Assignment 4

MAT 110

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Sec - 06

Assignment 4 Questions

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Maxima/Minima

0 points possible (ungraded)

(SKN 4) Find the critical points for the following function, and use the second derivative test to find the local extrema:

$$g(x,y) = \frac{1}{3}x^3 + y^2 + 2xy - 6x - 3y + 4$$

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Lagrange Multipliers

0 points possible (ungraded)

(FAB 3) Use Lagrange Multipliers to find the maximum and minimum values of $f(x,y,z) = 2x + y - 2z$ subject to the constraint $x^2 + y^2 + z^2 = 4$

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Conics

0 points possible (ungraded)

[AAN3] Find the equation of a hyperbola that has foci at $(-3,9)$, $(-3,-7)$ and vertices $(-3,-4)$, $(-3,6)$

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Vector Calculus

0 points possible (ungraded)

(NAS 1) Find the directional derivative of $f(x,y,z) = x^2y - yz^3$ at the point $(1,-2,0)$ in the direction of the vector $\mathbf{a} = 2\hat{i} + \hat{j} - 2\hat{k}$

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Problem 1:

$$g(x,y) = \frac{1}{3}x^3 + y^2 + 2xy - 6x - 3y + 4$$

$$g(x) = \frac{1}{3} \cdot 3x^2 + 2y - 6$$

$$= x^2 + 2y - 6$$

$$g(y) = 2y + 2x - 3$$

$g(x) = 0$

$\Rightarrow x^2 + 2y - 6 = 0$

$\Rightarrow x^2 + 2y = 6 \quad - (i)$

$g(y) = 0$

$\Rightarrow 2y + 2x - 3 = 0$

$\Rightarrow 2(y + x) = 3$

$\Rightarrow y = 3/2 - x \quad - (ii)$

we put the (ii) in (i),

$\Rightarrow x^2 + 2(3/2 - x) = 6$

$\Rightarrow x^2 + 3 - 2x - 6 = 0$

$\Rightarrow x^2 - 2x - 3 = 0 \quad - (iii)$

Comparing (iii) with $ax^2 + bx + c$. we find

x value $(3, -1)$

if, $x = -1$

$2y - 2(-1) - 3 = 0$

$\Rightarrow 2y = 1$

$\therefore y = (1/2)$

$\therefore (x,y) = (-1, 1/2)$

again, if $x=3$,

$$2y+2(3)-3=0$$

$$\Rightarrow 2y-3=0$$

$$\therefore y=-3/2$$

$$\therefore (x,y)=(3,-3/2)$$

$$g_{xx} = x^2+2y-6$$
$$= 2x$$

$$g_{yy} = 2y+2x-3=0$$
$$= 2$$

$$g_{xy} = 2y+2x-3$$
$$=2$$

$$D = g_{xx}\cdot g_{yy} - (g_{xy})^2$$
$$= 2x\cdot 2 - (2)^2$$
$$= 4x-4$$

Critical point	$D = g_{xx}.g_{yy}-(g_{xy})^2$	$g_{xx}=2x$	comment
$(-1, 1/2)$	$D=4(-1)-4 = -8<0$	$2\cdot-1=-2<0$	saddle point
$(3, -3/2)$	$D=4.3-4 = 8>0$	$2.3=6>0$	Local minima

Problem: 2

Given. $f(x,y,z) = 2x + y - 2z$

Subject to the constraint $x^2 + y^2 + z^2 = 4$

Here, $f_x = 2$, $f_y = 1$, $f_z = -2$

$g_x = 2x$, $g_y = 2y$, $g_z = 2z$

we know,

$f_x = \lambda g_x$	$f_y = \lambda g_y$	$f_z = \lambda g_z$
$\Rightarrow 2 = \lambda 2x$	$\Rightarrow 1 = \lambda 2y$	$\Rightarrow -2 = \lambda 2z$
$\therefore x = \frac{1}{\lambda}$	$\therefore y = \frac{1}{2\lambda}$	$\therefore z = -\frac{1}{\lambda}$

Now, $\left(\frac{1}{\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{\lambda}\right)^2 = 4$

$\Rightarrow \frac{1}{\lambda^2} + \frac{1}{4\lambda^2} + \frac{1}{\lambda^2} = 4$

$\Rightarrow \frac{4+1+4}{4\lambda^2} = 4$

$\Rightarrow \frac{9}{4\lambda^2} = 4$

$\Rightarrow \lambda^2 = 9/16$

$\therefore \lambda = \pm 3/4$

for, $\lambda = \frac{3}{4}$,

$x = \frac{4}{3}$, $y = \frac{2}{3}$, $z = \frac{-4}{3}$

for, $\lambda = -\frac{3}{4}$,

$x = -\frac{4}{3}$, $y = \frac{-2}{3}$, $z = \frac{4}{3}$

(x, y, z) in $f(x, y, z)$:

$\Rightarrow 2\left(\frac{4}{3}\right) + \frac{2}{3} - 2\left(-\frac{4}{3}\right)$

$\Rightarrow \frac{8}{3} + \frac{2}{3} + \frac{8}{3}$

$\Rightarrow \frac{8+2+8}{3}$

$\Rightarrow 6$

$\therefore$ 6 is the maximum value.

Again,

$f(x, y, z) = 2\left(-\frac{4}{3}\right) + \frac{-2}{3} - 2\left(\frac{4}{3}\right)$

$= -\frac{8}{3} - \frac{2}{3} - \frac{8}{3}$

$= -6$

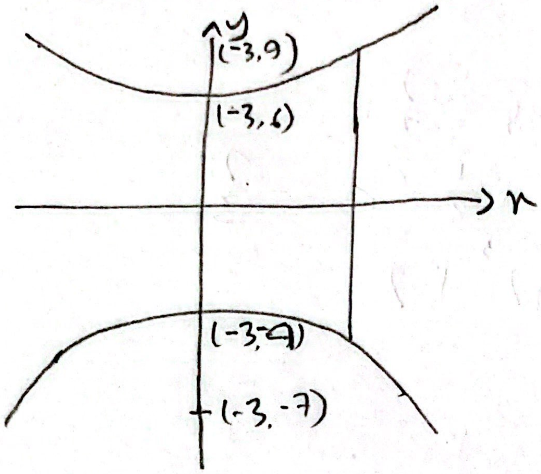
$\therefore$ -6 is the mimimum value.

Ans

problem: 3

given that, foci $(-3,9)$, $(-3,-7)$

ventices $(-3,-4)$, $(-3,6)$



foci $(h, k \pm e)$

ventices $(h, k \pm B)$

We have, $h = -3$

$$k = \frac{9-7}{2}$$

$$= 1$$

Again, $k + e = 9$

$\Rightarrow 1 + e = 9$

$\therefore e = 8$

$k + b = 6$

$\Rightarrow 1 + b = 6$

$\therefore b = 5$

$\therefore a^2 + b^2 = c^2$

$$a = \sqrt{c^2 - b^2} = \sqrt{8^2 - 5^2} = \sqrt{39}$$

$$\therefore \frac{(y-1)^2}{25} - \frac{(x+3)^2}{39} = 1$$

This is the equation of Hypenbola.

Problem 4:

$$U = \frac{a}{|a|} = \frac{2\hat{i}+\hat{j}-2\hat{k}}{\sqrt{2^2+1^2+(-2)^2}} = \frac{2\hat{i}+\hat{j}-2\hat{k}}{3}$$

$$\therefore U = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} - \frac{2}{3}\hat{k}$$

here, $a = \frac{2}{3}$, $b = \frac{1}{3}$, $c = -\frac{2}{3}$

$$f(x,y,z) = x^2y - yz^3 + z$$

$$f_x = 2xy$$

$$f_y = x^2 - z^3$$

$$f_z = -3yz^2 + 1$$

$$D_uf(x,y,z) = f_xa + f_yb + f_zc$$

$$= (2xy)\left(\frac{2}{3}\right) + (x^2-z^3)\left(\frac{1}{3}\right) + (-3yz^2+1)\left(-\frac{2}{3}\right)$$

$$= \frac{4}{3}xy + \frac{x^2}{3} - \frac{z^3}{3} + 2yz^2 - \frac{2}{3}$$

$$D_uf(1,-2,0) = \frac{4}{3}(1)(-2) + \frac{1^2}{3} - \frac{0^3}{3} + 2(-2)(0)^2 - \frac{2}{3}$$

$$= \frac{-8}{3} + \frac{1}{3} - \frac{2}{3}$$

$$= \frac{-9}{3}$$

(Ans)