Mathematical problem of probability-01 for final quiz

Mutually exclusive, non-mutually exclusive, dependent and independent event related:

1) The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will get an electric contract is 4/9. If the probability of getting both the contract is 14/15, what is the probability that he will get at least one contract?
2) In a survey of 100 readers, it was found 40 read The Daily Ittefaq and 15 read The Daily Star and 10 read both. What is the probability of a person reading at least one of the newspaper?
3) Suppose 35% of the students failed English, 25% of the students failed Statistics and 15% of the students failed both English and Statistics in a certain college. A student is selected at random.
   i) If failed Statistics, what is the probability that he failed English?
   ii) If failed English, what is the probability that he failed Statistics?
   iii) What is the probability that he failed English or Statistics?
4) Of 1000 assembled components, 10 have a working defect and 20 have a structural defect. There is a good reason to assume that no component has both defects. What is the probability that randomly chosen component will have either type of defects?
5) The probability that a management trainee will remain with a company is 0.60. The probability that an earns more than tk. 60,000 per month is 0.50. The probability that an employee is a management trainee who remained with the company or who earns more than Tk. 60,000 per month is 0.70. What is the probability that an employee earns more than Tk. 60,000 per month, given that he is a management trainee who stayed with the company?
6) The probability that a new marketing approach will be successful is 0.6. The probability that the expenditure for developing the approach can be kept within the original budget is 0.50. The probability that both of these objectives will be achieved is 0.30. What is the probability that at least one of these objectives will be achieved? For the two events describe above, determine whether the events are independent or dependent.
7) The personnel manager of a large manufacturing firm finds that 15% of the firm's employees are junior executives and 25% of the firm's employees are MBAs. He also discovers that 5% of the firm's employees are both junior executives and MBAs. What is the probability of selecting a junior executive if the selection is confined to MBAs?
8) During a research of auto accidents, the highway safety council found that that 60% of all accidents occur at night, 52% are alcohol related, and 37% occur at night and are alcohol related.
   i) What is the probability that an accident was alcohol related, given that it occurred at night?
   ii) What is the probability that an accident was at night, given that it was alcohol related?

Contingency table related/join & marginal probabilities problem:

1) A lot of 10,000 parts produced on four machines were inspected and classified into three grades. The results were given in the following table.

Grades	Machine				
	I	II	III	IV	V
Satisfactory	2400	1600	2400	1600	8000
Rework	450	300	450	300	1500
Scrap	150	100	150	100	500
Total	3000	2000	3000	2000	10000

If a part is selected at random from this lot, then find the following probabilities that-

i) It is produced by machine III.
ii) It is produced on machine I given that it is scrapped
iii) It is reworked given that it is produced on machine IV
iv) A satisfactory part is produced on machine II

2) Friendly's Department store has been the target of many shoplifters during the past month, but owing to increased security precautions, 250 shoplifters have been caught. Each shoplifter's gender is noted, also noted is whether the perpetrator was a first-time or repeated offender. The data are summarized in the table.

Gender	First-time offender	Repeated offender	Total
Male	60	70	130
Female	44	76	120
Total	104	146	250

Assuming that an apprehended shoplifter is chosen at random, find

i) The probability that the shoplifter is male
ii) The probability that the shoplifter is a first-time offender given that the shoplifter is male.
iii) The probability that the shoplifter is a female given that the shoplifter is repeated offender.
iv) The probability that the shoplifter is a male given that the shoplifter is first-time offender.
v) The probability that the shoplifter is both male and a repeated offender.

3) A study of job satisfaction was conducted for four occupations- Cabin maker, lawyer, doctor and system analyst. Job satisfaction was measured on a scale of 0-100. The data obtained are summarized in the following cross tabulation:

Occupation	**Under 50**	**50-59**	**60-69**	**70-79**	**80-89**	**Total**
Cabin maker	0	2	4	3	1	10
Lawyer	6	2	1	1	0	10
Doctor	0	5	2	1	2	10
System Analyst	2	1	4	3	0	10
Total	8	10	11	8	3	40

i) Develop a joint probability table.
ii) What is the probability of one of the participant studies had a satisfaction score in the 80's?
iii) What is the probability of a satisfaction score in the 80's given the study participant was a doctor?
iv) What is the probability that one of the participant studied was a lawyer?

v) What is the probability that one of the participant was a lawyer and received a score under 50?

vi) What is the probability of a satisfaction score under 50 given a person is a lawyer?

vii) What is the probability of a satisfaction score of 70 or higher?

4) A market survey conducted in four cities pertained to preference for keya soap. The responses are shown below:

	Dhaka	**Rajshahi**	**Khulna**	**Barishal**
Yes	45	55	60	50
No	35	45	35	45
No opinion	5	5	5	5

i) What is the probability that a consumer selected at random preferred Keya soap?

ii) What is the probability that a consumer preferred Keya soap and was from Khulna?

iii) What is the probability that a consumer preferred Keya soap, given that he was from Khulna?

iv) Given that a consumer preferred Keya soap, what is the probability that he was from Dhaka?

5) Shown here are the raw values matrix and corresponding probability matrix for the results of a national survey of 200 executives who were asked to identify the geographic locale of their company and their company's industry type. The executives were only allowed to select one locale and one industry type.

RAW VALUES MATRIX

		Geographic Location				
		Northeast D	*Southeast E*	*Midwest F*	*West G*	
Industry Type	*Finance A*	24	10	8	14	56
	Manufacturing B	30	6	22	12	70
	Communications C	28	18	12	16	74
		82	34	42	42	200

Suppose a respondent is selected randomly from these data.

a. What is the probability that the respondent is from the Midwest (F)?

b. What is the probability that the respondent is from the communications industry (C) or from the Northeast (D)?

c. What is the probability that the respondent is from the Southeast (E) or from the finance industry (A)?

Total probability theorem:

1) Mr. Ali wants to build a house this year. He applied for a bank loan. The probability that he will get it is 2/3. If he will get the bank loan, the probability that he will build a house is 3/4. However, if he will not get the bank loan, the probability that he will build the house is 1/4. What is the probability that Mr. Ali will build a house this year?

2) A certain disease is present in about 1 out of 1000 persons in a given population. Suppose that there is simple blood test which gives a positive reading with probability 0.99 for a diseased person and with 0.005 for a healthy person.

i) A person is selected at random from this population, what is the probability that the blood test of the selected person will give the positive reading?

ii) If the blood test pf the selected person gives positive reading, what is the probability that he does have the disease?

3) Two sets of candidates are competing for the position on the board of directors of a company. The probabilities that the first and second sets will be win are 0.6 and 0.4, respectively. If the first set wins, the probability of introducing a new product is 0.8 and the corresponding probability of the second set wins is 0.3. What is the probability that the new product will be introduced?

4) Three machines ***A, B*** and ***C*** produce respectively **50%, 30%** and **20%** of the total number of items of a factory. The percentages of defective output of these machines are **3%, 4%** and **5%.** If an item is selected at random, find the probability that the item is defective.

5) Three persons A, B and C are being considered for the appointment as Vice-Chancellor of a University whose chance of being selected for the post are in the proportion 5:3:2, respectively. The probability that A, if selected, will introduce democratization in the University structure is 0.3, the corresponding probabilities for B and C doing the same are, respectively 0.6, and 0.8. What is the probability that democratization would be introduced in the University?

Bayes Theorem:

1) In a certain college, **4%** of the men and 1% of the women are taller than **6** feet. Furthermore, **60%** of the students are women. Now if a student is selected at random and is taller than **6** feet, what is the probability that the student is a woman?
2) In a city, 60% of the people moves by bus, 25% by rickshaw, and 15% by car. 3% of the accident committed by bus, 5% by rickshaw and 1% by car. A person of the city falls in an accident. What is the probability that the accident has committed by rickshaw?
3) In a bank 45% and 55% of the monthly statements are prepared by Mrs. Ali and Miss Karim respectively. These employees are very reliable. However, they are in error sometimes. The probabilities of committing their errors are 0.05% and 0.01% respectively. A monthly statement was found to be erroneous, what is the probability that it was done by Miss Karim?
4) A manufacturing firm produces steel pipes in the three plants with daily production 500, 1000, and 2000 pipes respectively. It is known that the fractions of defective output produced by three plants are respectively 0.005, 0.008, and 0.010. If a pipe is selected from a day's total production and is found to be defective, find the probability that it came from the first plant?

Random Variable:

The pressure measured in pounds per cm^2 at a certain valve is a random variable X whose probability density function is

$$f(x) = \begin{cases} \frac{2}{9}(3x - x^2), & \text{if } 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that the pressure at this valve is

i) Not more than 2 pounds per cm^2
ii) Greater than 2 pounds per cm^2
iii) Between 1.5 and 2.5 per cm^2 and
iv) Less than 1.5 per cm^2