

This file contains many demonstrations of theorems and facts about vectors in  $R^n$ .

## 1 Proof that $\|\hat{v}\| = 1$

$$\begin{aligned}
\hat{v} &= \frac{v}{\|v\|} \\
v &= \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix} \\
\hat{v} = \frac{v}{\|v\|} &= \frac{1}{\|v\|} \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix} \\
\Rightarrow \hat{v} &= \begin{bmatrix} \frac{v_1}{\|v\|} & \frac{v_2}{\|v\|} & \dots & \frac{v_n}{\|v\|} \end{bmatrix} \\
\|v\| &= \sqrt{v_1^2 + v_2^2 + \dots + v_n^2} \\
\Rightarrow \hat{v} &= \begin{bmatrix} \frac{v_1}{\sqrt{v_1^2 + v_2^2 + \dots + v_n^2}} & \frac{v_2}{\sqrt{v_1^2 + v_2^2 + \dots + v_n^2}} & \dots & \frac{v_n}{\sqrt{v_1^2 + v_2^2 + \dots + v_n^2}} \end{bmatrix} \\
\|\hat{v}\| &= \sqrt{\left(\frac{v_1}{\sqrt{v_1^2 + v_2^2 + \dots + v_n^2}}\right)^2 + \left(\frac{v_2}{\sqrt{v_1^2 + v_2^2 + \dots + v_n^2}}\right)^2 + \dots + \left(\frac{v_n}{\sqrt{v_1^2 + v_2^2 + \dots + v_n^2}}\right)^2} \\
&= \sqrt{\frac{v_1^2}{v_1^2 + v_2^2 + \dots + v_n^2} + \frac{v_2^2}{v_1^2 + v_2^2 + \dots + v_n^2} + \dots + \frac{v_n^2}{v_1^2 + v_2^2 + \dots + v_n^2}} \\
&= \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_n^2}{v_1^2 + v_2^2 + \dots + v_n^2}} \\
&= \sqrt{1} \\
\Rightarrow \|\hat{v}\| &= 1
\end{aligned} \tag{1}$$

Example:

$$\begin{aligned}
\hat{v} &= \begin{bmatrix} \frac{1}{\sqrt{30}} & \frac{-3}{\sqrt{30}} & \frac{4}{\sqrt{30}} & \frac{2}{\sqrt{30}} \end{bmatrix} \\
\|\hat{v}\| &= \sqrt{\left(\frac{1}{\sqrt{30}}\right)^2 + \left(\frac{-3}{\sqrt{30}}\right)^2 + \left(\frac{4}{\sqrt{30}}\right)^2 + \left(\frac{2}{\sqrt{30}}\right)^2} \\
&= \sqrt{\frac{1}{30} + \frac{9}{30} + \frac{16}{30} + \frac{4}{30}} \\
&= \sqrt{\frac{30}{30}} \\
&= \sqrt{1} \\
\Rightarrow \|\hat{v}\| &= 1
\end{aligned} \tag{2}$$