Practice Sheet Solve

Monday, March 11, 2024

3. Compute the upper bound of interpolation error for $f(x) = 2sin(x) - 3cos(x)$ where $x \varepsilon \{-\pi/4,\ 0,\ \pi/4\}$ within [-1, 1]. Consider up to 4 significant figures.

Ans:

$n=2$ (as 3 nodes)

$$f(x) - P_2(x) = \frac{f^{(3)}(\xi)}{3!}\left(x+\frac{\pi}{4}\right)(x-0)\left(x-\frac{\pi}{4}\right)$$

$$= \left|\frac{-2\cos\xi - 3\sin\xi}{6}\right| \left|\left(x^3 - \frac{x\pi^2}{16}\right)\right|$$

$$= \left|\left(\left|\frac{-2\cos\xi}{6}\right| + \left|\frac{-3\sin\xi}{6}\right|\right)\right| \times |w(x)|$$

$f(x) = 2\sin(x) - 3\cos x$

$f'(x) = 2\cos x + 3\sin x$

$f''(x) = -2\sin x + 3\cos x$

$f'''(x) = -2\cos x - 3\sin x$

$x/\xi \in [-1, 1]$

Now, maximize each of the functions seperately!

$\xi \in [-1, 1]$

$$= \left|\left(\frac{2\cos 0}{6} + \frac{3\sin 1}{6}\right)\right| \times 0.383$$

$$= 0.2888 \ [4sf]$$

$$w(x) = x^3 - \frac{x\pi^2}{16}$$

$$\therefore w'(x) = 3x^2 - \frac{\pi^2}{16}$$

Solving $w'(x) = 0$, we get, $x = \pm\frac{\pi}{4\sqrt{3}}$

x	$\lvert w(x)\rvert$
$\frac{\pi}{4\sqrt{3}}$	0.186
$-\frac{\pi}{4\sqrt{3}}$	0.186
1	0.383
-1	0.383

pick max

4. Compute the upper bound of interpolation error for $f(x) = 2sin(x) - 3cos(x)$ where $x \varepsilon \{-\pi/4,\ 0,\ \pi/4\}$. Consider up to 4 significant figures.

Ans:

$n=2$ (as 3 nodes)

$$f(x) - P_2(x) = \frac{f^{(3)}(\xi)}{3!}\left(x+\frac{\pi}{4}\right)(x-0)\left(x-\frac{\pi}{4}\right)$$

$$= \left|\frac{-2\cos\xi - 3\sin\xi}{6}\right| \left|\left(x^3 - \frac{x\pi^2}{16}\right)\right|$$

$$= \left|\left(\left|\frac{-2\cos\xi}{6}\right| + \left|\frac{-3\sin\xi}{6}\right|\right)\right| \times |w(x)|$$

$f(x) = 2\sin(x) - 3\cos x$

$f'(x) = 2\cos x + 3\sin x$

$f''(x) = -2\sin x + 3\cos x$

$f'''(x) = -2\cos x - 3\sin x$

$$= \left|\left(\left|\frac{-2\cos\xi}{6}\right| + \left|\frac{-3\sin\xi}{6}\right|\right)\right| \times |w(x)|$$

Now, maximize each of the functions seperately!

No range!

$$= \left|\left(\frac{2\cos 0}{6} + \frac{3\sin\frac{\pi}{2}}{6}\right)\right| \times 0.186$$

$$= 0.1549 \ [4sf]$$

$$w(x) = x^3 - \frac{x\pi^2}{16}$$

$$\therefore w'(x) = 3x^2 - \frac{\pi^2}{16}$$

Solving $w'(x) = 0$, we get, $x = \pm\frac{\pi}{4\sqrt{3}}$

x	$\lvert w(x)\rvert$
$\frac{\pi}{4\sqrt{3}}$	0.186
$-\frac{\pi}{4\sqrt{3}}$	0.186

5. Consider the function $f(x) = e^{2x} + e^{-2x} - x^3 ln(x)$. Find the upper bound of interpolation error where $x \in \{2, 3, 4\}$ within [1.6, 2.3]. Consider up to 4 significant figures.

⑤

$$f(x) - P_2(x) = \frac{f^{(3)}(\xi)}{3!}(x-2)(x-3)(x-4)$$

$$= \left|\frac{8e^{2\xi} - 8e^{-2\xi} - 6\ln\xi - 11}{6}\right| \left|(x^3 - 9x^2 + 26x - 24)\right|$$

Seperate the functions!

$\xi \in [1.6, 2.3]$

$$= \left(\left|\frac{8e^{2\xi}}{6}\right| + \left|\frac{-8e^{-2\xi}}{6}\right| + \left|\frac{-6\ln\xi}{6}\right| + \left|\frac{-11}{6}\right|\right) \times w(x)$$

Maximize the functions

$$= \left(\left|\frac{8e^{2\times 2.3}}{6}\right| + \left|\frac{8e^{-2\times 1.6}}{6}\right| + \left|\frac{6\ln 2.3}{6}\right| + \left|\frac{11}{6}\right|\right) \times w(x)$$

$$= 135.4 \times 0.3849 = 52.12 \ [4sf]$$

$$f(x) = e^{2x} + e^{-2x} - x^3\ln x$$

$$f'(x) = 2e^{2x} - 2e^{-2x} - 3x^2\ln x - x^3 \cdot \frac{1}{x}$$

$$= 2e^{2x} - 2e^{-2x} - 3x^2\ln x - x^2$$

$$f''(x) = 4e^{2x} + 4e^{-2x} - 6x\ln x - 3x^2 \cdot \frac{1}{x} - 2x$$

$$= 4e^{2x} + 4e^{-2x} - 6x\ln x - 5x$$

$$f'''(x) = 8e^{2x} - 8e^{-2x} - 6\ln x - 6x \cdot \frac{1}{x} - 5$$

$$= 8e^{2x} - 8e^{-2x} - 6\ln x - 11$$

$$w(x) = x^3 - 9x^2 + 26x - 24$$

$$w'(x) = 3x^2 - 18x + 26$$

Solve $w'(x) = 0$, we get, $x = 3.577, 2.423$

x	$\lvert w(x)\rvert$
3.577	0.3849
2.423	0.3849

2·423	0·3849