MA227 (Assignment-7)

1. Use image from the given URL: https://tinyurl.com/yc2ryd8a and compress it by using built-in functions (e.g. `pca` in MATLAB.) [If you are using `pca` in Matlab, learn first about the built-in function from MATLAB help.] Execute the following steps:

   - Load the image and convert it into matrix/array.
   - Split the image into RGB (i.e. Red, Green and Blue). [You can use the functions (split(img) of cv2 library in python, imread and particular channel as red = 1, green = 2 and blue = 3 in matlab).]
   - Apply PCA on these components individually. (Test with three different component size i.e. 50, 100, 500).
   - Combine these RGB components back to original image (i.e. reconstruct the image back).
   - Plot the compressed images with names 'image using pca = 50' etc.

2. Write a function *SelfPower.m* that takes an $n \times n$ matrix A, initial vector x_0, integer $maxNumIter$ and positive small number $tol = 10^{-8}$, and returns dominant eigenvalue $lambda$ and corresponding eigenvector x of A. Use power method to find the outputs and apply the following stopping criteria

   - No. of iteration $k > maxNumIter$ OR
   - $|lambda^{(k)} - lambda^{(k-1)}| \leq tol * |lambda^{(k)}|$.

   Here $lambda^{(k)}$ represents the value of $lambda$ in the $k-$th iteration. Print outputs in the following format for $iter = 1, 2, 3, 4, 5, 10, 30, 50, \cdots$

$Iter$	λ^k
1	
2	
3	

3. Write a function *SelfQRIter.m* that takes an $n \times n$ matrix A, integer $maxNumIter$ and a positive small number $tol = 10^{-8}$, and returns the spectrum set of A. Use QR iterations to find the output and apply the following stopping criteria

   - No. of iteration $k > maxNumIter$ OR
   - $||A^{(k)}_{LowerTrig} - A^{(k-1)}_{LowerTrig}||_F \leq tol * ||A^{(k)}_{LowerTrig}||_F$

   For the above two problems, take $A = \begin{bmatrix} 17 & 24 & 1 & 8 & 15 \\ 23 & 5 & 7 & 14 & 16 \\ 4 & 6 & 13 & 20 & 22 \\ 10 & 12 & 19 & 21 & 3 \\ 11 & 18 & 25 & 2 & 8 \end{bmatrix}$. Check the answers by built-in functions, e.g. $[PD] = eig(A)$ in MATLAB.