Topology and Geometry (MA216)
Mid-Semester Examination, IIT Patna
Time: 10 am to 12 Noon

February 23, 2023 Maximum score: 30

Instruction: Please write your answer clearly, and give proper justifications for your arguments.

1. Show that the equation of the cone whose vertex is at $(0,0,0)$ and the guiding curve is the conic $\{f(x,y)=0;\ z=k\}$ is given by $f(\frac{xk}{z},\frac{yk}{z})=0$.

[5]

2. Show that the equation $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$ represents a cone if $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$.

[5]

3. Find the equation of the right circular cone whose vertex is at the point (α,β,γ), where the direction cosines of its axis is given by l,m,n and the semi-vertical angle is denoted by θ.

[5]

4. Show that the equation of a cone whose vertex is at $(0,0,0)$ is homogeneous. Also, show that the converse holds.

[8]

5. Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at the point $(1,-2,1)$ and cuts orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$.

[7]

Good luck