Discrete probability distribution

Bernoulli Distribution

A probability distribution describes the behavior of a random variable. Many times the observations obtained from different random experiments have a general type of behavior. Therefore, the random variable (s) associated with these experiments have a general type of probability distribution. So it can be represented by a single formula. This type of single formula about the behavior of a random variable is known as a probability distribution.

The probability distribution of a discrete random variable that arises from some statistical experiments is known as a discrete probability distribution. The following are some widely used discrete probability distributions-

1) Bernoulli distribution
2) Binomial distribution
3) Poisson distribution
4) Geometric distribution etc.

This distribution describes a natural phenomenon or a mechanical process in which you expect a particular event to appear or not. If the outcome of the random experiment is either a success with a fixed probability p or a failure with a probability $q = 1 - p$ and the random variable X takes either the value 1 in the case of success or a value of zero (0) in the case of failure, then the distribution of X is the Bernoulli distribution and the random variable is called Bernoulli random variable.

Jacob Bernoulli (1654-1705) Swiss mathematician first invented this probability distribution.

Bernoulli Trial

Definition: Bernoulli trial is a random experiment whose outcomes are classified as one of the two categories. (S , F) or (Success, Failure) or (1, 0)

$$\Rightarrow X(\text{Success}) = 1, \quad X(\text{Failure}) = 0$$

Example:

•Tossing a coin, observing Head or Tail

•Observing patient's status Died or Survived.

Therefore, if the probability of success in a Bernoulli trial is P and q= 1-P is the probability of failure in the same Bernoulli trial.

$$P(X = x) = \begin{cases} p & \text{if } x = 1 \\ 1 - p & \text{if } x = 0 \end{cases}$$

X=x	1	0
P(x)	p	q

then the probability mass function of the Bernoulli distribution is

$$P(x) = \begin{cases} P^x q^{1-x} \ for\ x = 0, 1 \\ 0\ otherwise \end{cases}$$

Where, p is the only parameter of the distribution satisfying $0 \le p \le 1\ and$ p+q =1
This distribution also called point binomial distribution. For example, the Bernoulli distribution can be used as the probability distribution of

- ✓ any single experiment that asks a yes-no question;
- ✓ the question results in a boolean-valued outcome;
- ✓ a (possibly biased) coin toss where 1 and 0 would represent “head” and “tail” (or vice versa).

Conditions:

i) Bernoulli trials are finite in number (have the dichotomous outcome)
ii) Trials are independent
iii) Probability of success and Failure remain the same from trial to trial

- ✓ Famous for Plata and plomo dialogue [Either you take the deal or you have to take the bullet) (Don Escober)
- ✓ **King of Cocaine**

We will generally call the two outcomes “success” and “failure”. Assign the value 1 to success and 0 to failure. Now mean of a Bernoulli distribution is $E(X) = \sum_{i=0}^{1} x\, p(x)$ =

$$\sum_{X=0}^{1} X \cdot p^X q^{1-X}$$

$$= 0 \cdot p^0 q^1 + 1 \cdot p^1 q^0 = p$$

$$\boxed{\mu = E(X) = p}$$

And variance of Bernoulli random variable is $V(X) = E(X^2) - [E(X)]^2$
Now,

$$E(X^2) = \sum_{X=0}^{1} X^2 \cdot p^X q^{1-X} = 0 \cdot p^0 q^1 + 1 \cdot p^1 q^0 = p$$

So that, $V(X) = P - P^2 = P(1-P) = Pq$

Example 01: In a class 70% students are boy. A student is selected at random from this class, then find the expected number of boy students.

Solution: Let X be random variable that a student be a boy, then

$$x = \begin{cases} 0, & if\ student\ is\ girl \\ 1, & if\ a\ studernt\ is\ boy \end{cases}$$

X=x	1	0
P(x)	P=0.7	q=1-p=0.3

$$E(X)=\sum_{x=0}^{1} xp(x) = 0 \times 0.3 + 1 \times 0.7 = 0.7$$

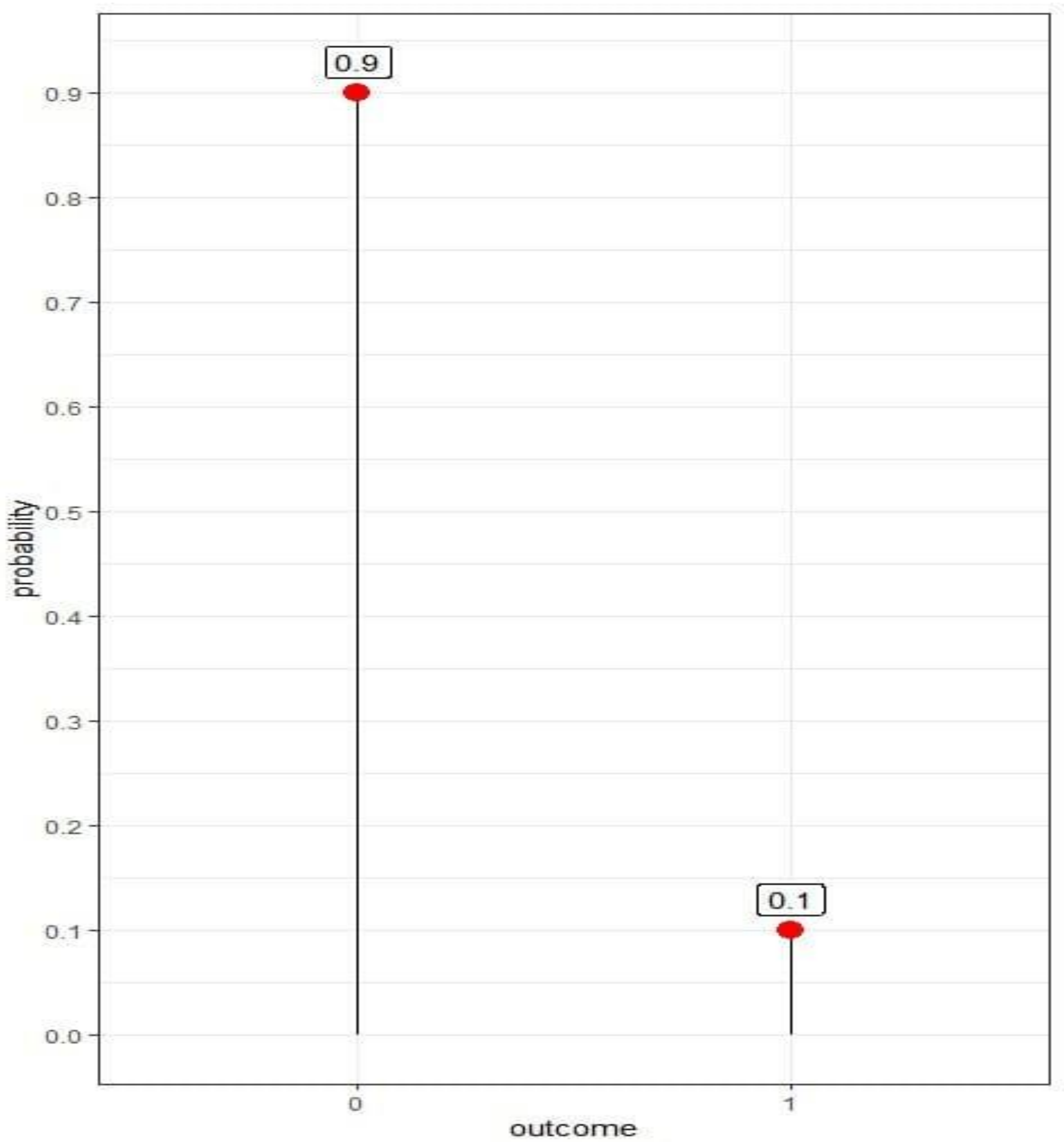
Bernoulli Distribution Example

Suppose there is an experiment where you flip a coin that is fair. If the outcome of the flip is heads then you will win. This means that the probability of getting heads is p = 1/2. If X is the random variable following a Bernoulli Distribution, we get P(X = 1) = p = 1/2.

Example 1: A basketball player can shoot a ball into the basket with a probability of 0.6. What is the probability that he misses the shot?
Solution: We know that success probability P (X = 1) = p = 0.6
Thus, probability of failure is P (X = 0) = 1 - p = 1 - 0.6 = 0.4
Answer: The probability of failure of the Bernoulli distribution is 0.4

Example 2- The prevalence of a certain disease in the general population is 10%. If we randomly select a person from this population, we can have only two possible outcomes (diseased or healthy person). We call one of these outcomes (diseased person) success and the other (healthy person), a failure.

The probability of success (p) or diseased person is 10% or 0.1. So, the probability of failure (q) or healthy person = 1-p = 1-0.1 = 0.9.
If we denote diseased person as 1 and healthy person as 0, we can plot this Bernoulli distribution as follows:



We have two outcomes:

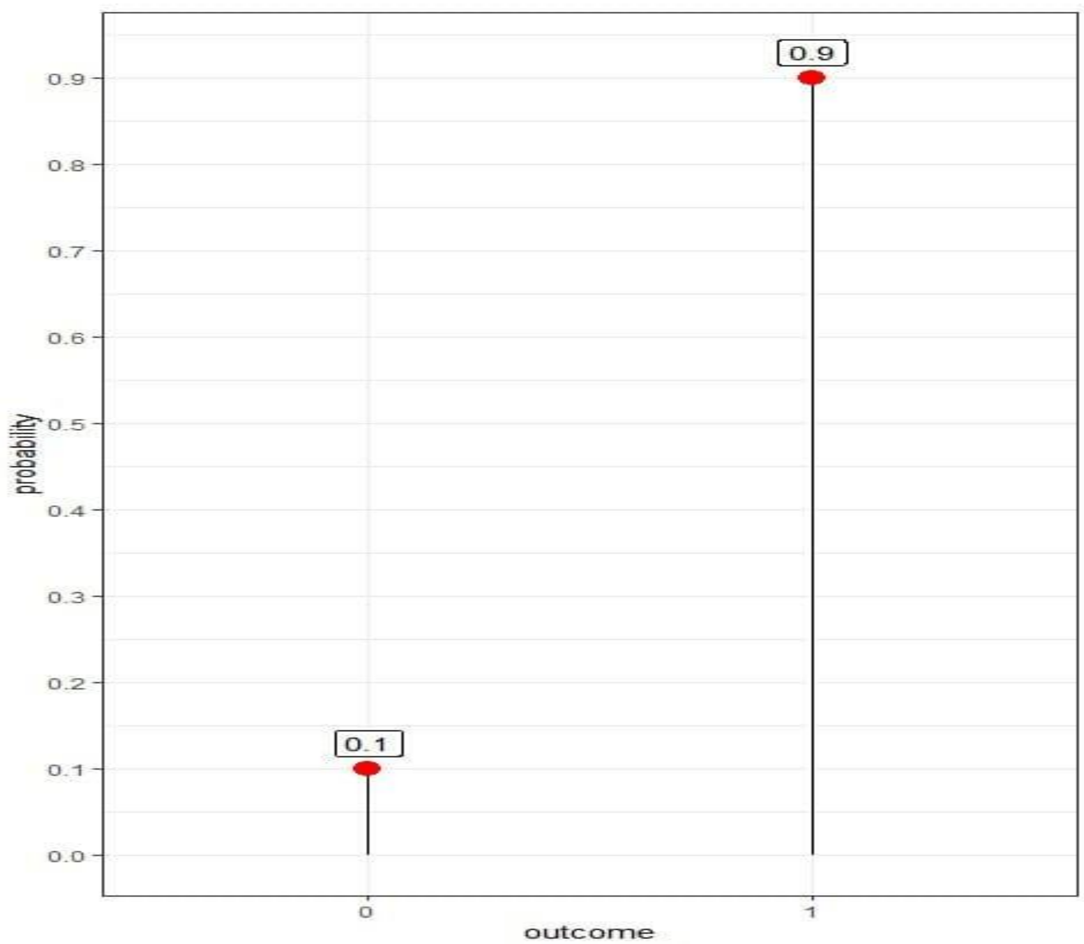
- A healthy person or 0 with a probability of 0.9.
- A diseased person or 1 with a probability of 0.1.

– Example 3

In the above example of disease prevalence of 10%, if We are interested in healthy persons and call the healthy person a success and the diseased person, a failure.

The probability of success (p) or healthy person is 90% or 0.9. So, the probability of failure (q) or diseased person = 1-p = 1-0.9 = 0.1.

If we denote a healthy person as 1 and diseased person as 0, we can plot this Bernoulli distribution as follows:



We have two outcomes:

- A healthy person or 1 with a probability of 0.9.
- A diseased person or 0 with a probability of 0.1.

There are only two outcomes of the following events-

i) Gender of babies born in a hospital.
ii) The outcome of a coin toss
iii) Outcome of a diagnostic test (+/-)
iv) Result of world cup final match of 2022
v) Selection of a person classified as male and female
vi) Selection of an industrial product (defective/non-defective)
vii) Identification of an employee (absent/present)
viii) Selection of worker (skilled/unskilled)
ix) Hit the target with a gun or bomb etc.