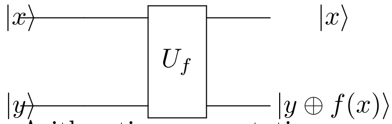
1 Introduction

This gate is used to apply a function $f(x)$ on a qubit.

2 Circuit



Arithmetic representation:

$$\begin{aligned} |\psi_0\rangle &= |x\rangle|y\rangle \\ |\psi_1\rangle &= |x\rangle|y \oplus f(x)\rangle \end{aligned} \tag{1}$$

3 Special Cases

3.1 y = 0

$$\begin{aligned} |\psi_0\rangle &= |x\rangle|0\rangle \\ \implies |\psi_1\rangle &= |x\rangle|f(x)\rangle \end{aligned} \tag{2}$$

4 Development

4.1 Development and simplification

$$\begin{aligned} |\psi_0\rangle &= |x\rangle|y\rangle \\ |x\rangle &= \alpha|0\rangle + \beta|1\rangle \\ |y\rangle &= \gamma|0\rangle + \delta|1\rangle \\ |\psi_1\rangle &= |x\rangle|y \oplus f(x)\rangle \\ \implies |\psi_1\rangle &= (\alpha|0\rangle + \beta|1\rangle)|(\gamma|0\rangle + \delta|1\rangle) \oplus f(x)\rangle \end{aligned} \tag{3}$$

4.2 Example

$$
\begin{aligned}
|\psi_0\rangle &= |x\rangle|y\rangle \\
|x\rangle &= \frac{|0\rangle + |1\rangle}{\sqrt{2}} + \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\
|y\rangle &= 1|0\rangle + 0|1\rangle \\
|\psi_1\rangle &= (\alpha|0\rangle + \beta|1\rangle)|(\gamma|0\rangle + \delta|1\rangle) \oplus f(x)\rangle \\
\implies |\psi_1\rangle &= (\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle)|0 \oplus f(x)\rangle \\
&= \frac{|0\rangle + |1\rangle}{\sqrt{2}}|f(x)\rangle \\
&= \frac{|0, f(x)\rangle + |1, f(x)\rangle}{\sqrt{2}}
\end{aligned} \tag{4}
$$