

ID:

Name:

Brac University

Semester: Spring 2023

Course Code: CSE250

Circuits And Electronics

Set

A

Assessment: *Midterm*

Duration: 1 hour 30 minutes

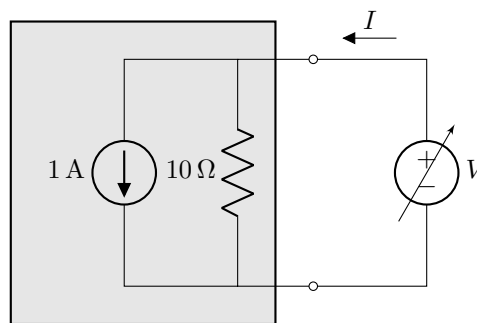
Date: March 5, 2023

Full Marks (incl. bonus 6): 56

- ✓ No washroom breaks. Phones must be turned off. Using/carrying any notes during the exam is not allowed.
- ✓ At the end of the exam, both the **answer script** and the **question paper** must be returned to invigilator.
- ✓ All **3 questions** are compulsory. Marks allotted for each question are mentioned beside each question.
- ✓ Write your answers inside the indicated boxes where applicable.
- ✓ Symbols have their usual meanings.

■ Question 1 of 3 [CO1, CO3] [20 marks]

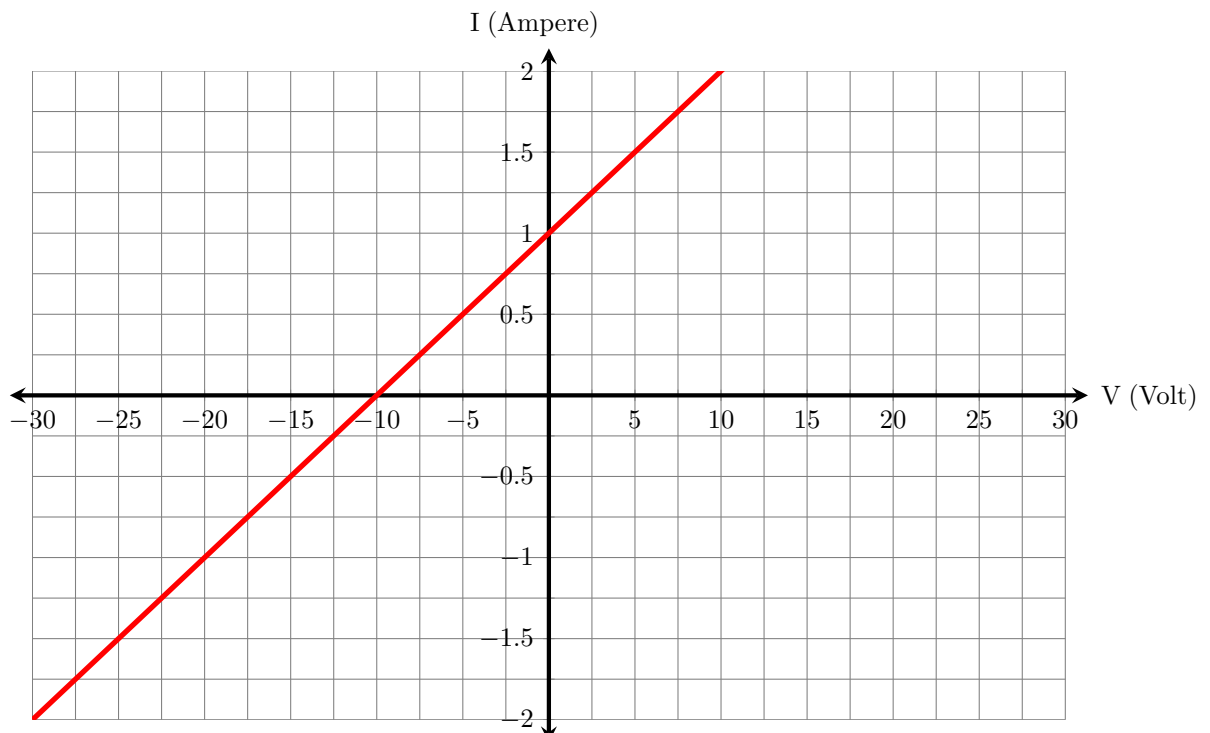
- (a) In order to test the $I - V$ characteristics of a two-terminal linear circuit (inside the gray box), the following circuit was constructed.



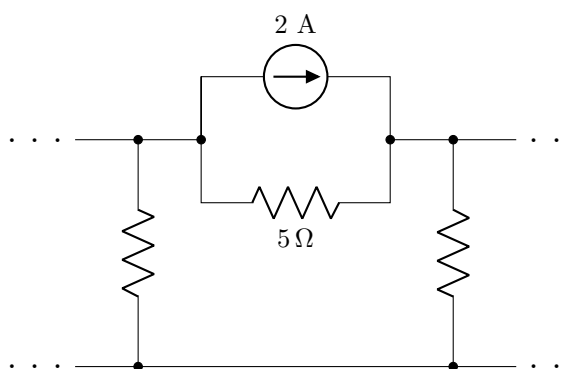
- (i) [1 mark] **Determine** the relationship between I and V , where V is the applied voltage difference across the test circuit that is varied and I is the current through it. In the following box write I in terms of V .

$$I = \frac{1}{10}V + 1$$

- (ii) [2 marks] Based on your answer in (i), plot the $I - V$ characteristics of the test circuit in the following grid.



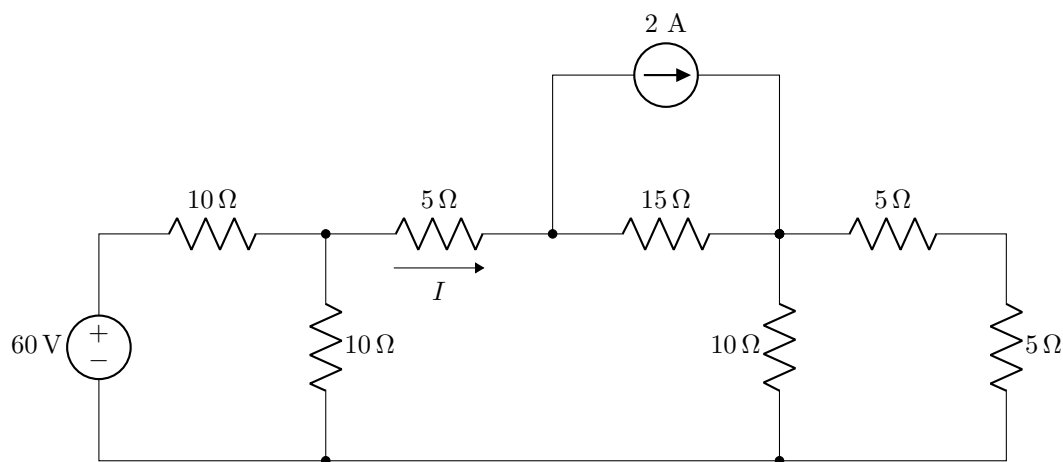
- (b) [2 marks] Which one is the correct **Source Transformation** of the following circuitry?



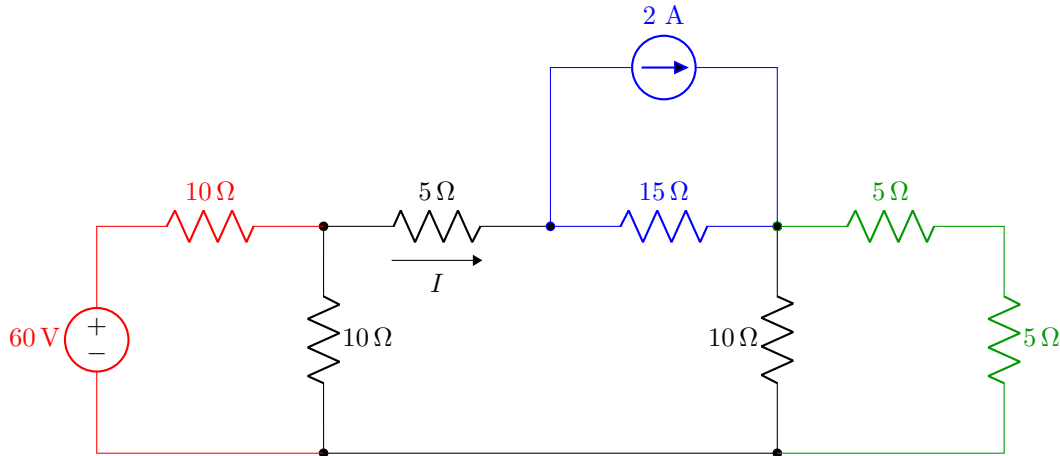
Cross-out or fill-in the checkbox (☐) at the top-left corner of the correct answer.

<input type="checkbox"/>	<input type="checkbox"/>
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- (c) [15 marks] Determine the current I as shown in the circuit below using **Superposition Principle** and/or **Source Transformation**.

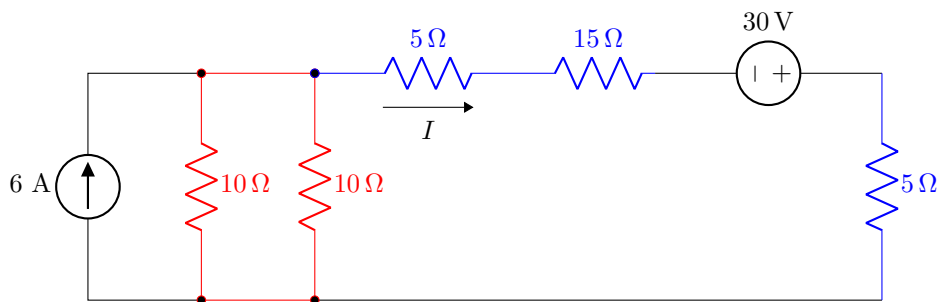


Solution:

Source Transformation Method:

- Transforming the 60 V voltage source in series with the 10 Ω resistor into a current source in parallel with a resistor.
- Transforming the 2 A current source in parallel with the 15 Ω resistor into a voltage source in series with a resistor.
- The two 5 Ω resistors in the rightmost part of the circuit are in series ($5 + 5 = 10 \Omega$) and the series combination is parallel with the 10 Ω resistor.

$$\Rightarrow 10 \parallel (5 + 5) = 5 \Omega$$

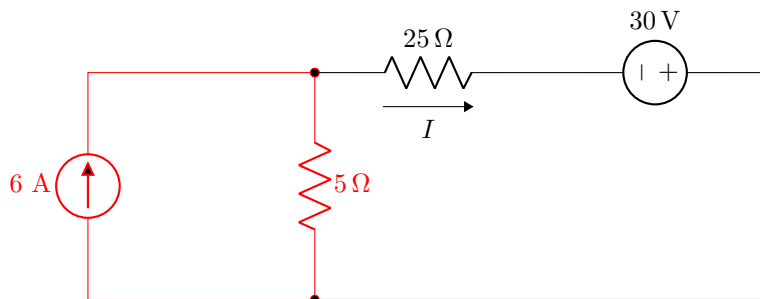


- The two 10 Ω resistors are parallel.

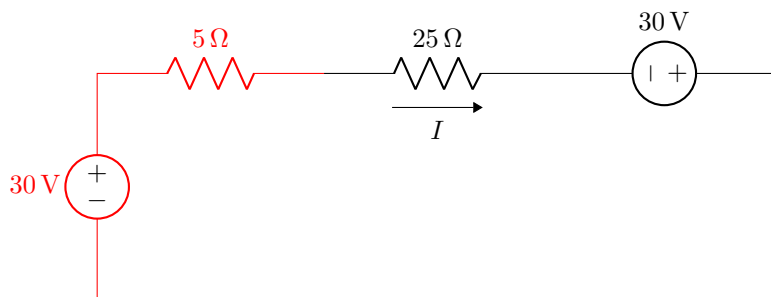
$$\Rightarrow 10 \parallel 10 = 5 \Omega \text{ and}$$

- The 5 Ω , 15 Ω , and 5 Ω resistors are parallel.

$$\Rightarrow 5 + 15 + 5 = 25 \Omega$$



- Transforming the 6 A current source in parallel with the 5 Ω resistor into a voltage source in series with a resistor.



- Replacing the two voltage sources by one: the value of the resultant source is,

$$\Rightarrow 30 + 30 = 60 \text{ V}$$

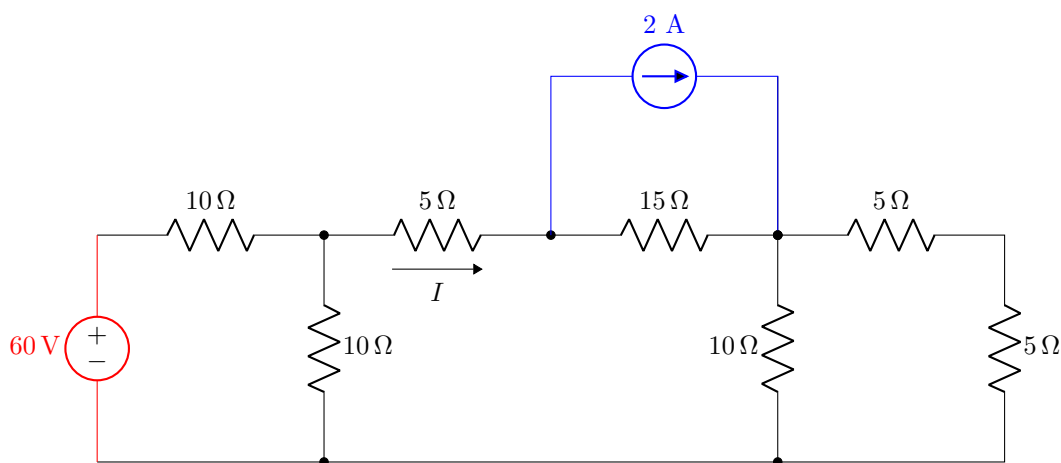
So, the current I can be calculated as,

$$I = \frac{60}{5+25} \text{ (A)}$$

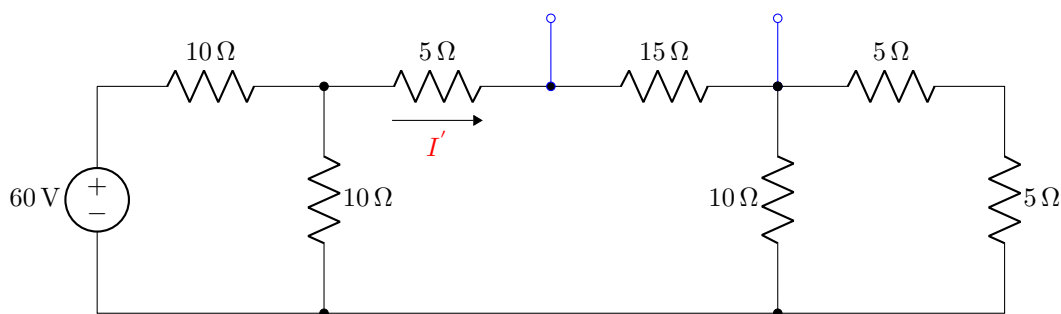
$$\Rightarrow \boxed{I = 2 \text{ A}}$$

Superposition Principle:

- There are two independent sources in the given circuit: the 60 V voltage source and the 2 A current source.



- Let's first calculate the contribution from the 60 V source only (I'). Turning off the 2 A source (open circuit), the circuit looks like the one shown below.



- The circuit can be solved in several ways. Let's do some series-parallel combination of resistors to reduce the circuit.

- $5\ \Omega$ and $5\ \Omega$ are in series.

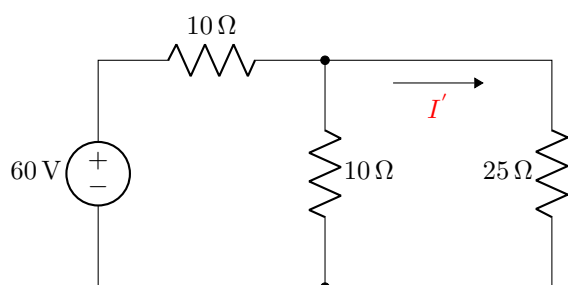
$$\Rightarrow 5\ \Omega + 5\ \Omega = 10\ \Omega$$

- Then their combination ($10\ \Omega$) is in parallel with the other $10\ \Omega$.

$$\Rightarrow 10\ \Omega \parallel 10\ \Omega = 5\ \Omega$$

- Then $5\ \Omega$, $15\ \Omega$, and $15\ \Omega$ are in series.

$$\Rightarrow 5\ \Omega + 15\ \Omega + 5\ \Omega = 25\ \Omega$$

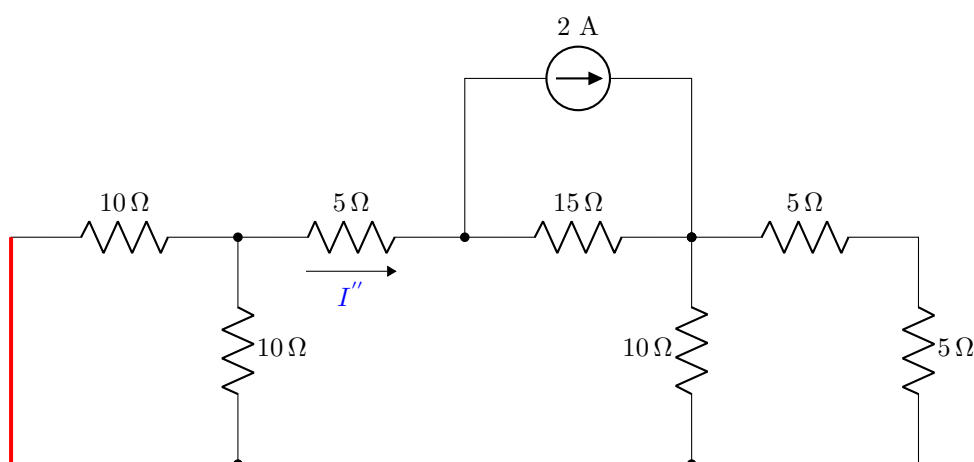


- The voltage across the parallel combination of the $10\ \Omega$ and $25\ \Omega$ can be found using the voltage divider rule as,

$$\frac{(10 \parallel 25)}{10 + (10 \parallel 25)} \times 60\ \text{V} = 25\ \text{V}$$

So, $I' = \frac{25}{25} = 1\ \text{A}$

- Now, for the 2 A current source, turning off the 60 V source (short circuit), the circuit looks like the one shown below.



- The circuit can be solved in several ways. Let's do some series-parallel combination of resistors to reduce the circuit.

- $5\ \Omega$ and $5\ \Omega$ are in series.

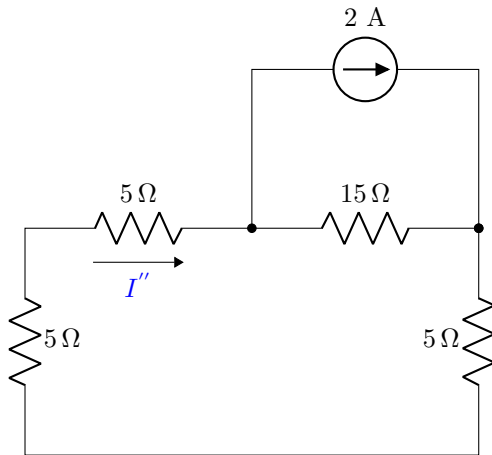
$$\Rightarrow 5\ \Omega + 5\ \Omega = 10\ \Omega$$

- Then their combination ($10\ \Omega$) is in parallel with the other $10\ \Omega$.

$$\Rightarrow 10\ \Omega \parallel 10\ \Omega = 5\ \Omega$$

- In the left side, $10\ \Omega$ and $10\ \Omega$ are in parallel.

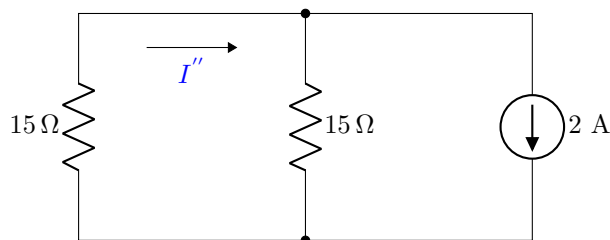
$$\Rightarrow 10\ \Omega \parallel 10\ \Omega = 5\ \Omega$$



- We may reduce further. The three $5\ \Omega$ resistors are in series where the current I'' flows.

$$\Rightarrow 5\ \Omega + 5\ \Omega + 5\ \Omega = 15\ \Omega$$

Now the circuit becomes,



- The current I'' will be halved through each of the $15\ \Omega$ resistors,

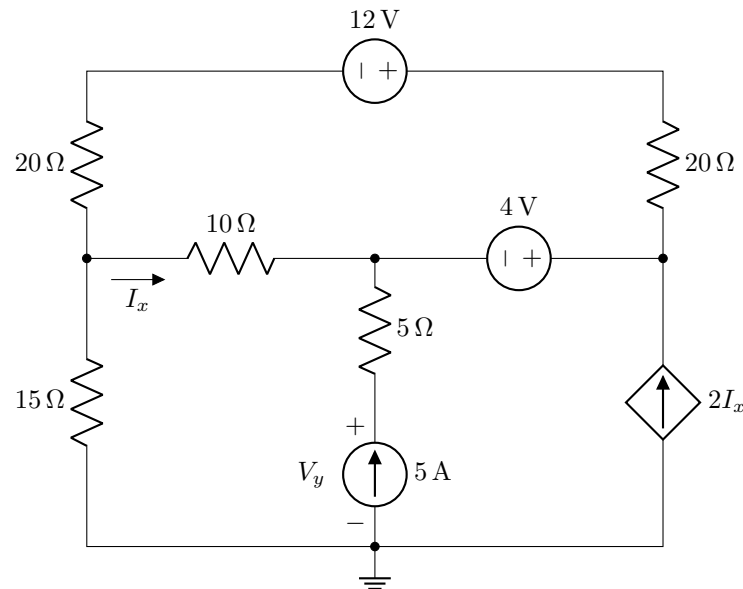
So, $I'' = 1\ A$

According to the Superposition Principle, the total current will be the algebraic summation of the two contributions of the two independent sources. That is,

$$I = I' + I'' = 1 + 1\ (A)$$

$$\Rightarrow I = 2\ A$$

■ Question 2 of 3 [CO2, CO4] [20 marks]



Apply Nodal/Mesh analysis to answer the following questions:

- (a) [1 mark] Which analysis method should be more advantageous in solving the above circuit?

Solution: Mesh analysis

- (b) [15 marks] Find all the node voltages/mesh currents in the circuit.

Solution: Mesh currents: $\pm 1.8\text{ A}$, $\mp 3.2\text{ A}$, $\pm 0.2\text{ A}$.
Node voltages: 27 V , 43 V , 47 V .

- (c) [2 marks] Find V_y , the voltage across the 5 A current source.

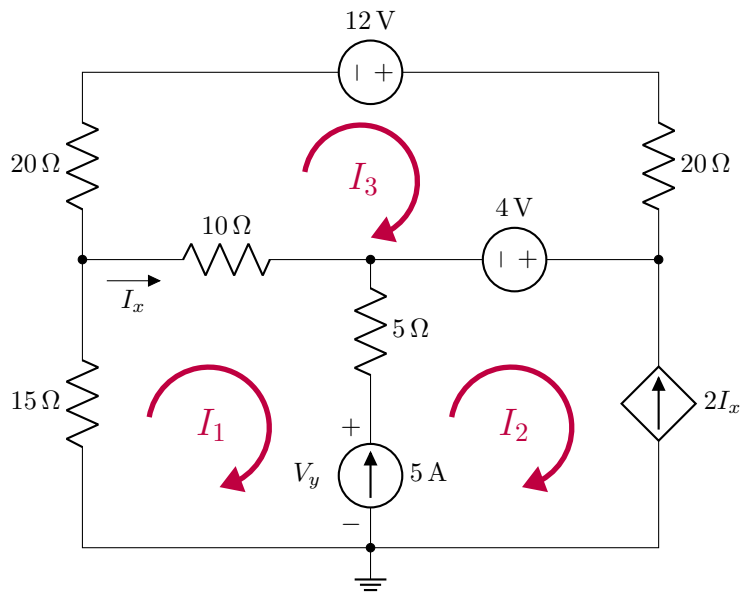
Solution: 68 V

- (d) [2 marks] How much **power** is the 5 A current source consuming/supplying to the circuit? Also mention whether the source is supplying or consuming power.

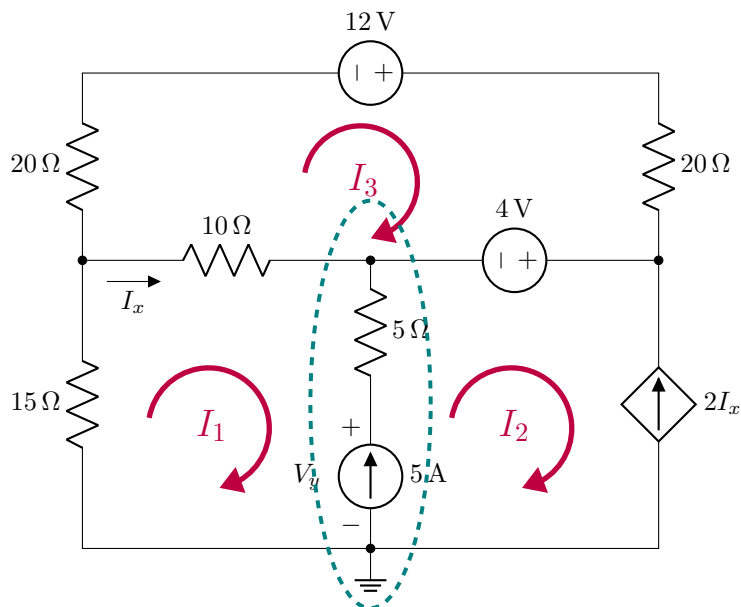
Solution: -340 W , supplying

Solution:

- (b) There are 3 meshes in the given circuit. Let's assign I_1 , I_2 , and I_3 , in ampere units, as the mesh currents, all taken in clockwise direction.



The 5 A current source forms a **Supermesh** between loops 1 and 2 as shown below.



- From loop 2, we can directly write,

$$I_2 = -2I_x$$

$$\text{where, } I_x = I_1 - I_3$$

$$\Rightarrow I_2 = -2(I_1 - I_3)$$

$$\Rightarrow 2I_1 + I_2 - 2I_3 = 0 \text{ ----- (eqn. 1)}$$

From the **Supermesh**, we can write using KCL,

$$I_2 - I_1 = 5$$

$$\Rightarrow I_1 - I_2 = -5 \text{ ----- (eqn. 2)}$$

- Applying KVL at loop 3,

$$-12 + 20I_3 + 4 + 10(I_3 - I_1) + 20I_3 = 0$$

$$\Rightarrow 10I_1 - 50I_3 = -8 \text{ --- (eqn. 3)}$$

Solving equations 1, 2, and 3, we get,

$$I_1 = -1.8 \text{ A}$$

$$I_2 = 3.2 \text{ A}$$

$$I_3 = -0.2 \text{ A}$$

(c) To find V_y , the voltage across the 5 A source, using KVL at loop 1,

$$15I_1 + 10(I_1 - I_3) + 5(I_1 - I_2) + V_y = 0$$

Substituting for I_1 , I_2 , and I_3 ,

$$V_y = 68 \text{ V}$$

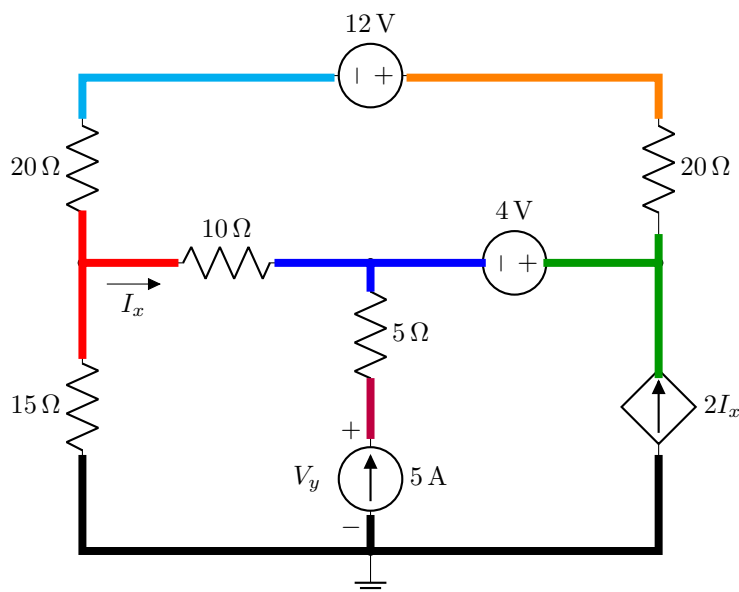
(d) The power of the 5 A current source according to passive sign convention is,

$$P_{5 \text{ A}} = -V_y \times 5 \text{ (Watt)}$$

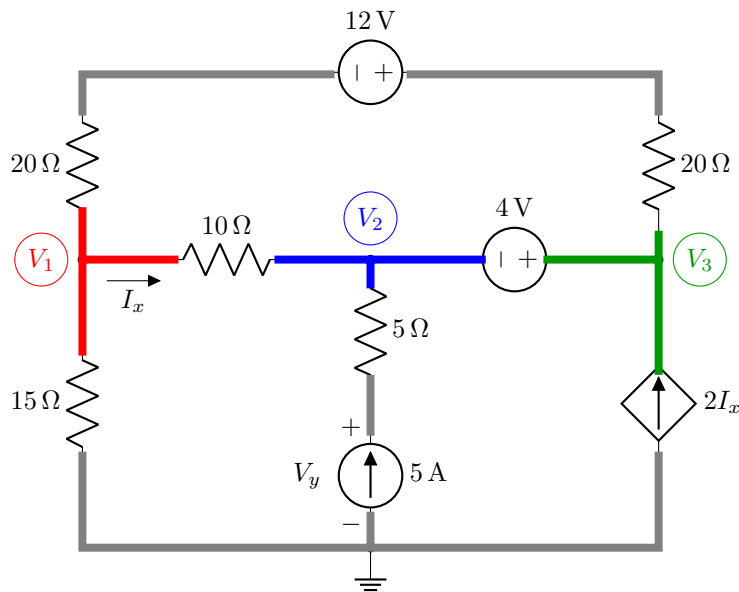
$$\Rightarrow P_{5 \text{ A}} = -68 \times 5 = -340 \text{ (W)}$$

Nodal Analysis Method: general approach

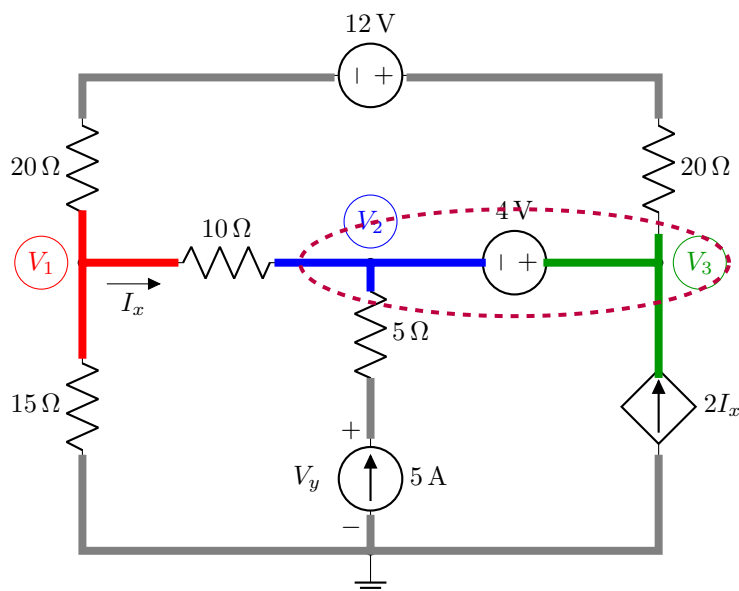
(b) There are 7 nodes in the given circuit as marked in the following diagram.



- But in the general approach of nodal analysis, we don't have to consider all the nodes. We have to consider only 3 nodes (red, blue, and green).



- The 4 V source forms a **Supernode** between nodes 2 (V_2) and 3 (V_3).



- From the **Supernode** we can write,

$$V_3 - V_2 = 4 \text{ V}$$

$$\Rightarrow V_2 - V_3 = -4 \text{ V} \text{ --- (eqn. 1)}$$

- Applying KCL at nodes 2 (V_2) and (V_3), → **Supernode**

$$5 + 2I_x = \frac{V_2 - V_1}{10} + \frac{V_3 - V_1 - 12}{20 + 20}$$

The current I_x through the 10 Ω resistor can be written as,

$$I_x = \frac{V_1 - V_2}{10}$$

Substituting for I_x ,

$$5 + 2 \left(\frac{V_1 - V_2}{10} \right) = \frac{V_2 - V_1}{10} + \frac{V_3 - V_1 - 12}{20 + 20}$$

$$\Rightarrow 13V_1 - 12V_2 - V_3 = -212 \text{ --- (eqn. 2)}$$

- Finally, applying KCL at node 1 (V_1),

$$\frac{V_1 - 0}{15} + \frac{V_1 - V_2}{10} + \frac{V_1 - V_3 + 12}{20 + 20} = 0$$

$$\Rightarrow 23V_1 - 12V_2 - 3V_3 = -36 \text{ --- (eqn. 3)}$$

Solving the 3 equations we get the node voltages.

$$\boxed{V_1 = 27 \text{ V}}$$

$$\boxed{V_2 = 43 \text{ V}}$$

$$\boxed{V_3 = 47 \text{ V}}$$

- (c) The voltage V_y can be found by applying KVL through the loop consisting of the 5 A source. That is, one way is,

$$V_2 - V_y + (5 \times 5) = 0$$

Substituting for $V_2 = 43 \text{ V}$,

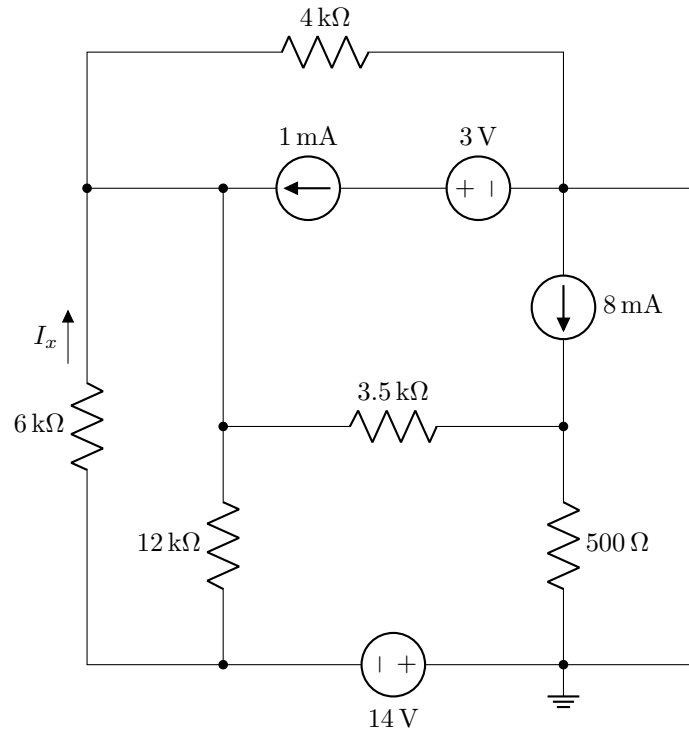
$$\Rightarrow \boxed{V_y = 68 \text{ V}}$$

- (d) The power of the 5 A current source according to passive sign convention is,

$$P_{5 \text{ A}} = -V_y \times 5 \text{ (Watt)}$$

$$\Rightarrow \boxed{P_{5 \text{ A}} = -68 \times 5 = -340 \text{ (W)}}$$

■ Question 3 of 3 [CO2, CO4] [16 marks]



Apply Nodal/Mesh analysis to answer the following questions:

- (a) [1 mark] Which analysis method should be more advantageous in solving the above circuit?

Solution: Nodal analysis

- (b) [14 marks] Find all the node voltages/mesh currents in the circuit.

Solution: Node voltages: -2 V , $\frac{2}{11}\text{ V}$.

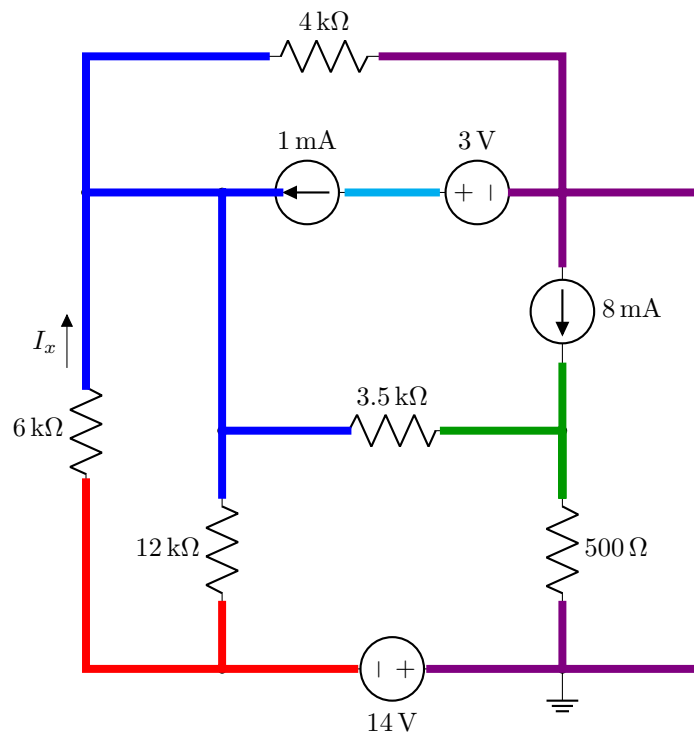
Mesh currents: $\pm 2\text{ mA}$, $\mp 3\text{ mA}$, $\pm 9.5\text{ mA}$, $\pm 1.5\text{ mA}$, $\pm 0.5\text{ mA}$

- (c) [1 mark] Find I_x , the amount of current through the $6\text{ k}\Omega$ resistor.

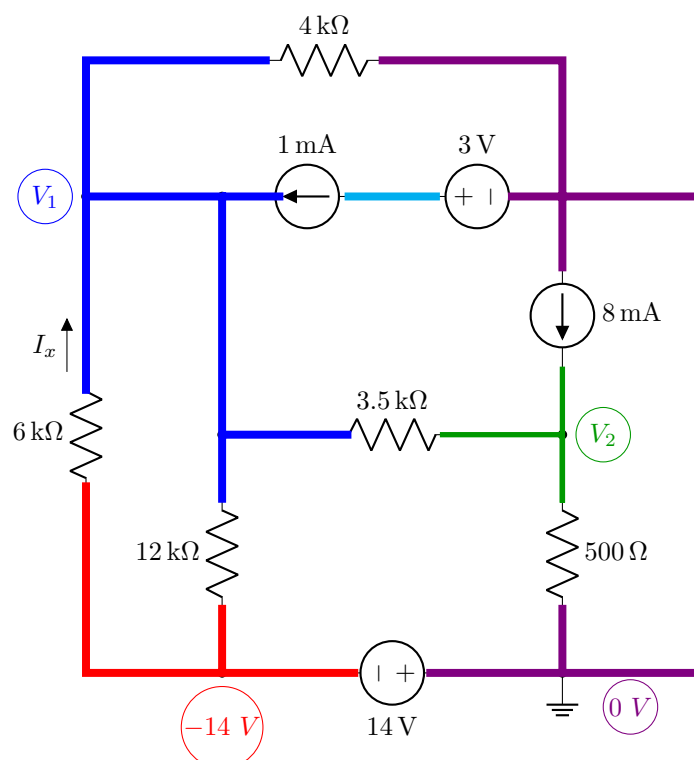
Solution: -2 mA

Solution: Nodal Analysis Method (General approach)

(b) There are 4 nodes in the given circuit apart from the ground as shown in the figure below.



- The red marked node voltage is -14 V , as can be seen from the figure.
- In the general approach of nodal analysis, we don't have to consider the node colored as cyan.
- Let's assign V_1 , V_2 as the remaining node variables (see the figure below).



- Applying KCL at node 1 (V_1),

$$1 = \frac{V_1 - 0}{4} + \frac{V_1 - (-14)}{6} + \frac{V_1 - (-14)}{12} + \frac{V_1 - V_2}{3.5}$$

$$\Rightarrow 24V_1 - 21V_2 = -1 \text{ --- (eqn. 1)}$$

- Applying KCL at node 2 (V_2),

$$8 = \frac{V_2 - V_1}{3.5} + \frac{V_2 - 0}{0.5}$$

$$\Rightarrow 7V_1 - 11V_2 = -16 \text{ --- (eqn. 2)}$$

Solving equations 1 and 2 we get,

$$V_1 = -2 \text{ V}$$

$$V_2 = \frac{2}{11} \text{ V}$$

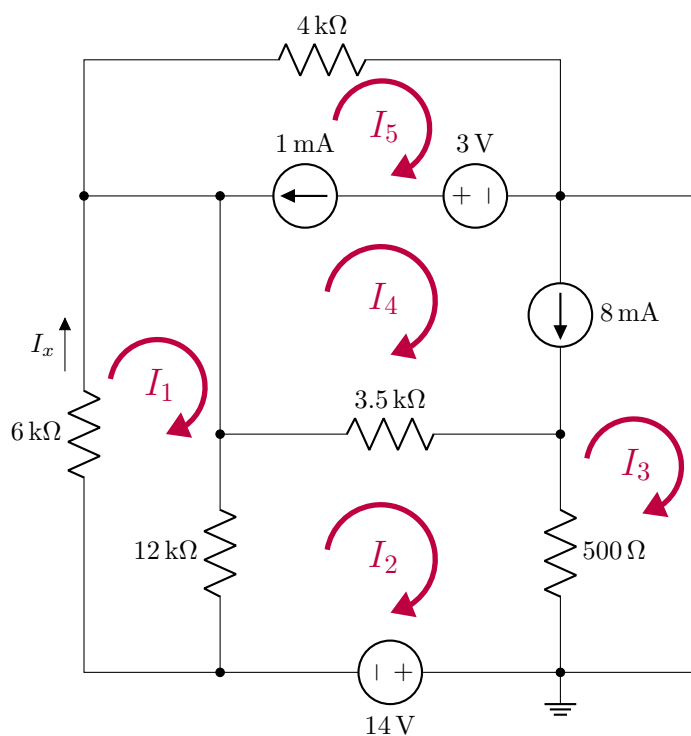
- (c) The current I_x through the $6 \text{ k}\Omega$ resistor is thus,

$$I_x = \frac{-14 - V_1}{6} \text{ (mA)}$$

$$\Rightarrow I_x = -2 \text{ mA}$$

Mesh Analysis Method:

- (b) There are 5 meshes in the given circuit. Let's assign I_1 , I_2 , I_3 , I_4 , and I_5 , in milliampere units, as the mesh currents.



-

$$I_5 - I_4 = 1 - \dots - (eqn. 1) \text{ and}$$

$$I_4 - I_3 = 8 \text{ --- --- --- --- --- (eqn. 2)}$$

- $$6I_1 + 12(I_1 - I_2) = 0$$

$$\Rightarrow 3I_1 - 2I_2 = 0 \text{ --- (eqn. 3)}$$

- $$14 + 12(I_2 - I_1) + 3.5(I_2 - I_4) + 0.5(I_2 - I_3) = 0$$

$$\Rightarrow 12I_1 - 16I_2 + 0.5I_3 + 3.5I_4 = 14 \text{ --- (eqn. 4)}$$

- $$4I_5 + 0.5(I_3 - I_2) + 3.5(I_4 - I_2) = 0$$

$$\Rightarrow 4I_2 - 0.5I_3 - 3.5I_4 - 4I_5 = 0 \text{ --- (eqn. 5)}$$

$$I_1 = -2 \text{ mA}$$

$$I_2 = -3 \text{ mA}$$

$$I_3 = -9.5 \text{ mA}$$

$$I_4 = -1.5 \text{ mA}$$

$$I_5 = -0.5 \text{ mA}$$

(c) It can be seen that, the current through the $6 \text{ k}\Omega$ resistor is I_1 .

So, $I_x = I_1 = -2 \text{ mA}$