

$$(Q1) D_1 = \{1, 2, 3, 4, 5, 6\}$$

$$D_2 = \{1, 2, 2, 4\}$$

SE = most likely  $\rightarrow$   
 8 = common  
 P = most .02

intersection $D_1 \cap D_2$		1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10		
2	3	5	6	7	8						
2	3	5	6	7	8						
4	5	6	7	8	9	10					

Now, Probability Distribution of  $X_i$ :

$x_i$	2	3	4	5	6	7	8	9	10
$P(x_i)$	$\frac{1}{24}$	$\frac{3}{24}$	$\frac{3}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{3}{24}$	$\frac{1}{24}$	$\frac{1}{24}$

$$(Q2) E(x) = 2 \times \frac{1}{24} + 3 \times \frac{3}{24} + 4 \times \frac{3}{24} + 5 \times \frac{4}{24} + 6 \times \frac{4}{24} + 7 \times \frac{4}{24} + 8 \times \frac{3}{24} + 9 \times \frac{1}{24} + 10 \times \frac{1}{24} \quad (\underline{\text{Ans}})$$

$$= 5.75$$

$$E(nx) = nE(x)$$

$$\Rightarrow 46 = n \times 5.75$$

$$\therefore n = 8 \quad (\underline{\text{Ans}})$$

$$\text{(Q3)} \quad E(7 \leq x \leq 5)$$

$$= E(5) + E(6) + E(7)$$

$$= (5 \times \frac{4}{24}) + (6 \times \frac{4}{24}) + (7 \times \frac{4}{24})$$

$$= 3$$

∴ So, when  $n \in \mathbb{N}$  and  $7 \geq n \geq 5$ , the number of expected trials is 3. (Ans)

$$\text{(Q4)} \quad P(\text{Exactly one } 2) = \frac{1}{6} \times \left(1 - \frac{1}{6}\right) + \left(1 - \frac{1}{6}\right) \times \frac{1}{4}$$

$$= \frac{1}{2}$$

$$P(\text{Exactly 3 out of 8}) = {}^8C_3 \times \left(\frac{1}{2}\right)^3 \times \left(1 - \frac{1}{2}\right)^{8-3}$$

$$= \frac{7}{32}$$

(Ans)

(Q.5) Given,  $P(X=n) = 0.1k$   $(0 \leq n \leq 5) \Rightarrow (0, 1, 2, 3, 4, 5)$

Therefore, we get probability mass function for  $X$  here as:

$$P(X=2) = 0.2$$

$$\therefore P(X=3) = 0.3 \quad (0 \leq n \leq 5) \text{ from } X \text{ has max value } 5$$

$$P(X=5) = 0.5 \quad (\text{E is standard notation})$$

$$\text{For } E(X) = \sum_n P(X=n)$$

$$= 0.2 \times 2 + 0.3 \times 3 + 0.5 \times 5$$

$$= 3.8 \quad (0 \leq n \leq 5) \quad (0, 1, 2, 3, 4, 5)$$

$$\text{For variance, } E(X^2) = \sum_n P(X=n)$$

$$= 0.2 \times 2^2 + 0.3 \times 3^2 + 0.5 \times 5^2$$

$$= 16$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= 16 - (3.8)^2$$

$$= 1.56$$

$$\therefore E(X) = 3.8$$

$$Var(X) = 1.56$$

(Ans)

6

$$n = 3$$

$P = \text{catching biza ball}$

$$1 - P = \text{not catching ball}$$

$$E(Y) = 1$$

$$n(1 - P) = 1$$

$$3(1 - P) = 1$$

$$1 - P = \frac{1}{3}$$

$$P = 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

(Ans)

0	2	1	5	4	1
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

For 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

For example consider

(Q.7) Given set,  $S = \{1, 2, 3, 4, 5, 7, \dots, 200\}$

For 7,

The lowest number and highest number that are divisible is 7 and 196.

$$\text{So, } n_7 = \frac{(196 - 7)}{7} + 1 \\ = 28$$

For 9,

The lowest and highest number that is divisible is 9 and 198.

$$\text{So, } n_9 = \frac{(198 - 9)}{9} + 1 \\ = 22$$

For 7 and 9, numbers that are divisible are

$$n_{7-9} = \left\lfloor \frac{200}{7 \times 9} \right\rfloor - 3$$

$\therefore$  Total numbers that are divisible by 7 or 9 but

$$\text{not both is } n = (n_7 + n_9) - 2(n_{7-9})$$

$$= (28 + 22) - 2 \times 3$$

$$= 44 \quad (\underline{\text{Ans}})$$

(Q-8) Given set,  $S = \{1, 2, 3, \dots, 200\}$  (Q. 8)

Now, amount of numbers,

$\therefore$  divisible by 3,  $n_1 = \left\lfloor \frac{200}{3} \right\rfloor = \left\lfloor \frac{200}{3} \right\rfloor = 66$

$\therefore$  divisible by 4,  $n_2 = \left\lfloor \frac{200}{4} \right\rfloor = \left\lfloor \frac{200}{4} \right\rfloor = 50$

$\therefore$  divisible by 12,  $n_3 = \left\lfloor \frac{200}{12} \right\rfloor = \left\lfloor \frac{200}{12} \right\rfloor = 16$

$\therefore$  Total number that are divisible by 3, 4, 12 are,

$$n = n_1 + n_2 - n_3$$

$$= 66 + 50 - 16$$

$$= 100$$

$\therefore$  Numbers that are not divisible =  $200 - 100$

$$\therefore \text{Ans} = \left\lfloor \frac{200}{12} \right\rfloor = 16$$

and Ans = 16

$$(e-f)(12) - (e(12) + f(12)) = 12$$

$$12e - 12f - (12e + 12f) = 12$$

$$12e - 12f = 12$$

(A)

(in)

5 couples can sit  $(5-1)! = 4!$

Q  $\frac{A, E, B, C, D, F}{1}$   
5

A, E can change themselves  $2!$

~~A E B C D F~~ = 24 ways

~~1, 2, 3, 4, 5, 6~~ = 2 ways

Chair All 6 can change in themselves,

$2! 2! 2! 2! 2! 2!$

= 64 ways.

Total =  $24 \times 2 \times 64$

= 3072 ways.

(Q10) Let  $x_1, x_2, x_3$  that denotes 1st, 2nd, 3rd dice rolls.

$$x = x_1 + x_2 + x_3$$

$$Y = x_1^L + x_2^L + x_3^L \quad (x_1^L) \oplus (x_2^L) \oplus (x_3^L) = (y)$$

Probability Distribution is,  $P(x_i = x) = \frac{1}{6}$

$$x = 1, 2, 3, 4, 5, 6 \quad \text{and} \quad i = 1, 2, 3$$

$$\text{We have, } E(x_i) = \sum_{n=1}^6 x P(x_i = n)$$

$$E(x_i) = \sum_{x=1}^6 x \left(\frac{1}{6}\right) \quad \text{Also } (x = x) \oplus \text{ (even)} = 2$$

$$\text{soft min sum} = \left(\frac{1}{6}\right) \sum_{n=1}^6 x \quad \text{for } x = 1, 2, 3, 4, 5, 6$$

$$= \frac{1}{6} (1+2+3+4+5+6)$$

$$= \frac{21}{6} = \frac{7}{2}$$

$$\text{Again, } E(x_i^L) = \sum_{n=1}^6 x^L P(x_i = x)$$

$$= \sum_{n=1}^6 x^L \left(\frac{1}{6}\right)$$

$$= \frac{1}{6} \sum_{n=1}^6 n^L \quad 2 \times 2^L + 3 \times 3^L + 4 \times 4^L + 5 \times 5^L + 6 \times 6^L$$

$$= \frac{1}{6} (1^L + 2^L + 3^L + 4^L + 5^L + 6^L)$$

$$= \frac{1}{6} \times 91$$

$$= \frac{91}{6}$$

$$\text{Now, } E(X) = E(x_1 + x_2 + x_3) \quad \text{as } x_1, x_2, x_3 \text{ are independent}$$

$$= 3 \times \frac{7}{2}$$

$$= \frac{21}{2}$$

$$E(Y) = E(x_1) + E(x_2) + E(x_3)$$

$$= 3 \times \frac{91}{6}$$

$$= \frac{91}{2}$$

$$(Any) x_i \sum_{i=1}^3 = (x) \sum_{i=1}^3$$

$$(1) \rightarrow \text{Ans}$$

(Q11) Total floors = 15

Mn. Raju is on the 11<sup>th</sup> floor.

floor above him =  $(15 - 11) = 4$

floor below him = 10

So, the probability of lift coming to Mr. Raju.,

$$\text{from below, } P_1 = \frac{10}{15} = \frac{2}{3}$$

$$\text{from above, } P_L = \frac{4}{15}$$

Since,  $P_1 > P_L$ , chances are greater that the lift will come from below to him. And as Mn. Raju wants to go down but the lift is going up, that's why he has to wait.

(Q12)

Total friends,  $n = 8$   $\Rightarrow$  most lot (8P8)  $\Rightarrow$  8 = group lot

Now take,

Tamim = T, Best friend = B, other friend = O

O B T all ~~except~~  $\Rightarrow$  ~~seating~~  $\Rightarrow$  ~~now~~ most  $\Rightarrow$  ~~as~~  
- O B T  $\Rightarrow$  ~~seating~~  
-- O B T  $\Rightarrow$  ~~seating~~  
-- - O B T  $\Rightarrow$  ~~seating~~  
-- - - O B T  $\Rightarrow$  ~~seating~~  
-- - - - O B T  $\Rightarrow$  ~~seating~~

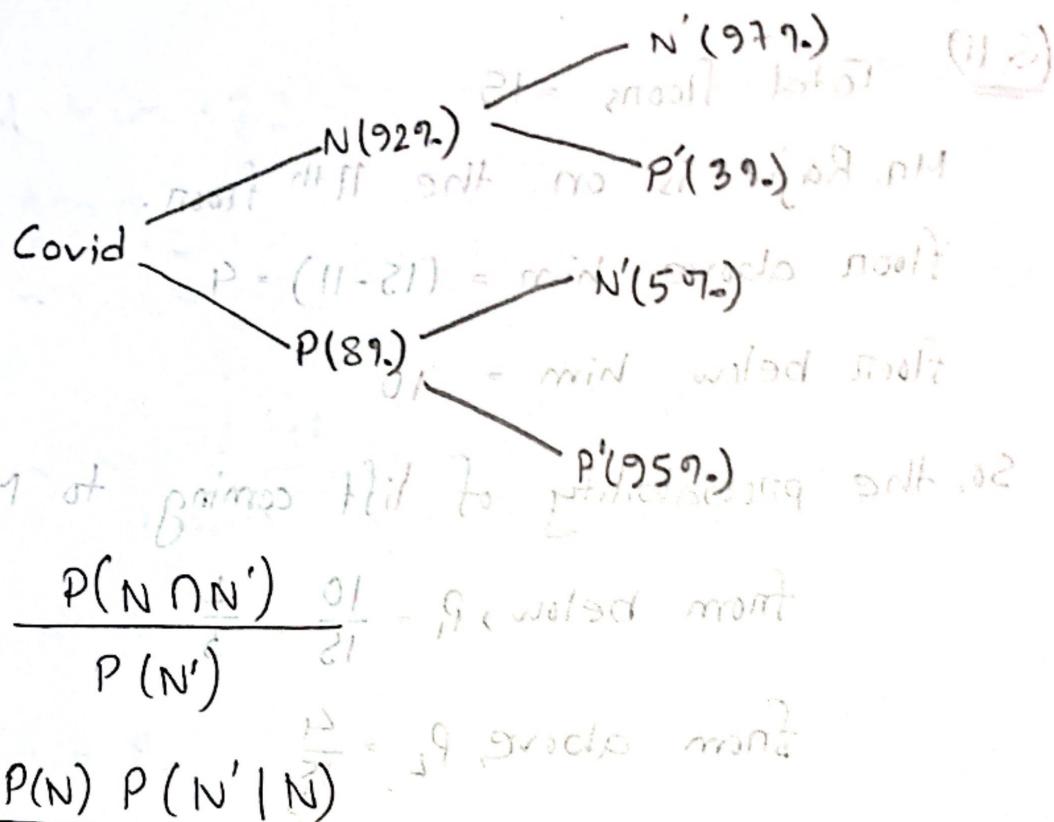
There will be 2 times the number  
of ways  $\Rightarrow$  ~~ways~~  $\Rightarrow$  ~~ways~~  
as T & O can be interchanged  
among themselves.

which is  $6 \times 2 = 12$   $\Rightarrow$  ~~ways~~

Now, Remaining 5 others can be seated  
in  $5!$  way.

$$\therefore \text{Total} = 5! \times 12 \\ = 1440 \quad (\text{Ans})$$

(Q13)



$$P(N|N') = \frac{P(N \cap N')}{P(N')} = \frac{P(N) P(N'|N)}{P(N)} = 0.92 \times 0.03 = 0.0276$$

$$= \frac{P(N) P(N'|N)}{P(N) P(N'|N) + P(P) P(N'|P)}$$

$$= \frac{0.92 \times 0.03}{0.92 \times 0.03 + 0.08 \times 0.92} = 0.0276 / 0.1616 = 0.1715$$

$$= \frac{0.92 \times 0.03}{0.92 \times 0.03 + 0.08 \times 0.92} = 0.0276 / 0.1616 = 0.1715$$

Ans

(Q14) Given that,  $(5x + \frac{3}{y} + \frac{4}{n} + \frac{5}{7}y)^8$   
 We know that,  $(5x + \frac{3}{y} + \frac{4}{n} + \frac{5}{7}y)^8 = \sum \frac{8!}{a!b!c!d!} (5x)^a \cdot (\frac{3}{y})^b \cdot (\frac{4}{n})^c \cdot (\frac{5}{7}y)^d$

where  $a, b, c, d$  are non-negative integers such  
 that  $a + b + c + d = 8$

Now,

$$\sum \frac{8!}{a!b!c!d!} (5)^{a+d} (x)^{a-c} (\frac{3}{y})^b (\frac{4}{n})^{d-b} (\frac{5}{7}y)^c$$

Hence in order to obtain the co-efficient  
 independent of  $x$  and  $y$ , we must have  $a=c$   
 and  $b=d$

Now, the possibilities are, — ①

$$a=0, b=0, c=4, d=4$$

$$a=1, b=1, c=3, d=3$$

$$a=2, b=2, c=2, d=2$$

$$a=3, b=3, c=1, d=1$$

$$a=4, b=4, c=0, d=0$$

for case ①,

$$\frac{8!}{0!0!4!4!} \times (5)^{0+4} \cdot 3^0 \cdot 4^4 \cdot 7^{-4}$$

$$= 9664.723$$

$$\text{for case } \textcircled{II}, \left(\frac{8!}{F} + \frac{8!}{A} + \frac{8!}{E} + \frac{8!}{D}\right) \text{ total no. of ways } \textcircled{II}$$

$$\frac{\left(\frac{8!}{F}\right) \cdot \left(\frac{8!}{A}\right) \cdot \left(\frac{8!}{E}\right) \cdot \left(\frac{8!}{D}\right)}{1! \cdot 1! \cdot 3! \cdot 3!} = \frac{18}{1+3} \cdot \left(\frac{8!}{F} + \frac{8!}{A} + \frac{8!}{E} + \frac{8!}{D}\right) \text{ total no. of ways}$$

above expression contains terms b, c, d, e  
 $= 391836.7347$

for case  $\textcircled{III}$ ,

$$\frac{8!}{2! \cdot 2! \cdot 2! \cdot 2!} \cdot \left(\frac{8!}{F}\right)^{3+2} \cdot \left(\frac{8!}{A}\right)^{3+2} \cdot \left(\frac{8!}{E}\right)^{3+2} \cdot \left(\frac{8!}{D}\right)^{3+2} \cdot \left(\frac{8!}{C}\right)^{3+2} \cdot \frac{18}{1+3+2+2} = 3$$

$\Rightarrow$   $b = d = 2$  and  $c = e = 1$  and  $a = f = 3$   
 $= 4628571.429$

for case  $\textcircled{IV}$ ,

$$\frac{8!}{3! \cdot 3! \cdot 1! \cdot 1!} \cdot \left(\frac{8!}{F}\right)^{3+1} \cdot \left(\frac{8!}{A}\right)^{3+1} \cdot \left(\frac{8!}{E}\right)^{3+1} \cdot \left(\frac{8!}{D}\right)^{3+1} \cdot \left(\frac{8!}{C}\right)^{3+1} \cdot \frac{18}{1+3+1+1} = 10800000$$

for case  $\textcircled{V}$ ,

$$\frac{8!}{4! \cdot 4! \cdot 0! \cdot 0!} \cdot \left(\frac{8!}{F}\right)^{4+0} \cdot \left(\frac{8!}{A}\right)^{4+0} \cdot \left(\frac{8!}{E}\right)^{4+0} \cdot \left(\frac{8!}{D}\right)^{4+0} \cdot \left(\frac{8!}{C}\right)^{4+0} \cdot \frac{18}{1+4+0+0} = 3543750$$

$$P = F \cdot P^F \cdot A^A \cdot E^E \cdot D^D \cdot C^C \cdot \frac{18}{1+4+0+0} = 3543750$$

EEF, FEE, EEF

(Q15) Total team = 32 8 - hr. duration total : (S1)

Total group = 8

∴ In 1 group there are 4 teams.  $T = \text{match}$

So, a team will play against 3 teams - in TQ

group stage, which can be arranged  
in  $3!$  ways

$$\therefore \text{Total match} = 8 \times 3!$$

$$= 48.$$

(Ans)