Pulak Deb Roy

23241078

Sec - 10

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Q: $L_1 = \{w \in (0,1)^* : 0^i 1^j \text{ where } i \leq j\}$

Let assume L_1 is a regular language.

Consider a string $w = 0^n 1^{n+1}$; where $w \in L_1$.

Here, n is the pumping constant of L_1.

Dividing string to such that
① $|y| \geq 0$
ⓘⓘ $|xy| \leq n$
ⓘⓘⓘ $xy^k z \in L_1$ for all $k \geq 0$

Let $n = 2$, $w = 00111$

Dividing w, we get $x = 0$, $y = 0$, $z = 111$

here $|y| \geq 0$ and $|xy| \leq n$ and for $k = 1$, $xy^k z \in L_1$

To prove for all $k \geq 0$, $xy^k z \in L_1$

if we keep pumping y,

for $k = 2$, $w = xy^2 z = 000111$; $w \in L_1$

for $k = 3$, $w = xy^3 z = 0000111$; $w \notin L_1$

So if we keep pumping y, w does not belong in L_1 as it doesn't follow $i \leq y$.

Therefore, L_1 is not regular.

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Q2 Assume L_2 is regular. Pumping length $= p$

According to pumping Lamma, for any strings in L_2 with length at least p it can splite into 3 parts xyz with this conditions,

① $|y| \geq 0$

② $|xy| \leq p$

③ For each $i \geq 0$, $xy^i z \in L_2$

Here, $s = a^3 b^3 c^{3+2}$, $i = j = k = 3$

① Let $s = a^3 b^3 c^{3+2}$ and $y = a^2$

$xy^2z = a^3 a^2 b^3 c^{3+2} = a^5 b^3 c^{3+2}$

which will violate the condition $i = k$ so

$xy^2 z \in L_2$.

case 2: y contains both a's and b's

Let $s = a^3 b^3 c^{3+2}$ and $y = (ab)^2$

$xy^2 z = a^3 (ab)^2 b^3 c^{3+2} = a^5 ab\, ab\, b^3 c^{3+2}$

which will violate the condition $i = k$

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Case 3: y contains only b's.

Let $S = a^3 b^3 c^{3+2}$ and $y = b^2$

$xy^2z = a^3 b^3 b^{3 \cdot 2} c^{3+2} = a^3 b^5 b^3 c^{3+2}$

which will also violate $i = k$.

case 4: y contains only c's.

Let $S = a^3 b^3 c^{3+2}$ and $y = c^2$

$xy^2z = a^3 b^3 (c^2)^2 b^3 c^{3+2}$

$= a^3 b^3 c^4 b^3 c^{3+2}$

which will violate $i = k$

so we can say the L_2 is not a regular language.

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Q3 Let's assume a pumping length P, so any strings in L_3 with length p can be divided into three parts xyz.

Now, (i) $i \geq 0$, $xy^iz \in L_3$

(ii) $|y| \geq 0$

(iii) $|xy| \leq p$

if it was a regular language, then the strings, $s = 0^p1^p$ where $s = 2p+1$ as $|xy| \leq p$, y consist only 0s and can only pump 0^p part of the string.

if $i = 2$, $s' = xy^2z$ so repeats the y part only.

It's in the form $0^{(p+|y|)}10^p$. As $y > 0$ the pumped string has more 0's in the first than 0.

A palindrome with more 0s in the first part than the second part is not in $L3$, so it doesn't satisfy xy^iz for any i

So L_3 is not regular.

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(Q4) Assume, L_4 is regular language.
There must exist a pumping constant p.

Let, $w \in L_4$

$w = 00\#0^n$

$x = 00\#$

$y = 0$

$z = 0^{n-1}$

pumping y twice or more will violate the condition of $|w_1| = 2 \times |w_2|$.

pumping twice will get $00\#0^{n+1}$

Here $|w_1|$ is no longer double the length of $|w_2|$

Thus, the pumping lemma is violated and it contradicts the assumption that L_4 is regular language.

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(Q5) Assume L_5 is regular language than there a 'w' string exists. which can be splited x, y, z which follows these rules,

(i) $xy^iz \in A$ for each $i \geq 0$

(ii) $|y| > 0$

(iii) $|xy| \leq P$

Now, Let pumping length - P

$\therefore w = a^{2P}$ $\qquad \therefore |y| \geq 1$

Let, $|y| = K$ [where $K \geq 1$]

Now. if $P = 2$,

$|xy^2z|$

$= |xyz| + |y|$

$= 2p + K$

but $2P + K > 2P$

which contradicts $|xy| \leq P$ rule

$\therefore L_5$ is not a regular language.

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