

Department of Mathematics,  
Indian Institute of Technology Patna

MA 218, Complex Analysis

Quiz-2. April 19, 2023

Maximum Marks: 10

Instruction: Please do not forget to write your **name and roll number** in the answer sheet.

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(1) Classify the nature of singularity of the function

$$f(z) = \frac{e^{-z}}{(z-2)^4}$$

and compute the residue.

2 marks.

(2) If  $f(z)$  is analytic at  $z_0$  such that  $f(z_0) \neq 0$  and  $g(z)$  has a zero of order 2 at  $z_0$ . Then show that

$$\operatorname{Res}_{z=z_0} \frac{f(z)}{g(z)} = \frac{6f'(z_0)g''(z_0) - 2f(z_0)g'''(z_0)}{3\{g''(z_0)\}^2}.$$

2 marks.

(3) Evaluate the integral

$$\int_C \frac{1}{(z+i)} dz.$$

where  $C$  is the square with vertices  $2 - 2i$ ,  $2 + 2i$ ,  $-2 + 2i$  and  $-2 - 2i$ .

2 marks.

(4) Evaluate the integral

$$\int_C \frac{4z - 5}{(z^2 - z)} dz,$$

where  $C$  is the positively oriented circle centered at origin with radius 2.

2 marks.

(5) Evaluate the integral

$$\int_0^{\infty} \frac{x \sin 2x}{(x^2 + 3)} dx$$

2 marks.