Topology and Geometry (MA216)
Department of Mathematics, IIT Patna
Time: 5 pm to 5:45 pm

February 17, 2023 Maximum score: 10

1. Find the equation of the sphere described on the line segment joining the points $A(2,-3,4)$ and $B(-5,6,-7)$ as diameter.

2. Determine the equation of the sphere passing through the points $(0,0,0)$, $(-a,b,c)$, $(a,-b,c)$, $(a,b,-c)$. What is its radius?

3. Show that the locus of the centers of all sections of the sphere $x^2+y^2+z^2 = r^2$ by planes which pass through the point (α,β,γ) will lie on the sphere

$$x(\alpha-x)+y(\beta-y)+z(\gamma-z)=0.$$

4. If the tangent plane to the sphere $x^2+y^2+z^2=r^2$ makes intercepts a,b,c on the x,y, and z axes respectively, then show that

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{r^2}.$$

5. Find the equation of a cone whose vertex is at the point $(1,2,3)$ and the guiding curve is the parabola $y^2=4ax;\ z=0$.