Stewart's Calculus

P 702 – 718

✓ Circles

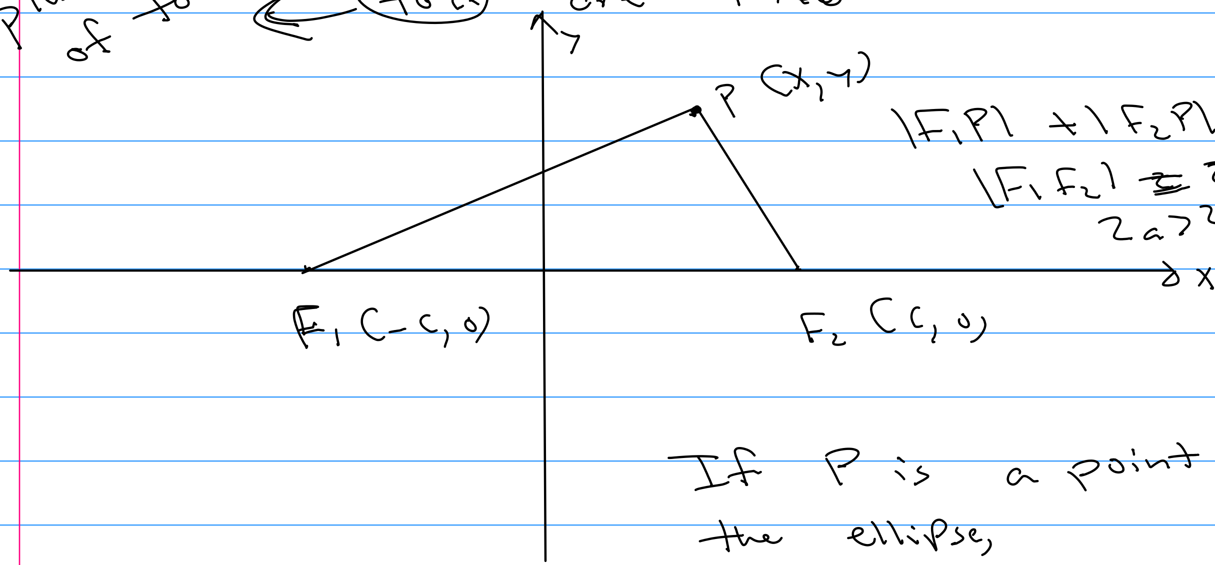
✓ Parabolas

✓ Ellipses

Hyperbolas

Ellipse: A set of points whose distances from two fixed points, called foci are fixed

plural of focus



$|F_1P| + |F_2P| = 2a$

$|F_1F_2| = 2c$

$2a > 2c \Rightarrow a > c$

If P is a point on the ellipse,

$$\underbrace{\sqrt{(x+c)^2 + y^2}}_{F_1P} + \underbrace{\sqrt{(x-c)^2+y^2}}_{F_2P} = 2a$$

$$\Rightarrow \sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$$

$$\Rightarrow (x+c)^2 + y^2 = 4a^2 - 2 \cdot 2a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$$

$$4a\sqrt{(x-c)^2 + y^2} = 4a^2 + \cancel{x^2} - 2cx + \cancel{c^2} + \cancel{y^2} - \cancel{x^2} - 2cx - \cancel{c^2} - \cancel{y^2}$$

$$= 4a^2 - 4cx$$

$$\Rightarrow a\sqrt{(x-c)^2 + y^2} = a^2 - cx$$

$$\Rightarrow a^2 [(x-c)^2 + y^2] = a^4 - 2a^2cx + c^2x^2$$

$$\Rightarrow x^2 - \cancel{2cx} + c^2 + y^2 = a^2 - \cancel{2cx} + \frac{c^2}{a^2}x^2$$

$$\Rightarrow \left(\frac{a^2 - c^2}{a^2}\right) x^2 + y^2 = a^2 - c^2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1$$

$a^2 - c^2$ is positive

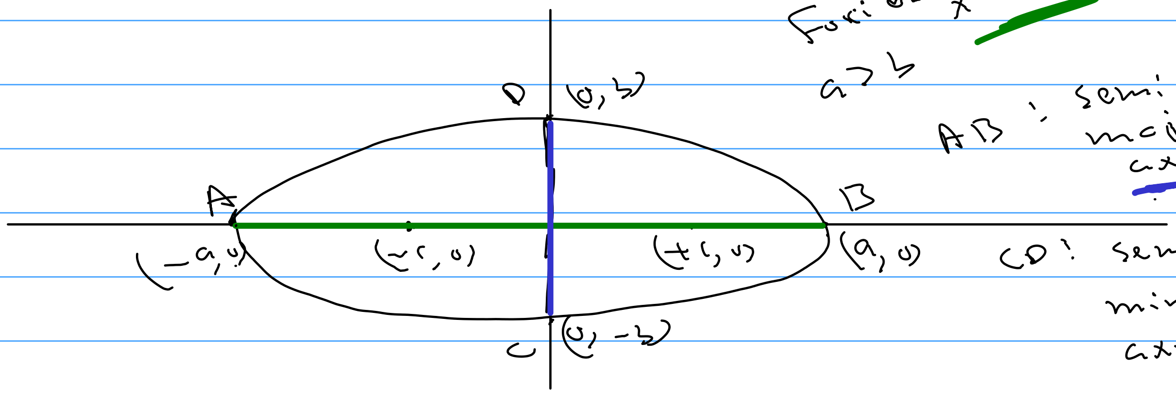
$b^2 := a^2 - c^2 \rightarrow a > b$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \rightarrow \text{general equation of the ellipse.}$$

The ellipse passes through $(\pm a, 0)$ and $(0, \pm b)$

$x = \pm a, \quad y = 0$

$y = \pm b, \quad x = 0$

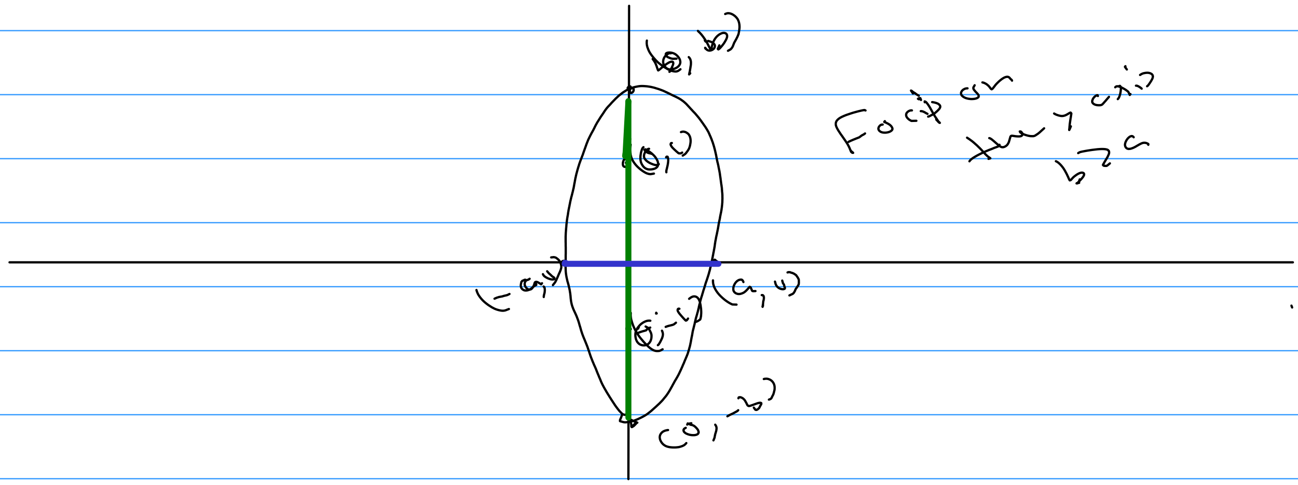


Foci on the x-axis

$a > b$

AB : semi major axis

CD : semi minor axis



Foci on the y axis

$b > a$

$$9x^2 + 16y^2 = 144 \Rightarrow \frac{9x^2}{144} + \frac{16y^2}{144} = 1$$

$$\Rightarrow \frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$$

$a = 4, \quad b = 3$

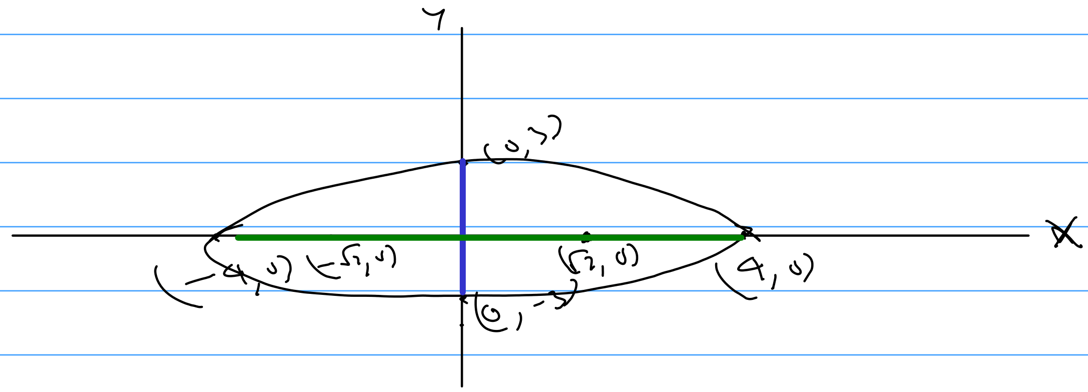
$c^2 = a^2 - b^2$

$c = \sqrt{16 - 9} = \sqrt{7}$

x intercepts: $(\pm 4, 0)$

y intercepts: $(0, \pm 3)$

foci : $(\pm\sqrt{7}, 0)$

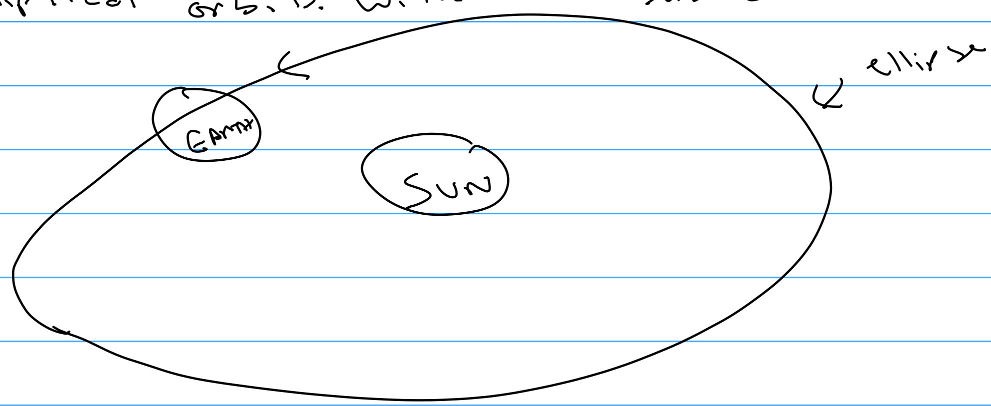


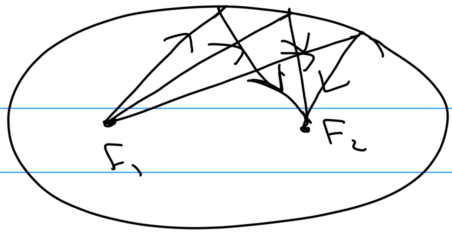
TERMS RELATED TO THE ELLIPSE.

Ellipse has two axes: ① Semi major axis
② Semi minor axis.

Vertices of the ellipse: Points at the ends of the semi major axis

The planets move around the sun in elliptical orbits with the sun at a focus.





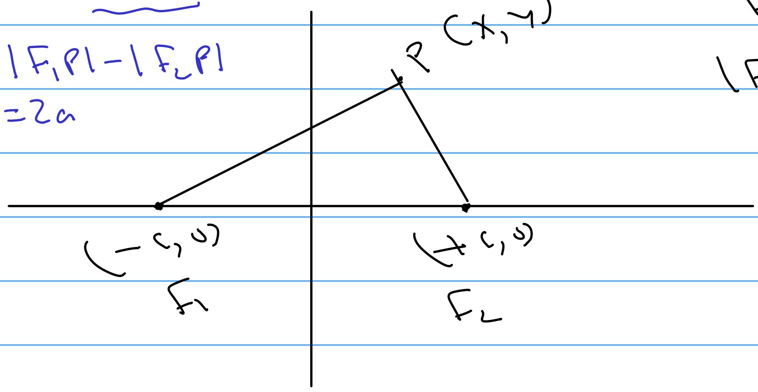
HYPERBOLA

Let's put the foci at $(\pm c, 0)$

$|F_2P| + |F_1F_2| > |F_1P|$

$2c > |F_1P| - |F_2P|$
$= 2a$

$2c > 2a$
$c > a$



$F_1F_2 = 2c$

$|F_1P| - |F_2P| = 2a$

$$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = 2a$$

$$\sqrt{(x+c)^2 + y^2} = 2a + \sqrt{(x-c)^2 + y^2}$$

$$\Rightarrow (x+c)^2 + y^2 = 4a^2 + 2 \cdot 2a \cdot \sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$$

$$4a\sqrt{(x-c)^2 + y^2} = \cancel{x^2} + 2cx + \cancel{c^2} + \cancel{y^2} - 4a^2 - \cancel{x^2} + 2cx - \cancel{c^2} - \cancel{y^2}$$

$$\Rightarrow a\sqrt{(x-c)^2 + y^2} = cx - a^2$$

$$\Rightarrow a^2\left[(x-c)^2 + y^2\right] = c^2x^2 + a^4 - 2a^2cx$$

$$\Rightarrow x^2 - 2\cancel{cx} + c^2 + y^2 = a^2 - 2\cancel{cx} + \frac{c^2}{a^2}x^2$$

$$\Rightarrow \left(1 - \frac{c^2}{a^2}\right)x^2 + y^2 = a^2 - c^2$$

$$\Rightarrow \left(\frac{a^2 - c^2}{a^2}\right)x^2 + y^2 = a^2 - c^2$$

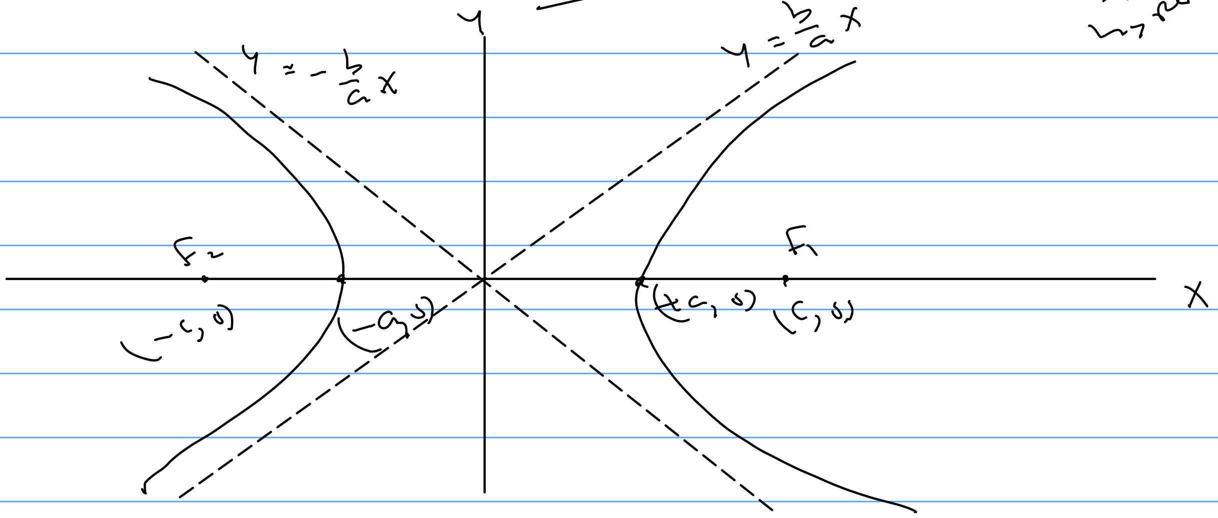
$b^2 = c^2 - a^2$

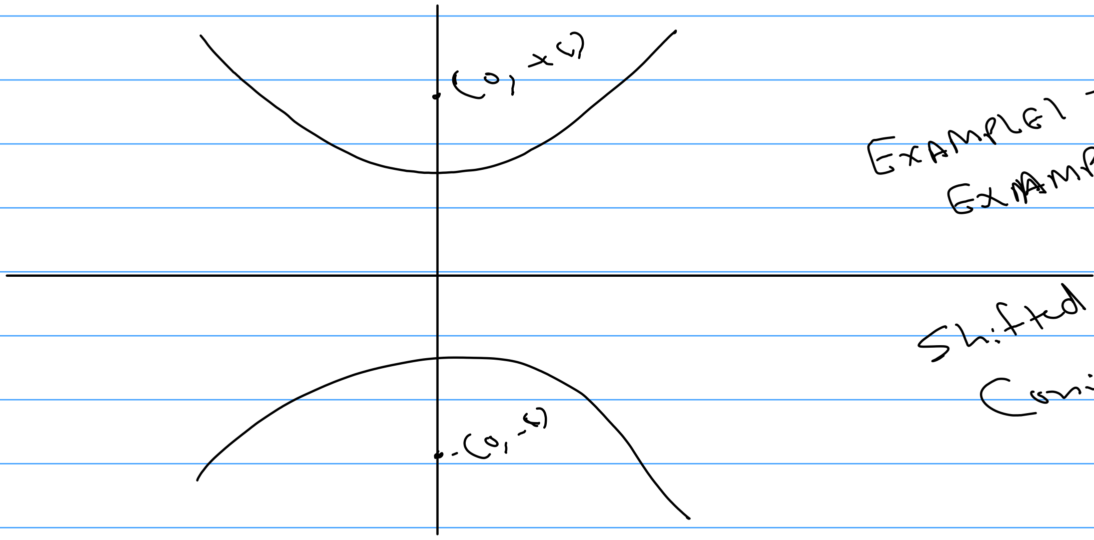
since $c > a$

then $-b^2 = a^2 - c^2$

$\Rightarrow \quad -\frac{b^2}{a^2}x^2 + y^2 = -b^2$

$\Rightarrow \quad \boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$ general equation of the hyperbola





EXAMPLE 1 -
EXAMPLE 7.

Shifted Conics.