Chapter - 2

(4) (a) If we want to construct a 3 degree polynomial, we would need to do Hermite Interpolation.

	x	f(x)	f'(x)
n = 1	0.1	-0.62	3.46
	0.2	-0.30	3.15

$P_{2n+1} = P_{2(1)+1} = P_3$

(b) $P_3(x) = h_0(x) f(x_0) + \hat{h}_0(x) f'(x_0) + h_1(x) f(x_1) + \hat{h}_1(x) f'(x_1)$

$h_k(x) = \{1 - 2(x - x_k) l'_k(x_k)\} (l_k(x))^2$

$\hat{h}_k(x) = (x - x_k)(l_k(x))^2$

$l_1(x) = \frac{x - x_0}{x_1 - x_0} = \frac{x - 0.1}{0.2 - 0.1} = \frac{1}{10}(x - 0.1)$

$l'_1(x) = \frac{1}{10}(1) = \frac{1}{10}$

$l_0(x) = \frac{x - x_1}{x_0 - x_1} = \frac{x - 0.2}{0.1 - 0.2} = -\frac{1}{10}(x - 0.2)$

$l'_0(x) = -\frac{1}{10}(1) = -\frac{1}{10}$

$\hat{h}_0(x) = (x - x_0)(l_0(x))^2$

$\hat{h}_0(x) = (x - 0.1)\left(-\frac{1}{10}(x - 0.2)\right)^2$

$\hat{h}_1(x) = (x - x_1)(l_1(x))^2$

$= (x - 0.2)\left(\frac{1}{10}(x - 0.1)\right)^2$

$$h_0(x) = \{1 - 2(x - x_0) l_0'(x_0)\} \cdot (l_0(x))^2$$

$$h_0(x) = \{1 - 2(x - 0.1)\left(\frac{-1}{10}\right)\}\left(\frac{-1}{10}(x - 0.2)\right)^2$$

$$h_1(x) = \{1 - 2(x - x_1) l_1'(x_1)\} (l_1(x))^2$$

$$h_1(x) = \{1 - 2(x - 0.2)\left(\frac{1}{10}\right)\}\left(\frac{1}{10}(x - 0.1)\right)^2$$

$$P_3(x) = \{1 - 2(x - 0.1)\left(\frac{-1}{10}\right)\}\left(\frac{-1}{10}(x - 0.2)\right)^2$$

$$(-0.62) + \{1 - 2(x - 0.2)\left(\frac{1}{10}\right)\}\left\{\frac{1}{10}(x - 0.1)\right\}^2$$

$$(-0.30) + (x - 0.1)\left(\frac{-1}{10}(x - 0.2)\right)^2$$

$$(3.46) + (x - 0.2)\left(\frac{1}{10}(x - 0.1)\right)^2 (3.15)$$

(c) At $x = 2$,

$$P_3(2) = \{1 - 2(2 - 0.1)\left(\frac{-1}{10}\right)\}\left(\frac{-1}{10}(2 - 0.2)\right)^2 (-0.62)$$

$$+ \{1 - 2(2 - 0.2)\left(\frac{1}{10}\right)\}\left\{\frac{1}{10}(2 - 0.1)\right\}^2 (-0.30)$$

$$+ (2 - 0.1)\left(\frac{-1}{10}(2 - 0.2)\right)^2 (3.46) +$$

$$(2 - 0.2)\left(\frac{1}{10}(2 - 0.1)\right)^2 (3.15)$$

$$= -0.02772 + (-6.9312 \times 10^{-3}) + 0.2129976$$

$$+ 0.204687$$

$$= 0.3830334$$

(5) $f(x) = e^{-x^2}$

(a) The way to overcome this problem is by choosing Chebyshev nodes. We choose more nodes on the edges for Runge to reduce the error caused by spikes on the edges for Runge functions.

(b) $I = [-2, 2]$ ~~$n = 5$~~ $n = 4$ [for 5 nodes]
↳ degree

$$x_j = r \cos\left[\frac{(2j+1)\pi}{2(n+1)}\right]$$

$j = 0, 1, 2, 3, 4, \cancel{5, 6}$ [5 nodes]

$$x_0 = 2\cos\left[\frac{(2\times0+1)\pi}{10}\right] = 2\cos\left(\frac{\pi}{10}\right)$$

$$x_1 = 2\cos\left[\frac{(2\times1+1)\pi}{10}\right] = 2\cos\left(\frac{3\pi}{10}\right)$$

$$x_2 = 2\cos\left[\frac{(2\times2+1)\pi}{10}\right] = 2\cos\left(\frac{5\pi}{10}\right)$$

$$x_3 = 2\cos\left[\frac{(2\times3+1)\pi}{10}\right] = 2\cos\left(\frac{7\pi}{10}\right)$$

$$x_4 = 2\cos\left[\frac{(2\times4+1)\pi}{10}\right] = 2\cos\left(\frac{9\pi}{10}\right)$$

(c) $I = [-2, 4]$ The range is assymetric so we need to shift the interval.

$$x_j = \frac{b-a}{2} \cos\left[\frac{(2j+1)\pi}{2(n+1)}\right] + \left(\frac{a+b}{2}\right)$$

$$= \frac{4-(-2)}{2} \cos\left[\frac{(2j+1)\pi}{2(n+1)}\right] + \left(\frac{-2+4}{2}\right)$$

$$= 3\cos\left[\frac{(2j+1)\pi}{10}\right] + 1$$

Find x_0, x_1, x_2, x_3, x_4, ~~x_5 and x_6~~ like part b using the above formula.