Assignment - 1

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Sec - 7

① $\left|\dfrac{2z-3}{2z+3}\right| = 1$

$\Rightarrow |2z-3| = |2z+3|$

$\Rightarrow |2(x+iy)-3| = |2(x+iy)+3|$

$\Rightarrow |2x+2iy-3| = |2x+2iy+3|$

$\Rightarrow \sqrt{(2x-3)^2+(2y)^2} = \sqrt{(2x+3)^2+(2y)^2}$

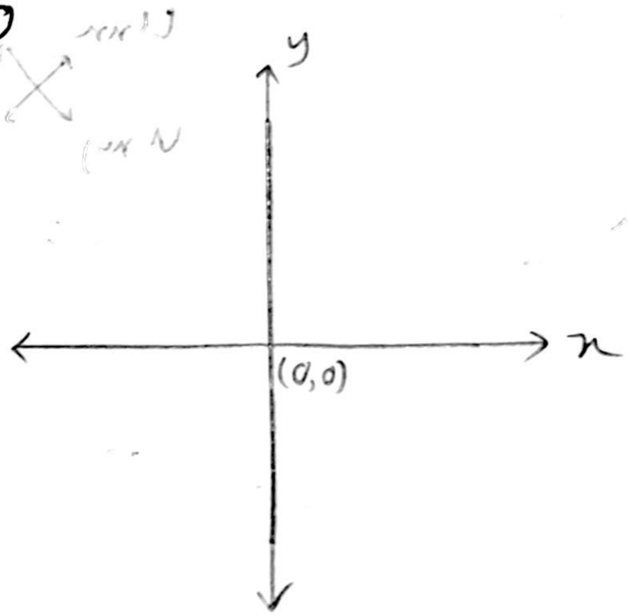
$\Rightarrow (2x-3)^2+(2y)^2 = (2x+3)^2+(2y)^2$

$\Rightarrow 4x^2-12x+9+4y^2 = 4x^2+12x+9+4y^2$

$\Rightarrow -12x = 12x$

$\Rightarrow -24x = 0$

$\therefore x = 0$



2. Prove that $|(2\bar{z}+5)(\sqrt{2}-i)| = \sqrt{3}\,|2z+5|$

$$LHS = |(2\bar{z}+5)(\sqrt{2}-i)$$

$$= |2\bar{z}+5| . |(\sqrt{2}-i)|$$

$\cancel{2\bar{z}+5 \Rightarrow 2(x-iy) \Rightarrow 2x-}$

$$2\bar{z}+5 \Rightarrow 2(x-iy)+5 \Rightarrow 2x-2iy+5$$

$$\therefore |2\bar{z}+5| \Rightarrow |2x-2iy+5|$$

$$2z+5 \Rightarrow 2(x+iy)+5 \Rightarrow 2x+2iy+5$$

$$\therefore |2z+5| \Rightarrow |2x+2iy+5|$$

Magnitude of $(\sqrt{2}-i)$:

$$|\sqrt{2}-i| = \sqrt{(\sqrt{2})^2+(-1)^2} = \sqrt{3}$$

Therefore,

$$|2\bar{z}+5| . |\sqrt{2}-i| = \sqrt{3} . |2\bar{z}+5|$$

Since the magnitude of $a+bi$ is the same as the magnitude of its complex conjungate $a-bi$,

$$|2x+2iy+5| = |2x-2iy+5|$$

$$\Rightarrow |2\bar{z}+5| = |2z+5|$$

So, $$|(2\bar{z}+5)(\sqrt{2}-i)| = \sqrt{3}\,|2z+5|$$

(Proved)

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3. Prove that $|z-i| = |z+i|$ represents a straight line.

$\Rightarrow |z-i| = |z+i|$

$\Rightarrow |x+iy-i| = |x+iy+i|$

$\Rightarrow |x+i(y-1)| = |x+i(y+1)|$

$\Rightarrow \sqrt{x^2+(y-1)^2} = \sqrt{x^2+(y+1)^2}$

$\Rightarrow x^2+(y-1)^2 = x^2+(y+1)^2$

$\Rightarrow x^2+y^2-2y+1 = x^2+y^2+2y+1$

$\Rightarrow -2y = 2y$

$\Rightarrow -4y = 0$

$\therefore y = 0$

This represents a horizontal line on the complex plane.

(Ans)

④ $z^4 = -16i$

$= (0-16i)$

$\therefore z = (0-16i)^{1/4}$

$x=0, \quad y=-16 \quad \therefore r = \sqrt{0^2+(-16)^2}$

$= 16$

$\theta_a = \tan^{-1}\left(\frac{-16}{0}\right) = -\frac{\pi}{2}$

$= 2\pi - \left|\frac{\pi}{2}\right|$

$= \frac{3\pi}{2}$

$\therefore (-16i)^{1/4} = (16)^{1/4}\left[\cos\left(\frac{1}{4}\right)\left(\frac{3\pi}{2}+2k\pi\right) + i\sin\left(\frac{1}{4}\right)\left(\frac{3\pi}{2}+2k\pi\right)\right]$

$= 2\left[\cos\left(\frac{3\pi}{8}+\frac{k\pi}{2}\right) + i\sin\left(\frac{3\pi}{8}+\frac{k\pi}{2}\right)\right]$

$= 2\left[\cos\left(\frac{3\pi+4k\pi}{8}\right) + i\sin\left(\frac{3\pi+4k\pi}{8}\right)\right]$

Now,

$k=0, \; z = 2\left[\cos\left(\frac{3\pi}{8}\right) + i\sin\left(\frac{3\pi}{8}\right)\right] \Rightarrow 0.765 + 1.848i$

$k=1, \; z = 2\left[\cos\left(\frac{3\pi+4\pi}{8}\right) + i\sin\left(\frac{3\pi+4\pi}{8}\right)\right] \Rightarrow -1.85 + 0.765i$

$k=2, \; z = 2\left[\cos\left(\frac{3\pi+4.2.\pi}{8}\right) + i\sin\left(\frac{3\pi+4.2.\pi}{8}\right)\right] \Rightarrow -0.765 - 1.848i$

$k=3, \; z = 2\left[\cos\left(\frac{3\pi+4.3.\pi}{8}\right) + i\sin\left(\frac{3\pi+4.3.\pi}{8}\right)\right]$

$\Rightarrow 1.848 - 0.765i$

(Ans)