Mathematical problem of probability for final quiz

1) The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will get an electric contract is 4/9. If the probability of getting both the contract is 14/15, what is the probability that he will get at least one contract?   8/45

2) In a survey of 100 readers, it was found 40 read The Daily Ittefaq and 15 read The Daily Star and 10 read both. What is the probability of a person reading at least one of the newspaper? 45

3) Suppose 35% of the students failed English, 25% of the students failed Statistics and 15% of the students failed both English and Statistics in a certain college. A student is selected at random.
    i)   If failed Statistics, what is the probability that he failed English?
    ii)   If failed English, what is the probability that he failed Statistics?
    iii)   What is the probability that he failed English or Statistics?

4) Of 1000 assembled components, 10 have a working defect and 20 have a structural defect. There is a good reason to assume that no component has both defects. What is the probability that randomly chosen component will have either type of defects?

5) The probability that a management trainee will remain with a company is 0.60. The probability that an earns more than tk. 60,000 per month is 0.50. The probability that an employee is a management trainee who remained with the company or who earns more than Tk. 60,000 per month is 0.70. What is the probability that an employee earns more than Tk. 60,000 per month, given that he is a management trainee who stayed with the company?

6) The probability that a new marketing approach will be successful is 0.6. The probability that the expenditure for developing the approach can be kept within the original budget is 0.50. The probability that both of these objectives will be achieved is 0.30. What is the probability that at least one of these objectives will be achieved? For the two events describe above, determine whether the events are independent or dependent.

7) The personnel manager of a large manufacturing firm finds that 15% of the firm's employees are junior executives and 25% of the firm's employees are MBAs. He also discovers that 5% of the firm's employees are both junior executives and MBAs. What is the probability of selecting a junior executive if the selection is confined to MBAs?

8) During a research of auto accidents, the highway safety council found that that 60% of all accidents occur at night, 52% are alcohol related, and 37% occur at night and are alcohol related.
    i)   What is the probability that an accident was alcohol related, given that it occurred at night?
    ii)   What is the probability that an accident was at night, given that it was alcohol related?

1) A lot of 10,000 parts produced on four machines were inspected and classified into three grades. The results were given in the following table.

Grades	Machine				
	I	II	III	IV	Total
Satisfactory	2400	1600	2400	1600	8000
Rework	450	300	450	300	1500
Scrap	150	100	150	100	500
Total	3000	2000	3000	2000	10000

If a part is selected at random from this lot, then find the following probabilities that-
   i)   It is produced by machine III. 3000/10,000
   ii)   It is produced on machine I given that it is scrapped 150/500
   iii)   It is reworked given that it is produced on machine IV   300/2000
   iv)   A satisfactory part is produced on machine II   1600/2000

2) Friendly's Department store has been the target of many shoplifters during the past month, but owing to increased security precautions, 250 shoplifters have been caught. Each shoplifter's gender is noted, also noted is whether the perpetrator was a first-time or repeated offender. The data are summarized in the table.

Gender	First-time offender	Repeated offender	Total
Male	60	70	130
Female	44	76	120
Total	104	146	250

Assuming that an apprehended shoplifter is chosen at random, find
   i)   The probability that the shoplifter is male
   ii)   The probability that the shoplifter is a first-time offender given that the shoplifter is male.
   iii)   The probability that the shoplifter is a female given that the shoplifter is repeated offender.
   iv)   The probability that the shoplifter is a male given that the shoplifter is first-time offender.
   v)   The probability that the shoplifter is both male and a repeated offender.

3) A study of job satisfaction was conducted for four occupations- Cabin maker, lawyer, doctor and system analyst. Job satisfaction was measured on a scale of 0-100. The data obtained are summarized in the following cross tabulation:

Occupation	Under 50	50-59	60-69	70-79	80-89	Total
Cabin maker	0	2	4	3	1	10
Lawyer	6	2	1	1	0	10
Doctor	0	5	2	1	2	10
System Analyst	2	1	4	3	0	10
Total	8	10	11	8	3	40

   i)   Develop a joint probability table.
   ii)   What is the probability of one of the participant studies had a satisfaction score in the 80's?

iii)   What is the probability of a satisfaction score in the 80's given the study participant was a doctor?

iv)   What is the probability that one of the participant studied was a lawyer?

v)   What is the probability that one of the participant was a lawyer and received a score under 50?

vi)   What is the probability of a satisfaction score under 50 given a person is a lawyer?

vii)   What is the probability of a satisfaction score of 70 or higher?

4) A market survey conducted in four cities pertained to preference for keya soap. The responses are shown below:

	Dhaka	Rajshahi	Khulna	Barishal
Yes	45	55	60	50
No	35	45	35	45
No opinion	5	5	5	5

i)   What is the probability that a consumer selected at random preferred Keya soap?

ii)   What is the probability that a consumer preferred Keya soap and was from Khulna?

iii)   What is the probability that a consumer preferred Keya soap, given that he was from Khulna?

iv)   Given that a consumer preferred Keya soap, what is the probability that he was from Dhaka?

5) Shown here are the raw values matrix and corresponding probability matrix for the results of a national survey of 200 executives who were asked to identify the geographic locale of their company and their company's industry type. The executives were only allowed to select one locale and one industry type.

RAW VALUES MATRIX

		Geographic Location				
		Northeast D	Southeast E	Midwest F	West G	
Industry Type	Finance A	24	10	8	14	56
	Manufacturing B	30	6	22	12	70
	Communications C	28	18	12	16	74
		82	34	42	42	200

Suppose a respondent is selected randomly from these data.

a. What is the probability that the respondent is from the Midwest (F)?

b. What is the probability that the respondent is from the communications industry (C) or from the Northeast (D)?

c. What is the probability that the respondent is from the Southeast (E) or from the finance industry (A)?

Total probability theorem:

1)   Mr. Ali wants to build a house this year. He applied for a bank loan. The probability that he will get it is 2/3. If he will get the bank loan, the probability that he will build a house is 3/4. However, if he will not get the bank loan, the probability that

he will build the house is 1/4. What is the probability that Mr. Ali will build a house this year?

2) A certain disease is present in about 1 out of 1000 persons in a given population. Suppose that there is simple blood test which gives a positive reading with probability 0.99 for a diseased person and with 0.005 for a healthy person.

    i) A person is selected at random from this population, what is the probability that the blood test of the selected person will give the positive reading?

    ii) If the blood test pf the selected person gives positive reading, what is the probability that he does have the disease?

3) Two sets of candidates are competing for the position on the board of directors of a company. The probabilities that the first and second sets will be win are 0.6 and 0.4, respectively. If the first set wins, the probability of introducing a new product is 0.8 and the corresponding probability of the second set wins is 0.3. What is the probability that the new product will be introduced?

4) Three machines *A, B* and *C* produce respectively **50%, 30%** and **20%** of the total number of items of a factory. The percentages of defective output of these machines are **3%, 4%** and **5%.** If an item is selected at random, find the probability that the item is defective.

5) Three persons A, B and C are being considered for the appointment as Vice-Chancellor of a University whose chance of being selected for the post are in the proportion 5:3:2, respectively. The probability that A, if selected, will introduce democratization in the University structure is 0.3, the corresponding probabilities for B and C doing the same are, respectively 0.6, and 0.8. What is the probability that democratization would be introduced in the University?

Bayes Theorem:

1) In a certain college, **4%** of the men and 1% of the women are taller than **6** feet. Furthermore, **60%** of the students are women. Now if a student is selected at random and is taller than **6** feet, what is the probability that the student is a woman?

2) In a city, 60% of the people moves by bus, 25% by rickshaw, and 15% by car. 3% of the accident committed by bus, 5% by rickshaw and 1% by car. A person of the city falls in an accident. What is the probability that the accident has committed by rickshaw?

3) In a bank 45% and 55% of the monthly statements are prepared by Mrs. Ali and Miss Karim respectively. These employees are very reliable. However, they are in error sometimes. The probabilities of committing their errors are 0.05% and 0.01% respectively. A monthly statement was found to be erroneous, what is the probability that it was done by Miss Karim?

4) A manufacturing firm produces steel pipes in the three plants with daily production 500, 1000, and 2000 pipes respectively. It is known that the fractions of defective output produced by three plants are respectively 0.005, 0.008, and 0.010. If a pipe is selected from a day's total production and is found to be defective, find out from which plant for this defective pipe, the probability is highest? And what is the probability that it came from the first plant?

5) In Dhaka city, male and female each form 50% of the population. It is known that 20% of the males and 5% of the females are unemployed, A research student studying the employment situation selects an unemployed person at random. Which is the probability that the person so selected is i) male ii) female?

6) In a Mohakhali post office, three clerks are assigned to process incoming mail. The first clerk processes 40%, the second clerk processes 35%, and the third clerk processes 25% of the mail. The first clerk has an error rate of 0.04, the second clerk has an error rate of 0.06 and the third clerk has an error rate of 0.03. A mail selected at random from a day's output is found to have an error. The post master wishes to know the probability that the mail was processed by the first, second and third clerk respectively.

7) In a winter of a country, it rains 45% of the days and shines 55% of the days. Mr. Saad has a barometer which wrongly predicts 5% of the time in rainy days and 10% of shiny days. Mr. Saad does not carry umbrella if the barometer predicts shine. One good shiny morning in winter his barometer predicts "shine" and he does not carry umbrella. What is the probability that Mr, Saad will suffer on that day due to rain?

8) A company has two plants to manufacture scooters. Plant I manufactures 80 per cent of the scooters and plant II manufactures 20 per cent. At plant I, 85 out of 100 scooters are rated standard quality or better. At plant II, only 65 out of 100 scooters are rated standard quality or better.
   (i)   What is the probability that scooter selected at random came from plant, I if it is known that the scooter is of standard quality ?
   (ii)   What is the probability that the scooter came from plant II, if it is known that the scooter is of standard quality ?

9) A company launches an advertising campaign of its new product on TV, radio and in print media in an area where 30% watch TV, 50% listen to the radio and the rest rely on newspapers for all information. It is estimated that a person who sees the advertisement on TV will buy the product with probability $0·6$. A person who has heard it on radio is expected to buy the product with probability $0·3$ and seeing the advertisement in print will convince a person to buy the product with probability $0·1$. A consumer, chosen at random, is found to have purchased the product. What is the probability she heard about the product on radio?

10) The results of an investigation by an expert on a fire accident in a skyscraper are summarised below :
    (i) Prob. (there could have been short circuit) $= 0·$
    (ii) Prob. (LPG cylinder explosion) $= 0·2$
    (iii) Chance of fire accident is 30% given a short circuit and 95% given an LPG explosion.
    Based on these, what do you think is the most probable cause of fire ? Statistically justify your answer.

11) Each of the three identical jewellery boxes has 2 drawers. In each drawer of the first box, there is a gold watch. In each drawer of the second box there is a silver watch. In one drawer of the third box, there is a gold watch while in the other drawer there is a silver watch. If we select a box at random, open one of the drawers and find it to contain a silver watch, what is the probability that the other drawer has the gold watch?

Random Variable:

1) The pressure measured in pounds per cm^2 at a certain valve is a random variable X whose probability density function is

$$f(x) = \begin{cases} \dfrac{2}{9}(3x - x^2), & \text{if } 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that the pressure at this valve is

i)   Not more than 2 pounds per cm^2
ii) Greater than 2 pounds per cm^2
iii) Between 1.5 and 2.5 per cm^2 and
iv) Less than 1.5 per cm^2

2) The time one has to wait for a bus at downtown out-stop is observes to be a random variable X with the following probability density function:

$$f(x) = \begin{cases} 0, & for\ X < 0 \\ \dfrac{1}{9}(x+1), & for\ 0 \le X < 1 \\ \dfrac{4}{9}\left(x - \dfrac{1}{2}\right), & for\ 1 \le X < \dfrac{3}{2} \\ \dfrac{4}{9}\left(\dfrac{5}{2} - x\right), & for\ \dfrac{3}{2} \le x < 2 \\ \dfrac{1}{9}(4-x), & for\ 2 \le X < 3 \\ \dfrac{1}{9}, & for\ 3 \le X < 6 \\ 0, & for\ X \ge 6 \end{cases}$$

Find the

i)   Probability that one waits between 0 and 2 minutes inclusive.
ii) Probability that one waits between 1 and 3 minutes inclusive.
iii) Probability that one waits between 2 and 6 minutes inclusive.

Mathematical expectation:

<mark>Discrete case:</mark>

1) Bangladesh Bank has six tellers available to serve customers. The number of tellers busy with customers at peak time, say, 11:00 a.m. varies from day to day. So it is a random variable denoted by X. It is known from the past records that the probability distribution of X is as follows.

Value of X=x	0	1	2	3	4	5	6
P(x)	.03	.05	.08	.15	k	0.26	0.22

i)   Find the value of K.
ii) Find the expected number of tellers busy with the customers at 11.00 a.m.
iii) Also find the variance and coefficient of variation of the number of tellers busy with the customers at 11.00 a.m.

2) Dan Desch, owner of college painters, studied his records for the past 20 weeks and reports the following number of houses pointed per week.

Number of houses pointed X:x	Probability : P(x)
10	.15
11	.20
12	.35
13	.20
14	.10

i)   Compute the mean number of houses pointed per week.   Expected value = mean
ii) Compute the variance of the number of houses pointed per week.

3) Eastern Apartments has a large number of units available to rent each month. A concern of management is the number of vacant apartments each month. The probability function of the number of vacant apartments in each month is

Number of vacant apartment	0	1	2	3	4	5
Probability	.15	.20	.35	.15	.10	.05

  i) Compute the mean number of vacant apartments each month.
  ii) Compute the variance and standard deviation of the number of vacant apartments each month.

Continuous case:

1) Suppose that in a certain region of a country the daily rainfall (in inches) is a continuous random variable X with probability density function f(x) given by-

$$f(x) = \frac{3}{4}(2x\text{-}x^2) \qquad for\ 0 < x < 2$$

Find the expected daily rainfall (in inches) in that region. Also find variance and standard deviation.

Probability distribution:

a) Bernoulli distribution:

1) In an office there are 10 Hp and 15 Dell computers. One computer is selected at random. Find the probability distribution of selecting a Dell computer. What is the expected number of selection of Dell computer? Also find its variance and standard deviation.

2) 20% vehicles running in a highway are operated by CNG. One day one vehicle is selected at random. Find the probability distribution and variance of the distribution of the number of vehicle operated by CNG.

b) Binomial Distribution:

1) An industrialist ensures that at best 10% of his product be defective. To investigate his assurance, it has been decided to conduct a survey on the understanding that, if at best 2 products out of 20 products are found defective, then the assurance is considered to be true. In practice, if the probability of producing a defective item is 0.08, what is probability of accepting the claim of industrialist?

2) Inoculation against Hepatitis-B has been done in an area. Previous experience says that the inoculation is effective in 80% cases. After 6 months 5 of the inoculated persons are randomly selected. Find the probability that, out of 5 persons
  i) Inoculation is not found effective for one person.
  ii) Inoculation is not found effective for at least one person

c) Poisson Distribution:

1) Between the hours 2 P.M. and 4 P.M. the average number of phone calls per minute coming into the switch board of a company is 2·35. Find the probability that during one particular minute, there will be at most 2 phone calls. [Given $e^{-2\cdot35} = 0\cdot095374$]

2) If 5% of the electric bulbs manufactured by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs
  i) none is defective,
  ii) 5 bulbs will be defective. (Given: $e^{-5} = 0\cdot007$).

3) The management of a photograph record company has discovered that the number of defects on records appears to follow a Poisson distribution with a mean equal to 0·4.
  i) What is the probability that a record selected at random will have three defects?
  ii) If management sets a policy that all photograph records sold to customers must not have any defects, what per cent of its records production will not be made available for sales because of defects? $e^{-4} = 0\cdot1832$, $e^{-0\cdot4} = 0\cdot6703$.

d) <mark>Geometric Distribution:</mark>

  1) A couple decides that they will take children until they have a male child or son. If the probability of a male child in their community is 1/3. What is the probability that the fourth child is a son?

e) Exponential distribution: practice lecture note

f) Normal distribution: practice lecture note