CSE331

ASSIGNMENT

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a. $L_1 = \{w \in (0,1)^* : 0^i 1^j \text{ where } i \leq j\}$

$\Rightarrow$ Let's assume L_1 is a regular language.

Consider a string $w = 0^n 1^{n+1}$; where $w \in L_1$.

Here n is the pumping constant/ pumping length of L_1.

Dividing string w such that
(i) $|y| \geq 0$
(ii) $|xy| \leq n$
(iii) $xy^k z \in L_1$ for all $k \geq 0$

Let $n = 2$, $w = 00111$

Dividing w, we get, $x = 0$, $y = 0$, $z = 111$

here ~~for~~ $|y| \geq 0$ and $|xy| \leq n$ and for $k = 1$, $xy^k z \in L_1$

To prove for all $k \geq 0$ $xy^k z \in L_1$

if we keep pumping y,

for $k = 2$, $w = xy^2 z = 000111$; $w \in L_1$

for $k = 3$, $w = xy^3 z$ 0000111 ; $w \notin L_1$

So if we keep pumping y, w does not belong in L_1 as it doesn't follow $i \leq j$. Therefore L_1 is not regular.

② Assume L_2 is regular.

pumping length $= p$

According to pumping Lamma

for any string s in L_2 with length at least p it can

splite in to 3 parts xyz. with this conditions.

1. $|y|>0$
2. $|xy| \leq p$
3. for each $i \geq 0$, $xy^iz \in L_2$

Here, $s = a^3 b^3 c^{3+2}$, $i = j = k = 3$

Case 1: y contains only a's

Let $s = a^3 b^3 c^{3+2}$ and $y = a^2$

$xy^2z = a^3 a^2 b^3 c^{3+2} = a^5 b^3 c^{3+2}$

which will violate the condition $i = k$.

so $xy^2z \in L_2$

Case 2: y contain's both a's and b's

Let $s = a^3 b^3 c^{3+2}$ and $y = (ab)^2$

$xy^2z = a^3 (ab)^2 b^3 c^{3+2} = a^5 abab\, b^3 c^{3+2}$

which will violate the condition $i = k$

Case 3: y contains only b's

Let $s = a^3 b^3 c^{3+2}$ and $y = b^2$.

$xy^2z = a^3 b^3 b^2 b^3 c^{3+2} = a^3 b^5 b^3 c^{3+2}$

which will also violate $i = k$

Case 4: y contains only c's

Let $s = a^3 b^3 c^{3+2}$ $y = c^2$

$xy^2z = a^3 b^3 (c^2)^2 b^3 c^{3+2} = a^3 b^3 c^4 b^3 c^{3+2}$

which will violate $i = k$

So we can say the L_2 is not a regular languag

c) $L_3 = \{ w \in \{0,1\}^* : w \text{ is a palindrome} \}$

Let's assume a pumping length P. so, any strin s in L_3 with length p can be divided into three parts, xyz.

Now,

$i \geq 0$, $xy^i z \in L$.

$|y| > 0$ or y non-empty

$|xy| \leq p$

if it was an regular language,

then the strings, $s = 0^p 1 0^p$ where $|s| = 2p+1$

As, $|xy| \leq P$, y consist only 0s and can only pump 0^p part of the string.

if $i = 2$,

$s' = xy^2 z$, so repeats the y part only,

It's in the form $0^{(p+|y|)} 1 0^p$. As $y > 0$ the pumped string has more 0s in the first than p.

A palindrome with more 0s in the ~~B~~ first part than the second part is not in L3. So, it doesn't satisfy xy^iz for any i.

So, L3 is not regular.

—x—

Assume that $L4$ is regular language.

There must exist a pumping constant p.

let, $w \in L4$

$w = 00\#0^n$

$x = 00\#$

$y = 0$

$z = 0^{n-1}$

pumping y twice or more will violate the condition of $|w1| = 2 \times |w2|$.

pumping twice we will get $00\#0^{n+2}$

here $|w1|$ is no longer double the length of $|w2|$

Thus the pumping lemma is violated and it contradicts the assumption that $L4$ is regular language

(e) .

$L_5 = \{ \omega \in \{a\}^* : a^{2n} \text{ where } n \geq 0 \}$

if it is regular then there a 'w' string exists which can be splited x, y, z which follows these rules

i) $xy^iz \in A$ for each $i \geq 0$

ii) $|y| > 0$

iii) $|xy| \leq P$

Now,

Let pumping length $= P$

$\therefore w = a^{2P}$ $\therefore |y| \geq 1$

let

$|y| = k$ [where $k \geq 1$]

Now if $p = 2$

~~so, $|w| = |xy^2z|$~~ $|xy^2z|$

~~$= 1x$~~ $= |xyz| + |y|$

$= 2p + k$

but $2p + k > 2p$

which contradicts $|xy| \leq p$ rule

$\therefore L_5$ is not a regular language.