(Q1) $D1 = \{1,2,3,4,5,6\}$

$D2 = \{1,2,2,4\}$

D1/D2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
2	3	4	5	6	7	8
4	5	6	7	8	9	10

Now, Probability Distribution of x_i:

x_i	2	3	4	5	6	7	8	9	10
$P(x_i)$	$1/24$	$3/24$	$3/24$	$4/24$	$4/24$	$4/24$	$3/24$	$1/24$	$1/24$

(Ans)

(Q2) $E(x) = 2\times\frac{1}{24} + 3\times\frac{3}{24} + 4\times\frac{3}{24} + 5\times\frac{4}{24} + 6\times\frac{4}{24} + 7\times\frac{4}{24} + 8\times\frac{3}{24} + 9\times\frac{1}{24} + 10\times\frac{1}{24}$

$= 5.75$

$E(nx) = nE(x)$

$\Rightarrow 46 = n\times 5.75$

$\therefore n = 8$

(Ans)

(Q3) $E(7 \geq x \geq 5)$

$= E(5) + E(6) + E(7)$

$= \left(5 \times \frac{4}{24}\right) + \left(6 \times \frac{4}{24}\right) + \left(7 \times \frac{4}{24}\right)$

$= 3$

$\therefore$ So, when $x \in X$ and $7 \geq x \geq 5$, the number of expected trials is 3.

(Ans)

(Q4) $P(\text{Exactly one } 2) = \frac{1}{6} \times \left(1 - \frac{2}{4}\right) + \left(1 - \frac{1}{6}\right) \times \frac{2}{4}$

$= \frac{1}{2}$

$P(\text{Exactly } 3 \text{ out of } 8) = {}^{8}C_{3} \times \left(\frac{1}{2}\right)^{3} \times \left(1 - \frac{1}{2}\right)^{8-3}$

$= \frac{7}{32}$

(Ans)

(Q.5) Given, $P(X=x) = 0.1K$

Therefore, we get probability mass function for X here as:

$P(X=2) = 0.2$

$P(X=3) = 0.3$

$P(X=5) = 0.5$

For $E(x) = \sum x P(X=x)$

$= 0.2\times 2 + 0.3\times 3 + 0.5\times 5$

$= 3.8$

For variance, $E(x^2) = \sum x^2 P(X=x)$

$= 0.2\times 2^2 + 0.3\times 3^2 + 0.5\times 5^2$

$= 16$

$var(x) = E(x^2) - [E(x)]^2$

$= 16 - (3.8)^2$

$= 1.56$

$\therefore E(X) = 3.8$

$V(X) = 1.56$

(Ans)

6

$n = 3$

p = catching a ball

$1-p$ = not " " "

$E(Y) = 1$

$n(1-p) = 1$

$3(1-p) = 1$

$1-p = \frac{1}{3}$

$p = 1 - \frac{1}{3}$

$= \frac{2}{3}$

(Ans)

(Q.7) Given set, $S = \{1, 2, 3, 4, 5, 7 \ldots 200\}$

For 7,

The lowest number and highest number that are divisible is 7 and 196.

So, $n_7 = \frac{(196-7)}{7} + 1$

$= 28$

For 9.

The lowest and highest number that is divisible is 9 and 198.

So, $n_9 = \frac{(198-9)}{9} + 1$

$= 22$

For 7 and 9, number that are divisible are

$n_{7\text{-}9} = \left| \frac{200}{7 \times 9} \right| = 3$

$\therefore$ Total numbers that are divisible by 7 or 9 but not both is $n = (n_7 + n_9) - 2(n_{7\text{-}9})$

$= (28 + 22) - 2 \times 3$

$= 44$ (Ans)

(Q-8) Given set, $S = \{1, 2, 3, \ldots 200\}$

Now, amount of numbers,

$\therefore$ divisible by 3, $n_1 = \left|\frac{S}{3}\right| = \left|\frac{200}{3}\right| = 66$

$\therefore$ divisible by 4, $n_2 = \left|\frac{S}{4}\right| = \left|\frac{200}{4}\right| = 50$

$\therefore$ divisible by 12, $n_3 = \left|\frac{S}{12}\right| = \left|\frac{200}{12}\right| = 16$

$\therefore$ Total number that are divisible by 3, 4, 12 are,

$$n = n_1 + n_2 - n_3$$

$$= 66 + 50 - 16$$

$$= 100$$

$\therefore$ Numbers that are not divisible $= 200 - 100$

$= 100$

(Ans)

(Ans)

9 $\underbrace{\underset{1}{\underline{A, E}}, B, C, DF}_{5}$

5 couples can sit $(5-1)! = 4!$

$= 24$ ways

A, E can change themselves $2!$

$= 2$ ways

All 6 can change in themselves

$2!\ 2!\ 2!\ 2!\ 2!\ 2!$

$= 64$ ways.

Total $= 24 \times 2 \times 64$

$= 3072$ ways.

(Q10) Let x_1, x_2, x_3 that denotes 1st, 2nd, 3rd dice rolls.

$$X = x_1 + x_2 + x_3$$

$$Y = x_1^2 + x_2^2 + x_3^2$$

Probability Distribution is. $P(x_i = x) = \frac{1}{6}$

$x = 1, 2, 3, 4, 5, 6$ and $i = 1, 2, 3$

We have, $E(x_i) = \sum_{n=1}^{6} x P(x_i = n)$

$$E(x_i) = \sum_{x=1}^{6} x\left(\frac{1}{6}\right)$$

$$= \left(\frac{1}{6}\right) \sum_{n=1}^{6} x$$

$$= \frac{1}{6}(1+2+3+4+5+6)$$

$$= \frac{21}{6} = \frac{7}{2}$$

Again, $E(x_i^2) = \sum_{n=1}^{6} x^2 P(x_i = x)$

$$= \sum_{n=1}^{6} x^2\left(\frac{1}{6}\right)$$

$$= \frac{1}{6} \sum_{n=1}^{6} n^2$$

$$= \frac{1}{6}(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2)$$

$$= \frac{1}{6} \times 91$$

$$= \frac{91}{6}$$

Now, $E(X) = E(X_1 + X_2 + X_3)$

$= 3 \times \frac{7}{2}$

$= 21/2$

$E(Y) = E(X_1^2) + E(X_2^2) + E(X_3^2)$

$= 3 \times \frac{91}{6}$

$= \frac{91}{2}$

(Ans)

(Q11) Total floors $= 15$

Mr. Raju is on the 11th floor.

floor above him $= (15-11) = 4$

floor below him $= 10$

So, the probability of lift coming to Mr. Raju.,

from below, $P_1 = \frac{10}{15} = \frac{2}{3}$

from above, $P_2 = \frac{4}{15}$

Since, $P_1 > P_2$, chances are greater that the lift will come from below to him. And as Mr. Raju wants to go down but the lift is going up. that's why he has to wait.

(Q 12) Total friends, $n = 8$

Now take,

Tamim = T, Best friend = B, other friend = O

O B T _ _ _ _ _
_ O B T _ _ _ _
_ _ O B T _ _ _
_ _ _ O B T _ _
_ _ _ _ O B T _
_ _ _ _ _ O B T

There will be 2times the number as T & O can be interchange among themselves.

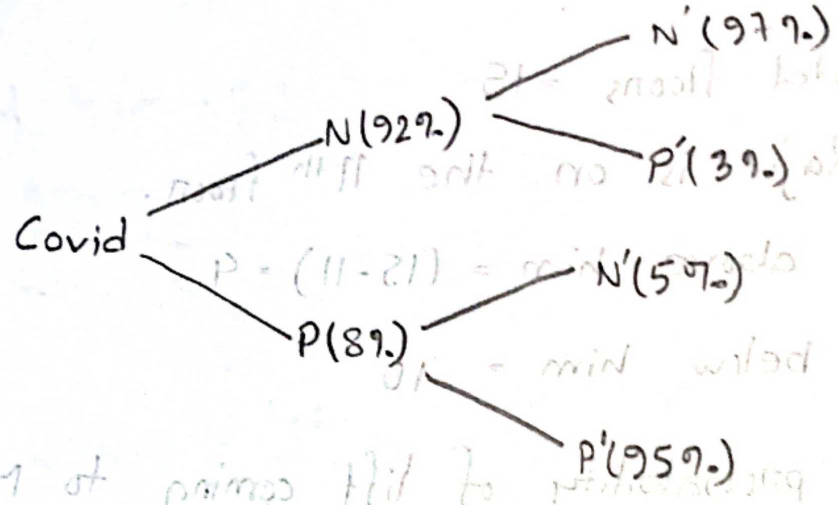
which is $6 \times 2 = 12$.

Now, Remaining 5 others can be seated in $5!$ way.

$\therefore$ Total $= 5! \times 12$

$= 1440$ (Ans)

(Q 13)



$$P(N \mid N') = \frac{P(N \cap N')}{P(N')}$$

$$= \frac{P(N)\,P(N' \mid N)}{P(N)\,P(N' \mid N) + P(P)\,P(N' \mid P)}$$

$$= \frac{0.92 \times 0.97}{0.92 \times 0.97 + 0.08 \times 0.05}$$

$$= 0.9955$$

(Ans)

(Q14) Given that, $\left(5x + \frac{3}{y} + \frac{4}{x} + \frac{5}{7}y\right)^8$

We know that, $\left(5x + \frac{3}{y} + \frac{4}{x} + \frac{5}{7}y\right)^8 = \sum \frac{8!}{a!b!c!d!}(5x)^a \cdot \left(\frac{3}{y}\right)^b \cdot \left(\frac{4}{x}\right)^c \cdot \left(\frac{5y}{7}\right)^d$

where a, b, c, d are non-negative integers such that $a + b + c + d = 8$

Now,

$$\sum \frac{8!}{a!b!c!d!}(5)^{a+d} \cdot (x)^{a-c} \cdot (3)^b \cdot (y)^{d-b} \cdot (4)^c \cdot (7)^{-d}$$

Hence in order to obtain the co-efficient independent of x and y, we must have $a = c$ and $b = d$

Now, the possibilities are,

$a = 0,\ b = 0,\ c = 4,\ d = 4$ — (I)

$a = 1,\ b = 1,\ c = 3,\ d = 3$ — (II)

$a = 2,\ b = 2,\ c = 2,\ d = 2$ — (III)

$a = 3,\ b = 3,\ c = 1,\ d = 1$ — (IV)

$a = 4,\ b = 4,\ c = 0,\ d = 0$ — (V)

for case (I),

$$\frac{8!}{0!0!4!4!} \times (5)^{0+4} \cdot 3^0 \cdot 4^4 \cdot 7^{-4}$$

$= 4664.723$

for case (ii),

$$\frac{8!}{1!\,1!\,3!\,3!} \times (5)^{1+3} \cdot 3^{1} \cdot 4^{3} \cdot 7^{-3}$$

$= 391836.7347$

for case (iii),

$$\frac{8!}{2!\,2!\,2!\,2!} \cdot (5)^{2+2} \cdot 3^{2} \cdot 4^{2} \cdot 7^{-2}$$

$= 4628571.429$

for case (iv),

$$\frac{8!}{3!\,3!\,1!\,1!} \cdot (5)^{3+1} \cdot 3^{3} \cdot 4^{1} \cdot 7^{-1}$$

$= 10800000$

for case (v),

$$\frac{8!}{4!\,4!\,0!\,0!} \cdot 5^{4+0} \cdot 3^{4} \cdot 4^{0} \cdot 7^{-0}$$

$= 3543750$

(Q15) Total team = 32

Total group = 8

∴ In 1 group there are 4 teams.

So, a team will plays against 3 teams in group stage, which can be arranged in 3! ways

∴ Total match = $8 \times 3!$

$= 48$.

(Ans)