$$
\begin{aligned}
|0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
|1\rangle &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
|0\rangle\langle 0| - |1\rangle\langle 1| &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}^{*} \begin{pmatrix} 1 \\ 0 \end{pmatrix}^{T} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}^{*} \begin{pmatrix} 0 \\ 1 \end{pmatrix}^{T} \\
&= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
\implies |0\rangle\langle 0| - |1\rangle\langle 1| &= Z
\end{aligned} \tag{1}
$$