MA227 (Assignment-10 (ESE))

1. Write a function *SelfFOM.m* that takes an $n \times n$ invertible matrix A, vector $b \in \mathbb{R}^n$ and an integer m. The function returns the solution of $Ax = b$ in Krylov subspace K_m by using the full orthogonalization method (FOM), the Arnoldi's output matrices H and V as discussed in the theory class. Stop Arnoldi if $h_{j+1,j} < 10^{-6}$ and do not apply any other stopping criteria.

   Test the above *SelfFOM.m* function for solution of $Ax = b$, where $A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Take initial guess $x_0 = 0$ and print the output (solution x, V, and H) for $m = 1, 2, 3$ and $m = 4$.

2. Write a function *SelfCG.m* that takes an $n \times n$ SPD matrix A, vector $b \in \mathbb{R}^n$, initial vector x_0, and positive small number $tol = 10^{-8}$, and returns solution to $Ax = b$. Use conjugate gradient (CG) method to find the solution and apply the stopping criteria $|r_k| \leq tol$. Here r_k represents the residual value in the $k-$th iteration. Test the function for $Ax = b$, where

$$A = \begin{bmatrix} 4 & -1 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 & 0 \\ 0 & 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & -1 & 4 \end{bmatrix} \text{ and } b = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 6 \\ -2 \\ 6 \end{bmatrix}.$$

3. Write a function *SelfSQRIter.m* that takes an $n \times n$ matrix A, a real number μ, and an integer $maxNumIter = 20$. The function returns the spectrum set of A. Use QR iterations with shift μ to find the output and apply the stopping criteria that the number of iteration $i > maxNumIter$.

   Test you function by finding all eigenvalues of $A = \begin{bmatrix} 17 & 24 & 1 & 8 & 15 \\ 23 & 5 & 7 & 14 & 16 \\ 4 & 6 & 13 & 20 & 22 \\ 10 & 12 & 19 & 21 & 3 \\ 11 & 18 & 25 & 2 & 8 \end{bmatrix}$. First take $\mu = 0$ and find error matrix E whose i^{th} column $e_i = abs(sort(ex) - sort(diag(A_i)))$ where ex is the vector of exact eigenvalues of A that you can calculate by built-in functions, e.g. $ex = eig(A)$ in MATLAB; A_i is the matrix in i^{th} iteration of the *SelfSQRIter.m* function; *sort* represents the sorting of array in increasing order of elements; and *abs* means the element-wise absolute values. Print the error matrix E. Repeat the exercise by taking shift $\mu = A_i(n, n)$ in the i^{th} iteration. Again print the error matrix E.