Topology and Geometry (MA216)
End-Semester Examination, IIT Patna
Time: 9 am to 12 pm

May 4, 2023 **Maximum score**: 50

Instruction: Please write your answer clearly, and give proper justifications for your arguments.

1. Reduce the conic $2x^2 + 3xy - 2y^2 - 10 = 0$ into its standard form.

[10]

2. Reduce the quadric surface $3x^2+2xy+4yz+2xz-2x-14y-2z-9 = 0$ into its standard form.

[10]

3. Let X be a non-empty set; let τ be the collection of all subsets A of X such that $X - A$ either is countable or is all of X. Show that τ defines a topology on X.

[10]

4. Let Y be a subspace of X; let S be a subset of Y; let $\overline{S}$ denote the closure of S in X. Show that the closure of S in Y equals $\overline{S} \cap Y$.

[10]

5. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous map; let $G_f : \mathbb{R} \to \mathbb{R}^2$ defined by $G_f(x) = (x, f(x))$, G_f is called the graph of f. Show that G_f is a continuous map and that its image (taken with the topology induced from $\mathbb{R}^2$) is homeomorphic to $\mathbb{R}$.

[10]