MA227 (Assignment-2)

1. Take $n = 2$ and $A = ones(n, n)$. Change the first element $A(1, 1) = 10^{-17}$. Find LU decomposition of A by your function *luSelfnP.m* that you have created in Assignment 1. Solve the system $Ax = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ by using functions *FdSubs.m* and *BdSubs.m*. Compare the solution with built-in function `backslash`. Are both the solutions same? What is wrong in this example? Repeat this problem by taking $A(1, 1) = 10^{-10}$, $A(1, 1) = 10^{-15}$, and $A(1, 1) = 10^{-16}$ and learn from the conclusion that MATLAB can work good up to $16th$ place of decimal. Moreover, learn the effect of rounding (also known as round-off) error if you do not apply partial pivoting in Gauss elimination.

2. Write a function *luSelfwP.m* that takes an invertible matrix A, and returns P, L and U such that $PA = LU$. Use partial pivoting before choosing each pivot. All the notations are the same as used in MA220. Now find the solution of the 2×2 system given in the above Problem 1. Compare the solution now with the exact solution by `backslash`.

   [Avoid small pivots to reduce the rounding error. You can do it by applying partial pivoting]

3. Take $n = 20, 40, 60, 100$. Take $A = rand(n, n)$. Change the first element $A(1, 1) = 10^{-20}$. Use built-in function `norm` to calculate the norm of matrix $A - LU$, where L and U are obtained by function *luSelfnP.m*. Also find the norm of $PA - LU$, where P, L and U are obtained by function *luSelfwP.m*. Learn the built-in function `lu` in MATLAB (or parallel in Python), find P, L and U factors by using `[P, L, U] = lu(A)` and now find the norm of $PA - LU$. Compare all the results.

   [In MATLAB, use `help norm` to learn the norm command]

4. Write a function *rrefSelfwP.m* that takes any $m \times n$ matrix A, and returns RREF(A). Use partial pivoting before choosing each pivot. Use your function to find rref of any 5 matrices of your choice of different sizes. Compare the results with built-in function `rref(A)`.