Suppose we have a *fixed* distribution of electric charges, $q_1, q_2, \ldots, q_n$. We assume that the charges are fixed in a way such that when we bring another charge, say q_0, in the vicinity of the previous charges their positions remain unchanged. Let us denote the position of the new charge by p. Using Coulomb's law and the superposition principle we can calculate the net force on charge q_0. It is given by

$$\begin{aligned}
\mathbf{F}_0 &= \mathbf{F}_{10} + \mathbf{F}_{20} + \cdots + \mathbf{F}_{n0} \\
&= k\left(\frac{q_1 q_0 \hat{\mathbf{r}}_{10}}{r_{10}^2} + \frac{q_2 q_0 \hat{\mathbf{r}}_{20}}{r_{20}^2} + \cdots + \frac{q_n q_0 \hat{\mathbf{r}}_{n0}}{r_{n0}^2}\right) \\
&= q_0 k\left(\frac{q_1 \hat{\mathbf{r}}_{10}}{r_{10}^2} + \frac{q_2 \hat{\mathbf{r}}_{20}}{r_{20}^2} + \cdots + \frac{q_n \hat{\mathbf{r}}_{n0}}{r_{n0}^2}\right) \\
\mathbf{F}_0 &= q_0 \mathbf{E}(p)
\end{aligned} \tag{2.4}$$

where we defined

$$\mathbf{E}(p) = k\left(\frac{q_1 \hat{\mathbf{r}}_{10}}{r_{10}^2} + \frac{q_2 \hat{\mathbf{r}}_{20}}{r_{20}^2} + \cdots + \frac{q_n \hat{\mathbf{r}}_{n0}}{r_{n0}^2}\right). \tag{2.5}$$

The quantity $\mathbf{E}$ in equation (2.5) is known as the electric field at the point p due to the sources $q_1, q_2, \ldots, q_n$. The interpretation of the electric field is as follows. The electric field $\mathbf{E}(p)$ at the point p due some distribution of point sources $q_1, q_2, \ldots, q_n$ gives the force *if a charge q_0 is placed at p via the relation*

$$\mathbf{F}_0 = q_0 \mathbf{E}(p). \tag{2.6}$$

Note that the definition of the electric field in *independent* of the charge q_0 used in the formula above. Another assumption that goes into the definition of the electric field is that when we put the charge q_0 (sometimes called the 'test charge') at the point p, the original distribution of charges $q_1, q_2, \ldots, q_n$ don't move from their original positions. If they do move due to the force by q_0 on them then the field at the point p changes to some new value. The units of electric field is newton/coulomb.

It is worthwhile to re-express the definition of the electric field in a bit more formal notation

$$\mathbf{E}(p) = k \sum_{i=1}^{n} \frac{q_i \hat{\mathbf{r}}_{i0}}{r_{i0}^2}. \tag{2.7}$$

Figure 2.1: The electric field $\mathbf{E}(p)$ at the point p due to the charges $+2$ coulomb and -1 coulomb is given by the vector sum of the electric fields due to the individual charges.

Since the point p taken to define the electric field was an arbitrary point we imagine the electric field due to some distribution of point charges permeating all of the space around those charges. The electric field is a somewhat abstract concept as we assume its existence even when there is no test charge present to feel its effect. The advantage of the concept of the electric field that it is a *local property*. If we know the electric field in some small region of space we can predict what would happen to a charge in that neighbourhood without any reference to the sources which created the electric field.

2.2 Visualizations of Electric Fields

An electric field is a special case of a more general physical concept known as a *vector field.* Put simply, a vector field is a mathematical construct where we attach a vector to each point in space. Vector fields describe a plethora of physical phenomena. For example, we could describe the flow of wind in a region of space by specifying the velocity of each air molecule at each point. Similarly, both laminar and turbulent flow of fluid can be described by specifying the velocity of the fluid at each point. Of course, for such dynamical systems we envision the vector field to change over time.

Perhaps the most familiar example of a vector field is acceleration due to gravity. As you know, the acceleration due to gravity around the earth at the sea-level is about $9.8\ \mathrm{m/s^2}$ and pointing towards the centre of the earth. The general formula for the acceleration due to gravity for a point around the earth which is at a distance d from

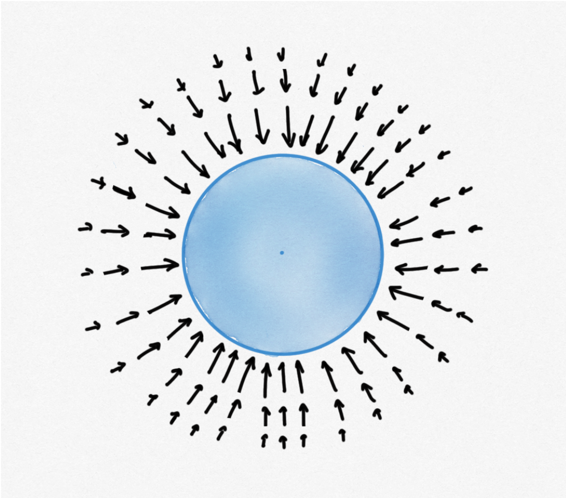

Figure 2.2: Visualizing the gravitational field around the earth. Notice how the length of the vectors **g** decreases as one moves away from the surface.

the centre of the earth is given by

$$\mathbf{g} = G_N \frac{M_E \hat{\mathbf{r}}}{d^2}, \tag{2.8}$$

where $\hat{\mathbf{r}}$ is the unit vector which points towards the centre of the earth and $M_E = 5.97237 \times 10^{24}$ kg is the mass of the earth. At the sea-level $d = 6378$ km and with $G_N = 6.674 \times 10^{-11}$ m^{3}kg^{-1}s^{-2} one gets $g = 9.8$ m/s^2. But as one goes away from the earth's surface the magnitude of $\mathbf{g}$ decreases as d increases. This vector field may be visualized in the way shown in figure 2.2.

In a similar way we may visualize the electric field due to various distributions of charge. For example, the electric field due to a point charge is very similar to the gravitational field. From Coulomb's law we can see that the electric field around a point charge points towards the charge and the magnitude of the field falls of as $1/r^2$ where r is the distance of the point from the point charge. However, because in the case of electrostatics we have two types of charge it means that electric field strength vectors point outwards from the source in case the source is a positive charge and they point towards the source in case the source is a negative charge.

Before we move onto the electric field configurations of other, more complicated charge distributions, let us introduce another way of visualizing the electric field. Another way of visualizing an electric field is by using *field lines*. These are lines (not necessarily straight

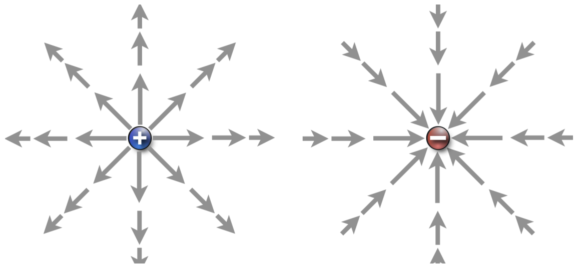

Figure 2.3: Electric fields for a point charge: The direction of the field depends on the sign of the charge.

lines) along which the vector field arrows we've drawing so far form tangents. The density of lines in some region denotes the *intensity* of the field in that region. For the electric field the field lines must necessarily originate at a positive charge and terminate at a negative charge, or they can go off to infinity.

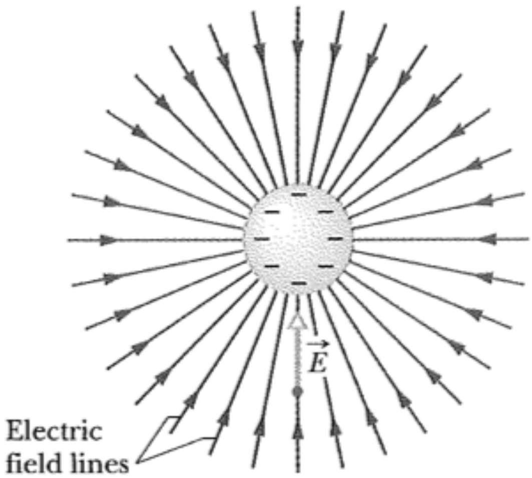

Figure 2.4: Electric field lines for a spherical distribution of charge.

In figure 2.4 we see the electric field lines for a spherical distribution of charges. In a future chapter when we introduce Gauss's law we shall be able to derive the electric field due to highly symmetric cases like this one with considerable ease. In figure 2.5 we see the electric field lines due to a configuration of charges of relative strengths $+3$ and -1. In the region behind the negative charge is a point where the electric field is zero (can you show it?) and so there are no electric field lines there.

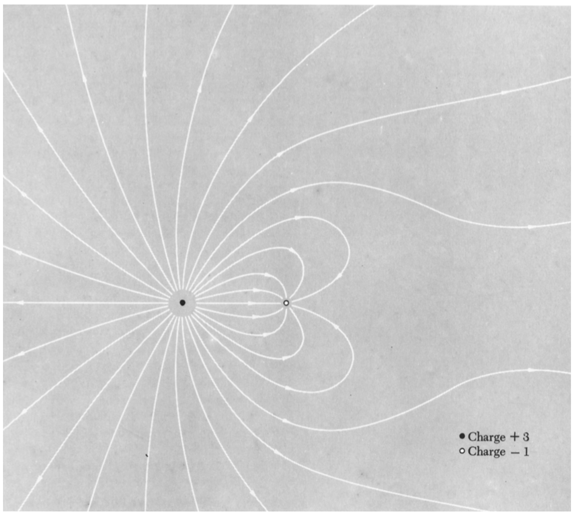

Figure 2.5: The electric field lines due to two charges of relative strengths $+3$ and -1.

Although figuring out the electric field lines for an arbitrary distribution of charge can be computationally intensive one can program a computer to it these days.

2.3 The Electric Field Due to an Electric Dipole

The *electric dipole* is the a configuration of two electric charges which have the same strength but opposite signs. The electric dipole is an important system to consider as it is the simplest example of a system which is electrically neutral (that is, the total charge of the system is zero) but its electric field is not zero. In nature we have an abundance of systems which are electrically neutral and the dipole is the simplest system with that property. The electric field lines due to the dipole is shown in figure 2.6.

Recall the electric field due to a point charge. According to Coulomb's law its strength falls off as $1/r^2$. We may ask, what about the field of the dipole? How does its field strength fall off as a function of distance? To answer this question, let us calculate the electric field of the dipole at a point on its axis of symmetry, that is, on the line joining the two charges. This is easier to do than finding out the electric field at an arbitrary point. Here we shall do this special case as we are interested in getting a rough idea of how the field falls off

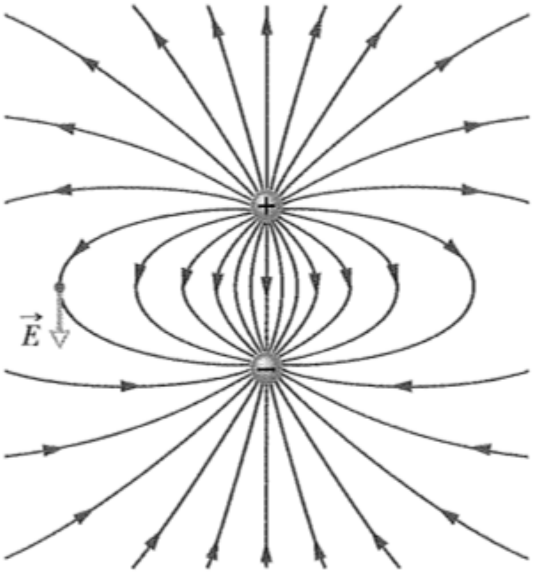

Figure 2.6: Electric field lines for a dipole.

as a function of the distance. The configuration is shown in 2.7.

We want to find out the electric field at the point P on the z-axis. It is given by

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_{(+)} - \mathbf{E}_{(-)} \\ &= \hat{\mathbf{k}} \left(k\frac{q}{r^2_{(+)}} - k\frac{q}{r^2_{(-)}} \right). \end{aligned} \tag{2.9}$$

After simplifying we get

$$E = \frac{2qk}{z^3} \frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}. \tag{2.10}$$

In the limit when $z \gg d$, that is when we are much further away than the size of the dipole, this formula becomes

$$E = \frac{2kqd}{z^3}. \tag{2.11}$$

A useful quantity for the dipole is the electric dipole moment $\mathbf{p}$ which is a vector that points from the negative to the positive charge and whose magnitude is give by dq, the product of the positive charge and the distance. The dipole moment measures the strength

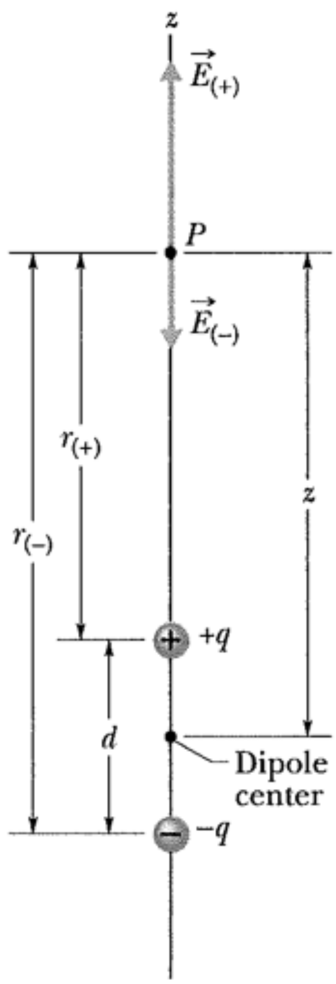

Figure 2.7: The electric field on the axis of a dipole.

of the dipole as well as its orientation in space. In terms of the magnitude of the dipole moment the last formula can be written as

$$E = 2k\frac{p}{z^3}. \tag{2.12}$$

Thus we conclude that the electric field strength of the dipole falls off as $1/r^3$ which is much faster than the fall off of the electric field due to a point charge.