Assignment 2

Pulak Deb Roy

23241078

Sec - 7

MAT 215

Scanned with CamScanner

Qw-1

$$\mathcal{L}^{-1}\left\{\frac{2s-5}{s^2-9}\right\}$$

$$\Rightarrow 2\mathcal{L}^{-1}\left\{\frac{s}{s^2-3^2}\right\}-\frac{5}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2-3^2}\right\}$$

$$\Rightarrow 2\cosh 3t-\frac{5}{3}\sinh 3t$$

$\therefore \frac{2s-5}{s^2-9}$

$\Rightarrow \frac{2s-5}{s^2-(3)^2}$

$\Rightarrow \frac{2s}{s^2-3^2}-\frac{5}{s^2-3^2}$

$\Rightarrow 2\left(\frac{s}{s^2-3^2}\right)-\frac{5}{3}\left(\frac{3}{s^2-3^2}\right)$

Ans

Scanned with CamScanner

4. $y'' + 3y' + 2y = e^{kt} - e^{8t}$ $\quad y(0) = 1$, $y'(0) = 0$

$\Rightarrow \mathcal{L}\{y'' + 3y' + 2y\} = \mathcal{L}\{e^{8t}\}$

$\Rightarrow \mathcal{L}\{y''\} + 3\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = \mathcal{L}\{e^{8t}\}$

$\Rightarrow [s^2\mathcal{L}\{y\} - s^1 y(0) - s^0 y'(0)] + 3[s^1\mathcal{L}\{y\} - s^0 y(0)]$

$+ 2[\mathcal{L}\{y\}] = \left(\frac{1}{s-8}\right)$

$\Rightarrow [s^2 y - s.1 - 0] + 3[sy - 1] + 2y = \frac{1}{s-8}$

$\Rightarrow s^2 y - s + 3sy - 3 + 2y = \frac{1}{s-8}$

$\Rightarrow s^2 y + 3sy + 2y = \frac{1}{s-8} + s + 3$

$\Rightarrow y(s^2 + 3s + 2) = \frac{1}{s-8} + s + 3$

$\Rightarrow y = \left(\frac{1}{s^2+3s+2}\right) \cdot \left(\frac{1}{s-8} + s + 3\right)$

$\mathcal{L}\{y\} = \left(\frac{1}{s^2+3s+2}\right)\left(\frac{1}{s-8} + s + 3\right)$

Scanned with CamScanner

$$Y = \mathcal{L}^{-1}\left\{ \frac{1}{s^2+3s+2}\left(\frac{1}{s-8}+s+3\right)\right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{(s-8)(s^2+3s+2)} + \frac{s}{s^2+3s+2} + \frac{3}{s^2+3s+2}\right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{(s-8)(s+2)(s+1)} + \frac{s}{(s+2)(s+1)} + \frac{3}{(s+2)(s+1)}\right\}$$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{(s-8)(s+2)(s+1)}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{(s+2)(s+1)}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{(s+2)(s+1)}\right\}$$

(1) $$\frac{1}{(s-8)(s+2)(s+1)} = \frac{A}{(s-8)} + \frac{B}{(s+2)} + \frac{C}{(s+1)}$$

$$1 = A(s+2)(s+1) + B(s-8)(s+1) + C(s-8)(s+2) \quad -(i)$$

$$= A(s^2+3s+2) + B(s^2-7s-8) + C(s^2-6s-16)$$

$$= As^2 + 3As + 2A + Bs^2 - 7Bs - 8B + Cs^2 - 6Cs - 16C$$

$$1 = s^2(A+B+C) + s(3A-7B-6C) + (2A-8B-16C) \quad -(ii)$$

Equating factors of like terms,

"s^2"	"s"	"constant"
$A+B+C=0$ — (*)	$3A-7B-6C=0$ — (**)	$2A-8B-16C=1$ — (***)

in eqn (i),

if $s=-1$, $\Rightarrow 1 = C(1+6-16)$

$$= C(-9)$$

$$\Rightarrow C = -\frac{1}{9}$$

Scanned with CamScanner

if $s=-2 \Rightarrow 1 - B(4+14-8) \Rightarrow B = \frac{1}{10}$

$\therefore$ substitute A, B into (***),

$$2A - 8\left(\frac{1}{10}\right) - 16\left(-\frac{1}{9}\right) = 1$$

$$\Rightarrow 2A - \frac{4}{5} + \frac{16}{9} = 1$$

$$\therefore A = \frac{1}{90}$$

$$\therefore \frac{1}{(s-8)(s+2)(s+1)} = \frac{1}{90(s-8)} + \frac{1}{10(s+2)} + \frac{-1}{9(s+1)}$$

(2) $$\frac{s}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$\Rightarrow s = A(s+1) + B(s+2) \quad -(i)$$

in eqn (i),

if $s=-1 \Rightarrow -1 = B(1) \Rightarrow B = -1$

if $s=-2 \Rightarrow -2 = A(-1) \Rightarrow A = 2$

$$\therefore \frac{s}{(s+2)(s+1)} = \frac{2}{(s+2)} + \frac{-1}{(s+1)}$$

Scanned with CamScanner

③ $\frac{1}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$

$1 = A(s+1) + B(s+2) \quad -①$

in eqn ①.

if $s = -1, \Rightarrow 1 = B(1) \Rightarrow B = 1$

if $s = -2 \Rightarrow 1 = A(-1) \Rightarrow A = -1$

$\therefore \frac{1}{(s+2)(s+1)} = \frac{-1}{s+2} + \frac{1}{s+1}$

$$y = \mathcal{L}^{-1}\left\{\frac{1}{(s-8)(s+2)(s+1)}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{(s+2)(s+1)}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{(s+2)(s+1)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{90(s-8)} + \frac{1}{10(s+2)} - \frac{1}{9(s+1)}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{(s+2)} - \frac{1}{(s+1)}\right\}$$

$$+ 3\mathcal{L}^{-1}\left\{\frac{-1}{(s+2)} + \frac{1}{(s+1)}\right\}$$

$$= \left[\frac{1}{90}\mathcal{L}^{-1}\left\{\frac{1}{(s-8)}\right\} + \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{1}{(s+2)}\right\} - \frac{1}{9}\mathcal{L}^{-1}\left\{\frac{1}{(s+1)}\right\}\right] +$$

$$\left[2\mathcal{L}^{-1}\left\{\frac{1}{(s+2)}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{(s+1)}\right\}\right] + 3\left[-1\mathcal{L}^{-1}\left\{\frac{1}{(s+2)}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)}\right\}\right]$$

$$\Rightarrow \left[\frac{1}{90} \cdot e^{8t} + \frac{1}{10} \cdot e^{-2t} - \frac{1}{9}e^{-t}\right] + 3\left[-e^{-2t} + e^{-t}\right] + \left[2e^{-2t} - e^{-t}\right]$$

Scanned with CamScanner

$$\Rightarrow \frac{1}{90}e^{8t} + \frac{1}{10}e^{-2t} - \frac{1}{9}e^{-t} - 3e^{-2t} + 3e^{-t} + 2e^{-2t} - e^{-t}$$

$$\Rightarrow \frac{1}{90}e^{8t} + \frac{1}{10}e^{-2t} - \frac{1}{9}e^{-t} - e^{-2t} + 2e^{-t}$$

$$\Rightarrow \frac{17}{9}e^{-t} - \frac{9}{10}e^{-2t} + \frac{1}{90}e^{8t}$$

(Ans)

Scanned with CamScanner