

Department of Mathematics,
Indian Institute of Technology Patna

MA 218, Complex Analysis

MSE, February 20, 2023

Maximum Marks: 30

Instruction: Please do not forget to write your **name and roll number** in the answer sheet.

- ✓ (1) Define the continuity and differentiability of complex valued function $f(z)$. If $f(z) = \operatorname{Re} z$, then show that f is nowhere differentiable. 4 marks.

- ✓ (2) Solve the equation

$$\sinh z = i.$$

3 marks.

- ✓ (3) Let

$$f(z) = \begin{cases} 0 & ; \quad \text{if } z = 0 \\ \frac{(\bar{z})^2}{z} & ; \quad \text{if } z \neq 0. \end{cases}$$

Prove that $C-R$ equations are satisfied at $z = (0, 0)$. Further also prove or disprove f is differentiable at $z = 0 + i.0$. 5 marks.

- ✓ (4) Prove or disprove

$$u(x, y) = \cos x \sinh y$$

is a harmonic function. If $u(x, y)$ is harmonic then find its harmonic conjugate and construct the corresponding analytic function. 4 marks.

- ✓ (5) Find the upper bound of the integral (without evaluate the integral)

$$\left| \int_C \frac{(z-1)e^{2z} \operatorname{Log} z}{z^2 - 7} dz \right|,$$

where $C = \{z : z = e^{i\theta}, \quad \frac{\pi}{3} \leq \theta \leq \frac{\pi}{2}\}$.

5 marks.

- ✓ (6) Show that

$$\log(i^2) \neq 2 \log(i),$$

when the branch $\log z = \ln r + i\theta$ $\left(r > 0, \frac{3\pi}{4} < \theta < \frac{11\pi}{4}\right)$.

4 marks.

- ✓ (7) Suppose that f is an analytic function in a domain D . If $|f(z)|$ is a constant throughout D , then prove that f is constant in D . 5 marks.