

The Gram-Schmidt method is a method to transform a set of vectors into an orthogonal set of vectors.

1 Definition

Let V be a vector space with an inner product $\langle \cdot, \cdot \rangle$ and $S = \{v_1, v_2, \dots, v_n\}$ a set of vectors of V . By definition the projection of V on U is:

$$P_u(v) = \frac{v \cdot u}{u \cdot u} u \quad (1)$$

The Gram-Schmidt method is defined as:

$$\begin{aligned} w_1 &= v_1 \\ w_2 &= v_2 - P_{w_1}(v_2) \\ w_3 &= v_3 - P_{w_1}(v_3) - P_{w_2}(v_3) \\ &\vdots \\ w_n &= v_n - P_{w_1}(v_n) - P_{w_2}(v_n) - \dots - P_{w_{n-1}}(v_n) \end{aligned} \quad (2)$$

The process can be summarized as:

$$w_n = v_n - \sum_{i=1}^{n-1} P_{w_i}(v_n) \quad (3)$$

2 Proof

2.1 Two vector orthogonal

Two vector A and B are orthogonal if $A \cdot B = 0$.

$$\begin{aligned} A &= \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}, B = \begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{pmatrix} \\ A \cdot B &= \sum_{i=1}^n x_i y_i \end{aligned} \quad (4)$$

If the two vectors are orthogonal then $\sum_{i=1}^n x_i y_i = 0$.

2.2 Gram-Schmidt method produce an orthonormal set

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