

Certainly! A normalized vector is obtained by scaling a non-zero vector by the reciprocal of its norm. The process of normalizing a vector produces a unit vector, which is a vector of length 1. Here's a step-by-step proof of why the norm of a normalized vector is equal to 1:

1. Definition of a Normalized Vector: Let \mathbf{v} be a non-zero vector, and $|\mathbf{v}|$ be its norm. A normalized vector \mathbf{u} is obtained by dividing \mathbf{v} by its norm:

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

2. Calculating the Norm of the Normalized Vector: Now, we'll calculate the norm of the normalized vector \mathbf{u} :

$$|\mathbf{u}| = \left| \frac{\mathbf{v}}{|\mathbf{v}|} \right|$$

3. Properties of Norms: The norm has the property that $|c\mathbf{a}| = |c| \cdot |\mathbf{a}|$ for any scalar c and vector \mathbf{a} . Applying this property, we get:

$$|\mathbf{u}| = \left| \frac{1}{|\mathbf{v}|} \right| \cdot |\mathbf{v}|$$

4. Simplification: Now, $|\mathbf{v}|$ is a positive quantity (since \mathbf{v} is a non-zero vector), so we can simplify further:

$$|\mathbf{u}| = \frac{1}{|\mathbf{v}|} \cdot |\mathbf{v}| = 1$$

So, we've shown that the norm of a normalized vector is equal to 1.