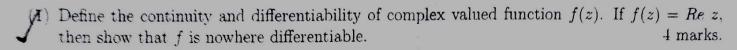
Department of Mathematics, Indian Institute of Technology Patna

MA 218, Complex Analysis

MSE, February 20, 2023

Maximum Marks: 30

Instruction: Please do not forget to write your name and roll number in the answer sheet.



(3) Solve the equation

$$sinh z = i$$
.

3 marks.

$$f(z) = \begin{cases} 0 & ; & \text{if } z = 0\\ \frac{(\bar{z})^2}{z} & ; & \text{if } z \neq 0. \end{cases}$$

Prove that C-R equations are satisfied at z=(0,0). Further also prove or disprove f is differentiable at z=0+i.0.

(A) Prove or disprove

$$u(x,y) = cosx \ sinhy$$

is a harmonic function. If u(x, y) is harmonic then find its harmonic conjugate and construct the corresponding analytic function.

4 marks.

Find the upper bound of the integral (without evaluate the integral)

$$\left| \int_C \frac{(z-1)e^{2z}Log\ z}{z^2-7} \, dz \right|,$$

where $C = \{z : z = e^{i\theta}, \quad \frac{\pi}{3} \le \theta \le \frac{\pi}{2}\}.$

5 marks.

(6) Show that

$$log(i^2) \neq 2 \ log(i),$$

when the branch
$$log z = lnr + i\theta$$
 $\left(r > 0, \frac{3\pi}{4} < \theta < \frac{11\pi}{4}\right)$.

4 marks.

Suppose that f is an analytic function in a domain D. If |f(z)| is a constant throughout D, then prove that f is constant in D.