

Department of Mathematics,
Indian Institute of Technology Patna

MA 218, Complex Analysis

ESE, May 06, 2023

Maximum Marks: 50

Roll No.:

Name:

Instruction: All questions are compulsory. **Mobiles, calculators are not allowed.** Please do not forget to write your **name and roll number** in the question paper.

- (1) Find the Cauchy principal value of the integral

$$\int_{-\infty}^{\infty} \frac{x dx}{(x^2 + 1)(x^2 + 2x + 2)}.$$

5 marks.

- ✓(2) Find the image of the strip $0 < y < \frac{1}{2c}$ under the transformation $w = \frac{1}{z}$.

5 marks.

- ✓(3) Prove the following.

(a) The inverse of a bilinear transformation $T(z)$ is also bilinear.

(b) The composition of two bilinear transformations is also bilinear.

[2 + 3] marks.

- ✓(4) State and prove the Argument Principle.

5 marks.

- ✓(5) State and prove the Cauchy integral formula. w

5 marks.

- ✓(6) Evaluate the following integrals.

(a)

$$\int_C \frac{\cos z}{(z^2 + 3z + 2)} dz,$$

where C is the positively oriented circle $|z + 1| = \frac{1}{2}$.

(b)

$$\int_C \frac{\sin z}{(z^2 + 9)^5} dz,$$

where C is closed contour lying in the open disk $|z| < 2$.

[3 + 3] marks.

- ✓(7) State and prove the principle of deformation of paths.

3 marks.

- ✓(8) Suppose f is analytic at z_0 . Then f has a zero of order m at z_0 if and only if there exists analytic and nonzero function g at z_0 such that

$$f(z) = (z - z_0)^m g(z).$$

5 marks.

- (9) Find the Laurent series for $f(z) = \frac{-1}{(z-1)(z-2)}$, $2 < |z| < \infty$.

5 marks.

- ✓(10) Find the image of the region $y > 1$ under the map $w = (1 - i)z$.

3 marks.

- (11) Find the value of the integral

$$\int_{|z|=2} \frac{1}{(z-3)(z^5-1)} dz,$$

where $|z| = 2$ is the positively oriented circle.

3 marks.