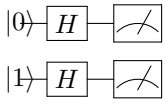
Calculate the result of the following circuit:



1 Method 1: Using tensor product

Step 0:

$$|\psi_0\rangle = |0\rangle \otimes |1\rangle = |01\rangle \tag{1}$$

Step 1:

$$\begin{aligned} |\psi_1\rangle &= H|0\rangle \otimes H|1\rangle \\ H|0\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ H|1\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \\ \implies |\psi_1\rangle &= \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle) \end{aligned} \tag{2}$$

2 Method 2: Using matrix multiplication

Step 0:

$$\begin{aligned} |\psi_0\rangle &= |0\rangle \otimes |1\rangle = |01\rangle \\ |\psi_0\rangle &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \end{aligned} \tag{3}$$

Step 1:

$$
\begin{aligned}
&|\psi_1\rangle = H|0\rangle \otimes H|1\rangle = (H \otimes H)(|0\rangle \otimes |1\rangle) \\
&H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \\
&(H \otimes H)(|0\rangle \otimes |1\rangle) = (H \otimes H) \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \\
&(H \otimes H) = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \\
&\implies (H \otimes H) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \\
&\implies (H \otimes H)(|0\rangle \otimes |1\rangle) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \\
&(H \otimes H)(|0\rangle \otimes |1\rangle) = \begin{bmatrix} 0 * \frac{1}{2} + 1 * \frac{1}{2} + 0 * \frac{1}{2} + 0 * \frac{1}{2} \\ 0 * \frac{1}{2} + 1 * -\frac{1}{2} + 0 * \frac{1}{2} + 0 * -\frac{1}{2} \\ 0 * \frac{1}{2} + 1 * \frac{1}{2} + 0 * -\frac{1}{2} + 0 * -\frac{1}{2} \\ 0 * \frac{1}{2} + 1 * -\frac{1}{2} + 0 * -\frac{1}{2} + 0 * \frac{1}{2} \end{bmatrix} \\
&(H \otimes H)(|0\rangle \otimes |1\rangle) = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} \\
&\implies |\psi_1\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)
\end{aligned}
\tag{4}
$$