2

$u_1 = (1,2,1) \qquad u_2 = (1,1,3) \qquad u_3 = (2,1,1)$

Step 1

$v_1 = u_1 = (1,2,1)$

Step 2

$$v_2 = u_2 - \frac{\langle u_2 . v_1 \rangle}{v_1 . v_1} v_1$$

$\langle u_2, v_1 \rangle = 1+2+3$
$= 6$

$v_1 . v_1 = 1+2+1$
$= 6$

$$= (1,1,3) - \frac{\cancel{6}\,3}{\cancel{6}\,2} (1,2,1)$$

$$= (1,1,3) - (\cancel{\tfrac{3}{2}, 3, \tfrac{3}{2}}) (1,2,1)$$

$$= (\cancel{-\tfrac{1}{2}, -2, \tfrac{3}{2}}) \; (0,-1,2)$$

$\langle u_3 . v_1 \rangle = 2+2+1$
$= 5$

$\cancel{v_2 . v_2 = -\tfrac{1}{2} + (-2) +}$
$= -1$

$\cancel{u_3 . v_2 = -1 - 2 + \tfrac{3}{2}}$
$= -\frac{3}{2}$

Step 3

$$v_3 = u_3 - \frac{\langle u_3 . v_1 \rangle}{v_1 : v_1} v_1 - \frac{\langle u_3 . v_2 \rangle}{v_2 . v_2} . v_2$$

$$= (2,1,1) - \frac{5}{6} (1,2,1) - \frac{-1}{5} (\cancel{-\tfrac{3}{2}, -3, \tfrac{9}{2}}) \, v_2$$

$$= (2,1,1) - \left(\frac{5}{6}, \frac{5}{3}, \frac{5}{6}\right) - (\cancel{-\tfrac{3}{4}, -\tfrac{3}{2}, \tfrac{9}{4}}) \left(0, -\tfrac{1}{5}, \tfrac{2}{5}\right)$$

$$\cancel{= \left(\frac{3}{4}, -\frac{3}{2}, -\frac{1}{4}\right) - \left(-\frac{3}{4}, -3, \frac{9}{4}\right)}$$

$$\Rightarrow (2,1,1) - (5/6, 5/3, 5/6) - (0, -1/5, 2/5)$$

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$$\Rightarrow \left(\frac{3}{2}, \frac{3}{2}, -\frac{5}{2}\right) \Rightarrow \left(\frac{7}{6}, -\frac{7}{5}, -\frac{7}{30}\right)$$

$$\therefore \text{Onthogonal basis} = \left\{ \begin{vmatrix} 1 \\ 2 \\ 1 \end{vmatrix}, \begin{vmatrix} -\frac{1}{2} \\ -2 \\ \frac{3}{2} \end{vmatrix}, \begin{vmatrix} \frac{3}{2} \\ \frac{3}{2} \\ -\frac{5}{2} \end{vmatrix}, \begin{vmatrix} \frac{7}{6} \\ -\frac{7}{5} \\ -\frac{7}{30} \end{vmatrix} \right\}$$

Again,

$$g_1 = \frac{v_1}{\|v_1\|} = \frac{(1,2,1)}{\sqrt{1^2+2^2+1^2}} = \frac{(1,2,1)}{\sqrt{5}}$$

$$= \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$$

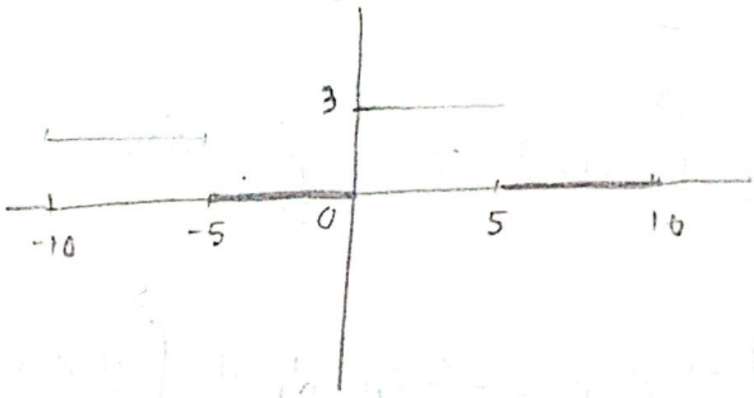
$$g_2 = \frac{v_2}{\|v_2\|} = \frac{(-\frac{1}{2}, -2, \frac{3}{2})}{\sqrt{(-\frac{1}{2})^2+(-2)^2+(\frac{3}{2})^2}} = \frac{(-\frac{1}{2}, -2, \frac{3}{2})}{\frac{\sqrt{26}}{2}}$$

$$= \left(-\frac{\sqrt{26}}{26},\right.$$

1

$$f(x) = \begin{cases} 0 , & -5 < x < 0 \\ 3 , & 0 < x < 5 \end{cases}$$

Period = 10



if $f(x) = -f(-x)$ then the function is odd.

let $x = 3$,

$f(3) = 3$, $f(-3) = 0$

$\therefore f(x) \neq -f(-x)$ $\therefore$ Not odd

if $f(x) = f(-x)$ then even,

$f(3) = 3$. $f(-3) = 0$

$\therefore f(x) \neq f(-x)$ $\therefore$ Not even.

$\therefore$ the function is neither odd nor even

Now, $\sin(n\pi) = 0$

Finding a_n,

$$a_n = \frac{1}{2L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} \, dx$$

$$= \frac{1}{5} \int_{-5}^{5} f(x) \cos \frac{n\pi x}{5} \, dx$$

$$= \frac{1}{5} \left[\int_{-5}^{0} 0 \cdot \cos \frac{n\pi}{5}(x) \, dx + \int_{0}^{5} 3 \cos \frac{n\pi}{5}(x) \, dx \right]$$

$$= \frac{1}{5} \left[0 + \left[3 \cdot \frac{\sin \frac{n\pi}{5}(x)}{\frac{n\pi}{5}} \right]_0^5 \right]$$

$$= \frac{1}{5} \left[\frac{15}{n\pi} \sin \frac{n\pi}{5}(x) \right]_0^5$$

$$= \frac{3}{n\pi} \left[\sin \frac{n\pi}{5} 5 - \sin \frac{\pi n}{5} \times 0 \right]$$

$$= \frac{3}{n\pi} [0 - 0]$$

$$= 0$$